Analytical axicon

```
xref = 47.5;
yref = 47.5;
zref = 40;
z1 = 25;
z0 = 5;
h = z1 - z0;
rr = 75;
flin[z_] := rr/h (z1 - z);

b1 = 0.15;
fexp[z_] := (rr (Exp[-b1 * (z - z1)] - 1))/(Exp[b1 * h] - 1);

b2 = 0.0005;
flog[z_] := Sqrt[1/b2 * Log[h/(h + (z - z1) * (1 - Exp[-b2 * rr^2]))]];

fcos[z_] := rr * Cos[(π * (z - z0))/(2 * h)];
```
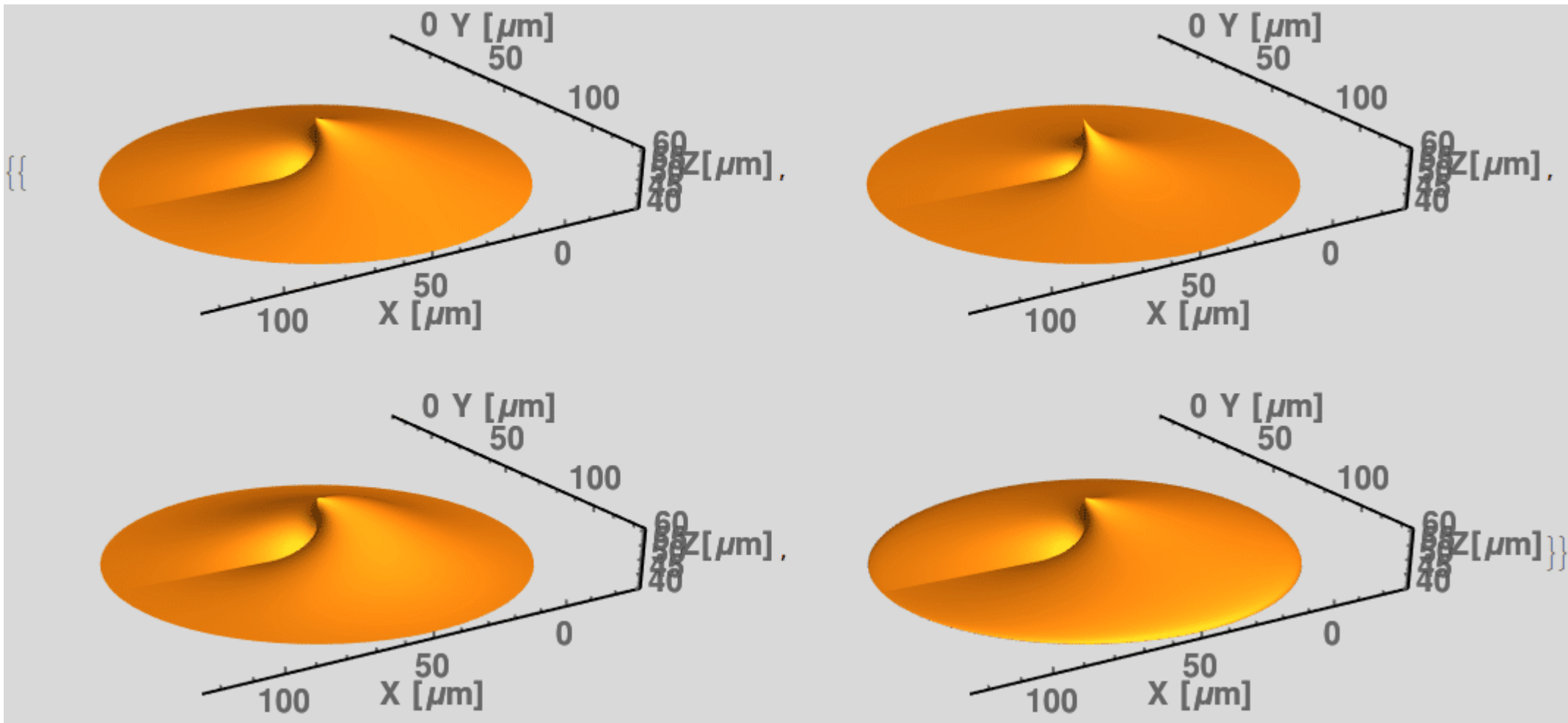
One Layer (symmetric)

```
imageSize = 510;
```

```
a1 = ParametricPlot3D[{flin[z] * Cos[θ] + xref, flin[z] * Sin[θ] + yref, z + zref}, {z, z0, z1}, {θ, 0, 2 * π}, Mesh → None, PlotPoints → 70, LabelStyle → Directive[Bold, FontSize → 20], AxesStyle → Thick,
    AxesLabel → {"X [μm]", "Y [μm]", "Z[μm]"}, Boxed → False, ImageSize → imageSize, ViewPoint → {1.3, 2.4, 1.}];
a2 = ParametricPlot3D[{fexp[z] * Cos[θ] + xref, fexp[z] * Sin[θ] + yref, z + zref}, {z, z0, z1}, {θ, 0, 2 * π}, Mesh → None, PlotPoints → 70, LabelStyle → Directive[Bold, FontSize → 20], AxesStyle → Thick,
    AxesLabel → {"X [μm]", "Y [μm]", "Z[μm]"}, Boxed → False, ImageSize → imageSize, ViewPoint → {1.3, 2.4, 1.}];
a3 = ParametricPlot3D[{flog[z] * Cos[θ] + xref, flog[z] * Sin[θ] + yref, z + zref}, {z, z0, z1}, {θ, 0, 2 * π}, Mesh → None, PlotPoints → 70, LabelStyle → Directive[Bold, FontSize → 20], AxesStyle → Thick,
    AxesLabel → {"X [μm]", "Y [μm]", "Z[μm]"}, Boxed → False, ImageSize → imageSize, ViewPoint → {1.3, 2.4, 1.}];
a4 = ParametricPlot3D[{fcos[z] * Cos[θ] + xref, fcos[z] * Sin[θ] + yref, z + zref}, {z, z0, z1}, {θ, 0, 2 * π}, Mesh → None, PlotPoints → 70, LabelStyle → Directive[Bold, FontSize → 20], AxesStyle → Thick,
    AxesLabel → {"X [μm]", "Y [μm]", "Z[μm]"}, Boxed → False, ImageSize → imageSize, ViewPoint → {1.3, 2.4, 1.}];
{{a1, a2, a3, a4}}
```
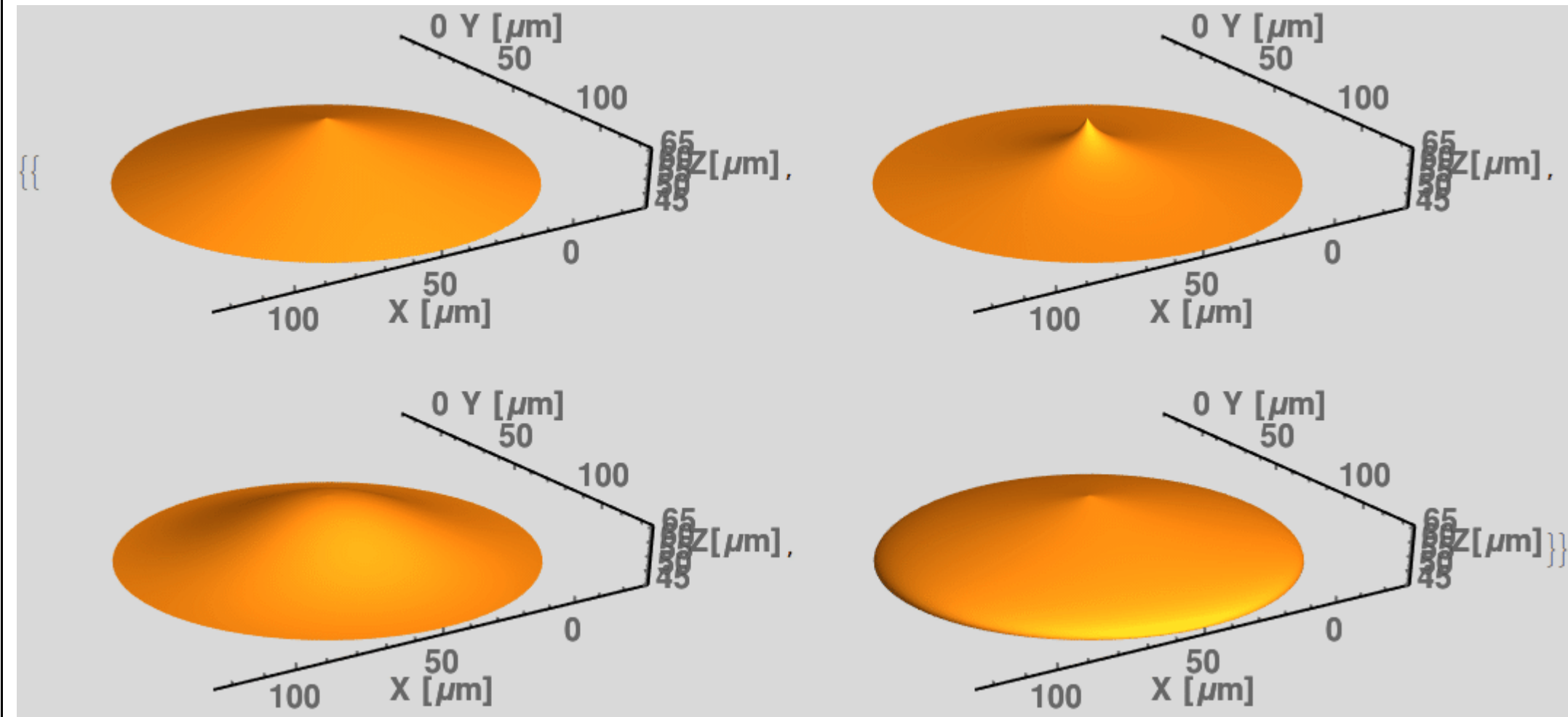
One Layer (asymmetrical axicon continuous)

```
η = 0.5;
g[z_, ϑ_] := (z - z0) * Sin[η ϑ];
```

```
a5 = ParametricPlot3D[{flin[z] * Cos[ϑ] + xref, flin[z] * Sin[ϑ] + yref, g[z, ϑ] + zref}, {z, z0, z1}, {ϑ, 0, 2 * π}, Mesh → None, PlotPoints → 70, LabelStyle → Directive[Bold, FontSize → 20], AxesStyle → Thick,
     AxesLabel → {"X [μm]", "Y [μm]", "Z[μm]"}, Boxed → False, ImageSize → imageSize, ViewPoint → {1.3, 2.4, 1.}];
a6 = ParametricPlot3D[{fexp[z] * Cos[ϑ] + xref, fexp[z] * Sin[ϑ] + yref, g[z, ϑ] + zref}, {z, z0, z1}, {ϑ, 0, 2 * π}, Mesh → None, PlotPoints → 70, LabelStyle → Directive[Bold, FontSize → 20], AxesStyle → Thick,
     AxesLabel → {"X [μm]", "Y [μm]", "Z[μm]"}, Boxed → False, ImageSize → imageSize, ViewPoint → {1.3, 2.4, 1.}];
a7 = ParametricPlot3D[{flog[z] * Cos[ϑ] + xref, flog[z] * Sin[ϑ] + yref, g[z, ϑ] + zref}, {z, z0, z1}, {ϑ, 0, 2 * π}, Mesh → None, PlotPoints → 70, LabelStyle → Directive[Bold, FontSize → 20], AxesStyle → Thick,
     AxesLabel → {"X [μm]", "Y [μm]", "Z[μm]"}, Boxed → False, ImageSize → imageSize, ViewPoint → {1.3, 2.4, 1.}];
a8 = ParametricPlot3D[{fcos[z] * Cos[ϑ] + xref, fcos[z] * Sin[ϑ] + yref, g[z, ϑ] + zref}, {z, z0, z1}, {ϑ, 0, 2 * π}, Mesh → None, PlotPoints → 70, LabelStyle → Directive[Bold, FontSize → 20], AxesStyle → Thick,
     AxesLabel → {"X [μm]", "Y [μm]", "Z[μm]"}, Boxed → False, ImageSize → imageSize, ViewPoint → {1.3, 2.4, 1.}];
{{a5, a6, a7, a8}}
```


One Layer (asymmetrical axicon discontinuous)

```
η = 0.2;
```

```
a9 = ParametricPlot3D[{flin[z] * Cos[ϑ] + xref, flin[z] * Sin[ϑ] + yref, g[z, ϑ] + zref}, {z, z0, z1}, {ϑ, 0, 2 * π}, Mesh → None, PlotPoints → 70, LabelStyle → Directive[Bold, FontSize → 20], AxesStyle → Thick,
     AxesLabel → {"X [μm]", "Y [μm]", "Z[μm]"}, Boxed → False, ImageSize → imageSize, ViewPoint → {1.3, 2.4, 1.}];
a10 = ParametricPlot3D[{fexp[z] * Cos[ϑ] + xref, fexp[z] * Sin[ϑ] + yref, g[z, ϑ] + zref}, {z, z0, z1}, {ϑ, 0, 2 * π}, Mesh → None, PlotPoints → 70, LabelStyle → Directive[Bold, FontSize → 20], AxesStyle → Thick,
     AxesLabel → {"X [μm]", "Y [μm]", "Z[μm]"}, Boxed → False, ImageSize → imageSize, ViewPoint → {1.3, 2.4, 1.}];
a11 = ParametricPlot3D[{flog[z] * Cos[ϑ] + xref, flog[z] * Sin[ϑ] + yref, g[z, ϑ] + zref}, {z, z0, z1}, {ϑ, 0, 2 * π}, Mesh → None, PlotPoints → 70, LabelStyle → Directive[Bold, FontSize → 20], AxesStyle → Thick,
     AxesLabel → {"X [μm]", "Y [μm]", "Z[μm]"}, Boxed → False, ImageSize → imageSize, ViewPoint → {1.3, 2.4, 1.}];
a12 = ParametricPlot3D[{fcos[z] * Cos[ϑ] + xref, fcos[z] * Sin[ϑ] + yref, g[z, ϑ] + zref}, {z, z0, z1}, {ϑ, 0, 2 * π}, Mesh → None, PlotPoints → 70, LabelStyle → Directive[Bold, FontSize → 20], AxesStyle → Thick,
     AxesLabel → {"X [μm]", "Y [μm]", "Z[μm]"}, Boxed → False, ImageSize → imageSize, ViewPoint → {1.3, 2.4, 1.}];
{{a9, a10, a11, a12}}
```

```
SurfaceNumber = 1;
ExportResolution = {2550};
errorInterfaces = 10^-2;
round1 = {errorInterfaces};
round2 = {2 π};
sizeFile = {};
SetDirectory["/home/enrique/workspace/"]
Do[
 p = ParametricPlot3D[{flog[z] * Cos[ϑ] + xref, flog[z] * Sin[ϑ] + yref, g[z, ϑ] + zref}, {z, z0, z1}, {ϑ, round1[[fileNumber]], round2[[fileNumber]]}, PlotPoints → ExportResolution[[fileNumber]]];
 (* Extracción de la lista de puntos usados para la representación 3D *)
 p1 = Join @@ Cases[Normal @ p, Line[x1__] ⧴ x1, Infinity];
p =.;
 sizeFile = Append[sizeFile, p1 // Length];
 p2 = Table[{p1[[i, 1]], p1[[i, 2]], p1[[i, 3]]}, {i, 1, Dimensions[p1][[1]]}];
 p1 =.;
 filename = StringJoin["surf", IntegerString[fileNumber], ".dat"];
 Export[filename, N[p2], "TABLE"]
  filename =.;
 p2 =.;
 , {fileNumber, 1, SurfaceNumber}]
sizeFile
```

⋯ SetDirectory : Cannot set current directory to /home/enrique/workspace/.

$Failed

Multi Layers (twisted axicon - discontinuous top surface)

```
cir = ParametricPlot3D[{r * Cos[ϑ] + xref, r * Sin[ϑ] + yref, zref}, {r, 0, rr}, {ϑ, 0, 2 * π}, Mesh → None];
```

```
η = 0.25;
c = 0.1;
jump = 1.1;
```

```
a13 = Show[cir, ParametricPlot3D[{flin[z]*Cos[ø]*Exp[-c*ø]+xref, flin[z]*Sin[ø]*Exp[-c*ø]+yref, Sqrt[z^jump*ø+zref]}, {z, z0, z1}, {ø, 0, 6*π}, Mesh → None, PlotPoints → 70], PlotRange -> All,
    LabelStyle → Directive[Bold, FontSize → 20], AxesStyle → Thick, AxesLabel → {"X [μm]", "Y [μm]", "Z[μm]"}, Boxed → False, ImageSize → imageSize, ViewPoint → {1.3, 2.4, 1.}];

a14 = Show[cir, ParametricPlot3D[{fexp[z]*Cos[ø]*Exp[-c*ø]+xref, fexp[z]*Sin[ø]*Exp[-c*ø]+yref, Sqrt[z^jump*ø+zref]}, {z, z0, z1}, {ø, 0, 4*π}, Mesh → None, PlotPoints → 70], PlotRange -> All,
    LabelStyle → Directive[Bold, FontSize → 20], AxesStyle → Thick, AxesLabel → {"X [μm]", "Y [μm]", "Z[μm]"}, Boxed → False, ImageSize → imageSize, ViewPoint → {1.3, 2.4, 1.}];

a15 = Show[cir, ParametricPlot3D[{flog[z]*Cos[ø]*Exp[-c*ø]+xref, flog[z]*Sin[ø]*Exp[-c*ø]+yref, Sqrt[z^jump*ø+zref]}, {z, z0, z1}, {ø, 0, 4*π}, Mesh → None, PlotPoints → 70], PlotRange -> All,
    LabelStyle → Directive[Bold, FontSize → 20], AxesStyle → Thick, AxesLabel → {"X [μm]", "Y [μm]", "Z[μm]"}, Boxed → False, ImageSize → imageSize, ViewPoint → {1.3, 2.4, 1.}];

a16 = Show[cir, ParametricPlot3D[{fcos[z]*Cos[ø]*Exp[-c*ø]+xref, fcos[z]*Sin[ø]*Exp[-c*ø]+yref, Sqrt[z^jump*ø+zref]}, {z, z0, z1}, {ø, 0, 4*π}, Mesh → None, PlotPoints → 70], PlotRange -> All,
    LabelStyle → Directive[Bold, FontSize → 20], AxesStyle → Thick, AxesLabel → {"X [μm]", "Y [μm]", "Z[μm]"}, Boxed → False, ImageSize → imageSize, ViewPoint → {1.3, 2.4, 1.}];
{{a13, a14, a15, a16}}
```
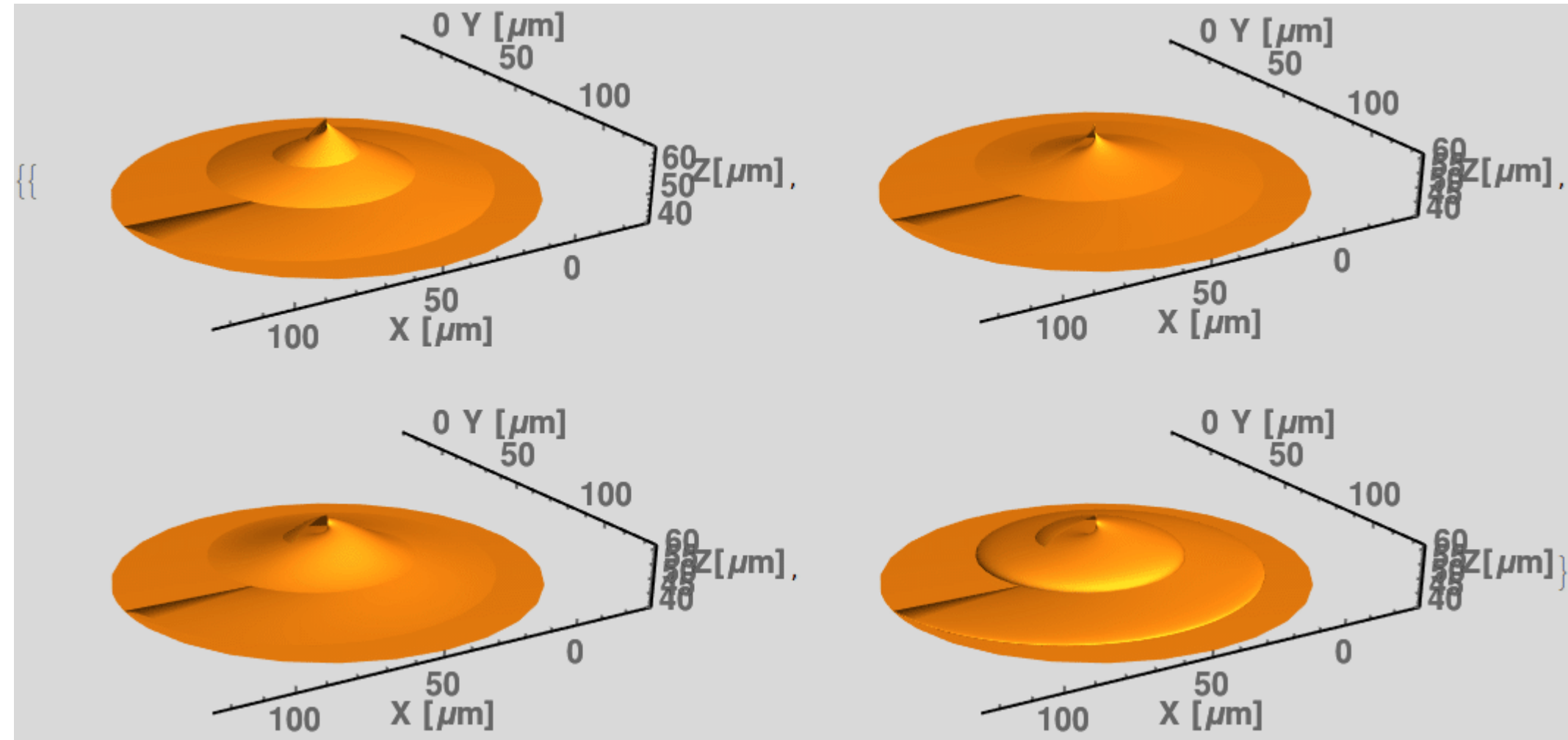

Multi Layers (twisted axicon - continuous top surface)
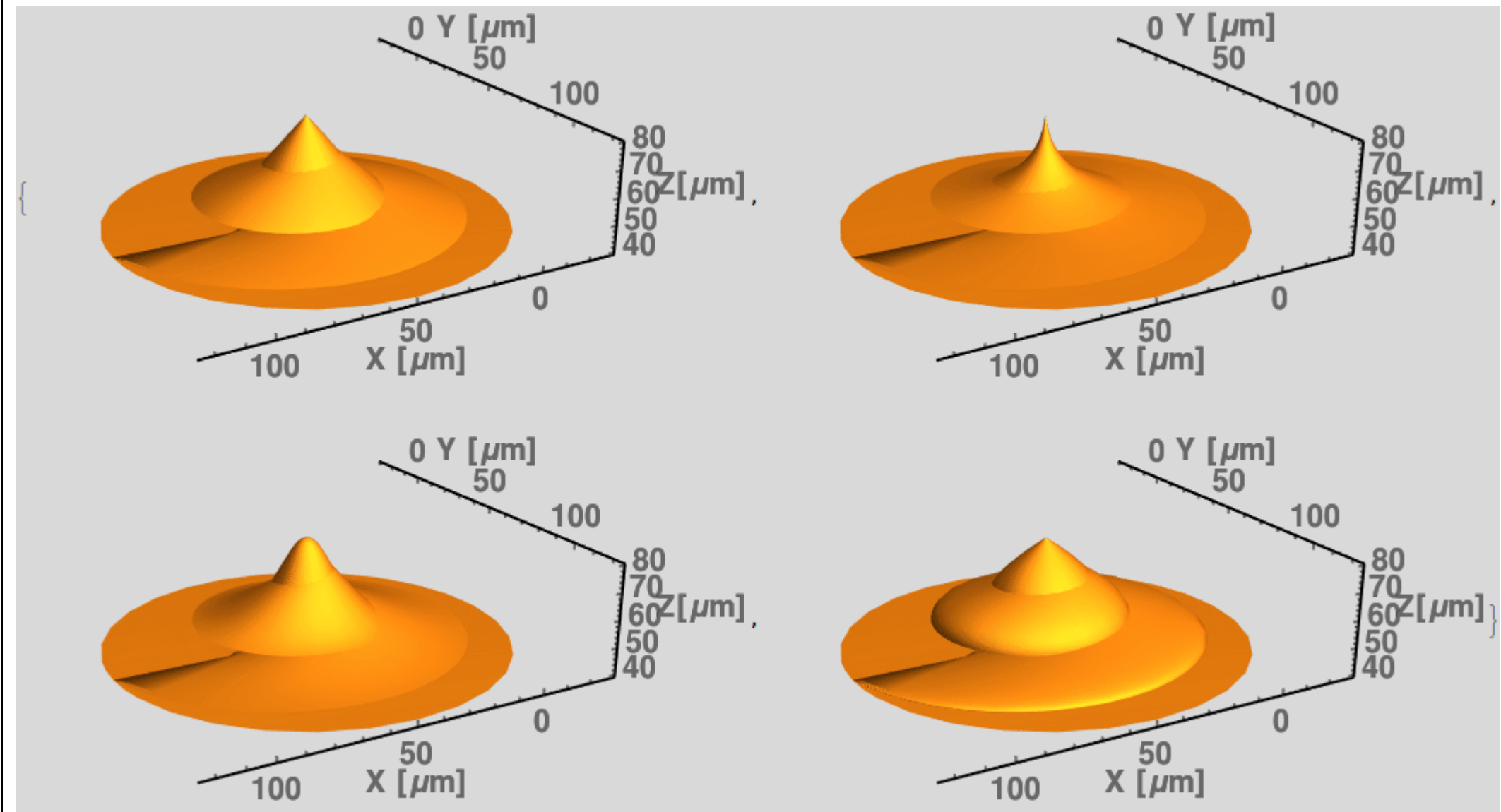
```
ølimit = 4 π;
jump = 1.5;
a17 = Show[cir, ParametricPlot3D[{flin[z]*Cos[ø]*If[ø < ølimit, Exp[-c*ø], Exp[-c*ølimit]]+xref, flin[z]*Sin[ø]*If[ø < ølimit, Exp[-c*ø], Exp[-c*ølimit]]+yref, If[ø < ølimit, Sqrt[z^jump*ø+zref], Sqrt[z^jump*ølimit+zref]]},
    {z, z0, z1}, {ø, 0, 6.1*π}, Mesh → None, PlotPoints → 70], PlotRange -> All, LabelStyle → Directive[Bold, FontSize → 20], AxesStyle → Thick, AxesLabel → {"X [μm]", "Y [μm]", "Z[μm]"}, Boxed → False,
    ImageSize → imageSize, ViewPoint → {1.3, 2.4, 1.}];

a18 = Show[cir, ParametricPlot3D[{fexp[z]*Cos[ø]*If[ø < ølimit, Exp[-c*ø], Exp[-c*ølimit]]+xref, fexp[z]*Sin[ø]*If[ø < ølimit, Exp[-c*ø], Exp[-c*ølimit]]+yref, If[ø < ølimit, Sqrt[z^jump*ø+zref], Sqrt[z^jump*ølimit+zref]]},
    {z, z0, z1}, {ø, 0, 6.1*π}, Mesh → None, PlotPoints → 70], PlotRange -> All, LabelStyle → Directive[Bold, FontSize → 20], AxesStyle → Thick, AxesLabel → {"X [μm]", "Y [μm]", "Z[μm]"}, Boxed → False,
    ImageSize → imageSize, ViewPoint → {1.3, 2.4, 1.}];

a19 = Show[cir, ParametricPlot3D[{flog[z]*Cos[ø]*If[ø < ølimit, Exp[-c*ø], Exp[-c*ølimit]]+xref, flog[z]*Sin[ø]*If[ø < ølimit, Exp[-c*ø], Exp[-c*ølimit]]+yref, If[ø < ølimit, Sqrt[z^jump*ø+zref], Sqrt[z^jump*ølimit+zref]]},
    {z, z0, z1}, {ø, 0, 6.1*π}, Mesh → None, PlotPoints → 70], PlotRange -> All, LabelStyle → Directive[Bold, FontSize → 20], AxesStyle → Thick, AxesLabel → {"X [μm]", "Y [μm]", "Z[μm]"}, Boxed → False,
    ImageSize → imageSize, ViewPoint → {1.3, 2.4, 1.}];

a20 = Show[cir, ParametricPlot3D[{fcos[z]*Cos[ø]*If[ø < ølimit, Exp[-c*ø], Exp[-c*ølimit]]+xref, fcos[z]*Sin[ø]*If[ø < ølimit, Exp[-c*ø], Exp[-c*ølimit]]+yref, If[ø < ølimit, Sqrt[z^jump*ø+zref], Sqrt[z^jump*ølimit+zref]]},
    {z, z0, z1}, {ø, 0, 6.1*π}, Mesh → None, PlotPoints → 70], PlotRange -> All, LabelStyle → Directive[Bold, FontSize → 20], AxesStyle → Thick, AxesLabel → {"X [μm]", "Y [μm]", "Z[μm]"}, Boxed → False,
    ImageSize → imageSize, ViewPoint → {1.3, 2.4, 1.}];
{a17, a18, a19, a20}
```
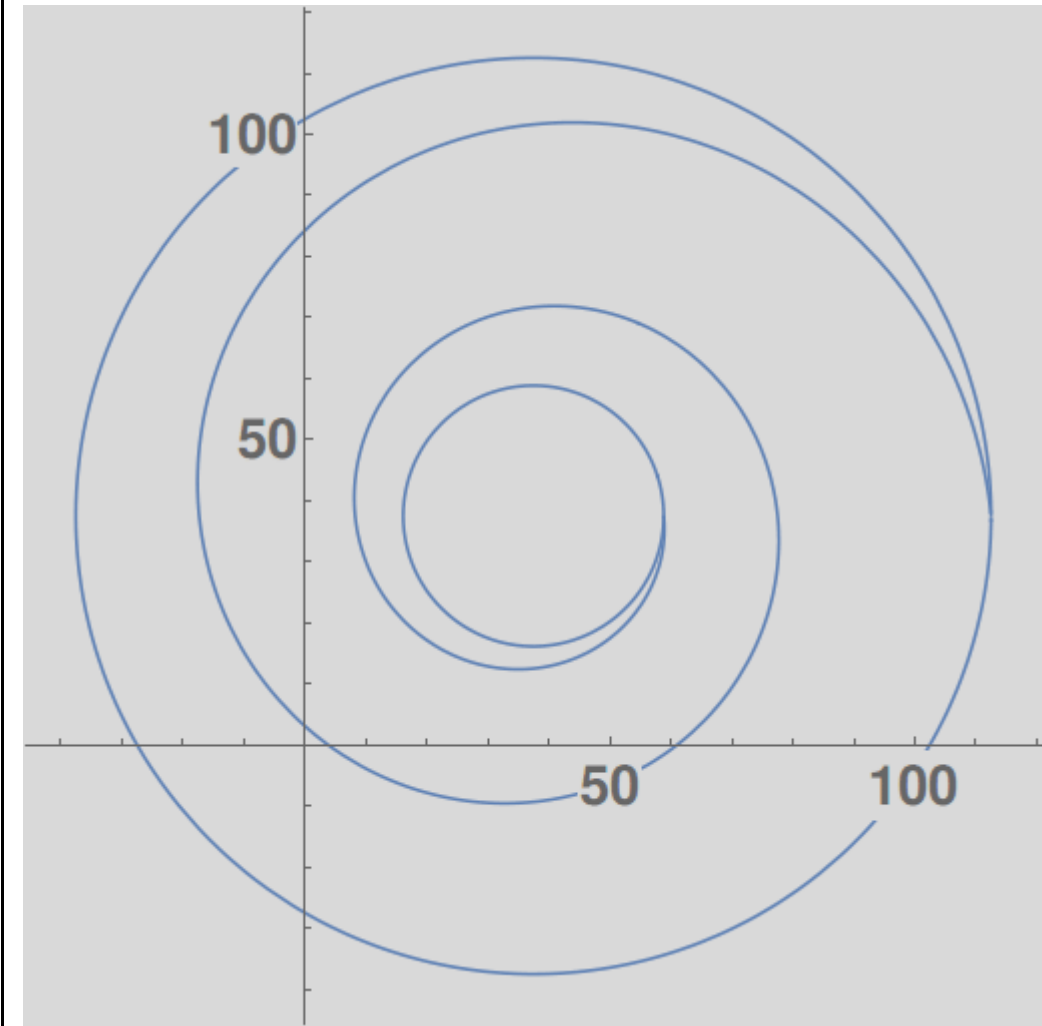

Surface boundaries


```
ParametricPlot[{rr*Cos[σ]*If[σ<0, 1, If[σ<σlimit, Exp[-c*σ], Exp[-c*σlimit]]] + rr/2, rr*Sin[σ]*If[σ<0, 1, If[σ<σlimit, Exp[-c*σ], Exp[-c*σlimit]]] + rr/2}, {σ, -2 π, 6.1*π},
 ImageSize → imageSize, LabelStyle → Directive[Bold, FontSize → 26]]
```

Export surfaces

```
SurfaceNumber = 3;
errorInterfaces = 10^-2;
ExportResolution = Table[3500, SurfaceNumber];
round1 = Table[i*π+errorInterfaces, {i, 0, SurfaceNumber-1}]
round2 = Table[i*π*2, {i, 1, SurfaceNumber}]
round2[[SurfaceNumber]] = round2[[SurfaceNumber]]+ π/2;
sizeFile = {};
SetDirectory["/media/enrique/workspace/"]
Do[
  p = ParametricPlot3D[{fbra[z]*Cos[θ]*If[θ < θlimit, Exp[-c*θ], Exp[-c*θlimit]]+xref, fbra[z]*Sin[θ]*If[θ < θlimit, Exp[-c*θ], Exp[-c*θlimit]]+yref, If[θ < θlimit, √(z^jump*θ)+zref, √(z^jump*θlimit)+zref]},
    {z, z0, z1}, {θ, round1[[fileNumber]], round2[[fileNumber]]}, PlotPoints → ExportResolution[[fileNumber]]];
  (* Extracción de la lista de puntos usados para la representación 3D *)
  p1 = Join @@ Cases[Normal @ p, Line[x1__] :> x1, Infinity];
  p =.;
  sizeFile = Append[sizeFile, p1 // Length];
  p2 = Table[{p1[[i, 1]], p1[[i, 2]], p1[[i, 3]]}, {i, 1, Dimensions[p1][[1]]}];
  p1 =.;
  filename = StringJoin["surf", IntegerString[fileNumber], "ele", IntegerString[sizeFile], ".dat"];
  Export[filename, N[p2], "TABLE"]
    filename =.;
  p2 =.;
  , {fileNumber, 1, SurfaceNumber}]
sizeFile
```
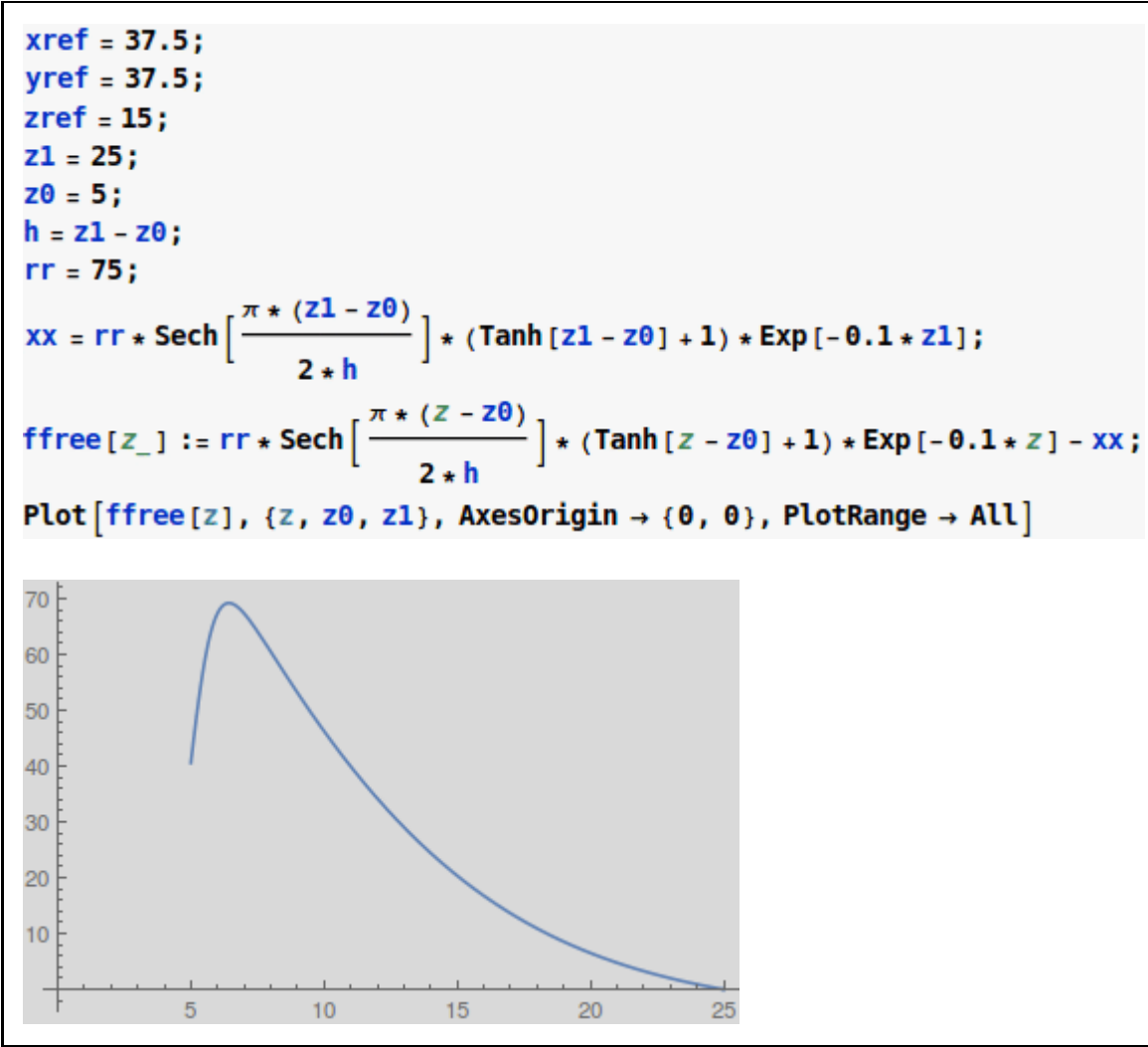
```
/media/enrique/TargetSimulation
```
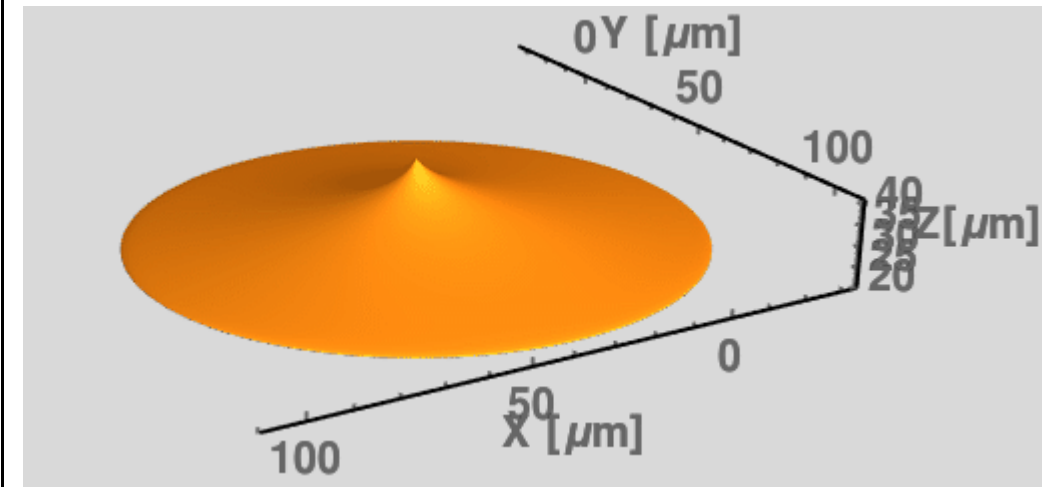
```
{210420, 210420, 210420}
```

Extra surface

```
xref = 37.5;
yref = 37.5;
zref = 15;
z1 = 25;
z0 = 5;
h = z1 - z0;
rr = 75;
xx = rr*Sech[ (π*(z1-z0))/(2*h) ]*(Tanh[z1-z0]+1)*Exp[-0.1*z1];
ffree[z_] := rr*Sech[ (π*(z-z0))/(2*h) ]*(Tanh[z-z0]+1)*Exp[-0.1*z]-xx;
Plot[ffree[z], {z, z0, z1}, AxesOrigin → {0, 0}, PlotRange → All]
```


One Layer (symmetric)

```
imageSize = 510;
```

```
ParametricPlot3D[{ffree[z] * Cos[ϴ] + xref, ffree[z] * Sin[ϴ] + yref, z + zref}, {z, z0, z1}, {ϴ, 0, 2 * π}, Mesh → None, PlotPoints → 70, LabelStyle → Directive[Bold, FontSize → 20], AxesStyle → Thick,
  AxesLabel → {"X [μm]", "Y [μm]", "Z[μm]"}, Boxed → False, ImageSize → imageSize, ViewPoint → {1.3, 2.4, 1.}]
```


One Layer (asymmetrical axicon continuous)

```
η = 0.5;
g[z_, ϴ_] := (z - z0) * Sin[η ϴ];
```

```
a5 = ParametricPlot3D[{ffree[z] * Cos[ϴ] + xref, ffree[z] * Sin[ϴ] + yref, g[z, ϴ] + zref}, {z, z0, z1}, {ϴ, 0, 2 * π}, Mesh → None, PlotPoints → 70, LabelStyle → Directive[Bold, FontSize → 20],
  AxesStyle → Thick, AxesLabel → {"X [μm]", "Y [μm]", "Z[μm]"}, Boxed → False, ImageSize → imageSize, ViewPoint → {1.3, 2.4, 1.}]
```
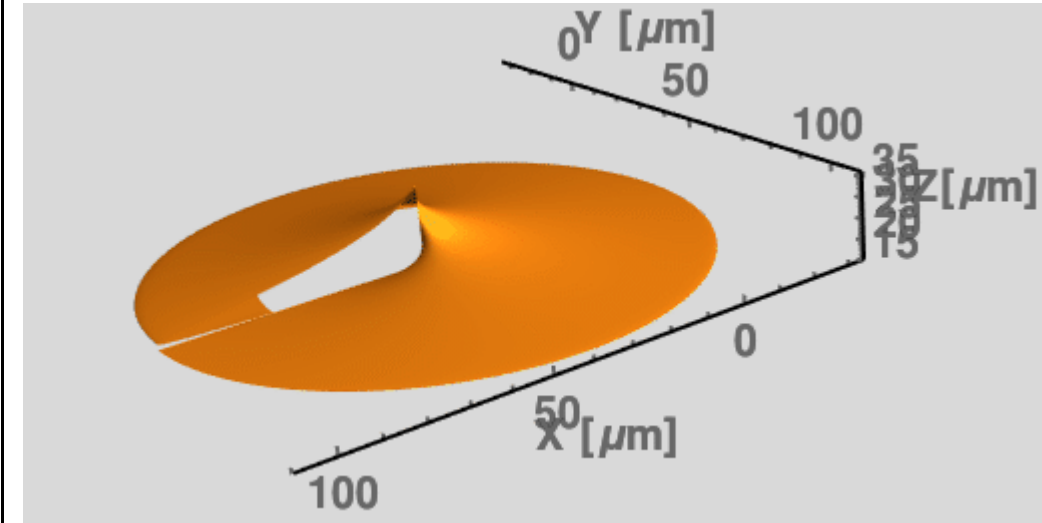

One Layer (asymmetrical axicon discontinuous)

```
η = 0.25;
```

```
a12 = ParametricPlot3D[{ffree[z] * Cos[ϴ] + xref, ffree[z] * Sin[ϴ] + yref, g[z, ϴ] + zref}, {z, z0, z1}, {ϴ, 0, 2 * π}, Mesh → None, PlotPoints → 70, LabelStyle → Directive[Bold, FontSize → 20],
  AxesStyle → Thick, AxesLabel → {"X [μm]", "Y [μm]", "Z[μm]"}, Boxed → False, ImageSize → imageSize, ViewPoint → {1.3, 2.4, 1.}]
```

```
∅limit = 4 π;
jump = 1.5;
c = 0.1;
cir = ParametricPlot3D[{r * Cos[∅] + xref, r * Sin[∅] + yref, zref}, {r, 0, rr}, {∅, 0, 2 * π}, Mesh → None];

Show[cir, ParametricPlot3D[{ffree[z] * Cos[∅] * If[∅ < ∅limit, Exp[-c * ∅], Exp[-c * ∅limit]] + xref, ffree[z] * Sin[∅] * If[∅ < ∅limit, Exp[-c * ∅], Exp[-c * ∅limit]] + yref, If[∅ < ∅limit, √(z^jump * ∅) + zref, √(z^jump * ∅limit) + zref]},
  {z, z0, z1}, {∅, 0, 6.1 * π}, Mesh → None, PlotPoints → 70], PlotRange -> All, LabelStyle → Directive[Bold, FontSize → 20], AxesStyle → Thick, AxesLabel → {"X [μm]", "Y [μm]", "Z[μm]"}, Boxed → False,
 ImageSize → imageSize, ViewPoint → {1.3, 2.4, 1.}]
```