

QTinRRAMs	
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Contents

1. Global Definitions
1.1. Parameters
1.2. Functions
1.3. Shared Properties
1.4. Material
2. Component 2D
2.1. Definitions
2.2. Geometry 2
2.3. Materials
2.4. Wall Distance
2.5. Electric Currents entire RRAM cell
2.6. Heat Transfer in Solids
2.7. Schrödinger Equation
2.8. Electrostatics: Potential on HfO₂
2.9. Multiphysics
2.10. Mesh 2
3. Classic approach
3.1. Stationary Classic approach
3.2. Solver Configurations
4. Semi-Classic approach
4.1. Stationary-Semi-Classic approach
4.2. Solver Configurations
5. Full quantum approach (High-Precision and slow, Matlab function)
5.1. Stationary
5.2. Schrödinger-Poisson
5.3. Solver Configurations
6. Results
6.1. Own plots from: Full quantum approach
6.2. Datasets
6.3. Derived Values
6.4. Tables
6.5. Classic approach plots
6.6. Semi-Classic approach plots
6.7. Full quantum approach plots
6.8. Plot Groups

1. Global Definitions

Date Oct 19, 2022, 10:04:52 AM

Global settings

Name	QTinRRAMs.mph
Path	C:\Users\EM\QTinRRAMs.mph
Version	COMSOL Multiphysics 6.0 (Build: 318)
Unit system	SI

Used products

COMSOL Multiphysics
Heat Transfer Module
LiveLink™ for MATLAB®
AC/DC Module
Semiconductor Module

Computer information

CPU	Intel64 Family 6 Model 58 Stepping 9, 4 cores
Operating system	Windows 7

1.1. Parameters

Parameters 1

Name	Expression	Value	Description
sigmaCF	1.0e5[S/m]	1E5 S/m	Filament conductivity
alphaT	1.0e-3[1/K]	0.001 1/K	Resistivity temperature coefficient α (SI unit: 1/K)
tref	300.0[K]	300 K	Reference temperature Tref
LBase	20[nm]	2E-8 m	Width cells
L_metal1	15[nm]	1.5E-8 m	Titanium thickness
L_metal2	15[nm]	1.5E-8 m	Tungsten thickness
Linsulator	2[nm]	2E-9 m	Insulator thickness (HfO ₂)



Rtop	3.5[nm]	3.5E-9 m	Filemant top radii
Rbottom	2[nm]	2E-9 m	Filemant bottom radii
Vapp	1[V]	1 V	Applied voltage
mm	1	1	Azimutal quantum number
mass_eff	0.25*me_const	2.2773E-31 kg	Electron effective mass of HfO2
Nd	2*10^13[cm^-3]	2E19 1/m^3	Doping
Ef	1.5[eV]	2.4033E-19 J	
qVapp	e_const*Vapp	1.6022E-19 J	

1.2. Functions

1.2.1. MATLAB : Quantum current density function

Function names	QuantumCurrent
Function type	MATLAB
Functions	
Function name	Arguments
QuantumCurrent Thickness Bias Temperature1 Temperature2	
Plot parameters	
Lower limit	Upper limit
0	5.0e-9
0	5
300	500
300	1200

1.3. Shared Properties

1.3.1. Default Model Inputs

Tag	cminpt
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1.4. Material

1.4.1. HfO2 (insulator)

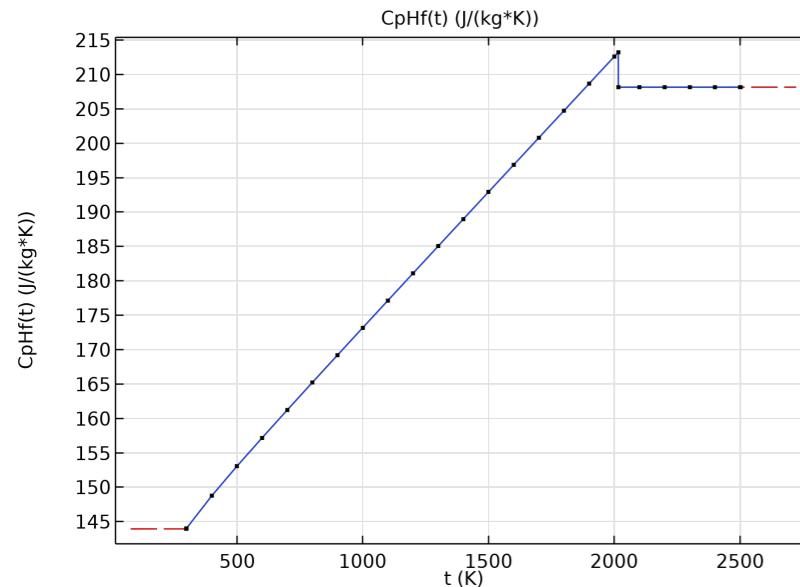
Material parameters		
Name	Value	Unit
Electrical conductivity	9.2592[1/(ohm*m)]	S/m
Relative permittivity	25	1
Thermal conductivity	{1, 1, 0.5}	W/(m·K)
Density	9680.0[kg/(m^3)]	kg/m^3
Heat capacity at constant pressure	120[J/(kg*K)]	J/(kg·K)
Basic		
Description	Value	
Electrical conductivity	{9.2592[1/(ohm*m)], 0, 0}, {0, 9.2592[1/(ohm*m)], 0}, {0, 0, 9.2592[1/(ohm*m)]}}	
Relative permittivity	{25, 0, 0}, {0, 25, 0}, {0, 0, 25}	
Thermal conductivity	{1, 0, 0}, {0, 1, 0}, {0, 0, 0.5}	
Density	9680.0[kg/(m^3)]	
Heat capacity at constant pressure	120[J/(kg*K)]	

1.4.2. Filament (Hafnium)

Material parameters		
Name	Value	Unit
Density	0.99*13.31[g/(cm^3)]	kg/m^3
Thermal conductivity	{0.99*kHf(T), 0.99*kHf(T), 0.99*0.9*kHf(T)}	W/(m·K)
Heat capacity at constant pressure	0.99*CpHf(T)	J/(kg·K)
Relative permittivity	1	1
Basic		
Description	Value	
Density	0.99*13.31[g/(cm^3)]	
Thermal conductivity	{0.99*kHf(T), 0, 0}, {0, 0.99*kHf(T), 0}, {0, 0, 0.99*0.9*kHf(T)}	
Heat capacity at constant pressure	0.99*CpHf(T)	
Relative permittivity	{1, 0, 0}, {0, 1, 0}, {0, 0, 1}	
Functions		
Function name	Type	
CpHf	Interpolation	
kHf	Interpolation	

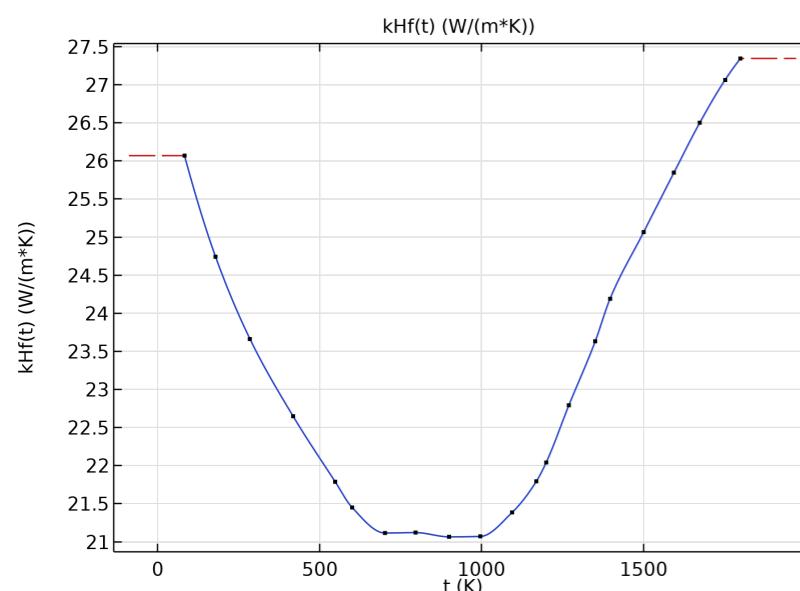
CpAna | Analytic |

Specific heat vs temperature (DOI: 10.1007/s11669-014-0319-5)



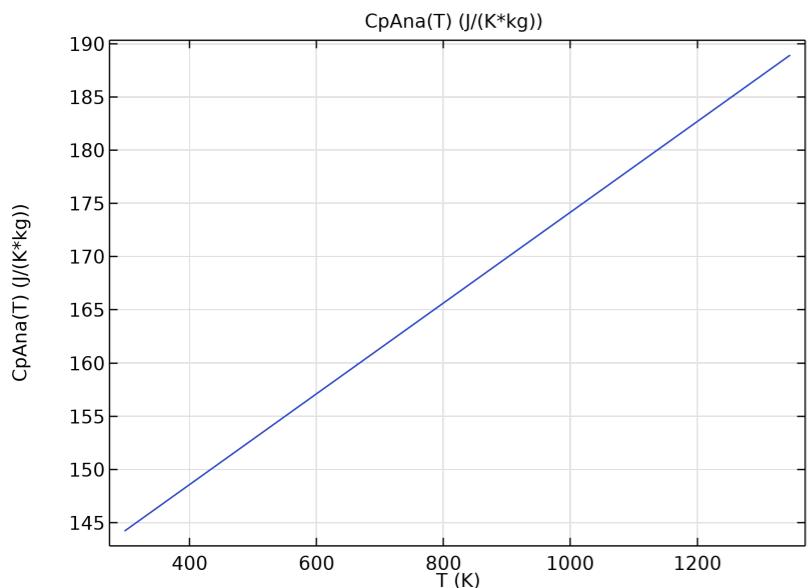
CpHf

Thermal conductivity vs temperature



kHf

Specific heat vs temperature (DOI: 10.1361/asmhba0005240)

*CpAna***1.4.3. Ti - Titanium**

Material parameters

Name	Value	Unit
Electrical conductivity	1e8/ReTi(T)	S/m
Heat capacity at constant pressure	CpTi(T)	J/(kg·K)
Density	rhoTi(T)	kg/m ³
Thermal conductivity	kTi(T)	W/(m·K)
Relative permittivity	1	1

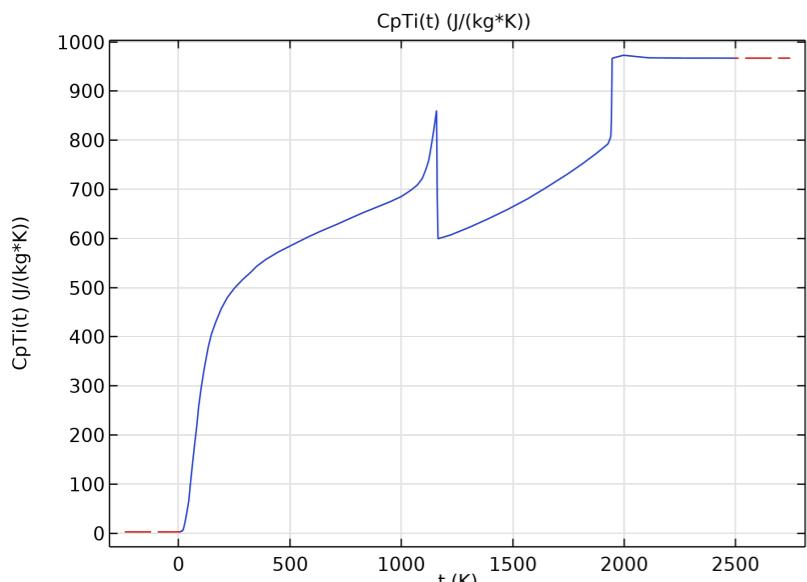
Basic

Description	Value
Electrical conductivity	{1e8/ReTi(T), 0, 0}, {0, 1e8/ReTi(T), 0}, {0, 0, 1e8/ReTi(T)}
Coefficient of thermal expansion	{8.60e-6[1/K], 0, 0}, {0, 8.60e-6[1/K], 0}, {0, 0, 8.60e-6[1/K]}
Heat capacity at constant pressure	CpTi(T)
Density	rhoTi(T)
Thermal conductivity	{kTi(T), 0, 0}, {0, kTi(T), 0}, {0, 0, kTi(T)}
Relative permittivity	{1, 0, 0}, {0, 1, 0}, {0, 0, 1}

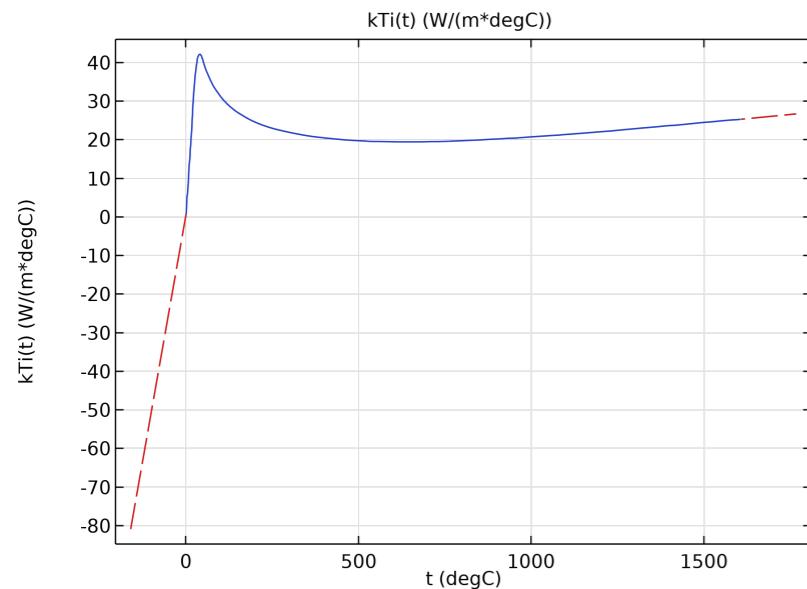
Functions

Function name	Type
CpTi	Interpolation
kTi	Interpolation
rhoTi	Interpolation
ReTi	Interpolation

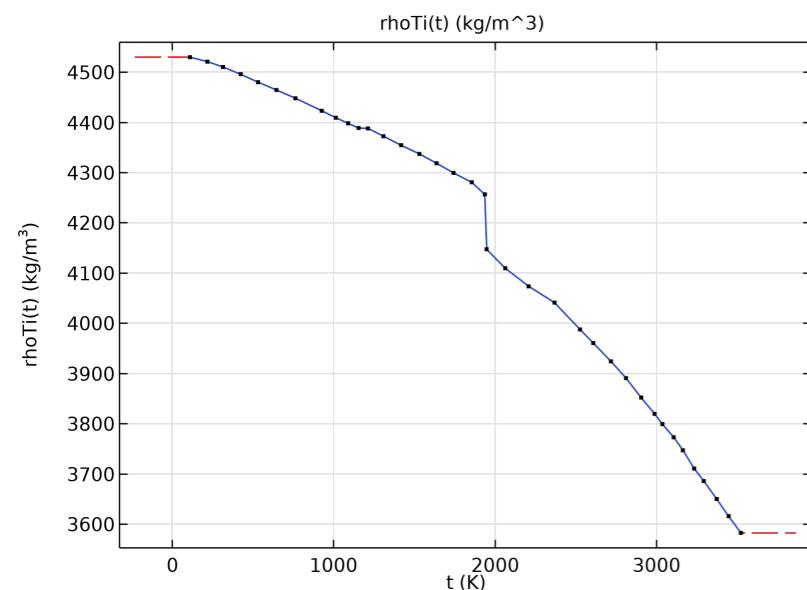
Specific heat vs temperature



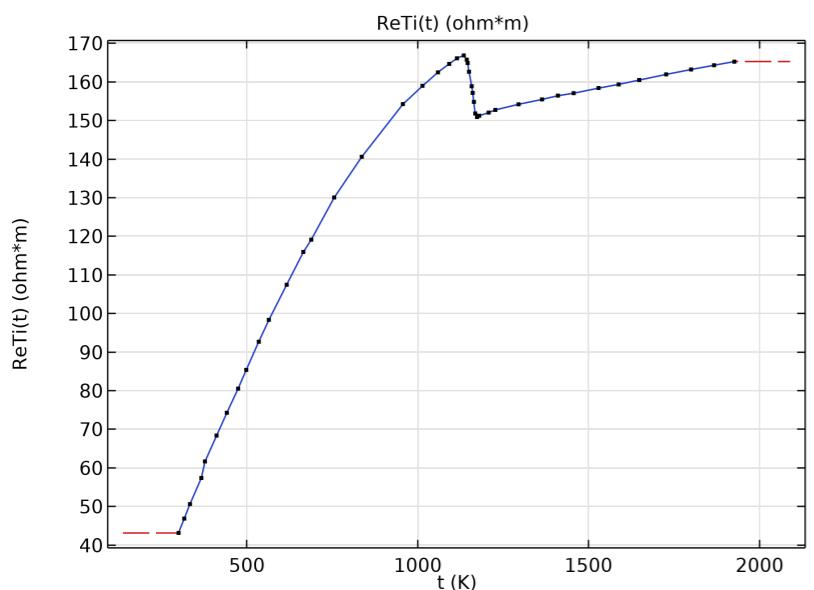
Thermal conductivity vs temperature

*kTi*

Density vs temperature

*rhoTi*

Resistivity vs temperature

*ReTi*

Young's modulus and Poisson's ratio

Description	Value
Young's modulus	115.7e9[Pa]
Poisson's ratio	0.321

1.4.4. W - Tungsten

Material parameters

Name	Value	Unit
Electrical conductivity	1e8/ResTu(T)	S/m
Heat capacity at constant pressure	CpTu(T)	J/(kg·K)
Density	rhoTu(T)	kg/m³
Thermal conductivity	kTu(T)	W/(m·K)
Relative permittivity	1	1

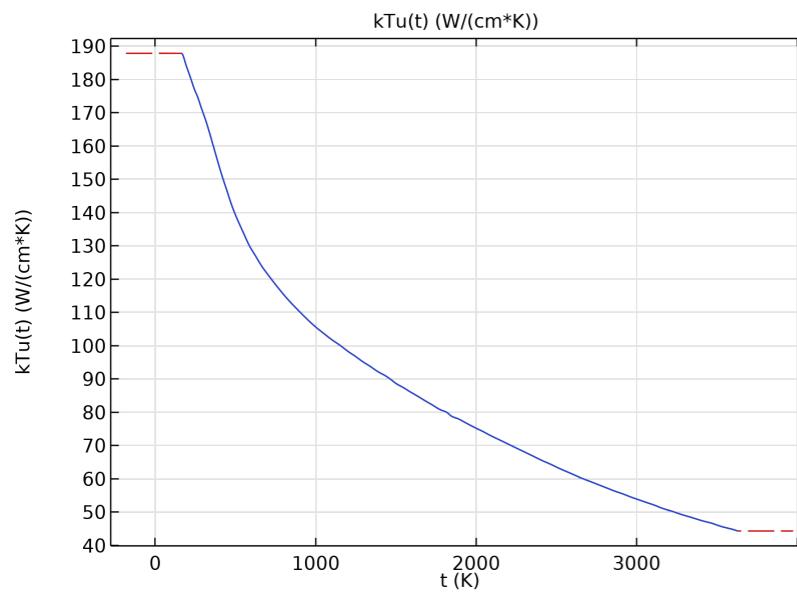
Basic

Description	Value
Electrical conductivity	{1e8/ResTu(T), 0, 0}, {0, 1e8/ResTu(T), 0}, {0, 0, 1e8/ResTu(T)}
Coefficient of thermal expansion	{4.5e-6[1/K], 0, 0}, {0, 4.5e-6[1/K], 0}, {0, 0, 4.5e-6[1/K]}
Heat capacity at constant pressure	CpTu(T)
Density	rhoTu(T)
Thermal conductivity	{kTu(T), 0, 0}, {0, kTu(T), 0}, {0, 0, kTu(T)}
Relative permittivity	{1, 0, 0}, {0, 1, 0}, {0, 0, 1}

Functions

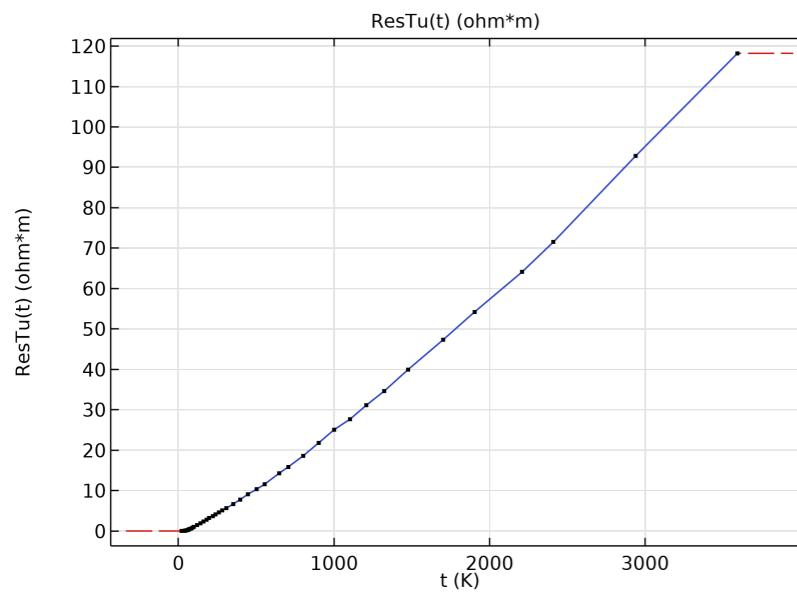
Function name	Type
kTu	Interpolation
ResTu	Interpolation
CpTu	Piecewise
rhoTu	Analytic

Thermal conductivity vs temperature



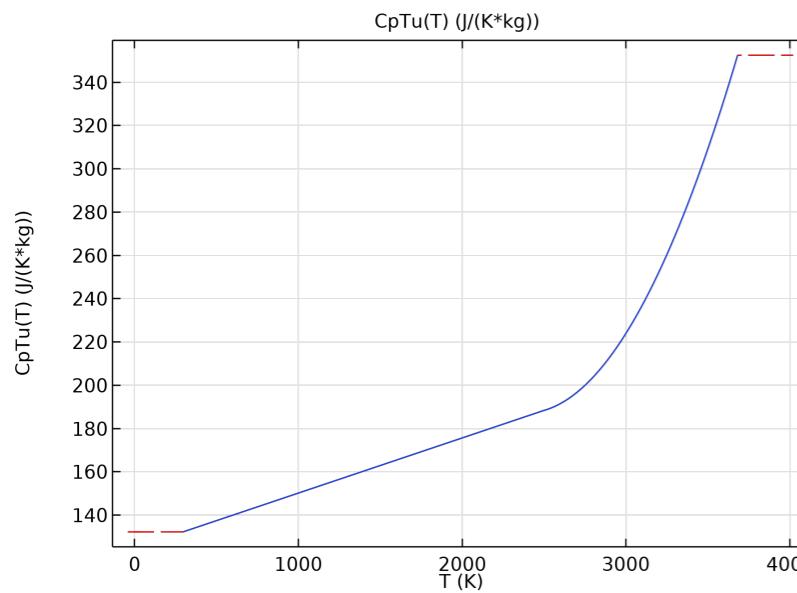
kTu

Resistivity vs temperature

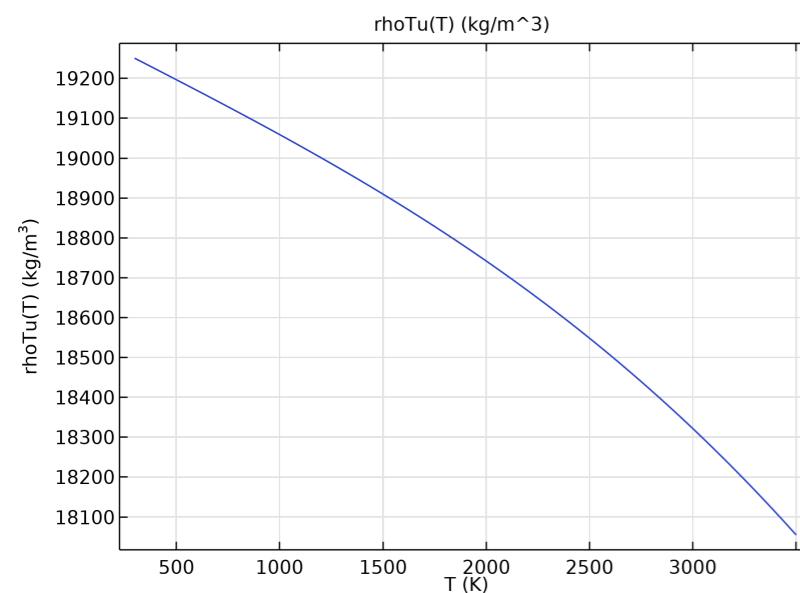


ResTu

Specific heat vs temperature



Density vs temperature

*rhoTu*Young's modulus and
Poisson's ratio

Description	Value
Young's modulus	411e9[Pa]
Poisson's ratio	0.28

2. Component 2D

Date Feb 2, 2022, 9:17:56 PM

Settings

Description	Value
Unit system	Same as global system (SI)
Geometry shape function	Automatic

Spatial frame
coordinates

First	Second	Third
r	phi	z

Material frame
coordinates

First	Second	Third
R	PHI	Z

Geometry frame
coordinates

First	Second	Third
Rg	PHig	Zg

Mesh frame
coordinates

First	Second	Third
Rm	PHim	Zm

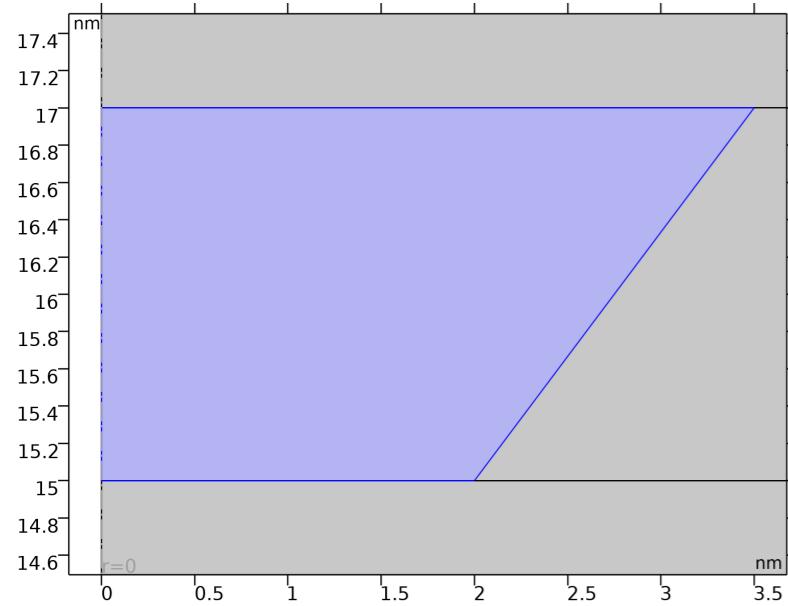
2.1. Definitions**2.1.1. Probes**

Maximum temperature in the filament

Probe type Domain probe

Selection

Geometric entity level	Domain
Selection	Geometry geom2: Dimension 2: Domain 2

*Selection*

Probe type

Description	Value
Type	Maximum

Expression

Description	Value
Expression	T
Table and plot unit	K
Description	Temperature

Table and window settings

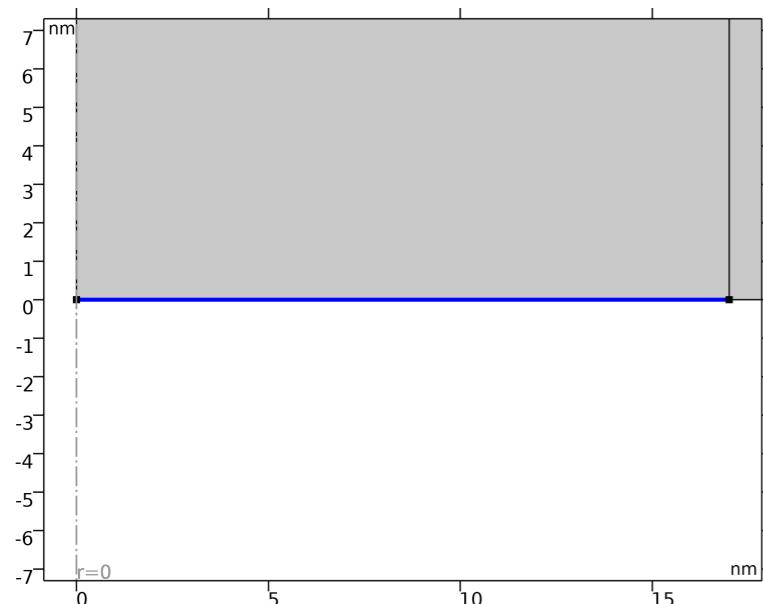
Description	Value
Output table	Probe Table 5
Plot window	Probe Plot 4

Density of current measured

Probe type	Boundary probe
------------	----------------

Selection

Geometric entity level	Boundary
Selection	Geometry geom2: Dimension 1: Boundary 2

*Selection*

Probe type

Description	Value
Type	Integral

Expression

Description	Value
Expression	ec.Jz
Table and plot unit	A
Description	Current density, z component

Table and window settings

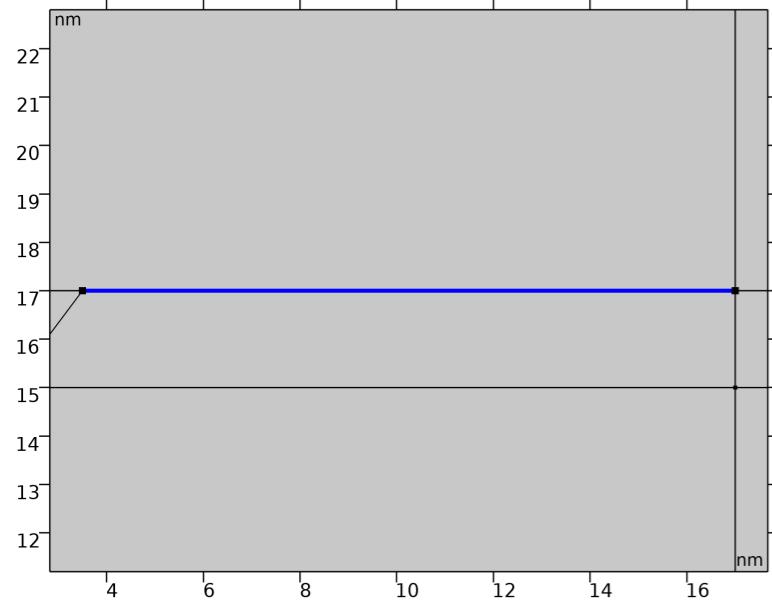
Description	Value
Output table	Probe Table 5
Plot window	Probe Plot 4

Average temperature at the metal 1 edge

Probe type	Boundary probe
------------	----------------

Selection

Geometric entity level	Boundary
Selection	Geometry geom2: Dimension 1: Boundary 10



Selection

Expression

Description	Value
Expression	T
Table and plot unit	K
Description	Temperature

Table and window settings

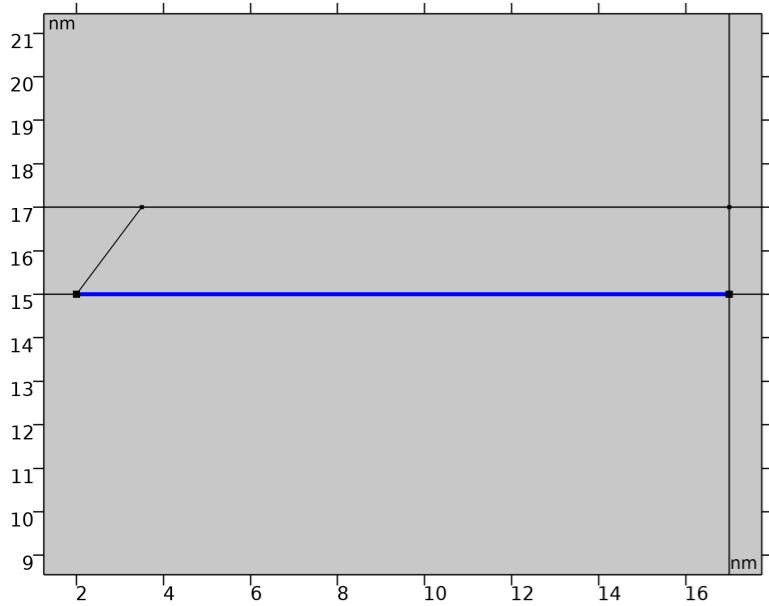
Description	Value
Output table	Probe Table 5
Plot window	Probe Plot 7

Average temperature at the metal 2 edge

Probe type	Boundary probe
------------	----------------

Selection

Geometric entity level	Boundary
Selection	Geometry geom2: Dimension 1: Boundary 9

*Selection*

Expression

Description	Value
Expression	T
Table and plot unit	K
Description	Temperature

Table and window settings

Description	Value
Output table	Probe Table 5
Plot window	Probe Plot 7

2.1.2. Coordinate Systems

Boundary System 2

Coordinate system type	Boundary system
Tag	sys2

Coordinate names

First	Second	Third
t1	to	n

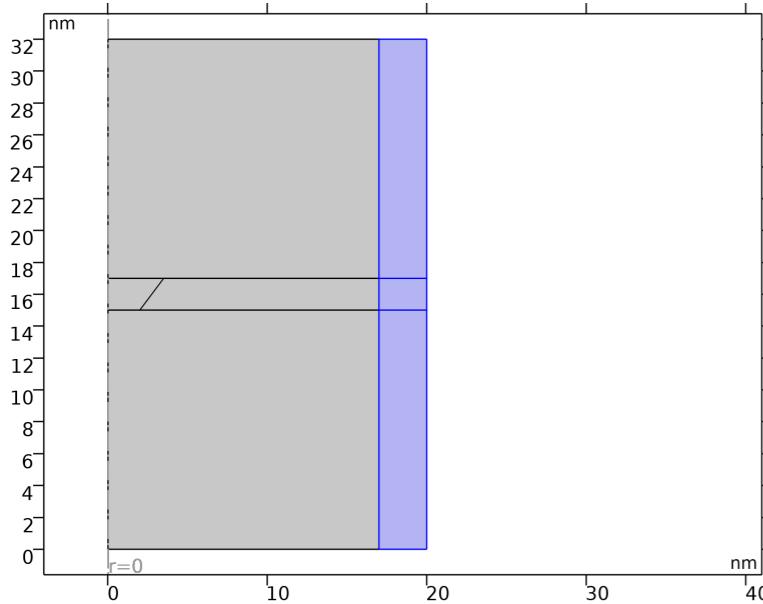
2.1.3. Artificial Domains

Infinite Element Domain 1

Tag	ie1
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Selection

Geometric entity level	Domain
Selection	Geometry geom2: Dimension 2: Domains 5–7

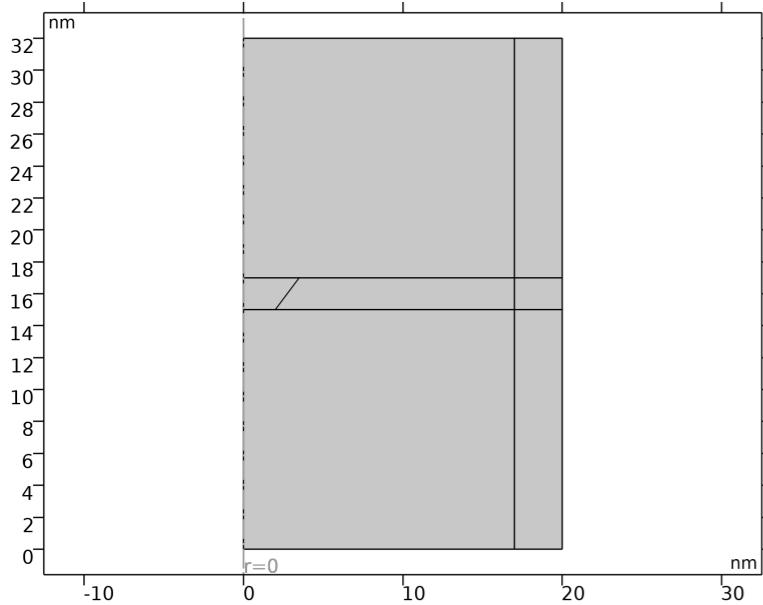
*Selection*

Geometry

Description	Value
Coordinate names	{ r , ϕ , z }
Type	Cylindrical

Scaling

Description	Value
Coordinate stretching type	Rational

2.2. Geometry 2*Geometry 2*

Units

Length unit	nm
Angular unit	deg

Geometry statistics

Description	Value
Space dimension	2
Number of domains	7
Number of boundaries	20
Number of vertices	14

2.2.1. Rectangle 1 (r1)

Position

Description	Value

Position	{0, 0}
Layers to the right	On
Layers on bottom	Off

Position

Layer name	Thickness (nm)
Layer 1	3

Size

Description	Value
Width	LBase
Height	L_metal1

2.2.2. Rectangle 2 (r2)

Position

Description	Value
Position	{0, L_metal1}
Layers to the right	On
Layers on bottom	Off

Position

Layer name	Thickness (nm)
Layer 1	3

Size

Description	Value
Width	LBase
Height	Linsulator

2.2.3. Rectangle 3 (r3)

Position

Description	Value
Position	{0, Linsulator + L_metal1}
Layers to the right	On
Layers on bottom	Off

Position

Layer name	Thickness (nm)
Layer 1	3

Size

Description	Value
Width	LBase
Height	L_metal2

2.2.4. Polygon 1 (pol1)

Object type

Description	Value
Type	Solid

Coordinates

Description	Value
Data source	Table

Coordinates

r (nm)	z (nm)
0	L_metal1
Rbottom	L_metal1
Rtop	Linsulator+L_metal1
0	Linsulator+L_metal1
0	L_metal1

2.2.5. Rectangle 4 (r4)

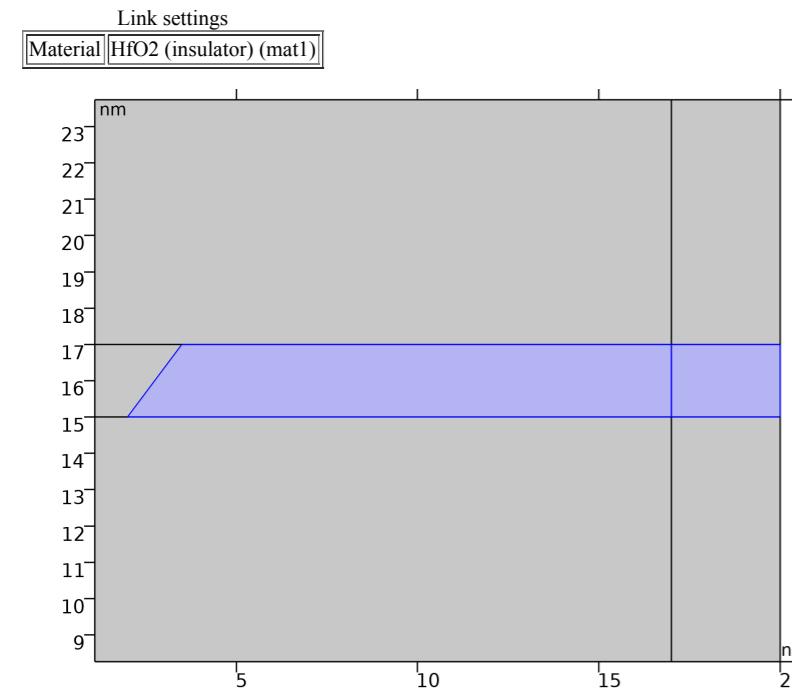
Position

Description	Value
Position	{0, L_metal1 - 1}

Size

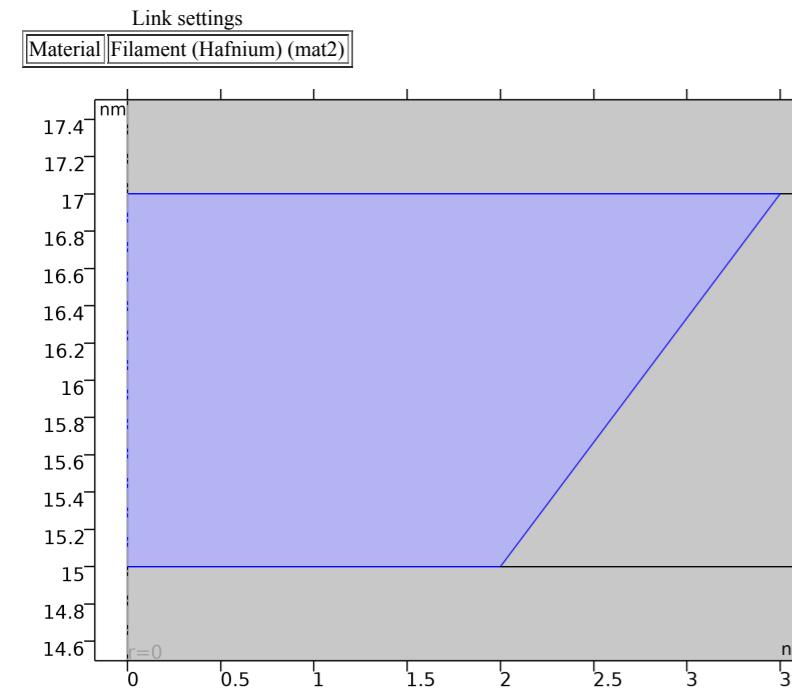
Description	Value
Width	Rbottom + Rtop
Height	Linsulator + 2

2.3. Materials

2.3.1. HfO₂HfO₂

Selection

Geometric entity level	Domain
Selection	Geometry geom2: Dimension 2: Domains 4, 6

2.3.2. Filament (Hafnium) (mat2)

Filament (Hafnium) (mat2)

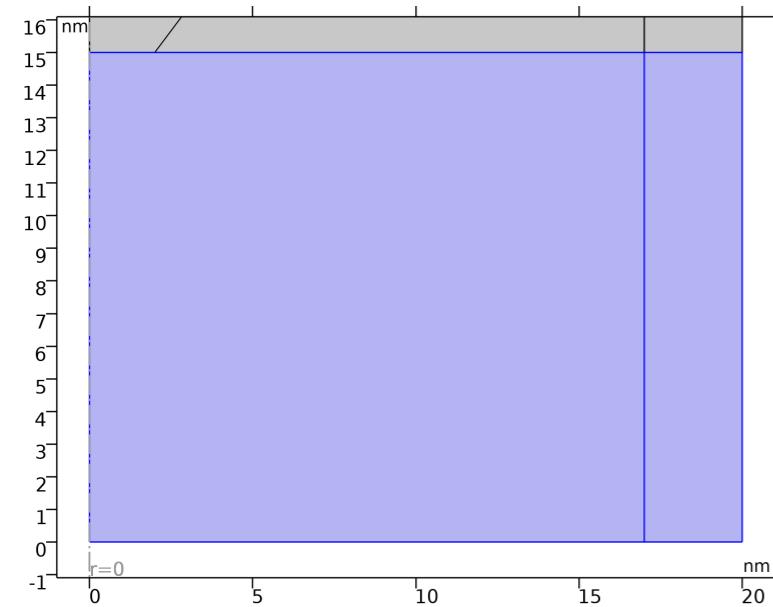
Selection

Geometric entity level	Domain
Selection	Geometry geom2: Dimension 2: Domain 2

2.3.3. Ti - Titanium

Link settings

Material	Ti - Titanium (mat3)
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*Ti - Titanium*

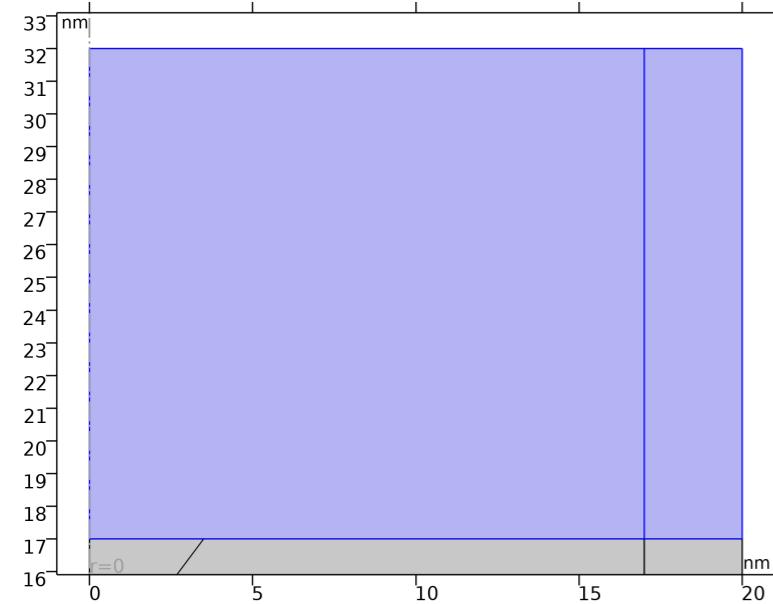
Selection

Geometric entity level	Domain
Selection	Geometry geom2: Dimension 2: Domains 1, 5

2.3.4. W - Tungsten

Link settings

Material	W - Tungsten (mat4)

*W - Tungsten*

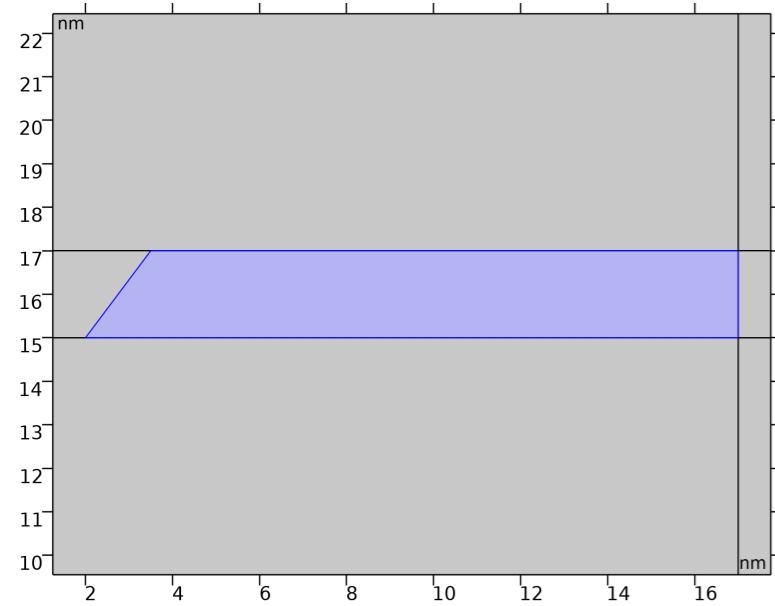
Selection

Geometric entity level	Domain
Selection	Geometry geom2: Dimension 2: Domains 3, 7

2.4. Wall Distance

Used products

COMSOL Multiphysics

*Wall Distance***Selection**

Geometric entity level	Domain
Selection	Geometry geom2: Dimension 2: Domain 4

2.4.1. Interface Settings**Discretization****Settings**

Description	Value
Element order	Quadratic

Settings

Description	Value
Equation form	Study controlled

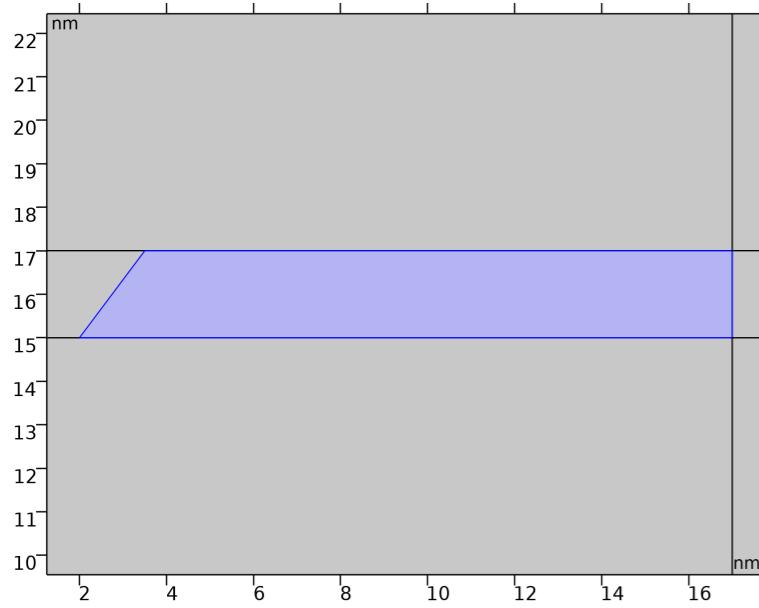
Settings

Description	Value
Smoothing parameter	0.2

2.4.2. Variables

Name	Expression	Unit	Description	Selection
wd.sigmax	0.2	1	Smoothing parameter	Domain 4
wd.nr	unr	1	Normal vector, r component	Boundaries 8–9, 13
wd.nphi	0	1	Normal vector, phi component	Boundaries 8–9, 13
wd.nz	unz	1	Normal vector, z component	Boundaries 8–9, 13
wd.nr	dnr	1	Normal vector, r component	Boundary 10
wd.nphi	0	1	Normal vector, phi component	Boundary 10
wd.nz	dnz	1	Normal vector, z component	Boundary 10
wd.nrmesh	unrmesh	1	Normal vector, r component	Boundaries 8–9, 13
wd.nphimesh	0	1	Normal vector, phi component	Boundaries 8–9, 13
wd.nzmesh	unzmesh	1	Normal vector, z component	Boundaries 8–9, 13
wd.nrmesh	dnmesh	1	Normal vector, r component	Boundary 10
wd.nphimesh	0	1	Normal vector, phi component	Boundary 10
wd.nzmesh	dnzmesh	1	Normal vector, z component	Boundary 10

2.4.3. Distance Equation

*Distance Equation*

Selection

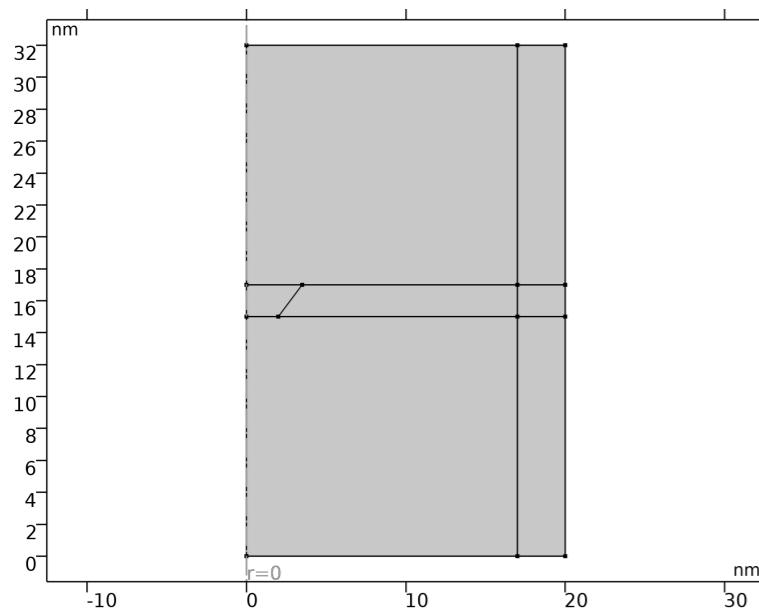
Geometric entity level	Domain
Selection	Geometry geom2: Dimension 2: All domains

Variables

Name	Expression	Unit	Description	Selection
wd.G0	nojac(2/wd.lref)	1/m	Auxiliary wall variable	Domain 4
wd.Ddirr	wd.DdirVarr	1	Direction toward nearest wall, r component	Domain 4
wd.Ddirphi	wd.DdirVarphi	1	Direction toward nearest wall, phi component	Domain 4
wd.Ddirz	wd.DdirVarz	1	Direction toward nearest wall, z component	Domain 4
wd.Dw	wd.DwVar	m	Wall distance	Domain 4
wd.DdirVarr	Gr/sqrt(max(eps,Gr^2+Gz^2))	1	Help variable, r component	Domain 4
wd.DdirVarphi	0	1	Help variable, phi component	Domain 4
wd.DdirVarz	Gz/sqrt(max(eps,Gr^2+Gz^2))	1	Help variable, z component	Domain 4
wd.DwVar	1/G-1/wd.G0	m	Help variable	Domain 4
wd.lref	1.99999999999997E-9	m	Reference length	Domain 4

Shape functions

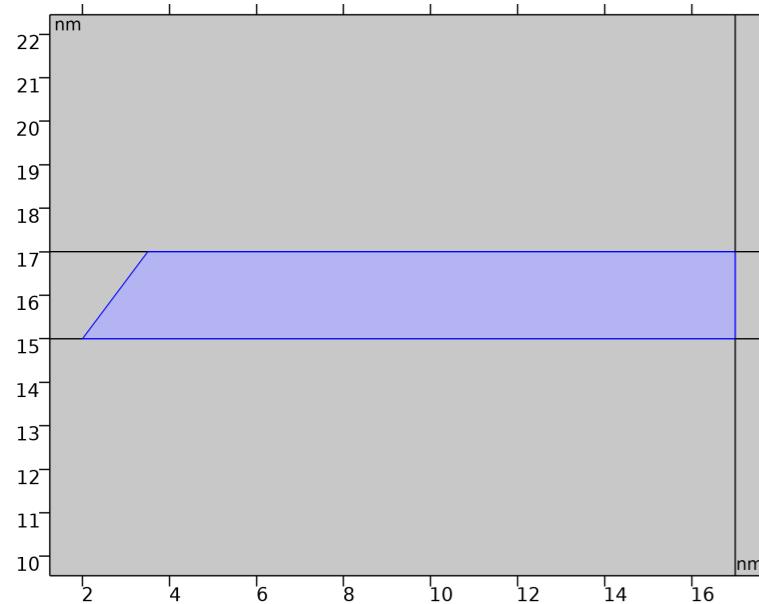
Name	Shape function	Unit	Description	Shape frame	Selection
G	Lagrange (Quadratic)	1/m	Reciprocal wall distance	Spatial	Domain 4

2.4.4. Axial Symmetry

Selection

Geometric entity level	Boundary
Selection	Geometry geom2: Dimension 1: All boundaries

2.4.5. Initial Values



Initial Values

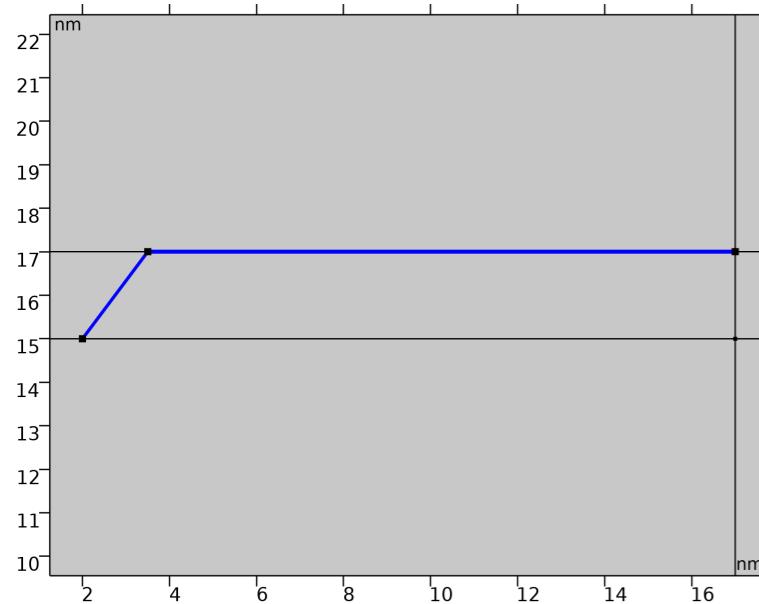
Selection

Geometric entity level	Domain
Selection	Geometry geom2: Dimension 2: All domains

Settings

Description	Value
Reciprocal wall distance	wd.G0

2.4.6. Wall (distance origin)



Wall (distance origin)

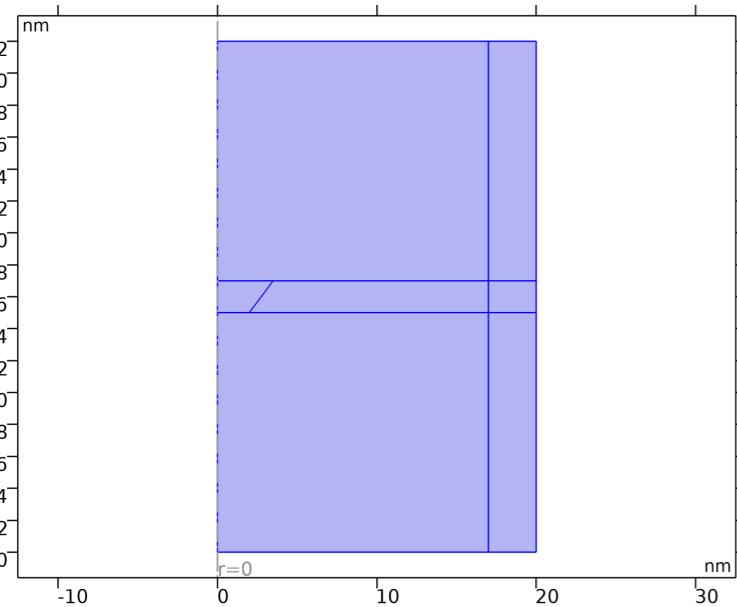
Selection

Geometric entity level	Boundary
Selection	Geometry geom2: Dimension 1: Boundaries 8, 10

2.5. Electric Currents entire RRAM cell

Used products

COMSOL Multiphysics



Electric Currents entire RRAM cell

Selection

Geometric entity level	Domain
Selection	Geometry geom2: Dimension 2: All domains

Equations

$$\nabla \cdot \mathbf{J} = Q_{j,v}$$

$$\mathbf{J} = \sigma \mathbf{E} + \mathbf{J}_e$$

$$\mathbf{E} = -\nabla V$$

2.5.1. Interface Settings

Discretization

Settings

Description	Value
Electric potential	Quadratic

Settings

Description	Value
Equation form	Study controlled

Manual Terminal Sweep Settings

Settings

Description	Value
Use manual terminal sweep	Off
Reference impedance	50[ohm]

2.5.2. Variables

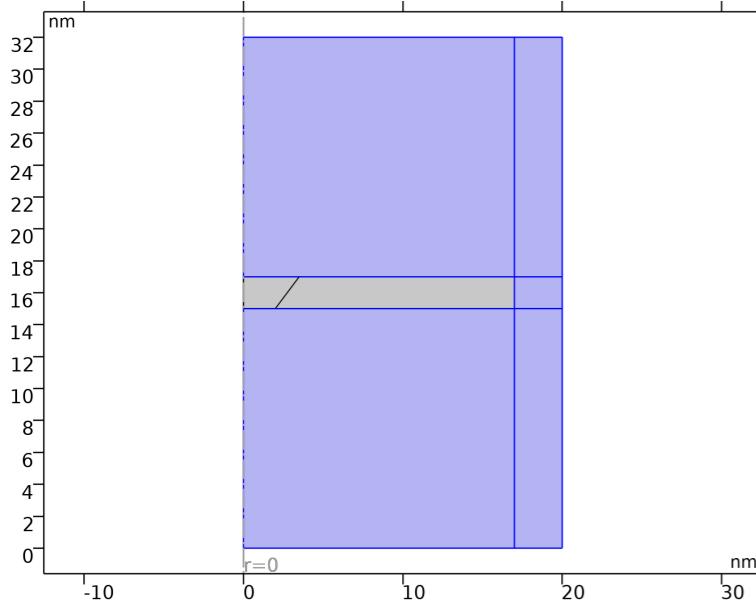
Name	Expression	Unit	Description	Selection	Details
ec.d	1	1	Contribution	Domains 1–4	
ec.d	1	1	Contribution	Domains 5–7	
ec.I_sRR	(spatial.invF11*(spatial.invF11*ec.I_srr+spatial.invF31*ec.I_srz)+spatial.invF31*(spatial.invF11*ec.I_srz+spatial.invF31*ec.I_szz))*spatial.detF	1	Spatial identity matrix, material frame, RR component	Domains 1–4	
ec.I_sPHIR	if(Rg>0.001*h,R/r,Rr)*(spatial.invF11*ec.I_sphir+spatial.invF31*ec.I_spviz)*spatial.detF	1	Spatial identity matrix, material frame, PHIR component	Domains 1–4	
ec.I_sZR	(spatial.invF11*(spatial.invF13*ec.I_srr+spatial.invF33*ec.I_srz)+spatial.invF33*(spatial.invF13*ec.I_srz+spatial.invF33*ec.I_szz))*spatial.detF	1	Spatial identity matrix, material frame, ZR component	Domains 1–4	
ec.I_sRPHI	if(Rg>0.001*h,R/r,Rr)*(spatial.invF11*ec.I_srphi+spatial.invF31*ec.I_szphi)*spatial.detF	1	Spatial identity matrix, material frame, RPHI component	Domains 1–4	
ec.I_sPHIPHI	if(Rg>0.001*h,R/r,Rr)*2*ec.I_sphiphi*spatial.detF	1	Spatial identity matrix, material frame, PHIPHI component	Domains 1–4	
ec.I_sZPHI	if(Rg>0.001*h,R/r,Rr)*(spatial.invF13*ec.I_srphi+spatial.invF33*ec.I_szphi)*spatial.detF	1	Spatial identity matrix, material frame, ZPHI component	Domains 1–4	
ec.I_sRZ	(spatial.invF13*(spatial.invF11*ec.I_srr+spatial.invF31*ec.I_srz)+spatial.invF33*(spatial.invF11*ec.I_srz+spatial.invF31*ec.I_szz))*spatial.detF	1	Spatial identity matrix, material frame, RZ component	Domains 1–4	
ec.I_sPHIZ	if(Rg>0.001*h,R/r,Rr)*(spatial.invF13*ec.I_sphir+spatial.invF33*ec.I_spviz)*spatial.detF	1	Spatial identity matrix, material frame, PHIZ component	Domains 1–4	
ec.I_sZZ	(spatial.invF13*(spatial.invF13*ec.I_srr+spatial.invF33*ec.I_srz)+spatial.invF33*(spatial.invF13*ec.I_srz+spatial.invF33*ec.I_szz))*spatial.detF	1	Spatial identity matrix, material frame, ZZ component	Domains 1–4	
ec.I_sRR	(spatial.invF11*(spatial.invF11*ec.I_srr+spatial.invF31*ec.I_srz)+spatial.invF31*(spatial.invF11*ec.I_srz+spatial.invF31*ec.I_szz))*spatial.detF	1	Spatial identity matrix, material frame, RR component	Domains 5–7	
ec.I_sPHIR	if(Rg>0.001*h,R/r,Rr)*(spatial.invF11*ec.I_sphir+spatial.invF31*ec.I_spviz)*spatial.detF	1	Spatial identity matrix, material frame, PHIR component	Domains 5–7	
ec.I_sZR	(spatial.invF11*(spatial.invF13*ec.I_srr+spatial.invF33*ec.I_srz)+spatial.invF33*(spatial.invF13*ec.I_srz+spatial.invF33*ec.I_szz))*spatial.detF	1	Spatial identity matrix, material frame, ZR component	Domains 5–7	

ec.I_sRPHI	if(Rg>0.001*h,R/r,Rr)*(spatial.invF11*ec.I_srphi+spatial.invF31*ec.I_szphi)*spatial.detF	1	Spatial identity matrix, material frame, RPHI component	Domains 5–7
ec.I_sPHIPHI	if(Rg>0.001*h,R/r,Rr)^2*ec.I_sphiphi*spatial.detF	1	Spatial identity matrix, material frame, PHIPHI component	Domains 5–7
ec.I_sZPHI	if(Rg>0.001*h,R/r,Rr)*(spatial.invF13*ec.I_srphi+spatial.invF33*ec.I_szphi)*spatial.detF	1	Spatial identity matrix, material frame, ZPHI component	Domains 5–7
ec.I_sRZ	(spatial.invF13*(spatial.invF11*ec.I_srr+spatial.invF31*ec.I_srz)+spatial.invF33*(spatial.invF11*ec.I_srz+spatial.invF31*ec.I_szz))*spatial.detF	1	Spatial identity matrix, material frame, RZ component	Domains 5–7
ec.I_sPHIZ	if(Rg>0.001*h,R/r,Rr)*(spatial.invF13*ec.I_sphir+spatial.invF33*ec.I_sphiz)*spatial.detF	1	Spatial identity matrix, material frame, PHIZ component	Domains 5–7
ec.I_sZZ	(spatial.invF13*(spatial.invF13*ec.I_srr+spatial.invF33*ec.I_srz)+spatial.invF33*(spatial.invF13*ec.I_srz+spatial.invF33*ec.I_szz))*spatial.detF	1	Spatial identity matrix, material frame, ZZ component	Domains 5–7
ec.I_srr	1	1	Spatial identity matrix, rr component	Domains 1–4
ec.I_sphir	0	1	Spatial identity matrix, phir component	Domains 1–4
ec.I_szr	0	1	Spatial identity matrix, zr component	Domains 1–4
ec.I_srphi	0	1	Spatial identity matrix, rphi component	Domains 1–4
ec.I_sphiphi	1	1	Spatial identity matrix, phiphi component	Domains 1–4
ec.I_szphi	0	1	Spatial identity matrix, zphi component	Domains 1–4
ec.I_srz	0	1	Spatial identity matrix, rz component	Domains 1–4
ec.I_sphiz	0	1	Spatial identity matrix, phiz component	Domains 1–4
ec.I_szz	1	1	Spatial identity matrix, zz component	Domains 1–4
ec.I_srr	1	1	Spatial identity matrix, rr component	Domains 5–7
ec.I_sphir	0	1	Spatial identity matrix, phir component	Domains 5–7
ec.I_szr	0	1	Spatial identity matrix, zr component	Domains 5–7
ec.I_srphi	0	1	Spatial identity matrix, rphi component	Domains 5–7
ec.I_sphiphi	1	1	Spatial identity matrix, phiphi component	Domains 5–7
ec.I_szphi	0	1	Spatial identity matrix, zphi component	Domains 5–7
ec.I_szr	0	1	Spatial identity matrix, rz component	Domains 5–7
ec.I_sphiz	0	1	Spatial identity matrix, phiz component	Domains 5–7
ec.I_szz	1	1	Spatial identity matrix, zz component	Domains 5–7
ec.nr	nr		Normal vector, r component	Boundaries 4, 6, 8–11, 13, 15
ec.nphi	0		Normal vector, phi component	Boundaries 4, 6, 8–11, 13, 15
ec.nz	nz		Normal vector, z component	Boundaries 4, 6, 8–11, 13, 15
ec.nr	dnr		Normal vector, r component	Boundaries 1–3, 5, 7
ec.nphi	0		Normal vector, phi component	Boundaries 1–3, 5, 7
ec.nz	dnz		Normal vector, z component	Boundaries 1–3, 5, 7
ec.nr	nr		Normal vector, r component	Boundaries 14, 16
ec.nphi	0		Normal vector, phi component	Boundaries 14, 16
ec.nz	nz		Normal vector, z component	Boundaries 14, 16
ec.nr	dnr		Normal vector, r component	Boundaries 12, 17–20
ec.nphi	0		Normal vector, phi component	Boundaries 12, 17–20
ec.nz	dnz		Normal vector, z component	Boundaries 12, 17–20
ec.nmeshr	nrmesh		Mesh normal vector, r component	Boundaries 4, 6, 8–11, 13, 15
ec.nmeshphi	0		Mesh normal vector, phi component	Boundaries 4, 6, 8–11, 13, 15
ec.nmeshz	nzmesh		Mesh normal vector, z component	Boundaries 4, 6, 8–11, 13, 15
ec.nmeshr	dfrmesh		Mesh normal vector, r component	Boundaries 1–3, 5, 7
ec.nmeshphi	0		Mesh normal vector, phi component	Boundaries 1–3, 5, 7
ec.nmeshz	dnzmesh		Mesh normal vector, z component	Boundaries 1–3, 5, 7
ec.nmeshr	nrmesh		Mesh normal vector, r component	Boundaries 14, 16
ec.nmeshphi	0		Mesh normal vector, phi component	Boundaries 14, 16
ec.nmeshz	nzmesh		Mesh normal vector, z component	Boundaries 14, 16
ec.nmeshr	dfrmesh		Mesh normal vector, r component	Boundaries 12, 17–20
ec.nmeshphi	0		Mesh normal vector, phi component	Boundaries 12, 17–20
ec.nmeshz	dnzmesh		Mesh normal vector, z component	Boundaries 12, 17–20
ec.unmeshr	unrmesh		Mesh normal vector, upside, r component	Boundaries 1–11, 13, 15
ec.unmeshphi	0		Mesh normal vector, upside, phi component	Boundaries 1–11, 13, 15
ec.unmeshz	unzmesh		Mesh normal vector, upside, z component	Boundaries 1–11, 13, 15
ec.unmeshr	unrmesh		Mesh normal vector, upside, r component	Boundaries 12, 14, 16–20
ec.unmeshphi	0		Mesh normal vector, upside, phi component	Boundaries 12, 14, 16–20
ec.unmeshz	unzmesh		Mesh normal vector, upside, z component	Boundaries 12, 14, 16–20
ec.dnmeshr	dfrmesh		Mesh normal vector, downside, r component	Boundaries 1–11, 13, 15
ec.dnmeshphi	0		Mesh normal vector, downside, phi component	Boundaries 1–11, 13, 15
ec.dnmeshz	dnzmesh		Mesh normal vector, downside, z component	Boundaries 1–11, 13, 15
ec.dnmeshr	dfrmesh		Mesh normal vector, downside, r component	Boundaries 12, 14, 16–20
ec.dnmeshphi	0		Mesh normal vector, downside, phi component	Boundaries 12, 14, 16–20
ec.dnmeshz	dnzmesh		Mesh normal vector, downside, z component	Boundaries 12, 14, 16–20

ec.unTr	ec.unTer		Pa	Maxwell upward surface stress tensor, r component	Boundaries 1–11, 13, 15
ec.unTphi	ec.unTephi		Pa	Maxwell upward surface stress tensor, phi component	Boundaries 1–11, 13, 15
ec.unTz	ec.unTez		Pa	Maxwell upward surface stress tensor, z component	Boundaries 1–11, 13, 15
ec.unTr	ec.unTer		Pa	Maxwell upward surface stress tensor, r component	Boundaries 12, 14, 16–20
ec.unTphi	ec.unTephi		Pa	Maxwell upward surface stress tensor, phi component	Boundaries 12, 14, 16–20
ec.unTz	ec.unTez		Pa	Maxwell upward surface stress tensor, z component	Boundaries 12, 14, 16–20
ec.dnTr	ec.bnTer		Pa	Maxwell downward surface stress tensor, r component	Boundaries 1–11, 13, 15
ec.dnTphi	ec.bnTephi		Pa	Maxwell downward surface stress tensor, phi component	Boundaries 1–11, 13, 15
ec.dnTz	ec.bnTez		Pa	Maxwell downward surface stress tensor, z component	Boundaries 1–11, 13, 15
ec.bnTr	ec.bnTer		Pa	Maxwell downward surface stress tensor, r component	Boundaries 12, 14, 16–20
ec.bnTphi	ec.bnTephi		Pa	Maxwell downward surface stress tensor, phi component	Boundaries 12, 14, 16–20
ec.bnTz	ec.bnTez		Pa	Maxwell downward surface stress tensor, z component	Boundaries 12, 14, 16–20
ec.unr	unr			Normal vector up direction, r component	Boundaries 1–11, 13, 15
ec.unphi	0			Normal vector up direction, phi component	Boundaries 1–11, 13, 15
ec.unz	unz			Normal vector up direction, z component	Boundaries 1–11, 13, 15
ec.unr	unr			Normal vector up direction, r component	Boundaries 12, 14, 16–20
ec.unphi	0			Normal vector up direction, phi component	Boundaries 12, 14, 16–20
ec.unz	unz			Normal vector up direction, z component	Boundaries 12, 14, 16–20
ec.dnr	dnr			Normal vector down direction, r component	Boundaries 1–11, 13, 15
ec.dnphi	0			Normal vector down direction, phi component	Boundaries 1–11, 13, 15
ec.dnz	dnz			Normal vector down direction, z component	Boundaries 1–11, 13, 15
ec.bnTr	dnr			Normal vector down direction, r component	Boundaries 12, 14, 16–20
ec.bnphi	0			Normal vector down direction, phi component	Boundaries 12, 14, 16–20
ec.bnz	dnz			Normal vector down direction, z component	Boundaries 12, 14, 16–20
ec.unTer	-0.5*ec.dnr*(real(up(ec.Dr))*real(up(ec.Er))+real(up(ec.Dphi))*real(up(ec.Ephi))+real(up(ec.Dz))*real(up(ec.Ez)))+real(up(ec.Dr))*(real(up(ec.Er))*ec.dnr+real(up(ec.Ephi))*ec.bnphi+real(up(ec.Ez))*ec.dnz)		Pa	Maxwell upward electric surface stress tensor, r component	Boundaries 4, 6, 8–11, 13, 15
ec.unTephi	-0.5*ec.bnphi*(real(up(ec.Dr))*real(up(ec.Er))+real(up(ec.Dphi))*real(up(ec.Ephi))+real(up(ec.Dz))*real(up(ec.Ez)))+real(up(ec.Dphi))*(real(up(ec.Er))*ec.bnphi+real(up(ec.Ephi))*ec.bnphi+real(up(ec.Ez))*ec.bnphi)		Pa	Maxwell upward electric surface stress tensor, phi component	Boundaries 4, 6, 8–11, 13, 15
ec.unTez	-0.5*ec.bnz*(real(up(ec.Dr))*real(up(ec.Er))+real(up(ec.Dphi))*real(up(ec.Ephi))+real(up(ec.Dz))*real(up(ec.Ez)))*(real(up(ec.Er))*ec.bnphi+real(up(ec.Ephi))*ec.bnphi+real(up(ec.Ez))*ec.bnphi)		Pa	Maxwell upward electric surface stress tensor, z component	Boundaries 4, 6, 8–11, 13, 15
ec.unTer	0		Pa	Maxwell upward electric surface stress tensor, r component	Boundaries 1–3, 5, 7
ec.unTephi	0		Pa	Maxwell upward electric surface stress tensor, phi component	Boundaries 1–3, 5, 7
ec.unTez	0		Pa	Maxwell upward electric surface stress tensor, z component	Boundaries 1–3, 5, 7
ec.unTer	-0.5*ec.bnphi*(real(up(ec.Dr))*real(up(ec.Er))+real(up(ec.Dphi))*real(up(ec.Ephi))+real(up(ec.Dz))*real(up(ec.Ez)))+real(up(ec.Dr))*(real(up(ec.Er))*ec.bnphi+real(up(ec.Ephi))*ec.bnphi+real(up(ec.Ez))*ec.bnphi)		Pa	Maxwell upward electric surface stress tensor, r component	Boundaries 14, 16
ec.bnTephi	-0.5*ec.bnphi*(real(up(ec.Dr))*real(up(ec.Er))+real(up(ec.Dphi))*real(up(ec.Ephi))+real(up(ec.Dz))*real(up(ec.Ez)))+real(up(ec.Dphi))*(real(up(ec.Er))*ec.bnphi+real(up(ec.Ephi))*ec.bnphi+real(up(ec.Ez))*ec.bnphi)		Pa	Maxwell upward electric surface stress tensor, phi component	Boundaries 14, 16
ec.bnTez	-0.5*ec.bnz*(real(up(ec.Dr))*real(up(ec.Er))+real(up(ec.Dphi))*real(up(ec.Ephi))+real(up(ec.Dz))*real(up(ec.Ez)))*(real(up(ec.Er))*ec.bnphi+real(up(ec.Ephi))*ec.bnphi+real(up(ec.Ez))*ec.bnphi)		Pa	Maxwell upward electric surface stress tensor, z component	Boundaries 14, 16
ec.bnTer	0		Pa	Maxwell upward electric surface stress tensor, r component	Boundaries 12, 17–20
ec.bnTephi	0		Pa	Maxwell upward electric surface stress tensor, phi component	Boundaries 12, 17–20
ec.bnTez	0		Pa	Maxwell upward electric surface stress tensor, z component	Boundaries 12, 17–20
ec.bnTer	-0.5*ec.unr*(real(down(ec.Dr))*real(down(ec.Er))+real(down(ec.Dphi))*real(down(ec.Ephi))+real(down(ec.Dz))*real(down(ec.Ez)))+real(down(ec.Dr))*(real(down(ec.Er))*ec.unr+real(down(ec.Ephi))*ec.unr+real(down(ec.Ez))*ec.unr)		Pa	Maxwell downward electric surface stress tensor, r component	Boundaries 1–11, 13, 15
ec.bnTephi	-0.5*ec.unphi*(real(down(ec.Dr))*real(down(ec.Er))+real(down(ec.Dphi))*real(down(ec.Ephi))+real(down(ec.Dz))*real(down(ec.Ez)))+real(down(ec.Dphi))*(real(down(ec.Er))*ec.unphi+real(down(ec.Ephi))*ec.unphi+real(down(ec.Ez))*ec.unphi)		Pa	Maxwell downward electric surface stress tensor, phi component	Boundaries 1–11, 13, 15
ec.bnTez	-0.5*ec.unz*(real(down(ec.Dr))*real(down(ec.Er))+real(down(ec.Dphi))*real(down(ec.Ephi))+real(down(ec.Dz))*real(down(ec.Ez)))+real(down(ec.Dz))*(real(down(ec.Er))*ec.unz+real(down(ec.Ephi))*ec.unz+real(down(ec.Ez))*ec.unz)		Pa	Maxwell downward electric surface stress tensor, z component	Boundaries 1–11, 13, 15
ec.bnTer	-0.5*ec.unr*(real(down(ec.Dr))*real(down(ec.Er))+real(down(ec.Dphi))*real(down(ec.Ephi))+real(down(ec.Dz))*real(down(ec.Ez)))+real(down(ec.Dr))*(real(down(ec.Er))*ec.unr+real(down(ec.Ephi))*ec.unr+real(down(ec.Ez))*ec.unr)		Pa	Maxwell downward electric surface stress tensor, r component	Boundaries 12, 14, 16–20
ec.bnTephi	-0.5*ec.unphi*(real(down(ec.Dr))*real(down(ec.Er))+real(down(ec.Dphi))*real(down(ec.Ephi))+real(down(ec.Dz))*real(down(ec.Ez)))+real(down(ec.Dphi))*(real(down(ec.Er))*ec.unphi+real(down(ec.Ephi))*ec.unphi+real(down(ec.Ez))*ec.unphi)		Pa	Maxwell downward electric surface stress tensor, phi component	Boundaries 12, 14, 16–20
ec.bnTez	-0.5*ec.unz*(real(down(ec.Dr))*real(down(ec.Er))+real(down(ec.Dphi))*real(down(ec.Ephi))+real(down(ec.Dz))*real(down(ec.Ez)))+real(down(ec.Dz))*(real(down(ec.Er))*ec.unz+real(down(ec.Ephi))*ec.unz+real(down(ec.Ez))*ec.unz)		Pa	Maxwell downward electric surface stress tensor, z component	Boundaries 12, 14, 16–20
ec.intWe	ec.int_We(ec.d*ec.dWe)		J	Total electric energy	Global
ec.Qh	0		W/m ³	Volumetric loss density, electromagnetic	Domains 1–4
ec.Qh	0		W/m ³	Volumetric loss density, electromagnetic	Domains 5–7
ec.Qsh	0		W/m ²	Surface loss density, electromagnetic	Boundaries 1–11, 13, 15
ec.Qsh	0		W/m ²	Surface loss density, electromagnetic	Boundaries 12, 14, 16–20
ec.Qlh	0		W/m	Line loss density, electromagnetic	Boundaries 1–11, 13, 15
ec.Qlh	0		W/m	Line loss density, electromagnetic	Boundaries 12, 14, 16–20
ec.R11	NaN+NaN*i			Resistance, 11 component	Global
ec.R21	NaN+NaN*i			Resistance, 21 component	Global
ec.R12	NaN+NaN*i			Resistance, 12 component	Global
ec.R22	NaN+NaN*i			Resistance, 22 component	Global

ec.G11	ec.I0_1/ec.V0_1	S	Conductance, 11 component	Global
ec.G21	ec.I0_2/ec.V0_1	S	Conductance, 21 component	Global
ec.G12	ec.I0_1/ec.V0_2	S	Conductance, 12 component	Global
ec.G22	ec.I0_2/ec.V0_2	S	Conductance, 22 component	Global
ec.S11	NaN+NaN*i		S-parameter, 11 component	Global
ec.S21	NaN+NaN*i		S-parameter, 21 component	Global
ec.S12	NaN+NaN*i		S-parameter, 12 component	Global
ec.S22	NaN+NaN*i		S-parameter, 22 component	Global
ec.S11dB	10*log10(realdot(ec.S11,ec.S11))	dB	S-parameter, dB, 11 component	Global
ec.S21dB	10*log10(realdot(ec.S21,ec.S21))	dB	S-parameter, dB, 21 component	Global
ec.S12dB	10*log10(realdot(ec.S12,ec.S12))	dB	S-parameter, dB, 12 component	Global
ec.S22dB	10*log10(realdot(ec.S22,ec.S22))	dB	S-parameter, dB, 22 component	Global
ec.zref	50[ohm]	Ω	Reference impedance	Global

2.5.3. Current Conservation Cell-RRAM/Filament



Current Conservation Cell-RRAM/Filament

Selection

Geometric entity level	Domain
Selection	Geometry geom2: Dimension 2: All domains

Equations

$$\nabla \cdot \mathbf{J} = Q_{\text{v}}$$

$$\mathbf{J} = \sigma \mathbf{E} + \mathbf{J}_e$$

$$\mathbf{E} = -\nabla V$$

Constitutive Relation Jc-E

Settings

Description	Value
Conduction model	Electrical conductivity
Electrical conductivity	From material

Constitutive Relation D-E

Settings

Description	Value
Dielectric model	Relative permittivity
Relative permittivity	From material

Coordinate System Selection

Settings

Description	Value
Coordinate system	Global coordinate system

Used products

COMSOL Multiphysics

Properties from material

Property	Material	Property group
Electrical conductivity	HfO ₂ (insulator)	Basic
Relative permittivity	HfO ₂ (insulator)	Basic
Electrical conductivity	Ti - Titanium	Basic
Relative permittivity	Ti - Titanium	Basic
Electrical conductivity	W - Tungsten	Basic
Relative permittivity	W - Tungsten	Basic

Variables

Name	Expression	Unit	Description	Selection	Details
ec.Qh	ec.Qrh	W/m ³	Volumetric loss density, electromagnetic	Domains 1, 3	
ec.Qh	ec.Qrh	W/m ³	Volumetric loss density, electromagnetic	Domains 5–7	
ec.Jir	ec.sigmarr*ec.Er+ec.sigmarphi*ec.Ephi+ec.sigmarz*ec.Ez	A/m ²	Conduction current density, r component	Domains 1, 3	
ec.Jiphi	ec.sigmaphir*ec.Er+ec.sigmaphiphi*ec.Ephi+ec.sigmaphiz*ec.Ez	A/m ²	Conduction current density, phi component	Domains 1, 3	
ec.Jiz	ec.sigmarz*ec.Er+ec.sigmarphi*ec.Ephi+ec.sigmazz*ec.Ez	A/m ²	Conduction current density, z component	Domains 1, 3	
ec.Jir	ec.sigmarr*ec.Er+ec.sigmarphi*ec.Ephi+ec.sigmarz*ec.Ez	A/m ²	Conduction current density, r component	Domains 5–7	
ec.Jiphi	ec.sigmaphir*ec.Er+ec.sigmaphiphi*ec.Ephi+ec.sigmaphiz*ec.Ez	A/m ²	Conduction current density, phi component	Domains 5–7	
ec.Jiz	ec.sigmarz*ec.Er+ec.sigmarphi*ec.Ephi+ec.sigmazz*ec.Ez	A/m ²	Conduction current density, z component	Domains 5–7	
ec.Jdr	0	A/m ²	Displacement current density, r component	Domains 1, 3	
ec.Jdphi	0	A/m ²	Displacement current density, phi component	Domains 1, 3	
ec.Jdz	0	A/m ²	Displacement current density, z component	Domains 1, 3	
ec.Jdr	0	A/m ²	Displacement current density, r component	Domains 5–7	
ec.Jdphi	0	A/m ²	Displacement current density, phi component	Domains 5–7	
ec.Jdz	0	A/m ²	Displacement current density, z component	Domains 5–7	
ec.Jer	0	A/m ²	External current density, r component	Domains 1, 3	+ operation
ec.Jephi	0	A/m ²	External current density, phi component	Domains 1, 3	+ operation
ec.Jez	0	A/m ²	External current density, z component	Domains 1, 3	+ operation
ec.Jer	0	A/m ²	External current density, r component	Domains 5–7	+ operation
ec.Jephi	0	A/m ²	External current density, phi component	Domains 5–7	+ operation
ec.Jez	0	A/m ²	External current density, z component	Domains 5–7	+ operation
ec.Jr	ec.Jir+ec.Jdr+ec.Jer	A/m ²	Current density, r component	Domains 1, 3	
ec.Jphi	ec.Jiphi+ec.Jdphi+ec.Jephi	A/m ²	Current density, phi component	Domains 1, 3	
ec.Jz	ec.Jiz+ec.Jdz+ec.Jez	A/m ²	Current density, z component	Domains 1, 3	
ec.Jr	ec.Jir+ec.Jdr+ec.Jer	A/m ²	Current density, r component	Domains 5–7	
ec.Jphi	ec.Jiphi+ec.Jdphi+ec.Jephi	A/m ²	Current density, phi component	Domains 5–7	
ec.Jz	ec.Jiz+ec.Jdz+ec.Jez	A/m ²	Current density, z component	Domains 5–7	
ec.normJ	sqrt(realdot(ec.Jr,ec.Jr)+realdot(ec.Jphi,ec.Jphi)+realdot(ec.Jz,ec.Jz))	A/m ²	Current density norm	Domains 1, 3	
ec.normJ	sqrt(realdot(ec.Jr,ec.Jr)+realdot(ec.Jphi,ec.Jphi)+realdot(ec.Jz,ec.Jz))	A/m ²	Current density norm	Domains 5–7	
ec.rhoq	ppr(d(ec.Dr,r)+if(abs(r)<0.001*h_spatial,d(ec.Dr,r),ec.Dr/r)+d(ec.Dz,z))	C/m ³	Space charge density	Domains 1, 3	
ec.rhoq	ppr(iel.T11*d(ec.Dr,r)-iel.T12*if(abs(r)<0.001*h_spatial,d(ec.Dphi,r),ec.Dphi/iel.r)+iel.T13*d(ec.Dr,z)+iel.T21*d(ec.Dphi,r)+iel.T22*if(abs(r)<0.001*h_spatial,d(ec.Dr,r),ec.Dr/iel.r)+iel.T23*d(ec.Dphi,z)+iel.T31*d(ec.Dz,r)+iel.T33*d(ec.Dz,z))	C/m ³	Space charge density	Domains 5–7	
ec.sigmarr	material.sigmal1	S/m	Electrical conductivity, rr component	Domains 1, 3	Meta
ec.sigmaphir	material.sigmal2	S/m	Electrical conductivity, phir component	Domains 1, 3	Meta
ec.sigmarz	material.sigmal3	S/m	Electrical conductivity, zr component	Domains 1, 3	Meta
ec.sigmarphi	material.sigmal4	S/m	Electrical conductivity, rphi component	Domains 1, 3	Meta

ec.sigmaphiphi	material.sigma22	S/m	Electrical conductivity, phiphi component	Domains 1, 3	Meta
ec.sigmarphi	material.sigma32	S/m	Electrical conductivity, zphi component	Domains 1, 3	Meta
ec.sigmarz	material.sigma13	S/m	Electrical conductivity, rz component	Domains 1, 3	Meta
ec.sigmaphiz	material.sigma23	S/m	Electrical conductivity, phiz component	Domains 1, 3	Meta
ec.sigmazz	material.sigma33	S/m	Electrical conductivity, zz component	Domains 1, 3	Meta
ec.sigmarr	material.sigma11	S/m	Electrical conductivity, rr component	Domains 5–7	Meta
ec.sigmaphir	material.sigma21	S/m	Electrical conductivity, phir component	Domains 5–7	Meta
ec.sigmarzr	material.sigma31	S/m	Electrical conductivity, zr component	Domains 5–7	Meta
ec.sigmarphi	material.sigma12	S/m	Electrical conductivity, rphi component	Domains 5–7	Meta
ec.sigmaphiphi	material.sigma22	S/m	Electrical conductivity, phiphi component	Domains 5–7	Meta
ec.sigmarphi	material.sigma32	S/m	Electrical conductivity, zphi component	Domains 5–7	Meta
ec.sigmarz	material.sigma13	S/m	Electrical conductivity, rz component	Domains 5–7	Meta
ec.sigmaphiz	material.sigma23	S/m	Electrical conductivity, phiz component	Domains 5–7	Meta
ec.sigmazz	material.sigma33	S/m	Electrical conductivity, zz component	Domains 5–7	Meta
ec.epsilonrrr	material.epsilonr11	1	Relative permittivity, rr component	Domains 1, 3	Meta
ec.epsilonrphir	material.epsilonr21	1	Relative permittivity, phir component	Domains 1, 3	Meta
ec.epsilonrzs	material.epsilonr31	1	Relative permittivity, zr component	Domains 1, 3	Meta
ec.epsilonrrphi	material.epsilonr12	1	Relative permittivity, rphi component	Domains 1, 3	Meta
ec.epsilonrphiphi	material.epsilonr22	1	Relative permittivity, phiphi component	Domains 1, 3	Meta
ec.epsilonrzphi	material.epsilonr32	1	Relative permittivity, zphi component	Domains 1, 3	Meta
ec.epsilonrrz	material.epsilonr13	1	Relative permittivity, rz component	Domains 1, 3	Meta
ec.epsilonrphiz	material.epsilonr23	1	Relative permittivity, phiz component	Domains 1, 3	Meta
ec.epsilonrzz	material.epsilonr33	1	Relative permittivity, zz component	Domains 1, 3	Meta
ec.epsilonrrr	material.epsilonr11	1	Relative permittivity, rr component	Domains 5–7	Meta
ec.epsilonrphir	material.epsilonr21	1	Relative permittivity, phir component	Domains 5–7	Meta
ec.epsilonrzs	material.epsilonr31	1	Relative permittivity, zr component	Domains 5–7	Meta
ec.epsilonrrphi	material.epsilonr12	1	Relative permittivity, rphi component	Domains 5–7	Meta
ec.epsilonrphiphi	material.epsilonr22	1	Relative permittivity, phiphi component	Domains 5–7	Meta
ec.epsilonrzphi	material.epsilonr32	1	Relative permittivity, zphi component	Domains 5–7	Meta
ec.epsilonrrz	material.epsilonr13	1	Relative permittivity, rz component	Domains 5–7	Meta
ec.epsilonrphiz	material.epsilonr23	1	Relative permittivity, phiz component	Domains 5–7	Meta
ec.epsilonrzz	material.epsilonr33	1	Relative permittivity, zz component	Domains 5–7	Meta
ec.epsilonr_iso	material.epsilonr_iso	1	Relative permittivity, isotropic value	Domains 1, 3	Meta
ec.epsilonr_iso	material.epsilonr_iso	1	Relative permittivity, isotropic value	Domains 5–7	Meta
ec.Dr	epsilon0_const*ec.I_srr*ec.Er+epsilon0_const*ec.I_srphi*ec.Ephi+epsilon0_const*ec.I_srz*ec.Ez+ec.Pr+ec.Per	C/m ²	Electric displacement field, r component	Domains 1, 3	
ec.Dphi	epsilon0_const*ec.I_sphir*ec.Er+epsilon0_const*ec.I_sphiphi*ec.Ephi+epsilon0_const*ec.I_sphiz*ec.Ez+ec.Pphi+ec.Pephi	C/m ²	Electric displacement field, phi component	Domains 1, 3	
ec.Dz	epsilon0_const*ec.I_srz*ec.Er+epsilon0_const*ec.I_szphi*ec.Ephi+epsilon0_const*ec.I_szz*ec.Ez+ec.Pz+ec.Pez	C/m ²	Electric displacement field, z component	Domains 1, 3	

ec.Dr	$\epsilon_{\text{const}} * \text{ec.I_srr} * \text{ec.Er} + \epsilon_{\text{const}} * \text{ec.I_srphi} * \text{ec.Ephi} + \epsilon_{\text{const}} * \text{ec.I_srz} * \text{ec.Ez} + \text{ec.Pr} + \text{ec.Per}$	C/m ²	Electric displacement field, r component	Domains 5–7
ec.Dphi	$\epsilon_{\text{const}} * \text{ec.I_sphir} * \text{ec.Er} + \epsilon_{\text{const}} * \text{ec.I_sphiphi} * \text{ec.Ephi} + \epsilon_{\text{const}} * \text{ec.I_sphiz} * \text{ec.Ez} + \text{ec.Pphi} + \text{ec.Pephi}$	C/m ²	Electric displacement field, phi component	Domains 5–7
ec.Dz	$\epsilon_{\text{const}} * \text{ec.I_srz} * \text{ec.Er} + \epsilon_{\text{const}} * \text{ec.I_szphi} * \text{ec.Ephi} + \epsilon_{\text{const}} * \text{ec.I_szz} * \text{ec.Ez} + \text{ec.Pz} + \text{ec.Pez}$	C/m ²	Electric displacement field, z component	Domains 5–7
ec.Pr	$\epsilon_{\text{const}} * (\text{ec.chirr} * \text{ec.Er} + \text{ec.chirphi} * \text{ec.Ephi} + \text{ec.chirz} * \text{ec.Ez})$	C/m ²	Polarization, r component	Domains 1, 3
ec.Pphi	$\epsilon_{\text{const}} * (\text{ec.chiphir} * \text{ec.Er} + \text{ec.chiphiphi} * \text{ec.Ephi} + \text{ec.chiphiz} * \text{ec.Ez})$	C/m ²	Polarization, phi component	Domains 1, 3
ec.Pz	$\epsilon_{\text{const}} * (\text{ec.chizr} * \text{ec.Er} + \text{ec.chizphi} * \text{ec.Ephi} + \text{ec.chizz} * \text{ec.Ez})$	C/m ²	Polarization, z component	Domains 1, 3
ec.Per	$\epsilon_{\text{const}} * (\text{ec.chirr} * \text{ec.Er} + \text{ec.chirphi} * \text{ec.Ephi} + \text{ec.chirz} * \text{ec.Ez})$	C/m ²	Polarization, r component	Domains 5–7
ec.Pphi	$\epsilon_{\text{const}} * (\text{ec.chiphir} * \text{ec.Er} + \text{ec.chiphiphi} * \text{ec.Ephi} + \text{ec.chiphiz} * \text{ec.Ez})$	C/m ²	Polarization, phi component	Domains 5–7
ec.Pz	$\epsilon_{\text{const}} * (\text{ec.chizr} * \text{ec.Er} + \text{ec.chizphi} * \text{ec.Ephi} + \text{ec.chizz} * \text{ec.Ez})$	C/m ²	Polarization, z component	Domains 5–7
ec.normD	$\sqrt{\text{realdot}(\text{ec.Dr}, \text{ec.Dr}) + \text{realdot}(\text{ec.Dphi}, \text{ec.Dphi}) + \text{realdot}(\text{ec.Dz}, \text{ec.Dz})}}$	C/m ²	Electric displacement field norm	Domains 1, 3
ec.normD	$\sqrt{\text{realdot}(\text{ec.Dr}, \text{ec.Dr}) + \text{realdot}(\text{ec.Dphi}, \text{ec.Dphi}) + \text{realdot}(\text{ec.Dz}, \text{ec.Dz})}}$	C/m ²	Electric displacement field norm	Domains 5–7
ec.normP	$\sqrt{\text{realdot}(\text{ec.Pr}, \text{ec.Pr}) + \text{realdot}(\text{ec.Pphi}, \text{ec.Pphi}) + \text{realdot}(\text{ec.Pz}, \text{ec.Pz})}}$	C/m ²	Polarization norm	Domains 1, 3
ec.normP	$\sqrt{\text{realdot}(\text{ec.Pr}, \text{ec.Pr}) + \text{realdot}(\text{ec.Pphi}, \text{ec.Pphi}) + \text{realdot}(\text{ec.Pz}, \text{ec.Pz})}}$	C/m ²	Polarization norm	Domains 5–7
ec.Per	0	C/m ²	Polarization contribution, r component	Domains 1, 3
ec.Pephi	0	C/m ²	Polarization contribution, phi component	Domains 1, 3
ec.Pez	0	C/m ²	Polarization contribution, z component	Domains 1, 3
ec.Per	0	C/m ²	Polarization contribution, r component	Domains 5–7
ec.Pephi	0	C/m ²	Polarization contribution, phi component	Domains 5–7
ec.Pez	0	C/m ²	Polarization contribution, z component	Domains 5–7
ec.chirr	-1+ec.epsilonrrr	1	Electric susceptibility, rr component	Domains 1, 3
ec.chiphir	ec.epsilonrphir	1	Electric susceptibility, phir component	Domains 1, 3
ec.chizr	ec.epsilonrzs	1	Electric susceptibility, zr component	Domains 1, 3
ec.chirphi	ec.epsilonrrphi	1	Electric susceptibility, rphi component	Domains 1, 3
ec.chiphiphi	-1+ec.epsilonrphiphi	1	Electric susceptibility, phiphi component	Domains 1, 3
ec.chizphi	ec.epsilonrzphi	1	Electric susceptibility, zphi component	Domains 1, 3
ec.chirz	ec.epsilonrrz	1	Electric susceptibility, rz component	Domains 1, 3
ec.chiphiz	ec.epsilonrphiz	1	Electric susceptibility, phiz component	Domains 1, 3
ec.chizz	-1+ec.epsilonrzz	1	Electric susceptibility, zz component	Domains 1, 3
ec.chirr	-1+ec.epsilonrrr	1	Electric susceptibility, rr component	Domains 5–7
ec.chiphir	ec.epsilonrphir	1	Electric susceptibility, phir component	Domains 5–7
ec.chizr	ec.epsilonrzs	1	Electric susceptibility, zr component	Domains 5–7
ec.chirphi	ec.epsilonrrphi	1	Electric susceptibility, rphi component	Domains 5–7
ec.chiphiphi	-1+ec.epsilonrphiphi	1	Electric susceptibility, phiphi component	Domains 5–7
ec.chizphi	ec.epsilonrzphi	1	Electric susceptibility, zphi component	Domains 5–7
ec.chirz	ec.epsilonrrz	1	Electric susceptibility, rz component	Domains 5–7
ec.chiphiz	ec.epsilonrphiz	1	Electric susceptibility, phiz component	Domains 5–7
ec.chizz	-1+ec.epsilonrzz	1	Electric susceptibility, zz component	Domains 5–7
ec.Er	-V_all_Cellr	V/m	Electric field, r component	Domains 1, 3
ec.Ephi	0	V/m	Electric field, phi component	Domains 1, 3
ec.Ez	-V_all_Cellz	V/m	Electric field, z component	Domains 1, 3
ec.Er	-ie1.T11*V_all_Cellr-ie1.T13*V_all_Cellz	V/m	Electric field, r component	Domains 5–7
ec.Ephi	-ie1.T21*V_all_Cellr-ie1.T23*V_all_Cellz	V/m	Electric field, phi component	Domains 5–7
ec.Ez	-ie1.T31*V_all_Cellr-ie1.T33*V_all_Cellz	V/m	Electric field, z component	Domains 5–7
ec.tEr	-V_all_CellTr	V/m	Tangential electric field, r component	Boundaries 1–2, 4–7, 9–11, 15

ec.tEphi	0	V/m	Tangential electric field, phi component	Boundaries 1–2, 4–7, 9–11, 15
ec.tEz	-V_all_CellTz	V/m	Tangential electric field, z component	Boundaries 1–2, 4–7, 9–11, 15
ec.tEr	-ie1.T11*V_all_CellTr-ie1.T13*V_all_CellTz	V/m	Tangential electric field, r component	Boundaries 12–14, 16–20
ec.tEphi	-ie1.T21*V_all_CellTr-ie1.T23*V_all_CellTz	V/m	Tangential electric field, phi component	Boundaries 12–14, 16–20
ec.tEz	-ie1.T31*V_all_CellTr-ie1.T33*V_all_CellTz	V/m	Tangential electric field, z component	Boundaries 12–14, 16–20
ec.normE	sqrt(realdot(ec.Er,ec.Er)+realdot(ec.Ephi,ec.Ephi)+realdot(ec.Ez,ec.Ez))	V/m	Electric field norm	Domains 1, 3
ec.normE	sqrt(realdot(ec.Er,ec.Er)+realdot(ec.Ephi,ec.Ephi)+realdot(ec.Ez,ec.Ez))	V/m	Electric field norm	Domains 5–7
ec.Qrh	ec.Jr*ec.Er+ec.Jphi*ec.Ephi+ec.Jz*ec.Ez	W/m ³	Volumetric loss density, electric	Domains 1, 3 + operation
ec.Qrh	ec.Jr*ec.Er+ec.Jphi*ec.Ephi+ec.Jz*ec.Ez	W/m ³	Volumetric loss density, electric	Domains 5–7 + operation
ec.W	ec.We	J/m ³	Energy density	Domains 1, 3 + operation
ec.W	ec.We	J/m ³	Energy density	Domains 5–7 + operation
ec.dWe	2*ec.We*pi*r	J/m ²	Integrand for total electric energy	Domains 1, 3 Meta
ec.dWe	2*ec.We*pi*ie1.r*ie1.detInvT	J/m ²	Integrand for total electric energy	Domains 5–7 Meta
ec.We	0.5*epsilon0_const*((ec.I_srr+ec.chirr)*ec.Er+(ec.I_srphi+ec.chirphi)*ec.Ephi+(ec.I_srz+ec.chirz)*ec.Ez)*ec.Er+((ec.I_sphir+ec.chiphir)*ec.Er+(ec.I_sphiphi+ec.chiphiphi)*ec.Ephi+(ec.I_sphiz+ec.chiphiz)*ec.Ez)*ec.Ephi+((ec.I_srz+ec.chizr)*ec.Er+(ec.I_szphi+ec.chizphi)*ec.Ephi+(ec.I_szz+ec.chizz)*ec.Ez)*ec.Ez)	J/m ³	Electric energy density	Domains 1, 3
ec.We	0.5*epsilon0_const*((ec.I_srr+ec.chirr)*ec.Er+(ec.I_srphi+ec.chirphi)*ec.Ephi+(ec.I_srz+ec.chirz)*ec.Ez)*ec.Er+((ec.I_sphir+ec.chiphir)*ec.Er+(ec.I_sphiphi+ec.chiphiphi)*ec.Ephi+(ec.I_sphiz+ec.chiphiz)*ec.Ez)*ec.Ephi+((ec.I_srz+ec.chizr)*ec.Er+(ec.I_szphi+ec.chizphi)*ec.Ephi+(ec.I_szz+ec.chizz)*ec.Ez)*ec.Ez)	J/m ³	Electric energy density	Domains 5–7
ec.rhoqs	ec.dnr*(up(ec.Dr)-down(ec.Dr))+ec.dnphi*(up(ec.Dphi)-down(ec.Dphi))+ec.dnz*(up(ec.Dz)-down(ec.Dz))	C/m ²	Surface charge density	Boundaries 4, 6, 9–11, 15
ec.rhoqs	ec.dnr*(up(ec.Dr)-down(ec.Dr))+ec.dnphi*(up(ec.Dphi)-down(ec.Dphi))+ec.dnz*(up(ec.Dz)-down(ec.Dz))	C/m ²	Surface charge density	Boundaries 13–14, 16
ec.rhoqs	-ec.dnr*down(ec.Dr)-ec.dnphi*down(ec.Dphi)-ec.dnz*down(ec.Dz)	C/m ²	Surface charge density	Boundaries 1–2, 5, 7
ec.rhoqs	-ec.dnr*down(ec.Dr)-ec.dnphi*down(ec.Dphi)-ec.dnz*down(ec.Dz)	C/m ²	Surface charge density	Boundaries 12, 17–20
ec.sigma_iso	material.sigma_iso	S/m	Electrical conductivity, isotropic value	Domains 1, 3 Meta
ec.sigma_iso	material.sigma_iso	S/m	Electrical conductivity, isotropic value	Domains 5–7 Meta

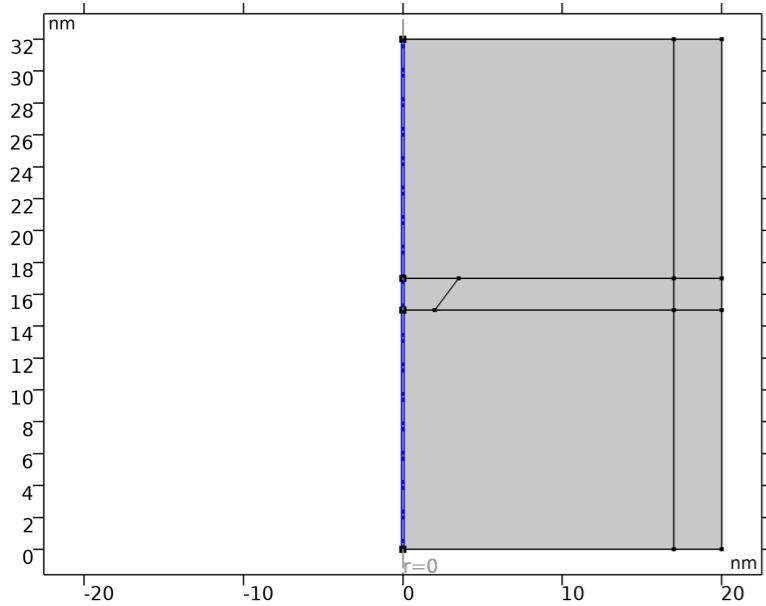
Shape functions

Name	Shape function	Unit	Description	Shape frame	Selection
V_all_Cell	Lagrange (Quadratic)	V	Electric potential	Spatial	Domains 1, 3
V_all_Cell	Lagrange (Quadratic)	V	Electric potential	Material	Domains 1, 3
V_all_Cell	Lagrange (Quadratic)	V	Electric potential	Geometry	Domains 1, 3
V_all_Cell	Lagrange (Quadratic)	V	Electric potential	Mesh	Domains 1, 3
V_all_Cell	Lagrange (Quadratic)	V	Electric potential	Spatial	Domains 5–7
V_all_Cell	Lagrange (Quadratic)	V	Electric potential	Material	Domains 5–7
V_all_Cell	Lagrange (Quadratic)	V	Electric potential	Geometry	Domains 5–7
V_all_Cell	Lagrange (Quadratic)	V	Electric potential	Mesh	Domains 5–7

Weak Expressions

Weak expression	Integration order	Integration frame	Selection
2*(ec.Jr*test(V_all_Cellr)+ec.Jz*test(V_all_Cellz))*ec.d*pi*r	4	Spatial	Domains 1, 3
2*(ec.Jr*test(ie1.T11*V_all_Cellr+ie1.T13*V_all_Cellz)+ec.Jphi*test(ie1.T21*V_all_Cellr+ie1.T23*V_all_Cellz)+ec.Jz*test(ie1.T31*V_all_Cellr+ie1.T33*V_all_Cellz))*ec.d*pi*ie1.r*ie1.detInvT	4	Spatial	Domains 5–7

2.5.4. Axial Symmetry



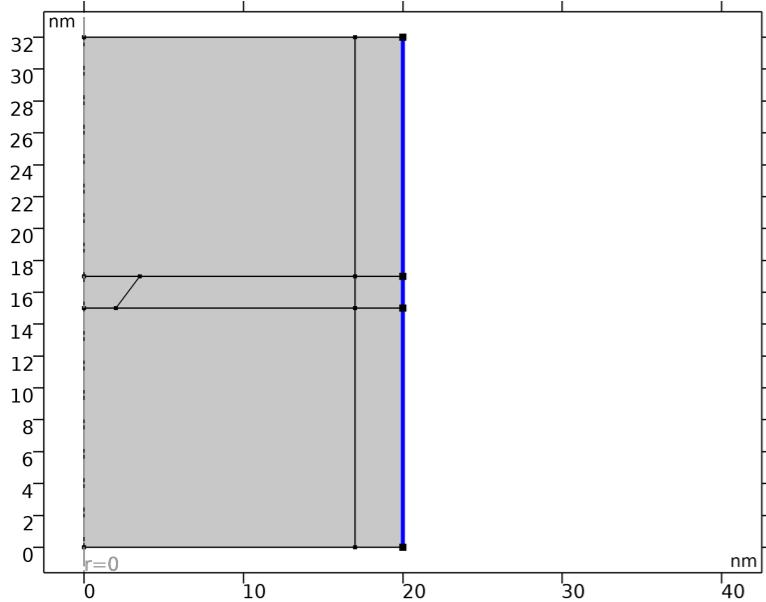
Selection

Geometric entity level	Boundary
Selection	Geometry geom2: Dimension 1: All boundaries

Used products

COMSOL Multiphysics

2.5.5. Electric Insulation



Electric Insulation

Selection

Geometric entity level	Boundary
Selection	Geometry geom2: Dimension 1: All boundaries

Equations

$$\mathbf{n} \cdot \mathbf{J} = 0$$

Used products

COMSOL Multiphysics

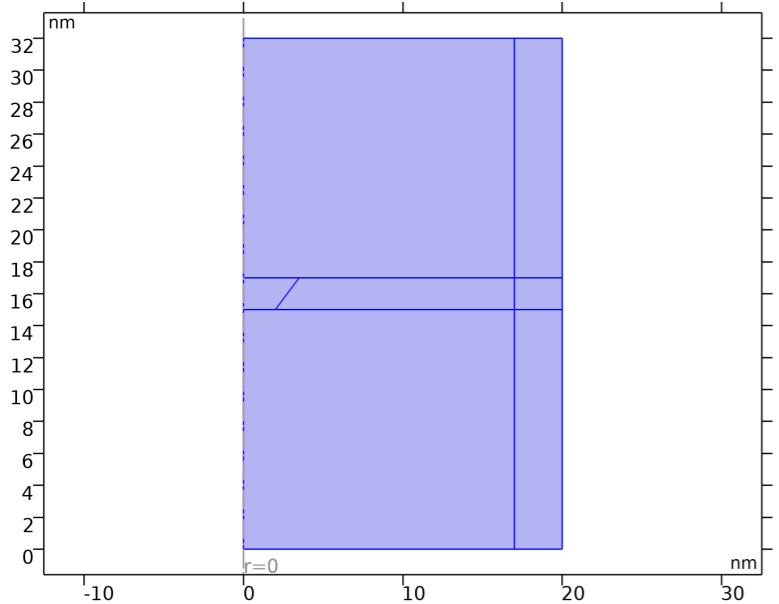
Variables

Name	Expression	Unit	Description	Selection	Details
ec.nJ	0	A/m ²	Normal current density	Boundaries 18–20	+ operation

Shape functions



Name	Shape function	Unit	Description	Shape frame	Selection	Details
V_all_Cell	Lagrange (Quadratic)	V	Electric potential	Spatial	No boundaries	Slit
V_all_Cell	Lagrange (Quadratic)	V	Electric potential	Material	No boundaries	Slit
V_all_Cell	Lagrange (Quadratic)	V	Electric potential	Geometry	No boundaries	Slit
V_all_Cell	Lagrange (Quadratic)	V	Electric potential	Mesh	No boundaries	Slit
V_all_Cell	Lagrange (Quadratic)	V	Electric potential	Spatial	No boundaries	Slit
V_all_Cell	Lagrange (Quadratic)	V	Electric potential	Material	No boundaries	Slit
V_all_Cell	Lagrange (Quadratic)	V	Electric potential	Geometry	No boundaries	Slit
V_all_Cell	Lagrange (Quadratic)	V	Electric potential	Mesh	No boundaries	Slit

2.5.6. Initial Values*Initial Values*

Selection

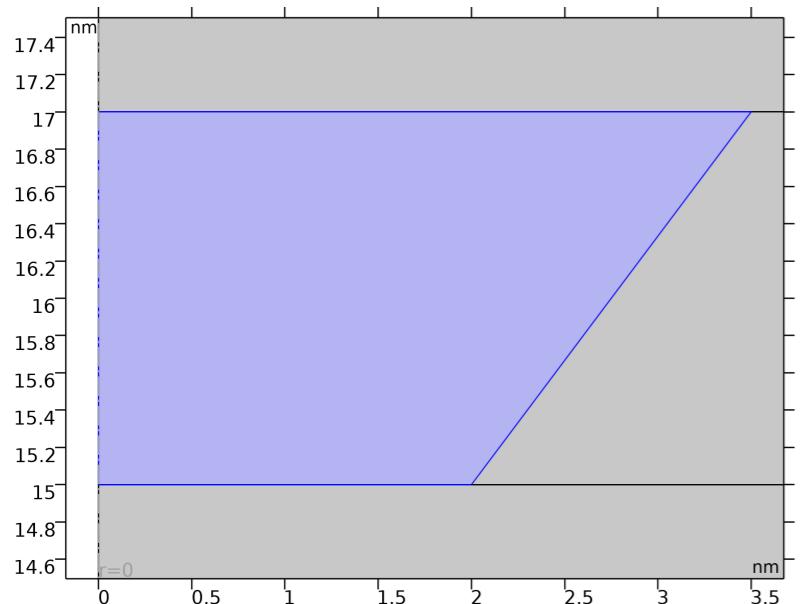
Geometric entity level	Domain
Selection	Geometry geom2: Dimension 2: All domains

Settings

Description	Value
Electric potential	0

Used products

COMSOL Multiphysics

2.5.7. Current Conservation Filament*Current Conservation Filament*

Selection

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Geometric entity level	Domain
Selection	Geometry geom2: Dimension 2: Domain 2

Equations

$$\nabla \cdot \mathbf{J} = Q_{j,v}$$

$$\mathbf{J} = \sigma \mathbf{E} + \mathbf{J}_e$$

$$\mathbf{E} = -\nabla V$$

Constitutive Relation Jc-E

Settings

Description	Value
Conduction model	Linearized resistivity
Reference resistivity	User defined
Reference resistivity	1/sigmaCF
Reference temperature	User defined
Reference temperature	tref
Resistivity temperature coefficient	User defined
Resistivity temperature coefficient	alphaT

Constitutive Relation D-E

Settings

Description	Value
Dielectric model	Relative permittivity
Relative permittivity	From material

Coordinate System Selection

Settings

Description	Value
Coordinate system	Global coordinate system

Used products

COMSOL Multiphysics

Properties from material

Property	Material	Property group
Relative permittivity	Filament (Hafnium)	Basic

Variables

Name	Expression	Unit	Description	Selection	Details
ec.Qh	ec.Qrh	W/m³	Volumetric loss density, electromagnetic	Domain 2	
ec.Jir	ec.sigmar*ec.Er+ec.sigmarphi*ec.Ephi+ec.sigmarz*ec.Ez	A/m²	Conduction current density, r component	Domain 2	
ec.Jiphi	ec.sigmar*ec.Er+ec.sigmarphi*ec.Ephi+ec.sigmarz*ec.Ez	A/m²	Conduction current density, phi component	Domain 2	
ec.Jiz	ec.sigmar*ec.Er+ec.sigmarphi*ec.Ephi+ec.sigmarz*ec.Ez	A/m²	Conduction current density, z component	Domain 2	
ec.Jdr	0	A/m²	Displacement current density, r component	Domain 2	
ec.Jdphi	0	A/m²	Displacement current density, phi component	Domain 2	
ec.Jdz	0	A/m²	Displacement current density, z component	Domain 2	
ec.Jer	0	A/m²	External current density, r component	Domain 2	+ operation
ec.Jephi	0	A/m²	External current density, phi component	Domain 2	+ operation
ec.Jez	0	A/m²	External current density, z component	Domain 2	+ operation
ec.Jr	ec.Jir+ec.Jdr+ec.Jer	A/m²	Current density, r component	Domain 2	
ec.Jphi	ec.Jiphi+ec.Jdphi+ec.Jephi	A/m²	Current density, phi component	Domain 2	
ec.Jz	ec.Jiz+ec.Jdz+ec.Jez	A/m²	Current density, z component	Domain 2	
ec.normJ	sqrt(realdot(ec.Jr,ec.Jr)+realdot(ec.Jphi,ec.Jphi)+realdot(ec.Jz,ec.Jz)))	A/m²	Current density norm	Domain 2	
ec.rhoq	ppr(d(ec.Dr,r)+if(abs(r)<0.001*h_spatial,d(ec.Dr,r),ec.Dr/r)+d(ec.Dz,z))	C/m³	Space charge density	Domain 2	
ec.sigmar	1/(ec.rho0*(1+ec.alpha*(ec.cucn2.minput_temperature-ec.Tref)))	S/m	Electrical conductivity, rr component	Domain 2	
ec.sigmarphi	0	S/m	Electrical conductivity, phir component	Domain 2	

ec.sigmazr	0		S/m	Electrical conductivity, zr component	Domain 2	
ec.sigmarphi	0		S/m	Electrical conductivity, rphi component	Domain 2	
ec.sigmaphiphi	$1/(ec.rho0*(1+ec.alpha*(ec.cucn2.minput_temperature-ec.Tref)))$		S/m	Electrical conductivity, phiphi component	Domain 2	
ec.sigmazphi	0		S/m	Electrical conductivity, zphi component	Domain 2	
ec.sigmarz	0		S/m	Electrical conductivity, rz component	Domain 2	
ec.sigmaphiz	0		S/m	Electrical conductivity, phiz component	Domain 2	
ec.sigmazz	$1/(ec.rho0*(1+ec.alpha*(ec.cucn2.minput_temperature-ec.Tref)))$		S/m	Electrical conductivity, zz component	Domain 2	
ec.epsilonrrr	material.epsilonr11		1	Relative permittivity, rr component	Domain 2	Meta
ec.epsilonrphir	material.epsilonr21		1	Relative permittivity, phir component	Domain 2	Meta
ec.epsilonrzz	material.epsilonr31		1	Relative permittivity, zr component	Domain 2	Meta
ec.epsilonrrphi	material.epsilonr12		1	Relative permittivity, rphi component	Domain 2	Meta
ec.epsilonrphiphi	material.epsilonr22		1	Relative permittivity, phiphi component	Domain 2	Meta
ec.epsilonrzphi	material.epsilonr32		1	Relative permittivity, zphi component	Domain 2	Meta
ec.epsilonrrz	material.epsilonr13		1	Relative permittivity, rz component	Domain 2	Meta
ec.epsilonrphiz	material.epsilonr23		1	Relative permittivity, phiz component	Domain 2	Meta
ec.epsilonrzzz	material.epsilonr33		1	Relative permittivity, zz component	Domain 2	Meta
ec.epsilonr_iso	material.epsilonr_iso		1	Relative permittivity, isotropic value	Domain 2	Meta
ec.Dr	$\epsilon_0 \text{const} * ec.I_srr * ec.Er + \epsilon_0 \text{const} * ec.I_srphi * ec.Ephi + \epsilon_0 \text{const} * ec.I_srz * ec.Ez + ec.Pr + ec.Per$		C/m ²	Electric displacement field, r component	Domain 2	
ec.Dphi	$\epsilon_0 \text{const} * ec.I_sphir * ec.Er + \epsilon_0 \text{const} * ec.I_sphiphi * ec.Ephi + \epsilon_0 \text{const} * ec.I_sphiz * ec.Ez + ec.Pphi + ec.Pephi$		C/m ²	Electric displacement field, phi component	Domain 2	
ec.Dz	$\epsilon_0 \text{const} * ec.I_szr * ec.Er + \epsilon_0 \text{const} * ec.I_szphi * ec.Ephi + \epsilon_0 \text{const} * ec.I_szz * ec.Ez + ec.Pz + ec.Pez$		C/m ²	Electric displacement field, z component	Domain 2	
ec.Pr	$\epsilon_0 \text{const} * (ec.chirr * ec.Er + ec.chirphi * ec.Ephi + ec.chirz * ec.Ez)$		C/m ²	Polarization, r component	Domain 2	
ec.Pphi	$\epsilon_0 \text{const} * (ec.chiphr * ec.Er + ec.chiphipi * ec.Ephi + ec.chiphiz * ec.Ez)$		C/m ²	Polarization, phi component	Domain 2	
ec.Pz	$\epsilon_0 \text{const} * (ec.chizr * ec.Er + ec.chizphi * ec.Ephi + ec.chizz * ec.Ez)$		C/m ²	Polarization, z component	Domain 2	
ec.normD	$\sqrt{(\text{realdot}(ec.Dr, ec.Dr) + \text{realdot}(ec.Dphi, ec.Dphi) + \text{realdot}(ec.Dz, ec.Dz))}$		C/m ²	Electric displacement field norm	Domain 2	
ec.normP	$\sqrt{(\text{realdot}(ec.Pr, ec.Pr) + \text{realdot}(ec.Pphi, ec.Pphi) + \text{realdot}(ec.Pz, ec.Pz))}$		C/m ²	Polarization norm	Domain 2	
ec.Per	0		C/m ²	Polarization contribution, r component	Domain 2	+ operation
ec.Pephi	0		C/m ²	Polarization contribution, phi component	Domain 2	+ operation
ec.Pez	0		C/m ²	Polarization contribution, z component	Domain 2	+ operation
ec.chirr	-1+ec.epsilonrrr		1	Electric susceptibility, rr component	Domain 2	
ec.chiphr	ec.epsilonrphir		1	Electric susceptibility, phir component	Domain 2	
ec.chirz	ec.epsilonrzz		1	Electric susceptibility, rz component	Domain 2	
ec.chirphi	ec.epsilonrrphi		1	Electric susceptibility, rphi component	Domain 2	
ec.chiphipi	-1+ec.epsilonrphiphi		1	Electric susceptibility, phiphi component	Domain 2	
ec.chizphi	ec.epsilonrzphi		1	Electric susceptibility, zphi component	Domain 2	
ec.chirz	ec.epsilonrrz		1	Electric susceptibility, rz component	Domain 2	
ec.chiphiz	ec.epsilonrphiz		1	Electric susceptibility, phiz component	Domain 2	
ec.chizz	-1+ec.epsilonrzz		1	Electric susceptibility, zz component	Domain 2	
ec.Er	-V_all_Cellr		V/m	Electric field, r component	Domain 2	
ec.Ephi	0		V/m	Electric field, phi component	Domain 2	
ec.Ez	-V_all_Cellz		V/m	Electric field, z component	Domain 2	
ec.tEr	-V_all_CellTr		V/m	Tangential electric field, r component	Boundaries 3–4, 6, 8	
ec.tEphi	0		V/m	Tangential electric field, phi	Boundaries 3–	

		component	4, 6, 8
ec.tEz	-V_all_CellTz	V/m	Tangential electric field, z component Boundaries 3-4, 6, 8
ec.normE	sqrt(realdot(ec.Er,ec.Er)+realdot(ec.Ephi,ec.Ephi)+realdot(ec.Ez,ec.Ez))	V/m	Electric field norm Domain 2
ec.Qrh	ec.Jr*ec.Er+ec.Jphi*ec.Ephi+ec.Jz*ec.Ez	W/m ³	Volumetric loss density, electric Domain 2 + operation
ec.W	ec.We	J/m ³	Energy density Domain 2 + operation
ec.dWe	2*ec.We*pi*r	J/m ²	Integrand for total electric energy Domain 2 Meta
ec.We	0.5*epsilon0_const*((ec.I_srr+ec.chirr)*ec.Er+(ec.I_srphi+ec.chirphi)*ec.Ephi+(ec.I_srz+ec.chirz)*ec.Ez)*ec.Er+((ec.I_sphir+ec.chiphir)*ec.Er+(ec.I_sphiphi+ec.chiphiphi)*ec.Ephi+(ec.I_sphiz+ec.chiphiz)*ec.Ez)*ec.Ephi+((ec.I_srz+ec.chizr)*ec.Er+(ec.I_szphi+ec.chizphi)*ec.Ephi+(ec.I_szz+ec.chizz)*ec.Ez)*ec.Ez)	J/m ³	Electric energy density Domain 2
ec.rhoqs	ec.dnr*(up(ec.Dr)-down(ec.Dr))+ec.dnphi*(up(ec.Dphi)-down(ec.Dphi))+ec.dnz*(up(ec.Dz)-down(ec.Dz))	C/m ²	Surface charge density Boundaries 4, 6, 8
ec.rhoqs	-ec.dnr*down(ec.Dr)-ec.dnphi*down(ec.Dphi)-ec.dnz*down(ec.Dz)	C/m ²	Surface charge density Boundary 3
ec.rho0	1/sigmaCF	Ω·m	Reference resistivity Domain 2
ec.Tref	tref	K	Reference temperature Domain 2
ec.alpha	alphaT	1/K	Resistivity temperature coefficient Domain 2

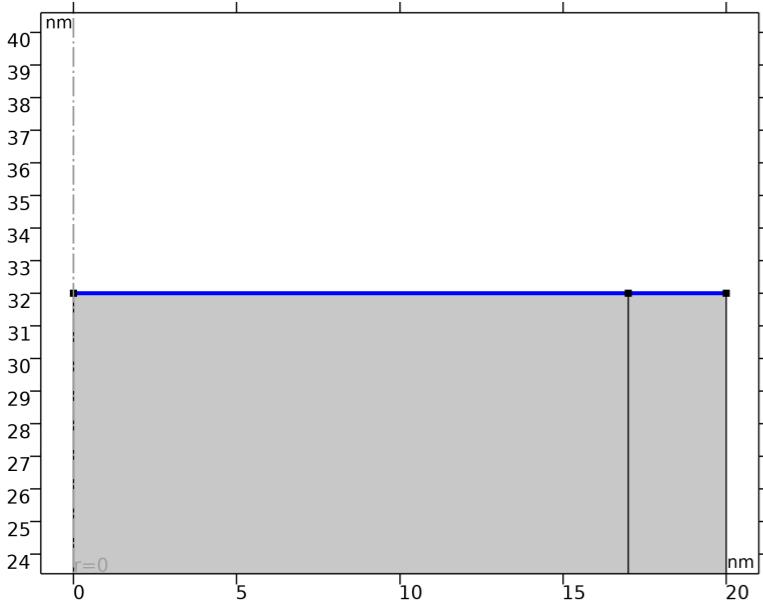
Shape functions

Name	Shape function	Unit	Description	Shape frame	Selection
V_all_Cell	Lagrange (Quadratic)	V	Electric potential	Spatial	Domain 2
V_all_Cell	Lagrange (Quadratic)	V	Electric potential	Material	Domain 2
V_all_Cell	Lagrange (Quadratic)	V	Electric potential	Geometry	Domain 2
V_all_Cell	Lagrange (Quadratic)	V	Electric potential	Mesh	Domain 2

Weak Expressions

Weak expression	Integration order	Integration frame	Selection
2*(ec.Jr*test(V_all_Cellr)+ec.Jz*test(V_all_Cellz))*ec.d*pi*r	4	Spatial	Domain 2

2.5.8. Ground



Ground

Selection

Geometric entity level	Boundary
Selection	Geometry geom2: Dimension 1: Boundaries 7, 17

Equations

$$V=0$$

Constraint Settings

Settings

Description	Value
Apply reaction terms on	All physics (symmetric)
Use weak constraints	Off
Constraint method	Elemental

Used products

COMSOL Multiphysics

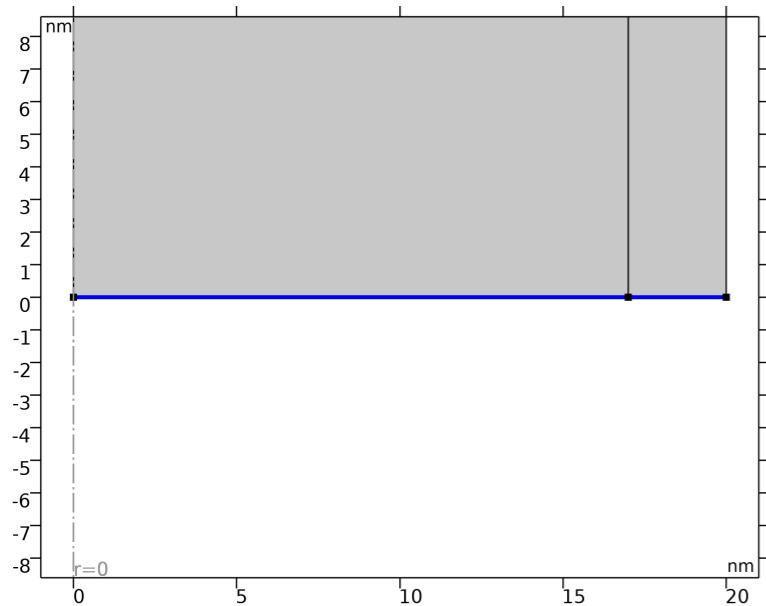
Variables

Name	Expression	Unit	Description	Selection	Details
ec.nJ	ec.unr*down(ec.Jr)+ec.unphi*down(ec.Jphi)+ec.unz*down(ec.Jz)	A/m ²	Normal current density	Boundary 7 + operation	
ec.nJ	ec.unr*down(ec.Jr)+ec.unphi*down(ec.Jphi)+ec.unz*down(ec.Jz)	A/m ²	Normal current density	Boundary 17 + operation	
ec.V0	0	V	Electric potential	Boundary 7	
ec.V0	0	V	Electric potential	Boundary 17	

Constraints

Constraint	Constraint force	Shape function	Selection	Details
ec.V0-V_all_Cell	test(ec.V0-V_all_Cell)	Lagrange (Quadratic)	Boundary 7	Elemental
ec.V0-V_all_Cell	test(ec.V0-V_all_Cell)	Lagrange (Quadratic)	Boundary 17	Elemental

2.5.9. Terminal



Terminal

Selection

Geometric entity level	Boundary
Selection	Geometry geom2: Dimension 1: Boundaries 2, 12

Equations

$$V = V_0$$

Terminal

Settings

Description	Value
Terminal name	I
Terminal type	Voltage
Voltage	Vapp

Advanced Settings

Settings

Description	Value
Current scaling type	Automatic

Constraint Settings

Settings

Description	Value
Apply reaction terms on	All physics (symmetric)
Use weak constraints	Off
Constraint method	Elemental

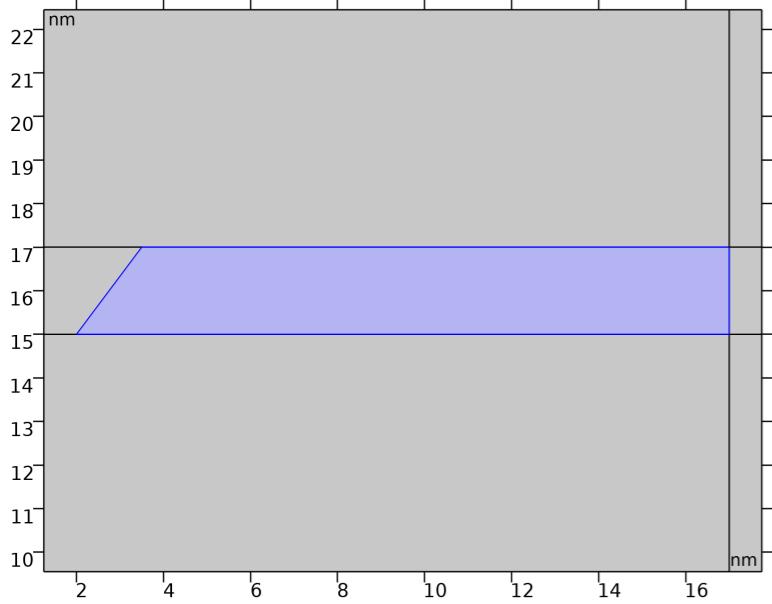
Variables

Name	Expression	Unit	Description	Selection	Details

ec.nJ	ec.unr*down(ec.Jr)+ec.unphi*down(ec.Jphi)+ec.unz*down(ec.Jz)	A/m ²	Normal current density	Boundary 2	+ operation
ec.nJ	ec.unr*down(ec.Jr)+ec.unphi*down(ec.Jphi)+ec.unz*down(ec.Jz)	A/m ²	Normal current density	Boundary 12	+ operation
ec.V0	ec.term1.V0	V	Electric potential	Boundary 2	
ec.V0	ec.term1.V0	V	Electric potential	Boundary 12	
ec.Vterm	ec.V0	V	Voltage	Boundary 2	
ec.Vterm	ec.V0	V	Voltage	Boundary 12	
ec.term1.V0	Vapp	V	Voltage	Boundary 2	
ec.term1.V0	Vapp	V	Voltage	Boundary 12	
ec.term1.Vinit	0	V	Initial value for voltage	Global	
ec.I0_1	ec.term1.sum(reacf(V_all_Cell))	A	Terminal current	Global	
ec.V0_1	ec.term1.int(V_all_Cell)/ec.term1.int(1)	V	Terminal voltage	Global	

Constraints

Constraint	Constraint force	Shape function	Selection	Details
ec.Vterm-V_all_Cell	test(ec.Vterm-V_all_Cell)	Lagrange (Quadratic)	Boundary 2	Elemental
ec.Vterm-V_all_Cell	test(ec.Vterm-V_all_Cell)	Lagrange (Quadratic)	Boundary 12	Elemental

2.5.10. Terminal : Dirichlet quantum potential*Terminal : Dirichlet quantum potential***Selection**

Geometric entity level	Domain
Selection	Geometry geom2: Dimension 2: Domain 4

Equations

$$V = V_0$$

Terminal**Settings**

Description	Value
Terminal name	2
Terminal type	Voltage
Voltage	VQM

Advanced Settings**Settings**

Description	Value
Current scaling type	Automatic

Constraint Settings**Settings**

Description	Value
Apply reaction terms on	All physics (symmetric)
Use weak constraints	Off

Constraint method	Elemental
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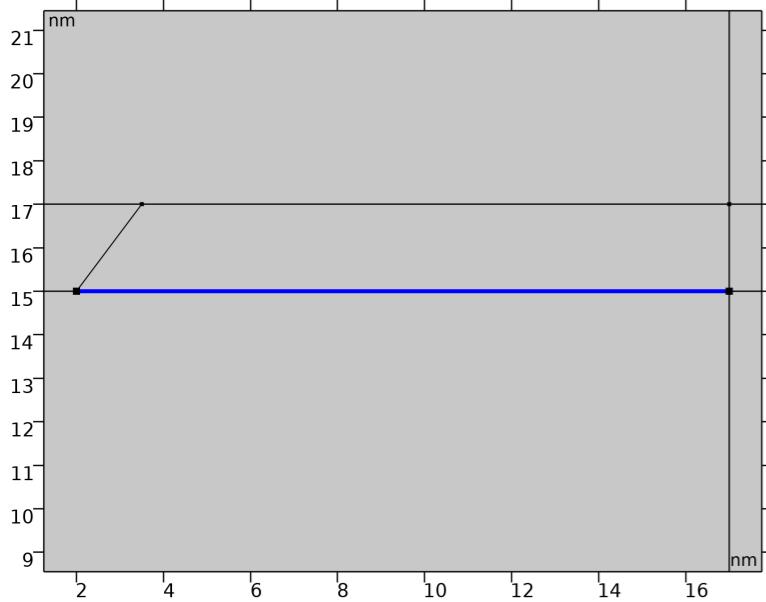
Variables

Name	Expression	Unit	Description	Selection	Details
V_all_Cell	ec.Vterm	V	Electric potential	Domain 4	
ec.nJ	ec.unr*down(ec.Jr)+ec.unphi*down(ec.Jphi)+ec.unz*down(ec.Jz)	A/m ²	Normal current density	Boundaries 8–9, 13	+ operation
ec.nJ	-ec.unr*up(ec.Jr)-ec.unphi*up(ec.Jphi)-ec.unz*up(ec.Jz)	A/m ²	Normal current density	Boundary 10	+ operation
ec.normE	0	V/m	Electric field norm	Domain 4	
ec.Qrh	0	W/m ³	Volumetric loss density, electric	Domain 4	+ operation
ec.dWe	0	J/m ²	Integrand for total electric energy	Domain 4	
ec.V0	ec.term2.V0	V	Electric potential	Domain 4	
ec.Vterm	ec.V0	V	Voltage	Domain 4	
ec.ER	0	V/m	Electric field, R component	Domain 4	
ec.EPHI	0	V/m	Electric field, PHI component	Domain 4	
ec.EZ	0	V/m	Electric field, Z component	Domain 4	
ec.term2.V0	VQM	V	Voltage	Domain 4	
ec.term2.Vinit	0	V	Initial value for voltage	Global	
ec.I0_2	ec.term2.sum(up(if(isdefined(V_all_Cell),reacf(V_all_Cell),0))+down(if(isdefined(V_all_Cell),reacf(V_all_Cell),0)))	A	Terminal current	Global	
ec.V0_2	ec.term2.int(V_all_Cell)/ec.term2.int(1)	V	Terminal voltage	Global	

Constraints

Constraint	Constraint force	Shape function	Selection	Details
ec.Vterm-V_all_Cell	test(ec.Vterm-V_all_Cell)	Lagrange (Quadratic)	Boundaries 8–10, 13	Elemental

2.5.11. Boundary Current Source : Quantum current density through a Matlab function



Boundary Current Source : Quantum current density through a Matlab function

Selection

Geometric entity level	Boundary
Selection	Geometry geom2: Dimension 1: Boundary 9

Equations

$$\mathbf{n} \cdot (\mathbf{J}_1 - \mathbf{J}_2) = Q_{j,s}$$

Coordinate System Selection

Settings

Description	Value
Coordinate system	Global coordinate system

Used products

COMSOL Multiphysics

Variables

Name	Expression	Unit	Description	Selection	Details

ec.nJ	ec.bcs3.Qjs	[A/m ²]	Normal current density	Boundary 9	+ operation
ec.bcs3.Qjs	QuantumCurrent(wd.Dw[1/m],VQM[1/V],bnd1[1/K],bnd2[1/K])[A/m^2]	[A/m ²]	Boundary current source	Boundary 9	
ec.Qjs	ec.bcs3.Qjs	[A/m ²]	Boundary current source	Boundary 9	+ operation

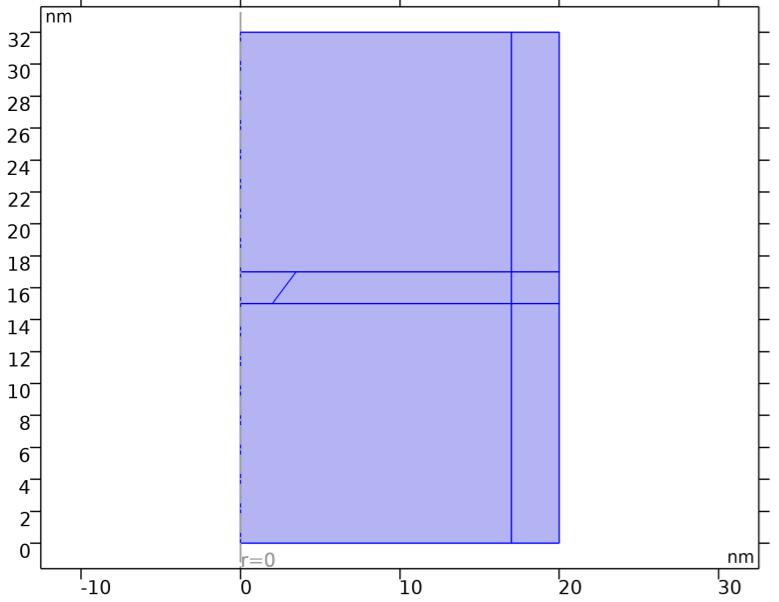
Weak Expressions

Weak expression	Integration order	Integration frame	Selection
$2*ec.d*ec.bcs3.Qjs*test(V_all_Cell)*pi*r$	4	Spatial	Boundary 9

2.6. Heat Transfer in Solids

Used products

COMSOL Multiphysics
Heat Transfer Module

*Heat Transfer in Solids*

Selection

Geometric entity level	Domain
Selection	Geometry geom2: Dimension 2: All domains

Equations

$$\rho C_p \mathbf{u} \cdot \nabla T + \nabla \cdot \mathbf{q} = Q + Q_{\text{ted}}$$

$$\mathbf{q} = -k \nabla T$$

2.6.1. Interface Settings

Discretization

Settings

Description	Value
Temperature	Quadratic Lagrange

Settings

Description	Value
Equation form	Study controlled

Physical Model

Settings

Description	Value
Isothermal domain	On
Reference temperature	User defined
Reference temperature	tref

Consistent Stabilization

Settings

Description	Value
Streamline diffusion	On

Crosswind diffusion On

Inconsistent Stabilization

Settings

Description	Value
Isotropic diffusion	Off

2.6.2. Variables

Name	Expression	Unit	Description	Selection	Details
ht.ndflux_u	-ht.ndflux_d	W/m ²	Internal normal conductive heat flux, upside	Boundaries 1–3, 5, 7	+ operation
ht.ndflux_u	0	W/m ²	Internal normal conductive heat flux, upside	Boundaries 4, 6, 8–11, 13, 15	+ operation
ht.ndflux_u	-ht.ndflux_d	W/m ²	Internal normal conductive heat flux, upside	Boundaries 12, 17–20	+ operation
ht.ndflux_u	0	W/m ²	Internal normal conductive heat flux, upside	Boundaries 14, 16	+ operation
ht.ndflux_d	0	W/m ²	Internal normal conductive heat flux, downside	Boundaries 1–11, 13, 15	+ operation
ht.ndflux_d	0	W/m ²	Internal normal conductive heat flux, downside	Boundaries 12, 14, 16–20	+ operation
ht.Q	0	W/m ³	Heat source	Domains 1–4	+ operation
ht.Q	0	W/m ³	Heat source	Domains 5–7	+ operation
ht.Qtot	0	W/m ³	Total heat source	Domains 1–4	+ operation
ht.Qtot	0	W/m ³	Total heat source	Domains 5–7	+ operation
ht.Qbtot	0	W/m ²	Total boundary heat source	Boundaries 2, 4, 6–11, 13, 15	+ operation
ht.Qbtot	0	W/m ²	Total boundary heat source	Boundaries 12, 14, 16–20	+ operation
ht.ntflux_contrib	0	W/m ²	Boundary sources and fluxes contribution	Domains 1–4	+ operation
ht.ntflux_contrib	0	W/m ²	Boundary sources and fluxes contribution	Domains 5–7	+ operation
ht.Tref	model.input.Tref	K	Reference temperature	Global	Meta
ht.d	1	m	Thickness	Domains 1–4	
ht.d	1	m	Thickness	Domains 5–7	
ht.HRef	0	J/kg	Reference enthalpy	Domains 1–4	
ht.HRef	0	J/kg	Reference enthalpy	Domains 5–7	
ht.alphap	0	1/K	Isobaric compressibility coefficient	Domains 1–4	
ht.alphap	0	1/K	Isobaric compressibility coefficient	Domains 5–7	
ht.chiT	0	1/Pa	Isothermal compressibility coefficient	Domains 1–4	
ht.chiT	0	1/Pa	Isothermal compressibility coefficient	Domains 5–7	
ht.DeltaH	ht.DeltaH_add	J/kg	Sensible enthalpy	Domains 1–4	
ht.DeltaH	ht.DeltaH_add	J/kg	Sensible enthalpy	Domains 5–7	
ht.DeltaH_cst	ht.DeltaH_add_cst	J/kg	Sensible enthalpy, constant material properties	Domains 1–4	
ht.DeltaH_cst	ht.DeltaH_add_cst	J/kg	Sensible enthalpy, constant material properties	Domains 5–7	
ht.DeltaH_add	0	J/kg	Sensible enthalpy	Domains 1–4	+ operation
ht.DeltaH_add	0	J/kg	Sensible enthalpy	Domains 5–7	+ operation
ht.DeltaH_add_cst	0	J/kg	Sensible enthalpy, constant material properties	Domains 1–4	+ operation
ht.DeltaH_add_cst	0	J/kg	Sensible enthalpy, constant material properties	Domains 5–7	+ operation
ht.H	0	J/kg	Enthalpy	Domains 1–4	+ operation
ht.H	0	J/kg	Enthalpy	Domains 5–7	+ operation
ht.H_cst	0	J/kg	Enthalpy, constant material properties	Domains 1–4	+ operation
ht.H_cst	0	J/kg	Enthalpy, constant material properties	Domains 5–7	+ operation
ht.H0	ht.H+ht.Ek	J/kg	Total enthalpy	Domains 1–4	
ht.H0	ht.H+ht.Ek	J/kg	Total enthalpy	Domains 5–7	
ht.H0_cst	ht.H_cst+ht.Ek	J/kg	Total enthalpy, constant material properties	Domains 1–4	
ht.H0_cst	ht.H_cst+ht.Ek	J/kg	Total enthalpy, constant material properties	Domains 5–7	
ht.Ei	0	J/kg	Internal energy	Domains 1–4	+ operation
ht.Ei	0	J/kg	Internal energy	Domains 5–7	+ operation
ht.Ei_cst	0	J/kg	Internal energy, constant material properties	Domains 1–4	+ operation
ht.Ei_cst	0	J/kg	Internal energy, constant material properties	Domains 5–7	+ operation
ht.Ei0	ht.Ei+ht.Ek	J/kg	Total internal energy	Domains 1–4	
ht.Ei0	ht.Ei+ht.Ek	J/kg	Total internal energy	Domains 5–7	
ht.Ei0_cst	ht.Ei_cst+ht.Ek	J/kg	Total internal energy, constant material properties	Domains 1–4	
ht.Ei0_cst	ht.Ei_cst+ht.Ek	J/kg	Total internal energy, constant material properties	Domains 5–7	
ht.Ek	0	J/kg	Kinetic energy	Domains 1–4	+ operation
ht.Ek	0	J/kg	Kinetic energy	Domains 5–7	+ operation
ht.dfluxr	0	W/m ²	Conductive heat flux, r component	Domains 1–4	+ operation
ht.dfluxphi	0	W/m ²	Conductive heat flux, phi component	Domains 1–4	+ operation
ht.dfluxz	0	W/m ²	Conductive heat flux, z component	Domains 1–4	+ operation
ht.dfluxr	0	W/m ²	Conductive heat flux, r component	Domains 5–7	+ operation

ht.dfluxphi	0		W/m ²	Conductive heat flux, phi component	Domains 5–7	+ operation
ht.dfluxz	0		W/m ²	Conductive heat flux, z component	Domains 5–7	+ operation
ht.dfluxr	mean(ht.dfluxr)		W/m ²	Conductive heat flux, r component	Boundaries 1–11, 13, 15	+ operation
ht.dfluxphi	mean(ht.dfluxphi)		W/m ²	Conductive heat flux, phi component	Boundaries 1–11, 13, 15	+ operation
ht.dfluxz	mean(ht.dfluxz)		W/m ²	Conductive heat flux, z component	Boundaries 1–11, 13, 15	+ operation
ht.dfluxr	mean(ht.dfluxr)		W/m ²	Conductive heat flux, r component	Boundaries 12, 14, 16–20	+ operation
ht.dfluxphi	mean(ht.dfluxphi)		W/m ²	Conductive heat flux, phi component	Boundaries 12, 14, 16–20	+ operation
ht.dfluxz	mean(ht.dfluxz)		W/m ²	Conductive heat flux, z component	Boundaries 12, 14, 16–20	+ operation
ht.dfluxtestr	0		W/m ²	Conductive heat flux, r component	Domains 1–4	+ operation
ht.dfluxtestphi	0		W/m ²	Conductive heat flux, phi component	Domains 1–4	+ operation
ht.dfluxtestz	0		W/m ²	Conductive heat flux, z component	Domains 1–4	+ operation
ht.dfluxtestr	mean(ht.dfluxtestr)		W/m ²	Conductive heat flux, r component	Boundaries 1–11, 13, 15	+ operation
ht.dfluxtestphi	mean(ht.dfluxtestphi)		W/m ²	Conductive heat flux, phi component	Boundaries 1–11, 13, 15	+ operation
ht.dfluxtestz	mean(ht.dfluxtestz)		W/m ²	Conductive heat flux, z component	Boundaries 1–11, 13, 15	+ operation
ht.dfluxtestr	mean(ht.dfluxtestr)		W/m ²	Conductive heat flux, r component	Boundaries 12, 14, 16–20	+ operation
ht.dfluxtestphi	mean(ht.dfluxtestphi)		W/m ²	Conductive heat flux, phi component	Boundaries 12, 14, 16–20	+ operation
ht.dfluxtestz	mean(ht.dfluxtestz)		W/m ²	Conductive heat flux, z component	Boundaries 12, 14, 16–20	+ operation
ht.dfluxMag	sqrt(ht.dfluxr^2+ht.dfluxphi^2+ht.dfluxz^2)		W/m ²	Conductive heat flux magnitude	Domains 1–4	
ht.dfluxMag	sqrt(ht.dfluxr^2+ht.dfluxphi^2+ht.dfluxz^2)		W/m ²	Conductive heat flux magnitude	Domains 5–7	
ht.cfluxr	0		W/m ²	Convective heat flux, r component	Domains 1–4	+ operation
ht.cfluxphi	0		W/m ²	Convective heat flux, phi component	Domains 1–4	+ operation
ht.cfluxz	0		W/m ²	Convective heat flux, z component	Domains 1–4	+ operation
ht.cfluxr	0		W/m ²	Convective heat flux, r component	Domains 5–7	+ operation
ht.cfluxphi	0		W/m ²	Convective heat flux, phi component	Domains 5–7	+ operation
ht.cfluxz	0		W/m ²	Convective heat flux, z component	Domains 5–7	+ operation
ht.cfluxMag	sqrt(ht.cfluxr^2+ht.cfluxphi^2+ht.cfluxz^2)		W/m ²	Convective heat flux magnitude	Domains 1–4	
ht.cfluxMag	sqrt(ht.cfluxr^2+ht.cfluxphi^2+ht.cfluxz^2)		W/m ²	Convective heat flux magnitude	Domains 5–7	
ht.tfluxr	ht.dfluxr+ht.cfluxr		W/m ²	Total heat flux, r component	Domains 1–4	
ht.tfluxphi	ht.dfluxphi+ht.cfluxphi		W/m ²	Total heat flux, phi component	Domains 1–4	
ht.tfluxz	ht.dfluxz+ht.cfluxz		W/m ²	Total heat flux, z component	Domains 1–4	
ht.tfluxr	ht.dfluxr+ht.cfluxr		W/m ²	Total heat flux, r component	Domains 5–7	
ht.tfluxphi	ht.dfluxphi+ht.cfluxphi		W/m ²	Total heat flux, phi component	Domains 5–7	
ht.tfluxz	ht.dfluxz+ht.cfluxz		W/m ²	Total heat flux, z component	Domains 5–7	
ht.tfluxMag	sqrt(ht.tfluxr^2+ht.tfluxphi^2+ht.tfluxz^2)		W/m ²	Total heat flux magnitude	Domains 1–4	
ht.tfluxMag	sqrt(ht.tfluxr^2+ht.tfluxphi^2+ht.tfluxz^2)		W/m ²	Total heat flux magnitude	Domains 5–7	
ht.tefluxr	0		W/m ²	Total energy flux, r component	Domains 1–4	+ operation
ht.tefluxphi	0		W/m ²	Total energy flux, phi component	Domains 1–4	+ operation
ht.tefluxz	0		W/m ²	Total energy flux, z component	Domains 1–4	+ operation
ht.tefluxr	0		W/m ²	Total energy flux, r component	Domains 5–7	+ operation
ht.tefluxphi	0		W/m ²	Total energy flux, phi component	Domains 5–7	+ operation
ht.tefluxz	0		W/m ²	Total energy flux, z component	Domains 5–7	+ operation
ht.teflux_cstr	0		W/m ²	Total energy flux, constant material properties, r component	Domains 1–4	+ operation
ht.teflux_cstphi	0		W/m ²	Total energy flux, constant material properties, phi component	Domains 1–4	+ operation
ht.teflux_cstz	0		W/m ²	Total energy flux, constant material properties, z component	Domains 1–4	+ operation
ht.teflux_cstr	0		W/m ²	Total energy flux, constant material properties, r component	Domains 5–7	+ operation
ht.teflux_cstphi	0		W/m ²	Total energy flux, constant material properties, phi component	Domains 5–7	+ operation
ht.teflux_cstz	0		W/m ²	Total energy flux, constant material properties, z component	Domains 5–7	+ operation
ht.tefluxMag	sqrt(ht.tefluxr^2+ht.tefluxphi^2+ht.tefluxz^2)		W/m ²	Total energy flux magnitude	Domains 1–4	
ht.tefluxMag	sqrt(ht.tefluxr^2+ht.tefluxphi^2+ht.tefluxz^2)		W/m ²	Total energy flux magnitude	Domains 5–7	
ht.thfluxr	0		W/m ²	Total enthalpy flux, r component	Domains 1–4	+ operation
ht.thfluxphi	0		W/m ²	Total enthalpy flux, phi component	Domains 1–4	+ operation
ht.thfluxz	0		W/m ²	Total enthalpy flux, z component	Domains 1–4	+ operation
ht.thfluxr	0		W/m ²	Total enthalpy flux, r component	Domains 5–7	+ operation
ht.thfluxphi	0		W/m ²	Total enthalpy flux, phi component	Domains 5–7	+ operation
ht.thfluxz	0		W/m ²	Total enthalpy flux, z component	Domains 5–7	+ operation
ht.thflux_cstr	0		W/m ²	Total enthalpy flux, constant material properties, r component	Domains 1–4	+ operation
ht.thflux_cstphi	0		W/m ²	Total enthalpy flux, constant material properties, phi component	Domains 1–4	+ operation
ht.thflux_cstz	0		W/m ²	Total enthalpy flux, constant material properties, z component	Domains 1–4	+ operation
ht.thflux_cstr	0		W/m ²	Total enthalpy flux, constant material properties, r component	Domains 5–7	+ operation
ht.thflux_cstphi	0		W/m ²	Total enthalpy flux, constant material properties, phi component	Domains 5–7	+ operation
ht.thflux_cstz	0		W/m ²	Total enthalpy flux, constant material properties, z component	Domains 5–7	+ operation
ht.thfluxMag	sqrt(ht.thfluxr^2+ht.thfluxphi^2+ht.thfluxz^2)		W/m ²	Total enthalpy flux magnitude	Domains 1–4	

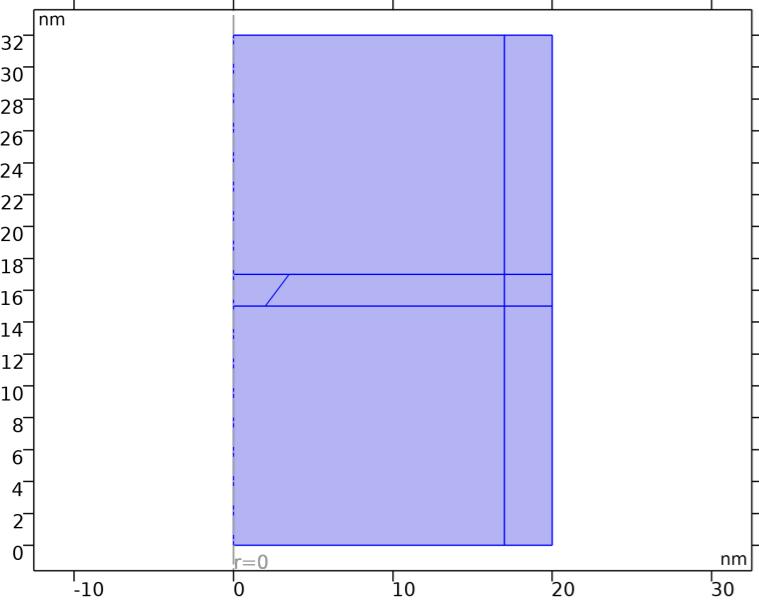
ht.thfluxMag	$\sqrt{ht.thfluxr^2 + ht.thfluxphi^2 + ht.thfluxz^2}$	W/m^2	Total enthalpy flux magnitude	Domains 5–7
ht.dflux_ur	$up(ht.dfluxr)$	W/m^2	Conductive heat flux, r component	Boundaries 1–11, 13, 15
ht.dflux_uphi	$up(ht.dfluxphi)$	W/m^2	Conductive heat flux, phi component	Boundaries 1–11, 13, 15
ht.dflux_uz	$up(ht.dfluxz)$	W/m^2	Conductive heat flux, z component	Boundaries 1–11, 13, 15
ht.dflux_ur	$up(ht.dfluxr)$	W/m^2	Conductive heat flux, r component	Boundaries 12, 14, 16–20
ht.dflux_uphi	$up(ht.dfluxphi)$	W/m^2	Conductive heat flux, phi component	Boundaries 12, 14, 16–20
ht.dflux_uz	$up(ht.dfluxz)$	W/m^2	Conductive heat flux, z component	Boundaries 12, 14, 16–20
ht.dflux_dr	$down(ht.dfluxr)$	W/m^2	Conductive heat flux, r component	Boundaries 1–11, 13, 15
ht.dflux_dphi	$down(ht.dfluxphi)$	W/m^2	Conductive heat flux, phi component	Boundaries 1–11, 13, 15
ht.dflux_dz	$down(ht.dfluxz)$	W/m^2	Conductive heat flux, z component	Boundaries 12, 14, 16–20
ht.dfluxtest_ur	$up(ht.dfluxtestr)$	W/m^2	Conductive heat flux, r component	Boundaries 1–11, 13, 15
ht.dfluxtest_uphi	$up(ht.dfluxtestphi)$	W/m^2	Conductive heat flux, phi component	Boundaries 1–11, 13, 15
ht.dfluxtest_uz	$up(ht.dfluxtestz)$	W/m^2	Conductive heat flux, z component	Boundaries 1–11, 13, 15
ht.dfluxtest_ur	$up(ht.dfluxtestr)$	W/m^2	Conductive heat flux, r component	Boundaries 12, 14, 16–20
ht.dfluxtest_uphi	$up(ht.dfluxtestphi)$	W/m^2	Conductive heat flux, phi component	Boundaries 12, 14, 16–20
ht.dfluxtest_uz	$up(ht.dfluxtestz)$	W/m^2	Conductive heat flux, z component	Boundaries 12, 14, 16–20
ht.dfluxtest_dr	$down(ht.dfluxtestr)$	W/m^2	Conductive heat flux, r component	Boundaries 1–11, 13, 15
ht.dfluxtest_dphi	$down(ht.dfluxtestphi)$	W/m^2	Conductive heat flux, phi component	Boundaries 1–11, 13, 15
ht.dfluxtest_dz	$down(ht.dfluxtestz)$	W/m^2	Conductive heat flux, z component	Boundaries 12, 14, 16–20
ht.rflux	0	W/m^2	Radiative heat flux	Boundaries 1–11, 13, 15
ht.rflux	0	W/m^2	Radiative heat flux	Boundaries 12, 14, 16–20
ht.ncflux	$mean(ht.cfluxr)*ht.nrmesh + mean(ht.cfluxphi)*ht.nphimesh + mean(ht.cfluxz)*ht.nzmesh$	W/m^2	Normal convective heat flux	Boundaries 1–11, 13, 15
ht.ncflux	$mean(ht.cfluxr)*ht.nrmesh + mean(ht.cfluxphi)*ht.nphimesh + mean(ht.cfluxz)*ht.nzmesh$	W/m^2	Normal convective heat flux	Boundaries 12, 14, 16–20
ht.ncflux_u	$up(ht.cfluxr)*ht.unrmesh + up(ht.cfluxphi)*ht.unphimesh + up(ht.cfluxz)*ht.unzmesh$	W/m^2	Internal normal convective heat flux, upside	Boundaries 1–11, 13, 15
ht.ncflux_u	$up(ht.cfluxr)*ht.unrmesh + up(ht.cfluxphi)*ht.unphimesh + up(ht.cfluxz)*ht.unzmesh$	W/m^2	Internal normal convective heat flux, upside	Boundaries 12, 14, 16–20
ht.ncflux_d	$down(ht.cfluxr)*ht.dnrmesh + down(ht.cfluxphi)*ht.dnphimesh + down(ht.cfluxz)*ht.dnzmesh$	W/m^2	Internal normal convective heat flux, downside	Boundaries 1–11, 13, 15
ht.ncflux_d	$down(ht.cfluxr)*ht.dnrmesh + down(ht.cfluxphi)*ht.dnphimesh + down(ht.cfluxz)*ht.dnzmesh$	W/m^2	Internal normal convective heat flux, downside	Boundaries 12, 14, 16–20
ht.ndflux	$0.5*(ht.ndflux_d - ht.ndflux_u)$	W/m^2	Normal conductive heat flux	Boundaries 1–11, 13, 15
ht.ndflux	$0.5*(ht.ndflux_d - ht.ndflux_u)$	W/m^2	Normal conductive heat flux	Boundaries 12, 14, 16–20
ht.ntflux	$ht.ndflux + ht.ncflux$	W/m^2	Normal total heat flux	Boundaries 1–11, 13, 15
ht.ntflux	$ht.ndflux + ht.ncflux$	W/m^2	Normal total heat flux	Boundaries 12, 14, 16–20
ht.ntflux_cst	$ht.ndflux + ht.ncflux$	W/m^2	Normal total heat flux, constant material properties	Boundaries 1–11, 13, 15
ht.ntflux_cst	$ht.ndflux + ht.ncflux$	W/m^2	Normal total heat flux, constant material properties	Boundaries 12, 14, 16–20
ht.ntflux_u	$ht.ndflux_u + ht.ncflux_u$	W/m^2	Internal normal total flux, upside	Boundaries 1–11, 13, 15
ht.ntflux_u	$ht.ndflux_u + ht.ncflux_u$	W/m^2	Internal normal total flux, upside	Boundaries 12, 14, 16–20
ht.ntflux_est_u	$ht.ndflux_u + ht.ncflux_u$	W/m^2	Internal normal total heat flux, constant material properties, upside	Boundaries 1–11, 13, 15
ht.ntflux_est_u	$ht.ndflux_u + ht.ncflux_u$	W/m^2	Internal normal total heat flux, constant material properties, upside	Boundaries 12, 14, 16–20
ht.ntflux_d	$ht.ndflux_d + ht.ncflux_d$	W/m^2	Internal normal total flux, downside	Boundaries 1–11, 13, 15
ht.ntflux_d	$ht.ndflux_d + ht.ncflux_d$	W/m^2	Internal normal total flux, downside	Boundaries 12, 14, 16–20
ht.ntflux_cst_d	$ht.ndflux_d + ht.ncflux_d$	W/m^2	Internal normal total heat flux, constant material properties, downside	Boundaries 1–11, 13, 15
ht.ntflux_cst_d	$ht.ndflux_d + ht.ncflux_d$	W/m^2	Internal normal total heat flux, constant material properties, downside	Boundaries 12, 14, 16–20
ht.nteflux	$mean(ht.tefluxr)*ht.nrmesh + mean(ht.tefluxphi)*ht.nphimesh + mean(ht.tefluxz)*ht.nzmesh - mean(ht.dfluxr)*ht.nrmesh - mean(ht.dfluxphi)*ht.nphimesh - mean(ht.dfluxz)*ht.nzmesh + ht.ndflux$	W/m^2	Normal total energy flux	Boundaries 1–11, 13, 15
ht.nteflux	$mean(ht.tefluxr)*ht.nrmesh + mean(ht.tefluxphi)*ht.nphimesh + mean(ht.tefluxz)*ht.nzmesh - mean(ht.dfluxr)*ht.nrmesh - mean(ht.dfluxphi)*ht.nphimesh - mean(ht.dfluxz)*ht.nzmesh + ht.ndflux$	W/m^2	Normal total energy flux	Boundaries 12, 14, 16–20
ht.nteflux_cst	$mean(ht.teflux_cstr)*ht.nrmesh + mean(ht.teflux_cstrphi)*ht.nphimesh + mean(ht.teflux_cstz)*ht.nzmesh - mean(ht.dfluxr)*ht.nrmesh - mean(ht.dfluxphi)*ht.nphimesh - mean(ht.dfluxz)*ht.nzmesh + ht.ndflux$	W/m^2	Normal total energy flux, constant material properties	Boundaries 1–11, 13, 15
ht.nteflux_cst	$mean(ht.teflux_cstr)*ht.nrmesh + mean(ht.teflux_cstrphi)*ht.nphimesh + mean(ht.teflux_cstz)*ht.nzmesh - mean(ht.dfluxr)*ht.nrmesh - mean(ht.dfluxphi)*ht.nphimesh - mean(ht.dfluxz)*ht.nzmesh + ht.ndflux$	W/m^2	Normal total energy flux, constant material properties	Boundaries 12, 14, 16–20
ht.nteflux_u	$up(ht.tefluxr)*ht.unrmesh + up(ht.tefluxphi)*ht.unphimesh + up(ht.tefluxz)*ht.unzmesh - up(ht.dfluxr)*ht.unrmesh - up(ht.dfluxphi)*ht.unphimesh - up(ht.dfluxz)*ht.unzmesh + ht.ndflux_u$	W/m^2	Internal normal total energy flux, upside	Boundaries 1–11, 13, 15
ht.nteflux_u	$up(ht.tefluxr)*ht.unrmesh + up(ht.tefluxphi)*ht.unphimesh + up(ht.tefluxz)*ht.unzmesh - up(ht.dfluxr)*ht.unrmesh - up(ht.dfluxphi)*ht.unphimesh - up(ht.dfluxz)*ht.unzmesh + ht.ndflux_u$	W/m^2	Internal normal total energy flux, upside	Boundaries 12, 14, 16–20
ht.nteflux_cst_u	$up(ht.teflux_cstr)*ht.unrmesh + up(ht.teflux_cstrphi)*ht.unphimesh + up(ht.teflux_cstz)*ht.unzmesh - up(ht.dfluxr)*ht.unrmesh - up(ht.dfluxphi)*ht.unphimesh - up(ht.dfluxz)*ht.unzmesh + ht.ndflux_u$	W/m^2	Internal normal total energy flux, constant material properties, upside	Boundaries 1–11, 13, 15
ht.nteflux_cst_u	$up(ht.teflux_cstr)*ht.unrmesh + up(ht.teflux_cstrphi)*ht.unphimesh + up(ht.teflux_cstz)*ht.unzmesh - up(ht.dfluxr)*ht.unrmesh - up(ht.dfluxphi)*ht.unphimesh - up(ht.dfluxz)*ht.unzmesh + ht.ndflux_u$	W/m^2	Internal normal total energy flux, constant material properties, upside	Boundaries 12, 14, 16–20
ht.nteflux_d	$down(ht.tefluxr)*ht.dnrmesh + down(ht.tefluxphi)*ht.dnphimesh + down(ht.tefluxz)*ht.dnzmesh - down(ht.dfluxr)*ht.dnrmesh - down(ht.dfluxphi)*ht.dnphimesh - down(ht.dfluxz)*ht.dnzmesh + ht.ndflux_d$	W/m^2	Internal normal total energy flux, downside	Boundaries 1–11, 13, 15
ht.nteflux_d	$down(ht.tefluxr)*ht.dnrmesh + down(ht.tefluxphi)*ht.dnphimesh + down(ht.tefluxz)*ht.dnzmesh - down(ht.dfluxr)*ht.dnrmesh - down(ht.dfluxphi)*ht.dnphimesh - down(ht.dfluxz)*ht.dnzmesh + ht.ndflux_d$	W/m^2	Internal normal total energy flux, downside	Boundaries 12, 14, 16–20
ht.nteflux_cst_d	$down(ht.teflux_cstr)*ht.dnrmesh + down(ht.teflux_cstrphi)*ht.dnphimesh + down(ht.teflux_cstz)*ht.dnzmesh - down(ht.dfluxr)*ht.dnrmesh - down(ht.dfluxphi)*ht.dnphimesh - down(ht.dfluxz)*ht.dnzmesh + ht.ndflux_d$	W/m^2	Internal normal total energy flux, constant material properties, downside	Boundaries 1–11, 13, 15
ht.nteflux_cst_d	$down(ht.teflux_cstr)*ht.dnrmesh + down(ht.teflux_cstrphi)*ht.dnphimesh + down(ht.teflux_cstz)*ht.dnzmesh - down(ht.dfluxr)*ht.dnrmesh - down(ht.dfluxphi)*ht.dnphimesh - down(ht.dfluxz)*ht.dnzmesh + ht.ndflux_d$	W/m^2	Internal normal total energy flux, constant material properties, downside	Boundaries 12, 14, 16–20
ht.nthflux	$mean(ht.thfluxr)*ht.nrmesh + mean(ht.thfluxphi)*ht.nphimesh + mean(ht.thfluxz)*ht.nzmesh$	W/m^2	Normal total enthalpy flux	Boundaries 1–11, 13, 15

ht.nthflux	mean(ht.thfluxr)*ht.nrmesh+mean(ht.thfluxphi)*ht.nphimesh+mean(ht.thfluxz)*ht.nzmesh	W/m ²	Normal total enthalpy flux	Boundaries 12, 14, 16–20	
ht.nthflux_cst	mean(ht.thflux_cstr)*ht.nrmesh+mean(ht.thflux_cstphi)*ht.nphimesh+mean(ht.thflux_cstz)*ht.nzmesh	W/m ²	Normal total enthalpy flux, constant material properties	Boundaries 1–11, 13, 15	
ht.nthflux_cst	mean(ht.thflux_cstr)*ht.nrmesh+mean(ht.thflux_cstphi)*ht.nphimesh+mean(ht.thflux_cstz)*ht.nzmesh	W/m ²	Normal total enthalpy flux, constant material properties	Boundaries 12, 14, 16–20	
ht.nthflux_u	up(ht.thfluxr)*ht.unrmesh+up(ht.thfluxphi)*ht.unphimesh+up(ht.thfluxz)*ht.unzmesh	W/m ²	Internal normal total enthalpy flux, upside	Boundaries 1–11, 13, 15	
ht.nthflux_u	up(ht.thfluxr)*ht.unrmesh+up(ht.thfluxphi)*ht.unphimesh+up(ht.thfluxz)*ht.unzmesh	W/m ²	Internal normal total enthalpy flux, upside	Boundaries 12, 14, 16–20	
ht.nthflux_cst_u	up(ht.thflux_cstr)*ht.unrmesh+up(ht.thflux_cstphi)*ht.unphimesh+up(ht.thflux_cstz)*ht.unzmesh	W/m ²	Internal normal total enthalpy flux, constant material properties, upside	Boundaries 1–11, 13, 15	
ht.nthflux_cst_u	up(ht.thflux_cstr)*ht.unrmesh+up(ht.thflux_cstphi)*ht.unphimesh+up(ht.thflux_cstz)*ht.unzmesh	W/m ²	Internal normal total enthalpy flux, constant material properties, upside	Boundaries 12, 14, 16–20	
ht.nthflux_d	down(ht.thfluxr)*ht.dnrmesh+down(ht.thfluxphi)*ht.dnphimesh+down(ht.thfluxz)*ht.dnzmesh	W/m ²	Internal normal total enthalpy flux, downside	Boundaries 1–11, 13, 15	
ht.nthflux_d	down(ht.thfluxr)*ht.dnrmesh+down(ht.thfluxphi)*ht.dnphimesh+down(ht.thfluxz)*ht.dnzmesh	W/m ²	Internal normal total enthalpy flux, downside	Boundaries 12, 14, 16–20	
ht.nthflux_cst_d	down(ht.thflux_cstr)*ht.dnrmesh+down(ht.thflux_cstphi)*ht.dnphimesh+down(ht.thflux_cstz)*ht.dnzmesh	W/m ²	Internal normal total enthalpy flux, constant material properties, downside	Boundaries 1–11, 13, 15	
ht.nthflux_cst_d	down(ht.thflux_cstr)*ht.dnrmesh+down(ht.thflux_cstphi)*ht.dnphimesh+down(ht.thflux_cstz)*ht.dnzmesh	W/m ²	Internal normal total enthalpy flux, constant material properties, downside	Boundaries 12, 14, 16–20	
ht.Qm	0	kg/(m ³ ·s)	Mass source	Domains 1–4	
ht.Qm	0	kg/(m ³ ·s)	Mass source	Domains 5–7	
ht.Qoop	0	W/m ³	Out-of-plane heat source	Domains 1–4	+ operation
ht.Qoop	0	W/m ³	Out-of-plane heat source	Domains 5–7	+ operation
ht.Qtot	0	W/m ²	Total interface source	Domains 1–4	+ operation
ht.Qtot	0	W/m ²	Total interface source	Domains 5–7	+ operation
ht.qs	0	W/(m ³ ·K)	Production/absorption coefficient	Domains 1–4	+ operation
ht.qs	0	W/(m ³ ·K)	Production/absorption coefficient	Domains 5–7	+ operation
ht.qs_oop	0	W/(m ³ ·K)	Out-of-plane production/absorption coefficient	Domains 1–4	+ operation
ht.qs_oop	0	W/(m ³ ·K)	Out-of-plane production/absorption coefficient	Domains 5–7	+ operation
ht.Qltot	0	W/m	Total line heat source	Boundaries 1, 3, 5	+ operation
ht.Qltot	0	W/m	Total line heat source	Points 5–10	+ operation
ht.Qltot	0	W/m	Total line heat source	Points 11–14	+ operation
ht.Qlrltot	0	W/m	Total line heat source with radius	Boundaries 1, 3, 5	+ operation
ht.Qlrltot	0	W/m	Total line heat source with radius	Points 5–10	+ operation
ht.Qlrltot	0	W/m	Total line heat source with radius	Points 11–14	+ operation
ht.Qptot	0	W	Total point heat source	Points 1–4	+ operation
ht.Qprtot	0	W	Total point heat source with radius	Points 1–4	+ operation
ht.Tvar	T	K	Temperature	Domains 1–4	
ht.Tvar	T	K	Temperature	Boundaries 1–11, 13, 15	
ht.Tvar	T	K	Temperature	Points 1–10	
ht.Tvar	T	K	Temperature	Domains 5–7	
ht.Tvar	T	K	Temperature	Boundaries 12, 14, 16–20	
ht.Tvar	T	K	Temperature	Points 11–14	
ht.Tu	up(T)	K	Temperature	Boundaries 4, 6, 8–11, 13, 15	
ht.Tu	T	K	Temperature	Boundaries 2, 7	
ht.Tu	up(T)	K	Temperature	Boundaries 14, 16	
ht.Tu	T	K	Temperature	Boundaries 12, 17–20	
ht.Td	down(T)	K	Temperature	Boundaries 4, 6, 8–11, 13, 15	
ht.Td	T	K	Temperature	Boundaries 2, 7	
ht.Td	down(T)	K	Temperature	Boundaries 14, 16	
ht.Td	T	K	Temperature	Boundaries 12, 17–20	
ht.TuIsDown	0	I	Help variable	Boundaries 4, 6, 8–11, 13, 15	
ht.TuIsDown	0	I	Help variable	Boundaries 2, 7	
ht.TuIsDown	0	I	Help variable	Boundaries 14, 16	
ht.TuIsDown	0	I	Help variable	Boundaries 12, 17–20	
ht.TdIsUp	0	I	Help variable	Boundaries 4, 6, 8–11, 13, 15	
ht.TdIsUp	0	I	Help variable	Boundaries 2, 7	
ht.TdIsUp	0	I	Help variable	Boundaries 14, 16	
ht.TdIsUp	0	I	Help variable	Boundaries 12, 17–20	
ht.du	up(ht.d)	I	Thickness	Boundaries 4, 6, 8–11, 13, 15	
ht.du	ht.d	I	Thickness	Boundaries 2, 7	
ht.du	up(ht.d)	I	Thickness	Boundaries 14, 16	
ht.du	ht.d	I	Thickness	Boundaries 12, 17–20	
ht.dd	down(ht.d)	I	Thickness	Boundaries 4, 6, 8–11, 13, 15	
ht.dd	ht.d	I	Thickness	Boundaries 2, 7	

ht.dd	down(ht.d)	1	Thickness	Boundaries 14, 16
ht.dd	ht.d	1	Thickness	Boundaries 12, 17–20
ht.q0	0	W/m ²	Inward heat flux	Boundaries 1–3, 5, 7 + operation
ht.q0	0	W/m ²	Inward heat flux	Boundaries 12, 17–20 + operation
ht.nr	nr	1	Normal vector, r component	Boundaries 4, 6, 8–11, 13, 15
ht.nphi	0	1	Normal vector, phi component	Boundaries 4, 6, 8–11, 13, 15
ht.nz	nz	1	Normal vector, z component	Boundaries 4, 6, 8–11, 13, 15
ht.nr	dnr	1	Normal vector, r component	Boundaries 1–3, 5, 7
ht.nphi	0	1	Normal vector, phi component	Boundaries 1–3, 5, 7
ht.nz	dnz	1	Normal vector, z component	Boundaries 1–3, 5, 7
ht.nr	nr	1	Normal vector, r component	Boundaries 14, 16
ht.nphi	0	1	Normal vector, phi component	Boundaries 14, 16
ht.nz	nz	1	Normal vector, z component	Boundaries 14, 16
ht.nr	dnr	1	Normal vector, r component	Boundaries 12, 17–20
ht.nphi	0	1	Normal vector, phi component	Boundaries 12, 17–20
ht.nz	dnz	1	Normal vector, z component	Boundaries 12, 17–20
ht.nrmesh	nrmesh	1	Normal vector (mesh), r component	Boundaries 4, 6, 8–11, 13, 15
ht.nphimesh	0	1	Normal vector (mesh), phi component	Boundaries 4, 6, 8–11, 13, 15
ht.nzmesh	nzmesh	1	Normal vector (mesh), z component	Boundaries 4, 6, 8–11, 13, 15
ht.nrmesh	dfrmesh	1	Normal vector (mesh), r component	Boundaries 1–3, 5, 7
ht.nphimesh	0	1	Normal vector (mesh), phi component	Boundaries 1–3, 5, 7
ht.nzmesh	dnzmesh	1	Normal vector (mesh), z component	Boundaries 1–3, 5, 7
ht.nrmesh	nrmesh	1	Normal vector (mesh), r component	Boundaries 14, 16
ht.nphimesh	0	1	Normal vector (mesh), phi component	Boundaries 14, 16
ht.nzmesh	nzmesh	1	Normal vector (mesh), z component	Boundaries 14, 16
ht.nrmesh	dfrmesh	1	Normal vector (mesh), r component	Boundaries 12, 17–20
ht.nphimesh	0	1	Normal vector (mesh), phi component	Boundaries 12, 17–20
ht.nzmesh	dnzmesh	1	Normal vector (mesh), z component	Boundaries 12, 17–20
ht.dnr	dnr	1	Normal vector down direction, r component	Boundaries 1–11, 13, 15
ht.dnphi	0	1	Normal vector down direction, phi component	Boundaries 1–11, 13, 15
ht.dnz	dnz	1	Normal vector down direction, z component	Boundaries 1–11, 13, 15
ht.dnr	dnr	1	Normal vector down direction, r component	Boundaries 12, 14, 16–20
ht.dnphi	0	1	Normal vector down direction, phi component	Boundaries 12, 14, 16–20
ht.dnz	dnz	1	Normal vector down direction, z component	Boundaries 12, 14, 16–20
ht.dnrmesh	dfrmesh	1	Normal vector down direction (mesh), r component	Boundaries 1–11, 13, 15
ht.dnphimesh	0	1	Normal vector down direction (mesh), phi component	Boundaries 1–11, 13, 15
ht.dnzmesh	dnzmesh	1	Normal vector down direction (mesh), z component	Boundaries 1–11, 13, 15
ht.dnrmesh	dfrmesh	1	Normal vector down direction (mesh), r component	Boundaries 12, 14, 16–20
ht.dnphimesh	0	1	Normal vector down direction (mesh), phi component	Boundaries 12, 14, 16–20
ht.dnzmesh	dnzmesh	1	Normal vector down direction (mesh), z component	Boundaries 12, 14, 16–20
ht.unr	unr	1	Normal vector up direction, r component	Boundaries 1–11, 13, 15
ht.unphi	0	1	Normal vector up direction, phi component	Boundaries 1–11, 13, 15
ht.unz	unz	1	Normal vector up direction, z component	Boundaries 1–11, 13, 15
ht.unr	unr	1	Normal vector up direction, r component	Boundaries 12, 14, 16–20
ht.unphi	0	1	Normal vector up direction, phi component	Boundaries 12, 14, 16–20
ht.unz	unz	1	Normal vector up direction, z component	Boundaries 12, 14, 16–20
ht.unrmesh	unrmesh	1	Normal vector up direction (mesh), r component	Boundaries 1–11, 13, 15
ht.unphimesh	0	1	Normal vector up direction (mesh), phi component	Boundaries 1–11, 13, 15
ht.unzmesh	unzmesh	1	Normal vector up direction (mesh), z component	Boundaries 1–11, 13, 15
ht.unrmesh	unrmesh	1	Normal vector up direction (mesh), r component	Boundaries 12, 14, 16–20
ht.unphimesh	0	1	Normal vector up direction (mesh), phi component	Boundaries 12, 14, 16–20
ht.unzmesh	unzmesh	1	Normal vector up direction (mesh), z component	Boundaries 12, 14, 16–20
ht.dEiInt	0	W	Total accumulated heat rate	Global + operation
ht.dEiInt_cst	0	W	Total accumulated heat rate, constant material properties	Global + operation
ht.dEi0Int	0	W	Total accumulated energy rate	Global + operation
ht.dEi0Int_cst	0	W	Total accumulated energy rate, constant material properties	Global + operation
ht.ntfluxInt	ht.intExtBnd(ht.ntflux*ht.varIntSpa)+ht.intIntBnd(ht.ncflux_u*up(ht.varIntSpa)+ht.ncflux_d*down(ht.varIntSpa))	W	Total net heat rate	Global
ht.ntfluxInt_cst	ht.intExtBnd(ht.ntflux*ht.varIntSpa)+ht.intIntBnd(ht.ncflux_u*up(ht.varIntSpa)+ht.ncflux_d*down(ht.varIntSpa))	W	Total net heat rate, constant material properties	Global
ht.ntefluxInt	ht.intExtBnd(ht.nteflux*ht.varIntSpa)+ht.intIntBnd(ht:nthflux_u*up(ht.varIntSpa)+ht:nthflux_d*down(ht.varIntSpa))	W	Total net energy rate	Global
ht.ntefluxInt_cst	ht.intExtBnd(ht.nteflux_cst*ht.varIntSpa)+ht.intIntBnd(ht:nthflux_cst_u*up(ht.varIntSpa)+ht:nthflux_cst_d*down(ht.varIntSpa))	W	Total net energy rate, constant material properties	Global

ht.QInt	ht.intDom(ht.Qtot*ht.varIntSpa)+ht.intIntLine(ht.Qltot*ht.varIntSpa)+ht.intLine(ht.Qlrtot*ht.varIntSpa)+ht.intAxis(ht.Qlrtot)+ht.intPnt(ht.Qprtot)-ht.intIntBnd(ht.ndflux_u*up(ht.varIntSpa)+ht.ndflux_d*down(ht.varIntSpa))	W	Total heat source	Global
ht.QInt_cst	ht.intDom(ht.Qtot*ht.varIntSpa)+ht.intIntLine(ht.Qltot*ht.varIntSpa)+ht.intLine(ht.Qlrtot*ht.varIntSpa)+ht.intAxis(ht.Qlrtot)+ht.intPnt(ht.Qprtot)-ht.intIntBnd(ht.ndflux_u*up(ht.varIntSpa)+ht.ndflux_d*down(ht.varIntSpa))	W	Total heat source, constant material properties	Global
ht.id	1	1	Physics indicator	Domains 1–4
ht.id	1	1	Physics indicator	Domains 5–7
ht.Wstr	0	W/m ³	Total stress power	Domains 1–4
ht.Wstr	0	W/m ³	Total stress power	Domains 5–7
ht.WstrInt	0	W	Total stress power	Global
ht.WstrInt_cst	0	W	Total stress power, constant material properties	Global
ht.Wtot	0	W/m ³	Total work source	Domains 1–4
ht.Wtot	0	W/m ³	Total work source	Domains 5–7
ht.WBndTot_u	0	W/m ²	Total work source, upside	Boundaries 4, 6, 8–11, 13, 15
ht.WBndTot_u	0	W/m ²	Total work source, upside	Boundaries 14, 16
ht.WBndTot_d	0	W/m ²	Total work source, downside	Boundaries 1–11, 13, 15
ht.WBndTot_d	0	W/m ²	Total work source, downside	Boundaries 12, 14, 16–20
ht.WInt	0	W	Total work source	Global
ht.WInt_cst	0	W	Total work source, constant material properties	Global
ht.heatBalance	ht.dEiInt+ht.ntfluxInt+ht.WstrInt-ht.QInt	W	Heat balance	Global
ht.heatBalance_cst	ht.dEiInt_cst+ht.ntfluxInt_cst+ht.WstrInt_cst-ht.QInt_cst	W	Heat balance, constant material properties	Global
ht.energyBalance	ht.dEi0Int+ht.ntefluxInt-ht.WInt-ht.QInt	W	Energy balance	Global
ht.energyBalance_cst	ht.dEi0Int_cst+ht.ntefluxInt_cst-ht.WInt_cst-ht.QInt_cst	W	Energy balance, constant material properties	Global
ht.varIntSpa	2*ht.d*pi*r	m	Intermediate variable	Domains 1–4
ht.varIntSpa	2*ht.d*pi*ie1.r*ie1.detInvT	m	Intermediate variable	Domains 5–7
ht.varIntSpa	ht.d	m	Intermediate variable	Boundaries 1, 3, 5

2.6.3. Solid



Solid

Selection

Geometric entity level	Domain
Selection	Geometry geom2: Dimension 2: All domains

Equations

$$\rho C_p \mathbf{u} \cdot \nabla T + \nabla \cdot \mathbf{q} = Q_{\text{ted}}$$

$$\mathbf{q} = -k \nabla T$$

Heat Conduction, Solid

Settings

Description	Value
Thermal conductivity	From material

Thermodynamics, Solid

Settings

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Description	Value
Density	From material
Heat capacity at constant pressure	From material

Coordinate System Selection

Settings	
Description	Value
Coordinate system	Global coordinate system

Model Input

Settings	
Description	Value
Volume reference temperature	Common model input

Properties from material

Property	Material	Property group
Thermal conductivity	HfO ₂ (insulator)	Basic
Density	HfO ₂ (insulator)	Basic
Heat capacity at constant pressure	HfO ₂ (insulator)	Basic
Thermal conductivity	Filament (Hafnium)	Basic
Density	Filament (Hafnium)	Basic
Heat capacity at constant pressure	Filament (Hafnium)	Basic
Thermal conductivity	Ti - Titanium	Basic
Density	Ti - Titanium	Basic
Heat capacity at constant pressure	Ti - Titanium	Basic
Thermal conductivity	W - Tungsten	Basic
Density	W - Tungsten	Basic
Heat capacity at constant pressure	W - Tungsten	Basic

Variables

Name	Expression	Unit	Description	Selection	Details
ht.ndflux_u	-uflux_spatial(T)/up(ht.varIntSpa)	W/m ²	Internal normal conductive heat flux, upside	Boundaries 4, 6, 8–11, 13, 15	+ operation
ht.ndflux_u	-uflux_spatial(T)/up(ht.varIntSpa)	W/m ²	Internal normal conductive heat flux, upside	Boundaries 14, 16	+ operation
ht.ndflux_d	-dflux_spatial(T)/down(ht.varIntSpa)	W/m ²	Internal normal conductive heat flux, downside	Boundaries 4, 6, 8–11, 13, 15	+ operation
ht.ndflux_d	-dflux_spatial(T)/down(ht.varIntSpa)	W/m ²	Internal normal conductive heat flux, downside	Boundaries 14, 16	+ operation
ht.ndflux_d	-dflux_spatial(T)/down(ht.varIntSpa)	W/m ²	Internal normal conductive heat flux, downside	Boundaries 2, 7	+ operation
ht.ndflux_d	-dflux_spatial(T)/down(ht.varIntSpa)	W/m ²	Internal normal conductive heat flux, downside	Boundaries 12, 17–20	+ operation
ht.ndflux_d	-dflux_spatial(T)/down(ht.d)	W/m ²	Internal normal conductive heat flux, downside	Boundaries 1, 3, 5	+ operation
domflux.Tr	2*ht.dfluxr*ht.d*pi*r	W/m	Domain flux, r component	Domains 1–4	
domflux.Tr	2*ht.dfluxr*ht.d*pi*r	W/m	Domain flux, r component	Domains 5–7	
domflux.Tz	2*ht.dfluxz*ht.d*pi*r	W/m	Domain flux, z component	Domains 1–4	

domflux.Tz	$2*ht.dfluxz*ht.d*pi*r$	W/m	Domain flux, z component	Domains 5–7	
ht.alphap	$-d(ht.rho,T)/max(ht.rho,eps)$	1/K	Isobaric compressibility coefficient	Domains 1–4	
ht.alphap	$-d(ht.rho,T)/max(ht.rho,eps)$	1/K	Isobaric compressibility coefficient	Domains 5–7	
ht.chiT	0	1/Pa	Isothermal compressibility coefficient	Domains 1–4	
ht.chiT	0	1/Pa	Isothermal compressibility coefficient	Domains 5–7	
ht.DeltaH_add	integrate(ht.CpInt,ht.TInt,ht.DeltaH_Tlow,T)	J/kg	Sensible enthalpy	Domains 1–4	+ operation
ht.DeltaH_add	integrate(ht.CpInt,ht.TInt,ht.DeltaH_Tlow,T)	J/kg	Sensible enthalpy	Domains 5–7	+ operation
ht.DeltaH_add_cst	$ht.Cp*(T-ht.DeltaH_Tlow)$	J/kg	Sensible enthalpy, constant material properties	Domains 1–4	+ operation
ht.DeltaH_add_cst	$ht.Cp*(T-ht.DeltaH_Tlow)$	J/kg	Sensible enthalpy, constant material properties	Domains 5–7	+ operation
ht.H	$ht.HRef+ht.DeltaH$	J/kg	Enthalpy	Domains 1–4	+ operation
ht.H	$ht.HRef+ht.DeltaH$	J/kg	Enthalpy	Domains 5–7	+ operation
ht.H_cst	$ht.HRef+ht.DeltaH_cst$	J/kg	Enthalpy, constant material properties	Domains 1–4	+ operation
ht.H_cst	$ht.HRef+ht.DeltaH_cst$	J/kg	Enthalpy, constant material properties	Domains 5–7	+ operation
ht.Ei	$ht.H$	J/kg	Internal energy	Domains 1–4	+ operation
ht.Ei	$ht.H$	J/kg	Internal energy	Domains 5–7	+ operation
ht.Ei_cst	$ht.H_cst$	J/kg	Internal energy, constant material properties	Domains 1–4	+ operation
ht.Ei_cst	$ht.H_cst$	J/kg	Internal energy, constant material properties	Domains 5–7	+ operation
ht.Ek	$0.5*(ht.ur^2+ht.uphi^2+ht.uz^2)$	J/kg	Kinetic energy	Domains 1–4	+ operation
ht.Ek	$0.5*(ht.ur^2+ht.uphi^2+ht.uz^2)$	J/kg	Kinetic energy	Domains 5–7	+ operation
ht.dfluxr	$-ht.k_effrr*Tr-ht.k_effrz*Tz$	W/m ²	Conductive heat flux, r component	Domains 1–4	+ operation
ht.dfluxphi	$-ht.k_effphir*Tr-ht.k_effphiz*Tz$	W/m ²	Conductive heat flux, phi component	Domains 1–4	+ operation
ht.dfluxz	$-ht.k_effzr*Tr-ht.k_effzz*Tz$	W/m ²	Conductive heat flux, z component	Domains 1–4	+ operation
ht.dfluxr	$-ht.k_effrr*(ie1.T11*Tr+ie1.T13*Tz)-ht.k_effrphi*(ie1.T21*Tr+ie1.T23*Tz)-ht.k_effrz*(ie1.T31*Tr+ie1.T33*Tz)$	W/m ²	Conductive heat flux, r component	Domains 5–7	+ operation
ht.dfluxphi	$-ht.k_effphir*(ie1.T11*Tr+ie1.T13*Tz)-ht.k_effphiphi*(ie1.T21*Tr+ie1.T23*Tz)-ht.k_effphiz*(ie1.T31*Tr+ie1.T33*Tz)$	W/m ²	Conductive heat flux, phi component	Domains 5–7	+ operation
ht.dfluxz	$-ht.k_effzr*(ie1.T11*Tr+ie1.T13*Tz)-ht.k_effzphi*(ie1.T21*Tr+ie1.T23*Tz)-ht.k_effzz*(ie1.T31*Tr+ie1.T33*Tz)$	W/m ²	Conductive heat flux, z component	Domains 5–7	+ operation
ht.dfluxtestr	$-ht.k_effrr*test(Tr)-ht.k_effrz*test(Tz)$	W/m ²	Conductive heat flux, r	Domains 1–4	+ operation

			component		
ht.dfluxtestphi	-ht.k_effphir*test(Tr)-ht.k_effphiz*test(Tz)		W/m ²	Conductive heat flux, phi component	Domains 1–4 + operation
ht.dfluxtestz	-ht.k_effzr*test(Tr)-ht.k_effzz*test(Tz)		W/m ²	Conductive heat flux, z component	Domains 1–4 + operation
ht.dfluxtestr	-ht.k_effrr*test(iel.T11*Tr+iel.T13*Tz)-ht.k_effrphi*test(iel.T21*Tr+iel.T23*Tz)-ht.k_effrz*test(iel.T31*Tr+iel.T33*Tz)		W/m ²	Conductive heat flux, r component	Domains 5–7 + operation
ht.dfluxtestphi	-ht.k_effphir*test(iel.T11*Tr+iel.T13*Tz)-ht.k_effphiphi*test(iel.T21*Tr+iel.T23*Tz)-ht.k_effphiz*test(iel.T31*Tr+iel.T33*Tz)		W/m ²	Conductive heat flux, phi component	Domains 5–7 + operation
ht.dfluxtestz	-ht.k_effzr*test(iel.T11*Tr+iel.T13*Tz)-ht.k_effzphi*test(iel.T21*Tr+iel.T23*Tz)-ht.k_effzz*test(iel.T31*Tr+iel.T33*Tz)		W/m ²	Conductive heat flux, z component	Domains 5–7 + operation
ht.cfluxr	ht.rho*ht.ur*ht.Ei		W/m ²	Convective heat flux, r component	Domains 1–4 + operation
ht.cfluxphi	ht.rho*ht.uphi*ht.Ei		W/m ²	Convective heat flux, phi component	Domains 1–4 + operation
ht.cfluxz	ht.rho*ht.uz*ht.Ei		W/m ²	Convective heat flux, z component	Domains 1–4 + operation
ht.cfluxr	ht.rho*ht.ur*ht.Ei		W/m ²	Convective heat flux, r component	Domains 5–7 + operation
ht.cfluxphi	ht.rho*ht.uphi*ht.Ei		W/m ²	Convective heat flux, phi component	Domains 5–7 + operation
ht.cfluxz	ht.rho*ht.uz*ht.Ei		W/m ²	Convective heat flux, z component	Domains 5–7 + operation
ht.tefluxr	ht.dfluxr+ht.thfluxr		W/m ²	Total energy flux, r component	Domains 1–4 + operation
ht.tefluxphi	ht.dfluxphi+ht.thfluxphi		W/m ²	Total energy flux, phi component	Domains 1–4 + operation
ht.tefluxz	ht.dfluxz+ht.thfluxz		W/m ²	Total energy flux, z component	Domains 1–4 + operation
ht.tefluxr	ht.dfluxr+ht.thfluxr		W/m ²	Total energy flux, r component	Domains 5–7 + operation
ht.tefluxphi	ht.dfluxphi+ht.thfluxphi		W/m ²	Total energy flux, phi component	Domains 5–7 + operation
ht.tefluxz	ht.dfluxz+ht.thfluxz		W/m ²	Total energy flux, z component	Domains 5–7 + operation
ht.teflux_cstr	ht.dfluxr+ht.thflux_cstr		W/m ²	Total energy flux, constant material properties, r component	Domains 1–4 + operation
ht.teflux_cstphi	ht.dfluxphi+ht.thflux_cstphi		W/m ²	Total energy flux, constant material properties, phi component	Domains 1–4 + operation
ht.teflux_cstz	ht.dfluxz+ht.thflux_cstz		W/m ²	Total energy flux, constant material properties, z component	Domains 1–4 + operation
ht.teflux_cstr	ht.dfluxr+ht.thflux_cstr		W/m ²	Total energy flux, constant material properties, r component	Domains 5–7 + operation
ht.teflux_cstphi	ht.dfluxphi+ht.thflux_cstphi		W/m ²	Total energy flux, constant material properties, phi component	Domains 5–7 + operation
ht.teflux_cstz	ht.dfluxz+ht.thflux_cstz		W/m ²	Total energy flux, constant material properties	Domains 5–7 + operation

			material properties, z component		
ht.thfluxr	ht.rho*ht.ur*ht.H0		W/m ²	Total enthalpy flux, r component	Domains 1–4 + operation
ht.thfluxphi	ht.rho*ht.uphi*ht.H0		W/m ²	Total enthalpy flux, phi component	Domains 1–4 + operation
ht.thfluxz	ht.rho*ht.uz*ht.H0		W/m ²	Total enthalpy flux, z component	Domains 1–4 + operation
ht.thfluxr	ht.rho*ht.ur*ht.H0		W/m ²	Total enthalpy flux, r component	Domains 5–7 + operation
ht.thfluxphi	ht.rho*ht.uphi*ht.H0		W/m ²	Total enthalpy flux, phi component	Domains 5–7 + operation
ht.thfluxz	ht.rho*ht.uz*ht.H0		W/m ²	Total enthalpy flux, z component	Domains 5–7 + operation
ht.thflux_cstr	ht.rho*ht.ur*ht.H0_cst		W/m ²	Total enthalpy flux, constant material properties, r component	Domains 1–4 + operation
ht.thflux_cstphi	ht.rho*ht.uphi*ht.H0_cst		W/m ²	Total enthalpy flux, constant material properties, phi component	Domains 1–4 + operation
ht.thflux_cstz	ht.rho*ht.uz*ht.H0_cst		W/m ²	Total enthalpy flux, constant material properties, z component	Domains 1–4 + operation
ht.thflux_cstr	ht.rho*ht.ur*ht.H0_cst		W/m ²	Total enthalpy flux, constant material properties, r component	Domains 5–7 + operation
ht.thflux_cstphi	ht.rho*ht.uphi*ht.H0_cst		W/m ²	Total enthalpy flux, constant material properties, phi component	Domains 5–7 + operation
ht.thflux_cstz	ht.rho*ht.uz*ht.H0_cst		W/m ²	Total enthalpy flux, constant material properties, z component	Domains 5–7 + operation
ht.dEiInt	ht.solid1.dEiInt		W	Total accumulated heat rate	Global + operation
ht.dEiInt_cst	ht.solid1.dEiInt_cst		W	Total accumulated heat rate, constant material properties	Global + operation
ht.dEi0Int	ht.solid1.dEi0Int		W	Total accumulated energy rate	Global + operation
ht.dEi0Int_cst	ht.solid1.dEi0Int_cst		W	Total accumulated energy rate, constant material properties	Global + operation
ht.Wstr	ht.pA*(d(ht.ur,r)+if(abs(r)<0.001*h_spatial,d(ht.ur,r),ht.ur/r)+d(ht.uz,z))		W/m ³	Total stress power	Domains 1–4 + operation
ht.Wstr	ht.pA*(ie1.T11*d(ht.ur,r)-ie1.T12*if(abs(r)<0.001*h_spatial,d(ht.uphi,r),ht.uphi/ie1.r)+ie1.T13*d(ht.ur,z)+ie1.T21*d(ht.uphi,r)+ie1.T22*if(abs(r)<0.001*h_spatial,d(ht.ur,r),ht.ur/ie1.r)+ie1.T23*d(ht.uphi,z)+ie1.T31*d(ht.uz,r)+ie1.T33*d(ht.uz,z))		W/m ³	Total stress power	Domains 5–7 + operation
ht.WstrInt	ht.solid1.WstrInt		W	Total stress power	Global + operation
ht.WstrInt_cst	ht.solid1.WstrInt_cst		W	Total stress power, constant material properties	Global + operation

ht.WInt	ht.solid1.WInt		W	Total work source	Global	+ operation
ht.WInt_cst	ht.solid1.WInt_cst		W	Total work source, constant material properties	Global	+ operation
ht.krr	material.k11		W/(m·K)	Thermal conductivity, rr component	Domains 1–4	Meta
ht.kphir	material.k21		W/(m·K)	Thermal conductivity, phir component	Domains 1–4	Meta
ht.kzr	material.k31		W/(m·K)	Thermal conductivity, zr component	Domains 1–4	Meta
ht.krphi	material.k12		W/(m·K)	Thermal conductivity, rphi component	Domains 1–4	Meta
ht.kphiphi	material.k22		W/(m·K)	Thermal conductivity, phiphi component	Domains 1–4	Meta
ht.kzphi	material.k32		W/(m·K)	Thermal conductivity, zphi component	Domains 1–4	Meta
ht.krz	material.k13		W/(m·K)	Thermal conductivity, rz component	Domains 1–4	Meta
ht.kphiz	material.k23		W/(m·K)	Thermal conductivity, phiz component	Domains 1–4	Meta
ht.kzz	material.k33		W/(m·K)	Thermal conductivity, zz component	Domains 1–4	Meta
ht.krr	material.k11		W/(m·K)	Thermal conductivity, rr component	Domains 5–7	Meta
ht.kphir	material.k21		W/(m·K)	Thermal conductivity, phir component	Domains 5–7	Meta
ht.kzr	material.k31		W/(m·K)	Thermal conductivity, zr component	Domains 5–7	Meta
ht.krphi	material.k12		W/(m·K)	Thermal conductivity, rphi component	Domains 5–7	Meta
ht.kphiphi	material.k22		W/(m·K)	Thermal conductivity, phiphi component	Domains 5–7	Meta
ht.kzphi	material.k32		W/(m·K)	Thermal conductivity, zphi component	Domains 5–7	Meta
ht.krz	material.k13		W/(m·K)	Thermal conductivity, rz component	Domains 5–7	Meta
ht.kphiz	material.k23		W/(m·K)	Thermal conductivity, phiz component	Domains 5–7	Meta
ht.kzz	material.k33		W/(m·K)	Thermal conductivity, zz component	Domains 5–7	Meta
ht.k_iso	material.k_iso		W/(m·K)	Thermal conductivity, isotropic value	Domains 1–4	Meta
ht.k_iso	material.k_iso		W/(m·K)	Thermal conductivity, isotropic value	Domains 5–7	Meta
ht.rho	material.rho		kg/m³	Density	Domains 1–4	Meta

ht.rho	material.rho		kg/m ³	Density	Domains 5–7	Meta
ht.Cp	material.Cp		J/(kg·K)	Heat capacity at constant pressure	Domains 1–4	Meta
ht.Cp	material.Cp		J/(kg·K)	Heat capacity at constant pressure	Domains 5–7	Meta
ht.solid1.pref	1[atm]		Pa	Reference pressure level	Domains 1–4	
ht.solid1.pref	1[atm]		Pa	Reference pressure level	Domains 5–7	
ht.res_T	-ht.k_effrr*Trr-ht.k_effrz*Trz-ht.k_effzr*Tzz-(ht.qs+ht.qs_oop)*T+ht.C_eff*(ht.ur*Tr+ht.uz*Tz)-ht.D_Hr*Tr-ht.D_Hz*Tz-ht.Q-ht.Qoop		W/m ³	Equation residual	Domains 1–7	+ operation
ht.pA	ht.pref		Pa	Absolute pressure	Domains 1–4	
ht.pA	ht.pref		Pa	Absolute pressure	Domains 5–7	
ht.Qmet	0		W/m ³	Metabolic heat source	Domains 1–4	+ operation
ht.Qmet	0		W/m ³	Metabolic heat source	Domains 5–7	+ operation
ht.pref	ht.solid1.pref		Pa	Reference pressure level	Domains 1–4	
ht.pref	ht.solid1.pref		Pa	Reference pressure level	Domains 5–7	
ht.DeltaH_Tlow	ht.Tref		K	Temperature lower bound for enthalpy evaluation	Domains 1–4	
ht.DeltaH_Tlow	ht.Tref		K	Temperature lower bound for enthalpy evaluation	Domains 5–7	
ht.DeltaH_plow	ht.pref		Pa	Pressure lower bound for enthalpy evaluation	Domains 1–4	
ht.DeltaH_plow	ht.pref		Pa	Pressure lower bound for enthalpy evaluation	Domains 5–7	
ht.dHdp	0		m ³ /kg	Intermediate variable	Domains 1–4	
ht.dHdp	0		m ³ /kg	Intermediate variable	Domains 5–7	
ht.alphapT	ht.alphap*T		1	Help variable	Domains 1–4	
ht.alphapT	ht.alphap*T		1	Help variable	Domains 5–7	
ht.mujtT	0		m·s ² ·K/kg	Isothermal Joule-Thomson coefficient	Domains 1–4	
ht.mujtT	0		m·s ² ·K/kg	Isothermal Joule-Thomson coefficient	Domains 5–7	
ht.cellPe	0.5*ht.rho*ht.Cp*h*sqrt(ht.ur^2+ht.uphi^2+ht.uz^2)/ht.kmean		1	Cell Péclet number	Domains 1–4	
ht.cellPe	0.5*ht.rho*ht.Cp*h*sqrt(ht.ur^2+ht.uphi^2+ht.uz^2)/ht.kmean		1	Cell Péclet number	Domains 5–7	
ht.ur	0		m/s	Velocity field, r component	Domains 1–4	+ operation
ht.uphi	0		m/s	Velocity field, phi component	Domains 1–4	+ operation
ht.uz	0		m/s	Velocity field, z component	Domains 1–4	+ operation
ht.ur	0		m/s	Velocity field, r component	Domains 5–7	+ operation
ht.uphi	0		m/s	Velocity field, phi component	Domains 5–7	+ operation
ht.uz	0		m/s	Velocity field, z component	Domains 5–7	+ operation
ht.rholnit	subst(ht.rho,T,ht.Tinit,minput.pA,ht.pref)		kg/m ³	Initial density	Domains 1–4	

ht.rhoInit	subst(ht.rho,T,ht.Tinit,minput.pA,ht.pref)	kg/m ³	Initial density	Domains 5–7	
ht.rho_eff	ht.rho	kg/m ³	Effective density	Domains 1–4	
ht.rho_eff	ht.rho	kg/m ³	Effective density	Domains 5–7	
ht.C_eff	ht.rho*ht.Cp	J/(m ³ ·K)	Effective volumetric heat capacity	Domains 1–4	
ht.C_eff	ht.rho*ht.Cp	J/(m ³ ·K)	Effective volumetric heat capacity	Domains 5–7	
ht.k_effrr	ht.krr	W/(m·K)	Effective thermal conductivity, rr component	Domains 1–4	
ht.k_effphir	ht.kphir	W/(m·K)	Effective thermal conductivity, phir component	Domains 1–4	
ht.k_effzr	ht.kzr	W/(m·K)	Effective thermal conductivity, zr component	Domains 1–4	
ht.k_effrphi	ht.krphi	W/(m·K)	Effective thermal conductivity, rphi component	Domains 1–4	
ht.k_effphiphi	ht.kphiphi	W/(m·K)	Effective thermal conductivity, phiphi component	Domains 1–4	
ht.k_effzphi	ht.kzphi	W/(m·K)	Effective thermal conductivity, zphi component	Domains 1–4	
ht.k_effrz	ht.krz	W/(m·K)	Effective thermal conductivity, rz component	Domains 1–4	
ht.k_effphiz	ht.kphiz	W/(m·K)	Effective thermal conductivity, phiz component	Domains 1–4	
ht.k_effzz	ht.kzz	W/(m·K)	Effective thermal conductivity, zz component	Domains 1–4	
ht.k_effrr	ht.krr	W/(m·K)	Effective thermal conductivity, rr component	Domains 5–7	
ht.k_effphir	ht.kphir	W/(m·K)	Effective thermal conductivity, phir component	Domains 5–7	
ht.k_effzr	ht.kzr	W/(m·K)	Effective thermal conductivity, zr component	Domains 5–7	
ht.k_effrphi	ht.krphi	W/(m·K)	Effective thermal conductivity, rphi component	Domains 5–7	
ht.k_effphiphi	ht.kphiphi	W/(m·K)	Effective thermal conductivity, phiphi component	Domains 5–7	
ht.k_effzphi	ht.kzphi	W/(m·K)	Effective thermal conductivity, zphi component	Domains 5–7	

			zphi component		
ht.k_effrz	ht.krz		Effective thermal conductivity, rz component	Domains 5-7	
ht.k_effphiz	ht.kphiz		Effective thermal conductivity, phiz component	Domains 5-7	
ht.k_effzz	ht.kzz		Effective thermal conductivity, zz component	Domains 5-7	
ht.kappaTrr	0		Turbulent thermal conductivity, rr component	Domains 1-4	
ht.kappaTphir	0		Turbulent thermal conductivity, phir component	Domains 1-4	
ht.kappaTzr	0		Turbulent thermal conductivity, zr component	Domains 1-4	
ht.kappaTrphi	0		Turbulent thermal conductivity, rphi component	Domains 1-4	
ht.kappaTphiphi	0		Turbulent thermal conductivity, phiphi component	Domains 1-4	
ht.kappaTzphi	0		Turbulent thermal conductivity, zphi component	Domains 1-4	
ht.kappaTrz	0		Turbulent thermal conductivity, rz component	Domains 1-4	
ht.kappaTphiz	0		Turbulent thermal conductivity, phiz component	Domains 1-4	
ht.kappaTzz	0		Turbulent thermal conductivity, zz component	Domains 1-4	
ht.kappaTrr	0		Turbulent thermal conductivity, rr component	Domains 5-7	
ht.kappaTphir	0		Turbulent thermal conductivity, phir component	Domains 5-7	
ht.kappaTzr	0		Turbulent thermal conductivity, zr component	Domains 5-7	
ht.kappaTrphi	0		Turbulent thermal conductivity, rphi component	Domains 5-7	
ht.kappaTphiphi	0		Turbulent thermal conductivity, phiphi component	Domains 5-7	
ht.kappaTzphi	0		Turbulent thermal conductivity, zphi component	Domains	

			thermal conductivity, zphi component	5-7		
ht.kappaTrz	0		W/(m·K)	Turbulent thermal conductivity, rz component	Domains 5-7	
ht.kappaTphiz	0		W/(m·K)	Turbulent thermal conductivity, phiz component	Domains 5-7	
ht.kappaTzz	0		W/(m·K)	Turbulent thermal conductivity, zz component	Domains 5-7	
ht.kmean	0.5*(ht.k_effrr+ht.k_effzz)		W/(m·K)	Mean effective thermal conductivity	Domains 1-4	
ht.kmean	0.5*(ht.k_effrr+ht.k_effzz)		W/(m·K)	Mean effective thermal conductivity	Domains 5-7	
ht.gradTr	Tr		K/m	Temperature gradient, r component	Domains 1-4	
ht.gradTphi	0		K/m	Temperature gradient, phi component	Domains 1-4	
ht.gradTz	Tz		K/m	Temperature gradient, z component	Domains 1-4	
ht.gradTr	ie1.T11*Tr+ie1.T13*Tz		K/m	Temperature gradient, r component	Domains 5-7	
ht.gradTphi	ie1.T21*Tr+ie1.T23*Tz		K/m	Temperature gradient, phi component	Domains 5-7	
ht.gradTz	ie1.T31*Tr+ie1.T33*Tz		K/m	Temperature gradient, z component	Domains 5-7	
ht.gradTmag	sqrt(ht.gradTr^2+ht.gradTphi^2+ht.gradTz^2)		K/m	Temperature gradient magnitude	Domains 1-4	
ht.gradTmag	sqrt(ht.gradTr^2+ht.gradTphi^2+ht.gradTz^2)		K/m	Temperature gradient magnitude	Domains 5-7	
ht.alphaTdrr	ht.k_effrr/ht.C_eff		m²/s	Thermal diffusivity, rr component	Domains 1-4	
ht.alphaTdphir	ht.k_effphir/ht.C_eff		m²/s	Thermal diffusivity, phir component	Domains 1-4	
ht.alphaTdzr	ht.k_effzr/ht.C_eff		m²/s	Thermal diffusivity, zr component	Domains 1-4	
ht.alphaTdrphi	ht.k_effrphi/ht.C_eff		m²/s	Thermal diffusivity, rphi component	Domains 1-4	
ht.alphaTdphiphi	ht.k_effphiphi/ht.C_eff		m²/s	Thermal diffusivity, phiphi component	Domains 1-4	
ht.alphaTdzphi	ht.k_effzphi/ht.C_eff		m²/s	Thermal diffusivity, zphi component	Domains 1-4	
ht.alphaTdrz	ht.k_effrz/ht.C_eff		m²/s	Thermal diffusivity, rz component	Domains 1-4	
ht.alphaTdpiz	ht.k_effphiz/ht.C_eff		m²/s	Thermal diffusivity, phiz component	Domains 1-4	
ht.alphaTdz	ht.k_effzz/ht.C_eff		m²/s	Thermal diffusivity, zz component	Domains 1-4	

ht.alphaTdrr	ht.k_effrr/ht.C_eff		m ² /s	Thermal diffusivity, rr component	Domains 5–7	
ht.alphaTdpfir	ht.k_effphir/ht.C_eff		m ² /s	Thermal diffusivity, phir component	Domains 5–7	
ht.alphaTdzr	ht.k_effzr/ht.C_eff		m ² /s	Thermal diffusivity, zr component	Domains 5–7	
ht.alphaTdrphi	ht.k_effrphi/ht.C_eff		m ² /s	Thermal diffusivity, rphi component	Domains 5–7	
ht.alphaTdpiphi	ht.k_effphiphi/ht.C_eff		m ² /s	Thermal diffusivity, phiphi component	Domains 5–7	
ht.alphaTdzphi	ht.k_effzphi/ht.C_eff		m ² /s	Thermal diffusivity, zphi component	Domains 5–7	
ht.alphaTdrz	ht.k_effrz/ht.C_eff		m ² /s	Thermal diffusivity, rz component	Domains 5–7	
ht.alphaTdpiz	ht.k_effphiz/ht.C_eff		m ² /s	Thermal diffusivity, phiz component	Domains 5–7	
ht.alphaTdzz	ht.k_effzz/ht.C_eff		m ² /s	Thermal diffusivity, zz component	Domains 5–7	
ht.alphaTdMean	ht.kmean/ht.C_eff		m ² /s	Mean thermal diffusivity	Domains 1–4	
ht.alphaTdMean	ht.kmean/ht.C_eff		m ² /s	Mean thermal diffusivity	Domains 5–7	
ht.Tradu	ht.Tu		K	Upside temperature	Domains 1–4	
ht.Tradu	ht.Tu		K	Upside temperature	Boundaries 1–11, 13, 15	
ht.Tradu	ht.Tu		K	Upside temperature	Domains 5–7	
ht.Tradu	ht.Tu		K	Upside temperature	Boundaries 12, 14, 16–20	
ht.Tradd	ht.Td		K	Downside temperature	Domains 1–4	
ht.Tradd	ht.Td		K	Downside temperature	Boundaries 1–11, 13, 15	
ht.Tradd	ht.Td		K	Downside temperature	Domains 5–7	
ht.Tradd	ht.Td		K	Downside temperature	Boundaries 12, 14, 16–20	
ht.solid1.dEiInt	ht.solid1.intDom((ht.dEi-ht.Qm*ht.Ei)*ht.varIntSpa)		W	Total accumulated heat rate	Global	
ht.dEi	0		W/m ³	Total accumulated heat rate density	Domains 1–4	
ht.dEi	0		W/m ³	Total accumulated heat rate density	Domains 5–7	
ht.solid1.dEiInt_cst	ht.solid1.intDom((ht.dEi_cst-ht.Qm*ht.Ei_cst)*ht.varIntSpa)		W	Total accumulated heat rate, constant material properties	Global	
ht.dEi_cst	0		W/m ³	Total accumulated heat rate density, constant	Domains 1–4	

			material properties		
ht.dEi_cst	0		W/m ³	Total accumulated heat rate density, constant material properties	Domains 5–7
ht.solid1.dEi0Int	ht.solid1.intDom((ht.dEi0-ht.Qm*ht.H)*ht.varIntSpa)		W	Total accumulated energy rate	Global
ht.dEi0	0		W/m ³	Total accumulated energy rate density	Domains 1–4
ht.dEi0	0		W/m ³	Total accumulated energy rate density	Domains 5–7
ht.solid1.dEi0Int_cst	ht.solid1.intDom((ht.dEi0_cst-ht.Qm*ht.H_cst)*ht.varIntSpa)		W	Total accumulated energy rate, constant material properties	Global
ht.dEi0_cst	0		W/m ³	Total accumulated energy rate density, constant material properties	Domains 1–4
ht.dEi0_cst	0		W/m ³	Total accumulated energy rate density, constant material properties	Domains 5–7
ht.solid1.ntfluxInt	ht.solid1.intExtBnd(ht.ntflux*ht.varIntSpa)+ht.solid1.intExtBndUp(ht.ntflux_u*up(ht.varIntSpa))+ht.solid1.intExtBndDown(ht.ntflux_d*down(ht.varIntSpa))+ht.solid1.intIntBnd(ht.ncflux_u*up(ht.varIntSpa)+ht.ncflux_d*down(ht.varIntSpa))		W	Total net heat rate	Global
ht.solid1.ntfluxInt_cst	ht.solid1.intExtBnd(ht.ntflux*ht.varIntSpa)+ht.solid1.intExtBndUp(ht.ntflux_u*up(ht.varIntSpa))+ht.solid1.intExtBndDown(ht.ntflux_d*down(ht.varIntSpa))+ht.solid1.intIntBnd(ht.ncflux_u*up(ht.varIntSpa)+ht.ncflux_d*down(ht.varIntSpa))		W	Total net heat rate, constant material properties	Global
ht.solid1.nthfluxInt	ht.solid1.intExtBnd(ht.nthflux*ht.varIntSpa)+ht.solid1.intExtBndUp(ht.nthflux_u*up(ht.varIntSpa))+ht.solid1.intExtBndDown(ht.nthflux_d*down(ht.varIntSpa))+ht.solid1.intIntBnd(ht.nthflux_u*up(ht.varIntSpa)+ht.nthflux_d*down(ht.varIntSpa))		W	Total net energy rate	Global
ht.solid1.nthfluxInt_cst	ht.solid1.intExtBnd(ht.nthflux_cst*ht.varIntSpa)+ht.solid1.intExtBndUp(ht.nthflux_cst_u*up(ht.varIntSpa))+ht.solid1.intExtBndDown(ht.nthflux_cst_d*down(ht.varIntSpa))+ht.solid1.intIntBnd(ht.nthflux_cst_u*up(ht.varIntSpa)+ht.nthflux_cst_d*down(ht.varIntSpa))		W	Total net energy rate, constant material properties	Global
ht.solid1.QInt	ht.solid1.intDom(ht.Qtot*ht.varIntSpa)+ht.solid1.intIntLine(ht.Qltot*ht.varIntSpa)+ht.intLine(subst(ht.Qlrltot,ht.id,isdefined(ht.solid1.id))*ht.varIntSpa)+ht.intAxis(subst(ht.Qlrltot,ht.id,isdefined(ht.solid1.id)))+ht.intPnt(subst(ht.Qprtot,ht.id,isdefined(ht.solid1.id)))-ht.solid1.intIntBnd(ht.ndflux_u*up(ht.varIntSpa)+ht.ndflux_d*down(ht.varIntSpa))		W	Total heat source	Global
ht.solid1.QInt_cst	ht.solid1.intDom(ht.Qtot*ht.varIntSpa)+ht.solid1.intIntLine(ht.Qltot*ht.varIntSpa)+ht.intLine(subst(ht.Qlrltot,ht.id,isdefined(ht.solid1.id))*ht.varIntSpa)+ht.intAxis(subst(ht.Qlrltot,ht.id,isdefined(ht.solid1.id)))+ht.intPnt(subst(ht.Qprtot,ht.id,isdefined(ht.solid1.id)))-ht.solid1.intIntBnd(ht.ndflux_u*up(ht.varIntSpa)+ht.ndflux_d*down(ht.varIntSpa))		W	Total heat source, constant material properties	Global
ht.solid1.WstrInt	ht.solid1.intDom(ht.Wstr*ht.varIntSpa)		W	Total stress power	Global
ht.solid1.WstrInt_cst	ht.solid1.intDom(ht.Wstr*ht.varIntSpa)		W	Total stress power, constant material properties	Global
ht.solid1.WInt	ht.solid1.intDom(ht.Wtot*ht.varIntSpa)+ht.solid1.intBndUp(ht.WBndTot_u*up(ht.varIntSpa))+ht.solid1.intBndDown(ht.WBndTot_d*down(ht.varIntSpa))		W	Total work source	Global
ht.solid1.WInt_cst	ht.solid1.intDom(ht.Wtot*ht.varIntSpa)+ht.solid1.intBndUp(ht.WBndTot_u*up(ht.varIntSpa))+ht.solid1.intBndDown(ht.WBndTot_d*down(ht.varIntSpa))		W	Total work source, constant material properties	Global
ht.solid1.heatBalance	ht.solid1.dEiInt+ht.solid1.ntfluxInt+ht.solid1.WstrInt-ht.solid1.QInt		W	Heat balance	Global
ht.solid1.heatBalance_cst	ht.solid1.dEiInt_cst+ht.solid1.ntfluxInt_cst+ht.solid1.WstrInt_cst-ht.solid1.QInt_cst		W	Heat balance, constant material properties	Global
ht.solid1.energyBalance	ht.solid1.dEi0Int+ht.solid1.nthfluxInt-ht.solid1.WInt-ht.solid1.QInt		W	Energy balance	Global

ht.solid1.energyBalance_cst	ht.solid1.dEi0Int_cst+ht.solid1.nefluxInt_cst-ht.solid1.WInt_cst-ht.solid1.QInt_cst		W	Energy balance, constant material properties	Global	
ht.D_Hr	0		W/(m ² ·K)	Enthalpy diffusion coefficient, r component	Domains 1–4	
ht.D_Hphi	0		W/(m ² ·K)	Enthalpy diffusion coefficient, phi component	Domains 1–4	
ht.D_Hz	0		W/(m ² ·K)	Enthalpy diffusion coefficient, z component	Domains 1–4	
ht.D_Hr	0		W/(m ² ·K)	Enthalpy diffusion coefficient, r component	Domains 5–7	
ht.D_Hphi	0		W/(m ² ·K)	Enthalpy diffusion coefficient, phi component	Domains 5–7	
ht.D_Hz	0		W/(m ² ·K)	Enthalpy diffusion coefficient, z component	Domains 5–7	
ht.Q_H	ht.D_Hr*Tr+ht.D_Hz*Tz		W/m ³	Enthalpy diffusion flux	Domains 1–4	
ht.Q_H	ht.D_Hr*(ie1.T11*Tr+ie1.T13*Tz)+ht.D_Hphi*(ie1.T21*Tr+ie1.T23*Tz)+ht.D_Hz*(ie1.T31*Tr+ie1.T33*Tz)		W/m ³	Enthalpy diffusion flux	Domains 5–7	
ht.timeDerivative	0		K/s	Temperature, first time derivative	Domains 1–4	
ht.timeDerivative	0		K/s	Temperature, first time derivative	Domains 5–7	
ht.gamma	1		1	Ratio of specific heats	Domains 1–4	
ht.gamma	1		1	Ratio of specific heats	Domains 5–7	
ht.Trho	ht.Tref		K	Temperature for density evaluation	Domains 1–4	
ht.Trho	ht.Tref		K	Temperature for density evaluation	Domains 5–7	
ht.dfltopaque	1		1	Default opacity	Domains 1–4	
ht.dfltopaque	1		1	Default opacity	Domains 5–7	
ht.CpInt	subst(material.Cp,ht.solid1.minput_pressure,ht.pref,ht.solid1.minput_temperature,ht.TInt)		J/(kg·K)	Specific heat capacity for integration	Domains 1–4	Meta
ht.CpInt	subst(material.Cp,ht.solid1.minput_pressure,ht.pref,ht.solid1.minput_temperature,ht.TInt)		J/(kg·K)	Specific heat capacity for integration	Domains 5–7	Meta
ht.Cp_ref	subst(material.Cp,ht.solid1.minput_pressure,ht.pref,ht.solid1.minput_temperature,ht.Tref)		J/(kg·K)	Specific heat capacity	Domains 1–4	Meta
ht.Cp_ref	subst(material.Cp,ht.solid1.minput_pressure,ht.pref,ht.solid1.minput_temperature,ht.Tref)		J/(kg·K)	Specific heat capacity	Domains 5–7	Meta
ht.helem	h_spatial		m	Element size	Domains 1–7	

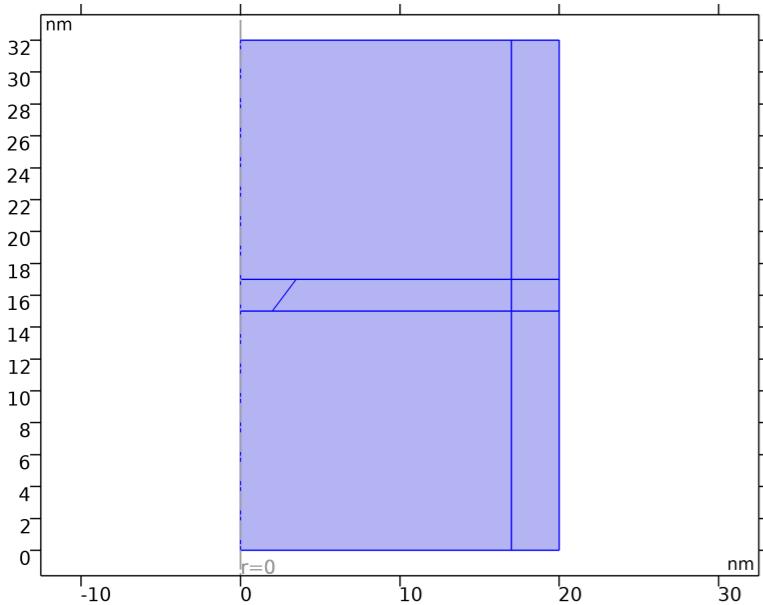
Shape functions

Name	Shape function	Unit	Description	Shape frame	Selection
T	Lagrange (Quadratic)	K	Temperature	Spatial	Domains 1–4
T	Lagrange (Quadratic)	K	Temperature	Spatial	Domains 5–7

Weak Expressions

Weak expression	Integration order	Integration frame	Selection
2*ht.streamline*pi*r	4	Spatial	Domains 1–7

SupplementaryMaterialI			
$2*(ht.dfluxr*test(Tr)+ht.dfluxz*test(Tz))*ht.d*pi*r$	4	Spatial	Domains 1-4
$2*(ht.dfluxr*test(ic1.T11*Tr+ie1.T13*Tz)+ht.dfluxphi*test(ic1.T21*Tr+ie1.T23*Tz)+ht.dfluxz*test(ic1.T31*Tr+ie1.T33*Tz))*ht.d*pi*ie1.r*ie1.detInvT$	4	Spatial	Domains 5-7
$-2*ht.C_eff*(ht.ur*Tr+ht.uz*Tz)*test(T)*ht.d*pi*r$	4	Spatial	Domains 1-4
$-2*ht.C_eff*(ht.ur*(ie1.T11*Tr+ie1.T13*Tz)+ht.uphi*(ie1.T21*Tr+ie1.T23*Tz)+ht.uz*(ie1.T31*Tr+ie1.T33*Tz))*test(T)*ht.d*pi*ie1.r*ie1.detInvT$	4	Spatial	Domains 5-7

2.6.4. Initial Values*Initial Values*

Selection

Geometric entity level	Domain
Selection	Geometry geom2: Dimension 2: All domains

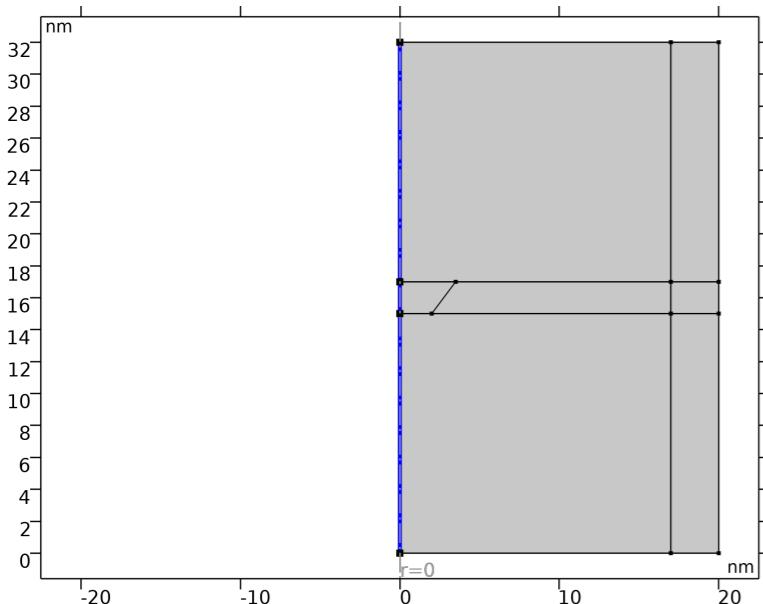
Initial Values

Settings

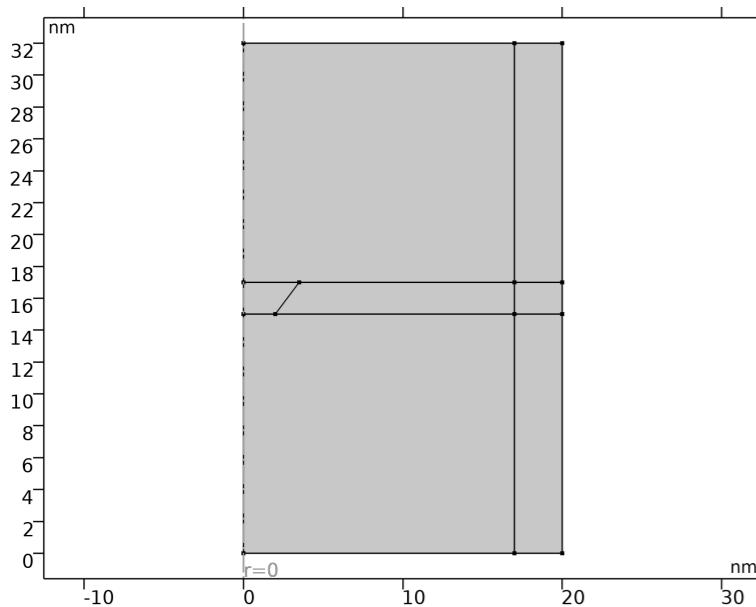
Description	Value
Temperature	User defined
Temperature	tref

Variables

Name	Expression	Unit	Description	Selection
ht.Tinit	tref	K	Initial temperature	Domains 1-4
ht.Tinit	tref	K	Initial temperature	Domains 5-7

2.6.5. Axial Symmetry

Selection	
Geometric entity level	Boundary
Selection	Geometry geom2: Dimension 1: All boundaries

2.6.6. Thermal Insulation*Thermal Insulation*

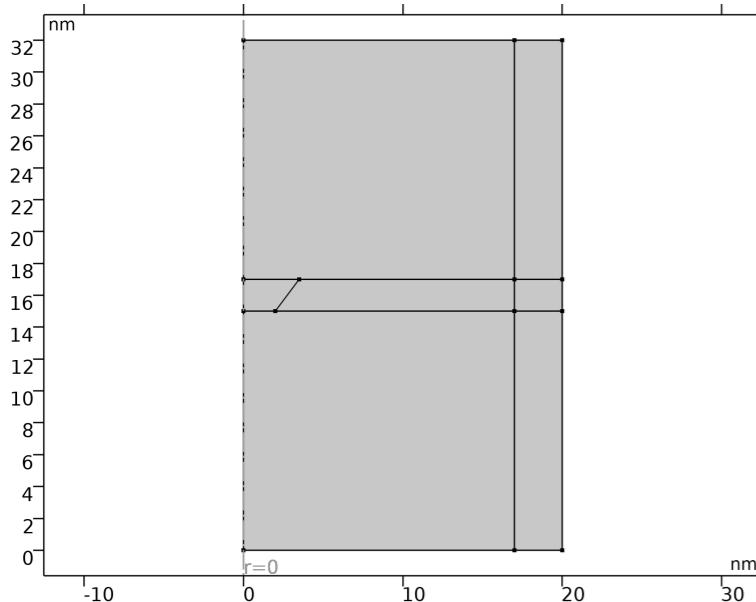
Selection	
Geometric entity level	Boundary
Selection	Geometry geom2: Dimension 1: All boundaries

Equations

$$-\mathbf{n} \cdot \mathbf{q} = 0.$$

Shape functions

Name	Shape function	Unit	Description	Shape frame	Selection	Details
T	Lagrange (Quadratic)	K	Temperature	Spatial	No boundaries	Slit
T	Lagrange (Quadratic)	K	Temperature	Material	No boundaries	Slit
T	Lagrange (Quadratic)	K	Temperature	Geometry	No boundaries	Slit
T	Lagrange (Quadratic)	K	Temperature	Mesh	No boundaries	Slit

2.6.7. Isothermal Domain Interface*Isothermal Domain Interface*

Selection	

Geometric entity level	Boundary
Selection	Geometry geom2: Dimension 1: All boundaries

Equations

$$-\mathbf{n} \cdot \mathbf{q} = 0.$$

Isothermal Domain Interface

Settings

Description	Value
Interface type	Thermal insulation

Constraint Settings

Settings

Description	Value
Apply reaction terms on	All physics (symmetric)
Use weak constraints	Off
Constraint method	Elemental

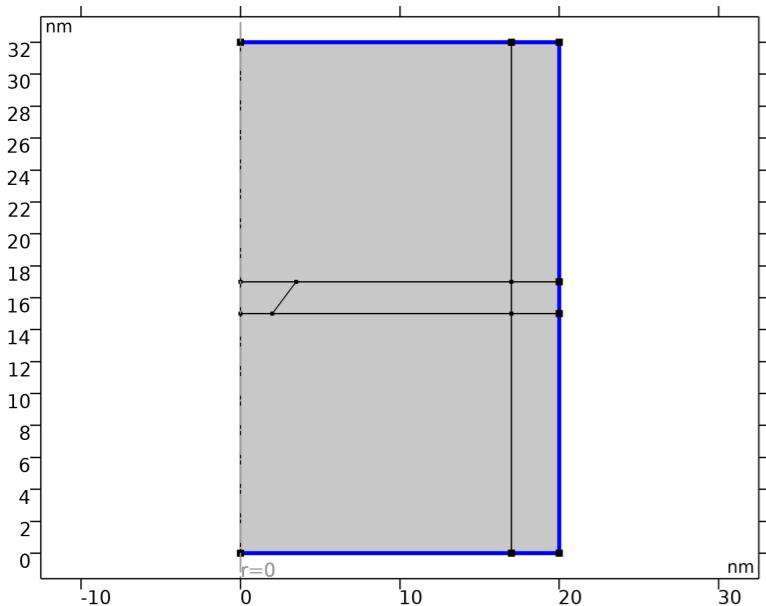
Coordinate System Selection

Settings

Description	Value
Coordinate system	Global coordinate system

Shape functions

Name	Shape function	Unit	Description	Shape frame	Selection	Details
T	Lagrange (Quadratic)	K	Temperature	Spatial	No boundaries	Slit
T	Lagrange (Quadratic)	K	Temperature	Material	No boundaries	Slit
T	Lagrange (Quadratic)	K	Temperature	Geometry	No boundaries	Slit
T	Lagrange (Quadratic)	K	Temperature	Mesh	No boundaries	Slit

2.6.8. Temperature**Temperature**

Selection

Geometric entity level	Boundary
Selection	Geometry geom2: Dimension 1: Boundaries 2, 7, 12, 17–20

Equations

$$T = T_0$$

Temperature

Settings

Description	Value
Temperature	User defined

Temperature | tref

Constraint Settings

Settings

Description	Value
	Classic constraints
Apply reaction terms on	All physics (symmetric)
Use weak constraints	Off
Constraint method	Elemental

Variables

Name	Expression	Unit	Description	Selection	Details
ht.temp1.T0	tref	K	Temperature	Boundaries 2, 7	+ operation
ht.temp1.T0	tref	K	Temperature	Boundaries 12, 17–20	+ operation
ht.temp1.ntfluxInt	ht.temp1.intExtBnd(ht.ntflux*ht.varIntSpa)+ht.temp1.intIntBnd(ht.ncflux_u*up(ht.varIntSpa)+ht.ncflux_d*down(ht.varIntSpa))	W	Total net heat rate	Global	
ht.temp1.ntfluxInt_cst	ht.temp1.intExtBnd(ht.ntflux*ht.varIntSpa)+ht.temp1.intIntBnd(ht.ncflux_u*up(ht.varIntSpa)+ht.ncflux_d*down(ht.varIntSpa))	W	Total net heat rate, constant material properties	Global	
ht.temp1.ntrfluxInt	ht.temp1.intExtBnd(ht.ntrflux*ht.varIntSpa)+ht.temp1.intIntBnd(ht.ntrflux_u*up(ht.varIntSpa)+ht.ntrflux_d*down(ht.varIntSpa))	W	Total net energy rate	Global	
ht.temp1.ntrfluxInt_cst	ht.temp1.intExtBnd(ht.ntrflux_cst*ht.varIntSpa)+ht.temp1.intIntBnd(ht.ntrflux_cst_u*up(ht.varIntSpa)+ht.ntrflux_cst_d*down(ht.varIntSpa))	W	Total net energy rate, constant material properties	Global	
ht.temp1.ntfluxInt_u	ht.temp1.intIntBnd(ht.ntflux_u*up(ht.varIntSpa))	W	Total net heat rate, upside	Global	
ht.temp1.ntfluxInt_cst_u	ht.temp1.intIntBnd(ht.ntflux_cst_u*up(ht.varIntSpa))	W	Total net heat rate, constant material properties, upside	Global	
ht.temp1.ntrfluxInt_u	ht.temp1.intIntBnd(ht.ntrflux_u*up(ht.varIntSpa))	W	Total net energy rate, upside	Global	
ht.temp1.ntrfluxInt_cst_u	ht.temp1.intIntBnd(ht.ntrflux_cst_u*up(ht.varIntSpa))	W	Total net energy rate, constant material properties, upside	Global	
ht.temp1.ntfluxInt_d	ht.temp1.intIntBnd(ht.ntflux_d*down(ht.varIntSpa))	W	Total net heat rate, downside	Global	
ht.temp1.ntfluxInt_cst_d	ht.temp1.intIntBnd(ht.ntflux_d*down(ht.varIntSpa))	W	Total net heat rate, constant material properties, downside	Global	
ht.temp1.ntrfluxInt_d	ht.temp1.intIntBnd(ht.ntrflux_d*down(ht.varIntSpa))	W	Total net energy rate, downside	Global	
ht.temp1.ntrfluxInt_cst_d	ht.temp1.intIntBnd(ht.ntrflux_cst_d*down(ht.varIntSpa))	W	Total net energy rate, constant material properties, downside	Global	
ht.temp1.Tave	nojac(ht.temp1.intBnd(ht.varIntSpa*ht.rho*ht.Cp*T*max(abs(ht.ur*ht.nrmesh+ht.uphi*ht.nphimesh+ht.uz*ht.nzmesh),eps))/nojac(ht.temp1.intBnd(ht.varIntSpa*ht.rho*ht.Cp*max(abs(ht.ur*ht.nrmesh+ht.uphi*ht.nphimesh+ht.uz*ht.nzmesh),eps)))	K	Weighted average temperature	Global	

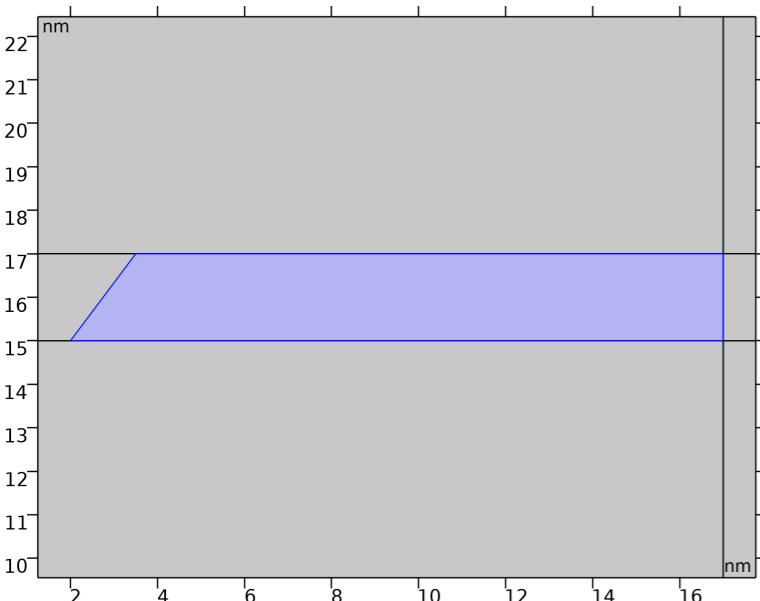
Constraints

Constraint	Constraint force	Shape function	Selection	Details
ht.temp1.T0-h.tvar	test(ht.temp1.T0-h.tvar)	Lagrange (Quadratic)	Boundaries 2, 7	Elemental
ht.temp1.T0-h.tvar	test(ht.temp1.T0-h.tvar)	Lagrange (Quadratic)	Boundaries 12, 17–20	Elemental

2.7. Schrödinger Equation

Used products

COMSOL Multiphysics
Semiconductor Module



Selection

Geometric entity level	Domain
Selection	Geometry geom2: Dimension 2: Domain 4

Equations

$$\sum_{n=1}^N \mathbf{H}_m \psi_n^{(i)}(\mathbf{r}) = E \psi_m^{(i)}(\mathbf{r}), \quad \mathbf{H}_{mm} + V_{mm} = \psi_m^{(i)}(\mathbf{r}), \quad \psi_m^{(i)}(\mathbf{r}, t) = \psi_m^{(i)}(\mathbf{r}) e^{i E_i t / \hbar}, \quad E_i = \lambda_{\text{scale}} \lambda_i, \quad i = \text{eigenvalue index}$$

$m = 1, 2, 3, \dots, N$, $N = \text{Number of wave function components}$

$\psi_n(r, \phi, z) = \psi_n(r, z) e^{-im\phi}$, $n = 1, 2, 3, \dots, N$, $N = \text{Number of wave function components}$

2.7.1. Interface Settings

Discretization

Settings

Description	Value
Element order	Quadratic

Settings

Description	Value
Equation form	Study controlled

Settings

Description	Value
Particle type	Electrons
Azimuthal quantum number	mm
Eigenvalue scale	1[eV]
Energy	1[eV]
Charge number	Automatic

2.7.2. Variables

Name	Expression	Unit	Description	Selection	Details
schr.V_psi	schr.V	J	Potential energy	Domain 4	+ operation
schr.nR	unR	1	Normal vector, R component	Boundaries 8–9, 13	
schr.nPHI	0	1	Normal vector, PHI component	Boundaries 8–9, 13	
schr.nZ	unZ	1	Normal vector, Z component	Boundaries 8–9, 13	
schr.nR	dnR	1	Normal vector, R component	Boundary 10	
schr.nPHI	0	1	Normal vector, PHI component	Boundary 10	
schr.nZ	dnZ	1	Normal vector, Z component	Boundary 10	
schr.rR	R	m	Position vector, R component	Domain 4	
schr.rPHI	0	m	Position vector, PHI component	Domain 4	
schr.rZ	Z	m	Position vector, Z component	Domain 4	
schr.zq	model.input.zq	1	Charge number	Domain 4	Meta
schr.q	-e_const	C	Charge of particle	Domain 4	
schr.m	mm	1	Azimuthal quantum number	Global	
schr.lambda_scale	1[eV]	J	Eigenvalue scale	Global	
schr.E	1[eV]	J	Energy	Global	
schr.Npsi	1	1	Number of wave function components	Global	
schr.vol_unit	1[m^3]	m³	Volume unit	Global	
schr.Pr	schr.Pr_psi	1/m³	Probability density	Domain 4	+ operation
schr.norm_fac2	schr.norm_fac2_psi	1	Normalization factor	Domain 4	+ operation
schr.norm_fac2_psi	schr.int(2*realdot(psi, psi)*pi*R)/schr.vol_unit	1	Normalization factor	Global	Meta
schr.Psi_psi	schr.norm_fac*psi	1	Normalized wave function	Domain 4	
schr.Pr_psi	conj(schr.Psi_psi)*schr.Psi_psi/schr.vol_unit	1/m³	Probability density	Domain 4	
schr.norm_fac	1/sqrt(schr.norm_fac2)	1	Normalization factor	Global	
schr.Psi	schr.norm_fac*psi	1	Normalized wave	Domain 4	

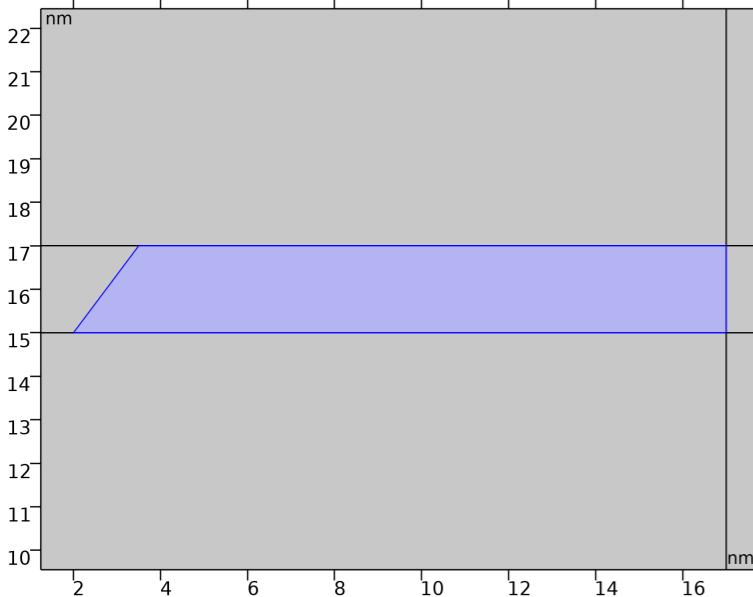
			function	
schr.Ei	schr.lambda_scale*lambda		J	Eigenenergy
schr.plot_fac	sqrt(schr.max(schr.Pr*schr.vol_unit))		I	Plot factor
schr.r_avR	schr.r_av_psiR		m	Expectation value of position, R component
schr.r_avPHI	schr.r_av_psiPHI		m	Expectation value of position, PHI component
schr.r_avZ	schr.r_av_psiZ		m	Expectation value of position, Z component
schr.pR	schr.p_psiR		kg/(m^2·s)	Canonical momentum density, R component
schr.pPHI	schr.p_psiPHI		kg/(m^2·s)	Canonical momentum density, PHI component
schr.pZ	schr.p_psiZ		kg/(m^2·s)	Canonical momentum density, Z component
schr.p_avR	schr.p_av_psiR		N·s	Expectation value of momentum, R component
schr.p_avPHI	schr.p_av_psiPHI		N·s	Expectation value of momentum, PHI component
schr.p_avZ	schr.p_av_psiZ		N·s	Expectation value of momentum, Z component
schr.L_avR	schr.int(2*(schr.p_psiZ*schr.rPHI*schr.p_psiPHI*schr.rZ)*pi*R)*l[m^3]/schr.vol_unit		J·s	Canonical angular momentum, R component
schr.L_avPHI	schr.int(2*(-schr.p_psiZ*schr.rR+schr.p_psiR*schr.rZ)*pi*R)*l[m^3]/schr.vol_unit		J·s	Canonical angular momentum, PHI component
schr.L_avZ	schr.int(2*(schr.p_psiPHI*schr.rR*schr.p_psiR*schr.rPHI)*pi*R)*l[m^3]/schr.vol_unit		J·s	Canonical angular momentum, Z component
schr.L2_av	(schr.int(2*(realdot(d(schr.Psi_psi,Z)*i*hbar_const*schr.rPHI,d(schr.Psi_psi,Z)*i*hbar_const*schr.rPHI)+realdot(i*hbar_const*(-d(schr.Psi_psi,Z)*schr.rR+d(schr.Psi_psi,R)*schr.rZ),i*hbar_const*(-d(schr.Psi_psi,Z)*schr.rR+d(schr.Psi_psi,R)*schr.rZ))+realdot(-d(schr.Psi_psi,R)*i*hbar_const*schr.rPHI,-d(schr.Psi_psi,R)*i*hbar_const*schr.rPHI)*pi*R)+schr.int(2*realdot(schr.Psi_psi,(1+(Z/R)^2)*schr.Psi_psi*(hbar_const*schr.m)^2*pi*R))/schr.vol_unit		kg^2·m^4/s^2	Canonical angular momentum
schr.E_av	schr.Ei		J	Expectation value of energy
schr.r_av_psiR	schr.int(2*realdot(psi,schr.rR*psi)*pi*R/schr.norm_fac2)/schr.vol_unit		m	Expectation value of position, R component
schr.r_av_psiPHI	schr.int(2*realdot(psi,schr.rPHI*psi)*pi*R/schr.norm_fac2)/schr.vol_unit		m	Expectation value of position, PHI component
schr.r_av_psiZ	schr.int(2*realdot(psi,schr.rZ*psi)*pi*R/schr.norm_fac2)/schr.vol_unit		m	Expectation value of position, Z component
schr.p_av_psiR	schr.int(2*schr.p_psiR*pi*R)*l[m^3]/schr.vol_unit		N·s	Expectation value of momentum, R component
schr.p_av_psiPHI	schr.int(2*schr.p_psiPHI*pi*R)*l[m^3]/schr.vol_unit		N·s	Expectation value of momentum, PHI component
schr.p_av_psiZ	schr.int(2*schr.p_psiZ*pi*R)*l[m^3]/schr.vol_unit		N·s	Expectation value of momentum, Z component
schr.p_psiR	realdot(schr.Psi_psi,i*hbar_const*d(schr.Psi_psi,R))/l[m^3]		kg/(m^2·s)	Canonical momentum density, R component
schr.p_psiPHI	(realdot(schr.Psi_psi,0)+realdot(schr.Psi_psi,hbar_const*schr.m*schr.Psi_psi/R))/l[m^3]		kg/(m^2·s)	Canonical momentum density, PHI component
schr.p_psiZ	realdot(schr.Psi_psi,i*hbar_const*d(schr.Psi_psi,Z))/l[m^3]		kg/(m^2·s)	Canonical momentum density, Z component
schr.invmeff_psiRR	(schr.meff_psiPHIPHI*schr.meff_psiZZ*schr.meff_psiPHIZ*schr.meff_psiZPHI)/(schr.meff_psiRR*schr.meff_psiPHIPHI*schr.meff_psiPHIZ*schr.meff_psiRPHI*schr.meff_psiPHIZ*schr.meff_psiZR+schr.meff_psiPHIR*schr.meff_psiZPHI-schr.meff_psiRR*schr.meff_psiPHIZ*schr.meff_psiZPHI-schr.meff_psiRPHI*schr.meff_psiPHIR*schr.meff_psiZR)		1/kg	Inverse effective mass, RR component
schr.invmeff_psiPHIR	(schr.meff_psiPHIZ*schr.meff_psiZR*schr.meff_psiPHIR*schr.meff_psiZZ)/(schr.meff_psiRR*schr.meff_psiPHIPHI*schr.meff_psiZZ+schr.meff_psiRPHI*schr.meff_psiPHIZ*schr.meff_psiZR+schr.meff_psiPHIR*schr.meff_psiZPHI-schr.meff_psiRR*schr.meff_psiPHIZ*schr.meff_psiZPHI-schr.meff_psiRPHI*schr.meff_psiPHIR*schr.meff_psiZR)		1/kg	Inverse effective mass, PHIR component
schr.invmeff_psiIZR	(schr.meff_psiPHIR*schr.meff_psiZPHI*schr.meff_psiPHIPHI*schr.meff_psiZR)/(schr.meff_psiRR*schr.meff_psiPHIPHI*schr.meff_psiZZ+schr.meff_psiPHIZ*schr.meff_psiZR+schr.meff_psiPHIR*schr.meff_psiZPHI-schr.meff_psiRR*schr.meff_psiPHIZ*schr.meff_psiZPHI-schr.meff_psiRPHI*schr.meff_psiPHIR*schr.meff_psiZR)		1/kg	Inverse effective mass, ZR component
schr.invmeff_psiRPHI	(schr.meff_psiRZ*schr.meff_psiZPHI*schr.meff_psiPHIPHI*schr.meff_psiZZ)/(schr.meff_psiRR*schr.meff_psiPHIPHI*schr.meff_psiPHIZ*schr.meff_psiRPHI*schr.meff_psiPHIZ*schr.meff_psiZR+schr.meff_psiPHIR*schr.meff_psiZPHI-schr.meff_psiRR*schr.meff_psiPHIZ*schr.meff_psiZPHI-schr.meff_psiRPHI*schr.meff_psiPHIR*schr.meff_psiZR)		1/kg	Inverse effective mass, RPHI component
schr.invmeff_psiPHIPHI	(schr.meff_psiRR*schr.meff_psiZPHI*schr.meff_psiPHIPHI*schr.meff_psiPHIZ*schr.meff_psiRPHI*schr.meff_psiPHIPHI*schr.meff_psiPHIPHI*schr.meff_psiZR)/(schr.meff_psiRR*schr.meff_psiPHIPHI*schr.meff_psiPHIPHI*schr.meff_psiPHIPHI*schr.meff_psiPHIPHI*schr.meff_psiZR)		1/kg	Inverse effective mass, PHIPHI component
schr.invmeff_psiZPHI	(schr.meff_psiRR*schr.meff_psiZPHI*schr.meff_psiPHIPHI*schr.meff_psiPHIPHI*schr.meff_psiPHIPHI*schr.meff_psiPHIPHI*schr.meff_psiPHIPHI*schr.meff_psiZR)/(schr.meff_psiRR*schr.meff_psiPHIPHI*schr.meff_psiPHIPHI*schr.meff_psiPHIPHI*schr.meff_psiPHIPHI*schr.meff_psiZR)		1/kg	Inverse effective mass, ZPHI component
schr.invmeff_psiRZ	(schr.meff_psiRPHI*schr.meff_psiPHIPHI*schr.meff_psiPHIPHI*schr.meff_psiPHIPHI*schr.meff_psiPHIPHI*schr.meff_psiPHIPHI*schr.meff_psiPHIPHI*schr.meff_psiZR)/(schr.meff_psiRR*schr.meff_psiPHIPHI*schr.meff_psiPHIPHI*schr.meff_psiPHIPHI*schr.meff_psiPHIPHI*schr.meff_psiZR)		1/kg	Inverse effective mass, RZ component
schr.invmeff_psiPHIZ	(schr.meff_psiRZ*schr.meff_psiPHIPHI*schr.meff_psiPHIPHI*schr.meff_psiPHIPHI*schr.meff_psiPHIPHI*schr.meff_psiPHIPHI*schr.meff_psiPHIPHI*schr.meff_psiZR)/(schr.meff_psiRR*schr.meff_psiPHIPHI*schr.meff_psiPHIPHI*schr.meff_psiPHIPHI*schr.meff_psiPHIPHI*schr.meff_psiZR)		1/kg	Inverse effective mass, PHIZ component

schr.invmeff_psiZZ	(schr.meff_psiRR*schr.meff_psiPHI*schr.meff_psiRPHI*schr.meff_psiPHIR)/(schr.meff_psiRR*schr.meff_psiPHI*schr.meff_psiZZ+schr.meff_psiRPHI*schr.meff_psiPHIZ*schr.meff_psiZR+schr.meff_psiRZ*schr.meff_psiPHIR*schr.meff_psiZPHI-schr.meff_psiRR*schr.meff_psiPHIZ*schr.meff_psiZPHI*schr.meff_psiRPHI*schr.meff_psiZZ*schr.meff_psiRZ*schr.meff_psiPHI*schr.meff_psiZR)	1/kg	Inverse effective mass, ZZ component	Domain 4
schr.ninvmeff_psi	(schr.nR*schr.invmeff_psiRR+schr.nPHI*schr.invmeff_psiPHI+schr.nZ*schr.invmeff_psiZR)*schr.nR+(schr.nR*schr.invmeff_psiRPHI+schr.nPHI*schr.invmeff_psiPHI*schr.nZ*schr.invmeff_psiZPHI)*schr.nPHI+(schr.nR*schr.invmeff_psiRZ+schr.nPHI*schr.invmeff_psiPHIZ+schr.nZ*schr.invmeff_psiZZ)*schr.nZ	1/kg	Inverse effective mass on boundary	Boundaries 8–10, 13
schr.invmeffRR	(schr.meffPHIPHI*schr.meffZZ*schr.meffPHIZ*schr.meffZPHI)/(schr.meffRR*schr.meffPHI*schr.meffZZ+schr.meffRPHI*schr.meffPHIZ*schr.meffZR+schr.meffRPHI*schr.meffRR*schr.meffPHI*schr.meffZPHI-schr.meffRPHI*schr.meffPHI*schr.meffZZ*schr.meffRPHI*schr.meffZR)	1/kg	Inverse effective mass, RR component	Domain 4
schr.invmeffPHIR	(schr.meffPHIZ*schr.meffZR*schr.meffPHIR*schr.meffZZ)/(schr.meffRR*schr.meffPHI*schr.meffZZ+schr.meffRPHI*schr.meffPHIZ*schr.meffZR+schr.meffPHIR*schr.meffZPHI-schr.meffRPHI*schr.meffPHI*schr.meffZZ*schr.meffRPHI*schr.meffZR)	1/kg	Inverse effective mass, PHIR component	Domain 4
schr.invmeffZR	(schr.meffPHIR*schr.meffZPHI*schr.meffPHI*schr.meffZR)/(schr.meffRR*schr.meffPHI*schr.meffZZ+schr.meffRPHI*schr.meffPHIZ*schr.meffZR+schr.meffPHIR*schr.meffZPHI-schr.meffRPHI*schr.meffPHI*schr.meffZZ*schr.meffRPHI*schr.meffZR)	1/kg	Inverse effective mass, ZR component	Domain 4
schr.invmeffRPHI	(schr.meffRZ*schr.meffZPHI*schr.meffRPHI*schr.meffZZ)/(schr.meffRR*schr.meffPHI*schr.meffZZ+schr.meffRPHI*schr.meffPHIZ*schr.meffZR+schr.meffPHIR*schr.meffZPHI-schr.meffRPHI*schr.meffPHI*schr.meffZZ*schr.meffRPHI*schr.meffZR)	1/kg	Inverse effective mass, RPHI component	Domain 4
schr.invmeffPHIPHI	(schr.meffRR*schr.meffZZ*schr.meffRPHI*schr.meffZR)/(schr.meffRR*schr.meffPHI*schr.meffZZ+schr.meffRPHI*schr.meffPHIZ*schr.meffZR+schr.meffPHIR*schr.meffZPHI-schr.meffRPHI*schr.meffPHI*schr.meffZZ*schr.meffRPHI*schr.meffZR)	1/kg	Inverse effective mass, PHIPHI component	Domain 4
schr.invmeffZPHI	(schr.meffRPHI*schr.meffZR*schr.meffZPHI)/(schr.meffRR*schr.meffPHI*schr.meffZZ+schr.meffRPHI*schr.meffPHIZ*schr.meffZR+schr.meffPHIR*schr.meffZPHI-schr.meffRPHI*schr.meffPHI*schr.meffZZ*schr.meffRPHI*schr.meffZR)	1/kg	Inverse effective mass, ZPHI component	Domain 4
schr.invmeffRZ	(schr.meffRPHI*schr.meffPHIZ*schr.meffRPHI*schr.meffZZ)/(schr.meffRR*schr.meffPHI*schr.meffZZ+schr.meffRPHI*schr.meffPHIZ*schr.meffZR+schr.meffPHIR*schr.meffZPHI-schr.meffRPHI*schr.meffPHI*schr.meffZZ*schr.meffRPHI*schr.meffZR)	1/kg	Inverse effective mass, RZ component	Domain 4
schr.invmeffPHIZ	(schr.meffRZ*schr.meffPHIR*schr.meffRR*schr.meffPHIZ)/(schr.meffRR*schr.meffPHI*schr.meffZZ+schr.meffRPHI*schr.meffPHIZ*schr.meffZR+schr.meffPHIR*schr.meffZPHI-schr.meffRPHI*schr.meffPHI*schr.meffZZ*schr.meffRPHI*schr.meffZR)	1/kg	Inverse effective mass, PHIZ component	Domain 4
schr.invmeffZZ	(schr.meffRR*schr.meffPHI*schr.meffRPHI*schr.meffPHIR)/(schr.meffRR*schr.meffPHI*schr.meffZZ+schr.meffRPHI*schr.meffPHIZ*schr.meffZR+schr.meffPHIR*schr.meffZPHI-schr.meffRPHI*schr.meffPHI*schr.meffZZ*schr.meffRPHI*schr.meffZR)	1/kg	Inverse effective mass, ZZ component	Domain 4
schr.ninvmeff	(schr.nR*schr.invmeffRR+schr.nPHI*schr.invmeffPHI+schr.nZ*schr.invmeffZR)*schr.nR+(schr.nR*schr.invmeffRPHI+schr.nPHI*schr.invmeffPHI*schr.nZ*schr.invmeffZPHI)*schr.nPHI+(schr.nR*schr.invmeffRZ+schr.nPHI*schr.invmeffPHIZ+schr.nZ*schr.invmeffZZ)*schr.nZ	1/kg	Inverse effective mass on boundary	Boundaries 8–10, 13
schr.omega	schr.omega/i	rad/s	Angular frequency	Global
schr.iomega	-root.lambda	rad/s	Complex angular frequency	Global
schr.freq	0.5*schr.omega/pi	Hz	Frequency	Global

2.7.3. Shape functions

Name	Shape function	Unit	Description	Shape frame	Selection
psi	Lagrange (Quadratic)	1	Wave function	Material	Domain 4

2.7.4. Effective Mass



Effective Mass

Selection	
Geometric entity level	Domain
Selection	Geometry geom2: Dimension 2: All domains

Equations

$$\mathbf{m}_{\text{eff},mm} = \mathbf{m}_{\text{eff},e,mm}, \quad \mathbf{H}_{mm} + = \frac{-\hbar^2}{2} \nabla \cdot (\mathbf{m}_{\text{eff},mm}(\mathbf{r})^{-1} \cdot \nabla)$$

$m = 1, 2, 3, \dots, N$, $N = \text{Number of wave function components}$

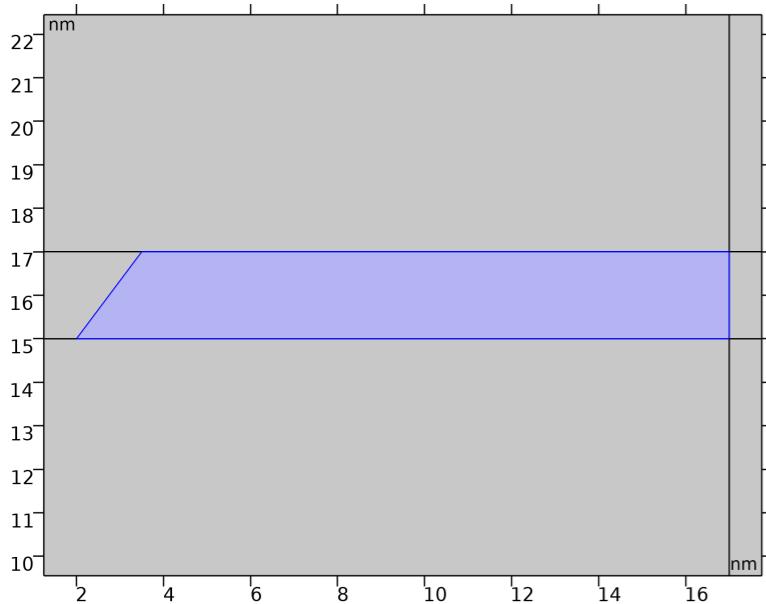
Settings

Description	Value
Electron effective mass	User defined
Electron effective mass	{ {mass_eff, 0, 0}, {0, mass_eff, 0}, {0, 0, mass_eff} }

Variables

Name	Expression	Unit	Description	Selection
schr.meff_psiRR	schr.meffe_psiRR	kg	Effective mass, RR component	Domain 4
schr.meff_psiPHIR	schr.meffe_psiPHIR	kg	Effective mass, PHIR component	Domain 4
schr.meff_psiZR	schr.meffe_psiZR	kg	Effective mass, ZR component	Domain 4
schr.meff_psiRPHI	schr.meffe_psiRPHI	kg	Effective mass, RPHI component	Domain 4
schr.meff_psiPHIPHI	schr.meffe_psiPHIPHI	kg	Effective mass, PHIPHI component	Domain 4
schr.meff_psiZPHI	schr.meffe_psiZPHI	kg	Effective mass, ZPHI component	Domain 4
schr.meff_psiRZ	schr.meffe_psiRZ	kg	Effective mass, RZ component	Domain 4
schr.meff_psiPHIZ	schr.meffe_psiPHIZ	kg	Effective mass, PHIZ component	Domain 4
schr.meff_psiZZ	schr.meffe_psiZZ	kg	Effective mass, ZZ component	Domain 4
schr.meffRR	schr.meff_psiRR	kg	Effective mass, RR component	Domain 4
schr.meffPHIR	schr.meff_psiPHIR	kg	Effective mass, PHIR component	Domain 4
schr.meffZR	schr.meff_psiZR	kg	Effective mass, ZR component	Domain 4
schr.meffRPHI	schr.meff_psiRPHI	kg	Effective mass, RPHI component	Domain 4
schr.meffPHIPHI	schr.meff_psiPHIPHI	kg	Effective mass, PHIPHI component	Domain 4
schr.meffZPHI	schr.meff_psiZPHI	kg	Effective mass, ZPHI component	Domain 4
schr.meffRZ	schr.meff_psiRZ	kg	Effective mass, RZ component	Domain 4
schr.meffPHIZ	schr.meff_psiPHIZ	kg	Effective mass, PHIZ component	Domain 4
schr.meffZZ	schr.meff_psiZZ	kg	Effective mass, ZZ component	Domain 4
schr.meffe_psiRR	mass_eff	kg	Electron effective mass, RR component	Domain 4
schr.meffe_psiPHIR	0	kg	Electron effective mass, PHIR component	Domain 4
schr.meffe_psiZR	0	kg	Electron effective mass, ZR component	Domain 4
schr.meffe_psiRPHI	0	kg	Electron effective mass, RPHI component	Domain 4
schr.meffe_psiPHIPHI	mass_eff	kg	Electron effective mass, PHIPHI component	Domain 4
schr.meffe_psiZPHI	0	kg	Electron effective mass, ZPHI component	Domain 4
schr.meffe_psiRZ	0	kg	Electron effective mass, RZ component	Domain 4
schr.meffe_psiPHIZ	0	kg	Electron effective mass, PHIZ component	Domain 4
schr.meffe_psiZZ	mass_eff	kg	Electron effective mass, ZZ component	Domain 4

2.7.5. Electron Potential Energy



Electron Potential Energy

Selection

Geometric entity level	Domain
Selection	Geometry geom2: Dimension 2: All domains

Equations

$$V_{mm} = V_{emm}, \quad m = 1, 2, \dots, N, \quad N = \text{Number of wave function components}$$

61/121

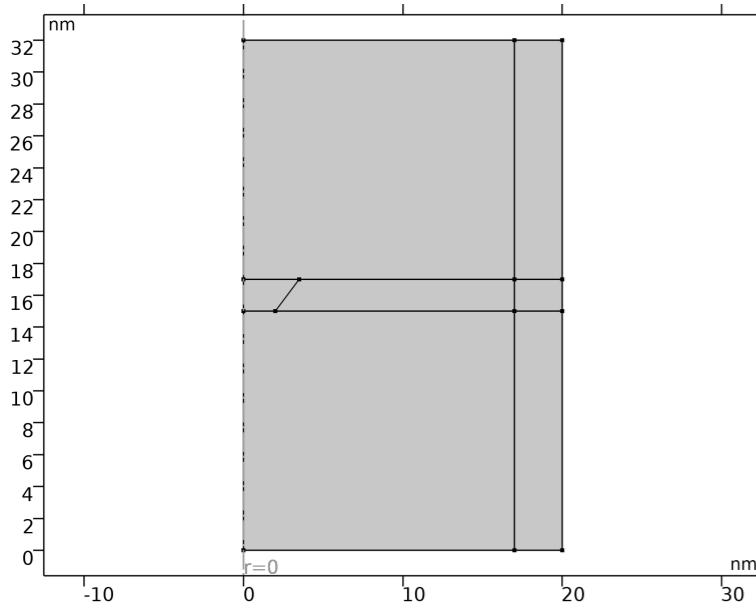
Electron Potential Energy

Settings

Description	Value
Electron potential energy	User defined
Electron potential energy	0

Variables

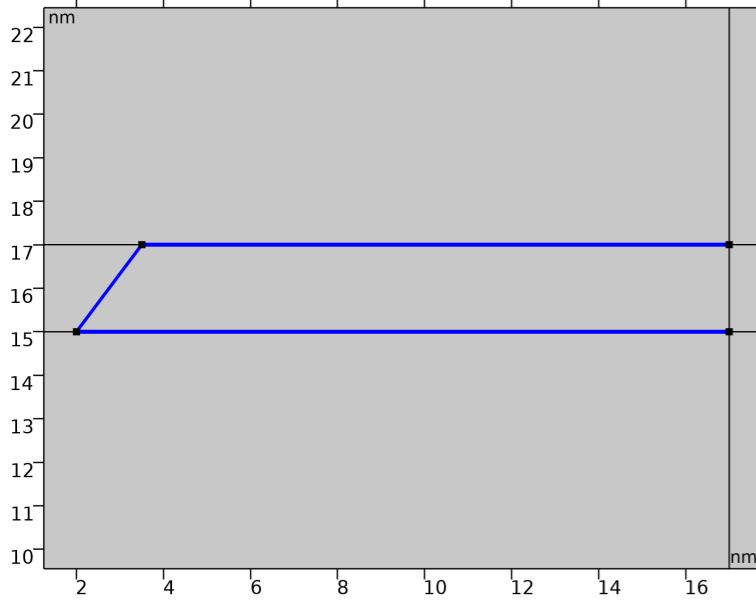
Name	Expression	Unit	Description	Selection	Details
schr.V	schr.ve1.Ve	J	Potential energy	Domain 4	+ operation
schr.ve1.Ve	0	J	Electron potential energy	Domain 4	
schr.ve1.r0R	0	m	Center of potential, R component	Domain 4	
schr.ve1.r0PHI	0	m	Center of potential, PHI component	Domain 4	
schr.ve1.r0Z	0	m	Center of potential, Z component	Domain 4	

2.7.6. Axial Symmetry

Axial Symmetry

Selection

Geometric entity level	Boundary
Selection	Geometry geom2: Dimension 1: All boundaries

2.7.7. Zero Flux

Zero Flux

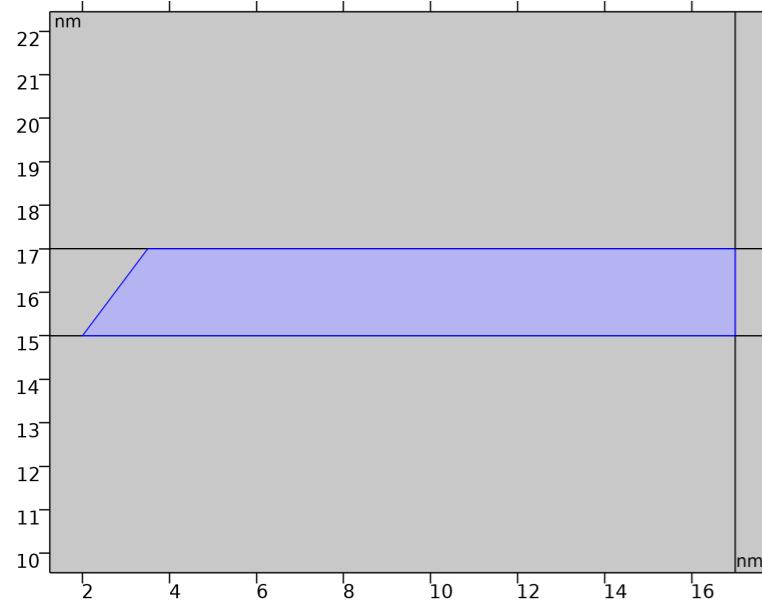
Selection

Geometric entity level	Boundary
Selection	Geometry geom2: Dimension 1: All boundaries

Equations

$$\mathbf{n} \cdot \nabla \psi = 0$$

2.7.8. Initial Values



Initial Values

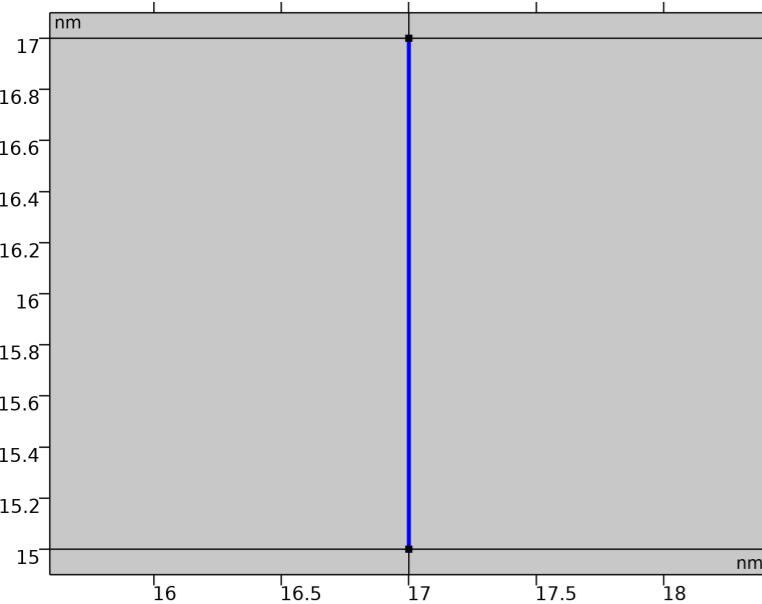
Selection

Geometric entity level	Domain
Selection	Geometry geom2: Dimension 2: All domains

Settings

Description	Value
Wave function	eps

2.7.9. Open Boundary : Plane waves



Open Boundary : Plane waves

Selection

Geometric entity level	Boundary
Selection	Geometry geom2: Dimension 1: Boundary 13

Equations

$$E = E_i$$

$$|\mathbf{k}| = \frac{1}{\hbar} \sqrt{\frac{2(E - V)}{\mathbf{n} \cdot \mathbf{m}_{\text{eff}}^{-1} \cdot \mathbf{n}}}$$

$$\mathbf{k}_{\text{eff}} = |\mathbf{k}|$$

$$\mathbf{n} \cdot \nabla \psi = -i \mathbf{k}_{\text{eff}} \psi$$

Open Boundary

Settings

Description	Value
Wave type	Plane wave
Incoming wave	Off

Open Boundary Type

Settings	
Description	Value
Open boundary type	Outgoing

Variables

Name	Expression	Unit	Description	Selection
schr.Ebnd	schr.Ei	J	Energy at boundary	Boundary 13
schr.k	if(schr.Ebnd>schr.V,sqrt(2*(schr.Ebnd-schr.V)/schr.ninvmeff)/hbar_const,0)	rad/m	Wave number at boundary	Boundary 13
schr.nk	schr.k	rad/m	Wave number at boundary	Boundary 13
schr.cos_th	if(realdot(psiR,psiR)+realdot(psiZ,psiZ))*psi==0.0,i*(psiR*schr.nR+psiZ*schr.nZ)*sqrt(realdot(psi,psi))/(psi*sqrt(realdot(psiR,psiR)+realdot(psiZ,psiZ))))	1	Direction cosine	Boundary 13

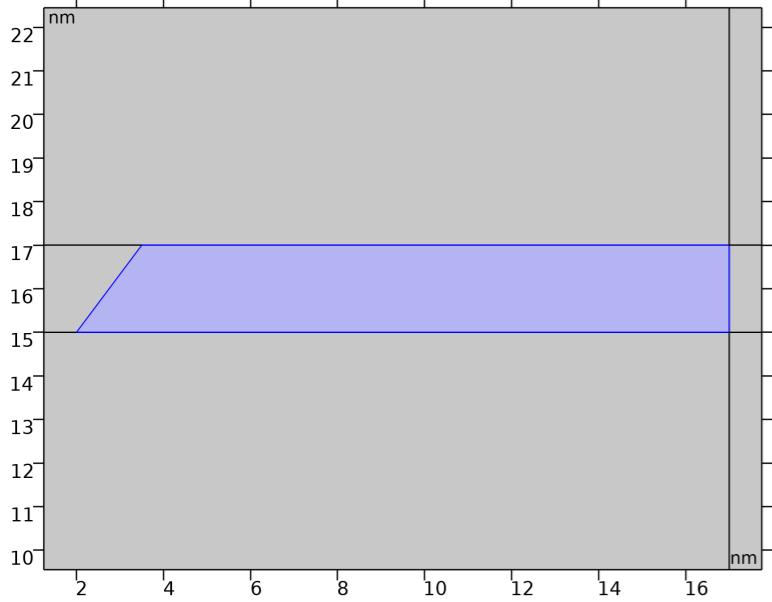
Weak Expressions

Weak expression	Integration order	Integration frame	Selection
-hbar_const^2*i*schr.nk*psi*test(psi)*schr.ninvmeff*pi*R	4	Material	Boundary 13

2.8. Electrostatics: Potential on HfO₂

Used products

COMSOL Multiphysics
AC/DC Module

Electrostatics: Potential on HfO₂

Selection

Geometric entity level	Domain
Selection	Geometry geom2: Dimension 2: Domain 4

Equations

$$\nabla \cdot \mathbf{D} = \rho_v$$

$$\mathbf{E} = -\nabla V$$

2.8.1. Interface Settings

Discretization

Settings	
Description	Value
Electric potential	Quadratic

Manual Terminal Sweep Settings

Settings	

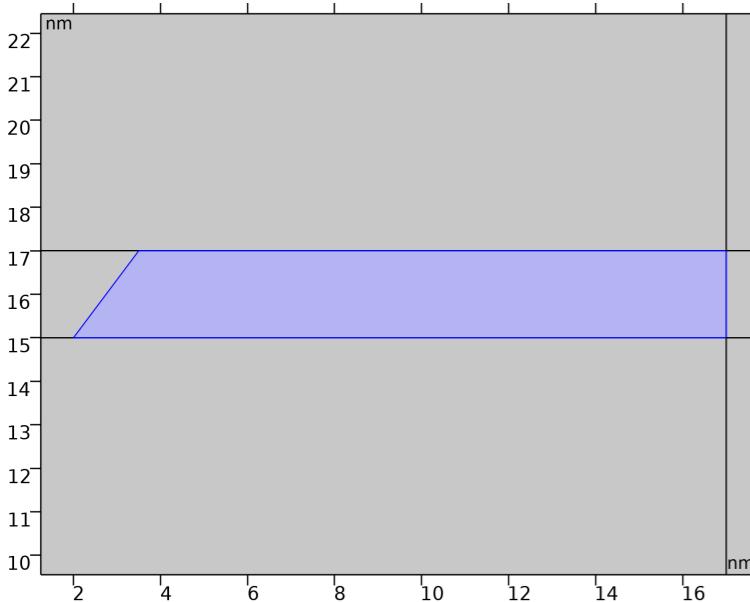
Description	Value
Use manual terminal sweep	Off
Reference impedance	50[ohm]

2.8.2. Variables

Name	Expression	Unit	Description	Selection	Details
es.d	1	1	Contribution	Domain 4	
es.nr	unr		Normal vector, r component	Boundaries 8–9, 13	
es.nphi	0		Normal vector, phi component	Boundaries 8–9, 13	
es.nz	unz		Normal vector, z component	Boundaries 8–9, 13	
es.nr	dnr		Normal vector, r component	Boundary 10	
es.nphi	0		Normal vector, phi component	Boundary 10	
es.nz	dnz		Normal vector, z component	Boundary 10	
es.nmeshr	unrmesh		Mesh normal vector, r component	Boundaries 8–9, 13	
es.nmeshphi	0		Mesh normal vector, phi component	Boundaries 8–9, 13	
es.nmeshz	unzmesh		Mesh normal vector, z component	Boundaries 8–9, 13	
es.nmeshr	dfrmesh		Mesh normal vector, r component	Boundary 10	
es.nmeshphi	0		Mesh normal vector, phi component	Boundary 10	
es.nmeshz	dnmesh		Mesh normal vector, z component	Boundary 10	
es.unmeshr	unrmesh		Mesh normal vector, upside, r component	Boundaries 8–10, 13	
es.unmeshphi	0		Mesh normal vector, upside, phi component	Boundaries 8–10, 13	
es.unmeshz	unzmesh		Mesh normal vector, upside, z component	Boundaries 8–10, 13	
es.dnmeshr	dfrmesh		Mesh normal vector, downside, r component	Boundaries 8–10, 13	
es.dnmeshphi	0		Mesh normal vector, downside, phi component	Boundaries 8–10, 13	
es.dnmeshz	dnmesh		Mesh normal vector, downside, z component	Boundaries 8–10, 13	
es.I_sRR	(spatial.invF11*(spatial.invF11*es.I_srr+spatial.invF31*es.I_srz)+spatial.invF31*(spatial.invF11*es.I_srz+spatial.invF31*es.I_szz))*spatial.detF	1	Spatial identity matrix, material frame, RR component	Domain 4	
es.I_sPHIR	if(Rg>0.001*h,R/r,Rr)*(spatial.invF11*es.I_sphir+spatial.invF31*es.I_spzphi)*spatial.detF	1	Spatial identity matrix, material frame, PHIR component	Domain 4	
es.I_sZR	(spatial.invF11*(spatial.invF13*es.I_srr+spatial.invF33*es.I_srz)+spatial.invF31*(spatial.invF13*es.I_srz+spatial.invF33*es.I_szz))*spatial.detF	1	Spatial identity matrix, material frame, ZR component	Domain 4	
es.I_sRPHI	if(Rg>0.001*h,R/r,Rr)*(spatial.invF11*es.I_srphi+spatial.invF31*es.I_szphi)*spatial.detF	1	Spatial identity matrix, material frame, RPHI component	Domain 4	
es.I_sPHIPHI	if(Rg>0.001*h,R/r,Rr)^2*es.I_sphiphi*spatial.detF	1	Spatial identity matrix, material frame, PHIPHI component	Domain 4	
es.I_sZPHI	if(Rg>0.001*h,R/r,Rr)*(spatial.invF13*es.I_srphi+spatial.invF33*es.I_szphi)*spatial.detF	1	Spatial identity matrix, material frame, ZPHI component	Domain 4	
es.I_sRZ	(spatial.invF13*(spatial.invF11*es.I_srr+spatial.invF31*es.I_srz)+spatial.invF33*(spatial.invF11*es.I_srz+spatial.invF31*es.I_szz))*spatial.detF	1	Spatial identity matrix, material frame, RZ component	Domain 4	
es.I_sPHIZ	if(Rg>0.001*h,R/r,Rr)*(spatial.invF13*es.I_sphir+spatial.invF33*es.I_spzphi)*spatial.detF	1	Spatial identity matrix, material frame, PHIZ component	Domain 4	
es.I_sZZ	(spatial.invF13*(spatial.invF13*es.I_srr+spatial.invF33*es.I_srz)+spatial.invF33*(spatial.invF13*es.I_srz+spatial.invF33*es.I_szz))*spatial.detF	1	Spatial identity matrix, material frame, ZZ component	Domain 4	
es.I_srr	1	1	Spatial identity matrix, rr component	Domain 4	
es.I_sphir	0	1	Spatial identity matrix, phir component	Domain 4	
es.I_srz	0	1	Spatial identity matrix, zr component	Domain 4	
es.I_srphi	0	1	Spatial identity matrix, rphi component	Domain 4	
es.I_sphiphi	1	1	Spatial identity matrix, phiphi component	Domain 4	
es.I_szphi	0	1	Spatial identity matrix, zphi component	Domain 4	
es.I_srz	0	1	Spatial identity matrix, rz component	Domain 4	
es.I_spzphi	0	1	Spatial identity matrix, phiz component	Domain 4	
es.I_szz	1	1	Spatial identity matrix, zz component	Domain 4	
es.unTr	es.unTer	Pa	Maxwell upward surface stress tensor, r component	Boundaries 8–10, 13	
es.unTphi	es.unTephi	Pa	Maxwell upward surface stress tensor, phi component	Boundaries 8–10, 13	
es.unTz	es.unTez	Pa	Maxwell upward surface stress tensor, z component	Boundaries 8–10, 13	
es.bnTr	es.bnTer	Pa	Maxwell downward surface stress tensor, r component	Boundaries 8–10, 13	
es.bnTphi	es.bnTephi	Pa	Maxwell downward surface stress tensor, phi component	Boundaries 8–10, 13	
es.bnTz	es.bnTez	Pa	Maxwell downward surface stress tensor, z component	Boundaries 8–10, 13	
es.unr	unr		Normal vector up direction, r component	Boundaries 8–10, 13	
es.unphi	0		Normal vector up direction, phi component	Boundaries 8–10, 13	
es.unz	unz		Normal vector up direction, z component	Boundaries 8–10, 13	
es.dnr	dnr		Normal vector down direction, r component	Boundaries 8–10,	

es.dnphi	0			Normal vector down direction, phi component	Boundaries 8–10, 13
es.dnz	dnz			Normal vector down direction, z component	Boundaries 8–10, 13
es.unTer	$-0.5 * \text{es.dnr} * (\text{real}(\text{up}(\text{es.Dr})) * \text{real}(\text{up}(\text{es.Er})) + \text{real}(\text{up}(\text{es.Dphi})) * \text{real}(\text{up}(\text{es.Ephi})) + \text{real}(\text{up}(\text{es.Dz})) * \text{real}(\text{up}(\text{es.Ez}))) + \text{real}(\text{up}(\text{es.Dr})) * (\text{real}(\text{up}(\text{es.Er})) * \text{es.dnr} + \text{real}(\text{up}(\text{es.Ephi})) * \text{es.dnphi} + \text{real}(\text{up}(\text{es.Ez})) * \text{es.dnz})$	Pa	Maxwell upward electric surface stress tensor, r component	Boundaries 8–9, 13	
es.unTeph	$-0.5 * \text{es.dnphi} * (\text{real}(\text{up}(\text{es.Dr})) * \text{real}(\text{up}(\text{es.Er})) + \text{real}(\text{up}(\text{es.Dphi})) * \text{real}(\text{up}(\text{es.Ephi})) + \text{real}(\text{up}(\text{es.Dz})) * \text{real}(\text{up}(\text{es.Ez}))) + \text{real}(\text{up}(\text{es.Dphi})) * (\text{real}(\text{up}(\text{es.Er})) * \text{es.dnr} + \text{real}(\text{up}(\text{es.Ephi})) * \text{es.dnphi} + \text{real}(\text{up}(\text{es.Ez})) * \text{es.dnz})$	Pa	Maxwell upward electric surface stress tensor, phi component	Boundaries 8–9, 13	
es.unTez	$-0.5 * \text{es.dnz} * (\text{real}(\text{up}(\text{es.Dr})) * \text{real}(\text{up}(\text{es.Er})) + \text{real}(\text{up}(\text{es.Dphi})) * \text{real}(\text{up}(\text{es.Ephi})) + \text{real}(\text{up}(\text{es.Dz})) * \text{real}(\text{up}(\text{es.Ez}))) + \text{real}(\text{up}(\text{es.Dz})) * (\text{real}(\text{up}(\text{es.Er})) * \text{es.dnr} + \text{real}(\text{up}(\text{es.Ephi})) * \text{es.dnphi} + \text{real}(\text{up}(\text{es.Ez})) * \text{es.dnz})$	Pa	Maxwell upward electric surface stress tensor, z component	Boundaries 8–9, 13	
es.unTer	0	Pa	Maxwell upward electric surface stress tensor, r component	Boundary 10	
es.unTeph	0	Pa	Maxwell upward electric surface stress tensor, phi component	Boundary 10	
es.unTez	0	Pa	Maxwell upward electric surface stress tensor, z component	Boundary 10	
es.bnTer	$-0.5 * \text{es.unr} * (\text{real}(\text{down}(\text{es.Dr})) * \text{real}(\text{down}(\text{es.Er})) + \text{real}(\text{down}(\text{es.Dphi})) * \text{real}(\text{down}(\text{es.Ephi})) + \text{real}(\text{down}(\text{es.Dz})) * \text{real}(\text{down}(\text{es.Ez}))) + \text{real}(\text{down}(\text{es.Dr})) * (\text{real}(\text{down}(\text{es.Er})) * \text{es.unr} + \text{real}(\text{down}(\text{es.Ephi})) * \text{es.unphi} + \text{real}(\text{down}(\text{es.Ez})) * \text{es.unz})$	Pa	Maxwell downward electric surface stress tensor, r component	Boundary 10	
es.bnTeph	$-0.5 * \text{es.unphi} * (\text{real}(\text{down}(\text{es.Dr})) * \text{real}(\text{down}(\text{es.Er})) + \text{real}(\text{down}(\text{es.Dphi})) * \text{real}(\text{down}(\text{es.Ephi})) + \text{real}(\text{down}(\text{es.Dz})) * \text{real}(\text{down}(\text{es.Ez}))) + \text{real}(\text{down}(\text{es.Dphi})) * (\text{real}(\text{down}(\text{es.Er})) * \text{es.unr} + \text{real}(\text{down}(\text{es.Ephi})) * \text{es.unphi} + \text{real}(\text{down}(\text{es.Ez})) * \text{es.unz})$	Pa	Maxwell downward electric surface stress tensor, phi component	Boundary 10	
es.bnTez	$-0.5 * \text{es.unz} * (\text{real}(\text{down}(\text{es.Dr})) * \text{real}(\text{down}(\text{es.Er})) + \text{real}(\text{down}(\text{es.Dphi})) * \text{real}(\text{down}(\text{es.Ephi})) + \text{real}(\text{down}(\text{es.Dz})) * \text{real}(\text{down}(\text{es.Ez}))) + \text{real}(\text{down}(\text{es.Dz})) * (\text{real}(\text{down}(\text{es.Er})) * \text{es.unr} + \text{real}(\text{down}(\text{es.Ephi})) * \text{es.unphi} + \text{real}(\text{down}(\text{es.Ez})) * \text{es.unz})$	Pa	Maxwell downward electric surface stress tensor, z component	Boundary 10	
es.bnTer	0	Pa	Maxwell downward electric surface stress tensor, r component	Boundaries 8–9, 13	
es.bnTeph	0	Pa	Maxwell downward electric surface stress tensor, phi component	Boundaries 8–9, 13	
es.bnTez	0	Pa	Maxwell downward electric surface stress tensor, z component	Boundaries 8–9, 13	
es.intWe	es.int_We(es.d*es.dWe)	J	Total electric energy	Global	+ operation
es.Cinv11	NaN+NaN*i	1/F	Inverse Maxwell capacitance, 11 component	Global	
es.Cinv21	NaN+NaN*i	1/F	Inverse Maxwell capacitance, 21 component	Global	
es.Cinv12	NaN+NaN*i	1/F	Inverse Maxwell capacitance, 12 component	Global	
es.Cinv22	NaN+NaN*i	1/F	Inverse Maxwell capacitance, 22 component	Global	
es.C11	es.Q0_1/es.V0_1	F	Maxwell capacitance, 11 component	Global	
es.C21	es.Q0_2/es.V0_1	F	Maxwell capacitance, 21 component	Global	
es.C12	es.Q0_1/es.V0_2	F	Maxwell capacitance, 12 component	Global	
es.C22	es.Q0_2/es.V0_2	F	Maxwell capacitance, 22 component	Global	
es.S11	NaN+NaN*i	S-parameter	11 component	Global	
es.S21	NaN+NaN*i	S-parameter	21 component	Global	
es.S12	NaN+NaN*i	S-parameter	12 component	Global	
es.S22	NaN+NaN*i	S-parameter	22 component	Global	
es.S11dB	$10 * \log_{10}(\text{realdot}(\text{es.S11}, \text{es.S11}))$	dB	S-parameter, dB, 11 component	Global	
es.S21dB	$10 * \log_{10}(\text{realdot}(\text{es.S21}, \text{es.S21}))$	dB	S-parameter, dB, 21 component	Global	
es.S12dB	$10 * \log_{10}(\text{realdot}(\text{es.S12}, \text{es.S12}))$	dB	S-parameter, dB, 12 component	Global	
es.S22dB	$10 * \log_{10}(\text{realdot}(\text{es.S22}, \text{es.S22}))$	dB	S-parameter, dB, 22 component	Global	
es.zref	50[ohm]	Ω	Reference impedance	Global	

2.8.3. Charge Conservation



Charge Conservation

Selection

Geometric entity level	Domain
Selection	Geometry geom2: Dimension 2: All domains

Equations

$$\mathbf{E} = -\nabla V$$

$$\nabla \cdot (\epsilon_0 \epsilon_r \mathbf{E}) = \rho_v$$

Constitutive Relation D-E

Settings

Description	Value
Dielectric model	Relative permittivity
Relative permittivity	From material

Coordinate System Selection

Settings

Description	Value
Coordinate system	Global coordinate system

Model Input

Settings

Description	Value
Temperature	Common model input

Used products

COMSOL Multiphysics

Properties from material

Property	Material	Property group
Relative permittivity	HfO2 (insulator)	Basic

Variables

Name	Expression	Unit	Description	Selection	Details
es.nD	0	C/m ²	Surface charge density	Boundaries 8–10, 13	+ operation
es.epsilonrr	material.epsilonr11	1	Relative permittivity, rr component	Domain 4	Meta
es.epsilonphir	material.epsilonr21	1	Relative permittivity, phir component	Domain 4	Meta
es.epsilonrz	material.epsilonr31	1	Relative permittivity, zr component	Domain 4	Meta
es.epsilonrrphi	material.epsilonr12	1	Relative permittivity, rphi component	Domain 4	Meta
es.epsilonphiphi	material.epsilonr22	1	Relative permittivity, phiphi component	Domain 4	Meta
es.epsilonrzphi	material.epsilonr32	1	Relative permittivity, zphi component	Domain 4	Meta
es.epsilonrrz	material.epsilonr13	1	Relative permittivity, rz component	Domain 4	Meta
es.epsilonrphiz	material.epsilonr23	1	Relative permittivity, phiz component	Domain 4	Meta
es.epsilonzz	material.epsilonr33	1	Relative permittivity, zz component	Domain 4	Meta
es.epsilonr_iso	material.epsilonr_iso	1	Relative permittivity, isotropic value	Domain 4	Meta
es.DrR	0	C/m ²	Remanent electric displacement, R component	Domain 4	
es.DrPHI	0	C/m ²	Remanent electric displacement, PHI component	Domain 4	
es.DrZ	0	C/m ²	Remanent electric displacement, Z component	Domain 4	
es.Dr	epsilon0_const*es.I_srr*es.Er+epsilon0_const*es.I_srphi*es.Ephi+epsilon0_const*es.I_srz*es.Ez+es.Pr+es.Per	C/m ²	Electric displacement field, r component	Domain 4	
es.Dphi	epsilon0_const*es.I_sphir*es.Er+epsilon0_const*es.I_sphiphi*es.Ephi+epsilon0_const*es.I_sphiz*es.Ez+es.Pphi+es.Pephi	C/m ²	Electric displacement field, phi component	Domain 4	
es.Dz	epsilon0_const*es.I_szr*es.Er+epsilon0_const*es.I_szphi*es.Ephi+epsilon0_const*es.I_szz*es.Ez+es.Pz+es.Pez	C/m ²	Electric displacement field, z component	Domain 4	
es.Pr	epsilon0_const*(es.chirr*es.Er+es.chirphi*es.Ephi+es.chirz*es.Ez)	C/m ²	Polarization, r component	Domain 4	
es.Pphi	epsilon0_const*(es.chiphir*es.Er+es.chiphiphi*es.Ephi+es.chiphiz*es.Ez)	C/m ²	Polarization, phi component	Domain 4	
es.Pz	epsilon0_const*(es.chizr*es.Er+es.chizphi*es.Ephi+es.chizz*es.Ez)	C/m ²	Polarization, z component	Domain 4	
es.normD	sqrt(realdot(es.Dr,es.Dr)+realdot(es.Dphi,es.Dphi)+realdot(es.Dz,es.Dz))	C/m ²	Electric displacement field norm	Domain 4	
es.normP	sqrt(realdot(es.Pr,es.Pr)+realdot(es.Pphi,es.Pphi)+realdot(es.Pz,es.Pz))	C/m ²	Polarization norm	Domain 4	
es.Per	0	C/m ²	Polarization contribution, r component	Domain 4	+ operation
es.Pephi	0	C/m ²	Polarization contribution, phi component	Domain 4	+ operation
es.Pez	0	C/m ²	Polarization contribution, z component	Domain 4	+ operation

es.chirr	-1+es.epsilonrrr		1	Electric susceptibility, rr component	Domain 4
es.chiphir	es.epsilonphir		1	Electric susceptibility, phir component	Domain 4
es.chizr	es.epsilonrzr		1	Electric susceptibility, zr component	Domain 4
es.chirphi	es.epsilonrrphi		1	Electric susceptibility, rphi component	Domain 4
es.chiphipi	-1+es.epsilonrphipi		1	Electric susceptibility, phipi component	Domain 4
es.chizphi	es.epsilonrzphi		1	Electric susceptibility, zphi component	Domain 4
es.chirz	es.epsilonrrz		1	Electric susceptibility, rz component	Domain 4
es.chiphiz	es.epsilonrphiz		1	Electric susceptibility, phiz component	Domain 4
es.chizz	-1+es.epsilonrzz		1	Electric susceptibility, zz component	Domain 4
es.Er	-VQMr		V/m	Electric field, r component	Domain 4
es.Ephi	0		V/m	Electric field, phi component	Domain 4
es.Ez	-VQMz		V/m	Electric field, z component	Domain 4
es.tEr	-VQMTr		V/m	Tangential electric field, r component	Boundaries 8–10, 13
es.tEphi	0		V/m	Tangential electric field, phi component	Boundaries 8–10, 13
es.tEz	-VQMTz		V/m	Tangential electric field, z component	Boundaries 8–10, 13
es.normE	sqrt(realdot(es.Er,es.Er)+realdot(es.Ephi,es.Ephi)+realdot(es.Ez,es.Ez))		V/m	Electric field norm	Domain 4
es.Jr	es.Jdr		A/m ²	Current density, r component	Domain 4
es.Jphi	es.Jdphi		A/m ²	Current density, phi component	Domain 4
es.Jz	es.Jdz		A/m ²	Current density, z component	Domain 4
es.JR	(spatial.invF11*es.Jdr+spatial.invF31*es.Jdz)*spatial.detF		A/m ²	Current density, R component	Domain 4
es.JPHI	if(Rg>0.001*h,R/r,Rr)*es.Jdphi*spatial.detF		A/m ²	Current density, PHI component	Domain 4
es.JZ	(spatial.invF13*es.Jdr+spatial.invF33*es.Jdz)*spatial.detF		A/m ²	Current density, Z component	Domain 4
es.Jdr	0		A/m ²	Displacement current density, r component	Domain 4
es.Jdphi	0		A/m ²	Displacement current density, phi component	Domain 4
es.Jdz	0		A/m ²	Displacement current density, z component	Domain 4
es.normJ	sqrt(realdot(es.Jr,es.Jr)+realdot(es.Jphi,es.Jphi)+realdot(es.Jz,es.Jz))		A/m ²	Current density norm	Domain 4
es.ccn1.nJ	es.dnr*up(es.Jr)+es.dnphi*up(es.Jphi)+es.dnz*up(es.Jz)		A/m ²	Inward current density	Boundaries 8–9, 13
es.cen1.nJ	es.unr*down(es.Jr)+es.unphi*down(es.Jphi)+es.unz*down(es.Jz)		A/m ²	Inward current density	Boundary 10
es.W	es.We		J/m ³	Energy density	Domain 4
es.dWe	2*es.We*pi*r		J/m ²	Integrand for total electric energy	Domain 4
es.We	0.5*epsilon0_const*((es.I_srr+es.chirr)*es.Er+(es.I_srphi+es.chirphi)*es.Ephi+(es.I_srz+es.chirz)*es.Ez)*es.Er+((es.I_sphir+es.chiphir)*es.Er+(es.I_sphipi+es.chiphipi)*es.Ephi+(es.I_sphiz+es.chiphiz)*es.Ez)*es.Ephi+((es.I_szr+es.chizr)*es.Er+(es.I_szphi+es.chizphi)*es.Ephi+(es.I_szz+es.chizz)*es.Ez)*es.Ez)		J/m ³	Electric energy density	Domain 4

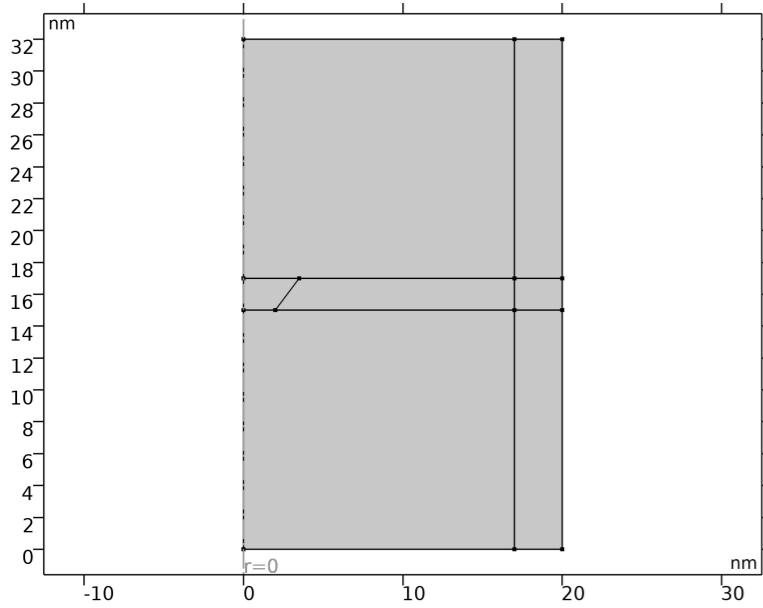
Shape functions

Name	Shape function	Unit	Description	Shape frame	Selection
VQM	Lagrange (Quadratic)	V	Electric potential	Spatial	Domain 4
VQM	Lagrange (Quadratic)	V	Electric potential	Material	Domain 4
VQM	Lagrange (Quadratic)	V	Electric potential	Geometry	Domain 4
VQM	Lagrange (Quadratic)	V	Electric potential	Mesh	Domain 4

Weak Expressions

Weak expression	Integration order	Integration frame	Selection
-2*(es.Dr*test(VQMr)+es.Dz*test(VQMz))*es.d*pi*r	4	Spatial	Domain 4

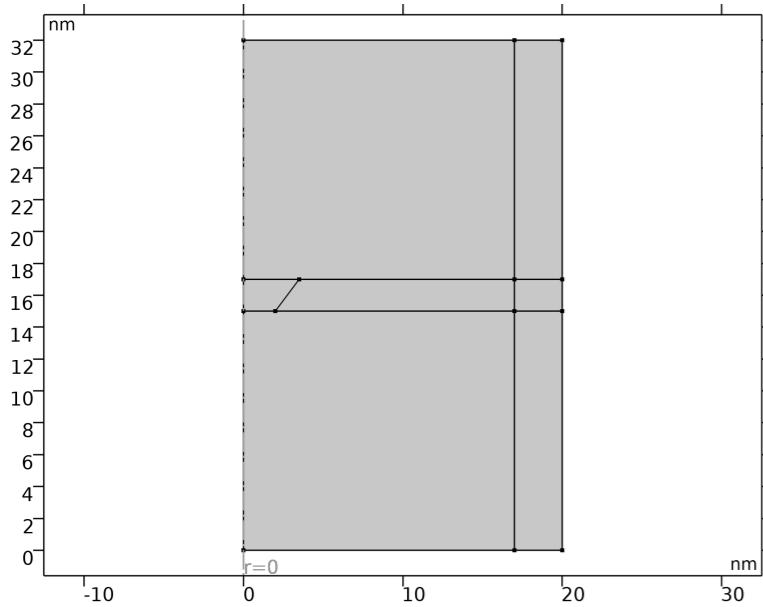
2.8.4. Axial Symmetry

*Axial Symmetry***Selection**

Geometric entity level	Boundary
Selection	Geometry geom2: Dimension 1: All boundaries

Used products

COMSOL Multiphysics

2.8.5. Zero Charge*Zero Charge***Selection**

Geometric entity level	Boundary
Selection	Geometry geom2: Dimension 1: All boundaries

Equations

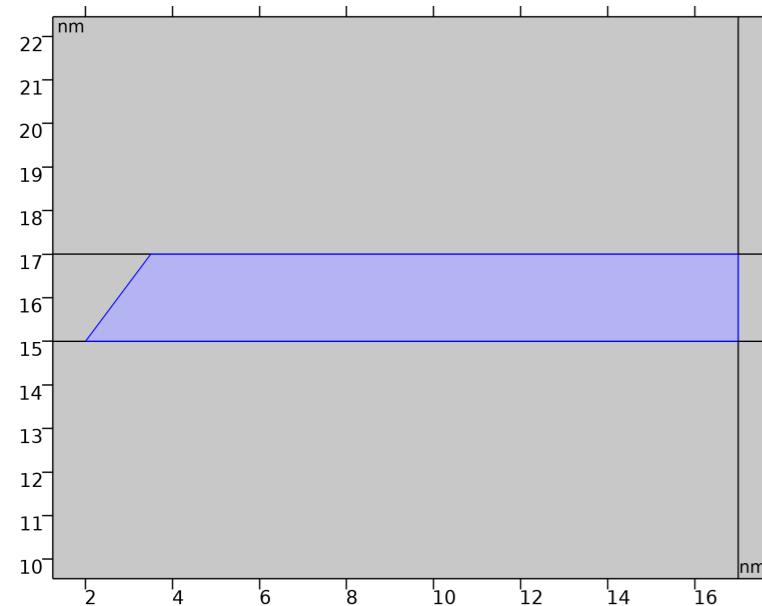
$$\mathbf{n} \cdot \mathbf{D} = 0$$

Used products

COMSOL Multiphysics

Shape functions

Name	Shape function	Unit	Description	Shape frame	Selection	Details
VQM	Lagrange (Quadratic)	V	Electric potential	Spatial	No boundaries	Slit
VQM	Lagrange (Quadratic)	V	Electric potential	Material	No boundaries	Slit
VQM	Lagrange (Quadratic)	V	Electric potential	Geometry	No boundaries	Slit
VQM	Lagrange (Quadratic)	V	Electric potential	Mesh	No boundaries	Slit

2.8.6. Initial Values*Initial Values*

Selection

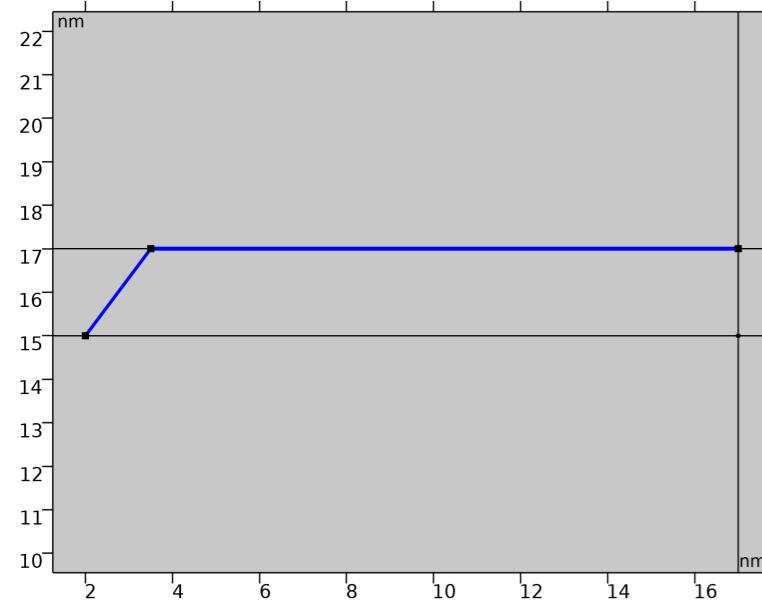
Geometric entity level	Domain
Selection	Geometry geom2: Dimension 2: All domains

Settings

Description	Value
Electric potential	0

Used products

COMSOL Multiphysics

2.8.7. Terminal Up/T1*Terminal Up/T1*

Selection

Geometric entity level	Boundary
Selection	Geometry geom2: Dimension 1: Boundaries 8, 10

Equations

$$V = V_0$$

Terminal

Settings

Description	Value
Terminal name	1

Terminal type	Voltage
Voltage	V_all_Cell

Advanced Settings

Settings	
Description	Value
Charge scaling type	Automatic

Constraint Settings

Settings	
Description	Value
Apply reaction terms on	All physics (symmetric)
Use weak constraints	Off
Constraint method	Elemental

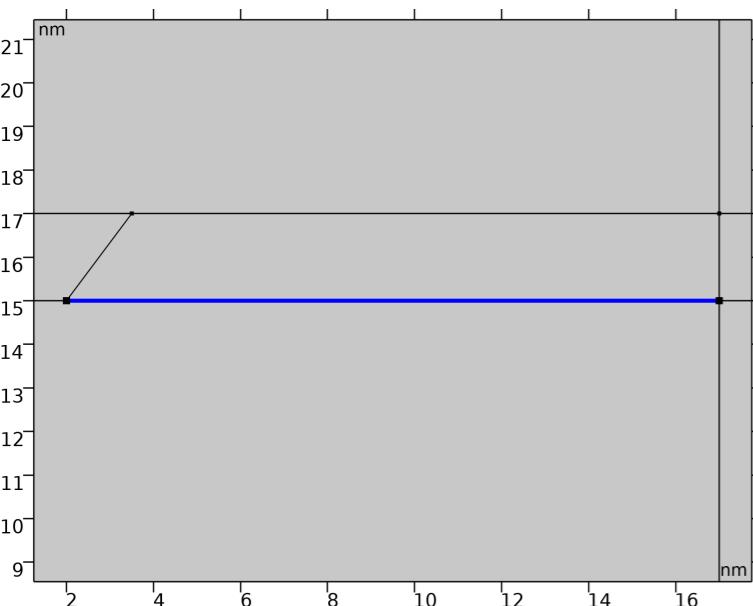
Variables

Name	Expression	Unit	Description	Selection	Details
es.nD	es.unr*(down(if(isdefined(es.Dr),es.Dr,0))-up(if(isdefined(es.Dr),es.Dr,0)))+es.unphi*(down(if(isdefined(es.Dphi),es.Dphi,0))-up(if(isdefined(es.Dphi),es.Dphi,0)))+es.unz*(down(if(isdefined(es.Dz),es.Dz,0))-up(if(isdefined(es.Dz),es.Dz,0)))	C/m ²	Surface charge density	Boundaries 8, 10	+ operation
es.Vterm	es.term1.V0	V	Voltage	Boundaries 8, 10	
es.term1.V0	V_all_Cell	V	Voltage	Global	
es.term1.I_cir	model.input.I_cir	A	Current	Global	Meta
es.Q0_1	-es.term1.sum(reacf(VQM))	C	Terminal charge	Global	
es.V0_1	es.term1.int(VQM)/es.term1.int(I)	V	Terminal voltage	Global	

Constraints

Constraint	Constraint force	Shape function	Selection	Details
es.Vterm-VQM	test(es.Vterm-VQM)	Lagrange (Quadratic)	Boundaries 8, 10	Elemental

2.8.8. Terminal Down/T2



Terminal Down/T2

Selection	
Geometric entity level	Boundary
Selection	Geometry geom2: Dimension 1: Boundary 9

Equations

$$V = V_0$$

Terminal

Settings	
Description	Value
Terminal name	2
Terminal type	Voltage

Voltage	<u>V_all_Cell</u>
---------	-------------------

Advanced Settings

Settings

Description	Value
Charge scaling type	Automatic

Constraint Settings

Settings

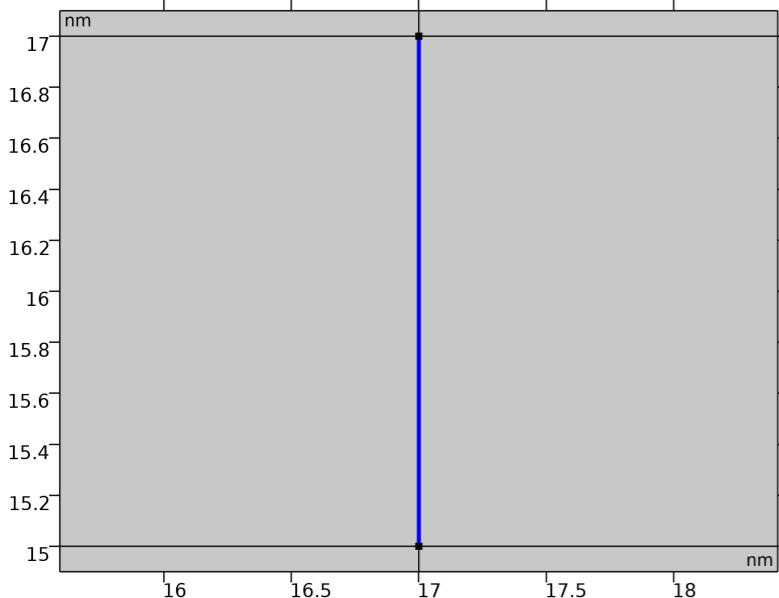
Description	Value
Apply reaction terms on	All physics (symmetric)
Use weak constraints	Off
Constraint method	Elemental

Variables

Name	Expression	Unit	Description	Selection	Details
es.nD	es.unr*(down(if(isdefined(es.Dr),es.Dr,0))-up(if(isdefined(es.Dr),es.Dr,0)))+es.unphi*(down(if(isdefined(es.Dphi),es.Dphi,0))-up(if(isdefined(es.Dphi),es.Dphi,0)))+es.unz*(down(if(isdefined(es.Dz),es.Dz,0))-up(if(isdefined(es.Dz),es.Dz,0)))	C/m ²	Surface charge density	Boundary 9	+ operation
es.Vterm	es.term2.V0	V	Voltage	Boundary 9	
es.term2.V0	<u>V_all_Cell</u>	V	Voltage	Global	
es.term2.I_cir	model.input.I_cir	A	Current	Global	Meta
es.Q0_2	-es.term2.sum(reacf(VQM))	C	Terminal charge	Global	
es.V0_2	es.term2.int(VQM)/es.term2.int(1)	V	Terminal voltage	Global	

Constraints

Constraint	Constraint force	Shape function	Selection	Details
es.Vterm-VQM	test(es.Vterm-VQM)	Lagrange (Quadratic)	Boundary 9	Elemental

2.8.9. Electric Displacement Field / Continuity

Electric Displacement Field / Continuity

Selection

Geometric entity level	Boundary
Selection	Geometry geom2: Dimension 1: Boundary 13

Equations

$$\mathbf{n} \cdot \mathbf{D} = \mathbf{n} \cdot \mathbf{D}_0$$

Electric Displacement Field

Settings

Description	Value
Boundary electric displacement field, r component	es.Dr
Boundary electric displacement field, phi component	0
Boundary electric displacement field, z component	es.Dz

Settings	
Description	Value
Coordinate system	Global coordinate system
Used products	
COMSOL Multiphysics	

Variables

Name	Expression	Unit	Description	Selection	Details
es.nD	-es.nr*es.df1.D0r-es.nphi*es.df1.D0phi-es.nz*es.df1.D0z	C/m ²	Surface charge density	Boundary 13	+ operation
es.df1.D0r	es.Dr	C/m ²	Boundary electric displacement field, r component	Boundary 13	
es.df1.D0phi	0	C/m ²	Boundary electric displacement field, phi component	Boundary 13	
es.df1.D0z	es.Dz	C/m ²	Boundary electric displacement field, z component	Boundary 13	

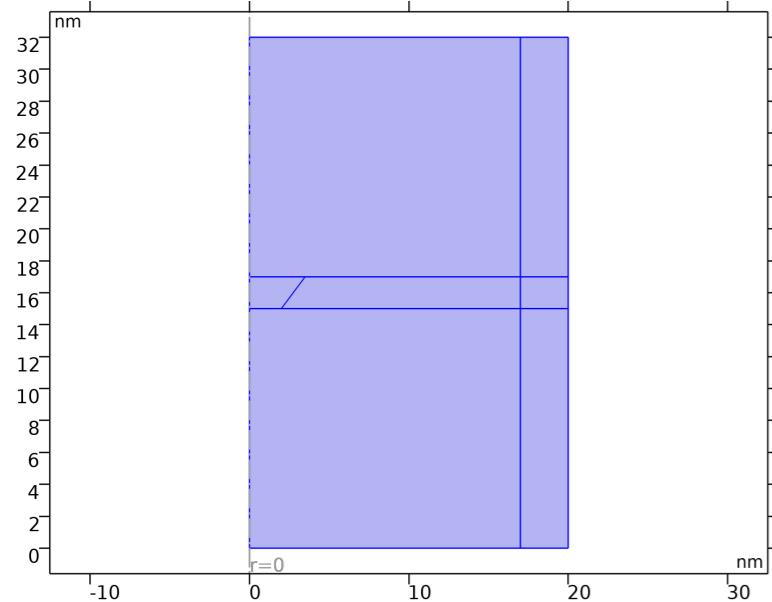
Weak Expressions

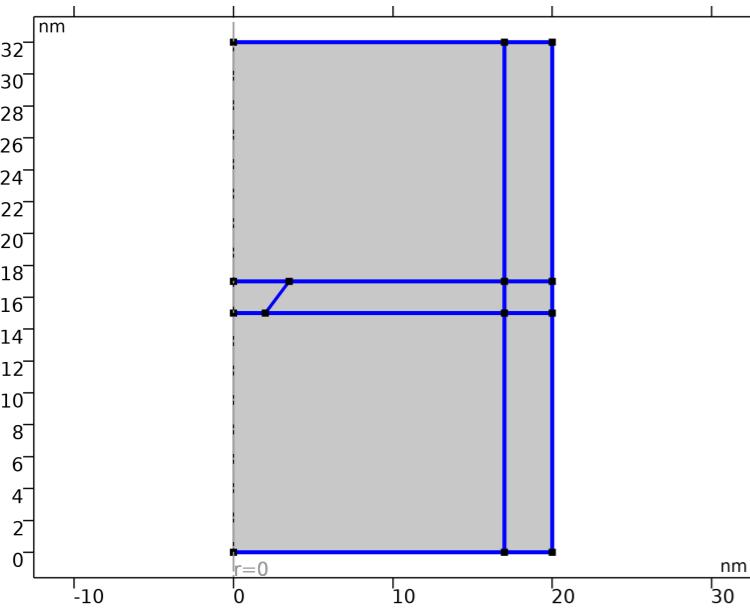
Weak expression	Integration order	Integration frame	Selection
$2*(es.nr*es.df1.D0r+es.nphi*es.df1.D0phi+es.nz*es.df1.D0z)*test(VQM)*es.d*pi*r$	4	Spatial	Boundary 13

2.9. Multiphysics**2.9.1. Electromagnetic Heating**

Used products

COMSOL Multiphysics

*Electromagnetic Heating*

**Boundary selection****Selection**

Geometric entity level	Domain
Selection	Geometry geom2: Dimension 2: Domains 1–7

Equations

$$\rho C_p \mathbf{u} \cdot \nabla T = \nabla \cdot (k \nabla T) + Q_e.$$

$$Q_e = \mathbf{J} \cdot \mathbf{E}$$

Coupled Interfaces**Settings**

Description	Value
Electromagnetic	Electric Currents entire RRAM cell (ec)
Heat transfer	Heat Transfer in Solids (ht)

Variables

Name	Expression	Unit	Description	Selection	Details
ht.ndflux_u	if(ht.isSolving,0,if(ht.TuIsDown,0,-0.5*emh.Qb)+if(ht.TdIsUp,-0.5*emh.Qb,0))	W/m ²	Internal normal conductive heat flux, upside	Boundaries 4, 6, 8–11, 13, 15	+ operation
ht.ndflux_u	if(ht.isSolving,0,if(ht.TuIsDown,0,-0.5*emh.Qb)+if(ht.TdIsUp,-0.5*emh.Qb,0))	W/m ²	Internal normal conductive heat flux, upside	Boundaries 14, 16	+ operation
ht.ndflux_d	if(ht.isSolving,0,if(ht.TdIsUp,0,-0.5*emh.Qb)+if(ht.TuIsDown,-0.5*emh.Qb,0))	W/m ²	Internal normal conductive heat flux, downside	Boundaries 4, 6, 8–11, 13, 15	+ operation
ht.ndflux_d	if(ht.isSolving,0,if(ht.TdIsUp,0,-0.5*emh.Qb)+if(ht.TuIsDown,-0.5*emh.Qb,0))	W/m ²	Internal normal conductive heat flux, downside	Boundaries 14, 16	+ operation
ht.Q	emh.Q	W/m ³	Heat source	Domains 1–4	+ operation
ht.Q	emh.Q	W/m ³	Heat source	Domains 5–7	+ operation
ht.Qtot	emh.Q	W/m ³	Total heat source	Domains 1–4	+ operation
ht.Qtot	emh.Q	W/m ³	Total heat source	Domains 5–7	+ operation
ht.Qbtot	emh.Qb	W/m ²	Total boundary heat source	Boundaries 2, 4, 6–11, 13, 15	+ operation
ht.Qbtot	emh.Qb	W/m ²	Total boundary heat source	Boundaries 12, 14, 16–20	+ operation
ht.ntflux_contrib	emh.Qb	W/m ²	Boundary sources and fluxes contribution	Boundaries 4, 6, 8–11, 13, 15	+ operation
ht.ntflux_contrib	emh.Qb	W/m ²	Boundary sources and fluxes contribution	Boundaries 14, 16	+ operation
emh.Q	ec.Qh	W/m ³	Heat source	Domains 1–4	
emh.Q	ec.Qh	W/m ³	Heat source	Domains 5–7	
emh.T	T	K	Temperature	Domains 1–4	
emh.T	T	K	Temperature	Domains 5–7	
emh.Qb	ec.Qsh	W/m ²	Boundary heat source	Global	
emh.varInt	2*pi*r	m	Intermediate variable	Boundaries 2, 4, 6–11, 13, 15	Meta
emh.varInt	2*pi*ie1.r*ie1.detInvT	m	Intermediate variable	Boundaries 12, 14, 16–20	Meta
emh.varInt	1	m	Intermediate variable	Boundaries 1, 3, 5	

emh.Tvar	$0.5 * (\text{ht.Tu} + \text{ht.Td})$	K	Temperature	Boundaries 2, 4, 6–11, 13, 15
emh.Tvar	$0.5 * (\text{ht.Tu} + \text{ht.Td})$	K	Temperature	Boundaries 12, 14, 16–20
emh.ntfluxInt	$\text{emh.intExtBnd}(\text{ht.ntflux} * \text{ht.varIntSpa}) + \text{emh.intIntBnd}(\text{ht.ncflux_u} * \text{up}(\text{ht.varIntSpa}) + \text{ht.ncflux_d} * \text{down}(\text{ht.varIntSpa}))$	W	Total net heat rate	Global
emh.ntfluxInt_cst	$\text{emh.intExtBnd}(\text{ht.ntflux} * \text{ht.varIntSpa}) + \text{emh.intIntBnd}(\text{ht.ncflux_u} * \text{up}(\text{ht.varIntSpa}) - \text{ht.ncflux_d} * \text{down}(\text{ht.varIntSpa}))$	W	Total net heat rate, constant material properties	Global
emh.ntrfluxInt	$\text{emh.intExtBnd}(\text{ht.ntrflux} * \text{ht.varIntSpa}) + \text{emh.intIntBnd}(\text{ht.ntrflux_u} * \text{up}(\text{ht.varIntSpa}) + \text{ht.ntrflux_d} * \text{down}(\text{ht.varIntSpa}))$	W	Total net energy rate	Global
emh.ntrfluxInt_cst	$\text{emh.intExtBnd}(\text{ht.ntrflux_cst} * \text{ht.varIntSpa}) + \text{emh.intIntBnd}(\text{ht.ntrflux_cst_u} * \text{up}(\text{ht.varIntSpa}) + \text{ht.ntrflux_cst_d} * \text{down}(\text{ht.varIntSpa}))$	W	Total net energy rate, constant material properties	Global
emh.ntfluxInt_u	$\text{emh.intIntBnd}(\text{ht.ntflux_u} * \text{up}(\text{ht.varIntSpa}))$	W	Total net heat rate, upside	Global
emh.ntfluxInt_cst_u	$\text{emh.intIntBnd}(\text{ht.ntflux_u} * \text{up}(\text{ht.varIntSpa}))$	W	Total net heat rate, constant material properties, upside	Global
emh.ntrfluxInt_u	$\text{emh.intIntBnd}(\text{ht.ntrflux_u} * \text{up}(\text{ht.varIntSpa}))$	W	Total net energy rate, upside	Global
emh.ntrfluxInt_cst_u	$\text{emh.intIntBnd}(\text{ht.ntrflux_cst_u} * \text{up}(\text{ht.varIntSpa}))$	W	Total net energy rate, constant material properties, upside	Global
emh.ntfluxInt_d	$\text{emh.intIntBnd}(\text{ht.ntflux_d} * \text{down}(\text{ht.varIntSpa}))$	W	Total net heat rate, downside	Global
emh.ntrfluxInt_cst_d	$\text{emh.intIntBnd}(\text{ht.ntrflux_d} * \text{down}(\text{ht.varIntSpa}))$	W	Total net heat rate, constant material properties, downside	Global
emh.ntrfluxInt_d	$\text{emh.intIntBnd}(\text{ht.ntrflux_d} * \text{down}(\text{ht.varIntSpa}))$	W	Total net energy rate, downside	Global
emh.ntrfluxInt_cst_d	$\text{emh.intIntBnd}(\text{ht.ntrflux_cst_d} * \text{down}(\text{ht.varIntSpa}))$	W	Total net energy rate, constant material properties, downside	Global
emh.Tave	$\text{nojac}(\text{emh.intBnd}(\text{ht.varIntSpa} * \text{ht.rho} * \text{ht.Cp} * \text{T} * \max(\text{abs}(\text{ht.ur} * \text{ht.nrmesh} + \text{ht.uphi} * \text{ht.nphimesh} + \text{ht.uz} * \text{ht.nzmesh}), \text{eps}))) / \text{nojac}(\text{emh.intBnd}(\text{ht.varIntSpa} * \text{ht.rho} * \text{ht.Cp} * \max(\text{abs}(\text{ht.ur} * \text{ht.nrmesh} + \text{ht.uphi} * \text{ht.nphimesh} + \text{ht.uz} * \text{ht.nzmesh}), \text{eps})))$	K	Weighted average temperature	Global
emh.T_cmi	model.input.T_cmi	K	Temperature	Global

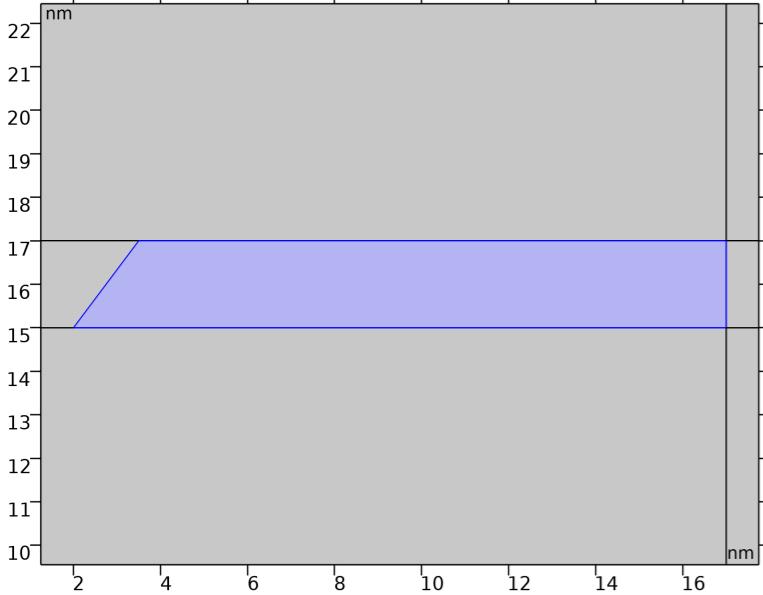
Weak Expressions

Weak expression	Integration order	Integration frame	Selection
$2 * \text{emh.Q} * \text{test}(\text{T}) * \text{ht.d} * \text{pi} * \text{r}$	4	Spatial	Domains 1–4
$2 * \text{emh.Q} * \text{test}(\text{T}) * \text{ht.d} * \text{pi} * \text{ie1.r} * \text{ie1.detInvT}$	4	Spatial	Domains 5–7
$2 * \text{emh.Qb} * \text{test}(\text{emh.Tvar}) * \text{pi} * \text{r}$	4	Spatial	Boundaries 2, 4, 6–11, 13, 15
$2 * \text{emh.Qb} * \text{test}(\text{emh.Tvar}) * \text{pi} * \text{ie1.r} * \text{ie1.detInvT}$	4	Spatial	Boundaries 12, 14, 16–20

2.9.2. Schrödinger-Poisson Coupling

Used products

COMSOL Multiphysics
Semiconductor Module

**Schrödinger-Poisson Coupling**

Selection

Geometric entity level	Domain
Selection	Geometry geom2: Dimension 2: Domain 4

Equations

$$n_{\text{sum}} = \sum_i g_i \frac{2}{1 + \exp\left(\frac{E_f + E_i}{k_B T}\right)} |\Psi|^2$$

$$q = z_q e, \quad e = \text{elementary charge}$$

$V_e = qV$ contribute to potential energy for Schrodinger Equation

$$\rho_v = q n_{\text{sum}} \exp\left(e^{-\alpha} \frac{-q(V - V_{\text{dd}})}{k_B T}\right) \text{ contribute to space charge density for Electrostatics}$$

Particle Density Computation

Settings

Description	Value
Particle density computation	Fermi - Dirac statistics, parabolic band
Fermi energy level	Ef
Degeneracy factor	1 + (mm>0)

Charge Density Computation

Settings

Description	Value
Charge number	-1
Charge density computation	Modified Gummel iteration
Tuning parameter (positive values: accelerate, negative values: more damping)	4
Global error variable	schrp.max((abs(schrp.V - schrp.V_old))/1[V])

Coupled Interfaces

Settings

Description	Value
Schrödinger Equation	Schrödinger Equation (schr)
Electrostatics	Electrostatics: Potential on HfO ₂ (es)

Model Input

Settings

Description	Value
Temperature	User defined
Temperature	tref

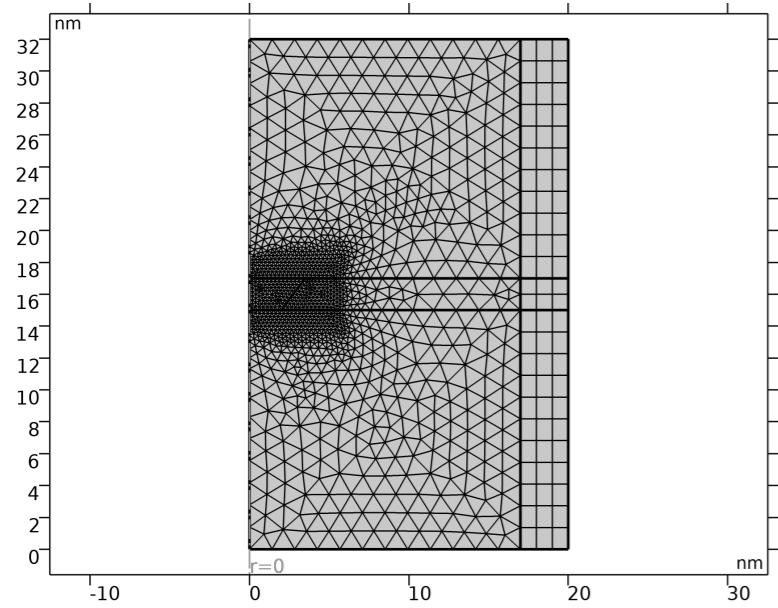
Variables

Name	Expression	Unit	Description	Selection	Details
schr.V_psi	schrp.q*schrp.V	J	Potential energy	Domain 4	+ operation
schrp.rhoq	schrp.q*schrp.n_sum*exp(-schrp.q*(schrp.V-schrp.V_old)*exp(-schrp.alpha1)/schrp.kT)	C/m ³	Space charge density	Domain 4	
schrp.V	VQM	V	Electric potential	Domain 4	
schrp.Ef	Ef	J	Fermi energy level	Domain 4	
schrp.gi	1+(mm>0)	I	Degeneracy factor	Domain 4	
schrp.kT	k_B_const*schrp.minput_temperature	J	Thermal energy	Domain 4	
schrp.ni	schrp.Ni*schrp.Pr	1/m ³	Charged particle density, individually weighted	Domain 4	
schrp.dEi	schrp.Ef-real(schrp.Ei)	J	Energy difference	Domain 4	
schrp.Ni	2*schrp.gi*if(abs(schrp.dEi)<40*schrp.kT,1/(1+exp(-schrp.dEi/schrp.kT)),if(schrp.dEi>0,1,0))	I	Weighted effective density of states	Domain 4	
schrp.zq	-1	I	Charge number	Domain 4	
schrp.q	schrp.zq*e_const	C	Charge of particle	Domain 4	
schrp.alpha1	4	I	Tuning parameter (positive values: accelerate, negative values: more damping)	Domain 4	
schrp.global_err	schrp.max(abs(schrp.V-schrp.V_old)/1[V])	I	Global error variable	Global	

Shape functions

Name	Shape function	Unit	Description	Shape frame	Selection
schrp.n_sum	Lagrange (Quadratic)	1/m ³	Charged particle density	Material	Domain 4
schrp.rhoq_old	Lagrange (Quadratic)	C/m ³	Space charge density from previous iteration	Material	Domain 4
schrp.V_old	Lagrange (Quadratic)	V	Electric potential from previous iteration	Material	Domain 4

2.10. Mesh 2



Mesh 2

Mesh statistics

Description	Value
Status	Complete mesh
Mesh vertices	1367
Triangles	2488
Quads	72
Edge elements	210
Vertex elements	14
Number of elements	2560
Minimum element quality	0.6115
Average element quality	0.876
Element area ratio	0.007522
Mesh area	640 nm ²

2.10.1. Size (size)

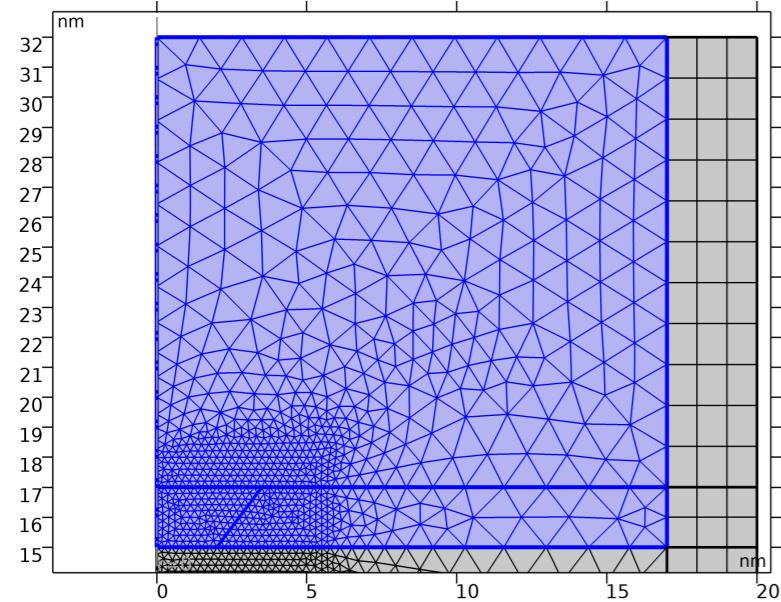
Settings

Description	Value
Maximum element size	1.41
Minimum element size	0.0114
Curvature factor	0.25
Maximum element growth rate	1.2
Predefined size	Finer
Custom element size	Custom

2.10.2. Size 1 (size1)

Selection

Geometric entity level	Domain
Selection	Geometry geom2: Dimension 2: Domains 2–4, 8



Size I

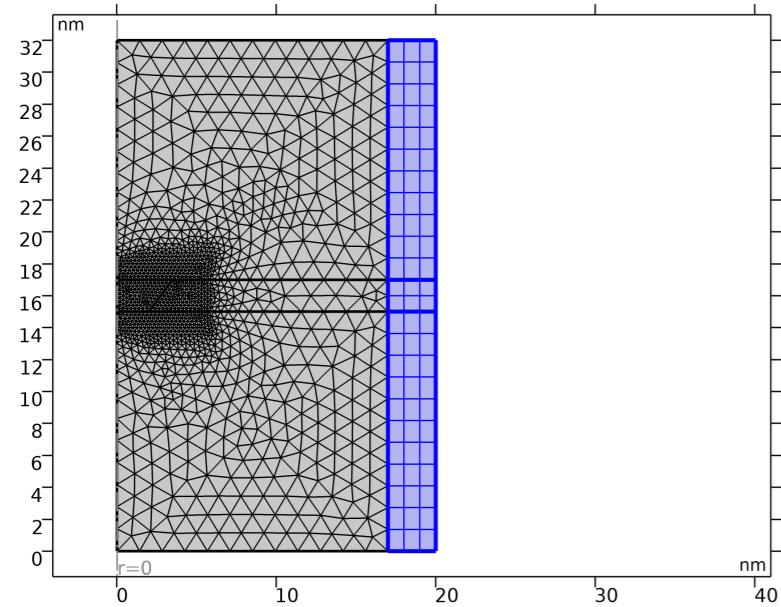
Settings

Description	Value
Maximum element size	0.25
Minimum element size	0.00475
Curvature factor	0.3
Maximum element growth rate	1.3
Custom element size	Custom

2.10.3. Mapped 1 (map1)

Selection

Geometric entity level	Domain
Selection	Geometry geom2: Dimension 2: Domains 5–7



Mapped I

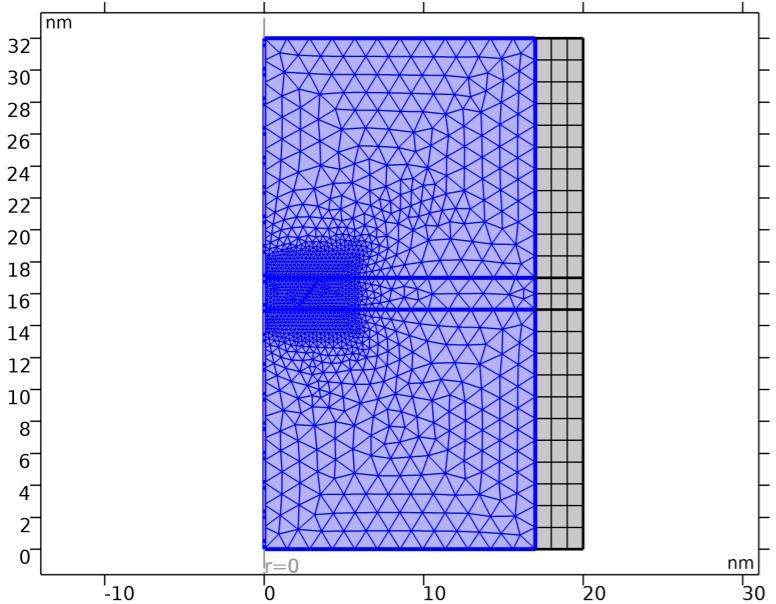
Settings

Description	Value
Number of iterations	4
Maximum element depth to process	4
Last build time	0
Built with	COMSOL 6.0.0.318 (win64) 2022 - 05 - 22T22:56:29.550594900

2.10.4. Free Triangular 1 (ftri1)

Selection

Geometric entity level	Domain
Selection	Remaining



Free Triangular I

Settings

Description	Value
Number of iterations	4
Maximum element depth to process	4
Last build time	0
Built with	COMSOL 6.0.0.318 (win64) 2022 - 05 - 22T22:56:29.672601800
Information	Removed_mesh_control_entities

3. Classic approach

Computation information

Computation time	18 s
------------------	------

3.1. Stationary Classic approach

Study settings

Description	Value
Include geometric nonlinearity	Off

Physics and variables selection

Physics interface	Discretization
Electric Currents entire RRAM cell (ec)	physics
Heat Transfer in Solids (ht)	physics
Electrostatics: Potential on HfO ₂ (es)	physics

Mesh selection

Geometry	Mesh
Geometry 2 (geom2)	mesh2

3.2. Solver Configurations**3.2.1. Solution 1**

Compile Equations: Stationary Classic approach (st1)

Study and step

Description	Value
Use study	Classic approach
Use study step	Stationary Classic approach

Log

<---- Compile Equations: Stationary Classic approach in Classic

approach/Solution 1 (sol1) -----

Started at Jun 11, 2022, 11:03:12 AM.

Geometry shape function: Quadratic Lagrange

Running on Intel® Family 6 Model 58 Stepping 9, GenuineIntel.

Using 1 socket with 4 cores in total on EM-PC.

Available memory: 32.70 GB.

Time: 15 s.
 Physical memory: 1.47 GB
 Virtual memory: 1.47 GB
 Ended at Jun 11, 2022, 11:03:27 AM.
 ----- Compile Equations: Stationary Classic approach in Classic
 approach/Solution 1 (sol1) ----->

Dependent Variables 1 (v1)

General

Description	Value
Defined by study step	Stationary Classic approach

Log

<---- Dependent Variables 1 in Classic approach/Solution 1 (sol1) ----->
 Started at Jun 11, 2022, 11:03:27 AM.
 Solution time: 0 s.
 Physical memory: 1.48 GB
 Virtual memory: 1.47 GB
 Ended at Jun 11, 2022, 11:03:27 AM.
 ----- Dependent Variables 1 in Classic approach/Solution 1 (sol1) ----->

Temperature (comp.T) (comp_T)

General

Description	Value
Field components	comp.T
Internal variables	{comp.uflux.T, comp.dflux.T}

Electric potential (comp.V_all_Cell) (comp_V_all_Cell)

General

Description	Value
Field components	comp.V_all_Cell

Electric potential (comp.VQM) (comp_VQM)

General

Description	Value
Field components	comp.VQM

Stationary Solver 1 (s1)

General

Description	Value
Defined by study step	Stationary Classic approach

Log

<---- Stationary Solver 1 in Classic approach/Solution 1 (sol1) ----->
 Started at Jun 11, 2022, 11:03:27 AM.

Nonlinear solver

Number of degrees of freedom solved for: 11499 (plus 670 internal DOFs).

Nonsymmetric matrix found.

Scales for dependent variables:

Temperature (comp.T): 3e+02

Electric potential (comp.V_all_Cell): 8.6

Electric potential (comp.VQM): 1.2e+11

Orthonormal null-space function used.

Iter	SolEst	ResEst	Damping	Stepsiz	#Res	#Jac	#Sol	LinErr	LinRes
1	5.2	1.8e+08	0.0100000	5.2	2	1	2	9e-14	2.8e-15
2	4.4	1.8e+06	0.1000000	4.9	3	2	4	1.5e-13	3.7e-15
3	0.26	1.4e+08	1.0000000	0.64	4	3	6	4.3e-15	7.2e-14
4	0.017	1.6e+06	1.0000000	0.13	5	4	8	1.9e-15	7e-14
5	0.00068	3.6e+02	1.0000000	0.021	7	5	10	7.8e-15	1.9e-14

Solution time: 2 s.

Physical memory: 1.51 GB

Virtual memory: 1.5 GB

Ended at Jun 11, 2022, 11:03:29 AM.

<---- Stationary Solver 1 in Classic approach/Solution 1 (sol1) ----->

Fully Coupled 1 (fc1)

General

Description	Value
Linear solver	Direct 1

Method and termination

Description	Value
Initial damping factor	0.01
Minimum damping factor	1.0E-6

Maximum number of iterations	50
------------------------------	----

4. Semi-Classic approach

Computation
information

Computation time	31 s
------------------	------

4.1. Stationary-Semi-Classic approach

Study settings

Description	Value
Include geometric nonlinearity	Off

Values of dependent variables

Description	Value
Settings	User controlled
Method	Solution
Study	Classic approach

Physics and variables selection

Physics interface	Discretization
Wall Distance (wd)	physics
Electric Currents entire RRAM cell (ec)	physics
Heat Transfer in Solids (ht)	physics
Electrostatics: Potential on HfO ₂ (es)	physics

Mesh selection

Geometry	Mesh
Geometry 2 (geom2)	mesh2

4.2. Solver Configurations

4.2.1. Solution 2

Compile Equations: Stationary-Semi-Classic approach (st1)

Study and step

Description	Value
Use study	Semi - Classic approach
Use study step	Stationary - Semi - Classic approach

Log

<---- Compile Equations: Stationary-Semi-Classic approach in Semi-Classic
approach/Solution 2 (sol2) ----->

Started at Jun 11, 2022, 11:03:40 AM.

Geometry shape function: Quadratic Lagrange

Running on Intel64 Family 6 Model 58 Stepping 9, GenuineIntel.

Using 1 socket with 4 cores in total on EM-PC.

Available memory: 32.70 GB.

Time: 4 s.

Physical memory: 1.55 GB

Virtual memory: 1.54 GB

Ended at Jun 11, 2022, 11:03:44 AM.

----- Compile Equations: Stationary-Semi-Classic approach in Semi-Classic
approach/Solution 2 (sol2) ----->

Dependent Variables 1 (v1)

General

Description	Value
Defined by study step	Stationary - Semi - Classic approach

Values of variables not solved for

Description	Value
Method	Solution
Solution	Solution 1

Log

<---- Dependent Variables 1 in Semi-Classic approach/Solution 2 (sol2) ----->

Started at Jun 11, 2022, 11:03:44 AM.

Values of variables not solved for: Solution 1 (sol1).

Solution time: 0 s.

Physical memory: 1.56 GB

Virtual memory: 1.54 GB

Ended at Jun 11, 2022, 11:03:44 AM.

----- Dependent Variables 1 in Semi-Classic approach/Solution 2 (sol2) ----->

Reciprocal wall distance (comp.G) (comp_G)

General

Description	Value
Field components	comp.G

Temperature (comp.T) (comp_T)

General

Description	Value
Field components	comp.T
Internal variables	{comp.uflux.T, comp.dflux.T}

Electric potential (comp.V_all_Cell) (comp_V_all_Cell)

General

Description	Value
Field components	comp.V_all_Cell

Electric potential (comp.VQM) (comp_VQM)

General

Description	Value
Field components	comp.VQM

Stationary Solver 1 (s1)

General

Description	Value
Defined by study step	Stationary - Semi - Classic approach

Log

<---- Stationary Solver 1 in Semi-Classic approach/Solution 2 (sol2) -----

Started at Jun 11, 2022, 11:03:44 AM.

Nonlinear solver

Number of degrees of freedom solved for: 12268 (plus 670 internal DOFs).

Nonsymmetric matrix found.

Scales for dependent variables:

Reciprocal wall distance (comp.G): 1e+09

Temperature (comp.T): 2.6e+09

Electric potential (comp.V_all_Cell): 7.4e+07

Electric potential (comp.VQM): 1e+18

Orthonormal null-space function used.

Iter	SolEst	ResEst	Damping	Stepsize	#Res	#Jac	#Sol	LinErr	LinRes
1	4.5	1.1e+10	0.0100000	4.5	2	1	2	3.2e-16	4.1e-16
2	3.9	1.7e+06	0.1000000	4.3	3	2	4	3.5e-16	4.4e-16
3	0.24	1.4e+08	1.0000000	0.57	4	3	6	3.7e-16	4.7e-16
4	0.034	1.5e+06	1.0000000	0.16	5	4	8	5.1e-16	5.3e-16
5	0.016	1.3e+04	1.0000000	0.081	6	5	10	5.6e-16	6.7e-16
6	0.0014	1.9e+02	1.0000000	0.027	7	6	12	6.4e-16	3.8e-16
7	4.8e-05	0.7	1.0000000	0.0014	9	7	14	6.6e-16	8.7e-16

Solution time: 27 s.

Physical memory: 1.56 GB

Virtual memory: 1.55 GB

Ended at Jun 11, 2022, 11:04:10 AM.

----- Stationary Solver 1 in Semi-Classic approach/Solution 2 (sol2) ----->

Fully Coupled 1 (fc1)

General

Description	Value
Linear solver	Direct 1

Method and termination

Description	Value
Initial damping factor	0.01
Minimum damping factor	1.0E-6
Maximum number of iterations	50

5. Full quantum approach (High-Precision and slow, Matlab function)

Computation information

Computation time	2 min 38 s
------------------	------------

5.1. Stationary

Study settings

Description	Value
Include geometric nonlinearity	Off

Values of dependent variables

Description	Value
Settings	User controlled
Study	Semi - Classic approach

Physics and variables selection

Physics interface	Discretization
Wall Distance (wd)	physics
Electric Currents entire RRAM cell (ec)	physics
Heat Transfer in Solids (ht)	physics
Electrostatics: Potential on HfO ₂ (es)	physics

Mesh selection

Geometry	Mesh
Geometry 2 (geom2)	mesh2

5.2. Schrödinger-Poisson

Study settings

Description	Value
Include geometric nonlinearity	Off

Study settings

Description	Value
Desired number of eigenvalues	On
Search for eigenvalues around	On

Iterations

Description	Value
Current_iteration	6

Physics and variables selection

Physics interface	Discretization
Electric Currents entire RRAM cell (ec)	physics
Heat Transfer in Solids (ht)	physics
Schrödinger Equation (schr)	physics
Electrostatics: Potential on HfO ₂ (es)	physics

Mesh selection

Geometry	Mesh
Geometry 2 (geom2)	mesh2

5.3. Solver Configurations

5.3.1. Solution 3

Compile Equations: Stationary (st1)

Study and step

Description	Value
Use study	Full quantum approach (High - Precision and slow, Matlab function)
Use study step	Stationary

Log

<---- Compile Equations: Stationary in Full quantum approach (High-Precision and slow, Matlab function)/Solution 3 (sol3) ----->

Started at Jun 11, 2022, 11:04:36 AM.

Geometry shape function: Quadratic Lagrange

Running on Intel64 Family 6 Model 58 Stepping 9, GenuineIntel.

Using 1 socket with 4 cores in total on EM-PC.

Available memory: 32.70 GB.

Time: 3 s.

Physical memory: 1.58 GB

Virtual memory: 1.57 GB

Ended at Jun 11, 2022, 11:04:40 AM.

<---- Compile Equations: Stationary in Full quantum approach (High-Precision and slow, Matlab function)/Solution 3 (sol3) ----->

Dependent Variables 1 (v1)

General

Description	Value
Defined by study step	Stationary

Values of variables not solved for

Description	Value
Solution	Solution 2

Log

<---- Dependent Variables 1 in Full quantum approach (High-Precision and slow,

Matlab function)/Solution 3 (sol3) -----
 Started at Jun 11, 2022, 11:04:40 AM.
 Values of variables not solved for: Solution 2 (sol2).
 Solution time: 0 s.
 Physical memory: 1.58 GB
 Virtual memory: 1.57 GB
 Ended at Jun 11, 2022, 11:04:40 AM.
 ---- Dependent Variables 1 in Full quantum approach (High-Precision and slow,
 Matlab function)/Solution 3 (sol3) ----->

Reciprocal wall distance (comp.G) (comp_G)

General

Description	Value
Field components	comp.G

Wave function (comp.psi) (comp_psi)

General

Description	Value
Field components	comp.psi
Solve for this field	Off

Charged particle density (comp.schrp.n_sum) (comp_schrp_n_sum)

General

Description	Value
Field components	comp.schrp.n_sum
Solve for this field	Off

Space charge density from previous iteration (comp.schrp.rhoq_old) (comp_schrp_rhoq_old)

General

Description	Value
Field components	comp.schrp.rhoq_old
Solve for this field	Off

Electric potential from previous iteration (comp.schrp.V_old) (comp_schrp_V_old)

General

Description	Value
Field components	comp.schrp.V_old
Solve for this field	Off

Temperature (comp.T) (comp_T)

General

Description	Value
Field components	comp.T
Internal variables	{comp.uflux.T, comp.dflux.T}

Electric potential (comp.V_all_Cell) (comp_V_all_Cell)

General

Description	Value
Field components	comp.V_all_Cell

Electric potential (comp.VQM) (comp_VQM)

General

Description	Value
Field components	comp.VQM

Stationary Solver 1 (s1)

General

Description	Value
Defined by study step	Stationary

Log

<---- Stationary Solver 1 in Full quantum approach (High-Precision and slow,
 Matlab function)/Solution 3 (sol3) -----
 Started at Jun 11, 2022, 11:04:40 AM.
 Nonlinear solver

Number of degrees of freedom solved for: 11619 (plus 670 internal DOFs).

Nonsymmetric matrix found.

Scales for dependent variables:

Reciprocal wall distance (comp.G): 1e+09

Temperature (comp.T): 2.6e+09

Electric potential (comp.V_all_Cell): 6.9e+07

Electric potential (comp.VQM): 1e+18

Orthonormal null-space function used.

Iter	SolEst	ResEst	Damping	Stepsize	#Res	#Jac	#Sol	LinErr	LinRes
1	4.5	1.2e+10	0.0100000	4.6	2	1	2	3.9e-16	4.2e-16
2	3.9	1.8e+06	0.1000000	4.3	3	2	4	3.4e-16	4.5e-16
3	0.24	1.4e+08	1.0000000	0.56	4	3	6	3.7e-16	4.8e-16
4	0.034	1.5e+06	1.0000000	0.16	5	4	8	4e-16	5.3e-16
5	0.016	1.3e+04	1.0000000	0.081	6	5	10	4.4e-16	6.1e-16
6	0.0014	1.9e+02	1.0000000	0.027	7	6	12	4.1e-16	4.1e-16
7	4.8e-05	0.5	1.0000000	0.0014	9	7	14	3.9e-16	7e-16

Solution time: 26 s.

Physical memory: 1.59 GB

Virtual memory: 1.58 GB

Ended at Jun 11, 2022, 11:05:06 AM.

----- Stationary Solver 1 in Full quantum approach (High-Precision and slow,
Matlab function)/Solution 3 (sol3) ----->

Fully Coupled 1 (fc1)

General

Description	Value
Linear solver	Direct 1

Method and termination

Description	Value
Initial damping factor	0.01
Minimum damping factor	1.0E-6
Maximum number of iterations	50

Solution Store 1 (su1)

General

Description	Value
Solution	Solution Store 1

Compile Equations: Schrödinger-Poisson (st2)

Study and step

Description	Value
Use study	Full quantum approach (High - Precision and slow, Matlab function)
Use study step	Schrödinger - Poisson

Log

<---- Compile Equations: Schrödinger-Poisson in Full quantum approach
(High-Precision and slow, Matlab function)/Solution 3 (sol3) ----->

Started at Jun 11, 2022, 11:05:06 AM.

Geometry shape function: Quadratic Lagrange

Time: 3 s.

Physical memory: 1.58 GB

Virtual memory: 1.57 GB

Ended at Jun 11, 2022, 11:05:09 AM.

----- Compile Equations: Schrödinger-Poisson in Full quantum approach
(High-Precision and slow, Matlab function)/Solution 3 (sol3) ----->

Dependent Variables 2 (v2)

General

Description	Value
Defined by study step	Schrödinger - Poisson

Initial values of variables solved for

Description	Value
Method	Solution
Solution	Solution 3

Values of variables not solved for

Description	Value
Method	Solution
Solution	Solution 3

Log

<---- Dependent Variables 2 in Full quantum approach (High-Precision and slow,
Matlab function)/Solution 3 (sol3) ----->

Started at Jun 11, 2022, 11:05:09 AM.

Initial values of variables solved for: Solution 3 (sol3).
 Values of variables not solved for: Solution 3 (sol3).

Solution time: 0 s.

Physical memory: 1.59 GB

Virtual memory: 1.57 GB

Ended at Jun 11, 2022, 11:05:09 AM.

----- Dependent Variables 2 in Full quantum approach (High-Precision and slow,
 Matlab function)/Solution 3 (sol3) ----->

Reciprocal wall distance (comp.G) (comp_G)

General

Description	Value
Field components	comp.G
Solve for this field	Off

Wave function (comp.psi) (comp_psi)

General

Description	Value
Field components	comp.psi

Charged particle density (comp.schrp.n_sum) (comp_schrp_n_sum)

General

Description	Value
Field components	comp.schrp.n_sum

Space charge density from previous iteration (comp.schrp.rhoq_old) (comp_schrp_rhoq_old)

General

Description	Value
Field components	comp.schrp.rhoq_old

Electric potential from previous iteration (comp.schrp.V_old) (comp_schrp_V_old)

General

Description	Value
Field components	comp.schrp.V_old

Temperature (comp.T) (comp_T)

General

Description	Value
Field components	comp.T
Internal variables	{comp.uflux.T, comp.dflux.T, comp.ht.dt2Inv_T}

Electric potential (comp.V_all_Cell) (comp_V_all_Cell)

General

Description	Value
Field components	comp.V_all_Cell

Electric potential (comp.VQM) (comp_VQM)

General

Description	Value
Field components	comp.VQM

For 1 (for1)

General

Description	Value
Defined by study step	Schrödinger - Poisson

Dependent Variables: Copy Electric Potential from Previous Iteration (v3)

Initial values of variables solved for

Description	Value
Solution	Solution 3

Values of variables not solved for

Description	Value
Method	Solution
Solution	Solution 3

Log

<---- Dependent Variables: Copy Electric Potential from Previous Iteration in
 Full quantum approach (High-Precision and slow, Matlab function)/Solution
 3 (sol3) ----->

Started at Jun 11, 2022, 11:06:52 AM.

Initial values of variables solved for: Solution 3 (sol3).
 Values of variables not solved for: Solution 3 (sol3).

Solution time: 0 s.

Physical memory: 1.72 GB

Virtual memory: 1.7 GB

Ended at Jun 11, 2022, 11:06:52 AM.

----- Dependent Variables: Copy Electric Potential from Previous Iteration in
 Full quantum approach (High-Precision and slow, Matlab function)/Solution
 3 (sol3) ----->

Reciprocal wall distance (comp.G) (comp_G)

General

Description	Value
Field components	comp.G
Solve for this field	Off

Wave function (comp.psi) (comp_psi)

General

Description	Value
Field components	comp.psi
Solve for this field	Off

Charged particle density (comp.schrp.n_sum) (comp_schrp_n_sum)

General

Description	Value
Field components	comp.schrp.n_sum
Solve for this field	Off

Space charge density from previous iteration (comp.schrp.rhoq_old) (comp_schrp_rhoq_old)

General

Description	Value
Field components	comp.schrp.rhoq_old

Electric potential from previous iteration (comp.schrp.V_old) (comp_schrp_V_old)

General

Description	Value
Field components	comp.schrp.V_old

Temperature (comp.T) (comp_T)

General

Description	Value
Field components	comp.T
Internal variables	{comp.uflux.T, comp.dflux.T, comp.ht.dt2Inv_T}
Solve for this field	Off

Electric potential (comp.V_all_Cell) (comp_V_all_Cell)

General

Description	Value
Field components	comp.V_all_Cell
Solve for this field	Off

Electric potential (comp.VQM) (comp_VQM)

General

Description	Value
Field components	comp.VQM
Solve for this field	Off

General

Description	Value
Solution	Solution Store 2

Compile Equations 1: Schrödinger-Poisson (st3)

Study and step

Description	Value
Use study	Full quantum approach (High - Precision and slow, Matlab function)
Use study step	Schrödinger - Poisson

Log

<---- Compile Equations 1: Schrödinger-Poisson in Full quantum approach
 (High-Precision and slow, Matlab function)/Solution 3 (sol3) ----->

Started at Jun 11, 2022, 11:06:52 AM.

Geometry shape function: Quadratic Lagrange

Time: 3 s.

Physical memory: 1.72 GB

Virtual memory: 1.7 GB

Ended at Jun 11, 2022, 11:06:55 AM.

----- Compile Equations 1: Schrödinger-Poisson in Full quantum approach
 (High-Precision and slow, Matlab function)/Solution 3 (sol3) ----->

Dependent Variables 1: to Solve for Wave Function (v4)

Initial values of variables solved for

Description	Value
Method	Solution
Solution	Solution 3

Values of variables not solved for

Description	Value
Method	Solution
Solution	Solution 3

Log

<---- Dependent Variables 1: to Solve for Wave Function in Full quantum approach
 (High-Precision and slow, Matlab function)/Solution 3 (sol3) ----->

Started at Jun 11, 2022, 11:06:56 AM.

Initial values of variables solved for: Solution 3 (sol3).

Values of variables not solved for: Solution 3 (sol3).

Solution time: 0 s.

Physical memory: 1.72 GB

Virtual memory: 1.7 GB

Ended at Jun 11, 2022, 11:06:56 AM.

----- Dependent Variables 1: to Solve for Wave Function in Full quantum approach
 (High-Precision and slow, Matlab function)/Solution 3 (sol3) ----->

Reciprocal wall distance (comp.G) (comp_G)

General

Description	Value
Field components	comp.G
Solve for this field	Off

Wave function (comp.psi) (comp_psi)

General

Description	Value
Field components	comp.psi

Charged particle density (comp.schrp.n_sum) (comp_schrp_n_sum)

General

Description	Value
Field components	comp.schrp.n_sum
Solve for this field	Off

Space charge density from previous iteration (comp.schrp.rhoq_old) (comp_schrp_rhoq_old)

General

Description	Value
Field components	comp.schrp.rhoq_old
Solve for this field	Off

Electric potential from previous iteration (comp.schrp.V_old) (comp_schrp_V_old)

General	
Description	Value
Field components	comp.schrp.V_old
Solve for this field	Off

Temperature (comp.T) (comp_T)

General	
Description	Value
Field components	comp.T
Internal variables	{comp.uflux.T, comp.dflux.T, comp.ht.dt2Inv_T}
Solve for this field	Off

Electric potential (comp.V_all_Cell) (comp_V_all_Cell)

General	
Description	Value
Field components	comp.V_all_Cell
Solve for this field	Off

Electric potential (comp.VQM) (comp_VQM)

General	
Description	Value
Field components	comp.VQM
Solve for this field	Off

Eigenvalue Solver 1: Solve for Wave Function (e1)

General	
Description	Value
Defined by study step	Schrödinger - Poisson
Relative tolerance	1.0E-10
Unit	rad/s
Value of eigenvalue linearization point	0.1

Values of linearization point

Description	Value
Store linearization point and deviation in output	On

Log

<---- Eigenvalue Solver 1: Solve for Wave Function in Full quantum approach

(High-Precision and slow, Matlab function)/Solution 3 (sol3) ----->

Started at Jun 11, 2022, 11:06:56 AM.

Eigenvalue solver

Number of degrees of freedom solved for: 769.

Symmetric matrices found

Scales for dependent variables:

Wave function (comp.psi): 2.2e-16

Orthonormal null-space function used.

Nonsymmetric eigenvalue solver

Iter ErrEst Nconv

1 1e-07 4

2 6.6e-15 6

30 linear system solutions.

30 matrix multiplications.

28 re-orthogonalizations.

Solution time: 1 s.

Physical memory: 1.72 GB

Virtual memory: 1.7 GB

Ended at Jun 11, 2022, 11:06:56 AM.

<---- Eigenvalue Solver 1: Solve for Wave Function in Full quantum approach

(High-Precision and slow, Matlab function)/Solution 3 (sol3) ----->

Solution Store 3 (su3)

General	
Description	Value
Solution	Solution Store 3

Combine Solutions: Sum Particle Density (cms1)

General	
Description	Value

Defined by study step	Schrödinger - Poisson
Solution operation	General summation

Use

Field	Value
Reciprocal wall distance (comp.G)	off
Wave function (comp.psi)	off
Charged particle density (comp.schrp.n_sum)	on
Space charge density from previous iteration (comp.schrp.rhoq_old)	off
Electric potential from previous iteration (comp.schrp.V_old)	off
Temperature (comp.T)	off
Electric potential (comp.V_all_Cell)	off
Electric potential (comp.VQM)	off

Expression

Field	Value
Reciprocal wall distance (comp.G)	comp.G
Wave function (comp.psi)	comp.psi
Charged particle density (comp.schrp.n_sum)	comp.schrp.ni
Space charge density from previous iteration (comp.schrp.rhoq_old)	comp.schrp.rhoq_old
Electric potential from previous iteration (comp.schrp.V_old)	comp.schrp.V_old
Temperature (comp.T)	comp.T
Electric potential (comp.V_all_Cell)	comp.V_all_Cell
Electric potential (comp.VQM)	comp.VQM

Log

<---- Combine Solutions: Sum Particle Density in Full quantum approach
(High-Precision and slow, Matlab function)/Solution 3 (sol3) ----->

Started at Jun 11, 2022, 11:06:56 AM.

Using General summation operation in Solution 3 (sol3) [Current].

Solution time: 0 s.

Physical memory: 1.72 GB

Virtual memory: 1.7 GB

Ended at Jun 11, 2022, 11:06:57 AM.

----- Combine Solutions: Sum Particle Density in Full quantum approach
(High-Precision and slow, Matlab function)/Solution 3 (sol3) ----->**Dependent Variables: Collect Particle Density and Previous Electric Potential (v5)**

Initial values of variables solved for

Description	Value
Method	Solution
Solution	Solution 3

Values of variables not solved for

Description	Value
Method	Solution
Solution	Solution 3

Log

<---- Dependent Variables: Collect Particle Density and Previous Electric Potential in Full quantum approach (High-Precision and slow, Matlab function)/Solution 3 (sol3) ----->

Started at Jun 11, 2022, 11:06:57 AM.

Initial values of variables solved for: Solution 3 (sol3), lambda=1 [Automatic (single solution)].

Values of variables not solved for: Solution 3 (sol3), Solution Store 2 (sol15).

Solution time: 0 s.

Physical memory: 1.72 GB

Virtual memory: 1.7 GB

Ended at Jun 11, 2022, 11:06:57 AM.

----- Dependent Variables: Collect Particle Density and Previous Electric Potential in Full quantum approach (High-Precision and slow, Matlab function)/Solution 3 (sol3) ----->

Reciprocal wall distance (comp.G) (comp_G)

General

Description	Value
Field components	comp.G
Solve for this field	Off

Wave function (comp.psi) (comp_psi)

General

Description	Value
Field components	comp.psi

Solve for this field	Off
----------------------	-----

Charged particle density (comp.schrp.n_sum) (comp_schrp_n_sum)

General

Description	Value
Field components	comp.schrp.n_sum

Space charge density from previous iteration (comp.schrp.rhoq_old) (comp_schrp_rhoq_old)

General

Description	Value
Field components	comp.schrp.rhoq_old
Solve for this field	Off

Electric potential from previous iteration (comp.schrp.V_old) (comp_schrp_V_old)

General

Description	Value
Field components	comp.schrp.V_old
Solve for this field	Off

Temperature (comp.T) (comp_T)

General

Description	Value
Field components	comp.T
Internal variables	{comp.uflux.T, comp.dflux.T, comp.ht.dt2Inv_T}
Solve for this field	Off

Electric potential (comp.V_all_Cell) (comp_V_all_Cell)

General

Description	Value
Field components	comp.V_all_Cell
Solve for this field	Off

Electric potential (comp.VQM) (comp_VQM)

General

Description	Value
Field components	comp.VQM
Solve for this field	Off

Compile Equations 2: Schrödinger-Poisson (st4)

Study and step

Description	Value
Use study	Full quantum approach (High - Precision and slow, Matlab function)
Use study step	Schrödinger - Poisson

Log

<---- Compile Equations 2: Schrödinger-Poisson in Full quantum approach
(High-Precision and slow, Matlab function)/Solution 3 (sol3) ----->

Started at Jun 11, 2022, 11:06:57 AM.

Geometry shape function: Quadratic Lagrange

Time: 3 s.

Physical memory: 1.72 GB

Virtual memory: 1.7 GB

Ended at Jun 11, 2022, 11:07:00 AM.

----- Compile Equations 2: Schrödinger-Poisson in Full quantum approach
(High-Precision and slow, Matlab function)/Solution 3 (sol3) ----->

Dependent Variables 2: to Solve for Electric Potential (v6)

Initial values of variables solved for

Description	Value
Method	Solution
Solution	Solution 3

Values of variables not solved for

Description	Value

Method	Solution
Solution	Solution 3

Log

<---- Dependent Variables 2: to Solve for Electric Potential in Full quantum approach (High-Precision and slow, Matlab function)/Solution 3 (sol3) ----
Started at Jun 11, 2022, 11:07:00 AM.
Initial values of variables solved for: Solution 3 (sol3).
Values of variables not solved for: Solution 3 (sol3).
Solution time: 0 s.
Physical memory: 1.72 GB
Virtual memory: 1.7 GB
Ended at Jun 11, 2022, 11:07:00 AM.
----- Dependent Variables 2: to Solve for Electric Potential in Full quantum approach (High-Precision and slow, Matlab function)/Solution 3 (sol3) --->

Reciprocal wall distance (comp.G) (comp_G)

General

Description	Value
Field components	comp.G
Solve for this field	Off

Wave function (comp.psi) (comp_psi)

General

Description	Value
Field components	comp.psi
Solve for this field	Off

Charged particle density (comp.schrp.n_sum) (comp_schrp_n_sum)

General

Description	Value
Field components	comp.schrp.n_sum
Solve for this field	Off

Space charge density from previous iteration (comp.schrp.rhoq_old) (comp_schrp_rhoq_old)

General

Description	Value
Field components	comp.schrp.rhoq_old
Solve for this field	Off

Electric potential from previous iteration (comp.schrp.V_old) (comp_schrp_V_old)

General

Description	Value
Field components	comp.schrp.V_old
Solve for this field	Off

Temperature (comp.T) (comp_T)

General

Description	Value
Field components	comp.T
Internal variables	{comp.uflux.T, comp.dflux.T, comp.ht.dt2Inv_T}

Electric potential (comp.V_all_Cell) (comp_V_all_Cell)

General

Description	Value
Field components	comp.V_all_Cell

Electric potential (comp.VQM) (comp_VQM)

General

Description	Value
Field components	comp.VQM

Stationary Solver 2: Solve for Electric Potential (s2)

Log

<---- Stationary Solver 2: Solve for Electric Potential in Full quantum approach

(High-Precision and slow, Matlab function)/Solution 3 (sol3) -----
 Started at Jun 11, 2022, 11:07:00 AM.
 Nonlinear solver
 Number of degrees of freedom solved for: 10850 (plus 3230 internal DOFs).
 Nonsymmetric matrix found.
 Scales for dependent variables:
 Temperature (comp.T): 8e+02
 Electric potential (comp.V_all_Cell): 21
 Electric potential (comp.VQM): 1.6e+10
 Orthonormal null-space function used.
 Iter SolEst ResEst Damping Stepsize #Res #Jac #Sol LinErr LinRes
 1 3.8e-08 9.7e-05 0.0100000 3.8e-08 2 1 2 8.7e-15 6.8e-14
 2 1e-08 5.5e-05 1.0000000 3.8e-08 4 2 4 8.4e-15 6.9e-14
 Solution time: 14 s.
 Physical memory: 1.76 GB
 Virtual memory: 1.74 GB
 Ended at Jun 11, 2022, 11:07:14 AM.
 ----- Stationary Solver 2: Solve for Electric Potential in Full quantum approach
 (High-Precision and slow, Matlab function)/Solution 3 (sol3) ----->

Fully Coupled 1 (fc1)

General

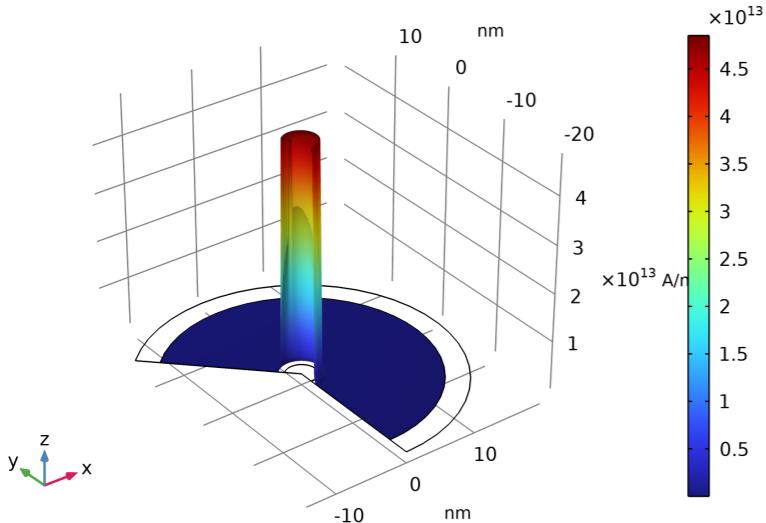
Description	Value
Linear solver	Direct 1

Method and termination

Description	Value
Initial damping factor	0.01
Minimum damping factor	1.0E-6
Maximum number of iterations	50

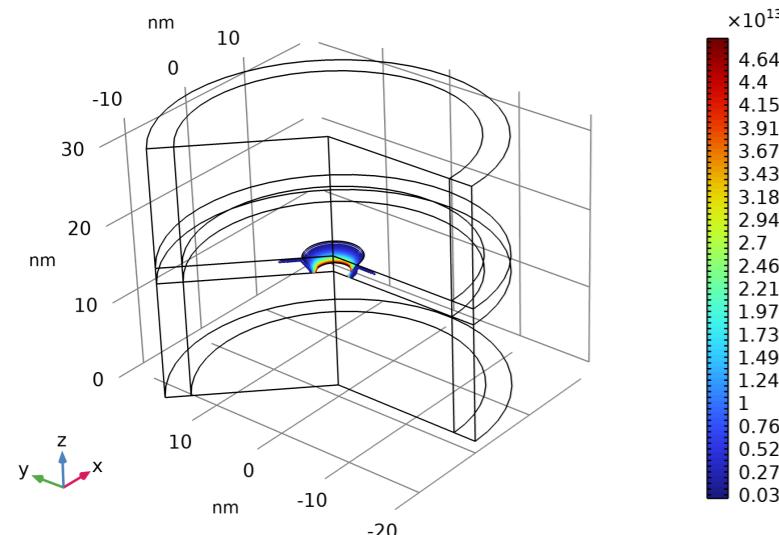
6. Results**6.1. Own plots from: Full quantum approach****6.1.1. Plot Groups**

Quantum tunneling current 2D

Surface: QuantumCurrent(wd.Dw[1/m],VQM[1/V],bnd1[1/K],bnd2[1/K])[A/m²] (A/mSurface: QuantumCurrent(wd.Dw[1/m],VQM[1/V],bnd1[1/K],bnd2[1/K])[A/m²] (A/m²)

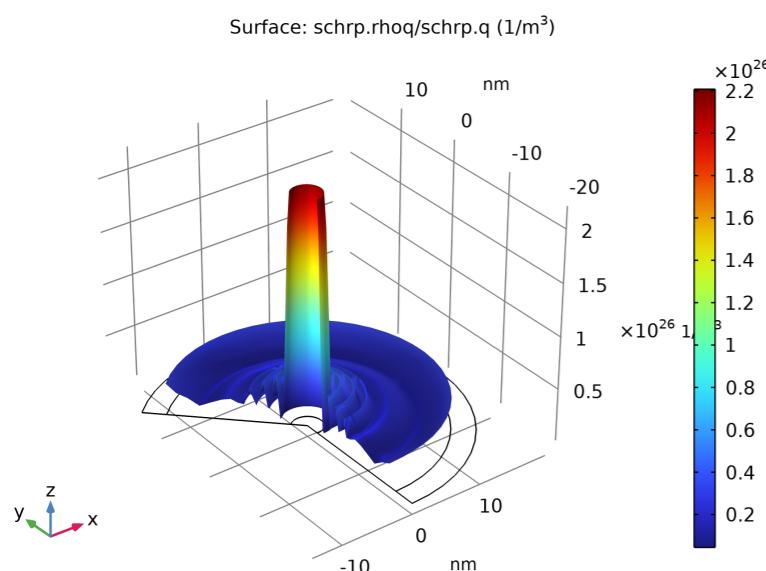
Quantum tunneling current 3D

Contour: QuantumCurrent(wd.Dw[1/m],VQM[1/V],bnd1[1/K],bnd2[1/K])[A/m²] (A/nm)



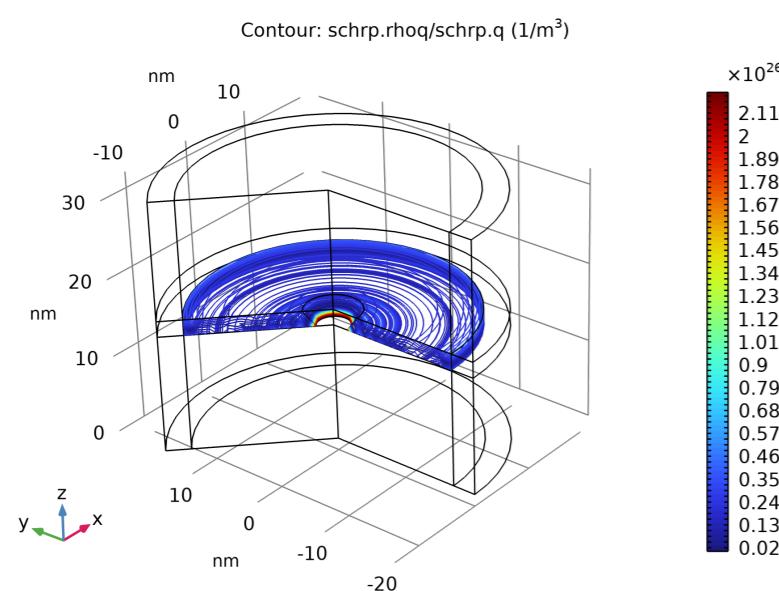
Contour: QuantumCurrent(wd.Dw[1/m],VQM[1/V],bnd1[1/K],bnd2[1/K])[A/m²] (A/m²)

Electron density 2D



Surface: schrp.rhoq/schrp.q (1/m³)

Electron density 3D



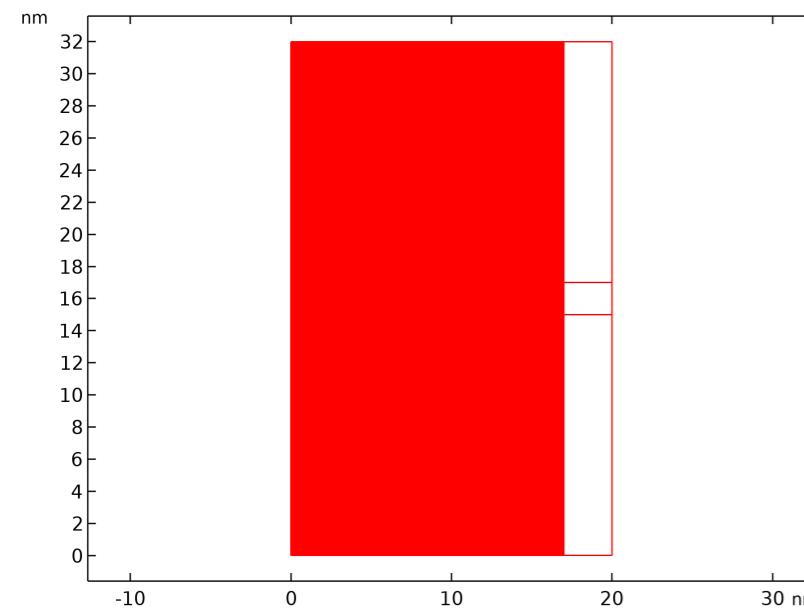
6.2. Datasets

6.2.1. Classic approach data set

Classic approach/Solution 1

Solution

Description	Value
Solution	Solution 1
Component	Component 2D (comp)



Dataset: Classic approach/Solution 1

Revolution 2D : Classic approach

Data

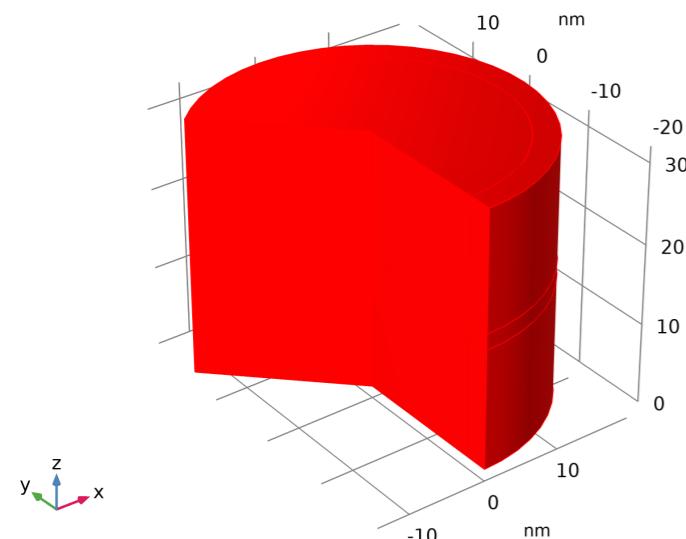
Description	Value
Dataset	Classic approach/Solution 1

Axis data

Description	Value
Axis entry method	Two points
Points	$\{\{0, 0\}, \{0, 1\}\}$

Revolution layers

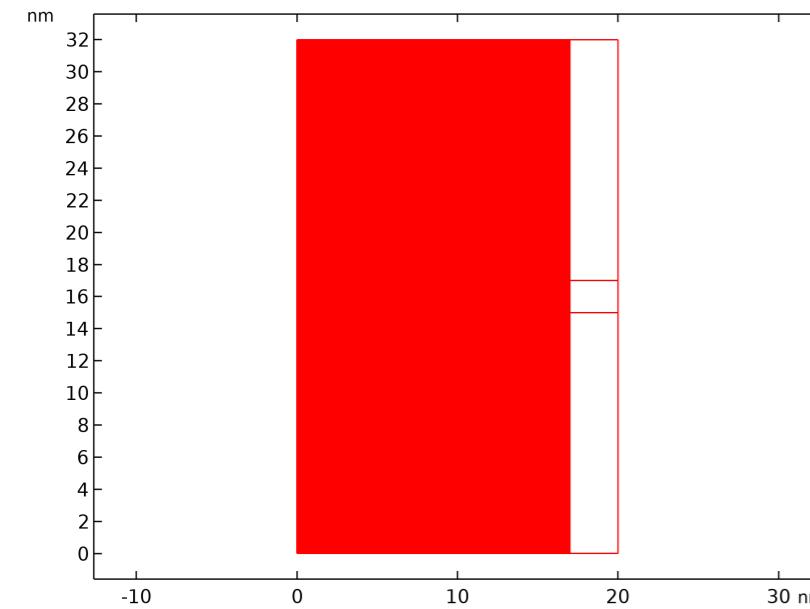
Description	Value
Start angle	-90
Revolution angle	225



Probe Solution 2

Solution

Description	Value
Solution	Solution 3
Component	Component 2D (comp)



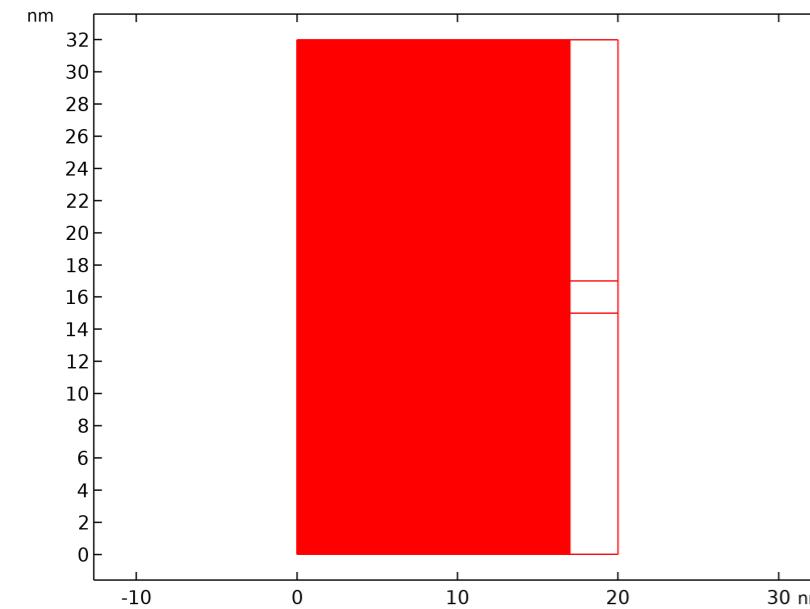
Dataset: Probe Solution 2

6.2.2. Semi-Classic approach data set

Semi-Classic approach/Solution 2

Solution

Description	Value
Solution	Solution 2
Component	Component 2D (comp)



Dataset: Semi-Classic approach/Solution 2

Revolution 2D : Semi-Classic approach

Data

Description	Value
Dataset	Semi - Classic approach/Solution 2

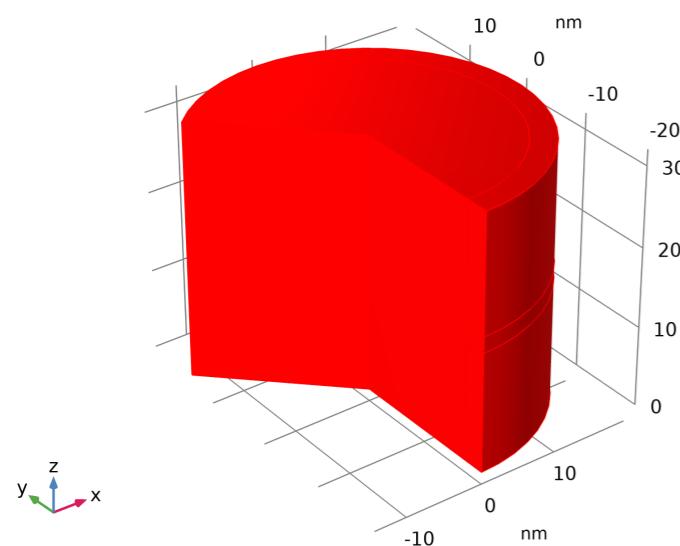
Axis data

Description	Value

Axis entry method	Two points
Points	<code>{ {0, 0}, {0, 1} }</code>

Revolution layers

Description	Value
Start angle	-90
Revolution angle	225



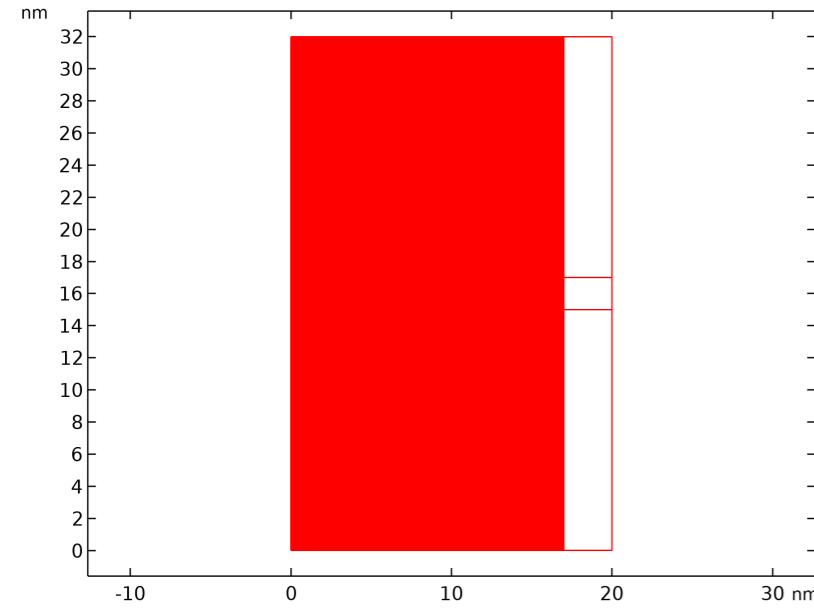
Dataset: Revolution 2D : Semi-Classic approach

6.2.3. Full quantum approach data set

Full quantum approach (High-Precision and slow, Matlab function)/Solution 3

Solution

Description	Value
Solution	Solution 3
Component	Component 2D (comp)



Dataset: Full quantum approach (High-Precision and slow, Matlab function)/Solution 3

Revolution 2D : Full quantum approach

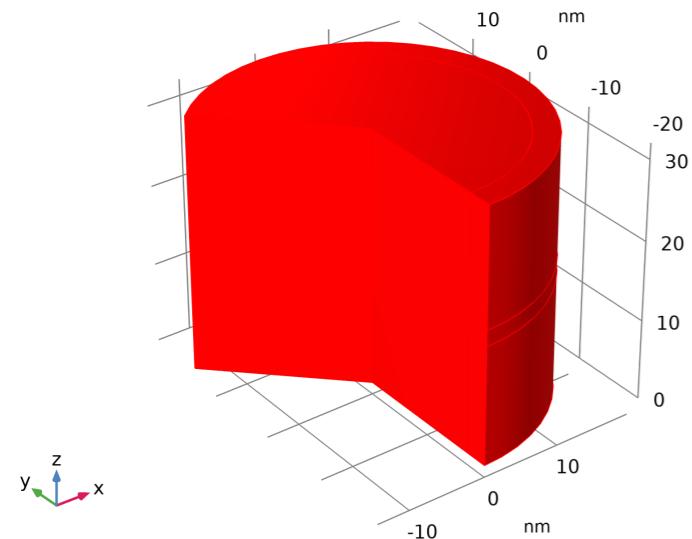
Data

Description	Value
Dataset	Full quantum approach (High - Precision and slow, Matlab function)/Solution 3

Axis data

Description	Value
Axis entry method	Two points
Points	<code>{ {0, 0}, {0, 1} }</code>

Revolution layers	
Description	Value
Start angle	-90
Revolution angle	225



Dataset: Revolution 2D : Full quantum approach

6.2.4. Own from: Full quantum approach

Cut Plane : Full quantum approach

Data

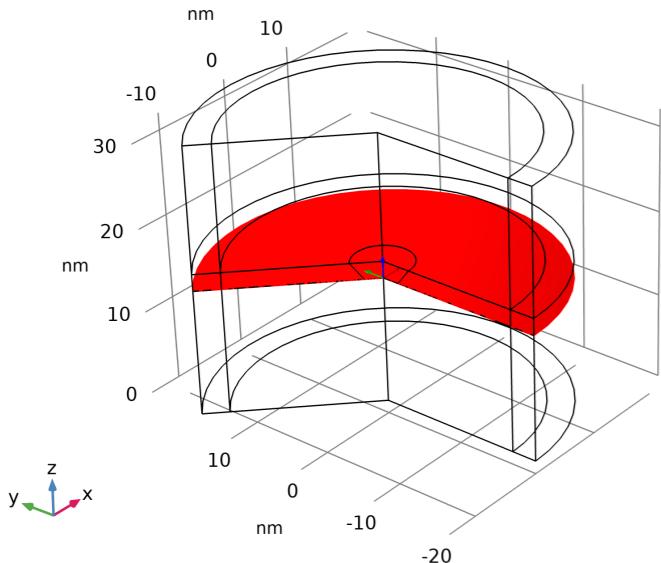
Description	Value
Dataset	Revolution 2D : Full quantum approach

Plane data

Description	Value
Plane type	Quick
Plane	xy - planes
z-coordinate	L_metal1

Advanced

Description	Value
Space variables	{cpl1x, cpl1y}
Normal variables	{cpl1nx, cpl1ny, cpl1nz}



Dataset: Cut Plane : Full quantum approach

6.2.5. Filter 1

Data	
Description	Value
Dataset	Revolution 2D - Classic approach

Expression

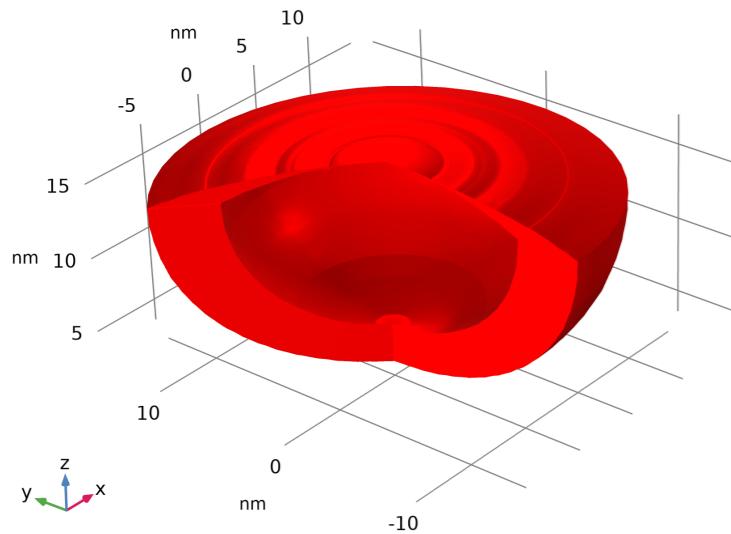
Description	Value
Expression	T
Unit	K
Description	Temperature

Filter

Description	Value
Bounds	Lower and upper
Lower bound	320
Upper bound	340
Geometry level	Take from dataset

Evaluation

Description	Value
Smoothing	Inside material domains



Dataset: Filter 1

6.2.6. Maximum temperature in the filament

Selection

Geometric entity level	Domain
Selection	Geometry geom2: Dimension 2: Domain 2

Data

Description	Value
Dataset	Probe Solution 2

Settings

Description	Value
Lagrange order	4

6.2.7. Density of current measured

Selection

Geometric entity level	Boundary
Selection	Geometry geom2: Dimension 1: Boundary 2

Data

Description	Value
Dataset	Probe Solution 2

Settings

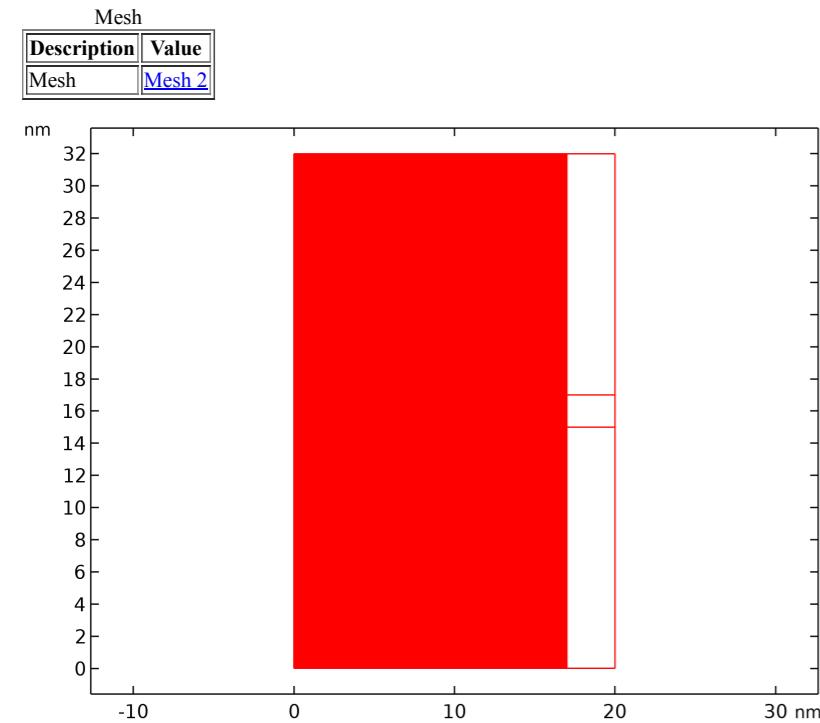
Description	Value
Method	Integration
Integration order	4
Integration order	On

6.2.8. Average temperature at the metal 1 edge

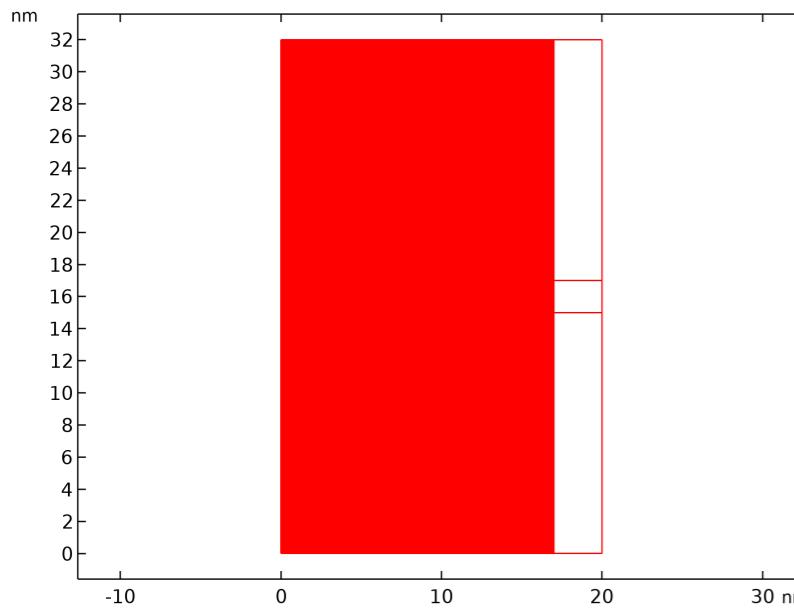
Selection	
Geometric entity level	Boundary
Selection	Geometry geom2: Dimension 1: Boundary 10
Data	
Description	Value
Dataset	Probe Solution 2
Settings	
Description	Value
Method	Integration
Integration order	4
Integration order	On

6.2.9. Average temperature at the metal 2 edge

Selection	
Geometric entity level	Boundary
Selection	Geometry geom2: Dimension 1: Boundary 9
Data	
Description	Value
Dataset	Probe Solution 2
Settings	
Description	Value
Method	Integration
Integration order	4
Integration order	On

6.2.10. Mesh 2*Dataset: Mesh 2***6.2.11. Full quantum approach (High-Precision and slow, Matlab function)/Solution Store 2**

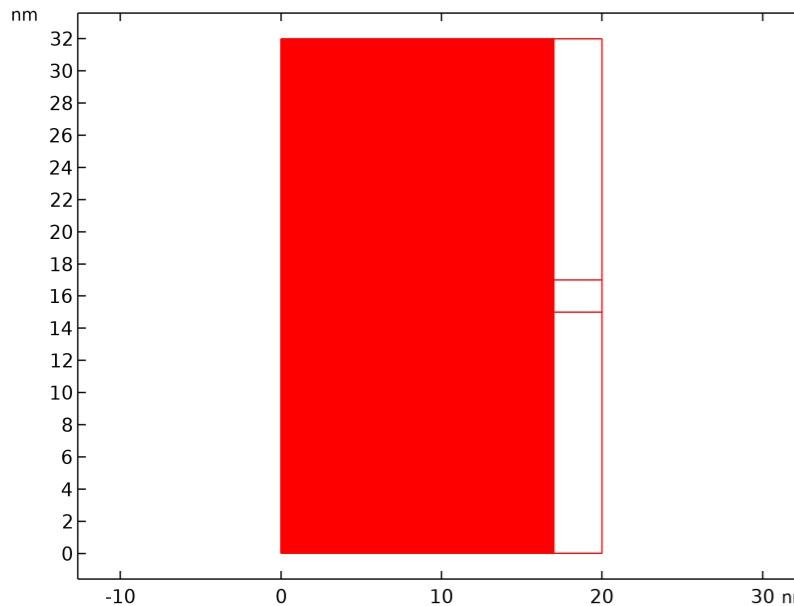
Solution	
Description	Value
Solution	Solution Store 2
Component	Component 2D (comp)



Dataset: Full quantum approach (High-Precision and slow, Matlab function)/Solution Store 2

6.2.12. Full quantum approach (High-Precision and slow, Matlab function)/Solution Store 3

Solution	
Description	Value
Solution	Solution Store 3
Component	Component 2D (comp)



Dataset: Full quantum approach (High-Precision and slow, Matlab function)/Solution Store 3

6.3. Derived Values

6.3.1. Classic approach derived values

Surface Maximum 1

Output	
Evaluated in	Table 1

Data

Description	Value
Dataset	Classic approach/Solution 1

Expressions

Expression	Unit	Description
T	K	Temperature

Line Average 1

Output		

Evaluated in [Table 2](#)

Data

Description	Value
Dataset	Classic approach/Solution 1

Expressions

Expression	Unit	Description
T	K	Temperature

Integration settings

Description	Value
Integration order	4
Compute surface integral	On

Line Average 2

Output

Evaluated in [Table 3](#)

Data

Description	Value
Dataset	Classic approach/Solution 1

Expressions

Expression	Unit	Description
T	K	Temperature

Integration settings

Description	Value
Integration order	4
Compute surface integral	On

Line Average 3

Output

Evaluated in [Table 4](#)

Data

Description	Value
Dataset	Classic approach/Solution 1

Expressions

Expression	Unit	Description
ec.Jiz	A/m^2	Conduction current density, z component

Integration settings

Description	Value
Integration order	4
Compute surface integral	On

6.3.2. Semi-Classic approach derived values

Surface Maximum 2

Output

Evaluated in [Table 6](#)

Data

Description	Value
Dataset	Semi - Classic approach/Solution 2

Expressions

Expression	Unit	Description
T	K	Temperature

Line Average 4

Output

Evaluated in [Table 7](#)

Data

Description	Value
Dataset	Semi - Classic approach/Solution 2

Expressions

Expression	Unit	Description
T	K	Temperature

Integration settings

Description	Value
Integration order	4
Compute surface integral	On

Line Average 5

Output						
Evaluated in Table 8						
Data						
<table border="1"> <thead> <tr><th>Description</th><th>Value</th></tr> </thead> <tbody> <tr><td>Dataset</td><td>Semi - Classic approach/Solution 2</td></tr> </tbody> </table>			Description	Value	Dataset	Semi - Classic approach/Solution 2
Description	Value					
Dataset	Semi - Classic approach/Solution 2					

Expressions

Expression	Unit	Description
T	K	Temperature

Integration settings

Description	Value
Integration order	4
Compute surface integral	On

Line Average 6

Output						
Evaluated in Table 9						
Data						
<table border="1"> <thead> <tr><th>Description</th><th>Value</th></tr> </thead> <tbody> <tr><td>Dataset</td><td>Classic approach/Solution 1</td></tr> </tbody> </table>			Description	Value	Dataset	Classic approach/Solution 1
Description	Value					
Dataset	Classic approach/Solution 1					

Expressions

Expression	Unit	Description
ec.Jiz	A/m^2	Conduction current density, z component

Integration settings

Description	Value
Integration order	4
Compute surface integral	On

6.3.3. Full quantum approach derived values

Surface Maximum 3

Output						
Evaluated in Table 10						
Data						
<table border="1"> <thead> <tr><th>Description</th><th>Value</th></tr> </thead> <tbody> <tr><td>Dataset</td><td>Full quantum approach (High - Precision and slow, Matlab function)/Solution 3</td></tr> </tbody> </table>			Description	Value	Dataset	Full quantum approach (High - Precision and slow, Matlab function)/Solution 3
Description	Value					
Dataset	Full quantum approach (High - Precision and slow, Matlab function)/Solution 3					

Expressions

Expression	Unit	Description
T	K	Temperature

Line Average 7

Output						
Evaluated in Table 11						
Data						
<table border="1"> <thead> <tr><th>Description</th><th>Value</th></tr> </thead> <tbody> <tr><td>Dataset</td><td>Full quantum approach (High - Precision and slow, Matlab function)/Solution 3</td></tr> </tbody> </table>			Description	Value	Dataset	Full quantum approach (High - Precision and slow, Matlab function)/Solution 3
Description	Value					
Dataset	Full quantum approach (High - Precision and slow, Matlab function)/Solution 3					

Expressions

Expression	Unit	Description
T	K	Temperature

Integration settings

Description	Value
Integration order	4
Compute surface integral	On

Line Average 8

Output						
Evaluated in Table 12						
Data						
<table border="1"> <thead> <tr><th>Description</th><th>Value</th></tr> </thead> <tbody> <tr><td>Dataset</td><td>Full quantum approach (High - Precision and slow, Matlab function)/Solution 3</td></tr> </tbody> </table>			Description	Value	Dataset	Full quantum approach (High - Precision and slow, Matlab function)/Solution 3
Description	Value					
Dataset	Full quantum approach (High - Precision and slow, Matlab function)/Solution 3					

Expressions

Expression	Unit	Description
T	K	Temperature

Integration settings

Description	Value
Integration order	4
Compute surface integral	On

Line Average 9

Output

Evaluated in [Table 13](#)

Data

Description	Value
Dataset	Full quantum approach (High - Precision and slow, Matlab function)/Solution 3

Expressions

Expression	Unit	Description
ec.Jz	A/m ²	Conduction current density, z component

Integration settings

Description	Value
Integration order	4
Compute surface integral	On

6.3.4. Volume Integration 1

Output

Evaluated in [Table 14](#)

Data

Description	Value
Dataset	Filter 1

Expressions

Expression	Unit	Description
l	nm ³	

Integration settings

Description	Value
Integration order	4

6.3.5. Maximum temperature in the filament

Output

Evaluated in [Probe Table 5](#)

Data

Description	Value
Dataset	Maximum temperature in the filament

Expressions

Expression	Unit	Description
T	K	Temperature

6.3.6. Density of current measured

Output

Evaluated in [Probe Table 5](#)

Data

Description	Value
Dataset	Density of current measured

Expressions

Expression	Unit	Description
ec.Jz	A	Current density, z component

6.3.7. Average temperature at the metal 1 edge

Output

Evaluated in [Probe Table 5](#)

Data

Description	Value
Dataset	Average temperature at the metal 1 edge

Expressions

Expression	Unit	Description
T	K	Temperature

6.3.8. Average temperature at the metal 2 edge

Output

Evaluated in [Probe Table 5](#)

Data

Description	Value	
Dataset	Average temperature at the metal 2 edge	
Expression	Unit	Description
T	K	Temperature

6.4. Tables**6.4.1. Classic approach tables**

Table 1

Surface Maximum 1

Temperature (K)
1178.6

Table 2

Line Average 1

Temperature (K)
300.21

Table 3

Line Average 2

Temperature (K)
349.59

Table 4

Line Average 3

Conduction current density, z component (A/m ²)
3.5568E11

6.4.2. Semi-Classic approach tables

Probe Table 5

Temperature (K), Maximum temperature in the filament	Current density, z component (A), Density of current measured	Temperature (K), Average temperature at the metal 1 edge	Temperature (K), Average temperature at the metal 2 edge
799.72	2.6386E-4	300.13	327.30

Table 6

Surface Maximum 2

Temperature (K)
1190.2

Table 7

Line Average 4

Temperature (K)
300.22

Table 8

Line Average 5

Temperature (K)
348.18

Table 9

Line Average 6

Conduction current density, z component (A/m ²)

3.5568E11

6.4.3. Full quantum approach tables

Table 10

Surface Maximum 3

Temperature (K)
799.75

Table 11

Line Average 7

Temperature (K)
300.13

Table 12

Line Average 8

Temperature (K)
327.32

Table 13

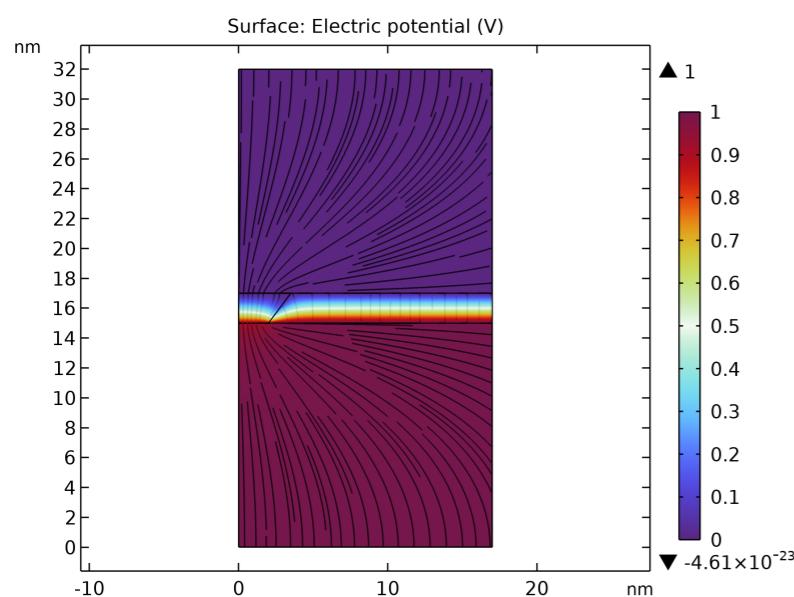
Line Average 9

Conduction current density, z component (A/m ²)
2.9063E11

6.4.4. Table 14

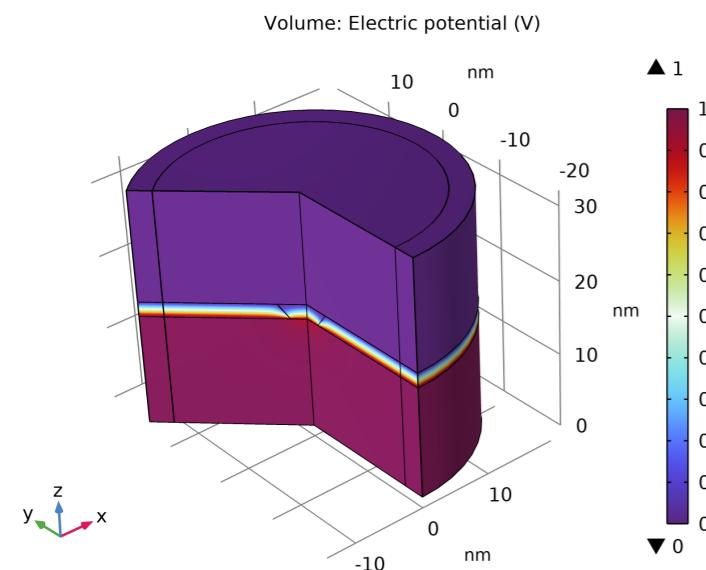
Volume Integration 1

1 (m ³)	1 (nm ³)
2.1201E-24	2120.1

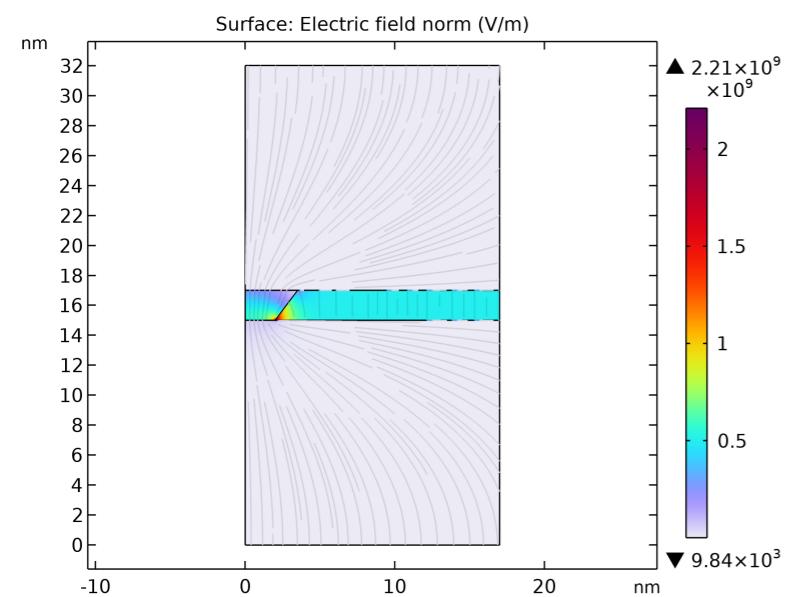
6.5. Classic approach plots**6.5.1. Electric Potential (ec)**

Surface: Electric potential (V)

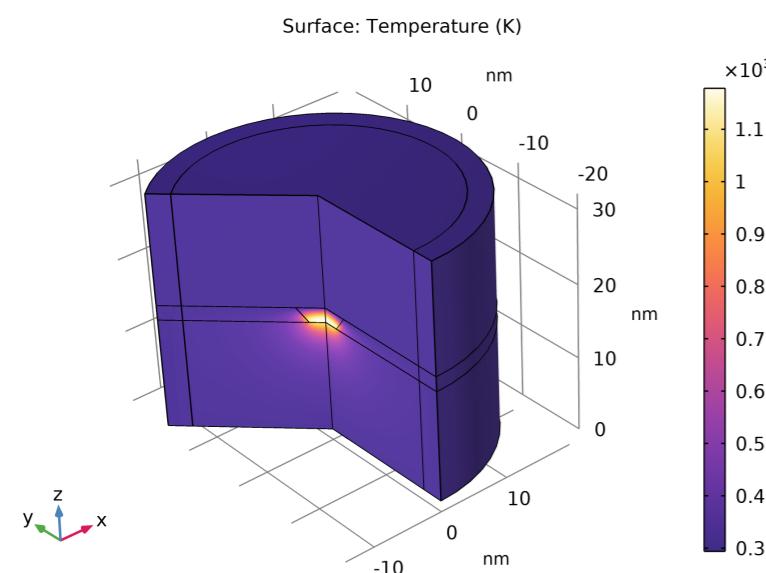
6.5.2. Electric Potential, Revolved Geometry (ec)

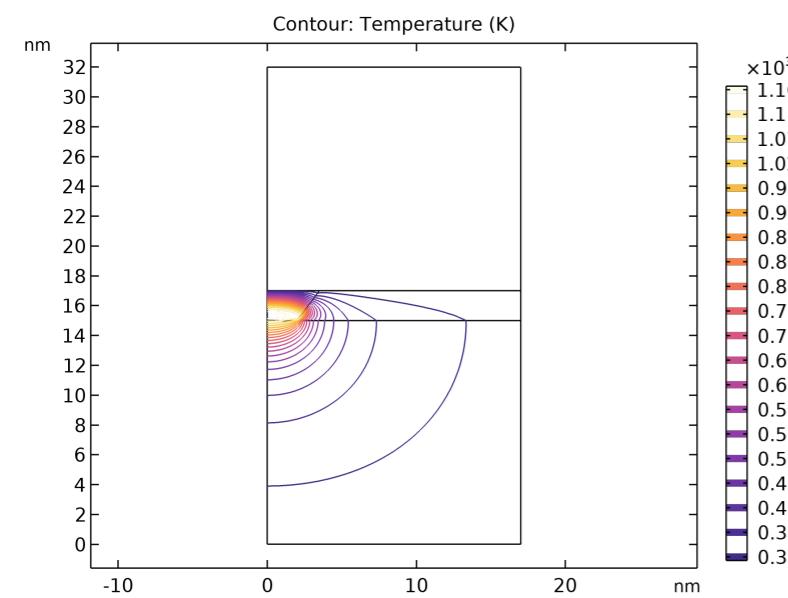


Volume: Electric potential (V)

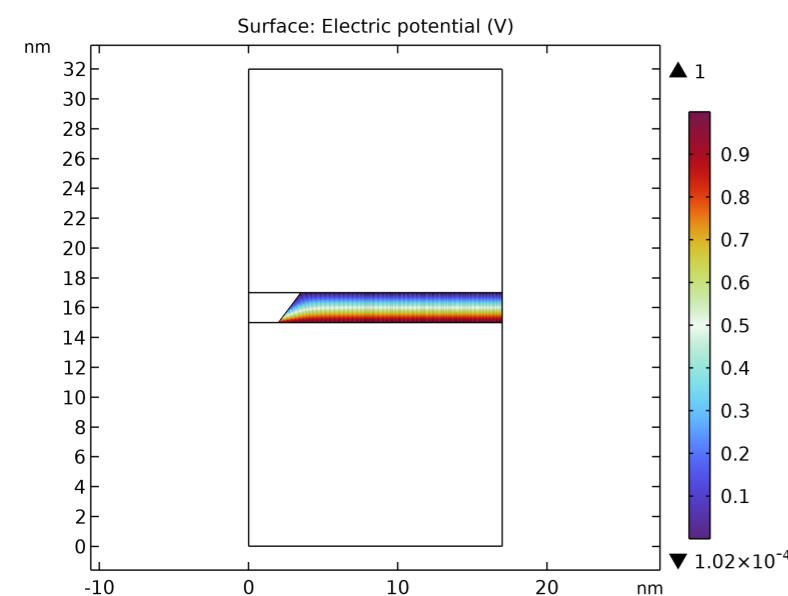
6.5.3. Electric Field Norm (ec)

Surface: Electric field norm (V/m)

6.5.4. Temperature, 3D (ht)

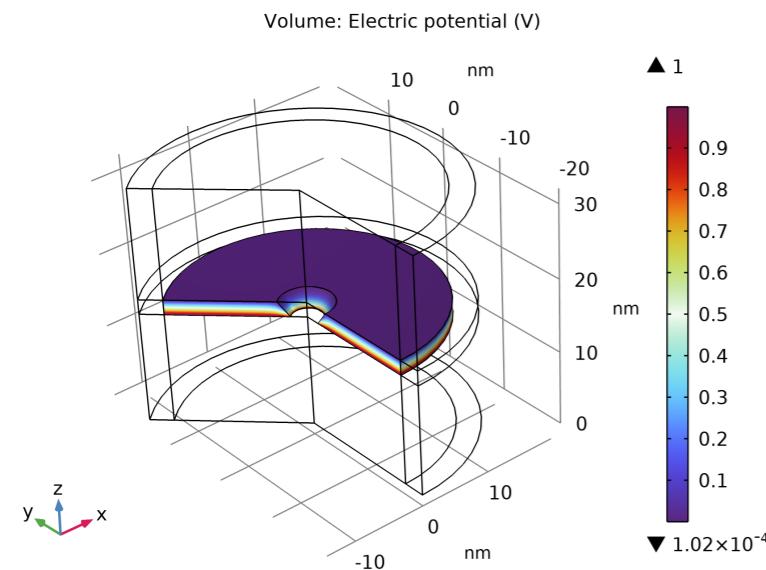
6.5.5. Isothermal Contours (ht)

Contour: Temperature (K)

6.5.6. Electric Potential (es)

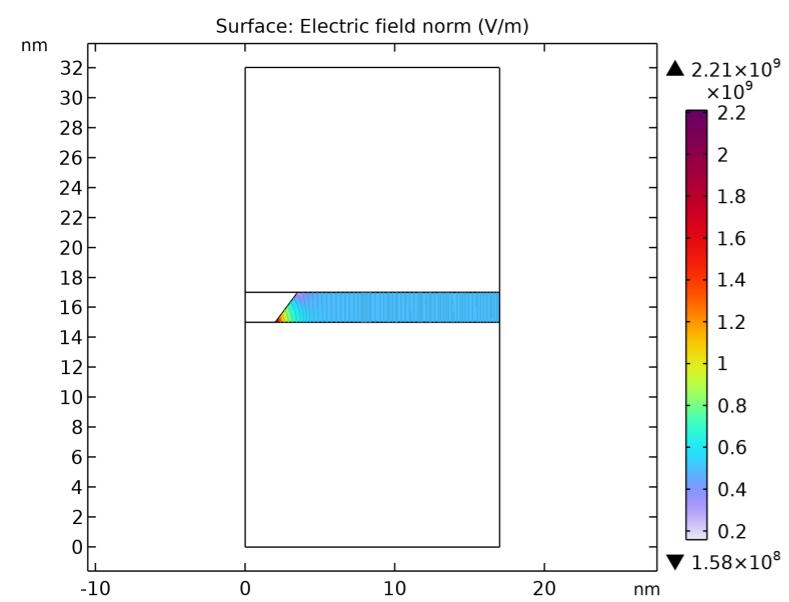
Surface: Electric potential (V)

6.5.7. Electric Potential, Revolved Geometry (es)



Volume: Electric potential (V)

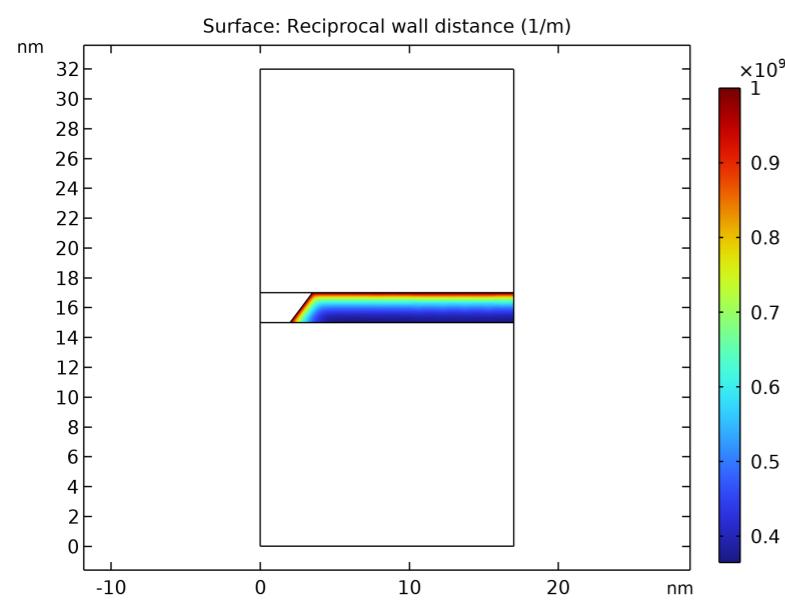
6.5.8. Electric Field Norm (es)



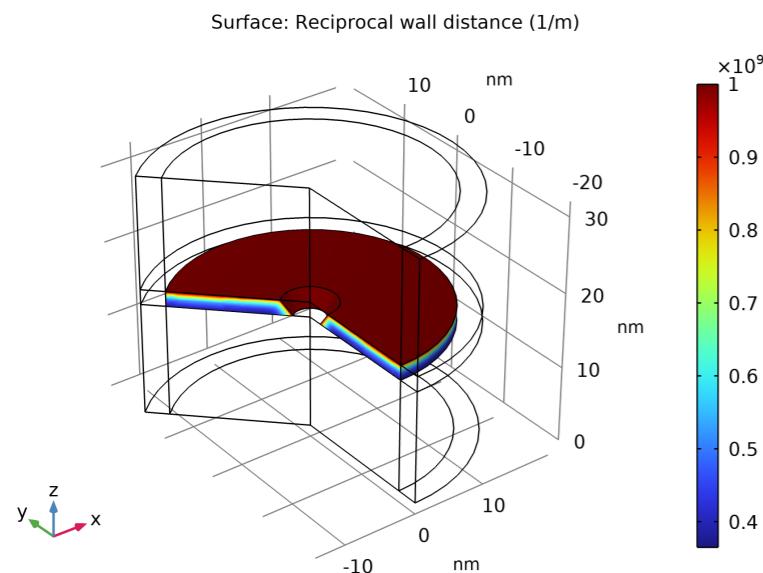
Surface: Electric field norm (V/m)

6.6. Semi-Classic approach plots

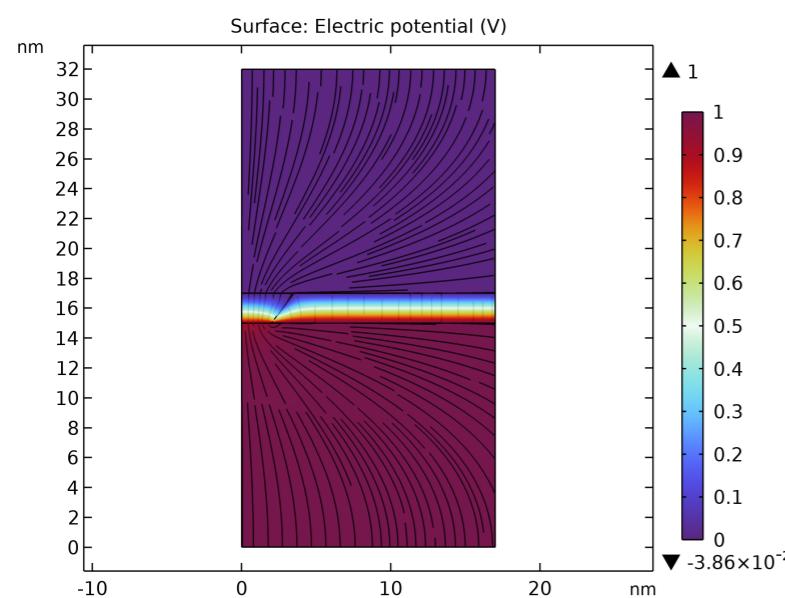
6.6.1. 2D Plot Group 9

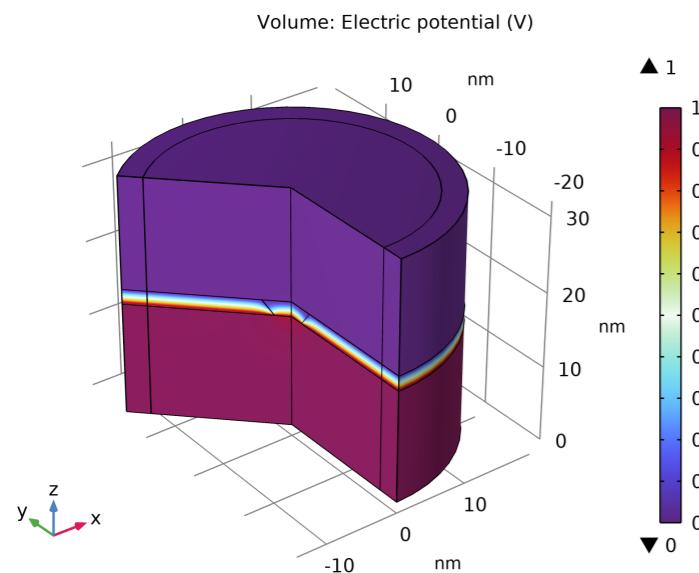


Surface: Reciprocal wall distance (1/m)

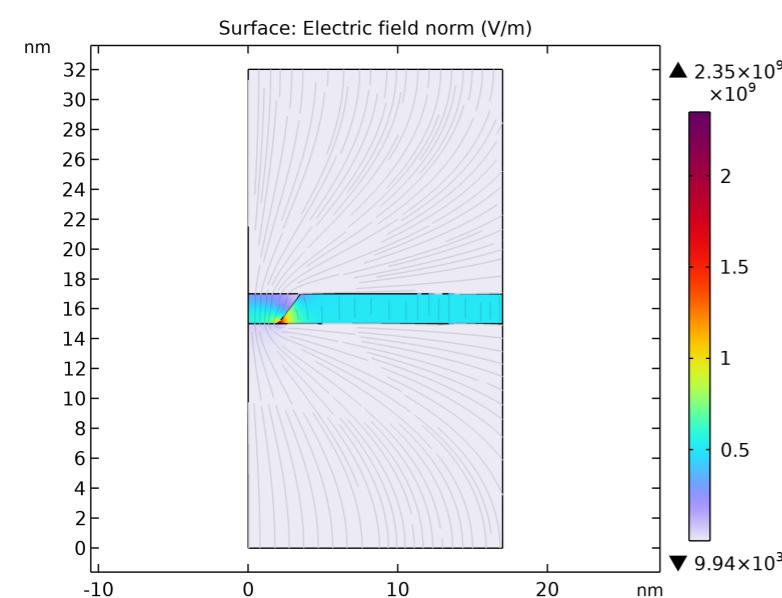
6.6.2. 3D Plot Group 10

Surface: Reciprocal wall distance (1/m)

6.6.3. Electric Potential (ec) 1

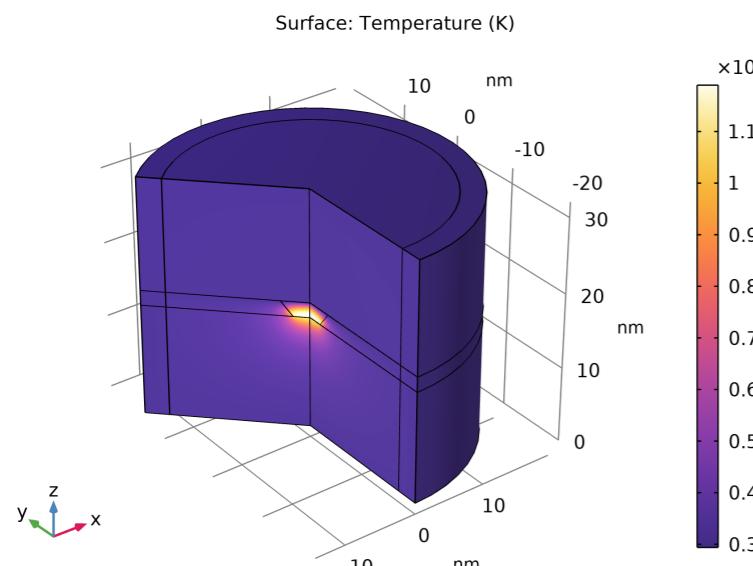
6.6.4. Electric Potential, Revolved Geometry (ec) 1

Volume: Electric potential (V)

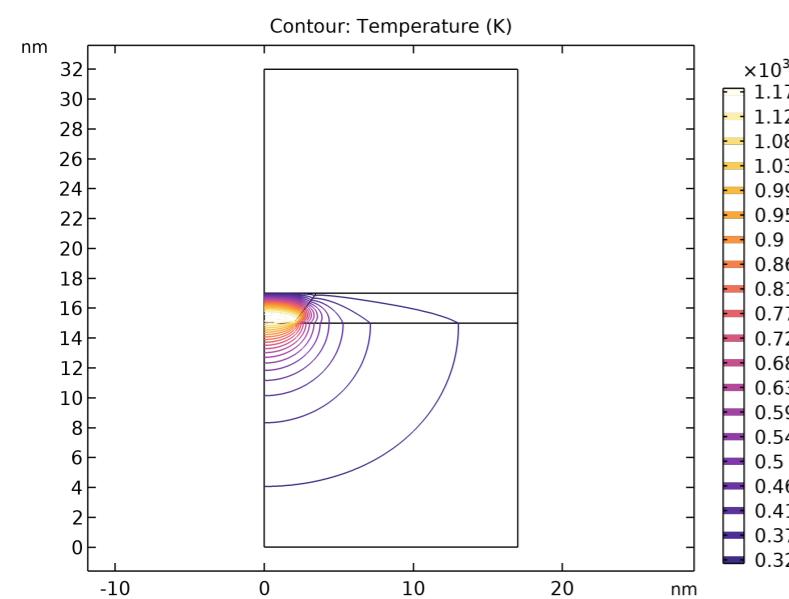
6.6.5. Electric Field Norm (ec) 1

Surface: Electric field norm (V/m)

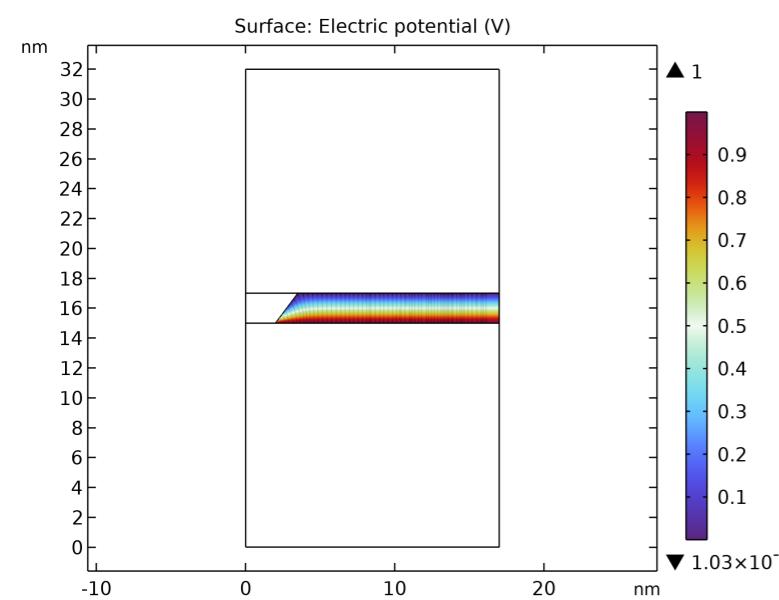
6.6.6. Temperature, 3D (ht) 1

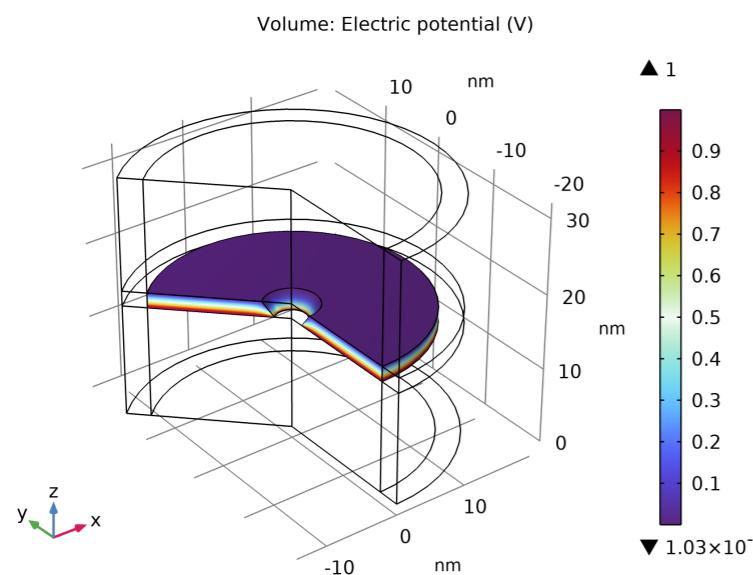


Surface: Temperature (K)

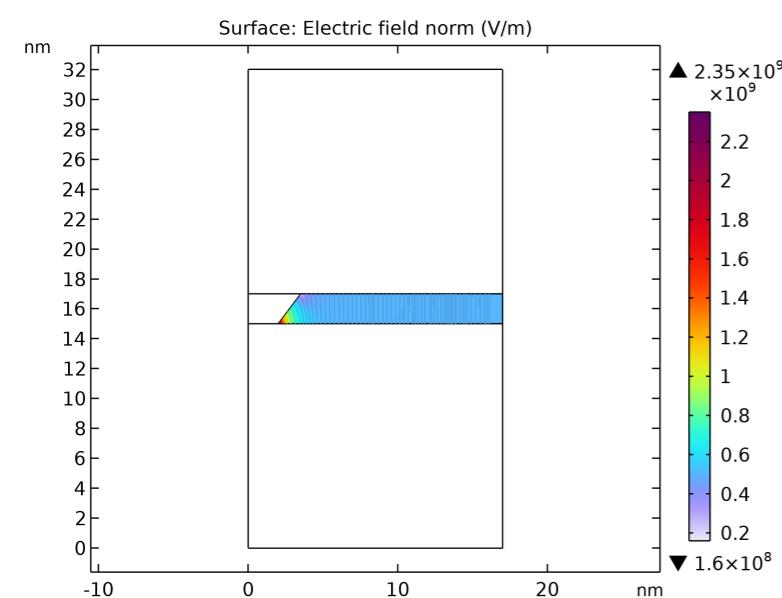
6.6.7. Isothermal Contours (ht) 1

Contour: Temperature (K)

6.6.8. Electric Potential (es) 1

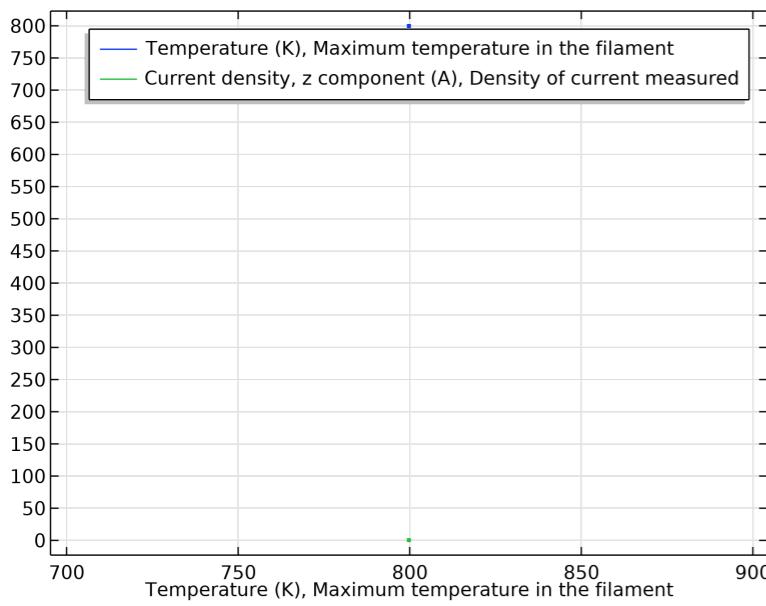
6.6.9. Electric Potential, Revolved Geometry (es) 1

Volume: Electric potential (V)

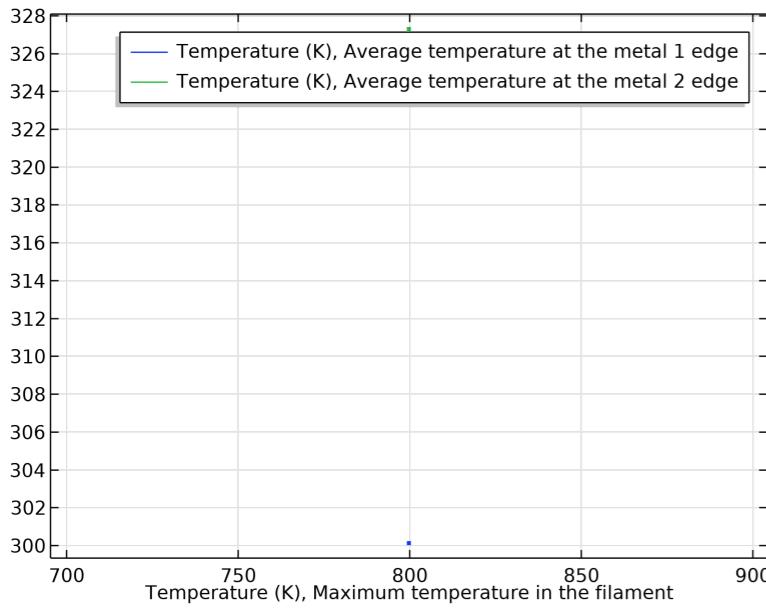
6.6.10. Electric Field Norm (es) 1

Surface: Electric field norm (V/m)

6.6.11. Probe Plot Group 19

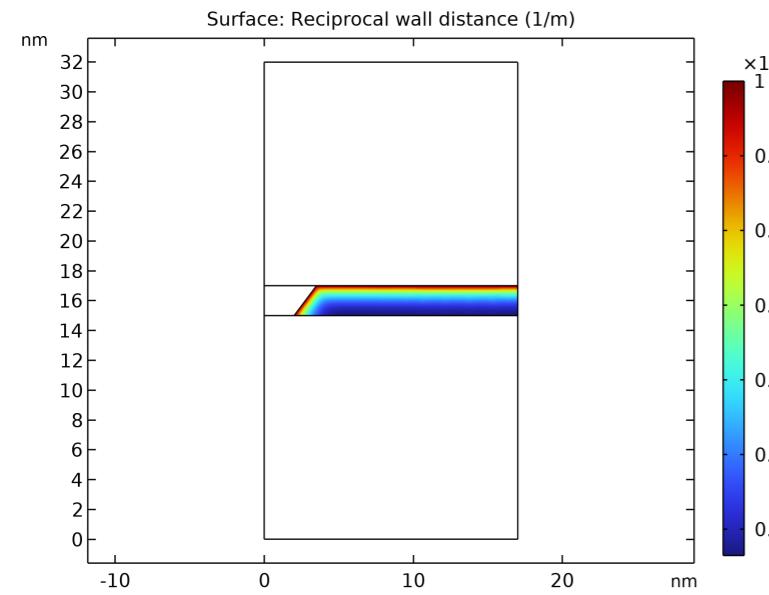


6.6.12. Probe Plot Group 20

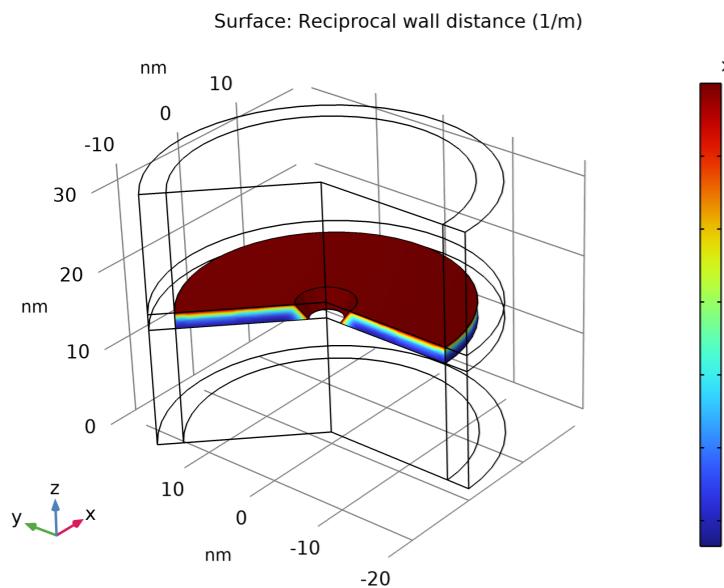


6.7. Full quantum approach plots

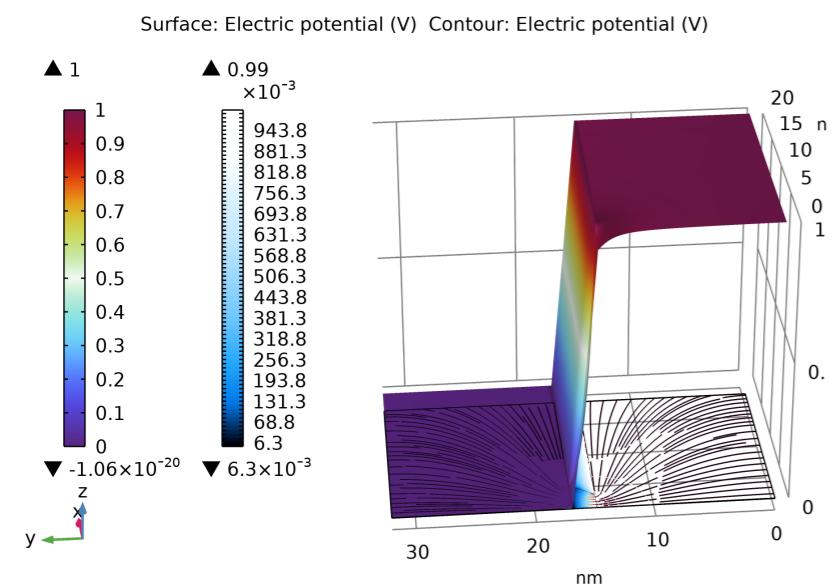
6.7.1. 2D Plot Group 21



Surface: Reciprocal wall distance (1/m)

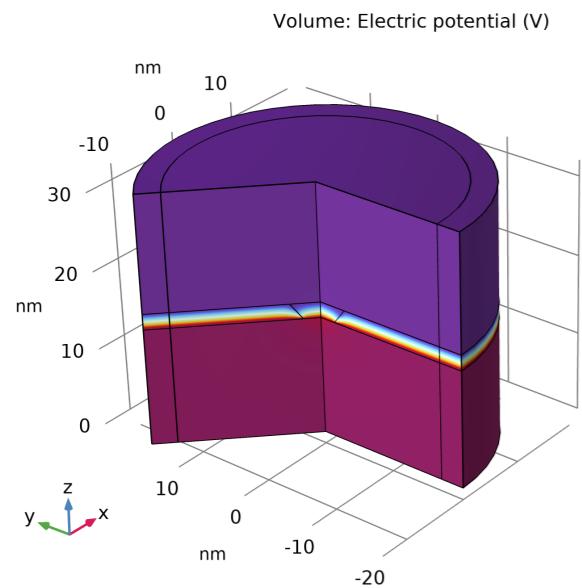
6.7.2. 3D Plot Group 22

Surface: Reciprocal wall distance (1/m)

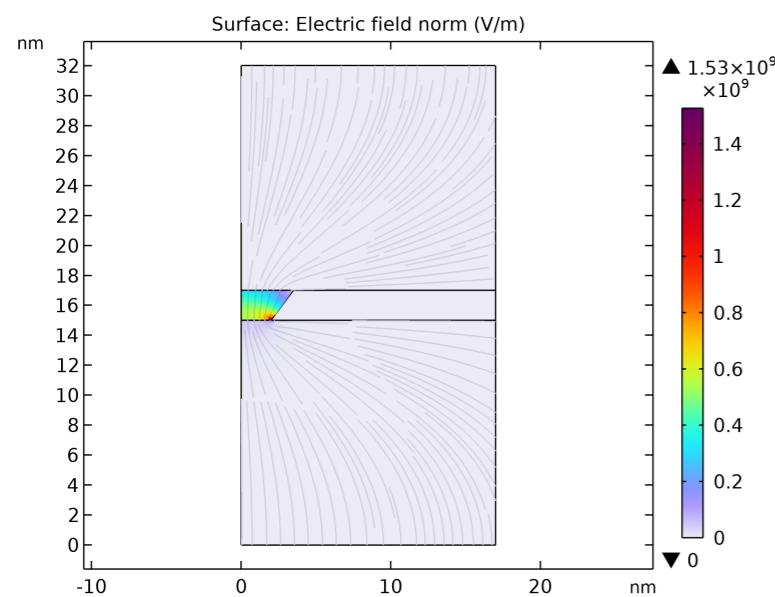
6.7.3. Electric Potential (ec) 2

Surface: Electric potential (V) Contour: Electric potential (V)

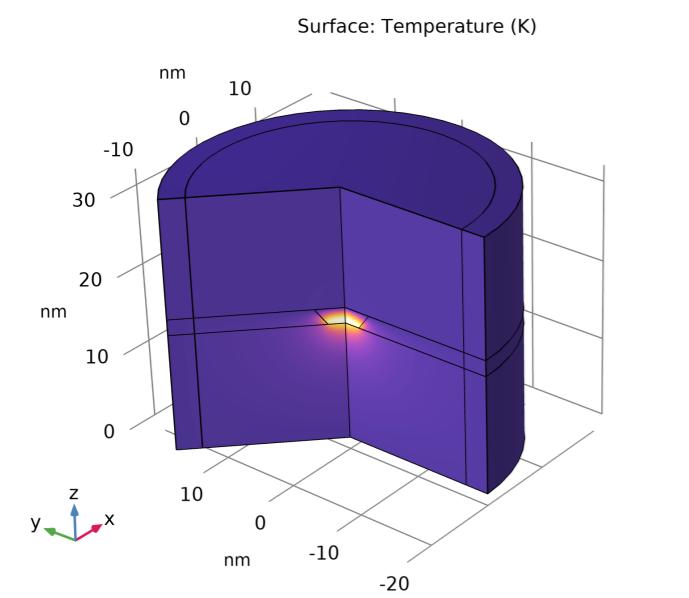
6.7.4. Electric Potential, Revolved Geometry (ec) 2



Volume: Electric potential (V)

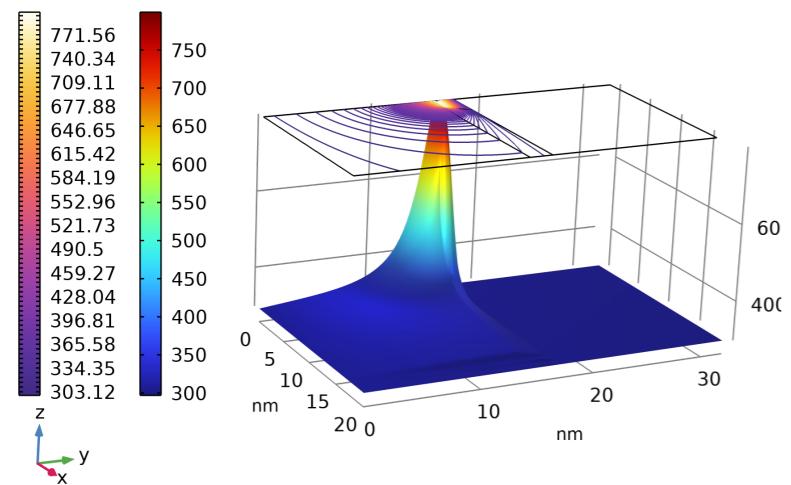
6.7.5. Electric Field Norm (ec) 2

Surface: Electric field norm (V/m)

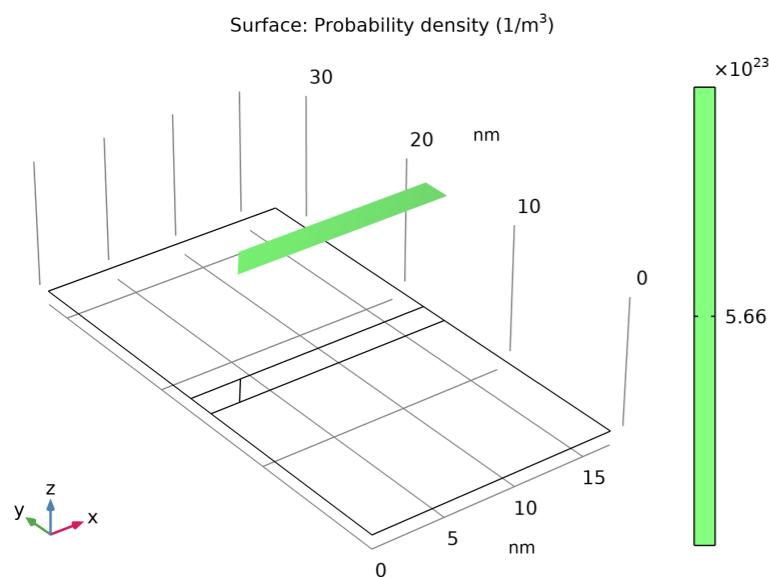
6.7.6. Temperature, 3D (ht) 2

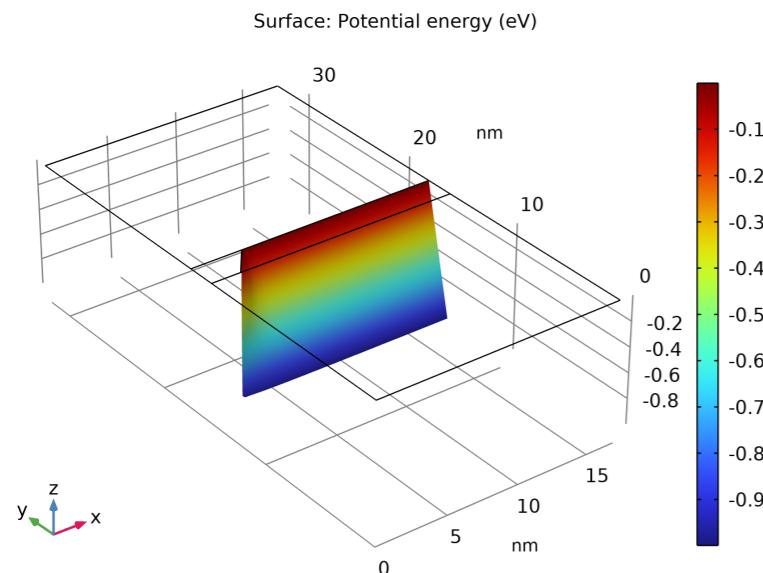
6.7. Isothermal Contours (ht) 2

Contour: Temperature (K) Surface: Temperature (K)



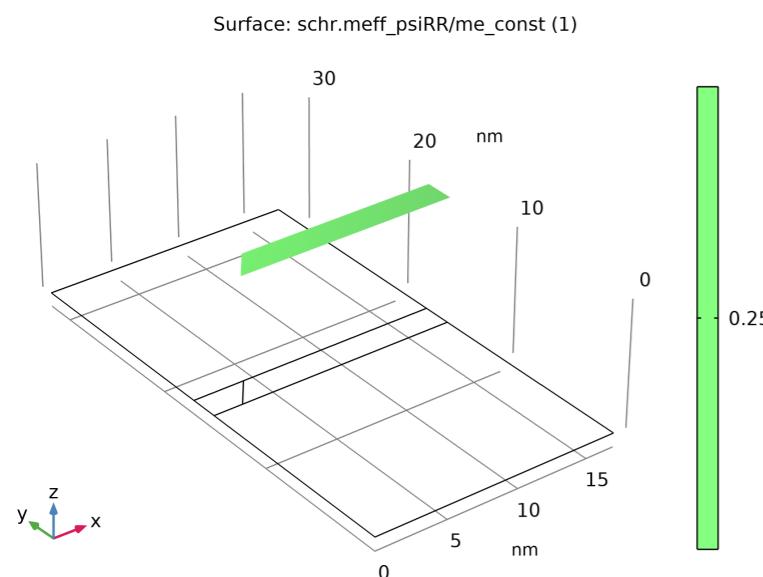
Contour: Temperature (K) Surface: Temperature (K)

6.7.8. Probability Density (schr)Surface: Probability density (1/m³)**6.7.9. Potential Energy (schr)**



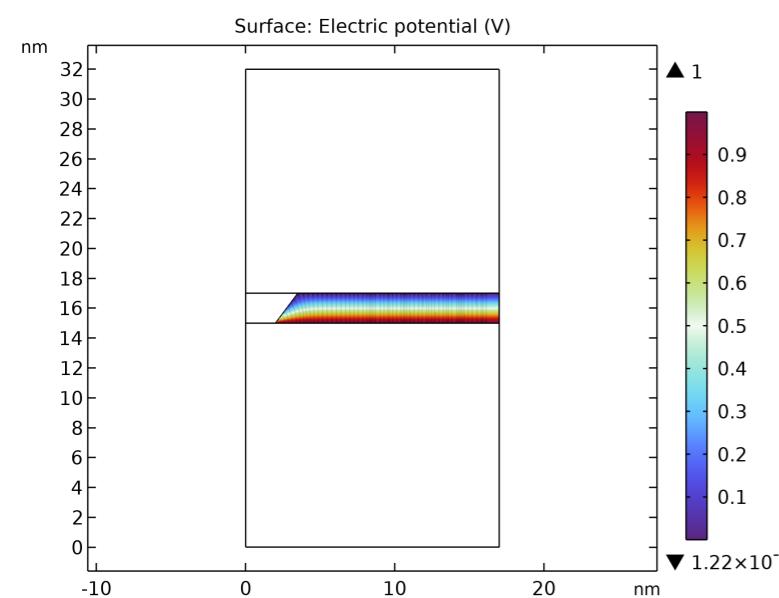
Surface: Potential energy (eV)

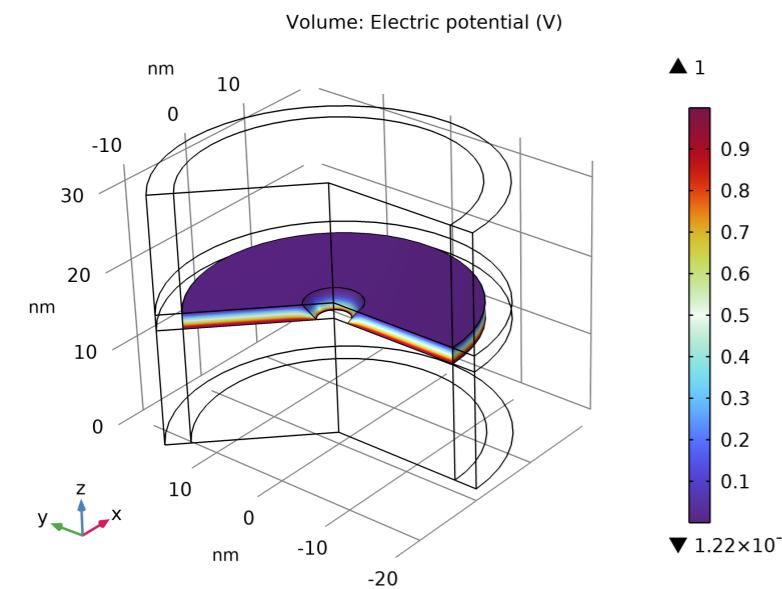
6.7.10. Effective Mass (schr)



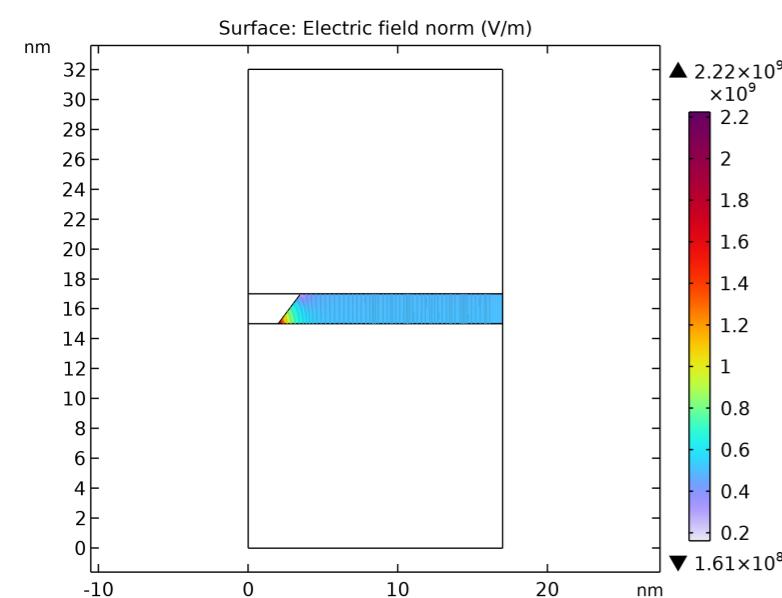
Surface: schr.meff_psiRR/me_const (1)

6.7.11. Electric Potential (es) 2



6.7.12. Electric Potential, Revolved Geometry (es) 2

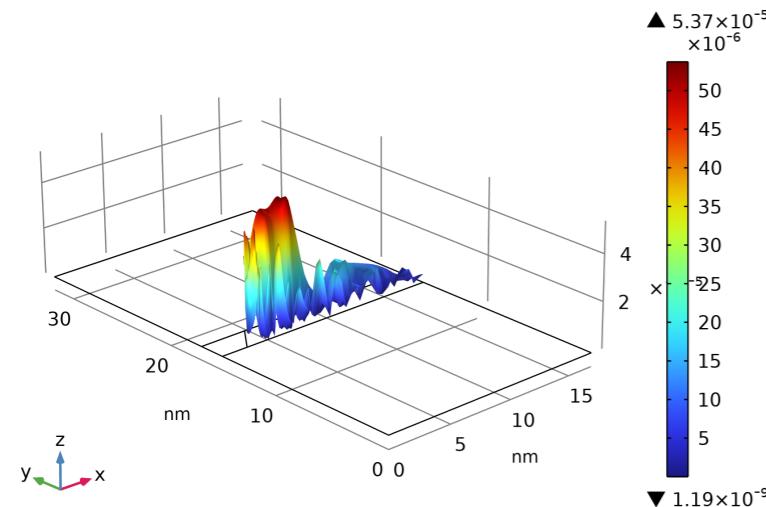
Volume: Electric potential (V)

6.7.13. Electric Field Norm (es) 2

Surface: Electric field norm (V/m)

6.7.14. Compare n with Previous Iteration (schrp)

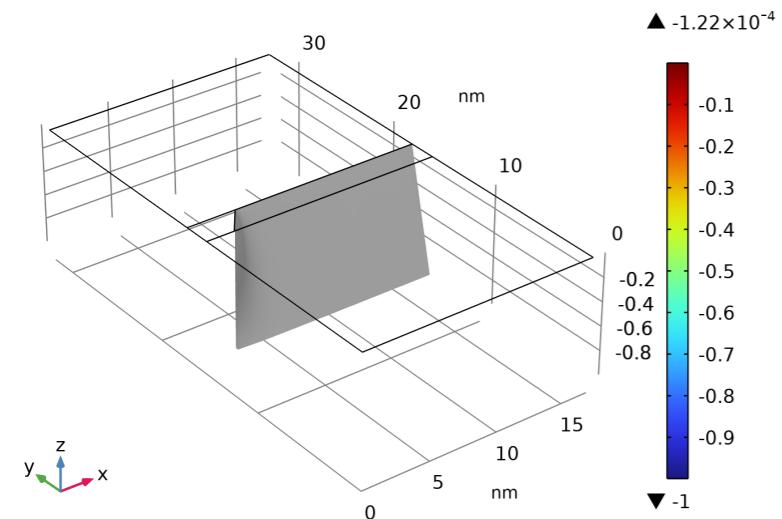
Error: Compare n with previous iteration



Error: Compare n with previous iteration

6.7.15. Compare V with Previous Iteration (schrp)

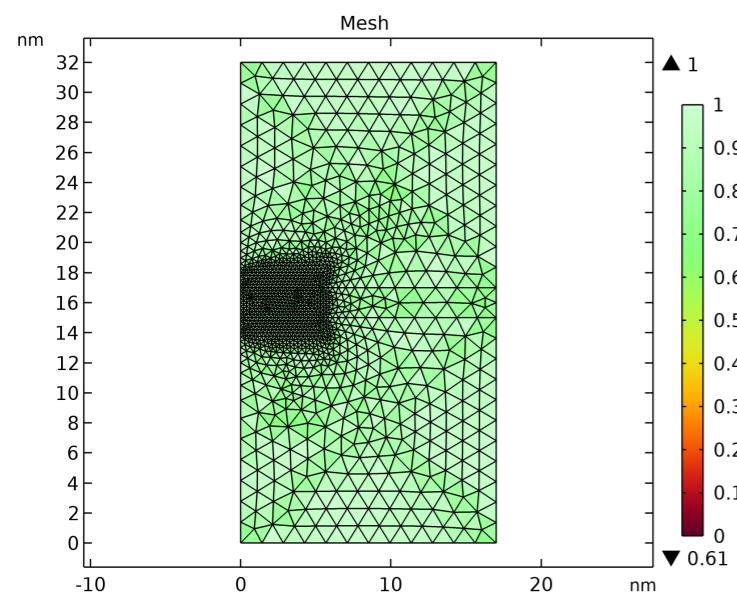
Compare V with previous iteration



Compare V with previous iteration

6.8. Plot Groups

6.8.1. Mesh Plot 40



Mesh