MATH 463 Topics in Biomathematics
Homework 7: Due Friday April 3 at Noon

Exercises:

1. For each of the two-dimensional linear systems, use the trace-determinant plane method to classify the type of phase-portrait the system exhibits. Find the eigenvalues for the linear system and explain why the values you obtain for them are consistent with the trace-determinant analysis. When possible, use the phaseR package in R to obtain a phase-portraits for each system and verify your previous conclusions.

   (a) $\dot{x} = 6x + 2y$, $\dot{y} = 2x + 3y$

   (b) $\dot{x} = x + 2y$, $\dot{y} = 4x + 3y$

   (c) $\dot{x} = -2x + 4y$, $\dot{y} = -x + y$

   (d) $\dot{x} = 2x + y$, $\dot{y} = x + 2y$

   (e) $\dot{x} = x + 5y$, $\dot{y} = x - 3y$

   (f) $\dot{x} = -x + ay$, $\dot{y} = ay$, $a \neq 0$

2. For each of the two-dimensional dynamical systems, find all steady-state values and classify their stability properties. Sketch any nullclines. Use the phaseR package in R to obtain phase-portraits for each system.

   (a) $\dot{x} = x + y - 2$, $\dot{y} = y - x$

   (b) $\dot{x} = x - y$, $\dot{y} = 1 - e^x$

   (c) $\dot{x} = y$, $\dot{y} = x(1 + y) - 1$

   (d) $\dot{x} = y - 2$, $\dot{y} = x^2 - 8y$

   (e) $\dot{x} = x(\lambda - ax - by)$, $\dot{y} = y(\mu - cx - dy)$, where $\lambda, \mu, a, b, c, d > 0$.