

MATH 463 Topics in Biomathematics
Homework 2: Due Friday February 14 at Noon

Exercises:

1. Let $N(t)$ be the population of an organism at time t . Suppose this function satisfies the differential equation

$$\frac{dN}{dt} = \kappa - rN,$$

where κ and r are positive constants.

- (a) Explain the meaning of the differential equation as a model for population growth.
 - (b) Based on the equation, what do you expect to happen to the size of the population in the long term?
 - (c) Use the method of separation to find a solution to the differential equation assuming some initial population N_0 .
 - (d) Does your solution of the differential confirm or otherwise your answer to part (b). Explain your answer.
2. The following function is used to describe a process:

$$X(t) = X_\infty + (X_0 - X_\infty)e^{-\lambda t}$$

Verify that $X(0) = X_0$ and that $\lim_{t \rightarrow \infty} X(t) = X_\infty$. Write down the ordinary differential equation corresponding to the given function, that is, complete the equation $\frac{dX}{dt} = \dots$ where in place of the dots some function of X is to be inserted.

3. Consider the differential equation

$$\frac{dC}{dt} = \frac{q_i(\gamma - C)}{V_0 + (q_i - q_0)t}, \quad C(0) = C_0. \quad (1)$$

Let $V_0 = 100$, $\gamma = 35$, $q_i = 5$ and $q_0 = 1$. Use R to obtain a numerical solution to this equation over the time interval $[0, 50]$ with initial condition $C_0 = 10$. Notice that the right hand side of this equation has explicit dependence on time t making this an example of a non-autonomous equation.

4. Lead enters the human body from the environment by inhalation, by eating, and by drinking. From the lungs and gut, lead is taken up by the blood and rapidly distributed to the liver and kidneys. It is slowly absorbed by other soft tissues and very slowly by the bones. Lead is excreted from the body primarily through the urinary system and through hair, nails, and sweat. Use figure 1 and compartment modeling techniques to write down a system of differential equations that models this situation.
5. Read the abstract and introduction sections of the paper *A Quantitative Model of Honey Bee Colony Population Dynamics* posted on D2L. Respond to the following prompts.

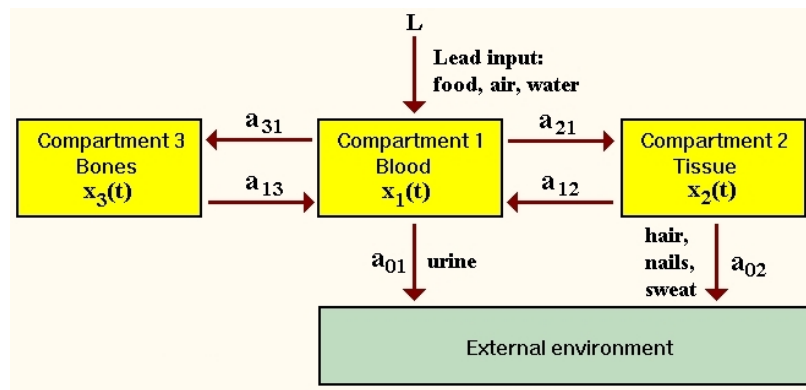


Figure 1: Compartment diagram for how lead affects the human body.

- (a) Based on the abstract of the paper, state in your own words the purpose of the article.
 - (b) What problem do the authors seek to address via a mathematical model?
 - (c) Explain in your own words why a mathematical model might be useful in order to address the problem(s) or question(s) that this paper seeks to resolve.
 - (d) What is a honey bee colony in the context of the paper?
 - (e) What are the principal features of a honey bee colony with regard to the work presented in the paper?
 - (f) What do the authors mean by social regulation of division of labour?
6. Read the abstract and introduction section of the paper *General Model of Inflammation* posted on D2L. Respond to the following prompts.
- (a) Based on the abstract of the paper, state in your own words the purpose of the article.
 - (b) What problem do the authors seek to address via a mathematical model?
 - (c) Explain in your own words why a mathematical model might be useful in order to address the problem(s) or question(s) that this paper seeks to resolve.
 - (d) Define the following terms as they relate to the paper: macrophage, cytokine, inflammation.
 - (e) Distinguish inflammation from chronic inflammation.