Biomath HW02

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#Problem #1 Consider the equation

$$\frac{dN}{dt} = \kappa - rN, \text{ where } \kappa, r > 0$$

#a) If the populaton is above $\frac{\kappa}{r}$, it will decrease until it reaches $\frac{\kappa}{r}$. If it below it will increase until it reaches $\frac{\kappa}{r}$. Notably, the maximum increase of the function is κ, whereas there is no limit to the rate of decrease. Equivalently,

$$\frac{dN}{dt} > 0 \text{ if } \kappa - rN > 0, \frac{\kappa}{r} > 0 \frac{dN}{dt} < 0 \text{ if } \kappa - rN < 0, \frac{\kappa}{r} < 0$$

This implies a constant birth rate, κ, and a death rate proportional to the populaton, scaled by the factor r.

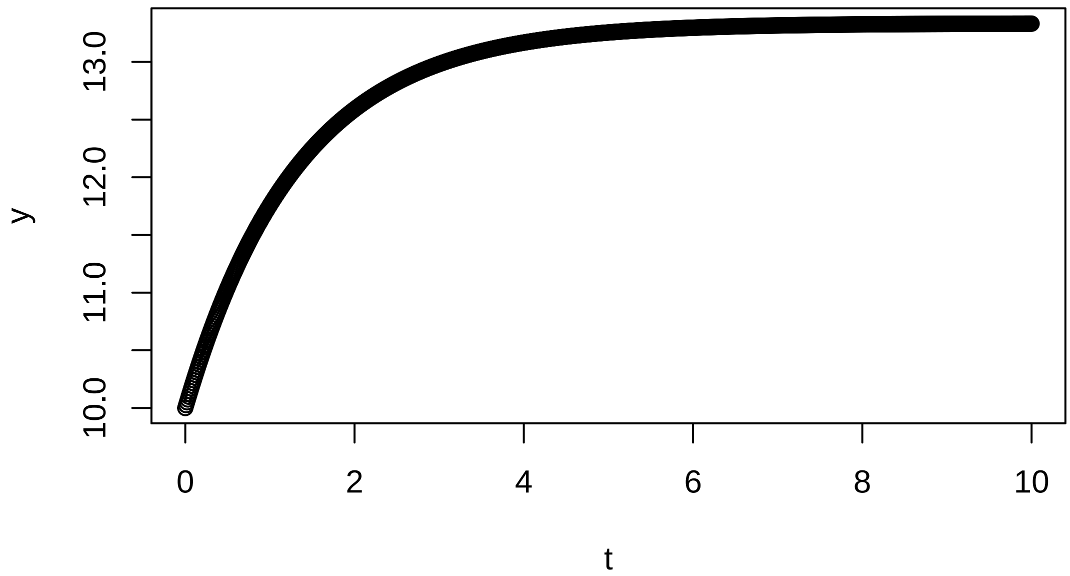
##b) The population will stabilize at $N = \frac{\kappa}{r}$ for the reasons in (a).

##c)

$$\begin{aligned}
\frac{dn}{dt} &= \kappa - rN \\
\int \frac{1}{\kappa - rN} dN &= \int dt \\
-\frac{1}{r} \ln(\kappa - rN) &= t + c \\
N(t) = -\frac{1}{r}\Big(e^{r(t+c)} - \kappa\Big), &\quad N(0) = N_0 \\
\therefore \quad c = -\frac{1}{r} \ln\Big(\kappa - rN\Big)
\end{aligned}$$

##d) Yes, this solution does fit the results calculated above. See below for justification

```
r <- .75; k <- 10; N0 <- 10; c <- -(1/r)*(log(k-r*N0));
t <- seq(0,10,length.out=1001)
y <- -(1/r)*(exp(-r*(t+c))-k)
plot(t,y)
```

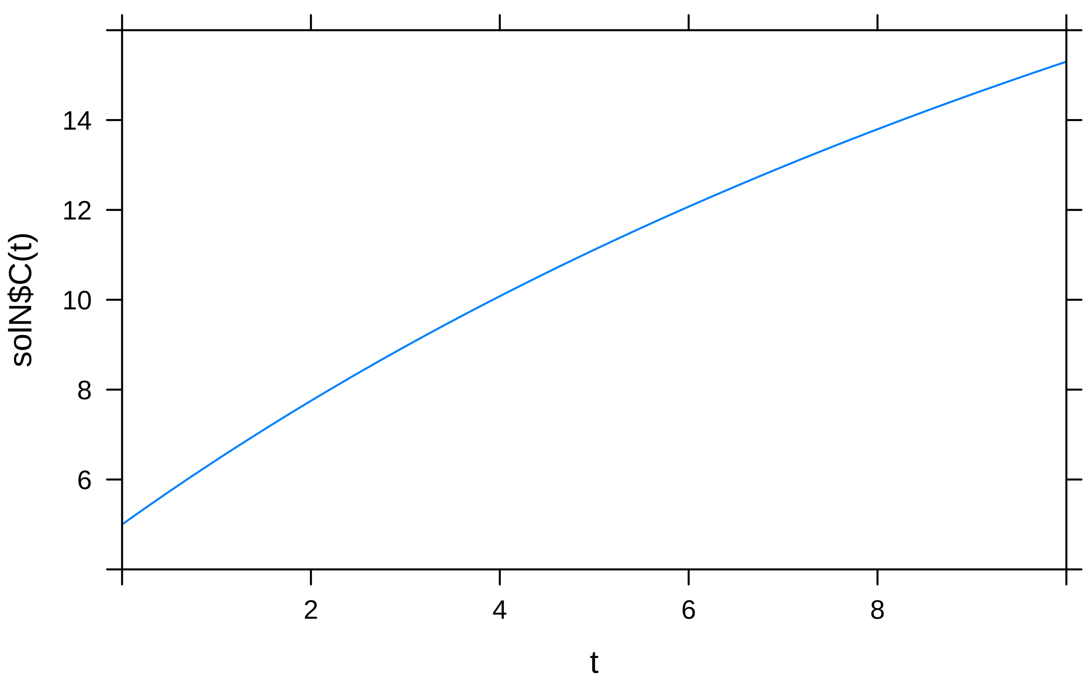


#Problem #2

$$X(t) = X_\infty + (X_0 - X_\infty)e^{-\lambda t}$$
$$X(0) = X_\infty + (X_0 - X_\infty)e^{0} = X_\infty + X_0 - X_\infty = X_0$$
$$\lim_{t\to\infty} X(t) = X_\infty - \lambda(X_0 - X_\infty)e^{-\infty} = X_\infty$$
$$\frac{dX}{dt} = X_\infty - \lambda(X_0 - X_\infty)e^{-\lambda t}$$

#Problem #3

```
func <- dC ~ qi*(v-C)/(v0+(qi-q0)*t)
solN <- integrateODE(func,v0=100,v=35,qi=5,q0=1,C=5,tdur=list(from=0,to=10))
plotFun(solN$C(t)~t,ylim=range(4,16),t.lim=range(0,10))
```



#Problem #4 System of equations for figure #1:

$$\frac{dX_1}{dt} = L - x_1(a_{01} + a_{21} + a_{31}) + x_3a_{13} + x_2a_{12} - x_1a_{01}$$
$$\frac{dX_2}{dt} = x_1a_{21} - x_2(a_{02} + a_{12})$$
$$\frac{dX_3}{dt} = x_1a_{31} - x_3a_{13}$$

#Problem #5

#Problem #6