MATH 463 Topics in Biomathematics
Homework 6: Due Friday March 13 at Noon

Exercises:

1. Consider the predator-prey system

$$\frac{dx}{dt} = \alpha x - \beta xy$$
$$\frac{dy}{dt} = \gamma xy - \delta y$$

   Nondimensionalize this system and obtain numerical solutions for a range of different parameter values and initial conditions. Explain the dynamics that you observe in the context of the model.

2. A two-dimensional linear autonomous system has the general form

$$\frac{dx}{dt} = ax + by,$$
$$\frac{dy}{dt} = cx + dy,$$

   where a, b, c, and d are real numbers. Compute a plot the numerical solution to the linear system corresponding to each of specified coefficient values and with initial condition $x_0 = 1$, and $y_0 = 1$.

   (a) $a = -1$, $b = 4$, $c = -3$, and $d = -1$

   (b) $a = 2$, $b = 3$, $c = 0$, and $d = -4$

   (c) $a = -1$, $b = 0$, $c = 0$, and $d = 1$

   (d) $a = 8$, $b = -11$, $c = 6$, and $d = -9$

3. Compute and plot numerical solutions to the nonlinear autonomous system for a variety of different initial conditions.

$$\frac{dx}{dt} = y - (x^2 + y^2)x,$$
$$\frac{dy}{dt} = -x - (x^2 + y^2)y.$$

   What characteristics do you observe about the solutions to this system?