MATH 463 Topics in Biomathematics
Homework 5: Due Wednesday March 4 at Noon

Exercises:

1. For each system, find the critical value of the parameter α_c that leads to a bifurcation and determine the type of bifurcation that occurs. Finally, sketch the appropriate bifurcation diagram.

   (a) $\dot{x} = \alpha + x - \ln(1+x)$

   (b) $\dot{x} = x - \alpha x(1-x)$

   (c) $\dot{x} = x + \frac{\alpha x}{1+x^2}$

   (d) $\dot{x} = \alpha x + \frac{x^3}{1+x^2}$

2. The first-order system $\dot{u} = au + bu^3 - cu^5$, where $b, c > 0$, has a subcritical pitchfork bifurcation at $a = 0$. Show that this equation can be rewritten as

$$\frac{dx}{d\tau} = \alpha x + x^3 - x^5,$$

   where $x = \frac{u}{U}$, $\tau = \frac{t}{T}$, and U, T, and α are to be determined in terms of a, b, and c.