

# Biomath HW06

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March 18, 2020

## Problem #1

Non-Dimensionalize the following system of equations:

$$\begin{aligned}\frac{dx}{dt} &= \alpha x - \beta xy \\ \frac{dy}{dt} &= \gamma xy - \delta y\end{aligned}$$

Start with setting non-dimensionalized values to arbitrary scalings of the original, dimensioned values.

$$\begin{aligned}X &= \frac{x}{x_1} & \frac{dX}{dT} &= \frac{dX}{dx} \frac{dx}{dt} \frac{dt}{dT} \\ Y &= \frac{y}{y_1} & \frac{dY}{dT} &= \frac{dY}{dy} \frac{dy}{dt} \frac{dt}{dT}\end{aligned}$$

This non-dimensionalized system becomes

$$\begin{aligned}\frac{dX}{dT} &= \left(\frac{t_1}{x_1}\right)(\alpha x - \beta xy) &= \left(\frac{t_1}{x_1}\right)(\alpha x_1 X - \beta x_1 X y_1 Y) \\ & &= [\alpha t_1] X - [\beta t_1 y_1] XY \\ \frac{dY}{dT} &= \left(\frac{t_1}{y_1}\right)(\gamma xy - \omega y) &= \left(\frac{t_1}{y_1}\right)(\gamma x_1 X y_1 Y - \beta y_1 Y) \\ & &= [\gamma t_1 x_1] XY - [\omega t_1] Y\end{aligned}$$

Now we must assign values to the parameters such that the bracketed terms will be simplified.  
The substitution

$$t_1 = \frac{1}{\alpha}, \quad y_1 = \frac{\alpha}{\beta} \quad x_1 = \frac{\alpha}{\gamma}$$

simplifies the parameters to this dimensionless system of equations:

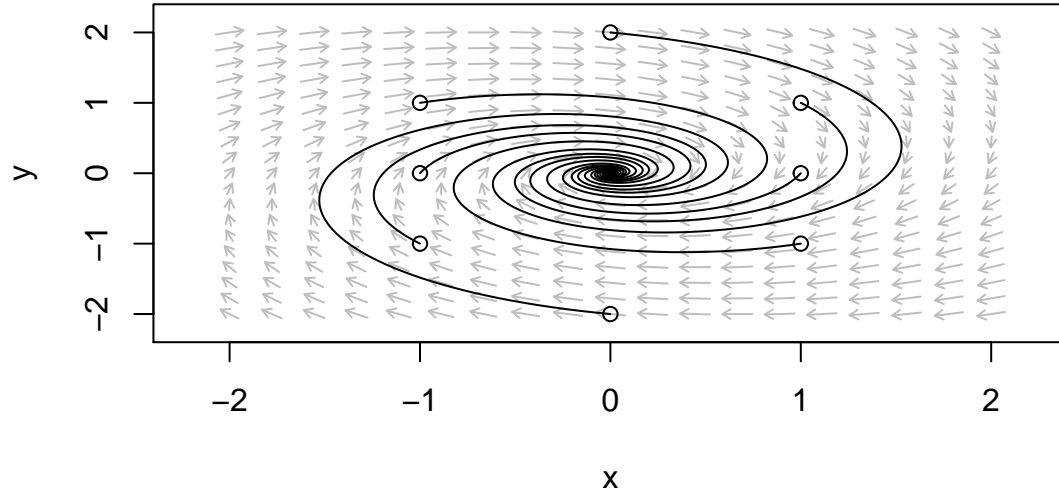
$$\begin{aligned}\frac{dX}{dT} &= X - XY \\ \frac{dY}{dT} &= XY - aY, \quad \text{where } a = \frac{\gamma}{\alpha}\end{aligned}$$

## Problem #2

(a)

```
## [1] "Tr(A) > 0 : FALSE   det(A) > 0 : TRUE"
```

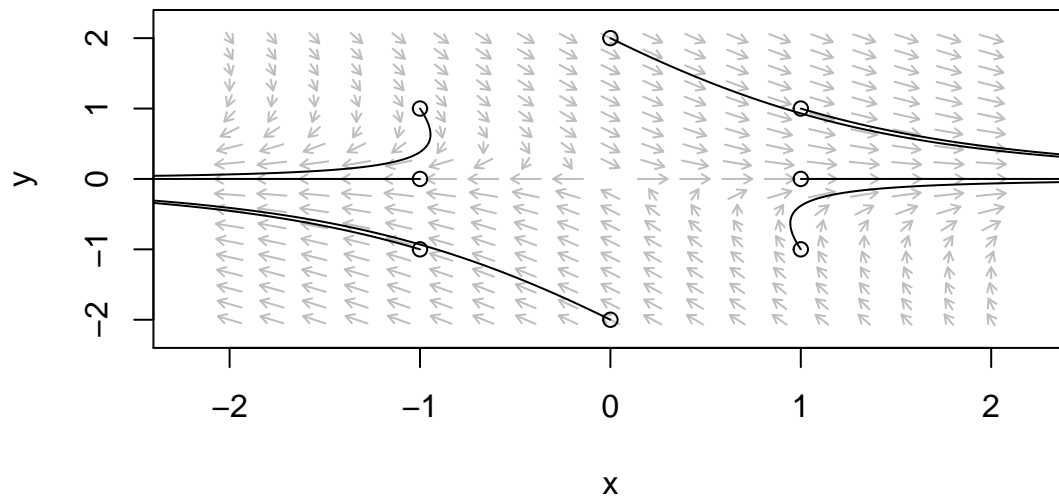
```
## [1] "det(A) > (Tr(A)^2)/4 : TRUE"
```



(b)

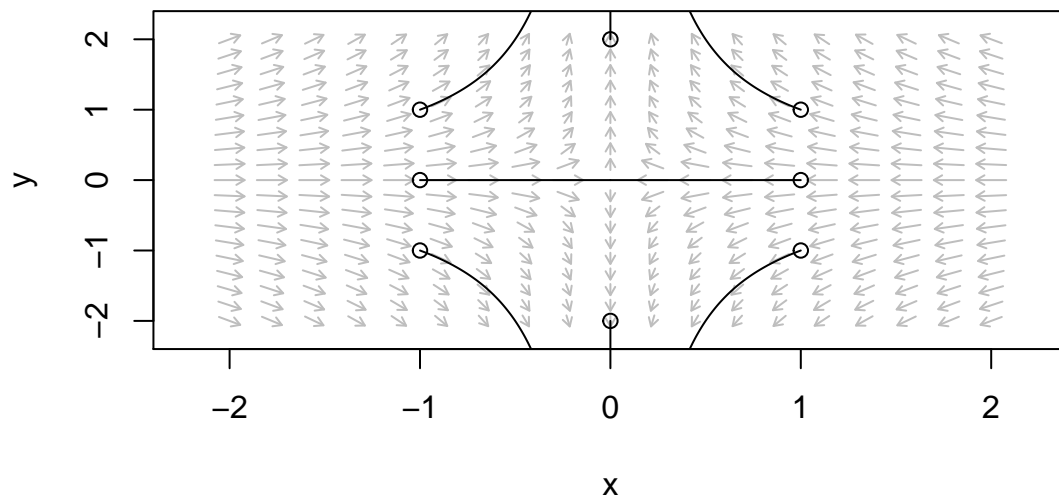
```
## [1] "Tr(B) > 0 : FALSE   det(B) > 0 : FALSE"
```

```
## [1] "det(B) > (Tr(B)^2)/4 : FALSE"
```



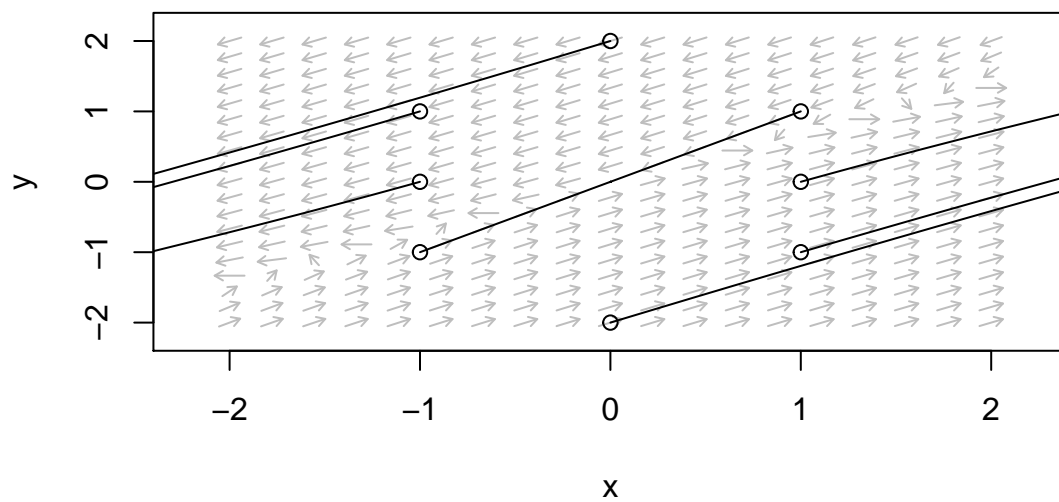
(c)

```
## [1] "Tr(C) > 0 : FALSE   det(C) > 0 : FALSE"
## [1] "det(C) > (Tr(A)^2)/4 : FALSE"
```



(d)

```
## [1] "Tr(D) > 0 : FALSE   det(D) > 0 : FALSE"
## [1] "det(D) > (Tr(A)^2)/4 : FALSE"
```



### Problem #3

$$\begin{aligned}\frac{dx}{dt} &= y - (x^2 + y^2)x \\ \frac{dy}{dt} &= -x - (x^2 + y^2)y\end{aligned}$$

