Biomath HW1

Maxwell Greene

January 29, 2020

Problem #1

$$
\begin{aligned}
N_{t+1} &= \rho N_t \\
N_{t'} &= \rho N_{t'-1} \\
&= \rho(\rho N_{t'-2}) \\
&= \rho^{t'} N_{t'-t'} \\
&= \rho^{t'} N_0
\end{aligned}
$$

Problem #2

$$
\begin{aligned}
&\text{For } 0 < \rho < 1: \\
N_t &= \rho^t N_0 \\
\lim_{t\to\infty} N_t &= \lim_{t\to\infty} \rho^t N_0 \\
N_\infty &= 0 * N_0 \\
N_\infty &= 0
\end{aligned}
\qquad
\begin{aligned}
&\text{For } \rho > 1: \\
N_t &= \rho^t N_0 \\
\lim_{t\to\infty} N_t &= \lim_{t\to\infty} \rho^t N_0 \\
N_\infty &= \infty * N_0 \\
N_\infty &= \infty
\end{aligned}
$$

Problem #3

Considering the following simplistic population model,

$$\frac{dN}{dt} = rN, N(0) = N_0$$

there are several pros and cons. First, it is simple and computationally efficient; we can easily determine a solution by hand and determine the population or rate of change for any given t. It also models a continuously growing population, which is biologically accurate for certain populations. However, it is not able to account for the dynamics of many (possibly most) populations that have limits like a carrying capacity or a non-zero death rate.

Problem #4

Use the Method of separation to solve the IVP:

$$
\frac{dN}{dt} = rN, N(0) = N_0
\qquad
\begin{aligned}
\frac{dN}{dt} &= rN \\
\frac{1}{N} dN &= r * dt \\
\int \frac{1}{N} dN &= \int r * dt \\
\log N &= rt + c \\
N &= e^c e^{rt} \\
N(0) = N_0 &= e^c e^{r(0)} = e^c \\
\text{Finally: } N(t) &= N_0 e^{rt}
\end{aligned}
$$

Problem #5

Consider the logistic growth model:

$$\frac{dN}{dt} = r\left(1 - \frac{N}{K}\right)N, N(0) = N_0.$$

##(a) When is $N(t)$ increasing/decreasing?
Increasing when $r\left(1 - \frac{N}{K}\right) > 1$, decreasing when $r\left(1 - \frac{N}{K}\right) < 1$.

For the increasing case:

$$\begin{aligned} 1 &< r\left(1 - \frac{N}{K}\right) \\ \frac{1}{r} &< 1 - \frac{N}{K} \\ \frac{1}{r} - 1 &< -\frac{N}{K} \\ K\left(1 - \frac{1}{r}\right) &> N \\ \text{Equivalently, } N &< K\left(\frac{r-1}{r}\right) \end{aligned}$$

For the decreasing case:

$$\begin{aligned} 1 &> r\left(1 - \frac{N}{K}\right) \\ \frac{1}{r} &> 1 - \frac{N}{K} \\ \frac{1}{r} - 1 &> -\frac{N}{K} \\ K\left(1 - \frac{1}{r}\right) &< N \\ \text{Equivalently, } N &> K\left(\frac{r-1}{r}\right) \end{aligned}$$

(b)

The rate of change of the population is zero when $\frac{dN}{dt} = 0$. Assuming $r, K, N \neq 0$:

$$\begin{aligned} 0 &= r\left(1 - \frac{N}{K}\right) \\ \text{Since } r \neq 0, \quad 0 &= 1 - \frac{N}{K} \\ 1 &= \frac{N}{K}, \boxed{N = K} \end{aligned}$$