Maxwell Greene Data Science HW 10, Question 1

1)

$$\ln\left(\frac{\tilde{\pi}_K \frac{1}{\sqrt{2\pi}\sigma} e^{\left(-\frac{(x-\mu_K)^2}{2\sigma^2}\right)}}{\sum_{l=1}^{K} \tilde{\pi}_l \frac{1}{\sqrt{2\pi}\sigma} e^{\left(-\frac{(x-\mu_l)^2}{2\sigma^2}\right)}}\right) = \ln\left(\tilde{\pi}_K \frac{1}{\sqrt{2\pi}\sigma}\right) + \ln\left(e^{\left(-\frac{(x-\mu_K)^2}{2\sigma^2}\right)}\right) - \ln\left(\tilde{\pi}_l \frac{1}{\sqrt{2\pi}\sigma}\right) - \ln\left(e^{\left(-\frac{(x-\mu_l)^2}{2\sigma^2}\right)}\right)$$

$$= \ln(\tilde{\pi}_K) - \cancel{\ln(\sqrt{2\pi}\sigma)} - \frac{(x-\mu_K)^2}{2\sigma^2} - \ln(\tilde{\pi}_l) + \cancel{\ln(\sqrt{2\pi}\sigma)} + \frac{(x-\mu_l)^2}{2\sigma^2}$$

$$= \ln(\tilde{\pi}_K) - \ln(\tilde{\pi}_l) + \frac{(x-\mu_l)^2 - (x-\mu_K)^2}{2\sigma^2}$$

$$= \ln(\tilde{\pi}_K) - \cancel{\ln(\tilde{\pi}_l)} + \frac{2x(\mu_K - \cancel{\mu_l}) + \cancel{\mu_l^2} - \mu_K^2}{2\sigma^2}$$

- all "l" terms can be omitted -

$$= \ln(\tilde{\pi}_K) + \frac{2x\mu_K}{2\sigma^2} - \frac{\mu_K^2}{2\sigma^2}$$

$$= \frac{x\mu_K}{\sigma^2} - \frac{\mu_K^2}{2\sigma^2} + \ln(\tilde{\pi}_K)$$

$(x-y)^2 = x^2 - 2xy + y^2$

$(x^2 - 2x\mu_i + \mu_i^2) - (x^2 - 2x\mu_K + \mu_K^2) =$

$-2x\mu_i + \mu_i^2 + 2x\mu_K + \mu_K^2 =$

$2x(\mu_K - \mu_i) + \mu_i^2 + \mu_K^2$