MATH 284 Mathematical Programming
Homework 4: Due Wednesday March 7 at 1:00pm

Exercises:

1. Write a user-defined MATLAB function for the following math function:

$$y(x) = 0.6x^3e^{-0.47x} + 1.5x^2e^{-0.6x}.$$

The input to the function is x and the output is y. Write the function such that x can be a vector (use element-by-element operations).

   (a) Use the function to calculate $y(-2)$ and $y(4)$.

   (b) Use the function to make a plot of the function $y(x)$ for $-4 \leq x \leq 8$.

2. Write a function `plot2fnhand` that will receive two function handles as input arguments, and will display in a Figure Window separate plots of these functions using subplots, with the function names in the titles. The function will create an x vector that ranges from 1 to n, where n is a random integer in the range from 4 to 10 (this can be done with the command `randi([4 10],1)`). For example, if the function is called as follows:

```
>> plot2fnhand(@sqrt,@exp)
```

and the random integer is 5, the Figure Window would display the graphs of the square root function from 1 to 5, and the exponential function from 1 to 5 plotted on separate axes but in the same window.

3. The first derivative $\frac{df}{dx}$ of a function f at a point $x = a$ can be approximated with the four-point central difference formula:

$$\frac{df}{dx}(a) = \frac{f(a-2h) - 8f(a-h) + 8f(a+h) - f(a+2h)}{12h},$$

where h is a small number relative to a. Write a user-defined function that inputs another user-defined function, a value for a, and a value for h that returns an approximation for $\frac{df}{dx}(a)$ obtained using the four-point central difference formula. For the user-defined function name, use `dfdx = FoPtder(Fun,a,h)` , where `Fun` is a name for the function passed into `FoPtder`, a is the value at which the derivative is to be approximation, and h is the "step-size". Use your function to calculate the following:

   (a) An approximation for the derivative of $f(x) = x^3e^{2x}$ at $a = 0.6$, with $h = \frac{a}{100}$, $h = \frac{a}{500}$. Write a script that plots f and an approximation to f' together on the same axes over the interval $-1 \leq x \leq 1$. For this, you should use your function for the four-point central difference formula to obtain an approximation to f' over the specified interval.

   (b) An approximation for the derivative of $f(x) = \frac{3^x}{x^2}$ at $a = 2.5$, with $h = \frac{a}{100}$, $h = \frac{a}{500}$. Write a script that plots f and an approximation to f' together on the same axes over the interval $1 \leq x \leq 3$. For this, you should use your function for the four-point central difference formula to obtain an approximation to f' over the specified interval.

(c) Write a function that inputs f and x_0 and x_1 which then plots f and an approximation to f' over the interval $[x_0, x_1]$ which uses the function `FoPtder` to obtain an approximation for f'.