Maxwell Greene Numerical Analysis Test #1

Problem 1:

Show that $xcos(x) - 2x^2 + 3x - 1 = 0$, has at least one solution in $[0.2, 0.3]$ and $[1.2, 1.3]$ and write a program to find roots.

```
syms x;
y = @(x) x.*cos(x)-2.*x.^2+3.*x-1;
interval1 = [0.2,0.3];
interval2 = [1.2,1.3];
```

The function has a guaranteed solution on $[a, b]$ by the intermediate value theorem if the funciton is continuous and differentiable on $[a, b]$ and $y(a)$ and $y(b)$ are of opposite sign. That is, that $0 \in [y(a), y(b)]$.

Intermediate Value Theorem:

If f is continuous on $[a, b]$ and y lies between $f(a)$ and $f(b)$

```
diff(y(x))
```

ans = $\cos(x) - 4\,x - x\sin(x) + 3$

The function has a derivative and no discontinuities, so we know that it holds true to those qualifications. Now we shall check the values at $y(a)$ and $y(b)$ for both intervals.

```
%y(a) and y(b) for first interval
y(interval1);
%Check if signs of y(a) and y(b) are the same for interval1
sign(y(interval1(1))) == sign(y(interval1(2)))
```

```
ans = logical
   0
```

```
%y(a) and y(b) for second interval
y(interval2);
%Check if signs of y(a) and y(b) are the same for interval2
sign(y(interval2(1))) == sign(y(interval2(2)))
```

```
ans = logical
   0
```

We can see that both intervals must cross the y-axis within the interval by the intermediate value theorem and therefore the equation given must hold true.

Now to find the two roots I will use the bisection method.

```
tol = 10e-10;
```

```
error = Inf;
xVal = 0;
a = interval1(1); b = interval1(2);
while abs(b-a) > tol
    xVal = (a+b)/2;
    if(sign(y(a))==sign(y(xVal)))
        a = xVal; else b=xVal;  end
end
xVal
```

```
xVal = 0.2975
```

```
a = interval2(1); b = interval2(2);
while abs(b-a) > tol
    xVal = (a+b)/2;
    if(sign(y(a))==sign(y(xVal)))
        a = xVal;
    else
        b=xVal;
    end
end
xVal
```

```
xVal = 1.2566
```

Problem #2:

Show that $f(x) = x^4 + 2x^2 - x - 3$ has a fixed point and find the fixed point.

First, to show there is a fixed point:

Let $g(x) = f(x) - x$. Then g is continuous and $g(a) \geq 0$ while $g(b) \leq 0$. By IVT, g has at least one zero on $[a, b]$.

```
syms x;
f = @(x) x.^4 + 2.*x.^2-x-3;
g = @(x) f(x) - x;
a=1;b=2;
```

Now, check if $g(a)$ is the opposite sign of $g(b)$:

```
sign(g(a))==sign(g(b))
```

```
ans = logical
   0
```

Therefore, the fixed point of $f(x)$ with $x \in [a, b]$.

Now we find the gixed point by finding the zero of $g(x)$ by using the bisection method.

```
while abs(b-a) > tol
```

```
    xVal = (a+b)/2;
    if(sign(g(a))==sign(g(xVal)))
        a = xVal;
    else
        b = xVal;
    end
end
xVal
```

```
xVal = 1.2438
```

Problem #3:

Find the LU decomposition of

```
A = [0,1,1;1,-2,-1;1,-1,1]
```

```
A =
     0     1     1
     1    -2    -1
     1    -1     1
```

Show elementary matrices used to do composition. Use "inv" for inverses.

Since all diagonal elements of A are zero, we must use a permutation matrix.

```
P = [0,1,0;1,0,0;0,0,1];%Switch rows 1&2
A1 = P*A;
E1 = [1,0,0;0,1,0;-1,0,1];  A2 = E1*A1;
E2 = [1,0,0;0,1,0;0,-1,1];   A3 = E2*A2;
```

At this stage, A_3 is an upper triangular matrix. So $U = A_3$ and $L = E_1^{-1}E_2^{-1}$.

```
U = A3;
L = inv(E1)*inv(E2)
```

```
L =
     1     0     0
     0     1     0
     1     1     1
```

```
%Check for correctness:
P*A == L*U
```

```
ans = 3×3 logical array
   1   1   1
   1   1   1
   1   1   1
```

Problem #4:

Find the singular value decomposition of:

```
A = [2,1;-1,1;1,1;2,-1]
```

```
A =
     2     1
    -1     1
     1     1
     2    -1
```

First, find A^T and A^TA

```
AT = transpose(A);
ATA = AT*A;
```

Then find the eigenvalues and eigenvectors of A^TA, normalize them, then combine to form V.

```
[ATA_EVect,ATA_EVal] = eig(ATA)
```

```
ATA_EVect =
     0     1
     1     0
ATA_EVal =
     4     0
     0    10
```

```
for i = 1:length(ATA_EVect)
    ATA_EVect(:,i) = ATA_EVect(i,:)/norm(ATA_EVect(:,i));
end
[sorted, order] = sort(ATA_EVal)
```

```
sorted =
     0     0
     4    10
order =
     2     1
     1     2
```

It just so happens that matlab already returns the matrix of eigenvalues in the form required, so normalizing and combining is not necessary. Therefore,

```
V = ATA_EVect(order(1,:),:)
```

```
V =
     1     0
     0     1
```

Now we do something similar to find U:

Find eigenvectors of AA^T, combine them, then normalize to form U.

```
AAT = A*AT
```

```
AAT =
     5    -1     3     3
    -1     2     0    -3
```

```
     3     0     2     1
     3    -3     1     5
```

```
[AAT_EVect,AAT_EVal] = eig(AAT);
for i = 1:length(AAT_EVect)
    AAT_EVect(:,i) = AAT_EVect(:,i)/norm(AAT_EVect(:,i));
end
U = AAT_EVect
```

```
U =
    0.5687    0.1631   -0.5000    0.6325
    0.1398   -0.7940   -0.5000   -0.3162
   -0.8048    0.0472   -0.5000    0.3162
   -0.0963   -0.5837    0.5000    0.6325
```

```
inv(U)*A*inv(transpose(V))
```

```
ans =
   -0.0000    0.0000
    0.0000   -0.0000
    0.0000   -2.0000
    3.1623    0.0000
```

Therefore, the singular values are

```
[ans(3,2),ans(4,1)]
```

```
ans =
   -2.0000    3.1623
```