Numerical Analysis Homework #1

Problem #1:

Solve $x^3 - 3x - 1 = 0$ using Newton's method of approximations:

The exuation to the tangent line is given by $f(x) = \text{f'}(x_0) * (x - x_0) + f(x_0)$.

We want to find x such that $f(x) = 0$. If x is a solution such that $f(x_0) = 0$, then $0 = \text{f'}(x_0) * (x - x_0) + f(x_0)$. Some algebra gives the equation

$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$. The intuition here being that x_1 will be a better guess than x_0.

In a generalized form we have $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$, the solution to which can be easily implemented in a loop.

```
tolerance = 1e-20;   %Something really small, assuring our answer will be close
xGuess = 0;          %Doesn't really matter what you choose...
fValue = 10;
xZero=0;

while abs(fValue) > tolerance
    %From the generalized equation above, since we know the derivative
    xZero = xGuess - (xGuess^3 - 3*xGuess - 1) / (3*xGuess - 3);
    fValue = xZero^3-3*xZero-1;
    xGuess=xZero;
end
xZero
```

```
xZero = -0.3473
```

Let's check to make sure this answer is correct.

```
xZero^3-3*xZero-1
```

```
ans = 0
```

Problem #2:

Solve $\int_0^{\frac{\pi}{4}} tan(x)dx$ using the Reimann sum method:

Reimann sum is fairly straightforward. Add all "slices" of $\tan(x)$ evaluated at the appropriate x value with a width of $\frac{\pi/4 - 0}{n}$.

```
n=1000;     %Should ne high enough to get a reasonable approximation
```

```
total=0;    %Start off at 0
for i = 1:n
    total = total + tan(i*pi/(4*n))*pi/(4*n);
end
total
```

```
total = 0.3470
```

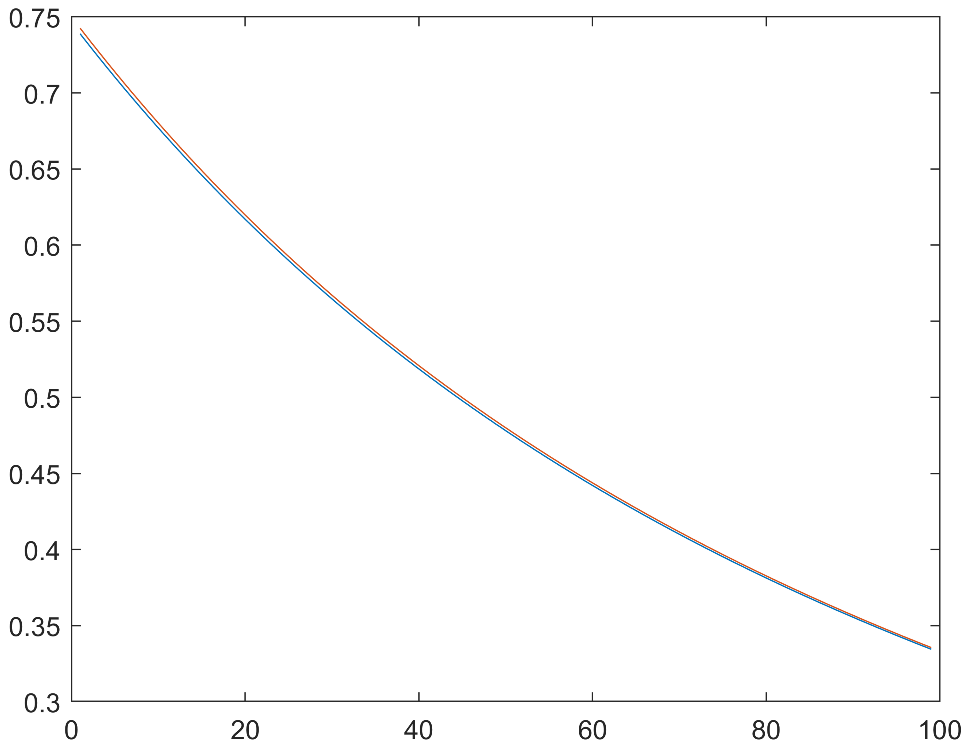
Problem #3:

Find $f'(x)$ for $f(x) = \frac{x-1}{x+2}$ on $0 < x < 1$.

With this approximation method, the derivative can be found by finding the difference between $f(x)$ and $f(x+h)$ for h small.

In loop form for this will look like $f'(x) = g(x) = \sum f(i) = n * \sum_{i=0}^{n} \frac{x_{i+1} * \frac{1}{n} - 1}{x_{i+1} * \frac{1}{n} + 2} - \frac{x_i * \frac{1}{n} - 1}{x_i * \frac{1}{n} + 2}$ for this particular equation.

```
n=100;
for i = 1:n-1
    x(i) = i;
    f(i) = (((i+1)/n-1)/((i+1)/n+2)-(i/n-1)/(i/n+2))*n;
    %Check using derivative taken by hand
    g(i) = (3/(i/n+2)^2);
end
plot(x,f,x,g)
```


0.75
0.7
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20
40
60
80
100