

# Maxwell Greene Numerical Analysis Test #1

## Problem 1:

Show that  $x\cos(x) - 2x^2 + 3x - 1 = 0$ , has at least one solution in  $[0.2, 0.3]$  and  $[1.2, 1.3]$  and write a program to find roots.

```
syms x;  
y = @(x) x.*cos(x)-2.*x.^2+3.*x-1;  
interval1 = [0.2,0.3];  
interval2 = [1.2,1.3];
```

The function has a guaranteed solution on  $[a, b]$  by the intermediate value theorem if the function is continuous and differentiable on  $[a, b]$  and  $y(a)$  and  $y(b)$  are of opposite sign. That is, that  $0 \in [y(a), y(b)]$ .

### Intermediate Value Theorem:

If  $f$  is continuous on  $[a, b]$  and  $y$  lies between  $f(a)$  and  $f(b)$

```
diff(y(x))
```

```
ans = cos(x) - 4 x - x sin(x) + 3
```

The function has a derivative and no discontinuities, so we know that it holds true to those qualifications. Now we shall check the values at  $y(a)$  and  $y(b)$  for both intervals.

```
%y(a) and y(b) for first interval  
y(interval1);  
%Check if signs of y(a) and y(b) are the same for interval1  
sign(y(interval1(1))) == sign(y(interval1(2)))
```

```
ans = logical  
0
```

```
%y(a) and y(b) for second interval  
y(interval2);  
%Check if signs of y(a) and y(b) are the same for interval2  
sign(y(interval2(1))) == sign(y(interval2(2)))
```

```
ans = logical  
0
```

We can see that both intervals must cross the y-axis within the interval by the intermediate value theorem and therefore the equation given must hold true.

Now to find the two roots I will use the bisection method.

```
tol = 10e-10;
```

```

error = Inf;
xVal = 0;
a = interval1(1); b = interval1(2);
while abs(b-a) > tol
    xVal = (a+b)/2;
    if(sign(y(a))==sign(y(xVal)))
        a = xVal; else b=xVal; end
end
xVal

```

xVal = 0.2975

```

a = interval2(1); b = interval2(2);
while abs(b-a) > tol
    xVal = (a+b)/2;
    if(sign(y(a))==sign(y(xVal)))
        a = xVal;
    else
        b=xVal;
    end
end
xVal

```

xVal = 1.2566

## Problem #2:

Show that  $f(x) = x^4 + 2x^2 - x - 3$  has a fixed point and find the fixed point.

First, to show there is a fixed point:

Let  $g(x) = f(x) - x$ . Then  $g$  is continuous and  $g(a) \geq 0$  while  $g(b) \leq 0$ . By IVT,  $g$  has at least one zero on  $[a, b]$ .

```

syms x;
f = @(x) x.^4 + 2.*x.^2-x-3;
g = @(x) f(x) - x;
a=1;b=2;

```

Now, check if  $g(a)$  is the opposite sign of  $g(b)$ :

```
sign(g(a))==sign(g(b))
```

```
ans = logical
0
```

Therefore, the fixed point of  $f(x)$  with  $x \in [a, b]$ .

Now we find the fixed point by finding the zero of  $g(x)$  by using the bisection method.

```
while abs(b-a) > tol
```

```

xVal = (a+b)/2;
if(sign(g(a))==sign(g(xVal)))
    a = xVal;
else
    b = xVal;
end
end
xVal

```

xVal = 1.2438

### Problem #3:

Find the LU decomposition of

```
A = [0,1,1;1,-2,-1;1,-1,1]
```

```

A =
     0     1     1
     1    -2    -1
     1    -1     1

```

Show elementary matrices used to do composition. Use "inv" for inverses.

Since all diagonal elements of  $A$  are zero, we must use a permutation matrix.

```

P = [0,1,0;1,0,0;0,0,1];%Switch rows 1&2
A1 = P*A;
E1 = [1,0,0;0,1,0;-1,0,1];  A2 = E1*A1;
E2 = [1,0,0;0,1,0;0,-1,1];   A3 = E2*A2;

```

At this stage,  $A_3$  is an upper triangular matrix. So  $U = A_3$  and  $L = E_1^{-1}E_2^{-1}$ .

```
U = A3
```

```

U =
     1    -2    -1
     0     1     1
     0     0     1

```

```
L = inv(E1)*inv(E2)
```

```

L =
     1     0     0
     0     1     0
     1     1     1

```

```

%Check for correctness:
P*A == L*U

```

```

ans = 3x3 logical array
     1     1     1
     1     1     1

```

1 1 1