Maxwell Greene Numerical Analysis Test #1

Problem 1:

Show that $xcos(x) - 2x^2 + 3x - 1 = 0$, has at least one solution in $[0.2, 0.3]$ and $[1.2, 1.3]$ and write a program to find roots.

```
syms x;
y = @(x) x.*cos(x)-2.*x.^2+3.*x-1;
interval1 = [0.2,0.3];
interval2 = [1.2,1.3];
```

The function has a guaranteed solution on $[a, b]$ by the intermediate value theorem if the funciton is continuous and differentiable on $[a, b]$ and $y(a)$ and $y(b)$ are of opposite sign. That is, that $0 \in [y(a), y(b)]$.

Intermediate Value Theorem:

If f is continuous on $[a, b]$ and y lies between $f(a)$ and $f(b)$

```
diff(y(x))
```

ans = $\cos(x) - 4\,x - x\sin(x) + 3$

The function has a derivative and no discontinuities, so we know that it holds true to those qualifications. Now we shall check the values at $y(a)$ and $y(b)$ for both intervals.

```
%y(a) and y(b) for first interval
y(interval1);
%Check if signs of y(a) and y(b) are the same for interval1
sign(y(interval1(1))) == sign(y(interval1(2)))
```

```
ans = logical
   0
```

```
%y(a) and y(b) for second interval
y(interval2);
%Check if signs of y(a) and y(b) are the same for interval2
sign(y(interval2(1))) == sign(y(interval2(2)))
```

```
ans = logical
   0
```

We can see that both intervals must cross the y-axis within the interval by the intermediate value theorem and therefore the equation given must hold true.

Now to find the two roots I will use the bisection method.

```
tol = 10e-10;
```

```
error = Inf;
xVal = 0;
a = interval1(1); b = interval1(2);
while abs(b-a) > tol
    xVal = (a+b)/2;
    if(sign(y(a))==sign(y(xVal)))
        a = xVal; else b=xVal;  end
end
xVal
```

```
xVal = 0.2975
```

```
a = interval2(1); b = interval2(2);
while abs(b-a) > tol
    xVal = (a+b)/2;
    if(sign(y(a))==sign(y(xVal)))
        a = xVal;
    else
        b=xVal;
    end
end
xVal
```

```
xVal = 1.2566
```

Problem #2:

Show that $f(x) = x^4 + 2x^2 - x - 3$ has a fixed point and find the fixed point.

First, to show there is a fixed point:

Let $g(x) = f(x) - x$. Then g is continuous and $g(a) \geq 0$ while $g(b) \leq 0$. By IVT, g has at least one zero on $[a, b]$.

```
syms x;
f = @(x) x.^4 + 2.*x.^2-x-3;
g = @(x) f(x) - x;
a=1;b=2;
```

Now, check if $g(a)$ is the opposite sign of $g(b)$:

```
sign(g(a))==sign(g(b))
```

```
ans = logical
   0
```

Therefore, the fixed point of $f(x)$ with $x \in [a, b]$.

Now we find the gixed point by finding the zero of $g(x)$ by using the bisection method.

```
while abs(b-a) > tol
```

```
    xVal = (a+b)/2;
    if(sign(g(a))==sign(g(xVal)))
        a = xVal;
    else
        b=xVal;
    end
end
xVal
```

```
xVal = 1.2438
```

Problem #3:

Find the LU decomposition of

```
A = [0,1,1;1,-2,-1;1,-1,1]
```