Numerical Analysis Homework 4

Questions from book:

4.6, 5 & 10

Section 4.6, Question #5

Determine whether true or false:

$$\|A\|^{-1} = \|A^{-1}\|$$

Not necessarily true. Here's a counter-example:

```
A = [1,7,3;8,2,7;0,7,4]
```

```
A =
     1     7     3
     8     2     7
     0     7     4
```

```
isequal(inv(norm(A)),norm(inv(A)))
```

```
ans = logical
   0
```

Section 4.6, Question 10a

Show that the following matrix is orthogonal if $c^2 + s^2 = 1$

```
syms c; syms s
A = [c,s;-s,c]
```

```
A =
```
$$\begin{pmatrix} c & s \\ -s & c \end{pmatrix}$$

```
AT = A.'
```

```
AT =
```
$$\begin{pmatrix} c & -s \\ s & c \end{pmatrix}$$

```
A*AT
```

```
ans =
```
$$\begin{pmatrix} c^2 + s^2 & 0 \\ 0 & c^2 + s^2 \end{pmatrix}$$

Therefore, this matrix would be orthogonal by the condition $A' * A = I$.

Section 4.6, Problem 10b

A

A =

$$\begin{pmatrix} c & s \\ -s & c \end{pmatrix}$$

```
syms a1; syms a2
a = [a1;a2]
```

a =

$$\begin{pmatrix} a_1 \\ a_2 \end{pmatrix}$$

A*a

ans =

$$\begin{pmatrix} a_1 c + a_2 s \\ a_2 c - a_1 s \end{pmatrix}$$

Supplementary Problem:

```
orthonormalBases
```

```
A =
    1.0000         0         0    2.0000
         0    1.0000         0   -0.5000
         0         0    1.0000    1.0000
Orthogonal Basis for Column Space
ans =
    1    0    0
ans =
    0    1    0
ans =
    0    0    1
AT =
    1.0000         0         0
         0    1.0000         0
         0         0    1.0000
    2.0000   -0.5000    1.0000
Orthogonal Basis for Row Space
ans =
    1    0    0
ans =
    0    1    0
ans =
```

```
        0    0    1
```

```
function orthonormalBases

    A = [1,0,-1,1
         0,2,1,0
         0,0,1,1];

    for j = 1:3
        A(j,:) = A(j,:)/A(j,j);
        for i = 1:3
            if i ~= j
                A(i,:) = -A(j,:)*A(i,j)+A(i,:);
            end
        end
    end

    A


    fprintf('Orthogonal Basis for Column Space\n');
    A(:,1).', A(:,2).', A(:,3).'

    AT = A.'

    for j = 1:3
        AT(j,:) = AT(j,:)/AT(j,j);
        for i = 1:4
            if i ~= j
                AT(i,:) = -AT(j,:)*AT(i,j)+AT(i,:);
            end
        end
    end


    fprintf('Orthogonal Basis for Row Space\n');
    AT(1,:), AT(2,:), AT(3,:)
end
```