Prob & Stat HW8

Question 1

X, Y independent random variables with X,Y ~ $geom(p)$: $p_X(k) = p_Y(y) = p(1-p)^{k-1}, 0 < p < 1$

If $W = X + Y$, show

$$p_W(w) = (w-1)p^2(1-p)^{w-2}, w \geq 1$$

Hint: $p_W(w) = \sum_{\text{all } k} p_X(k)p_y(w-k)$

$$\begin{aligned}
p_W(w) &= \sum_{\text{all } k} p_X(k)p_y(w-k) \\
&= \sum_{\text{all } k} p(1-p)^{1-k}p(1-p)^{w-k-1} \\
&= \sum_{\text{all } k} p^2(1-p)^{w-2} \\
&= \boxed{(w-1)p^2(1-p)^{w-2}}
\end{aligned}$$

Question 2

Use $\Gamma(r) = (r-1)\Gamma(r-1)$ to prove $\Gamma(n) = (n-1)!$ if n is a positive integer. Hint: prove by induction.

$\Gamma(1) = 1$ and $0! = 1$, which verifies the equation for $n = 1$. Assuming $\Gamma(k) = (k-1)!$, $\Gamma(k+1) = k\Gamma(k) = k(k-1)! = k!$.

Question 3

Prove that $\Gamma(\frac{1}{2}) = \sqrt{\pi}$

$$\begin{aligned}
1 &= E(Z^2) \\
&= \frac{\sqrt{2}}{\sqrt{\pi}} \int_0^\infty z^2 e^{\frac{-z^2}{2}} dz \\
&= \frac{\sqrt{2}}{\sqrt{\pi}} \int_0^\infty \frac{(2u)e^{-u}}{\sqrt{2}\sqrt{u}} du, u = \frac{z^2}{2} \\
&= \frac{2}{\sqrt{\pi}} \int_0^\infty u^{(\frac{3}{2}-1)} e^{-u} du \\
\sqrt{\pi} &= 2\Gamma(\frac{3}{2}) \\
\sqrt{\pi} &= 2 * \frac{1}{2}\Gamma(\frac{1}{2}) \\
&\boxed{\sqrt{\pi} = \Gamma(\frac{1}{2})}
\end{aligned}$$

Question 4

```
pnorm(1.33,0,1)-pnorm(-0.44,0,1)
```

```
## [1] 0.5782723
```

```r
pnorm(0.94,0,1)
```

```
## [1] 0.8263912
```

```r
1-pnorm(-1.48,0,1)
```

```
## [1] 0.9305634
```

```r
pnorm(-4.32,0,1)
```

```
## [1] 7.80146e-06
```

Question 5

Part (a)

```r
pnorm(2.07,0,1)-pnorm(0,0,1)
```

```
## [1] 0.4807738
```

Part (b)

```r
pnorm(-0.11,0,1)-pnorm(-0.64,0,1)
```

```
## [1] 0.1951184
```

Part (c)

```r
1-pnorm(-1.06,0,1)
```

```
## [1] 0.8554277
```

Part (d)

```r
pnorm(-2.33,0,1)
```

```
## [1] 0.009903076
```

Part (e)

```r
1-pnorm(4.61,0,1)
```

```
## [1] 2.013345e-06
```

Question 6

```r
qnorm(.75,0,1)-qnorm(.25,0,1)
```

```
## [1] 1.34898
```