Prob & Stat HW5

Maxwell Greene

Question #7

Rayleigh Distribution:

$$f(x) = \frac{x}{a^2} e^{\frac{x^2}{2a^2}}, \qquad \text{where} \qquad a > 0, 0 \leq x < \infty$$

(a)

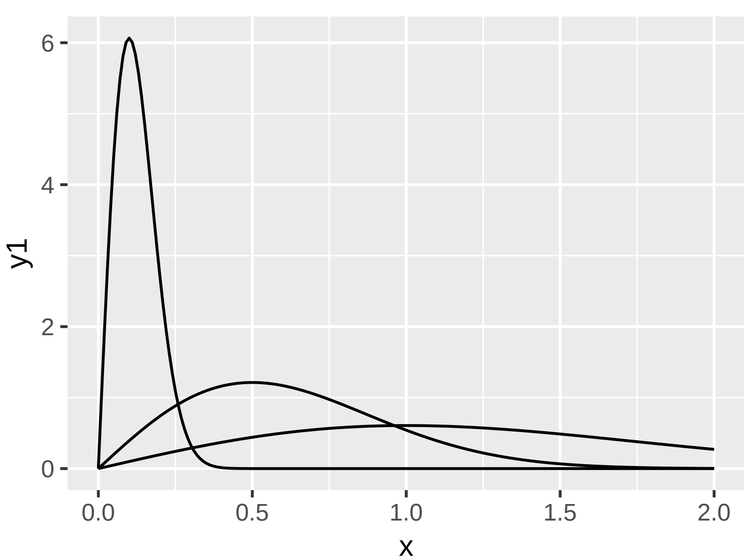
$$\int_0^\infty f(x)dx = \int_0^\infty \frac{x}{a^2} e^{-\frac{x^2}{2a^2}} dx$$

Performing the u-substitution $u = \frac{x^2}{2a^2}$, $du = \frac{x}{a^2}dx$ gives

$$\begin{aligned} \int_0^\infty f(x)dx &= \int_0^\infty e^{-u} du \\ &= -e^{-u}\Big|_0^\infty = -e^{-\frac{x^2}{2a^2}}\Big|_0^\infty = -e^{-\infty} + e^0 = \boxed{1} \end{aligned}$$

(d)

```r
x<-seq(0,2,.01)
y1<-drayleigh(x,scale=.1)
y2<-drayleigh(x,scale=.5)
y3<-drayleigh(x,scale=1)
data<-data.frame(x=rep(x,3),
  alpha=rep(c(1,2,3),3,each=length(x)),
  y1=y1,y2=y2,y3=y3)
ggplot(data) +
  geom_line(mapping=aes(x=x,y=y1))+
  geom_line(mapping=aes(x=x,y=y2))+
  geom_line(mapping=aes(x=x,y=y3))
```



Question #8

Expectation of Rayleigh Distribution given by

$$E(R) = \int_0^\infty (x * f_R)dx = \int_0^\infty x\frac{x}{a^2}e^{\frac{x^2}{2a^2}}dx$$

With $\sigma = 1$ the expected value is

```
a<-1
rayleighExpect <- function(x)
{return(x*drayleigh(x,scale=a))}
integrate(rayleighExpect,lower=0,upper=Inf)$value
```

```
## [1] 1.253314
```

Notably, this is approximately $\sigma\sqrt{\frac{\pi}{2}}$.

```
a*sqrt(pi/2)
```

```
## [1] 1.253314
```

Question #10

Probability mass function of dice roll given by

X	P(X)
1	1/6
2	1/6
3	1/6
4	1/6
5	1/6
6	1/6

I'll write this in R as

```
diceProb <- data.frame(value=1:6,prob=rep(1/6,6))
```

Now, we calculate the expacted value given by $\mu = \sum xP(x)$

```
diceExpected <- sum(diceProb$value*diceProb$prob)
diceExpected
```

```
## [1] 3.5
```

And the variance given by $\sigma = \sum P(x)(x-\mu)^2$

```
diceVariance<-sum(diceProb$prob*(diceProb$value-diceExpected)^2)
diceVariance
```

```
## [1] 2.916667
```

Question #11

Expected Value:

$$\begin{aligned}
E(Y) &= \int_{-\infty}^{\infty} f_x(\frac{x-\mu}{\sigma})dx \\
&= \frac{1}{\sigma}\int_{-\infty}^{\infty} x * f_x dx - \frac{1}{\sigma}\int_{-\infty}^{\infty} \mu * f_x dx \\
&= \frac{\mu}{\sigma} - \frac{\mu}{\sigma} = \boxed{0}
\end{aligned}$$

$$\text{because} \int_{-\infty}^{\infty} x * f_x dx = \mu \quad \text{and} \quad \int_{-\infty}^{\infty} f_x dx = 1$$

Variance:

$$\begin{aligned}
Var(Y) &= \int_{-\infty}^{\infty} f * (y-\mu)^2 dx \\
&= \frac{1}{\sigma^2}\int f * (x-\mu)^2 dx - \frac{2\mu}{\sigma}\int f * (x-\mu)dx + \mu^2 \int f dx \\
&= \frac{1}{\sigma^2}\int f * x^2 dx - 2\mu\int f * x dx + \mu^2 \int f dx - \frac{2\mu}{\sigma}(0) + \mu^2 \\
&= \frac{\sigma^2}{\sigma^2} - 2\mu^2 + \mu^2 + \mu^2 = \boxed{1}
\end{aligned}$$

Question 3.5.8, 3.5.14, 3.5.30, 3.6.8 on separate sheet.