MATH 310 Applied Probability and Mathematical Statistics
Homework 8: Due Friday March 22 at 3:00pm

Exercises:

1. Let X and Y be two independent random variables both satisfying $X, Y \sim \text{Geom}(p)$, that is,
$$p_X(k) = p_Y(k) = p(1-p)^{k-1},\ 0 < p < 1.$$
If $W = X + Y$ is the random variable that is the sum of X and Y, show that
$$p_W(w) = (w-1)p^2(1-p)^{w-2},\ w \geq 1.$$
Hint: Recall that, by Theorem 3.8.3 on page 176 we have
$$p_W(w) = \sum_{\text{all } k} p_X(k)p_Y(w-k).$$

2. Assume the following property of the gamma function
$$\Gamma(r) = (r-1)\Gamma(r-1).$$
Use this to prove that if n is a positive integer, then
$$\Gamma(n) = (n-1)!$$
Hint: Proof by induction.

3. Prove that $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$. **Hint:** Start by observing that if Z is a random variable with standard normal distribution, then $E(Z^2) = 1$. Now compute as follows:
$$\begin{aligned} 1 &= E(Z^2) \\ &= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} z^2 e^{-\frac{z^2}{2}}\, dz \\ &= \frac{\sqrt{2}}{\sqrt{\pi}} \int_{0}^{\infty} z^2 e^{-\frac{z^2}{2}}\, dz. \end{aligned}$$
Now compute this last integral using the following substitution, let $u = \frac{z^2}{2}$.

4. Do exercise 4.3.1 from the textbook, but instead of using the table A.1, use R to compute the required values.

5. Do exercise 4.3.2 from the textbook but again use R instead of a table to compute the values.

6. Do exercise 4.3.6 using R to carry out the necessary computations. **Hint:** The function `qnorm` is helpful here.

7. Do the following problems from the textbook: 4.4.2, 4.4.5, 4.4.7, 4.5.2, 4.5.6, 4.2.2, 4.2.10, 4.2.22, 4.2.26, 4.3.11, 4.3.15, 4.3.16, 4.3.18, 4.6.1, 4.6.5