Notes on calculating atom interferometer gyroscope sensitivity

A. Theory

The goal of this project is to allow us to explore how the sensitivity of a Mach-Zehnder atom interferometer gyroscope is determined by:

- choice of atom (and its associated van der Waals C_3 coefficient)
- atom velocity v
- grating period d_g
- grating open fractions $f = w/d_g$, where w is the width of the gaps between adjacent grating bars
- grating longitudinal thickness l.
- longitudinal distance between successive gratings L

Choosing the grating thickness l and grating-to-grating distance L is likely fairly simple: we always want smaller l and larger L, but l will probably be determined by the grating fabrication process and L is limited by the maximum size of the apparatus. The grating period d_g and open fraction w/d_g can be optimized for a choice of atom and atom velocity v. Therefore, we want to be able to plot the sensitivity of a Mach-Zehnder atom interferometer gyroscope as a function of v for different atoms.

The sensitivity S of a gyroscope is described as

$$S = \delta\Omega\sqrt{t} \tag{1}$$

where $\delta\Omega$ is the uncertainty in a typical measurement of rotation rate Ω and t is the amount of data acquisition time required to make that measurement. Smaller values of S are preferable. Values of S are usually expressed in rad/s/$\sqrt{\mathrm{Hz}}$. S also is a measure of the uncertainty in the gyroscope's angular orientation as a function of operating time and is therefore sometimes expressed in $^\circ/\sqrt{h}$ (i.e. the uncertainty in the gyroscope's angular orientation is $S\sqrt{T}$, where T is the operating time since last calibration).

This sensitivity S can be written in terms of the items listed previously. For an atom interferometer gyroscope, the phase ϕ can be written as

$$\phi = \frac{4\pi\vec{\Omega}\cdot\vec{A}}{\lambda_{dB}v} - \phi_0 \tag{2}$$

where ϕ_0 is an arbitrary refrence phase, λ_{dB} is the atoms' de Broglie wavelength, and $\vec{\Omega}\cdot\vec{A}$ is the projection of the gyroscope's rotation rate vector $\vec{\Omega}$ onto the plane of the interferometer with enclosed area A. If we let Ω be the component of $\vec{\Omega}$ normal to $\vec{A}$ and represent λ_{dB} as h/mv, we can rewrite the above equation as

$$\delta\phi = \frac{4\pi mA}{h}\delta\Omega \tag{3}$$

and solve it for $\delta\Omega$ to get

$$\delta\Omega = \frac{h}{4\pi mA}\delta\phi \tag{4}$$

According to [1], the uncertainty $\delta\phi$ on a single measurement of the phase of an atom interferometer is defined by

$$\delta\phi^2 = \frac{1}{|C|^2 N} \tag{5}$$

where N is the number of atoms detected. Substituting Eq. 5 into Eq. 4 and the result into Eq. 1, we get

$$S = \frac{h}{4\pi mA}\sqrt{\frac{t}{|C|^2 N}} \tag{6}$$

If we take t to be the unit time, we see N/t is the average atom beam intensity (i.e. flux) $\langle I\rangle$.

The interferometer's enclosed area A can be written as

$$A = L^2\tan\theta_d = L^2\tan\left(\arcsin\left(\frac{h}{mvd_g}\right)\right) \approx \frac{L^2 h}{mvd_g} \tag{7}$$

Substituting into Eq. 6, we get

$$S = \frac{vd_g}{4\pi L^2}\frac{1}{\sqrt{|C|^2\langle I\rangle}} \tag{8}$$

[2] describes in detail how to calculate $|C|^2\langle I\rangle$ from the nanograting open fraction, thickness, and period as well as atom velocity and atom-grating C_3 coefficient. This calculation can be done with and without considering van der Waals interactions between the atoms and the grating bars. Kapitza-Dirac gratings can be represented as gratings for which $C_3 = 0$ and the open fraction $f = 1/2$.

Note that the $\left(e_n^{Gi}\right)^2$ terms in Eq. 14 in [2] should actually be written as $\left|e_n^{Gi}\right|^2$.

van der Waals C_3 coefficients for alkali metals interacting with various media (including silicon nitride) are listed in atomic units in [3]. To convert the C_3 coefficients in atomic units to SI units (for the sake of the numerical calculations), multiply by 1 Hartree times the Bohr radius cubed. Useful C_3 coefficients are listed in atomic units in Table I.

TABLE I. van der Waals C_3 coefficients in atomic units for various combinations of atoms and dielectric media [3].

	SiN_x	SiO_2
Li	0.715	0.484
Na	0.794	0.5424
K	1.212	0.839
Rb	1.369	0.956

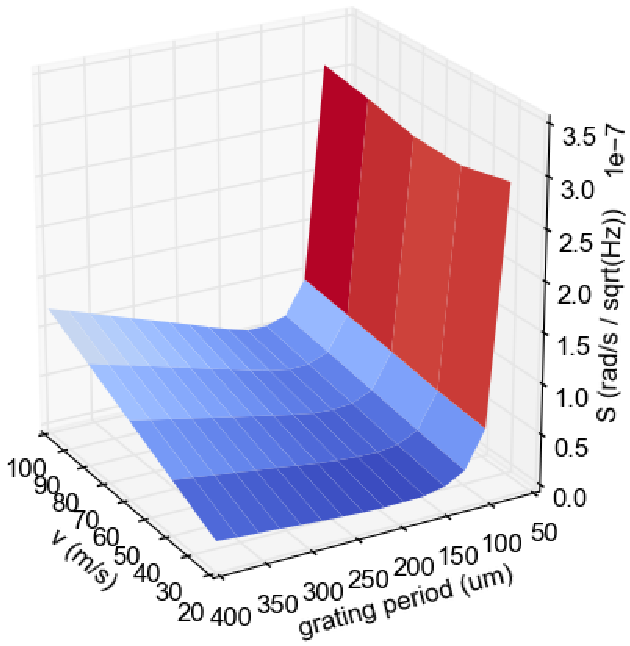

FIG. 1. Sensitivity of a Rb atom interferometer gyroscope vs velocity and SiN_x grating period assuming $L = 10$ cm. For each point on this plot, the optimal open fractions were calculated. The optimal open fractions were restricted to a grating-period-dependent maximum, which corresponds to a minimum grating bar width of 40 nm. Lower S is preferable.

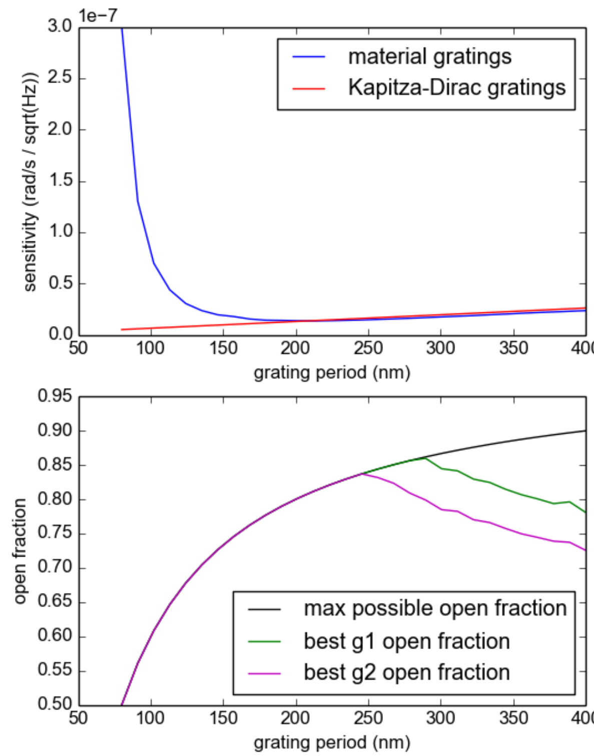

FIG. 2. Sensitivity of a Rb atom interferometer gyroscope vs SiN_x grating period for $v = 20$ m/s, $L = 10$ cm and the optimal open fractions that correspond to that sensitivity. The maximum open fractions were calculated assuming a minimum grating bar width of 40 nm. Lower S is preferable.

B. Results

Fig 1 shows the sensitivity of an atom interferometer gyroscope with material gratings as a function of v and d_g. For this calculation, it was assumed that $L = 10$ cm and the C_3 coefficient was that of Rb interacting with SiN_x. For each (v, d) coordinate on the plot, the sensitivity S was optimized with respect to the grating 1 and grating 2 open fractions f_1 and f_2. For this optimization, f_1 and f_2 were allowed to vary between 0 and $1-w_{min}/d$, where w_{min} is the minimum width of a single grating bar that is allowed by the nanograting fabrication process and that will not cause the grating to fall apart. For these calculations, $w_{min} = 40$ nm, although that number is essentially just a guess. Fig 1 indicates that sensitivity S always improves as v decreases, which is convenient for the low speeds associated with launching atoms out of a 2D MOT.

Fig 2 shows a slice of Fig 1 at $v = 20$ m/s. We can see that S is optimized when $d_g = 200$ nm. Fig 2 also shows the optimal open fractions, along with the maximum open fractions, as a function of d_g. At $d_g = 200$ nm, the optimal open fractions are equal to the maximum open fractions, which impies that S could be further improved by reducing w_{min}.

From Fig 2, we can also see that material gratings are preferable to Kapitza-Dirac gratings in terms of both SWP and sensitivity. While S appears to decrease monotonically as d_g decreases for K-D gratings, obtaining a CW laser, and the optics for that laser, much below 350 nm is essentially impossible. As a side note, the reason the material gratings sensitivity is slightly lower than the K-D sensitivity for $d_g > 200$ nm is because K-D gratings are restricted to 50% open fractions, whereas material gratings can have more optimal open fractions.

Fig 3 shows how sensitivity depends on the van der Waals C_3 coefficient associated with interaction between the atoms and the grating bars. For this calculation, it was assumed that $v = 20$ m/s, $d_g = 200$ nm, and $L = 10$. This calculation suggests that using K or Na rather

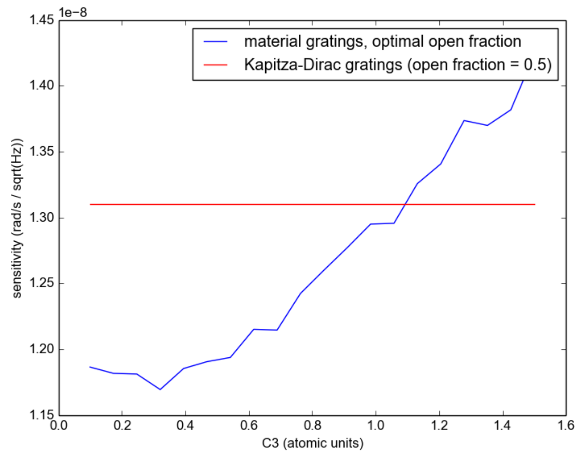

FIG. 3. Sensitivity of a Rb atom interferometer gyroscope vs the van der Waals C_3 coefficient for interaction between the atoms and the gratings at $d_g = 200$ nm, $v = 20$ m/s, $L = 10$ cm and optimal open fractions.

than Rb is probably not worth the effort of developing a new K source, which may involve temperatures that are unreasonably high for this application.

Note that L only appears outside the expression for $\sqrt{|C|^2 \langle I \rangle}$ in Eq. 8. This means that the length of the interferometer has no bearing on the optimal v, d_g, C_3, or open fractions. Therefore, we simply want L to be as long as possible given the size constraints of the apparatus. If the apparatus must exist in a box, then we could consider orienting the interferometer to stretch between opposite corners.

For these calculations, I used $l = 150$ nm because that is the longitudinal thickness of the grating bars in our atom interferometer at University of Arizona.

Based on the calculations presented here, I propose to build a nanograting atom interferometer gyroscope with the following specifications:

- atom: Rb (because we already have Rb source and optics)
- v: 20 m/s (typical 2D MOT output, lower is better)
- d_g: 200 nm
- open fractions $f_{g1} = 0.8$, $f_{g2} = 0.8$, $f_{g3} = 0.37$
- $l = 150$ nm (estimated lower limit)
- $L = 10$ nm (estimated upper limit)

An interferometer with these specifications would have an optimal sensitivity of $S = 2.0 \cdot 10^{-8}$ rad/s/$\sqrt{\text{Hz}}$, which corresponds to an ARW of $6.9 \cdot 10^{-5}$ °/$\sqrt{h}$. While the state-of-the-art, table-top, atom interferometer gyroscopes have a sensitivity that is an order of magnitude better [4], a portable, low-SWP system such as the one described here represents a significant advancement for the field.

[1] A. Lenef, T. D. Hammond, E. T. Smith, M. S. Chapman, R. A. Rubenstein, and D. E. Pritchard, "Rotation Sensing with an Atom Interferometer," Phys. Rev. Lett. **78**, 760 (1997).

[2] Alexander D. Cronin, Lu Wang, and John D. Perreault, "Limitations of Nanotechnology for Atom Interferometry," (2005), arXiv:0508032 [physics].

[3] B. Arora and B. K. Sahoo, "van der Waals coefficients for alkali-metal atoms in material media," Phys. Rev. A **89**, 022511 (2014).

[4] D. S. Durfee, Y. K. Shaham, and M. A. Kasevich, "Long-Term Stability of an Area-Reversible Atom-Interferometer Sagnac Gyroscope," Phys. Rev. Lett. **97**, 240801 (2006).