Notes on calculating atom interferometer gyroscope sensitivity

The goal of this project is to allow us to explore how the sensitivity of a Mach-Zehnder atom interferometer gyroscope is determined by:

- choice of atom (with mass m and van der Waals coefficient C_3)
- atom velocity v
- grating period d_g
- grating open fraction (defined as w/d_g, where w is the width of the gaps between adjacent grating bars)
- grating longitudinal thickness l
- longitudinal distance between successive gratings L

Choosing the grating thickness l and grating-to-grating distance L is likely fairly simple: we always want smaller l and larger L, but l will probably be determined by the grating fabrication process and L is limited by the maximum size of the apparatus. The grating period d_g and open fraction w/d_g can be optimized for a choice of atom and atom velocity v. Therefore, we want to be able to plot the sensitivity of a Mach-Zehnder atom interferometer gyroscope as a function of v for different atoms.

Short-term gyroscope sensitivity S is described as

$$S = \delta\Omega\sqrt{T} \tag{1}$$

where $\delta\Omega$ is the uncertainty in a typical measurement of rotation rate Ω and T is the amount of data acquisition time required to make that measurement. This sensitivity S can be written in terms of the items listed previously. For an atom interferometer gyroscope, the phase ϕ can be written as

$$\phi = \frac{4\pi\vec{\Omega}\cdot\vec{A}}{\lambda_{dB}v} - \phi_0 \tag{2}$$

where ϕ_0 is an arbitrary refrence phase, λ_{dB} is the atoms' de Broglie wavelength, and $\vec{\Omega}\cdot\vec{A}$ is the projection of the gyroscope's rotation rate $\vec{\Omega}$ onto the plane of the interferometer with enclosed area A. If we let Ω be the component of $\vec{\Omega}$ normal to $\vec{A}$ and represent λ_{dB} as h/mv, we can rewrite the above equation as

$$\delta\phi = \frac{4\pi mA}{h}\delta\Omega \tag{3}$$

and solve it for $\delta\Omega$ to get

$$\delta\Omega = \frac{h}{4\pi mA}\delta\phi \tag{4}$$

According to [1]

[1] I. Hromada, R. Trubko, W. F. Holmgren, M. D. Gregoire, and A. D. Cronin, "de Broglie wave-front curvature induced by electric-field gradients and its effect on precision measurements with an atom interferometer," Phys. Rev. A **89**, 033612 (2014).