Notes on calculating atom interferometer gyroscope sensitivity

The goal of this project is to allow us to explore how the sensitivity of a Mach-Zehnder atom interferometer gyroscope is determined by:

- choice of atom (and its associated van der Waals C_3 coefficient)

- atom velocity v

- grating period d_g

- grating open fraction (defined as w/d_g, where w is the width of the gaps between adjacent grating bars)

- grating longitudinal thickness l

- longitudinal distance between successive gratings L

Choosing the grating thickness l and grating-to-grating distance L is likely fairly simple: we always want smaller l and larger L, but l will probably be determined by the grating fabrication process and L is limited by the maximum size of the apparatus. The grating period d_g and open fraction w/d_g can be optimized for a choice of atom and atom velocity v. Therefore, we want to be able to plot the sensitivity of a Mach-Zehnder atom interferometer gyroscope as a function of v for different atoms.

The short-term sensitivity S of a gyroscope is described as

$$S = \delta\Omega\sqrt{t} \tag{1}$$

where $\delta\Omega$ is the uncertainty in a typical measurement of rotation rate Ω and t is the amount of data acquisition time required to make that measurement. Smaller values of S are preferable. This sensitivity S can be written in terms of the items listed previously. For an atom interferometer gyroscope, the phase ϕ can be written as

$$\phi = \frac{4\pi\vec{\Omega}\cdot\vec{A}}{\lambda_{dB}v} - \phi_0 \tag{2}$$

where ϕ_0 is an arbitrary refrence phase, λ_{dB} is the atoms' de Broglie wavelength, and $\vec{\Omega}\cdot\vec{A}$ is the projection of the gyroscope's rotation rate vector $\vec{\Omega}$ onto the plane of the interferometer with enclosed area A. If we let Ω be the

component of $\vec{\Omega}$ normal to $\vec{A}$ and represent λ_{dB} as h/mv, we can rewrite the above equation as

$$\delta\phi = \frac{4\pi mA}{h}\delta\Omega \tag{3}$$

and solve it for $\delta\Omega$ to get

$$\delta\Omega = \frac{h}{4\pi mA}\delta\phi \tag{4}$$

According to [1], the uncertainty $\delta\phi$ on a single measurement of the phase of an atom interferometer is defined by

$$\delta\phi^2 = \frac{1}{|C|^2 N} \tag{5}$$

where N is the number of atoms detected. Substituting Eq. 5 into Eq. 4 and the result into Eq. 1, we get

$$S = \frac{h}{4\pi mA}\sqrt{\frac{t}{|C|^2 N}} \tag{6}$$

If we take t to be the unit time, we see N/t is the average atom beam intensity (i.e. flux) $\langle I \rangle$.

The interferometer's enclosed area A can be written as

$$A = L^2\tan\theta_d = L^2\tan\left(\arcsin\left(\frac{h}{mvd_g}\right)\right) \approx \frac{L^2 h}{mvd_g} \tag{7}$$

Substituting into Eq. 6, we get

$$S = \frac{vd_g}{4\pi L^2}\frac{1}{\sqrt{|C|^2\langle I\rangle}} \tag{8}$$

[2] describes in detail how to calculate $|C|^2\langle I\rangle$ from the nanograting open fraction, thickness, and period. (Note that [2] refers to $|C|^2$ simply as C^2, although C may be complex.) This calculation can be done with and without considering van der Waals interactions between the atoms and the grating bars. The case without van der Waals interaction represents a Mach-Zehnder atom interferometer made with Kapitza-Dirac gratings.

We may also want to plot other properties vs v and atom species, such as the diffraction efficiencies themselves. We can add this functionality into the code as needed.

[1] A. Lenef, T. D. Hammond, E. T. Smith, M. S. Chapman, R. A. Rubenstein, and D. E. Pritchard, "Rotation Sensing with an Atom Interferometer," Phys. Rev. Lett. **78**, 760 (1997).

[2] Alexander D. Cronin, Lu Wang, and John D. Perreault, "Limitations of Nanotechnology for Atom Interferometry," (2005), arXiv:0508032 [physics].