Notes on calculating atom interferometer gyroscope sensitivity

A. Theory

The goal of this project is to allow us to explore how the sensitivity of a Mach-Zehnder atom interferometer gyroscope is determined by:

- choice of atom (and its associated van der Waals C_3 coefficient)
- atom velocity v
- grating period d_g
- grating open fraction (defined as w/d_g, where w is the width of the gaps between adjacent grating bars)
- grating longitudinal thickness l
- longitudinal distance between successive gratings L

Choosing the grating thickness l and grating-to-grating distance L is likely fairly simple: we always want smaller l and larger L, but l will probably be determined by the grating fabrication process and L is limited by the maximum size of the apparatus. The grating period d_g and open fraction w/d_g can be optimized for a choice of atom and atom velocity v. Therefore, we want to be able to plot the sensitivity of a Mach-Zehnder atom interferometer gyroscope as a function of v for different atoms.

The sensitivity S of a gyroscope is described as

$$S = \delta\Omega\sqrt{t} \tag{1}$$

where $\delta\Omega$ is the uncertainty in a typical measurement of rotation rate Ω and t is the amount of data acquisition time required to make that measurement. Smaller values of S are preferable. Values of S are usually expressed in $rad/s/\sqrt{Hz}$. S also is a measure of the uncertainty in the gyroscope's angular orientation as a function of operating time and is therefore sometimes expressed in $^\circ/\sqrt{h}$ (i.e. the uncertainty in the gyroscope's angular orientation is $S\sqrt{T}$, where T is the operating time since last calibration).

This sensitivity S can be written in terms of the items listed previously. For an atom interferometer gyroscope, the phase ϕ can be written as

$$\phi = \frac{4\pi\vec{\Omega}\cdot\vec{A}}{\lambda_{dB}v} - \phi_0 \tag{2}$$

where ϕ_0 is an arbitrary refrence phase, λ_{dB} is the atoms' de Broglie wavelength, and $\vec{\Omega}\cdot\vec{A}$ is the projection of the gyroscope's rotation rate vector $\vec{\Omega}$ onto the plane of the interferometer with enclosed area A. If we let Ω be the component of $\vec{\Omega}$ normal to $\vec{A}$ and represent λ_{dB} as h/mv, we can rewrite the above equation as

$$\delta\phi = \frac{4\pi m A}{h}\delta\Omega \tag{3}$$

and solve it for $\delta\Omega$ to get

$$\delta\Omega = \frac{h}{4\pi m A}\delta\phi \tag{4}$$

According to [1], the uncertainty $\delta\phi$ on a single measurement of the phase of an atom interferometer is defined by

$$\delta\phi^2 = \frac{1}{|C|^2 N} \tag{5}$$

where N is the number of atoms detected. Substituting Eq. 5 into Eq. 4 and the result into Eq. 1, we get

$$S = \frac{h}{4\pi m A}\sqrt{\frac{t}{|C|^2 N}} \tag{6}$$

If we take t to be the unit time, we see N/t is the average atom beam intensity (i.e. flux) $\langle I\rangle$.

The interferometer's enclosed area A can be written as

$$A = L^2\tan\theta_d = L^2\tan\left(\arcsin\left(\frac{h}{mvd_g}\right)\right) \approx \frac{L^2 h}{mvd_g} \tag{7}$$

Substituting into Eq. 6, we get

$$S = \frac{vd_g}{4\pi L^2}\frac{1}{\sqrt{|C|^2\langle I\rangle}} \tag{8}$$

[2] describes in detail how to calculate $|C|^2\langle I\rangle$ from the nanograting open fraction, thickness, and period as well as atom velocity and atom-grating C_3 coefficient. This calculation can be done with and without considering van der Waals interactions between the atoms and the grating bars. Kapitza-Dirac gratings can be represented as gratings for which $C_3 = 0$ and the open fraction $f = 1/2$.

Note that the $\left(e_n^{Gi}\right)^2$ terms in Eq. 14 in [2] should actually be written as $\left|e_n^{Gi}\right|^2$.

van der Waals C_3 coefficients for alkali metals interacting with various media (including silicon nitride) are listed in atomic units in [3]. To convert the C_3 coefficients in atomic units to SI units (for the sake of the numerical calculations), multiply by 1 Hartree times the Bohr radius cubed. Useful C_3 coefficients are listed in SI units in Table I.

TABLE I. van der Waals C_3 coefficients in units of 10^{-49} Jm^3 for various combinations of atoms and dielectric media [3].

	SiN_x	SiO_2
Li	4.61	3.10
Na	5.10	3.49
K	7.82	5.42
Rb	8.85	6.20

B. Results

Fig 1, top left, shows the sensitivity of an atom interferometer gyroscope with material gratings as a function of v and d. For this calculation, it was assumed that $L = 10$ cm and the C_3 coefficient was that of Rb interacting with SiN_x. This figure indicates that sensitivity S always improves as grating period decreases and that the optimal velocity v increases as d decreases.

[1] A. Lenef, T. D. Hammond, E. T. Smith, M. S. Chapman, R. A. Rubenstein, and D. E. Pritchard, "Rotation Sensing with an Atom Interferometer," Phys. Rev. Lett. **78**, 760 (1997).

[2] Alexander D. Cronin, Lu Wang, and John D. Perreault, "Limitations of Nanotechnology for Atom Interferometry," (2005), arXiv:0508032 [physics].

[3] B. Arora and B. K. Sahoo, "van der Waals coefficients for alkali-metal atoms in material media," Phys. Rev. A **89**, 022511 (2014).

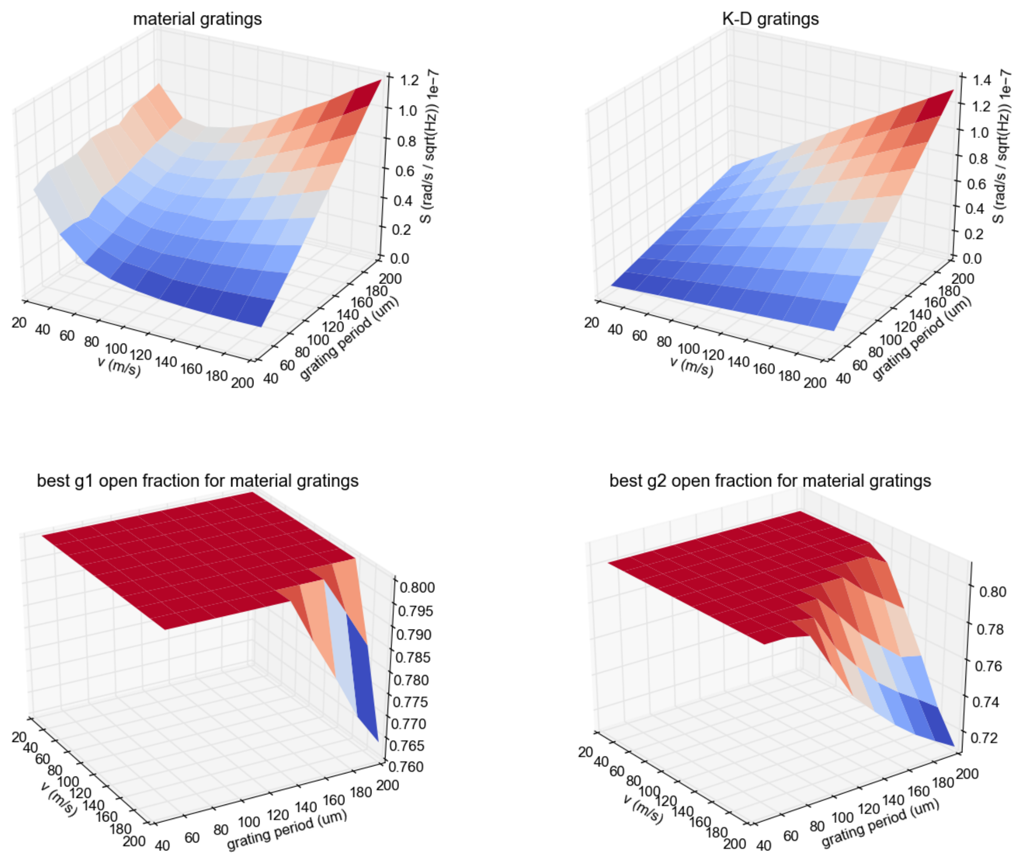

FIG. 1. TOP LEFT: Sensitivity S of an atom interferometer gyroscope as a function of atom velocity v and grating period d. For this calculation, it was assumed that $L = 10$ cm and the C_3 coefficient was that of Rb interacting with SiN_x. TOP RIGHT: Same as top left, but for Kapitza-Dirac gratings (open fractions set to 1/2, $C_3 = 0$). BOTTOM LEFT and BOTTOM RIGHT: optimal open fractions for gratings 1 and 2 as a function of v and d. The optimal open fractions were capped at 0.8, since there is a physical limit to how small the grating bars can be. 0.8 was simply a guess at this limit–the actual limit likely depends on d.