1. Instructions

After reading and completing the pre-lab assignment for chapter 1, complete the following problems. For each problem you will either have graph that appears or you an output that appears in the command line. If a graph appears, you will need to save the figure as shown in class. If an output appears in the command line that answers the question, then you will copy and paste this into a .txt file along with a brief description of which problem this corresponds to.

2. Problems

(1) Sometimes special functions will be presented to you in an output. Using `dsolve()` on the differential equation below is such an example. Research the special function and give a brief description of the special function(s) that MatLab outputs for the solution of the differential equation below.

$$y' = y^2 + t^2$$

(2) Another way to gain an explicit solution is to use the `diff()` command. For example, in MatLab, type `syms y(t)`. This is defining y as a function of t in terms of a symbolic variable. Next, type

```
ode = diff(y,t) == t*y;
```

You have now defined the differential equation $y' = ty$ and called that equation `ode`. You can solve this differential equation using `dsolve()`:

```
dsolve(ode)
```

The output is `C1*exp(t^2 /2)`. Use the command `help dsolve` to read about higher order derivatives. If there are other initial conditions, we can solve them by creating a $1 \times N$ vector of the form

```
conditions = [condition1, condition2, ..., conditionN].
```

Then, given a differential equation of order N, we can solve using the above method by typing

```
dsolve(ode,conditions)
```

To define a condition on the an Nth order derivative, use the command

```
DNy = diff(y,t,N)
```

where you replace N by whatever order it is. Use the `diff` command and create a `conditions` vector to arrive at an explicit solution to the following differential equation.

$$y'' = (\cos(2x))^2 - y,$$

with the initial conditions $y(0) = 1$ and $y'(0) = 0$.

(3) Plotting the direction field: Consider the differential equation given by

$$y'(x) = 3y - 10\sin(x).$$

Use the command `ezplot()` to graph $f(x,t) = 0$ over the interval $(-5,5) \times (0,10)$ by entering:

```
ezplot('3*y-10*sin(x)', [0,10,-5,5])
```

Which parts of the graph correspond to positive slopes ($y'(x) > 0$)? What about negative slopes?

Type of the following commands to produce the direction field for the differential equation.

```
>>[x,y]=meshgrid(0:.3:10,-5:.3:5);
>>dy = 3*y-10*sin(x);
>>dx=ones(size(dy));
>>quiver(x,y,dx,dy)
```

The first command creates a discretization in the xy-plane. To understand the third command, type `help quiver` and read the documentation. Essentially, you are writing your differential equation as

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{3y - 10\sin(x)}{1}.$$

Save your figure as `DirectionFieldLab1.fig`. This is the direction field and is useful in understanding how the solution behaves without actually knowing the solution itself.

Recall that vectors have both magnitude and direction. To make all the vectors the same length, do the following. Type

```
>>dyu = dy./sqrt(dy.^2+dx.^2);
>>dxu = dx./sqrt(dy.^2+dx.^2);
>>quiver(x,y,dxu,dyu)
```

Save your figure as `DirectionFieldNormalizedLab1.fig`.