Instructions

After reading and completing the in-class Matlab code for solving the second-order linear differential equation with distinct real roots given initial conditions, complete the following problems. For each problem,

(1) copy and paste your entire Matlab codes into a .txt file; add comments to your codes to describe the code as needed;
(2) provide your own example and inputs satisfying each problem's assumptions;
(3) import the Matlab figure to your report (i.e., text file).
(4) All figures must have a title, label, and legend as needed.
(5) If an output appears in the command line that answers the question, then you will copy and paste this into a .txt file along with a brief description.
(6) Upload your final solutions (codes, description, inputs, outputs, figures, etc) as a single file (preferably PDF format) on Canvas by March 20, 2022, at 11:59 pm.

PROBLEMS

Problem 1.
Solve a second-order linear differential equation with constant coefficients given initial conditions (i.e., y_0 and y_0') for the case with **complex roots** of the characteristic equation.
Assumptions:

1. $t_0 = 0$
2. characteristic equation has complex roots

Inputs:

1. $t_0 = 0$
2. $y_0 = y(t_0)$
3. $y_0' = y'(t_0)$
4. domain of the function:
   4.1. the minimum value of time (tmin)
   4.2. the maximum value of time (tmax)
5. The user-defined characteristic equation (i.e., second-order polynomial function) as a row vector with three entities.

Steps:

1. Find the roots of the characteristic equation using *roots.*
2. Find λ and μ using *real* and *imag* functions.
3. Define fundamental set of solutions y_1, y_2, and general solution $y(t) = y_1 + y_2 = C_1 e^{\lambda t}\cos(\mu t) + C_2 e^{\lambda t}\sin(\mu t)$.

4. Find the values of constants C_1 and C_2 (i.e., vector x). To do so, construct 2by2 matrix A(i.e., A=[1 0;λ μ]), vector b (initial conditions), and solve x=A\b.
5. Now that you have the values of λ, μ, C_1, and C_2, display the general solution in Command Window using *disp* function.
6. Plot the solution.

Problem 2.

Solve a second-order linear differential equation with constant coefficients given initial conditions (i.e., y_0 and y_0') for the case with **real repeated roots** of the characteristic equation.

Assumptions:

1. $t_0 = 0$
2. characteristic equation has real repeated roots

Inputs:

1. $t_0 = 0$
2. $y_0 = y(t_0)$
3. $y_0' = y'(t_0)$
4. domain of the function:
   4.1. minimum value of time (tmin)
   4.2. maximum value of time (tmax)

Steps:

1. Find the roots (i.e., $m_1 = m_2 = m$) of the characteristic equation using *roots.* Note that the characteristic equation is a user-input second-order polynomial function as a row vector with three entities.
2. Define fundamental set of solutions y_1, y_2, and general solution as follows $y(t) = y_1(t) + y_2(t) = C_1 e^{mt} + C_2 t e^{mt}$.
3. Find the values of constants C_1 and C_2 (i.e., vector x). To do so, construct 2by2 matrix A(i.e., A=[1 0; m 1]), vector b (initial conditions), and solve x=A\b.
4. Now that you have the values of C_1, and C_2, display the general solution in Command Window using *disp* function.
5. Plot the solution.