MXB101 Problem Solving Task 4 (10%)
Semester 1, 2025

This assignment covers material up to and including Topic 12. Please submit your solutions in a **single PDF file** using the Grade Scope submission link on the MXB101 Canvas site by **11:59 pm on Friday Week 13 (30th May, 2025)**. **Please ensure your responses to each question begin on a new page**.

Please justify all your solutions, and remember to: define events and probabilities; state all rules used; show all working; and, write or type neatly. A correct solution without justification will receive zero marks. This is an individual assignment, you must not share your working, or solutions, with your peers. Your solution must be your own. You are not permitted to copy, summarise, or paraphrase the work of others in your solution. **You are not permitted to use generative artificial intelligence tools for this assignment**.

Question 1. (6 marks)

The *infectivity* of a disease is a measure of how quickly a disease can spread. One measure of infectivity is the probability, P, that an infection is transmitted from an infected person to someone else in an interaction between the two. Suppose that the infectivity, P, varies person-to-person such that P is a random variable described by the probability density function

$$f(p) = \tfrac{3}{2}p(2 - p), \qquad 0 < p < 1. \tag{1}$$

Now suppose that infections are passed between individuals independently. This means, for example, that the number of people, N, that an infectious person interacts with before infecting someone with the disease has a geometric distribution, $N|P = p \sim \text{Geo}(p)$, with probability mass function

$$\Pr(N = n|P = p) = (1 - p)^{n-1}p, \qquad N \geq 1.$$

(a) Find $\mathbb{E}(N)$, the expected number of people an infectious person interacts with before infecting someone with the disease.

(b) What is the expected number of individuals infected from 100 interactions with the infectious person? *You may find it useful to know that $\mathbb{E}(P) = \tfrac{5}{8}$.*

Question 2. (8 marks)

Consider now that the duration of the infection (i.e., the length of time that a person has the disease), T, is dependent upon the infectivity such that

$$f(t|p) = \frac{2 - pt}{2(2 - p)}, \qquad 0 < p < 1, \quad 0 < t < 2.$$

Note that P is described by the probability density function given in eq. (1).

(a) Find $f(t, p)$.

(b) Show that $\text{Cov}(T, P) = -\tfrac{1}{48}$. *You may find it useful to use the hint given in Question 1b.*

(c) Are infections that last longer more infectious on average?

Question 3. (6 marks)

The disease is seasonal and whether or not a person is infected one year depends whether or not they were infected the previous year. By tracking people who were infected and recording whether they were infected the following year, a group of researchers has determined that if a person has an infection one year, then they will also be infected the following year with probability 0.4. The researchers denoted the probability that an individual who was not infected in a year was also not infected the following year as k, and decided to model the infection status of individuals year-to-year as a Markov chain.

(a) Write the transition probability matrix for the Markov chain.

(b) While the researchers could not measure k directly, they were able to determine that, in the long run, 25% of individuals are infected in any given year. By considering the steady-state of the Markov chain, determine k.

Criteria and Standards Guide

Question 1(a), 2(a), 3(a), 1(b), 2(b) 3(b)

- 3 marks: Fully correct with full working or, the question is incorrect due to a very minor error. A very minor error is defined as correct working, with an error in calculation.

- 2 marks: Incorrect due one or two minor errors. Minor errors are classified as errors relating to algebra or calculus.

- 1 mark: Incorrect due to three minor errors, or one major error. A major error is classified as an error related to probability rules.

- 0 marks: Question incorrect due to more than three minor errors, or more than two major errors; or, fully correct with no working shown.

Question 2(c)

- 2 mark: Correct with justification.

- 1 marks: Correct without justification.

- 0 marks: Incorrect.