MXB101 Problem Solving Task 2 (10%)
Semester 1, 2025

This assignment covers material up to and including Topic 5. Please submit your solutions in a **single PDF file** using the Grade Scope submission link on the MXB101 Canvas site by **11:59pm Friday Week 8 (25th April, 2025)**. Please ensure your responses to each question begin on a new page.

Please justify all your solutions, and remember to: define events and probabilities; state all rules used; show all working; and, write or type neatly. A correct solution without justification will receive zero marks. This is an individual assignment, you must not share your working, or solutions, with your peers. Your solution must be your own. You are not permitted to copy, summarise, or paraphrase the work of others in your solution. **You are not permitted to use generative artificial intelligence tools for this assignment**.

Question 1. (6 marks)

You have been commissioned by an online fashion store to test a new online sales system. The store specialises in unisex, eco-friendly, easy to match, mathematically themed clothes.

Currently, their product range comprises shirts and trousers of two colours: at present, they stock 12 different shirt designs (8 coloured brown, 4 coloured black) and 8 different types of trousers (3 coloured brown, 5 coloured black). The store has recently developed a tool that randomly produces a web page containing five complete outfits (i.e., shirt and trousers) to new users, and marks the first two outfits as "suggested" (Figure 1).

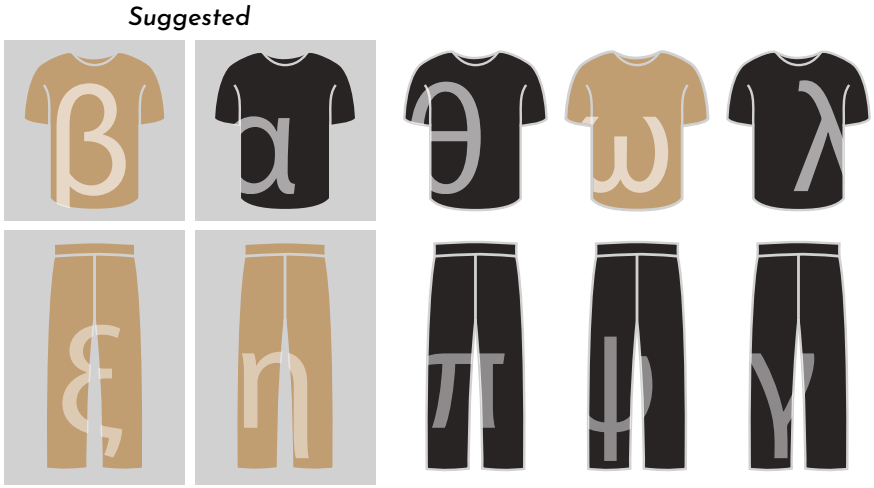

Figure 1: Example web page produced by the outfit suggester. The first two (in grey) are marked as "Suggested".

(a) How many unique ways can this web page be produced by the system?

(b) What is the probability that the two outfits that the system marks as suggested are entirely brown?

Question 2. (6 marks)

A transmission system sends messages 8 bits (i.e., 8 digits that take values of either 0 or 1) in length. Transmitted messages are often corrupted by errors, whereby bits that were originally 0 are switched to 1 or vice versa (Figure 2).

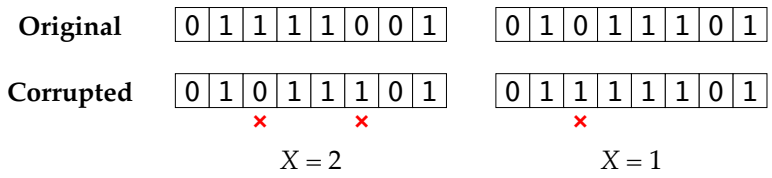

Figure 2: Two transmitted and corrupted messages with $X = 2$ and $X = 1$ errors, respectively.

The total number of such errors present in a message is given by the discrete random variable X with probability mass function $\Pr(X = x) = p(x)$ for $x \in \{0, 1, ..., 8\}$ where $p(x)$ is given by

$$p(x) = \begin{cases} \frac{1}{3}, & \text{if } x = 0, \\ \frac{k}{x(x+1)}, & \text{if } x \in \{1, 2, ..., 8\}. \end{cases}$$

(a) Find k such that $p(x)$ is a valid probability mass function.

(b) Calculate the standard deviation of X.

(c) The *entropy* of a random variable represents the average level of uncertainty in the variable. For a discrete random variable entropy is given by

$$\mathbb{E}[-\ln(p(X))] = \sum_{x \in \Omega} -\ln(p(x))p(x).$$

Calculate the entropy of X.

Question 3. (8 marks)

Consider the continuous random variable, X, defined by the quantile function

$$Q(p) = \sqrt{-\ln(1-p)}.$$

(a) Find the cumulative distribution function of X, $F(x) = \Pr(X \leq x)$, and the range of possible values X can take.

(b) Find $\mathbb{E}[X]$. You must validate the assumptions behind any rules you apply. *Note that you may find it useful to know that*

$$\int_{-\infty}^{\infty} e^{-x^2} \mathrm{d}x = \sqrt{\pi}.$$

(c) Calculate the interquartile range of X; that is, the difference between the upper and lower quartiles.

Criteria and Standards Guide

All sub-questions

- 2 marks: Fully correct with full working; or, the question is incorrect due to a carried error from a previous sub-question; or, the question is incorrect due to a very minor error. A very minor error is defined as correct working, with an error in calculation.
- 1 mark: Incorrect due to no more than one or two minor errors.
- 0 marks: Question incorrect due to more than two minor errors, or one major error; or, fully correct with no working shown.

Question 3(a)

- +1 mark: Correct range of possible values given (if the solution to 3(a) meets the criteria for 2 marks, but the correct range of possible values is not given, a total of 2 marks is awarded).

Question 3(b)

- +1 mark: Assumptions behind any rules applied are verified.