MXB101 Problem Solving Task 1 (10%)
Semester 1, 2025

This assignment covers material up to and including Topic 3. Please submit your solutions in a **single PDF file** using the Grade Scope submission link on the MXB101 Canvas site by **11:59pm Friday Week 5 (28th March, 2025)**. Please ensure your responses to each question begin on a new page.

Please justify all your solutions, and remember to: define events and probabilities; state all rules used; show all working; and, write or type neatly. A correct solution without justification will receive zero marks. This is an individual assignment, you must not share your working, or solutions, with your peers. Your solution must be your own. You are not permitted to copy, summarise, or paraphrase the work of others in your solution. **You are not permitted to use generative artificial intelligence tools for this assignment**.

Question 1. (8 marks)

Using rules of set theory and probability theory, prove the following statements are true:

(a) $\overline{(\overline{A} \cup \overline{B})} \cup \overline{(\overline{A} \cup B)} = A$.

(b) $(A \cup B) \cap \overline{(A \cap B)} = (A \cap \overline{B}) \cup (\overline{A} \cap B)$.

(c) If $A \cap B = \emptyset$, then $\Pr(A) \leq \Pr(\overline{B})$.

(d) If $\Pr(A) = \Pr(B) = \Pr(A \cap B)$, then $\Pr((A \cap \overline{B}) \cup (\overline{A} \cap B)) = 0$.

(e) If $\Pr(A) = \Pr(B) = 1$, then $\Pr(A \cap B) = 1$.

(f) Show that the addition rule for four events A, B, C, D is given by:

$$\begin{aligned}\Pr(A \cup B \cup C \cup D) = &\Pr(A) + \Pr(B) + \Pr(C) + \Pr(D) \\ &- \Pr(A \cap B) - \Pr(A \cap C) - \Pr(A \cap D) - \Pr(B \cap C) - \Pr(B \cap D) - \Pr(C \cap D) \\ &+ \Pr(A \cap B \cap C) + \Pr(A \cap B \cap D) + \Pr(B \cap C \cap D) + \Pr(A \cap C \cap D) \\ &- \Pr(A \cap B \cap C \cap D).\end{aligned}$$

Hint: You may use the addition rule for three events, given by

$$\Pr(A \cup B \cup C) = \Pr(A) + \Pr(B) + \Pr(C) - \Pr(A \cap B) - \Pr(A \cap C) - \Pr(B \cap C) + \Pr(A \cap B \cap C).$$

Then note that $\Pr(A \cup B \cup C \cup D) = \Pr(A) + \Pr(B \cup C \cup D) - \Pr(A \cap (B \cup C \cup D))$ *as a consequence of the addition rule for two events (you may use this result without proof).*

Question 2. (6 marks)

Your company has ordered electrical components from two different manufacturers. In the last delivery, your company received two boxes: one containing $1,000$ components from manufacturer 1, and the other containing $2,000$ components from manufacturer 2. Your quality control team tested every component and found that both boxes contain exactly 100 faulty components. Unfortunately they also removed all the labels from the boxes, so you cannot tell which box is from which manufacturer. Given this information, answer the following (justify your answers):

(a) Suppose you randomly pick a box by tossing a fair coin, then draw two components from that box. What is the probability that both components are faulty?

(b) Given the process in (a), if both components are faulty, what is the probability that the box selected is from manufacturer 1?

Question 3. (6 marks)

Consider the circuit below made of two types of components (type 1 are boxes, and type 2 are circles). Let W_i be the event that the i-th type 1 component functions, and let H_k be the event that the k-th type 2 component functions. Suppose the probability that a type 1 component functions is p, and the probability that a type 2 component functions is q. You may assume independence of all components regardless of type.

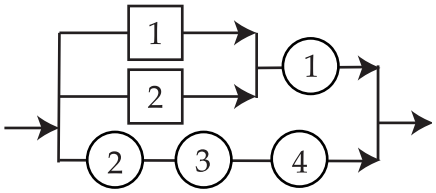

Figure 1: Two component circuit design. Type 1 are boxes and type 2 are circles.

(a) Let S be the event that the circuit functions. Describe the event S in terms of W_i's and H_k's using set operations.

(b) Show that $\Pr(S) = (p^2 - 2p)q^4 + q^3 + (2p - p^2)q$.

(c) Would $p > q$ or $p < q$ lead to a more stable circuit design? Explain why.

Criteria and Standards Guide

Question 1(a) to 1(e)

- +1 mark: For each correct proof with correct application of probability theory for justification.
- +0.75 mark: For each incorrect proof with correct application of probability theory for justification, but minor calculation errors.
- +0.5 mark: For each incorrect proof with minor errors in application of probability theory for justification.
- +0 marks: For each incorrect proof without justification, or justifcation with serious errors.

Question 1(f)

- +3 marks: For valid proof with correct application of probability theory for justification.
- +2 mark: For invalid proof with correct application of probability theory for justification, but minor calculation errors.
- +1 mark: For invalid proof due to minor errors in application of probability theory for justification.
- +0 marks: Invalid proof without justification, or justification with serious errors.

Question 2 and 3

- +2 marks: Correctly interpreting all probabilities and events. If a probability from the question is misinterpreted, a only single mark is awarded (i.e., a deduction of -1 mark).
- +3 marks: Mathematical results are correct with clear justification of steps using rules of set theory and probability theory. Incorrect application of theory will lead to a deduction (-1 mark). Unjustified steps (i.e, correct, but not reasons given) will lead to a deduction (up to -1 mark). Calculation and algebra errors can lead to deductions (up to -1 mark).
- +1 marks: Appropriate interpretation and communication of results. Incorrect interpretation will lead to a deduction (-0.5 marks). Poor communication will lead to a deduction (-0.5 marks).