MXB101 Problem Solving Task 3 (10%)
Semester 1, 2025

This assignment covers material up to and including Topic 8. Please submit your solutions in a **single PDF file** using the Grade Scope submission link on the MXB101 Canvas site by **11:59 pm on Friday Week 11 (16th May, 2025)**. **Please ensure your responses to each question begin on a new page**.

Please justify all your solutions, and remember to: define events and probabilities; state all rules used; show all working; and, write or type neatly. A correct solution without justification will receive zero marks. This is an individual assignment, you must not share your working, or solutions, with your peers. Your solution must be your own. You are not permitted to copy, summarise, or paraphrase the work of others in your solution. **You are not permitted to use generative artificial intelligence tools for this assignment**.

Question 1. (7 marks)

A wildlife conservation group is designing a monitoring study of wallaby behaviour in a remote Queensland national park. The group has decided to study several regions in the park, the boundary of which form squares with side lengths W km and areas X km^2. A statistician has decided to choose the regions such that the region area, X, is a uniformly distributed random variable on the interval $1 \leq x \leq a$ such that $X \sim \text{U}(1, a)$.

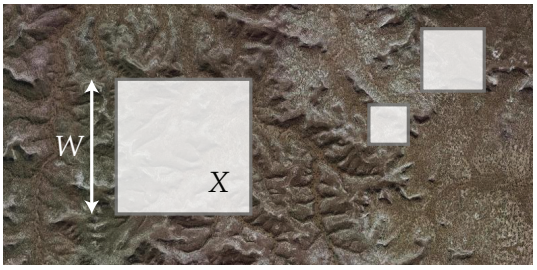

Figure 1: Schematic of randomly selected regions in a remote Queensland national park (Source: QImagery).

The statistician has deduced that $W = \sqrt{X}$ is a random variable that describes the side lengths of the regions. The statistician has also deduced that W has the cumulative distribution function

$$F_W(w) = b(w^2 - 1). \tag{1}$$

Here, the value of b and the range of W depends on a.

(a) Show that $b = \dfrac{1}{a-1}$.

(b) The group choose the **maximum allowable region area**, a, such that the **average region area** is equal to 10 km^2. What is the average region side length, $\mathbb{E}(W)$?

(c) The monthly monitoring cost comprises a base rate of \$700 plus \$80 per km^2.

i. Write an expression for the monitoring cost, C, in terms of the region area, X.

ii. Find the average monitoring cost.

iii. Find the variance of the monitoring cost.

If you did not solve (b), you can leave your answer in terms of the unknown quantity a.

Question 2. (13 marks)

The wildlife conservation group is interested in the health of reproducing female wallabies. The wildlife group has established that 78% of female wallabies have a joey in their pouch. The group also know that 62% of wallabies in the region are female. You may assume that the sex and joey status of wallabies are independent wallaby to wallaby.

Figure 2: Wallaby with a joey in a Queensland national park.

(a) Use the conditional probability rule to argue that the probability of a randomly chosen wallaby having a joey in their pouch is 0.4836.

(b) On average, how many wallabies does a team in the field have to check before finding six that have a joey?

(c) What is the probability that the team checks fewer than three wallabies before finding the first with a joey?

(d) Use an appropriate limiting distribution to estimate the probability that at least 700 wallabies from a sample of 1200 have a joey.

(e) The gestation period (that is, the time from conception to birth) of wallabies is, on average, 33 days, with a standard deviation of 2 days. Assuming that the gestation period is normally distributed, what is the probability that a particular joey had a gestation period of within 3 days of the mean?

(f) From birth, joeys spend an average of 280 days in their mothers pouch. To answer the following, you may assume that the time a joey spends in the pouch is exponentially distributed.

   i. What is the probability that a joey spends more than 320 days in their mothers pouch?

   ii. You have determined from the size of the joey that it has been in the pouch more than 120 days. What is the probability that it has been in the pouch for more than 160 days?

Criteria and Standards Guide

Questions 1(a), 1(b), 2(e), 2(f)(i), 2(f)(ii)

- 2 marks: Fully correct with full working; or, the question is incorrect due to a carried error from a previous sub-question; or, the question is incorrect due to a very minor error. A very minor error is defined as correct working, with an error in calculation.
- 1 mark: Incorrect due to no more than one or two minor errors.
- 0 marks: Question incorrect due to more than two minor errors, or one major error; or, fully correct with no working shown.

Questions 1(c)(i), 1(c)(ii), 1(c)(iii), 2(a)

- 1 mark: Fully correct with full working, or incorrect due to a very minor error. A very minor error is defined as correct working, with an error in calculation.
- 0 marks: Question incorrect due to more than two minor errors, or one major error; or, fully correct with no working shown.

Questions 2(b), 2(c), 2(d)

- 1 mark: Fully correct with full working, or incorrect due to a very minor error. A very minor error is defined as correct working, with an error in calculation.
- 0 marks: Question incorrect due to more than two minor errors, or one major error; or, fully correct with no working shown.
- +1 mark: Correct probability distribution identified.