Problem: Consider we have a relative depth map $f : [0, h] \times [0, w] \to \mathbb{R}$, as well as n points $(a_1, b_1), \ldots, (a_n, b_n)$ with absolute depth values $d_1, \ldots, d_n$. The goal is to find some scaling factor $c \in \mathbb{R}$ to minimize the distance between vectors

$$d_{rel} = [f(a_1, b_1), f(a_2, b_2), \ldots, f(a_n, b_n)]$$
$$d_{rel,scal} = [cf(a_1, b_1), cf(a_2, b_2), \ldots, cf(a_n, b_n)]$$

and

$$d_{abs} = [d_1, d_2, \ldots, d_n]$$

The first is the original relative depth positions, the second is the scaled depth predictions of the relative depth map and the second is the absolute depth values. We find the min via differentiation:

$$\begin{aligned}
&\min_{c\in\mathbb{R}} \|d_{rel,scal} - d_{abs}\|_2 \\
=&\min_{c\in\mathbb{R}} \|d_{rel,scal} - d_{abs}\|_2^2 \\
=&\min_{c\in\mathbb{R}} \sum_{i\in[n]} (cf(a_i, b_i) - d_i)^2 \\
&\text{Now we differentiate w.r.t. } c \text{ to find the min value} \\
&\frac{\partial f}{\partial c} = 0 \\
\iff &\sum_{i\in[n]} 2(c_{opt} f(a_i, b_i) - d_i) f(a_i, b_i) = 0 \\
\iff &\sum_{i\in[n]} 2d_i f(a_i, b_i) = \sum_{i\in[n]} 2c_{opt} f(a_i, b_i)^2 \\
&\text{Note that all coefficients of } c_{opt} \text{ are nonnegative,} \\
&\text{so the second derivative is nonegative and thus we have a minima} \\
\iff &\frac{\sum_{i\in[n]} d_i f(a_i, b_i)}{\sum_{i\in[n]} f(a_i, b_i)^2} = c_{opt} \\
\iff &\frac{d_{abs} \cdot d_{rel}}{\|d_{rel}\|_2^2} = c_{opt}
\end{aligned}$$

We have found that the optimal value of c is the dot product of the two original vectors over the squared magnitude of the relative depth vector.