The Sorcerer’s Gears Project Report

Team Jigglypuff

Tiffany Lau
Vivian Nguyen
Michael Si
Maxwell Wu

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Executive Summary

The objective of our assignment was to design and build a crawler machine that is able to ascend a ~1.5 meter 3-walled wooden shaft and place a small booklet on a shelf at the top while wasting as little energy as possible.

We built several design iterations, including 2- and 3-wheeled crawlers with various gear ratios using a combination of spur, crown, and worm gears.

Our final design consisted of four wheels - two for support and two for driving. To drive our crawler, a DC motor turned a worm gear engaged with a 24-tooth crown gear, which was connected to the driving axle. One of the main factors behind our decision to use the worm gear was its ability to lock the connecting gear and prevent the axle from turning, thus allowing our crawler to remain still in the shaft once voltage has been cut. Our design was v-shaped, with rubber bands providing the necessary tension between the arms to accommodate slightly varying shaft widths. The last component of our design was our book tossing mechanism, which consisted of a slanted shelf to hold the book and a lever that was triggered by hitting the top shelf as the crawler ascended the shaft.

We maximized our crawler's efficiency by testing various gear ratios, weights, and voltages, and eventually we settled on a design that minimized current. Furthermore, we measured motor torque and power, analyzed forces acting on the vehicle, and generated MATLAB plots to pinpoint the optimal current and gear ratio for our final design. We utilized this information and operated our crawler at the maximum voltage, 12 V, and used a larger gear ratio to achieve minimal current (0.11 A).

If we were to repeat this project, we would focus more on minimizing our gear ratio, and thereby minimizing travel time, rather than minimizing current. This would have cut our travel time by a third while only doubling our current, which would have resulted in significantly less energy wasted.

From this project, we learned three key points: 1) proper gear meshing is extremely important for an effective gear train, 2) you must always consider stresses on the building materials (in our case, Legos), and 3) the best way to approach a building problem is by first approaching the design process as a whole, and then troubleshooting small problems in iterations.

Background

Our objective was to deliver Nicholas Flamel's notebook onto a shelf with the use of gears and a motor. Our crawler could only be made using the materials provided in the Lego kits (which included gears, axles, Lego pieces, and a motor), rubber bands, springs, tape or glue, lubricants, and tie wraps. To deliver the notebook onto the shelf, our device had to first travel roughly 1.5 meters up a wooden 3-walled shaft and consume less than 32 Joules per meter traveled, or ideally, less than 8 Joules per meter. The input voltage supply could not exceed 12 V, and other than the power cables, no additional lead was permitted up the shaft. Additionally, the device had to be able to hold still in the wooden shaft without slipping if power was cut at any given time (and no external voltage is applied), however, while supplied, the power could be adjusted to modulate speed and direction. Due to variation in shaft and shelf size, our crawler had to be robust and flexible enough to accommodate any shaft.

We interpreted, from the design requirements, that the key features of the crawler were 1) ascending the shaft, 2) remaining still in the shaft when power is cut, and 3) placing the book on the shelf. While performing all of this within the energy limits was important, we decided to fulfill the other requirements first and then approach lowering our energy consumption afterwards.

Basic Description of Our Design

Before reaching our final design, we ran through several design iterations to determine the best structure and gear train that would allow our device to ascend, remain immobile in, and accommodate any shaft.

We began with a four-wheeled, two-wheeled drive device with various spur gear configurations (see Figure 1). After encountering difficulty with meshing gears (due to the limitations that the Lego blocks' sizes and heights presented) and realizing our crawler required normal force in order to remain still in the shaft, we began building our second prototype.

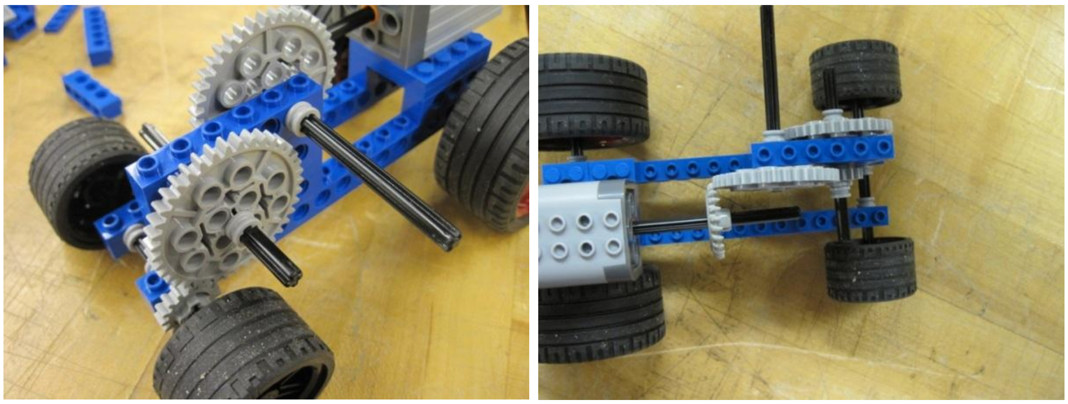

Figure 1. Prototype 1: Four wheeled car, two wheel drive, multi-spur-gear train with crown gear.

With our second prototype, we began by designing a structure that addressed our last prototype's lack of normal force to hold a constant position in the shaft. We alleviated this issue by building devices with three point support systems with one driving wheel and three support wheels (see Figure 2). After attempting this design with a large wheel, we realized the smaller wheels provided more surface area for traction and support (see Figure 3).

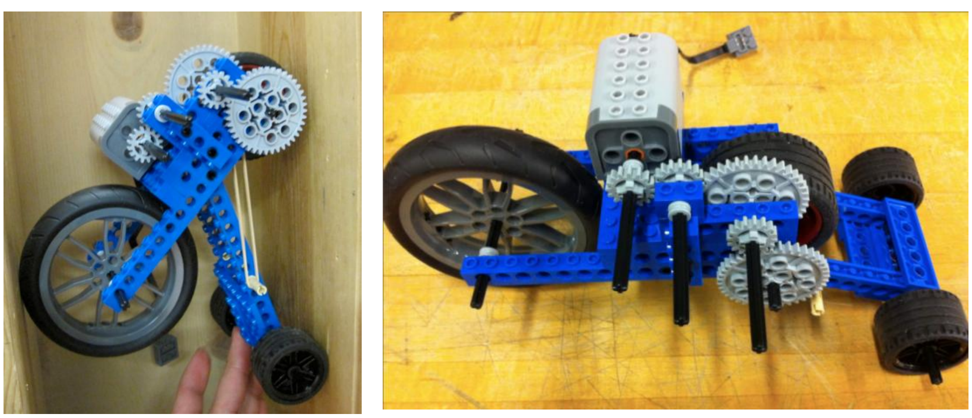

Figure 2. Prototype 2, Iteration 1: Attempted to use one of the large wheels, placed the motor parallel with the driving wheel axle, and used multiple gear train.

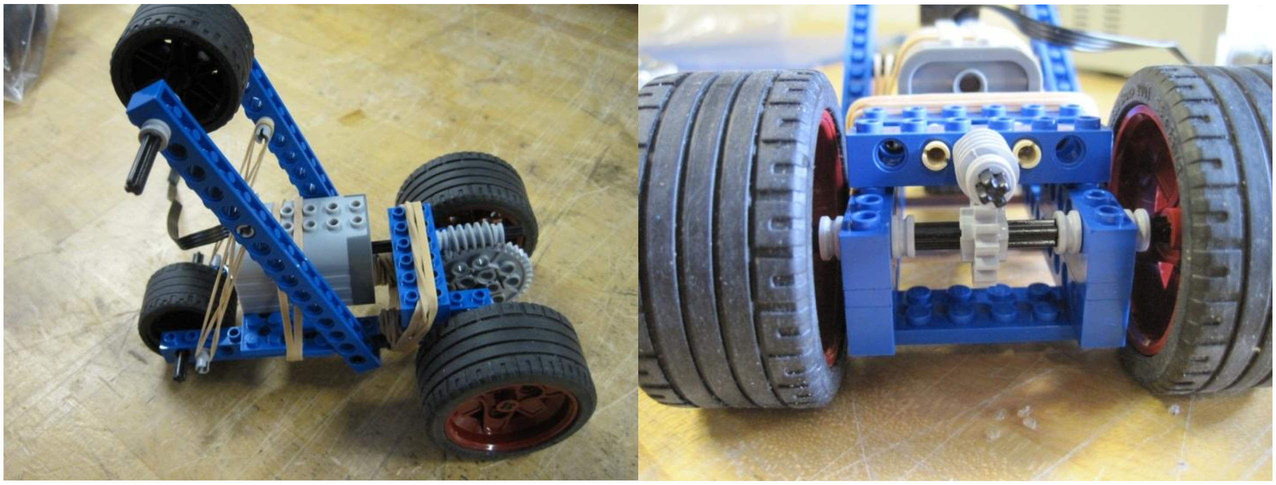

Figure 3. Prototype 2, Iteration 2/3: Moved location of motor to be perpendicular to driving axle, switched to one gear (we tried 40 and 16), used only small wheels, and used rubber bands to better bind together Lego pieces.

After several iterations, we decided upon the following design criteria:

1. Choosing the medium-sized and smallest wheels because they possessed the most surface area and traction,
2. Placing the driving wheels between the support wheels on the opposite side of the shaft for stability and efficiency,
3. Connecting the two arms of our device with rubber bands in tension to create normal force and fix the device in the shaft, as well as to help our device accommodate shafts that slightly vary in size,
4. “Binding” Lego pieces together more securely using rubber bands to provide structural stability,
5. Using fewer gears in order to reduce meshing issues (given the limitations of the Lego sizes provided),
6. Using a worm gear to “lock” the axle, preventing the driving wheels from rotating and ensuring the crawler would remain still in the shaft if voltage was cut,
7. Meshing the crown gear with the worm gear because they mesh better with the sizes of Lego pieces provided,
8. Placing the motor perpendicular to the wheel axle to better distribute the crawler’s weight, and,
9. Placing a gear at the end of one of the support wheel axles so that it would roll in the shaft’s groove and guide the wheels’ direction.

For the notebook-placing mechanism, used a lever system because it is simple, light, and reliable (see Figure 4). As the crawler traveled up the shaft, the lever hit the underside of the shelf and tossed the notebook onto the shelf. The only difficulties we encountered with this mechanism were 1) ensuring the lever was long enough to toss the notebook onto the shelf, and 2) ensuring our crawler traveled straight enough for the lever arm to consistently hit the shelf. We addressed these issues by elongating the lever arm and adding a guiding gear that helped out crawler traveled straight.

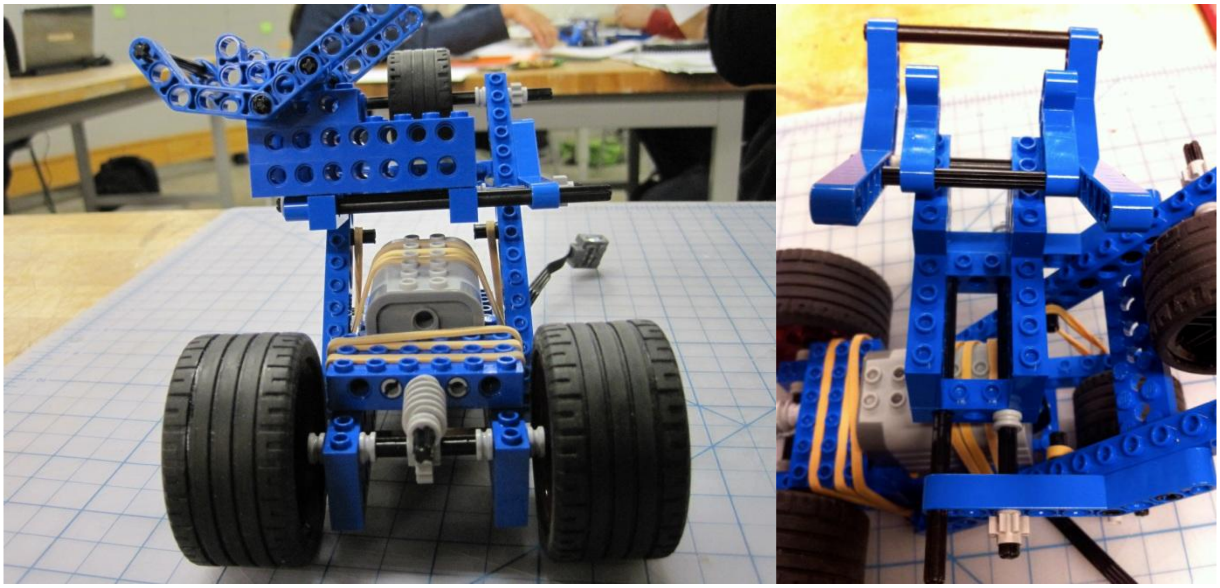

Figure 4. Prototype 2, Iteration 3 with tossing mechanism

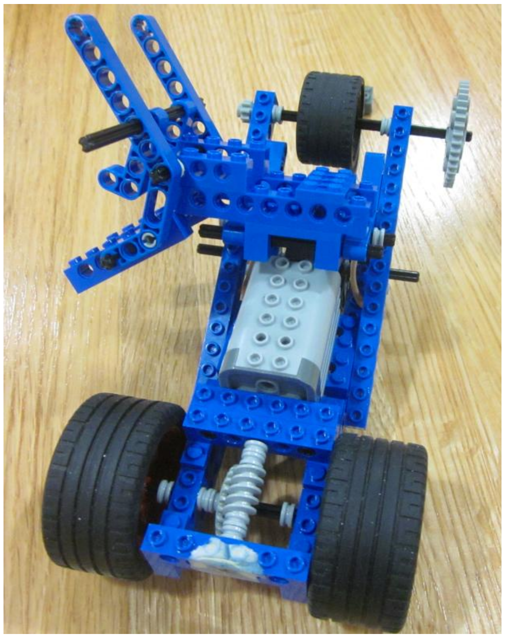

Figure 5. Prototype 2, Iteration 4: Final design: Guiding gear for direction, supported worm gear axle, tossing mechanism.

Our design was effective because it is a small, light and robust device that remained still in the shaft (due to the normal force provided by the rubber bands) and traveled straight (due to the guiding gear in the shaft groove). It also had secure meshing between the worm and crown gear while only placing a reasonable load on the worm gear axle, which was also supported on the other side (see Figure 5). The lever mechanism allowed the notebook to rest on its arm without sliding and tossed the notebook onto the wooden shelf when the lever arm came into contact with the underside of the shelf.

Analysis of Performance in the Shaft

Force analysis

The free body diagram of our crawler is shown in Figure 6.

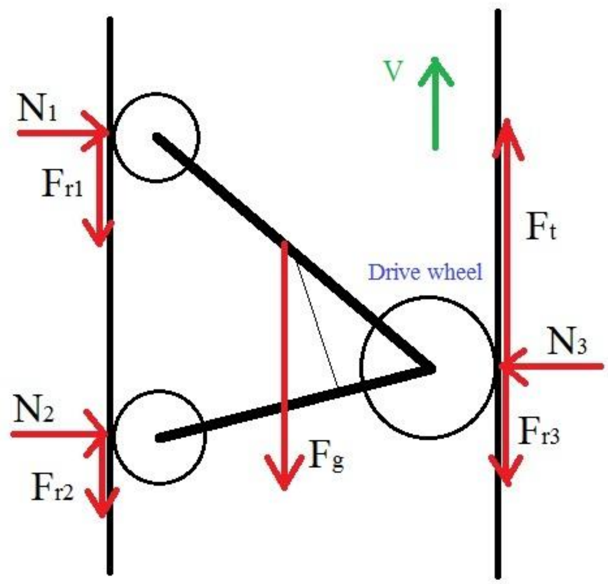

Figure 6. Free body diagram of the crawler design

The mass of our crawler is 224 g, and hence its weight is 2.195 N. With a weighing scale we measure the normal force exerted on the drive wheel, N_3, to be 1.61 N. In order to obtain the coefficient of rolling friction, we conduct an experiment in which we allow the crawler to roll down a ramp freely with drivetrain and motor detached (Figure 7.). We measure the angle of slope, the distance traveled, and the time taken, and calculate the coefficient of rolling friction.

Figure 7. Experiment setup for determining coefficient of rolling friction

The net force in the direction parallel to the slope is:

$$F_{net} = mg \cdot \sin\theta - f = mg \cdot \sin\theta - c \cdot N = mg \cdot \sin\theta - c \cdot mg \cdot \cos\theta = m \cdot a$$

Where

m is the mass of the crawler
θ is the angle of the slope
f is the rolling friction
N is the normal force
c is the coefficient of rolling friction
a is the acceleration of the crawler down the slope

We cancel the mass on both sides and obtain:

$$a = g \cdot (\sin\theta - c \cdot \cos\theta) \text{ ------ } (1)$$

Also, we analyze the kinematics and have:

$$\frac{1}{2}at^2 = L \text{ ------ } (2)$$

Where

L is the length of the slope, i.e., the distance crawler traveled down the slope

Therefore, substitute (1) into (2), we have:

$$g \cdot (\sin\theta - c \cdot \cos\theta) = \frac{2L}{t^2}$$

$$c = \frac{g \cdot sin\theta - \frac{2L}{t^2}}{g \cdot cos\theta} = \frac{9.8\ {}^{m}/_{s^2} \times \sin(5.8) - \frac{2 \times 1.684\ m}{(3.301\ s)^2}}{9.8\ {}^{m}/_{s^2}\ \times \cos(5.8)} = 0.07$$

The coefficient of rolling friction is determined to be 0.07, and hence $F_{rx} = 0.07 \cdot N_x$.

Since $N_1 + N_2 = N_3$ and $F_t = F_{r1} + F_{r2} + F_{r3} + F_g$, the driving force on the wheel is:

$$F_t = F_g + c \cdot 2N_3 = 2.195\ N + 0.07 \times 2 \times 1.61\ N = 2.42\ N.$$

Power and efficiency analysis

In our test runs, our average timing for traveling 1 m up the shaft is around 10.4 s, and hence the speed is 0.096 m/s. At average applied voltage and measured current of V = 12 V and i = 0.11 A, the average power input from the motor is:

$$P_{in} = V \cdot i$$

Where

V is the applied voltage
i is the measured current

And the average power output to the crawler is:

$$P_{out} = F_t \cdot v$$

Where

F_t is the driving force
v is the speed of the crawler

We plug in the values of voltage, current, driving force, and speed and obtain $P_{in} = 12 \times 0.11 = 1.32$ W, and $P_{out} = 2.42 \times 0.096 = 0.232$ W. Therefore, the total efficiency is:

$$\eta_{total} = P_{out}/P_{in} = 0.232 \text{ W} / 1.32 \text{ W} = 17.6\%$$

The total efficiency consists of the electrical efficiency of the motor, the mechanical efficiency of the internal gearbox, and the mechanical efficiency of the drivetrain. The efficiency of a motor can be calculated by:

$$\eta_{motor} = (1 - \frac{i_{nl}}{i})(1 - \frac{i}{i_s})$$

Where

i_{nl} is the measured no-load current
i_s is the measured stall current

At an operating voltage of 12 V, measured no-load current of 0.02 A, and measured stall current of 0.62 A (Appendix 1), the relationship between motor efficiency and measured current can be described by:

$$\eta_{motor} = -1.613\text{i} - 0.02\text{i}^{-1} + 1.032$$

Therefore, the motor efficiency is determined to be η_{motor} = 67.3%. In order to obtain the efficiency of the internal gearbox, $\eta_{gearbox}$, we perform an experiment in which we use a large wheel (with a radius of 0.0381 m) as a pulley to raise object powered by the motor. By knowing the weight and the speed of the object, the applied voltage, the current, and the torque constant of the motor (Appendix 1), we can determine $\eta_{gearbox}$.

The data from our two test runs are shown in Table 1. The experiment is run at the nominal voltage 9 V, and the relationship between motor efficiency and current can be described by:

$$\eta_{motor} = -2.128\text{i} - 0.02\text{i}^{-1} + 1.043 \text{ (Appendix 1)}$$

mass of the object (kg)	Distance traveled (m)	Time taken (s)	Current (A)	Torque input (W)	Torque output (W)	Gearbox efficiency
0.0567	0.50	0.45	0.30	0.0321	0.0212	66.0%
0.0851	0.50	1	0.37	0.0396	0.0318	80.3%

Table 1. Motor internal gearbox efficiency determination experiment data

Therefore, the average efficiency of the internal gearbox is 73.2%.

Since the drivetrain efficiency is:

$$\eta_{drivetrain} = \frac{\eta_{total}}{\eta_{motor} \cdot \eta_{gearbox}}$$

The drivetrain efficiency (with a gear ratio of 24:1) is calculated to be $\eta_{drivetrain}$= 0.176 / (0.673×0.732) = 35.7%.

Now, since we know η_{total} = 17.6%, $\eta_{drivetrain}$= 35.7%, $\eta_{gearbox}$= 73.2%, and η_{motor}= 67.3%, we are able to calculate the energy losses due to the inefficiencies of various components when the crawler is traveling up the shaft at V = 12 V, i = 0.11 A, and t = 10.4 s for 1 m distance. The energy loss in the motor is:

$$E_{motor,loss} = V \cdot i \cdot t \cdot (1 - \eta_{motor})$$

Thus, $E_{motor,loss}$ = 12 V·0.11 A·10.4 s· (1-0.673) = 4.49 J.

The energy loss in the internal gearbox is:

$$E_{gearbox,loss} = V \cdot i \cdot t \cdot \eta_{motor} \cdot (1 - \eta_{gearbox})$$

Thus, $E_{drivetrain,loss}$ = 12 V·0.11 A·10.4 s·0.673·(1-0.732) = 2.48J.

The energy loss in the drivetrain is:

$$E_{drivetrain,loss} = V \cdot i \cdot t \cdot \eta_{motor} \cdot \eta_{gearbox} \cdot (1 - \eta_{drivetrain})$$

Thus, $E_{drivetrain,loss}$ = 12 V·0.11 A·10.4 s·0.673·0.732·(1-0.357) = 4.35 J.

The total energy loss is:

$$E_{total,loss} = V \cdot i \cdot t \cdot (1 - \eta_{total}) = E_{motor,loss} + E_{gearbox,loss} + E_{drivetrain,loss}$$

Thus, $E_{total,loss}$ = 12 V·0.11 A·10.4 s· (1- 0.176) = 11.3 J

Analysis of Anticipated Performance with Gravity Changes

Normal gravity analysis

Under normal gravity ($g = 9.81\ m/s^2$), the motor torque and torque output can be calculated by:

$$\tau_m = K \cdot i$$

$$\tau_{out} = F_t \cdot R$$

Where

K is the torque constant of the motor
i is the measured current
R is the radius of drive wheel

At V = 12 V, i = 0.11 A, K = 0.1069 Nm/A, and R = 0.028 m, τ_m = 0.1069 Nm/A × 0.11 A = 0.0118 Nm, and τ_{out} = 2.42 N × 0.028 m= 0.0678 Nm.

The output speed v_{out}, which is the speed of the crawler in the shaft, is 0.096 m/s. The input speed (tangential speed of the wheel free spinning) v_{in} without gear reduction from the drivetrain can be calculated by:

$$v_{in} = \omega \cdot R = (-1675.77\tau_m + 112.25) \cdot R = (-1675.77 \times 0.0118 + 112.25) \times 0.028 = 2.59\ m/s$$

Therefore, we can obtain the ratios of v_{in}/ v_{out} = 27.17 and τ_{out}/ τ_m = 5.74. The difference between the speed ratio (27.17) and the drivetrain ratio (24) is likely due to slipping and other factors.

Modified gravity analysis

The following analysis is based on the assumption that the normal forces exerted on the wheels of the crawler, as well as the ratios of v_{out}/v_{in} and τ_{out}/τ_m, stay invariant under variable gravity conditions. The operating voltage remains at 12 V. Let the gravity now be g^*, and F_t^* would be:

$$F_t^* = F_g^* + c \cdot 2N_3 = 0.224\ kg \cdot g^* + 0.07 \times 2 \times 1.61\ N = 0.224 \cdot g^* + 0.225\ N$$

The torque output becomes:

$$\tau_{out}^* = F_t^* \cdot R = (0.224 \cdot g^* + 0.225) \cdot 0.028 = 0.00627 \cdot g^* + 0.00631\ Nm$$

The current becomes:

$$i^* = \tau_m/K = \tau_{out}/(5.74 \times K) = (0.00627 \cdot g^* + 0.00631)/(5.74 \times 0.1069) = 0.0102 \cdot g^* + 0.0103\ A$$

The motor torque becomes:

$$\tau_m{}^* = K \cdot i^* = 0.1069 \times (0.0102 \cdot g^* + 0.0103) = 0.00109 \cdot g^* + 0.00110 \text{ Nm}$$

The input speed before drivetrain becomes:

$$v_{in}{}^* = \omega^* \cdot R = (-1675.77\tau_m^* + 112.25) \cdot R$$

$$= (-1675.77 \times (0.00109 \cdot g^* + 0.00110) + 112.25) \times 0.028 = -0.05127 \cdot g^* + 3.091 \text{ m/s}$$

The output speed of the crawler becomes:

$$v_{out}{}^* = v_{in}{}^* / 27.17 = (-0.05127 \cdot g^* + 3.091) / 27.17 = -0.002422 \cdot g^* + 0.1138 \text{ m/s}$$

The time needed to ascend the shaft becomes:

$$t^* = 1 / v_{out}{}^* = 1/(-0.002422 \cdot g^* + 0.1138) \text{ s}$$

The total energy input would be:

$$E_{in} = V \cdot i^* \cdot t^* = 12 \times (0.0102 \cdot g^* + 0.0103) / (-0.002422 \cdot g^* + 0.1138)$$

$$= (0.1224 \cdot g^* + 0.1236) / (-0.002422 \cdot g^* + 0.1138) \text{ J}$$

The energy output to ascend the crawler is:

$$E_{out} = F_g{}^* \cdot 1 \text{ m} = 0.224 \cdot g^* \text{ J}$$

In order to limit the energy dissipation under 32 J/m, we have:

$$E_{loss} = E_{in} - E_{out} = (0.1224 \cdot g^* + 0.1236) / (-0.002422 \cdot g^* + 0.1138) - 0.224 \cdot g^* \leq 32 \text{ J/m}$$

Solve for g^*, we get:

$$g^* \leq 21.14 \text{ m/s}^2$$

Therefore, the crawler would be able to ascend the shaft while maintaining total energy dissipation below 32 J/m as long as the gravitational acceleration remains below 21.14 m/s^2.

Conclusions

In its final run, our crawler fulfilled all of its design requirements: ascending the shaft, remaining still in the shaft when power is cut, placing the small booklet on the top shelf, and all within the energy efficiency limits. With a mass of 224 grams, it climbed 1 meter in 10.453 seconds with a loss of 11.4 Joules. It had a gear ratio of 24:1 and a robust, yet flexible design.

Going into the final run, we were confident our design would work - it consistently ascended the shaft, remained still when power was cut, placed the booklet on the top shelf, and used less than 12 Joules. Our only concern was our crawler's ability to consistently travel in a straight line. With the guiding gear and proper initial crawler alignment, we knew our crawler would perform well, however, without proper alignment, our crawler did not consistently travel straight - which would either prevent our crawler from ascending the entire shaft or cause the lever arm to miss the shelf. In the end, we were able to perfect our initial alignment of our crawler, but ideally our crawler would have been functional despite small variations in alignment.

Another drawback to our design was that our efficiency varied based on factors such as friction between the wood and guiding gear, lack of lubrication on the worm gear and bending of the wheels and axles. We addressed these issues by heavily lubricating our worm gear with olive oil and supporting the worm gear axle, but we were not able to fully solve these efficiency problems.

Overall, our crawler had a good design and was pretty consistently effective, however, we do not think it would win 'best machine design' due to its non-optimal efficiency and its requirement that it be precisely aligned each run.