a)

The exact eigenvalues for harmonic oscillator potential is:

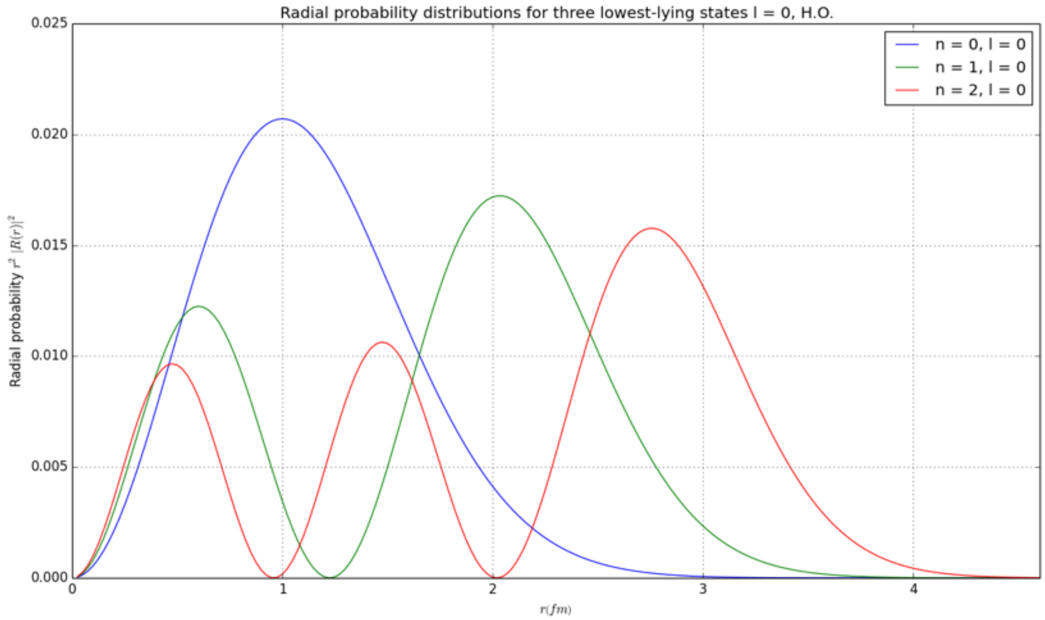
$$\lambda = 4n + 2l + 3$$

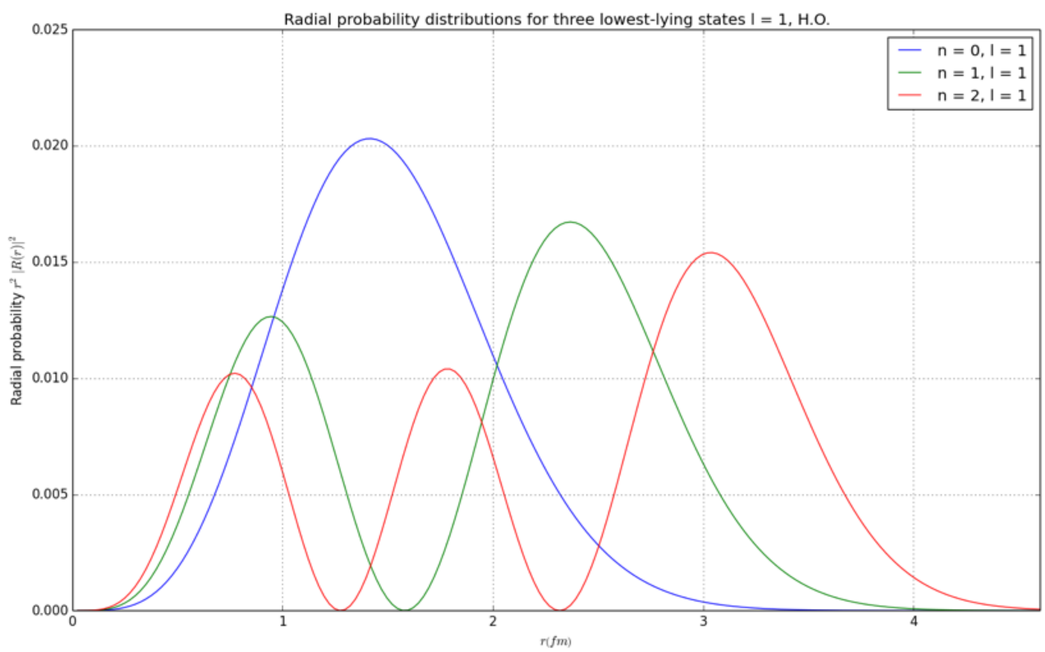
Angular momentum quantum number l = 0:
For a **fixed dimension** (resolution), Dim = 400, RMax = 10, the lowest 3 eigenvalues are initially close to the exact solutions: 3 (2.999806), 7 (6.996028), 11 (10.997628).
As we increases RMax, the eigenvalues become smaller and smaller deviating from the exact values. This is due to the fact that we're using a delta function basis to solve the Schrödinger's equation, however if we increase the dimension linearly with RMax, that is, we use the same step size, the numerical solutions become close to the exact ones again. But by doing so (increasing both Dim and RMax) we increase the computational cost, so it's important that we find a suitable cutoff radius in these problems.
As Rmax goes to infinity (together with Dim)like in the analytical expression, we should reproduce solutions closer and closer to the exact values.

As we add non-zero angular momentum quantum number l, the eigenvalues increment is ~2l, just as in the exact solution, other behaviors of the numerical solution are similar to the l = 0 situation described above.

Also worth noting is that with RMax fixed, increasing Dim will lower the overall probability scale, lowering Dim will do the exact opposite, it's also a symptom of using delta function basis.

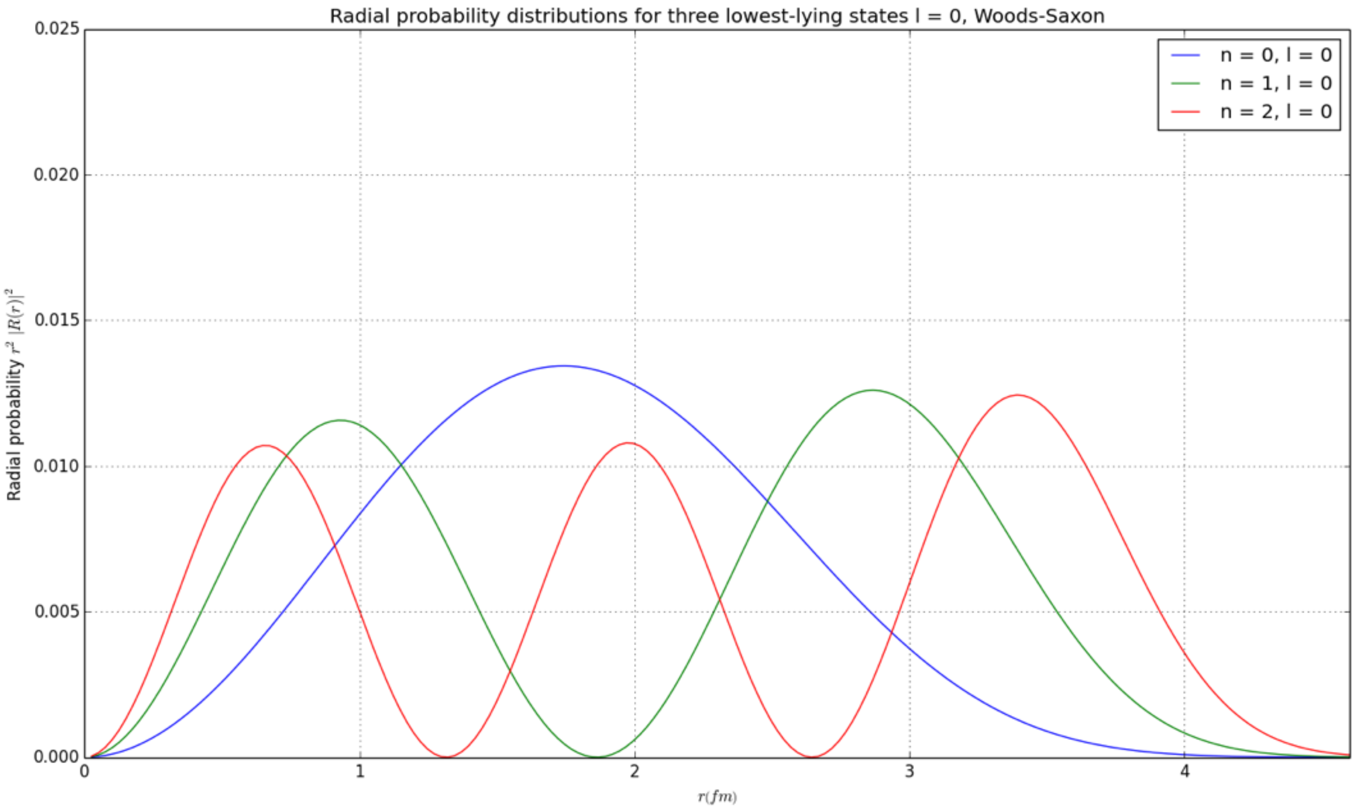
b)





As angular momentum increases we can see the wave functions shift to a larger r, which is intuitive as one would imagine while the angular momentum builds up, the centrifugal force increases which requires a larger radius to compensate it.

c)



Using Woods-Saxon potential with:
Parameters: v0 = 50 MeV, A = 100, a = 0.5 fm, r0 = 1.25 fm, R = r0*$A^{1/3}$.
Three lowest eigenvalues: -49.83 MeV, -49.41 MeV, -48.82 MeV.
The eigenfunctions of Woods-Saxon potential have similar shape as those of H.O. potential. It's worth noting that r is in units of fm, and the eigenfunctions are more 'spread', it's likely due to the large A (=100) used here, the Woods-Saxon potential has a flat bottom at large A, while H.O. potential increases quadratically.

If one would look at the detailed components of the eigenfunctions, one can see that, compared to H.O. potential, the sum of Woods-Saxon radial probability converges to 1 faster, this is due to the rapid decrease of Woods-Saxon potential at the surface R ~ 4.16fm for A = 100, which reproduces the short range nature of nuclear interaction.

p.s.
Due to time limitation this week, I'll be completing Ex. 3 & 4 during the weekends.