## a) 8 neutrons, without interaction:

```
[ 15.  15.  25.  25.  25.  25.  25.  25.  35.  35.  35.  35.  35.  35.  35.
  35.  35.  35.  35.  35.  45.  45.  45.  45.  45.  45.  45.  45.  45.  45.
  45.  45.  45.  45.  45.  45.  45.  45.  45.  45.]
```

calculation proves correct at this step, energy obeys:

$$E_{h.o.} = (2n + l + 1.5) \cdot \hbar\omega$$

## b)

Since the **interaction** given in this problem is **scalar**, it does not have the property to change angular momentum, basically **it's a spherical harmonic ~$Y_{00}$** which cannot link 2 different l or j values. Again since the **interaction** given is **spin-independent**, it doesn't involve the quantum number m. The harmonic potential halmitonian is diagonal on it's own with the Kronecker delta, thus the entire halmitonian, **$h_0$+V is diagonal in ljm** when added together.

In the number representation, we have the many-body energy:

$$E[\Psi] = \sum_{i=1}^{A} \sum_{\alpha\beta} C_{i\alpha}^* C_{i\beta} \langle\alpha| h |\beta\rangle + \frac{1}{2} \sum_{ij=1}^{A} \sum_{\alpha\beta\gamma\delta} C_{i\alpha}^* C_{j\beta}^* C_{i\gamma} C_{j\delta} \langle\alpha\beta| \hat{v} |\gamma\delta\rangle_{AS}$$

and for the coefficient C's, we have:

$$\langle i|j\rangle = \delta_{i,j} = \sum_{\alpha\beta} C_{i\alpha}^* C_{i\beta} \langle\alpha|\beta\rangle = \sum_{\alpha} C_{i\alpha}^* C_{i\alpha},$$

within the HF variational method, we sought to minimize the energy, which should converge to the true ground state energy if we've chosen a good set of basis.
Thus we'd want to minimize the below functional, w.r.t. the approximated wave-function (thus the coefficient vector) we choose:

$$F[\Phi^{HF}] = E[\Phi^{HF}] - \sum_{i=1}^{A} \epsilon_i \sum_{\alpha} C_{i\alpha}^* C_{i\alpha}$$

$$\frac{d}{dC_{i\alpha}^*} \left[ E[\Phi^{HF}] - \sum_{j} \epsilon_j \sum_{\alpha} C_{j\alpha}^* C_{j\alpha} \right] = 0$$

after bringing in explicitly the top expression of $E^{HF}$, we get:

$$\sum_{\beta} C_{i\beta} \langle\alpha|h|\beta\rangle + \sum_{j=1}^{A} \sum_{\beta\gamma\delta} C_{j\beta}^* C_{j\delta} C_{i\gamma} \langle\alpha\beta|\hat{v}|\gamma\delta\rangle_{AS} = \epsilon_i^{HF} C_{i\alpha}$$

we can now clearly see if we replace the hamiltonian with:

$$h_{n_1 n_3}^{lj} = \delta_{n_3 n_1} (2n_1 + l + 3/2)\hbar\omega + \sum_{n_2 n_4} \sum_{l'j'}^{occ} \langle n_1 l j n_2 l' j' | V | n_3 l j n_4 l' j' \rangle \rho_{n_4 n_2}^{l'j'}$$

we can easily arrive at:

$$\sum_{n_3} h_{n_1 n_3}^{lj} C_{n_3 \bar{n}}^{lj} = \epsilon_{\bar{n}lj} C_{n_3 \bar{n}}^{lj}$$

## c) 8 neutrons, with interaction:

```
Iteration: 14, diff = 6.1687343067e-08 MeV
```

```
[  0.31029368   0.31029387  14.70873554  14.70873565  14.70873989
  14.70873991  16.82953551  16.82953556  28.85066256  28.85066258
  28.85066401  28.85066404  28.85066601  28.85066603  29.24695785
  29.24695791  31.14975808  31.1497581   31.14975916  31.14975919
  40.47253942  40.47253945  40.47254123  40.47254124  41.04486896
  41.04486898  41.04486907  41.04486909  41.04487074  41.04487076
  41.04487094  41.04487095  41.06195329  41.0619533   43.04670674
  43.04670676  43.04670716  43.04670716  43.04670819  43.04670819]
```

Compared to no-interaction harmonic oscillator single-particle energies, all of the neutron s.p. energies with interaction are lowered, this is the work of these single particles binding with each other, forming a more stable collective entity, through nuclear forces (no Coulomb for neutron). We could also see the degeneracy for states with the same angular momentum J.
However, all the s.p. energies are positive, which means **under the given interaction, this 8-neutron system is not bound.**
Starting from 0s, 0p, 1s0d, 1p0f, we can determine the corresponding orbitals (energy from low to high), using also the degeneracy numbers, and also see the detailed shell structure:

**0s shell, magic #: 2**

$0s_{1/2}$ 2 states ~0.31 MeV

**0p shell, magic #: 2 + (4+2) = 8**

$0p_{3/2}$ 4 states ~14.71 MeV

$0p_{1/2}$ 2 states ~16.83 MeV

**1s0d shell, magic #: 8 + (6+2+4) = 20**

$0d_{5/2}$ 6 states ~28.85 MeV

$1s_{1/2}$ 2 states ~29.25 MeV

$0d_{3/2}$ 4 states ~31.15 MeV

**1p0f shell, magic #: 20 + (4+8+2+6) = 40****

**A more precise interaction should show that $1f_{7/2}$ state has lower energy intruding into the 1s0d shell, thus forming a shell of 28. In my calculation, $1p_{3/2}$ state has a lower energy.

$1p_{3/2}$ 4 states ~40.47 MeV

$0f_{7/2}$ 8 states ~41.04 MeV

$1p_{1/2}$ 2 states ~41.06 MeV

$0f_{5/2}$ 6 states ~43.05 MeV

## c) 8 protons, with interaction:

```
Iteration: 15, diff = 4.57021918088e-08 MeV
```

```
[  0.47931374   0.47931399  14.82113847  14.82113864  14.82114229
  14.82114238  16.92933316  16.92933321  28.90094561  28.90094568
  28.90094629  28.90094638  28.90094712  28.90094714  29.30152728
  29.30152736  31.19275532  31.19275537  31.19275551  31.19275552
  40.51646855  40.51646857  40.51647221  40.51647225  41.07108411
  41.07108416  41.07108436  41.07108439  41.07108454  41.0710846
  41.07108528  41.07108529  41.1054643   41.1054643   43.06893353
  43.06893354  43.06893362  43.06893363  43.06893395  43.06893397]
```

We could use similar analysis as in the above neutron case. The difference is, protons has Coulomb interaction, thus the s.p. energies are higher when you put a bunch of protons that has Coulomb repulsion with each other close together.

## d) 8 neutrons + 8 protons, with interaction:

```
Iteration: 13, diff = 8.92481359926e-07 MeV
```

```
[-40.64258531 -40.64258519 -40.46016272 -40.46016264 -11.72008582
 -11.72008581 -11.72008045 -11.72008042 -11.58864794 -11.5886479
 -11.58864506 -11.588645    -6.84033386  -6.84033384  -6.71334463
  -6.7133446   18.7588728   18.75887281  18.7588886   18.75888862
  18.75889643  18.75889644  18.80821479  18.80821479  18.80823189
  18.80823191  18.80823697  18.80823698  21.02472817  21.02472818
  21.06535442  21.06535444  22.91526075  22.91526077  22.9152619
  22.91526192  22.96218947  22.96218949  22.96219118  22.96219121
  35.13448888  35.13448888  35.13449126  35.13449126  35.15841728
  35.15841728  35.15841746  35.15841747  35.85257225  35.85257225
  35.87705735  35.87705736  36.03386508  36.03386509  36.03386531
  36.03386532  36.03386571  36.03386572  36.03387317  36.03387318
  36.05961653  36.05961653  36.05961726  36.05961727  36.05961842
  36.05961843  36.05962512  36.05962513  39.28250077  39.28250079
  39.2825019   39.28250191  39.28250316  39.28250317  39.30670738
  39.3067074   39.30670831  39.30670832  39.30670861  39.30670863]
```

It's quite obvious that the first 16 s.p. states are occupied, and have a significant lower energy (all negative) than those s.p. states above occupation level, which tells us **this system is bound under the given interaction.** I'm guessing that the 'twobody.dat' interaction was abstracted from real data, thus only with the full neutron-proton picture, the s.p. energies would be negative and thus showing that they're bound.

Given that we've discovered protons have slightly higher s.p. energy than the neutrons with the orbital quantum numbers, we can interpret these s.p. energy eigenvalues as neutrons first,

protons second within each set of orbital quantum numbers, with a little counting, we can identify the s.p. energies for different proton, neutron orbitals.

Now, we should be comparing:

1. **Proton** separation energy of **O-16** with the **$0p_{1/2}$** proton energy of our O-16 HF calculation,
2. **Proton** separation energy of **F-16** with the **$0d_{5/2}$** proton energy of our O-16 HF calculation,
3. **Neutron** separation energy of **O-16** with the **$0p_{1/2}$** neutron energy of our O-16 HF calculation,
4. **Neutron** separation energy of **O-17** with the **$0d_{5/2}$** neutron energy of ourO-16 HF calculation.

$$BE(^{16}\mathrm{O}) - BE(^{15}\mathrm{O}) \approx \epsilon^{\mathrm{HF}}_{0p^{\nu}_{1/2}}, \qquad BE(^{17}\mathrm{O}) - BE(^{16}\mathrm{O}) \approx \epsilon^{\mathrm{HF}}_{0d^{\nu}_{5/2}},$$

$$BE(^{16}\mathrm{O}) - BE(^{15}\mathrm{N}) \approx \epsilon^{\mathrm{HF}}_{0p^{\pi}_{1/2}}. \qquad BE(^{17}\mathrm{F}) - BE(^{16}\mathrm{O}) \approx \epsilon^{\mathrm{HF}}_{0d^{\pi}_{5/2}}.$$

comparing with experimental values on the right column (ignore the minus signs):

$$^{n}\epsilon^{HF}_{0p_{1/2}} \approx -6.84MeV \qquad S_n^{exp}\left(O^{16}\right) \approx 15.664MeV$$

$$^{n}\epsilon^{HF}_{0d_{5/2}} \approx 18.76MeV \qquad S_n^{exp}\left(O^{17}\right) \approx 4.143MeV$$

$$^{p}\epsilon^{HF}_{0p_{1/2}} \approx -6.71MeV \qquad S_p^{exp}\left(O^{16}\right) \approx 12.127MeV$$

$$^{p}\epsilon^{HF}_{0d_{5/2}} \approx 18.81MeV \qquad S_p^{exp}\left(F^{17}\right) \approx 16.801MeV$$

It's not ideal… except for Fluorine-17's proton separation energy.