**Types of Boundary conditions**

1. **Dirichlet boundary condition**
2. **Neumann boundary condition**
3. **Robin boundary condition**

## Dirichlet boundary condition

The Dirichlet (or first-type) boundary condition is a type of boundary condition, named after Peter Gustav Lejeune Dirichlet (1805–1859) that specifies the values that a solution needs to take along the boundary of the domain when imposed on an ordinary or a partial differential equation.

The question of finding solutions to such equations is known as the Dirichlet problem. In applied sciences, a Dirichlet boundary condition may also be referred to as a fixed boundary condition.

**Examples**

For each of the following ODE

The Dirichlet boundary conditions on the interval [x1, x2] take the form

, Where x1, x2 α and β are given numbers.

**For a partial differential equations**



Where 𝝯**2** and 𝝯 denotes the Laplace operator,

the Dirichlet boundary conditions on a domain A ⊂ Rn take the form

Where f is a known function defined on the boundary ∂A.

**Applications**

The following would be considered Dirichlet boundary conditions:

• In mechanical engineering and civil engineering (beam theory), where one end of a beam is held at a fixed position in space.

• In heat transfer, where a surface is held at a fixed temperature.

• In electrostatics, where a node of a circuit is held at a fixed voltage.

• In fluid dynamics, the no-slip condition for viscous fluids states that at a solid boundary, the fluid will have zero velocity relative to the boundary

## Dirichlet boundary condition

The Dirichlet (**second-type**) boundary condition is a type of boundary condition, named after **Carl Neumann** that specifies the values of the derivative applied at the boundary of the domain when imposed on an ordinary or a partial differential equation.

**Examples**

For each of the following ODE

The Dirichlet boundary conditions on the interval [x1, x2] take the form

, Where x1, x2 α and β are given numbers.

**For a partial differential equations**



Where 𝝯**2** and 𝝯 denotes the Laplace operator,

the Dirichlet boundary conditions on a domain A ⊂ Rn take the form

Where f is a known function defined on the boundary ∂A.

Where **n** denotes the (typically exterior) normal to the boundary ∂A, and *f*  is a given scalar function.

The [normal derivative](https://en.wikipedia.org/wiki/Normal_derivative), which shows up on the left side, is defined as

∂�∂�(�)=∇�(�)⋅�^(�),Where ∇*y*(**x**) represents the gradient vector of *y*(**x**),  is the unit normal and ⋅ represents the [inner product](https://en.wikipedia.org/wiki/Inner_product) operator.

**Applications**

The following applications involve the use of Neumann boundary conditions:

* In [thermodynamics](https://en.wikipedia.org/wiki/Thermodynamics), a prescribed heat flux from a surface would serve as boundary condition. For example, a perfect insulator would have no flux while an electrical component may be dissipating at a known power.
* In [magnetostatics](https://en.wikipedia.org/wiki/Magnetostatics), the [magnetic field](https://en.wikipedia.org/wiki/Magnetic_field) intensity can be prescribed as a boundary condition in order to find the [magnetic flux density](https://en.wikipedia.org/wiki/Magnetic_flux_density) distribution in a magnet array in space, for example in a permanent magnet motor. Since the problems in magnetostatics involve solving [Laplace's equation](https://en.wikipedia.org/wiki/Laplace%27s_equation) or [Poisson's equation](https://en.wikipedia.org/wiki/Poisson%27s_equation) for the [magnetic scalar potential](https://en.wikipedia.org/wiki/Magnetic_scalar_potential), the boundary condition is a Neumann condition.
* In [spatial ecology](https://en.wikipedia.org/wiki/Spatial_ecology), a Neumann boundary condition on a [reaction–diffusion system](https://en.wikipedia.org/wiki/Reaction%E2%80%93diffusion_system), such as [Fisher's equation](https://en.wikipedia.org/wiki/Fisher%27s_equation), can be interpreted as a reflecting boundary, such that all individuals encountering ∂Ω are reflected back onto A.

## Robin boundary condition

The Robin boundary condition (third type boundary condition), is a type of boundary condition, named after Victor Gustavo Robin (1855–1897) that is a specification of a linear combination of the values of a function and the values of its derivative on the boundary of the domain When imposed on an ordinary or a partial differential equation. Other equivalent names in use are Fourier-type condition and radiation condition.

**Examples**

Assume A is the domain on which the given equation is to be solved and ∂A denotes its boundary, the Robin boundary condition is:

��+�∂�∂�=�on ∂ΩFor some non-zero constants a and *b* and a given function *h* defined on ∂A. Here, *u* is the unknown solution defined on A and denotes the [normal derivative](https://en.wikipedia.org/wiki/Normal_derivative) at the boundary. More generally, *a* and *b* are allowed to be (given) functions, rather than constants.

In one dimension, if, for example, A = [0,1], the Robin boundary condition becomes the conditions:

**Application**

Robin boundary conditions are commonly used in solving **Sturm–Liouville problems** which appear in many contexts in science and engineering.

## Mixed boundary

A mixed boundary condition for a partial differential equation defines a boundary value problem in which the solution of the given equation is required to satisfy different boundary conditions on disjoint parts of the boundary of the domain where the condition is stated. Precisely, in a mixed boundary value problem, the solution is required to satisfy a Dirichlet or a Neumann boundary condition in a mutually exclusive way on disjoint parts of the boundary.

For example, given a solution u to a partial differential equation on a domain A with boundary ∂A, it is said to satisfy a mixed boundary condition if, consisting ∂Ω of two disjoint parts, Γ1 and Γ2, such that ∂A= Γ1 , u verifies the following equations:

Where u

0 and g are given functions defined on those portions of the boundary.[1]

The mixed boundary condition differs from the Robin boundary condition in that the latter requires a linear combination, possibly with point wise variable coefficients, of the Dirichlet and the Neumann boundary value conditions to be satisfied on the whole boundary of a given domain.

## Cauchy boundary condition

a Cauchy boundary condition augments an [ordinary differential equation](https://en.wikipedia.org/wiki/Ordinary_differential_equation) or a [partial differential equation](https://en.wikipedia.org/wiki/Partial_differential_equation) with conditions that the solution must satisfy on the boundary; ideally so as to ensure that a unique solution exists. A Cauchy boundary condition specifies both the function value and [normal derivative](https://en.wikipedia.org/wiki/Normal_derivative) on the [boundary](https://en.wikipedia.org/wiki/Boundary_(topology)) of the [domain](https://en.wikipedia.org/wiki/Domain_(mathematical_analysis)). This corresponds to imposing both a [Dirichlet](https://en.wikipedia.org/wiki/Dirichlet_boundary_condition) and a [Neumann boundary condition](https://en.wikipedia.org/wiki/Neumann_boundary_condition). It is named after the prolific 19th-century French mathematical analyst [Augustin-Louis Cauchy](https://en.wikipedia.org/wiki/Augustin-Louis_Cauchy" \o "Augustin-Louis Cauchy).

Cauchy boundary conditions are simple and common in second-order ordinary differential equations,

Where, in order to ensure that a unique solution exists, one may specify the value of the function and the value of the derivative y’ at a given point s=a, i.e.

and Where is a boundary or initial point. Since the parameter is usually time, Cauchy conditions can also be called initial value conditions or initial value data or simply Cauchy data. An example of such a situation is Newton's laws of motion, where the acceleration depends on position, velocity, and the time ; here, Cauchy data corresponds to knowing the initial position and velocity.

Partial differential equations

For partial differential equations, Cauchy boundary conditions specify both the function and the normal derivative on the boundary. To make things simple and concrete, consider a second-order differential equation in the plane

Where is the unknown solution, denotes derivative of with respect to etc. The functions specify the problem.

We now seek a that satisfies the partial differential equation in a domain, which is a subset of the plane, and such that the Cauchy boundary conditions

Hold for all boundary points. Here is the derivative in the direction of the normal to the boundary. The functions and are the Cauchy data.