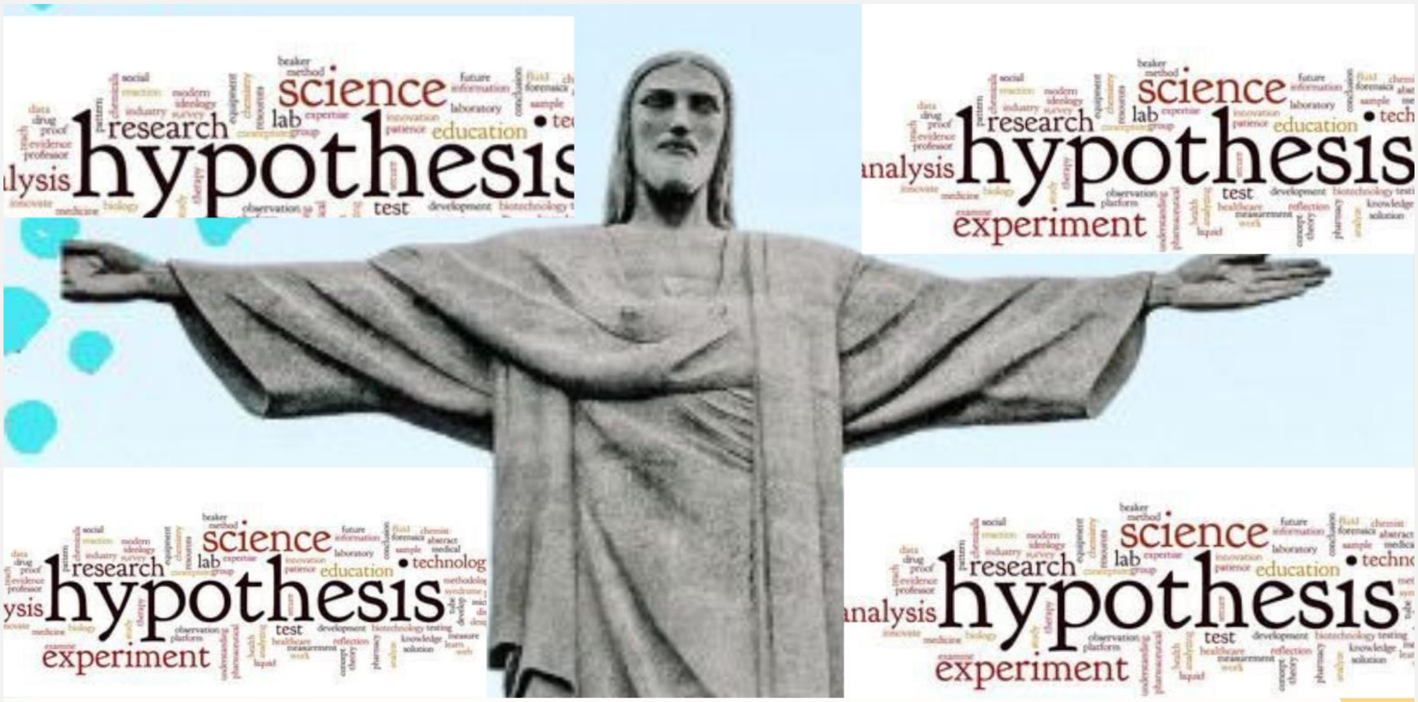
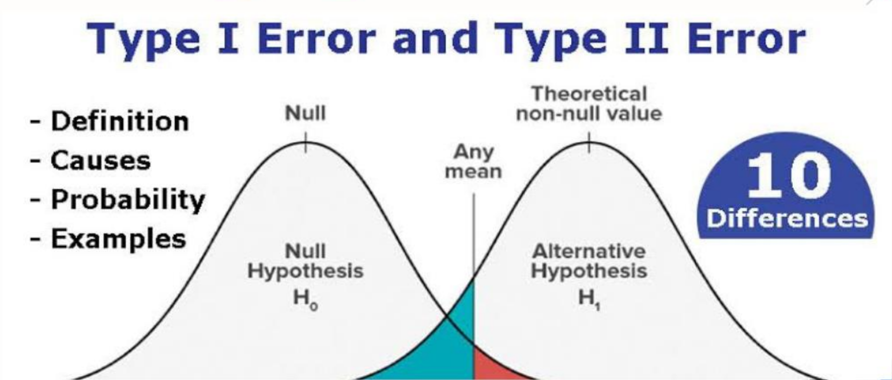


Angel of Hypothesis



.HOW TO:

State the null and alternative hypothes
Determine your risk
Gather your data
Run the appropriate test
Make right decisions
Make correct inferences

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1.0 Introduction to Hypothesis

What is hypothesis???

Hypothesis can be simply defined as a supposition or proposed explanation made on the basis of limited evidence as a starting point for further investigation. In other word a hypothesis is a speculation or theory based on insufficient evidence that lends itself to further testing and experimentation. With further testing, a hypothesis can usually be proven true or false.

Let's look at an example. My friend (James) hypothesizes to me that "the flowers I plants on my land will grow faster than flowers on your land". She plants on my land and his land and waters each plant daily for a 3month (experiment) and proves her hypothesis true!

Here, the statement made by my friend is called hypothesis.

What is Hypothesis testing???

Hypothesis testing is an act in statistics where by an analyst test an assumption regarding a population parameter. The methodology employed by the analyst depends on the nature of the data used and the reason for the analysis.

In the example above, the experiment perform by James is actually what we called hypothesis testing.

Hypothesis testing is used to assess the plausibility of a hypothesis by using sample data. Such data may come from a larger population, or from a data-generating process. The word "population" will be used for both of these cases in the following descriptions.

Hypothesis testing is used to assess the likelihood of a hypothesis by using sample data.

The test provides evidence concerning the likelihood of the hypothesis, given the data.

Statistical analysts test a hypothesis by measuring and examining a random sample of the population being analyzed.

How Hypothesis Testing Works???

In hypothesis testing, the statistician tests a statistical sample, with the aims to provide evidence to the likelihood of the null hypothesis. The test is carried out by measuring and examining a random sample of the population being analyzed. All analysts usually used a random population sample to test two different hypotheses: null and the alternative hypothesis.

What is null Hypothesis??

A null hypothesis is a hypothesis that says there is no statistical significance between the two variables in the hypothesis. It is the hypothesis that the researcher is trying to disprove. It is usually denoted by Ho

In the example above, null hypothesis would be something like this: There is no statistically significant relationship between the lands I used to plant the flowers and growth of the flowers.

A researcher is challenged by the null hypothesis and usually wants to disprove it, to demonstrate that there is a statistically-significant relationship between the two variables in the hypothesis.

What Is an Alternative Hypothesis?

An alternative hypothesis simply is the inverse, or opposite, of the null hypothesis. So, if we continue with the above example, the alternative hypothesis would be that "there is indeed a statistically-significant relationship between what type of land used to plant the flower and its growth. To make it more easily the following show the two hypothesis. Alternative Hypothesis is usually denoted by Ha

Null: *a flower plant on my land and the one planted in James' land will have no difference in growth rate. (Bear it in mind that null always means nothing extraordinary is going on.)*

Alternative*: a flower plant on my land and the one planted in James' land will be different in tern of growth. i.e. (the flower on James' land will grow faster than mine).*

Mathematically the same Hypothesis can be written as

H_o: $H_j = H_m$

H_a : $H_j > H_m$ (where H_j and H_m represent the mean height of James' land plant and my land plant respectively.

Note: *when no indicator for alternative hypothesis use 'not equal to '*

What are the steps to be taken for Hypothesis Testing?

All hypotheses can easily be tested using the following four steps:

1. State the two hypotheses i.e. (Determine H0 and Ha. Remember, they are contradictory)

2. Formulate an analysis plan, which only outlines how the data will be evaluated.

3. Calculate the test statistic.

4. Use the right table to calculate the critical value.

5. Compare the calculated test statistic with the Z critical value determined by the level of significance required by the test and make a decision (reject H_0 or fail to reject H_0), and write a clear conclusion using English sentences.

1.1 How to write the null and alternative hypothesis

The way your hypothesis will be written depend on what you are trying to test, so we can categorized all different ways you might come across in to 4 Categories

Category A: Test for the average of one a population

Category B: Test for the average of two populations.

Category C: Test *for significant difference between means of two population.*

Category D: Test *for significant difference between proportions of two population.*

1.1 Category A (Test for average of a population)

Here, all we are going to be dealing with will be on just one population and the mean will always be our estimator. In this category your null and alternative will always look like each of the following three patterns.

Right and left(2-tailed)	*Right(1-tailed)*	*Left(1-tailed)*
H_0: μ = *mean Value*	H_0: $\mu \leq$ *mean Value*	H_0: $\mu \geq$ *mean Value*
H_a: $\mu \neq$ *mean Value*	H_a: $\mu >$ *mean Value*	H_a: $\mu <$ *mean Value*

Note: equal to sign is common to all null hypothesis. Hence the words that will be helping you to know the null statement are: ‘is’, ‘is equal to’, ’not less than’, ’not more than’, ‘not greater than’, ‘at least’, ‘at most’, ‘equal to’, ’more than or equal to’, ‘less than or equal to’, ’not up to’.

Alternative hypothesis indicator are: ‘less than’, ‘greater than’, ‘not equal to’, ‘more than’, ‘is not’, ’higher than’, ‘lower than’ etc.

Example1.1.1

The average life of a car battery of a certain brand is six years. This information is gathered using data obtained from people who have purchased and used this brand of battery over a period of several years. A researcher at the battery company develops a new type of car battery and claims that the average life of this battery is more than six years. To determine whether this claim is true, one would need to do some hypothesis testing.

What would be the null hypothesis and the alternative hypothesis for this hypothesis test?

Assuming the researcher claimed that the average life is not five year. Write out the null and alternative hypothesis.

Solution

From the question above you can easily notice the null statement and the alternative. The researcher claim is the alternative as you can see alternative indicator “more than”. While the first statement in the question is the null for the null indicator “is” was used.

Hence:

Null hypothesis: *"The average life of the new car battery is six years."*

Alternative hypothesis: *"The average life of the new car battery is more than five years."*

Mathematically

H_0: μ = 6years

H_a: μ > 6years

(Where μ represent the average life of the new car battery)

Null hypothesis: "The average life of the new car battery is six years."

Alternative hypothesis: "The average life of the new car battery is not five years."

Mathematically

H0: μ = 6years

Ha: $\mu \neq$ 6years

Example1.1.2.

Past research data from a period of over several years states that the average life expectancy of whales is 85 years. A researcher at a laboratory wishes to test this hypothesis. To that end they procure a sample of life spans of several whales. What is the null hypothesis and the alternative hypothesis that this researcher will establish?

Solution

The null hypothesis for the researcher will state that, "The average life expectancy of whales is exactly equal to 85years."

The alternative hypothesis will read: "The average life expectancy of whales is not equal to 100 years."

Mathematically

H0: μ = 85years

Ha: $\mu \neq$ 85years

More Explanation: Here our null indicator is ‘is’ which was use in the first statement. We are not given any indicator for the alternative here. But we know that the new researcher only want to know if the statement propose is likely to be true. Since the previous research state It is 85 then opposite of that will be ‘it is not 85’.

Assuming in the question above that the researcher believes that it should more than 85 then it should be the following:

H0: μ = 85years

Ha: μ > 85years

Note: when no indicator for alternative hypothesis use ‘not equal to ‘

Example1.1.3

A company producing groundnut chip claims that each nylon contains 30 groundnut chips on average but I have a feeling that it is not up to that so what will be my null hypothesis and alternative hypothesis??

SOLUTION

The null hypothesis: "The average number of groundnut chips in a nylon is 30."

Alternative hypothesis will read: " the average number of groundnut chips in a nylon is less than 30." (Remember I said not up to)

Mathematically

$H0: \mu = 30$

$H_a: \mu < 30$

Null indicator here is 'contain' which is similar to 'is'

Alternative indicator is 'not up to' which is same as less than

Example1.1.4

A particular brand of tires claims that its deluxe tire averages at least 20,000 miles before it needs to be replaced. From past studies of this tire, the standard deviation is known to be 1,000. A researcher believes this is a lie and conduct survey.

Solution

The null hypothesis: "The average is at least 20,000." i.e more than or equal to 20,000

The alternative hypothesis will read: "it is less than 20,000

Mathematically

$H0: \mu \geq 20000$

$Ha: \mu < 20000$

Null Indicator: at least; Alternative indicator: Not stated

Here: the alternative will be the opposite of the null which is greater than.

Did you understand this?? Yeah, it is as easy as that.

Example1.1.5

Suppose that a recent article stated that the mean time spent in jail by a first–time convicted rapist is 14 years. Your team conducted a research to sees if the mean time has increased last year. What will be the null and alternative hypothesis for this research??

Solution

Null: *that the mean time spent in jail by a first–time convicted rapist is 14 years.*

Alternative: *the mean time has increased*

When something increased what does it mean?? Yeah it means it is more than before. Hence

Mathematically

H0: μ=*14years*

Ha: μ >*14years*

Summary

In short your null hypothesis is very easy to write as it will always include equal to or

Specific value. E.g H_0: μ = 85years, H_0: μ = 35kg. it can be in for any of the three below

Two-tailed test	one-tailed test	one-tailed test
H_0: μ = mean Value	H_0: $\mu \leq$ mean Value	H_0: $\mu \geq$ mean Value
H_a: $\mu \neq$ mean Value	H_a: μ > mean Value	H_a: μ < mean Value

The Alternative Hypothesis can be of any of the three below.

1. *H*a: $\mu \neq$ 85years (when we are only testing to say it is not equal i.e 2tails test.)
2. *H*a: μ > 85years (when we are testing if it is greater than i.e. 1tail test.)
3. *H*a: μ > 85years (when we are testing if it is less than i.e 1tail test)

Note: if the null is form of first pattern i.e H_0: μ = mean Value then its alternative can be any of the three pattern but if it is any of the remaining two, then it should go with specific alternative written above.

That is how to write the null and alternative hypothesis for the first Category.

Now try to deal with the exercise.

Exercise

1. *Imagine you were carrying out research for a fluorescent bulbs company, you are to test the hypothesis that the mean lifetime of the bulbs produced by the company is 506days. What will be your null and alternative hypothesis statement?*

2. *A school owner want to know if increasing the amount of light during the studying will increase the performance of the participant in the test score so he asked for your help to construct the hypothesis.*

3. *Your friend posted on his status that "on average Nigerian females lost their virginity at the latest by 15years or less. You think Nigerian females are not that worst hence you try to test the virginity of some females. What will be your hypothesis for this research?*

4. *A company has stated that their straw machine makes straws that are 4mm diameter. A worker believes the machine no longer make straws of this size. To perform the hypothesis test what will be the null and alternative hypothesis for this worker's belief*

5. *A doctor believes that the average teen sleeps on average no longer than 9hours per day. A researcher believes it should be that teens on average sleep longer than that. Write the null and alternative hypothesis for the research.*

6. *It is believed that a candy machine makes chocolate bars that are on average 5.5g. A worker claims that the machine after maintenance is no longer makes 5.5g bars. Write the null and alternative hypothesis for this claim.*

7. You are testing that the mean speed of your cable Internet connection is more than three Megabits per second. State the null and alternative hypotheses..

8. The mean entry level salary of an employee at a company is $58,000. You believe it is higher for IT professionals in the company. State the null and alternative hypotheses.

9. Suppose that a recent article stated that the mean time spent in jail by a first–time convicted burglar is 2.5 years. A study was then done to see if the mean time has increased in the new century. A random sample of 26 first-time convicted burglars in a recent year was picked. The mean length of time in jail from the survey was 3 years with a standard deviation of 1.8 years. Suppose that it is somehow known that the population standard deviation is 1.5. If you were conducting a hypothesis test to determine if the mean length of jail time has increased, what would the null and alternative hypotheses be? The distribution of the population is normal

   H_0: ________ H_a: ________

10. A random survey of 75 death row inmates revealed that the mean length of time on death row is 17.4 years with a standard deviation of 6.3 years. If you were conducting a hypothesis test to determine if the population mean time on death row could likely be 15 years, what would the null and alternative hypotheses be?

    H_0: ________ H_a: ________

1.2. How to write hypothesis of proportion (Category B)

Here we are going to be dealing with proportion and percentage our test here will focus on just one population. To write the test will be similar to the previous category, only that we are no more using mean instead we will be using proportion**.**

Example1.2.1

The school board claims that at least 60% of students brings a phone to school. A teacher believes this number is too high and randomly selected 25students. What will be the null and alternative hypothesis??

Solution

Null: at least 60% of students brings a phone to school.

Alt: Not at least 60% of students brings a phone to school.

Here it should be understand that at least in the statement is the null indicator also it can be interpreted as greater than or equal to. The opposite of at least will always be less than.

Hence;

H_0: $P \geq 60\%$

H_a: $P < 60\%$

Example1.2.2

Your friend wrote the following quote on his whatsapp status.

"Not less than 80% of beautiful ladies in Nigeria usually have low IQ". You decided to do research if this is true what will be the null and alternative hypothesis.

Solution

Null: N*ot less than 80% of beautiful ladies in Nigeria usually have low IQ*

Alt: N*ot less than 80% of beautiful ladies in Nigeria usually have low IQ*.

Here it should be understand that ***not less than*** in the statement is the null indicator also it can be interpreted as greater than or equal to. The opposite of ***not less than*** is ***less than***.

Hence;

H_0: $P \geq 80\%$

H_a: $P < 80\%$

Example1.2.3

Your friend wrote the following quote on his whatsApp status.

"Not less than 80% of beautiful ladies in Nigeria usually have low IQ". You decided to do research if this is true what will p be the null and alternative hypothesis.

Solution

Null: N*ot less than 80% of beautiful ladies in Nigeria have low IQ*

Alt: N*ot less than 80% of beautiful ladies in Nigeria usually have low IQ.*

Here it should be understand that ***not less than*** in the statement is the null indicator also it can be interpreted as greater than or equal to. The opposite of ***not less than*** is ***less than***.

Hence;

H_0: $P \geq 80\%$

H_a: $P<80\%$

Where P represent the percentage of beautiful ladies in Nigeria with low IQ

Example1.2.4

It was believe that out of 1950 people admitted to air force every year, 500 of them are female. Then a researcher want to test the claim. Establish the hypothesis for this research.

Solution

Here also we are dealing with proportion only that the figures are given. Hence we need to calculate the proportion by ourselves.

Total=1950

Female=500

Proportion of female (P)=500/1950=0.256

We are testing whether it is the same or not. This shows that our hypothesis we look like

Null: the proportion of female air force is 0.256

Alt: the proportion of female air force is not 0.256

Hence;

H_0: $P= 0.256$

H_a: $P\neq 0.256$

Example 1.2.5

The last article stated that the poll taken among 25faculty member at a university shows that 60% of those polled favor a longer break between semester (and a shorter summer vacation) As a researcher you think this is too high and decided to conduct a new poll to justify your belief what will be your null and alternative hypothesis.

Solution

This question is straight and direct, we already know what the alternative and the null will look like looking at the word “too high” the word mean we believe it should be less.

Null: the percentage of those who favor a longer break between semesters is 60%

Alt: the percentage of those who favor a longer break between semesters is less than 60%

H_0: P=60%

H_a: P<60%

Example 1.2.6

It was stated that *‘not more than 30% of Nigerian Youths have more than #10,000 in their bank account since 2018 till date’.* A business man believes Nigeria has not gotten worst to that extent and ask you to carry out research on that. Write out your Hypothesis.

Solution

Here the word “not more than” is null indicator here. And the opposite which the business man believed is “more than”

Null: *not more than 30% of Nigerian Youths have more than #10,000*

Alt: *more than 30% of Nigerian Youths have more than #10,000*

H_0: P≤30%

*Ha: P>*30%

Note: Not more than can be interpreted as less than or equal to.

Now try the following Exercise

Exercise1.2

1. Marketers believe that 92% of adults in the United States own a cell phone. A cell phone manufacturer believes that number is actually lower. Write out the hypothesis if you were to test for this claim.
2. The National Institute of Mental Health published an article stating that in any one-year period, approximately 9.5 percent of American adults suffer from depression or a depressive illness. Suppose that in a survey of 100 people in a certain town, seven of them suffered from depression or a depressive illness. If you were conducting a hypothesis test to determine if the true proportion of people in that town suffering from depression or a depressive illness is lower than the percent in the general adult American population, what would the null and alternative hypotheses be?

   H_0: _________

   H_a: _________
3. In a population of fish, approximately 42% are female. A test is conducted to see if, in fact, the proportion is less. State the null and alternative hypotheses
4. Read the following poem very well and answer the question in it.

   My dog has so many fleas,

   They do not come off with ease.
   As for shampoo, I have tried many types
   Even one called Bubble Hype
   Which only killed 25% of the fleas,
   Unfortunately I was not pleased.

   Until I had given up hope
   Until one day I saw
   An ad that put me in awe.

   A shampoo used for dogs
   called GOOD ENOUGH to Clean a Hog
   guaranteed to kill more fleas.

   With the shampoo I gave Fido a bath
   and started doing the math.
   But I need you to write me the Hypothesis

   Before I can do the analysis
   so that I can figure out,

   Whether to use the new shampoo or go without?

/

5. A research shows that about 35% of students of university of Ibadan pass the UTME on the first try. We want to test if more than 35% pass on the first try. Write out the Null and alternative hypothesis for the test with correct symbols.
6. Maxwizard (An analyst) claims that the proportion of students going to love garden in the University of Ibadan to meet someone He/she loves is 0.8. You want to test to see if the claim is correct. State the null and alternative hypotheses.
7. It was claimed long ago that in a population of fish, approximately 42% are female. A test is conducted to see if, in fact, the proportion is less. State the null and alternative hypotheses.
8. A recent global report on insurance claims payment says that, "a less than satisfactory claims experience prompts one in five customers to switch insurance providers."
9. The Pew Research center just reported that Muslims have an average birthrate of 2.9 children per woman. You want to test if the claim is true for your country what will be null and alternative hypothesis.
10. Dr. West Crenshaw published an article in "Your Teen Magazine "that indicated that 47% of high school students in the US have had sex. You want to test if this proportion is the same for high school students in Nigeria what will be the null and Alternative hypothesis
11. From Dr. West Crenshaw's article it was also stated that 70% teens in the world have had sex or a make out experience at least. You think the world has not bad to that extent what will be the null and alternative hypothesis

1.3 (Homogeneous or Heterogeneous).

Here we will be dealing with both mean and proportion of two different population or one population of two categories. Our research will focus on comparison between two things. The question of the research will be; is there a significant differences or not? If yes which one is higher or less?? To understand this well let's take a look at some examples.

Example 1.3.1

Suppose that a recent article stated that the mean time spent in jail by a first–time convicted rapist in US is 14 years. You client just has a case of raping, so He would either get justice in Nigeria or US that depends on his choice. Your team conducted a research to sees if the mean time spend in jail by first-time convicted rapist in US is the same as in Nigeria or not.

A) What will be your null and alternative hypothesis?

B) Assuming the research is to know if the mean time spent in US prison is less than Nigeria's. What will be the null and alternative?

Solution

A) H_0: $\mu_u = \mu_N = 14$**years** H_a: $\mu_u \neq \mu_N$

Explanation

Here we are considering two population, rapist in US and rapist in Nigeria. As you can see they are two different countries. The hypothesis indicator in the question above is 'is the same as' in the statement **the mean time spend in jail by first-time convicted rapist in US is the same as in Nigeria or not.** This shows that the alternative will be not equal to. In another word we can write it as the following

Null: the mean time spend in jail by first-time convicted rapist in US is the same as in Nigeria

Alt: the mean time spend in jail by first-time convicted rapist in US is not the same as in Nigeria

B) H_0: $\mu_u \geq \mu_N$ H_a: $\mu_u < \mu_N$

Explanation

The hypothesis indicator here has been changed to **less than**. So it is very easy to understand this.

Null: the mean time spend in jail by first-time convicted rapist in US is not less than mean time in Nigeria.

Alt: the mean time spend in jail by first-time convicted rapist in US is less than mean time in Nigeria.

Example 1.3.2

You were given a projects to do research on a project labelled “Do younger U.S. males weigh more on average than older males?” write out your null and alternative hypothesis for this research.

Solution

Hypothesis Indicator: ‘more’ more here is telling us that our alternative will be written with more than while the null will be written with the opposite. Which will be less than or equal to.

H0: $\mu_y \geq \mu_A$

Ha: $\mu_y > \mu_A$ (where μ_y, μ_A represent the weight of younger U.S male and older U.S male on average respectively)

Example 1.3.3

There was a belief that the proportion of students happened to be Yahoo boys in University of Ibadan is the same as that of LAUTEC. If you were to test the claim what will be your null and alternative hypothesis??

Solution

H0: $P_I = P_L$

Ha: $P_I = P_L$ (where P_I and P_L represent the proportion of yahoo boys in U.I and LAUTEC respectively)

Example 1.3.4

A tattoo magazine claimed that the percent of men that have at least one tattoo is greater than the percent of women with at least one tattoo. To test this claim what will be the hypothesis.

Solution

H0: $P_m = P_F$

Ha: $P_m > P_F$ (where P_m and P_F represent the percentage of men and women that have at least one tattoo respectively)

Example 1.3.5

A health magazine claims that marriage status is one of the most telling factors for a person's happiness. To test the claim that the percent of married people that are unhappy is lower than the percent of single or divorced people that are unhappy what will be your null and alternative.

Solution

H_0: $P_m = P_{sd}$

H_a: $P_m < P_{sd}$ (where P_m and P_{sd} represent the percentage of unhappy married people and unhappy single or divorced people respectively)

Exercise

Write the null and alternative hypothesis for the following

a. The mean number of years Americans work before retiring is 34.

b. At most 60% of Americans vote in presidential elections.

c. The mean starting salary for San Jose State University graduates is at least $100,000 per year.

d. Twenty-nine percent of high school seniors get drunk each month.

e. Fewer than 5% of adults ride the bus to work in Los Angeles.

f. The mean number of cars a person owns in her lifetime is not more than ten.

g. About half of Americans prefer to live away from cities, given the choice.

h. Europeans have a mean paid vacation each year of six weeks.

i. The chance of developing breast cancer is under 11% for women.

j. Private universities' mean tuition cost is more than $20,000 per year.

2.0 How to determine the type of test to use.

There are two important test commonly used although there are still more which we are not interested in for now they are:

1. Z-test or normal distribution test

2. T-test

Rule: Use the Z-test if and only if the data is normally distributed or the sample size is more than or equal to thirty i.e. when $n > 30$. Otherwise use T-test.

Rule2: use Z-test (normal distribution table) when the population standard deviation is known

Rule3: whenever you are dealing with proportion use Z-test (normal distribution table) if and only if $np \geq 30$ *and* $n(1-p) \geq 30$ *otherwise use Binomial distribution*

Exercise

1. Which distribution do you use when you are testing a population mean and the population standard deviation is known? Assume a normal distribution, with $n \geq 30$.
2. Which distribution do you use when the standard deviation is not known and you are testing one population mean? Assume sample size is large.
3. A population mean is 13. The sample mean is 12.8, and the sample standard deviation is two. The sample size is 20. What distribution should you use to perform a hypothesis test? Assume the underlying population is normal.
4. A population has a mean of 25 and a standard deviation of five. The sample mean is 24, and the sample size is 108. What distribution should you use to perform a hypothesis test?
5. It is thought that 42% of respondents in a taste test would prefer Brand *A*. In a particular test of 100 people, 39% preferred Brand *A*. What distribution should you use to perform a hypothesis test?
6. You are performing a hypothesis test of a single population mean using a Student's *t*-distribution. What must you assume about the distribution of the data?
7. You are performing a hypothesis test of a single population mean using a Student's *t*-distribution. The data are not from a simple random sample. Can you accurately perform the hypothesis test?

8. You are performing a hypothesis test of a single population proportion. What must be true about the quantities of *np* and *nq*?
9. That *np* is less than five. What must you do to be able to perform a valid hypothesis test?
10. You are performing a hypothesis test of a single population proportion. The data come from which distribution?

2.2 How to know which tail test to be used.

To choose between two tails or one tail test is very easy, all we need to inspect is the hypothesis statement written and follow the following principle.

Principle: Use one tail test when your alternative hypothesis has greater than or less than sign else use two tails.

What is the Principle trying to explain? If you have any of the following as your alternative hypothesis you are expected to use one tail

H_a: μ > mean Value H_a: μ < mean Value

H_a: P > proportion Value H_a: μ < proportion Value

Here greater than > denote the right tail test while less than < sign denote left tail test.

Note: The only time you should use two tails is when your alternative hypothesis is written with not equal to sign or you were instructed to use it.

Example 2.2

For each of the following question

a) Write the null and alternative hypothesis.

b) Label whether the null or the alternative is the original claim.

c) Tell whether this is a left tail test, a right tail test, or a two tail test.

1. A company producing groundnut chip claims that each nylon contains 30 groundnut chips on average but you have a feeling that it is not up to that so you want to test the claim.
2. A particular brand of tires claims that its deluxe tire averages at least 20,000 miles before it needs to be replaced. From past studies of this tire, the standard deviation is known to be 1,000. A researcher believes this is a lie and conduct survey.
3. *A health magazine claims that marriage status is one of the most telling factors for a person's happiness. Test the claim that the percent of married people that are unhappy is lower than the percent of single or divorced people that are unhappy last* year. You want to test if by chance it is possible that the proportion of females is still the same this year. Then establish the hypothesis for this research.

Solution

1. H_0: $\mu = 30$

H_a: $\mu<30$

b) The original claim is null. (As you can see here that what was written for null is the company claim)

c) This is left tail since the alternative hypothesis is less than.

2. H_0: $\mu \geq 20000$ Ha: $\mu <20000$

b) The null is the claim

c) It is left tail

3 **H_0:** $P_m=P_{sd}$ **H_a:** $P_m<P_{sd}$

(where P_m and P_{sd} represent the percentage of unhappy married people and unhappy single or divorced people respectively)

b) The alternative hypothesis is the claim.

c) It is left tail.

4. Since 500/960 =0.256

H_0: P= 0.256 H_a: $P\neq 0.256$

b) The null is the claim

c) It is two tail test (it the alternative has not equal to)

Exercise 2.2

a) Write the null and alternative hypothesis.

b) Label whether the null or the alternative is the original claim.

c) Tell whether this is a left tail test, a right tail test, or a two tail test.

1. According to a CNN report, besides cell phones, 93% of Americans also own a traditional phone. But has that percentage decreased as more and more Americans opt to only use a cellphone and throw away their traditional phones.

2. More and more Americans are becoming financially sound and opting to not own a credit card. According to an article in USA Today, 74% of Americans still have at least one credit card. But this claim seems a little on the low side. We think that more than 74% of Americans own a credit car

3. According to a recent Newspaper article, people in California spend 1.25 hours a day eating and drinking. Suppose we want to test the claim that the number of hours spent eating and drinking is really 1.25 hours.

4. The standard deviation for the heights of men was thought to be 2.9 inches. New studies disagree with this. Test the claim that the standard deviation for heights of men is not 2.9 inches.

5. It has long been thought that normal body temperature is really 98.6 degrees Fahrenheit. Recent study is now claiming that normal body temperature is really lower than 98.6 degrees.

6. Wikipedia suggests that at least 10% of the world population is left handed. Wikipedia may not be very accurate. Test the claim that at least 10% of the world population is left handed

7. The percent of women that hold CEO level jobs is lower than the percent of men that hold CEO level jobs

State the type of test to be used for the following (1tail, 2tails, right tail, left or tail)

8. Assume H_0: $\mu = 9$ and H_a: $\mu < 9$. Is this a left-tailed, right-tailed, or two-tailed test?

9. Assume H_0: $\mu \leq 6$ and H_a: $\mu > 6$. Is this a left-tailed, right-tailed, or two-tailed test?

10. Assume H_0: $p = 0.25$ and H_a: $p \neq 0.25$. Is this a left-tailed, right-tailed, or two-tailed test?

11. A bottle of water is labeled as containing 16 fluid ounces of water. You believe it is less than that. What type of test would you use?

12. Your friend claims that his mean golf score is 63. You want to show that it is higher than that. What type of test would you use?

13. A bathroom scale claims to be able to identify correctly any weight within a pound. You think that it cannot be that accurate. What type of test would you use?

14. You flip a coin and record whether it shows heads or tails. You know the probability of getting heads is 50%, but you think it is less for this particular coin. What type of test would you use? If the alternative hypothesis has a not equals ($\neq$) symbol, you know to use which type of test?

15. Assume the null hypothesis states that the mean is at least 18. Is this a left-tailed, right-tailed, or two-tailed test?

16. Assume the null hypothesis states that the mean is at most 12. Is this a left-tailed, right-tailed, or two-tailed test?

17. Assume the null hypothesis states that the mean is equal to 88. The alternative hypothesis states that the mean is not equal to 88. Is this a left-tailed, right-tailed, or two-tailed test?

3.0 How to calculate the Zstatistics

I believe you should already know how to calculate the Zstatistics but should you don't I am going to give you some example on it but it will be inform of the hypothesis questions. But before we go in to that let me give you some important formula you need to know when trying to calculate the Zstatistics and when to use them.

What are the most important formula used in calculating Zstatistics in hypothesis??

The most important formula needed for calculating the Zstatistics are of four category.

1. Using sample mean, sample size and standard Deviation

2. Using the proportion and sample size

3. Using two sample mean, size and standard deviation from two different population

4. Using two proportion and sample size from two different population

CASE 1

Formula for calculating Zstatistics when Sample mean, standard deviation and sample size are given.

$$Z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}}$$

$\bar{x}$ = Sample mean

μ = population mean

σ = Population standard deviation

n = Sample size

Note: S.E=$\sigma/\sqrt{n}$ where S.E represent the standard error. Hence we can also rewrite the formula as

$$Z = \frac{\bar{x} - \mu}{SE}$$

Formula for calculating Zstatistics when proportion and sample size is given

$$Z_{cal} = \frac{P - P_0}{\sqrt{\frac{P_0(1 - P_0)}{n}}}$$

Where P_0 is the population proportion

P is the proportion from sample

n is the size of the sample

In this case mean and standard deviation can be easily calculated as following;

$\mu_{X} =$ np and $sd = \sqrt{np(1-p)}$

CASE 3.

Formula for calculating Zstatistics when Sample mean, size and standard deviation from two different population is given.

$$Z_{cal} = \frac{\overline{x_1} - \overline{x_2}}{\sqrt{\frac{\sigma_1}{n_1} + \frac{\sigma_2}{n_2}}}$$

Where $\overline{x_1}$ is the mean of the first sample?

$\overline{x_2}$ Is the mean of the second sample?

n_1 Is the size of the first sample from the first population?

n_2 Is the size of the second sample from the second population?

$(\overline{x_1} - \overline{x_2})$ Is the difference between the two proportions?

CASE 4.

Formula for calculating Zstatistics when two proportion and sample size from two different population is given.

$$P = \frac{(P_1 * n_1) + (P_2 * n_2)}{n_1 + n_2}$$

$$Z = \frac{(P_1 - P_2)}{\sqrt{P(1-P)\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

Where P is the average calculated proportion

P_1 Is the proportion of the first sample?

P_2 Is the proportion of the second sample?

n_1 Is the size of the first sample from the first population?

n_2 Is the size of the second sample from the second population?

$P_1 - P_2$ Is the difference between the two proportions?

All of the formula above needed to be master well before attempting any question we shall work on their example later

4.0 How to check the critical value from statistical table

4.1 How to make use of statistical table to find the critical value from normal distribution table

Firstly it is important that you download the table here. Then move on to make use of it.

The Pdf you downloaded has two different normal distribution table in which one are labeled with negative numbers while the other is positive number. Now let make use of the negative one.

Example 4.1.1

What is the critical value when α is 5% assuming it is two tailed test.

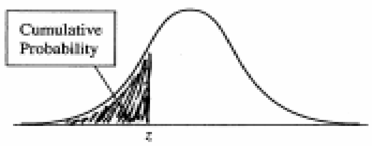
Solution

Step1

Here α=5% =0.05 α /2 = 0.05/2 =0.025 (since it is two tailed test)

Step2

Now that we already know the important value go in to the table and look for wherever you have (0.025) or any number very closed to that



Cumulative probability for z is the area under the standard normal curve to the left of z

z	.00
−5.0	.000000287
−4.5	.00000340
−4.0	.0000317
−3.5	.000233

TABLE A Standard Normal Cumulative Probabilities

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
−3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
−3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
−3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
−3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
−3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
−2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
−2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
−2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
−2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
−2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
−2.4	.0082	.0080	.0078	.0075	.0073	.007[illegible]	.0069	.0068	.0066	.0064
−2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
−2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
−2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
−2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
−1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
−1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
−1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
−1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
−1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
−1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
−1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
−1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
−1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
−1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
−0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
−0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
−0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
−0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
−0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
−0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
−0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
−0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
−0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
−0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641

Step3: As you can see above we see exactly the number we are looking for. Now match the row and column of the place where the number is located.

Here if we go to the leftmost we can see (-1.9) and to the topmost we can see (0.06).

Step 4: we merge both numbers together and neglect the negative i.e 1.9 + 0.06 = 1.96

Hence the critical value is ±1.96.

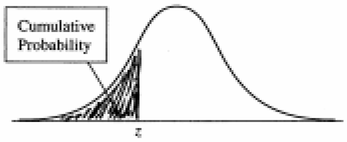
Example 4.1.2

What is the critical value of the question given in example1 assuming it is one tail test?

Solution

Here α=5% =0.05 (no need to divide b 2 since it is one tailed test)

Now that we already know the important value, go to the table and look for wherever you have (0.05) or any number very closed to that.



Cumulative probability for z is the area under the standard normal curve to the left of z

z	.00
−5.0	.000000287
−4.5	.00000340
−4.0	.0000317
−3.5	.000233

TABLE A Standard Normal Cumulative Probabilities

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
−3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
−3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
−3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
−3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
−3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
−2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
−2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
−2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
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−0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641

Step3: here we do not see the exact value (0.05), hence we will pick first number closest to it. Here we can pick either (0.0505) or (0.0495) but I prefer the first one.

Now match the row and column of the place where the number is located. If we go to the leftmost we can see (-1.6) and to the topmost we can see (0.04).

Step 4: we merge both numbers together and neglect the negative i.e 1.6+ 0.04 = 1.64

Hence the critical value is 1.64.

Note: only one value is gotten for one tailed test while plus or minus is written for two tailed test.

Example 4.1.3

What is the critical value of a normal distribution data given 98% as confidence level assuming you are doing left tail test??

Solution

Step1

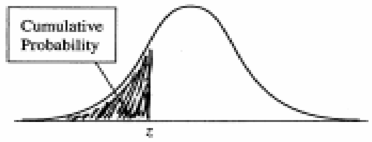
CL=98% =0.98

α =1-CL =1-0.98 =0.02

Here α=0.02 (no need to divide b 2 since it is one tailed test)

Step2

Now that we already know the important value, go to the table and look for wherever you have (0.02) or any number very closed to that.



Cumulative probability for z is the area under the standard normal curve to the left of z

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−5.0	.000000287
−4.5	.00000340
−4.0	.0000317
−3.5	.000233

TABLE A Standard Normal Cumulative Probabilities

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
−3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
−3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
−3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
−3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
−3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
−2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
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−2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
−2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
−2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
−2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
−2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
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−2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
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−1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
−1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
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−0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
−0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
−0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641

Step3: As you can see above we don't see exact number we are looking for. So we are choosing the leftmost value between the two closed value of our target i.e. (0.0202 and 0.0197), so we choose 0.0202. Now match the row and column of the place where the number is found. Here if we go to the leftmost we can see (-2.0) and to the topmost we can see (0.05).

Step 4: we merge both numbers together and neglect the negative i.e. 2.0 + 0.05=2.05

Hence the critical value is -2.05. (Whenever you are given left tail our answer will be negative)

Note: we would choose positive answer if the question has not told us that it is left tail.

Example 4.1.4

What is the critical value of a normal distribution data given 98% as confidence level??

Solution

CL=98% =0.98

α =1-CL =1-0.98 =0.02

Here α=0.02 /2 =0.01 (since the type of tail is not stated we will choose two tails)

Now that we already know the important value, go to the table and look for wherever you have (0.01) or a number very closed to that.

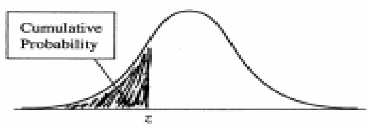

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TABLE A Standard Normal Cumulative Probabilities

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−3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
−3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
−3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
−3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
−3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
−2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
−2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
−2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
−2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
−2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
−2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
−2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
−2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
−2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
−2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
−1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
−1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
−1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
−1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
−1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
−1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
−1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
−1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
−1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
−1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
−0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
−0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
−0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
−0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
−0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
−0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
−0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
−0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
−0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
−0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641

Step3: As you can see above we don't see exact number we are looking for. So we are choosing the leftmost value between the two closed value of our target i.e. (0.0102 and 0.0099), so we choose 0.0102. Now match the row and column of the place where the number is found.

Here if we go to the leftmost we can see (-2.3) and to the topmost we can see (0.02).

Step 4: we merge both numbers together and neglect the negative i.e 2.3 + 0.02=2.32

Hence the critical value is ±2.05. (Remember two answers are gotten for 2tails)

4.2 How to check the critical value from t-table

Example 4.2.1

What is the critical value of a data with sample size 9 and level of significant of 1% for 1 tail test.

Solution

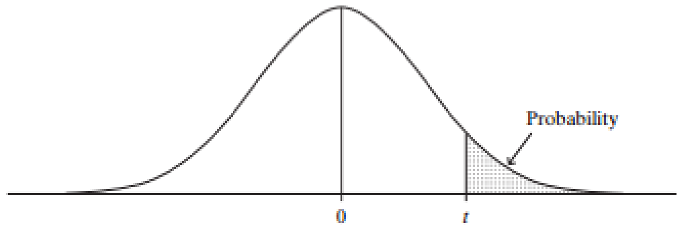
Here α = 1% =0.01

n=9, df=n-1

df=9-1=8

Now look for the 1% or 0.01 on the top of our table and 8 from 'df' column. Match it together as you can see below

TABLE B: ***t*** **Distribution Critical Values**



	Confidence Level					
	80%	90%	95%	98%	99%	99.8%
	Right-Tail Probability					
df	$t_{.100}$	$t_{.050}$	$t_{.025}$	$t_{.010}$	$t_{.005}$	$t_{.001}$
1	3.078	6.314	12.706	31.821	63.656	318.289
2	1.886	2.920	4.303	6.965	9.925	22.328
3	1.638	2.353	3.182	4.541	5.841	10.214
4	1.533	2.132	2.776	3.747	4.604	7.173
5	1.476	2.015	2.571	3.365	4.032	5.894
6	1.440	1.943	2.447	3.143	3.707	5.208
7	1.415	1.895	2.365	2.998	3.499	4.785
8	1.397	1.860	2.306	2.896	3.355	4.501
9	1.383	1.833	2.262	2.821	3.250	4.297
10	1.372	1.812	2.228	2.764	3.169	4.144
11	1.363	1.796	2.201	2.718	3.106	4.025
12	1.356	1.782	2.179	2.681	3.055	3.930
13	1.350	1.771	2.160	2.650	3.012	3.852
14	1.345	1.761	2.145	2.624	2.977	3.787
15	1.341	1.753	2.131	2.602	2.947	3.733
16	1.337	1.746	2.120	2.583	2.921	3.686
17	1.333	1.740	2.110	2.567	2.898	3.646
18	1.330	1.734	2.101	2.552	2.878	3.611
19	1.328	1.729	2.093	2.539	2.861	3.579
20	1.325	1.725	2.086	2.528	2.845	3.552
21	1.323	1.721	2.080	2.518	2.831	3.527
22	1.321	1.717	2.074	2.508	2.819	3.505
23	1.319	1.714	2.069	2.500	2.807	3.485
24	1.318	1.711	2.064	2.492	2.797	3.467
25	1.316	1.708	2.060	2.485	2.787	3.450
26	1.315	1.706	2.056	2.479	2.779	3.435
27	1.314	1.703	2.052	2.473	2.771	3.421
28	1.313	1.701	2.048	2.467	2.763	3.408
29	1.311	1.699	2.045	2.462	2.756	3.396
30	1.310	1.697	2.042	2.457	2.750	3.385

As you can see the answer is 2.896, please don't be deceived by 98% on the top, it is nothing what is written as decimal is what Is important if you have this kind of table with you.

Example 4.2.2.

Assuming we decide to do two-tail test for example1 above what will be the critical value??

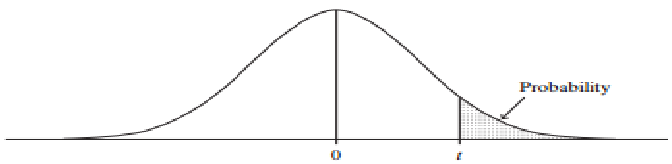
Solution

α = 1% α /2 = ½ %=0.5% = 0.5/100=0.005 (to Decimal)

n=9, df=n-1 df=9-1=8

Now look for the 0.5% or 0.005 on the top of our table and 8 from 'df' column. Match it together as you can see below

TABLE B: t **Distribution Critical Values**



	Confidence Level					
	80%	90%	95%	98%	99%	99.8%
	Right-Tail Probability					
df	$t_{.100}$	$t_{.050}$	$t_{.025}$	$t_{.010}$	$t_{.005}$	$t_{.001}$
1	3.078	6.314	12.706	31.821	63.656	318.289
2	1.886	2.920	4.303	6.965	9.925	22.328
3	1.638	2.353	3.182	4.541	5.841	10.214
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22	1.321	1.717	2.074	2.508	2.819	3.505
23	1.319	1.714	2.069	2.500	2.807	3.485
24	1.318	1.711	2.064	2.492	2.797	3.467
25	1.316	1.708	2.060	2.485	2.787	3.450
26	1.315	1.706	2.056	2.479	2.779	3.435
27	1.314	1.703	2.052	2.473	2.771	3.421
28	1.313	1.701	2.048	2.467	2.763	3.408
29	1.311	1.699	2.045	2.462	2.756	3.396
30	1.310	1.697	2.042	2.457	2.750	3.385

Now it is very clear that the result from the table is 3.355

Example 4.2.3

What will be the critical value of sample of size 18 with confidence level of 95%??

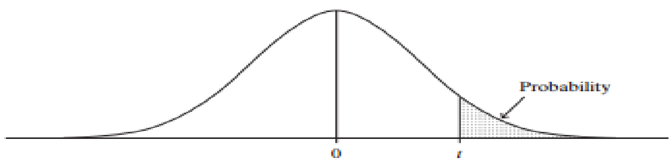
Solution

CL=95% =0.95 $\alpha = 1 - \mathrm{CL} = 1 - 0.95 = 0.05$ $\frac{\alpha}{2} = \frac{0.05}{2}$ =0.025 (to Decimal)

n=9, df=n-1 df=18-1=17

Now look for the 2.5% or 0.025 on the top of our table and 17 from 'df' column. Match it together as you can see below

TABLE B: *t* **Distribution Critical Values**



	Confidence Level					
	80%	90%	95%	98%	99%	99.8%
	Right-Tail Probability					
df	$t_{.100}$	$t_{.050}$	$t_{.025}$	$t_{.010}$	$t_{.005}$	$t_{.001}$
1	3.078	6.314	12.706	31.821	63.656	318.289
2	1.886	2.920	4.303	6.965	9.925	22.328
3	1.638	2.353	3.182	4.541	5.841	10.214
4	1.533	2.132	2.776	3.747	4.604	7.173
5	1.476	2.015	2.571	3.365	4.032	5.894
6	1.440	1.943	2.447	3.143	3.707	5.208
7	1.415	1.895	2.365	2.998	3.499	4.785
8	1.397	1.860	2.306	2.896	3.355	4.501
9	1.383	1.833	2.262	2.821	3.250	4.297
10	1.372	1.812	2.228	2.764	3.169	4.144
11	1.363	1.796	2.201	2.718	3.106	4.025
12	1.356	1.782	2.179	2.681	3.055	3.930
13	1.350	1.771	2.160	2.650	3.012	3.852
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21	1.323	1.721	2.080	2.518	2.831	3.527
22	1.321	1.717	2.074	2.508	2.819	3.505
23	1.319	1.714	2.069	2.500	2.807	3.485
24	1.318	1.711	2.064	2.492	2.797	3.467
25	1.316	1.708	2.060	2.485	2.787	3.450
26	1.315	1.706	2.056	2.479	2.779	3.435
27	1.314	1.703	2.052	2.473	2.771	3.421
28	1.313	1.701	2.048	2.467	2.763	3.408
29	1.311	1.699	2.045	2.462	2.756	3.396
30	1.310	1.697	2.042	2.457	2.750	3.385

Now it is very clear that the result from the table is 2.110

Example4.2.4

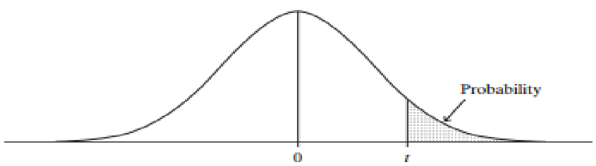
What will be the critical value of sample of size 28 with confidence level of 90% for one tail test??

Solution

CL=95% =0.90   α =1-CL= 1-0.90=0.1,   n=28, df =n-1   df=28-1=27

Now look for the 10% or 0.1 on the top of our table and 27 from 'df' column. Match it together as you can see below

TABLE B: *t* Distribution Critical Values



	Confidence Level					
	80%	90%	95%	98%	99%	99.8%
	Right-Tail Probability					
df	$t_{.100}$	$t_{.050}$	$t_{.025}$	$t_{.010}$	$t_{.005}$	$t_{.001}$
1	3.078	6.314	12.706	31.821	63.656	318.289
2	1.886	2.920	4.303	6.965	9.925	22.328
3	1.638	2.353	3.182	4.541	5.841	10.214
4	1.533	2.132	2.776	3.747	4.604	7.173
5	1.476	2.015	2.571	3.365	4.032	5.894
6	1.440	1.943	2.447	3.143	3.707	5.208
7	1.415	1.895	2.365	2.998	3.499	4.785
8	1.397	1.860	2.306	2.896	3.355	4.501
9	1.383	1.833	2.262	2.821	3.250	4.297
10	1.372	1.812	2.228	2.764	3.169	4.144
11	1.363	1.796	2.201	2.718	3.106	4.025
12	1.356	1.782	2.179	2.681	3.055	3.930
13	1.350	1.771	2.160	2.650	3.012	3.852
14	1.345	1.761	2.145	2.624	2.977	3.787
15	1.341	1.753	2.131	2.602	2.947	3.733
16	1.337	1.746	2.120	2.583	2.921	3.686
17	1.333	1.740	2.110	2.567	2.898	3.646
18	1.330	1.734	2.101	2.552	2.878	3.611
19	1.328	1.729	2.093	2.539	2.861	3.579
20	1.325	1.725	2.086	2.528	2.845	3.552
21	1.323	1.721	2.080	2.518	2.831	3.527
22	1.321	1.717	2.074	2.508	2.819	3.505
23	1.319	1.714	2.069	2.500	2.807	3.485
24	1.318	1.711	2.064	2.492	2.797	3.467
25	1.316	1.708	2.060	2.485	2.787	3.450
26	1.315	1.706	2.056	2.479	2.779	3.435
27	1.314	1.703	2.052	2.473	2.771	3.421
28	1.313	1.701	2.048	2.467	2.763	3.408
29	1.311	1.699	2.045	2.462	2.756	3.396
30	1.310	1.697	2.042	2.457	2.750	3.385

Now it is very clear that the result from the table is 1.314

5.0 How to make decision Using Rejection Rule.

To make your decision on whether to reject the null hypothesis or not you have to compare the Zcal and

Zstats. Or you compare the Pvalue and the level of significant.

Decision Using Zcal and Zstats

Rule1: if the alternative hypothesis is written with greater than then reject the null hypothesis if and only if the Zstatistics is greater than the Z-value gotten from the statistical table i.e. the critical value.

Rule2: if the alternative hypothesis is written with less than then reject the null hypothesis if and only if the Zstatistics is less than the Z-value gotten from the statistical table i.e. the critical value.

Rule3: if the alternative hypothesis is written with equal to then Reject the null hypothesis if and only if the absolute value of the Zstatistics is greater than the Z-value gotten from the statistical table i.e. the critical value.

Note: before you do comparison you must let your Z-value from table correspond with the Z-statistic i.e. negate the Z-value whenever your Z-statistics is negative value before you do the comparison.

Now that you have learned all the important procedure let's take a look at some examples

Decision using P-value

The P-value is another approach for reaching a conclusion in hypothesis testing. It is widely reported by most statistical software.

What is P-value?

P-value is the least significant value at which a null hypothesis mu be rejected. It gives the probability of obtaining a sample result that is at least as unlikely as what is observed

P-value = P(z > z∗) - right tailed test

P-value = P(z < z∗) - left tailed test

P-value = 2 × P(z > |z ∗ |) - two tailed test.

Where z∗ is the computed value for the test statistic.

Interpreting the P-values

P-value is used in hypothesis testing to help measure the strength of the evidence against the null hypothesis. The smaller the P-values, the stronger is the evidence against the null hypothesis.

P-value > 0.1, indicates little or weak evidence against H0

0.5 < p-value ≤ 0.1 indicates some evidence against H0

0.01 < p-value ≤ 0.05 indicates moderate evidence against H0

0.001 < p-value ≤ 0.01 indicates strong evidence against H0

P-value < 0.001, indicates very strong evidence against H0

Advantage of P-values

It can be used without statistical table.

By comparing directly with the level of significance (α).

When using or comparing with α, the decision is:

Reject H0, if p-value is < α. Then we say the test is significant!

Do not reject H0, if p-value is > α. Then, we say the test is not significant

Since the sample size is up to 30 then we will need to use Z-table also it must be two tails test and rule3 should be apply for decision since the alternative test is with “not equal to” sign.

Now we need to proceed to step3

Step3

$\boldsymbol{\mu_0 = 100 \quad \sigma = 15 \quad n = 30}\ \bar{x} = 150$

$$Z_{cal} = \frac{(\bar{x}-\mu)}{\frac{\sigma}{\sqrt{n}}}$$

$$Z_{cal} = \frac{(150-100)}{\frac{15}{\sqrt{30}}}$$

Zcal= 50/2.7386

Zcal=18.2574

Step4

α=5% = 0.05 (since the level of significant is not given.)

$Z_{\alpha/2} = Z_{0.05/2} = Z_{0.025}$ (since we are dealing with two tails test)

We need to look for the value of 0.025 from the normal distribution table and that give 1.96

Step5

Comparing the Z_{cal} and Z_{table} we see that $Z_{cal} > Z_{table}$ therefore:

Decision: we reject the null hypothesis.

Conclusion: The result of the test shows that at 5% level of significant the medication is consider to have effect on the IQ. (Since the null hypothesis is rejected)

Example 6.1.2

Past research data from a period of over several years states that the average life expectancy of whales is 85 years. A researcher at a laboratory wishes to test this hypothesis. To that end, they procure a sample of the life spans of 35 whales. If the mean and standard deviation of the 40 whales is 90 and 2.5 respectively is the past research still right at 1% level of significant?

Solution

Step1: The null hypothesis for the researcher will state that ''the average life expectancy of whales is exactly equal to 85 years.''

Alternative hypothesis will read: ''the average life expectancy of whales is not equal to 100 years.''

Mathematically

H_0: $\mu = 85$years

H_a: $\mu \neq 85$years

Step2:

α=0.01 (it is not given in the question as 1%)

Since the sample size is more than 30 then we will need to use Z-table also it must be two tails test since the alternative test is with "not equal to sign".

Step3

$\mu = 85$, $\sigma = 2.5$, $n = 40$, $\bar{x} = 90$

$$Z_{cal} = \frac{(\bar{x}-\mu)}{\sigma/\sqrt{n}} \qquad Z_{cal} = \frac{(90-85)}{2.5/\sqrt{40}}$$

Zcal= 5/0.3952

Zcal=12.6518

Step4

α= 1% = 0.01 (since the level of significant is not given.)

Zα/2=Z0.01/2 = Z0.005 (since we are dealing with two tails test)

We need to look for the value of0.025 from the normal distribution table and that give 2.57

Step5

Comparing the Zcal(12.6518) and Z-table (2.57) we see that Zcal >Ztable which follow rule3 of the Rejection rules, therefore :

Decision: we reject the null hypothesis.

Conclusion

The result above give solid evidence that the average life expectancy of whales is not 85 years at 1% level of significance therefore it can be stated that the past research is no more right

Example 6.1.3

A company producing groundnut chips claims that each nylon contains 30 groundnut chips on average but I have a feeling that it is not up to that so I inspected a package containing 60 nylons. If the mean and standard deviation of the package is 33 and 2 respectively should I sue the company at 99% confidence???

***S*olution**

Null hypothesis*: "The average number of groundnut chips in a nylon is 30."*

Alternative hypothesis: *" the average number of groundnut chips in nylon is less than 30." (Remember I said not up to)*

Mathematically

H_0: $\mu = 30$

Ha: $\mu < 30$

Step2

CL=99% =0.99 (this is what is given)

α=1-CL=1-0.99 =0.01

since the sample size is more than 30 then we will need to use Z_{table}

also it must be one tail test since the alterrnative test is with less than sign.

Step 3

$\mu = 30,\ \sigma = 2, n = 60, \bar{x} = 33$

$$Z_{cal} = \frac{(\bar{x}-\mu)}{\sigma/\sqrt{n}}$$

$$Z_{cal} = \frac{(33-30)}{2/\sqrt{60}}$$

$$Z_{cal} = \frac{3}{0.2582}$$

$$Z_{cal} = 11.618$$

Step4

α= 1% = 0.01

Zα=Z0.01 (since we are dealing with one tails test)

We need to look for the value of 0.01 from the normal distribution table and that gives 2.326

Step5

Comparing the Z_{cal} (11.6189) and Z_{-table} (2.326) we see that Zcal >Ztable which follows rule2 of the Rejection rules because we are using less than sign in alternative hypothesis, therefore:

Decision: we do not reject the null hypothesis.

Conclusion: There is not enough evidence to claim of that the chips on average is not up to 30. Therefore I would be advised not to sue the company.

Example 6.1.4

A researcher examined 200 students on academic performances. He discovered that the average score of the students in major courses is 68 with a variance of 8. It is believed that the average score of these students in major courses is 65 but the researcher thinks that is too high. Test the hypothesis for this claim at 1% level of significant.

Solution

The initial belief will be our null hypothesis i.e

Null: the average score of students in major courses is 65

Alternative: the average score of these students in major courses is less than 65.

Notice since it is not stated what the researcher thinks "that is too high, meaning it should be less than 65.

Mathematically

H_0: $\mu = 65$

Ha: $\mu < 65$

Step2

α=1% = 0.01

Since the sample size is more than 30 then we will need to use Z-table also it must be one tails test since the alternative test is with less than to sign.

step3

$\mu = 65$, $\sigma = 8$, $n= 200$, $\bar{x}= 68$

$$Z_{cal} = \frac{(\bar{x} - \mu)}{\sigma/\sqrt{n}}$$

$$Z_{cal} = \frac{(68 - 65)}{8/\sqrt{200}}$$

Z_{cal}= 3/1.7889

Z_{cal}=5.3033

Step4

α= 1% = 0.01 (since the level of significant is not given.)

$Z\alpha=Z0.01$ (since we are dealing with one tails test)

We need to look for the value of 0.01 from the normal distribution table and that gives 2.326

Step5

Comparing the Zcal (5.3033) and Z-table (2.326) we see that Zcal >Ztable which follows rule2 of the Rejection rules because we are using less than sign in alternative hypothesis, therefore:

Decision: we do not reject the null hypothesis.

Conclusion: we do not have sufficient evidence to state that the average score of the students in major courses is less than 65.

Note: we can also Use Pvalue to make decision too like b

Since our Zcal= 5.3033. we check for the Pvalue from the table

P (Z<5.3033) =0.000000287

Now comparing the P_{value} and the level of significant.

(0.000000287) P_{value} < alpha (0.01) Hence we do not reject the null hypothesis.

Example 6.1.5

The mean life of a sample of 13 fluorescent bulbs produced by a company is found to be 497days with standard deviation of 47days. Test the hypothesis that the mean lifetime of the bulbs produced by the company is 506days at 1% level of significance.

Solution

The hypothesis statement here is clear. The question state we should test that mean lifetime is 506days that means we will have the following as our hypothesis.

Null: the mean lifetime of the bulbs produced by the company is 506days

Alternative: the mean lifetime of the bulbs produced by the company is not 506days

Mathematically

H_0: $\mu = 506$

H_a: $\mu \neq 506$

Step2

α=1% = 0.01

Since the sample size (13) is less than 30 then we will need to use T-table also it must be two tails test since the alternative test is with no equal to sign.

step3

$\mu = 506$, $\sigma = 47$, $n = 13$, $\bar{x} = 497$

$$t_{cal} = \frac{(\bar{x} - \mu)}{\sigma/\sqrt{n}}$$

$$t_{cal} = \frac{(90 - 85)}{2.5/\sqrt{40}}$$

$t_{cal} = (497-506)/(47/\sqrt{13})$

$t_{cal} = -9/13.035$

$t_{cal} = -0.69$

Step4

α= 1% = 0.01 (since the level of significant is not given.)

$T\alpha/2$=T0.01/2=0.005 (since we are dealing with two tails test)

Now To get the value from the table we need the degree of freedom and the level of significance

Recall that df = n-1

df=13-1 =12

We need to look for the value of 12 under 0.005 or 0.5% from T-distribution table and that gives 4.221

Step5

Comparing the Zcal (0.69) and Z-table (3.055) we see that Zcal < Ztable applying rule3 of the Rejection rules because we are using not equal sign in alternative hypothesis, therefore:

Decision: we do not reject the null hypothesis.

Conclusion: we do not have sufficient evidence to state that the mean lifetime of the bulbs produced by the company is not 506days.

Note: we compare with 0.69 and not -0.69 as it is stated in rule3 that we should compare with absolute value

Example 6.1.6

The farmer claims that the weight of the yam on his farm on average is not less than 0.5 kg. A buyer took a random sample of 14 yams from the farm and observed. After a few observations, the buyer believes it shouldn't up to 0.5kg. if the weight of the samples are 0.5kg, 0.4kg, 0.55kg, 0.3kg, 0.45kg, 0.45kg, 0.35kg, 0.6kg, 0.5kg, 0.55kg, 0.4kg, 0.5kg, 0.4kg and 0.4kg. Carry out a hypothesis test on the claim.

Solution

Remember the farmer claim not less than 0.5 i.e it is either 0.5 or greater this will be our null hypothesis and the alternative will be that it is less than that.

Null: the weight of the yam on his farm on average is not less than 0.5 kg

Alternative: the weight of the yam on his farm on average is less than 0.5 kg

Mathematically

H_0: $\mu \geq 0.5$kg

Ha: $\mu < 0.5$kg

Step2

α=5% = 0.05

Since the sample size (14) is less than 30 then we will need to use T-table also it must be one tail test since the alternative test is with less than sign.

step3

Here the mean is not given but we are given the data itself hence we need to calculate the mean and standard deviation by ourselves.

For Mean

$$\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n}$$

$$\bar{x} = \frac{0.5+0.4+0.55+0.3+0.45+0.45+0.35+0.6+0.5+0.5+0.4+0.5+0.4+0.4}{14}$$

$$\bar{x} = \frac{6.3}{14} = 0.45\text{kg}$$

For standard Deviation

	x	x-x	$(x-x)^2$
	0.5	0.05	0.0025
	0.4	-0.05	0.0025
	0.55	0.1	0.01
	0.3	-0.15	0.0225
	0.45	0	0
	0.45	0	0
	0.35	-0.1	0.01
	0.6	0.15	0.0225
	0.5	0.05	0.0025
	0.5	0.05	0.0025
	0.4	-0.05	0.0025
	0.5	0.05	0.0025
	0.4	-0.05	0.0025
	0.4	-0.05	0.0025
Total	6.3	0	0.085

$$s = \sqrt{\frac{\sum(x-\bar{x})^2}{n}} = \sqrt{\frac{0.085}{14}}$$

$$s = \sqrt{0.0061}$$

s=0.078

S= 0.0781.

Now our parameters has been completed.

μ = 0.5kg σ = 0.0781, n= 14, x̄= 0.45kg

$$Z_{cal} = \frac{\bar{\bar{x}} - \mu}{\frac{\sigma}{\sqrt{n}}}$$

$$Z_{cal} = \frac{\overline{0.45} - 0.5}{\frac{0.0781}{\sqrt{14}}} = -\frac{0.05}{0.0209}$$

Zcal=- -2.3923

Step4

α= 5% = 0.05 (since the level of significant is not given.)

T0.05 (since we are dealing with two tails test)

Now To get the value from the table we need the degree of freedom and the level of significance

Recall that df = n-1

df =14-1 =13

We need to look for the value of 13 under 0.05 or 5% from T-distribution table and that gives 4.221

Step5

Comparing the Zcal (0.69) and Z-table (1.771) we see that Zcal < Z-table. By applying rule2 of the Rejection rules therefore:

Decision: we do not reject the null hypothesis.

Conclusion: we do not have sufficient evidence to state that the mean lifetime of the bulbs produced by the company is not 506days.

Note: we compare with 0.69 and not -0.69 as it is stated in rule3 that we should compare with absolute value.

Example6.1.7

Doctor believes that the average teen sleeps on average no longer than 10hrs per day. A researcher believes that teens on average sleep longer. To prove the claim the researcher sample 40 teens and found out that they sleep for 9hrs on average and their standard deviation is 2. Is the doctor claim right at 1% level of significant??

Solution

Null: the average teen sleeps on average no longer than 10hrs per day

Alternative; the average teen sleeps on average no longer than 10hrs per day

Notice since it is not stated that “no longer than 10”, meaning it should be less than or equal to 10.

Mathematically

H_0: μ ≤ 10hrs per day

H_a: μ > 10hr per day

Step2

α=1% = 0.01

Since the sample size is more than 30 then we will need to use Z-table also it must be one tails test since the alternative test is with greater than to sign.

step3

μ = 10, σ = 2, n= 40, x̄= 9

$$Z_{\text{cal}} = \frac{\bar{x}-\mu}{\frac{\sigma}{\sqrt{n}}} \qquad Z_{\text{cal}} = \frac{9-10}{\frac{2}{\sqrt{40}}}$$

$Z_{\text{cal}} = \frac{-1}{0.3162}$= -3.1626

Z_{cal}=-3.1626

Step4

α= 1% = 0.01 (since the level of significant is not given.)

$Z\alpha=Z_{0.01}$ (since we are dealing with one tails test)

We need to look for the value of 0.01 from the normal distribution table and that gives 2.326

Step5

Comparing the Zcal (3.1626) and Z-table (2.326) we see that $Z_{cal} > Z_{table}$ using Rejection rules

Decision: we do not reject the null hypothesis.

Conclusion: we do not have sufficient evidence to state that the average score of the students in major courses is less than 65

6.2 Testing for proportion of a population

Example 6.2.1

School board claims that at least 60% of student brings a phone to school. A teacher believes this number is too high and randomly selected 25students to test at a level of significance of 0.02 if 15 of them brings phone to school. Does the school claim likely to be truth at 99% confident??

Solution

Here the percentage given is our proportions i.e that 60% means 60/100 which is 0.6. The following are our hypothesis

Null : at least 60% of student brings a phone to school i.e starting from 60% upward which means it can be 60% or greater

Alternative: it is less than 60% (not at least 60% of student brings a phone to school)

H_0: $P \geq 0.6$
H_a: $P < 0.6$

Step2

α=1-0.99=0.01 (whenever it is not given we use this)

From the question, we have n=25 and α=0.05

To determine the distribution to be used, we check n×P and n×(1−p).

For the value of p, we use the claim from the null hypothesis (p=0.6).

$n\times p=25\times 0.6=15>5$

$n\times(1-p)=25\times(1-0.6)=10>5$

Since both $n\times p\geq 5$ and $n(1-p)\geq 5$ we use a normal distribution to calculate the critical value.

also it must be one tails test since the alternative test is with less than to sign.

Now we need to proceed to step3

Step3

n=25, P=15/25 =0.6(since 15 brings phone out of 25), α=0.01, Po=0.6

$$\mathbf{Z}_{\text{cal}} = \frac{\text{P} - \text{P}_\text{o}}{\sqrt{\frac{\text{P}_\text{o}(1 - \text{P}_\text{o})}{\text{n}}}}$$

$$\mathbf{Z}_{\text{cal}} = \frac{0.6-0.6}{\sqrt{\frac{0.6(1-0.6)}{25}}} \qquad \mathbf{Z}_{\text{cal}} = \frac{0}{\sqrt{\left(0.6*\frac{0.4}{25}\right)}} \qquad Z_{cal}=0$$

Step4

α= 1% = 0.01

We need to look for the value 0.01 from the tabe and that gives 2.325

Step5

Comparing the Zcal(0) and Ztable(2.325) we see that Zcal <Ztable By applying rule1 of the Rejection rules

Decision: we do not reject the null hypothesis.

Conclusion: at 99% confidence level, we don’t have enough evidence to state that the school board claim is unlikely to be truth. In fact seeing the result of the test there is high chances that the school claims is truth.

Example 6.2.2

A new flu vaccine claims to prevent a certain type of flu in 75% of people who are vaccinated. If out of 50 people tested 40 of them were prevented is this claim too high at 2% level of significance??

Solution

Step1.

Null: it prevents 75%

Alternative: 75% is too high i.e. it should be less than 75%

Mathematically

H_0: P = 0.75

H_a: P < 0.75

Step2

α=0.5 (whenever it is not given we use this)

To determine the distribution to be used, we check n×P and n× (1−p).

For the value of p, we use the claim from the null hypothesis (p=0.75).

$n \times p=50\times0.75=37.5>5$

$n \times (1-p) =50\times(1-0.75)=12.5>5$

Since both $n\times p\geq5$ and $n (1-p) \geq 5$ we use a normal distribution to calculate the critical value.

Also it must be one tails test since the alternative test is with less than to sign.

Now we need to proceed to step3

Step3

$P_o = 0.75$, $n = 50$, $P=40/50=0.8$

Zcal= $(P- P_o)/ \sqrt{(P_o(1-P_o)/n)}$

$Z_{cal}= (0.8-0.75)/\sqrt{(0.75(1-0.75)/50)}$

$Z_{cal}= (0.05)/\sqrt{(0.75*0.25/50)}$

Zcal= 0.05/0.0612

Z_{cal}=0.8165

Step4

α= 2% = 0.02 (since the level of significant is not given.)

$Z\alpha=Z_{0.02}$ (since we are dealing with one tails test)

We need to look for the value of 0.02 from the normal distribution table and that give 2.05

Step5

Comparing the Z_{cal} (0.8165) and Z_{table} (2.05) we see that $Z_{cal} < Z_{table}$ which follows rule2 of the Rejection rules therefore:

Decision: we reject the null hypothesis.

Conclusion: There is enough evidence that the claim is high.

Example 6.2.3

Prof Kuye believes that 40% of first-time brides in the Nigeria are younger than their grooms. He performs a hypothesis test to determine if the percentage is the same or different from 40%. He samples 100 first-time brides and 50 reply that they are younger than their grooms. Use a 5% significance level.

Solution

H_0: 40% of first-time brides are younger than the groom

H_a: it is not 40% of first-time brides that are younger than the groom

H_0: $P = 40\%$

Ha: $\neq 40\%$

Step2: $\alpha = 0.05$

To determine the distribution to be used, we check n×P and n×(1−p).

From the question, we have n=100

For the value of p, we use the claim from the null hypothesis (p=0.4).

$n \times p = 100 \times 0.4 = 40 > 5$

$n \times (1 - p) = 100 \times (1 - 0.4) = 60 > 5$

Since both $n \times p \geq 5$ and $n(1 - p) \geq 5$ we use a normal distribution table will be used.

n=25, P=15/25 =0.6 (since 15 brings phone out of 25)

$$\mathbf{Z}_{\text{cal}} = \frac{P - P_o}{\sqrt{\frac{P_o(1 - P_o)}{n}}}$$

$$\mathbf{Z}_{\text{cal}} = \frac{0.5 - 0.4}{\sqrt{\frac{0.5(1 - 0.5)}{100}}}$$

$$\mathbf{Z}_{\text{cal}} = \frac{0.1}{\sqrt{\left(0.5 * \frac{0.5}{100}\right)}}$$

$Z_{cal} = 2$

α= 5% so, $\alpha/2$= 0.05/2 =0.25

We need to look for the value 0.025 from the table and that gives 1.96

Step5

Comparing the Zcal (2) and Z_{table} (1.96) we see that $Z_{cal} > Z_{\textbf{table}}$ By applying rejection rule

We need to reject the null

Decision: we reject the null hypothesis

Conclusion: . At the 5% significance level there is not enough evidence to suggest that the proportion of first-time brides that are younger than the groom is different from 50%.

Note: It should be noted that we can also make the decision using P-value like below.

We check the P(Z=2) from the table we get 0.0228. Since it is two tail test we will multiply the result by 2. So Pvalue= 0.0228*2 = 0.0456.

Now comparing the Pvalue and level of significant we see that **Pvalue (0.0456) < α (0.05)** Hence **we reject the Null hypothesis.**

Example 6.2.4

A group of researchers claimed that not less than 80% of adults in a community have BMI greater than 28kg/m2. A young Professor believes that this is not true and sampled 50peoples in the community then take their measurement. The following table shows the summary of all the measurements.

BMI(kg/m2)	No of People
0 – 13	10
14 – 27	10
28 – 41	30

Does the sample support the claim of the researchers at 1% alpha level?

Solution

Hypothesis: H_0: $P \geq 80\%$, H_a: $P < 80\%$

We need to get the proportion of adults that have BMI greater than 28kg/m2 from the table.

No of People greater than 28kg/m2= 30

Proportion of People greater than 28kg/m2= 30/50 =0.6

Step2: $\alpha=0.05$

To determine the distribution to be used, we check $n\times P$ and $n\times(1-p)$.

From the question, we have n=100

For the value of p, we use the claim from the null hypothesis (p=8%=0.8).

$n\times p=50\times 0.8=40 > 5$

$n \times(1-p)=50\times(1-0.8)=10>5$

Since both $n\times p\geq 5$ and $n(1-p)\geq 5$ we use a normal distribution table will be used.

Step3

n=50, P=30/50 =0.6 (since 15 brings phone out of 25)

$$\mathbf{Z}_{\text{cal}} = \frac{P - P_o}{\sqrt{\frac{P_o(1 - P_o)}{n}}} \qquad = \frac{0.6 - 0.8}{\sqrt{\frac{0.8(1 - 0.8)}{100}}}$$

$$\mathbf{Z}_{\text{cal}} = \frac{-0.2}{\sqrt{\left(0.8 * \frac{0.2}{100}\right)}} = -5$$

Step4:

α= 1% so, α=0.01

We need to look for the value 0.025 from the table and that gives 1.96

Step5

Comparing the Zcal (-5) and Z_{table} (-2.325) we see that $Z_{cal} < Z_{\mathbf{table}}$ By applying rejection rule

We need to reject the null

Decision: we reject the null hypothesis

Conclusion: At the 1% significance level there is not enough evidence to suggest that the proportion of first-time brides that are younger than the groom is different from 50%.

Making decision using P-value

We check the P(Z<-5) from the table we get 000000287, So P_{value} = 000000287

Now comparing the Pvalue and level of significant we see that **Pvalue (**000000287**) < α (0.01)** Hence **we reject the Null hypothesis.**

Example 6.2.5

Marketers believe that 90% of UI student own an android phone. A cell phone manufacturer believes that number is actually lower. In a sample of 400 students, 87% own an android phone. At the 1% significance level, determine if the proportion of students that own an android phone is lower than the marketers' claim.

Solution

Step1: writing hypothesis:

H_0: $P \geq 90\%$,

H_a: $P < 90\%$

Step2: α=0.05

To determine the distribution to be used, we check n×P and n×(1−p).

From the question, we have n=500

For the value of p, we use the claim from the null hypothesis (p=90%=0.9).

$n \times p = 400 \times 0.9 = 360 > 5$

$n \times (1-p) = 400 \times (1-0.9) = 40 > 5$

Since both n×p≥5 and n(1−p)≥5 we use a normal distribution table will be used.

Step3

n=500, P=87% =0.87 P_0=0.9

$$\mathbf{Z}_{cal} = \frac{P - P_o}{\sqrt{\frac{P_o(1-P_o)}{n}}} \qquad = \frac{0.87 - 0.9}{\sqrt{\frac{0.9(1-0.9)}{400}}}$$

$$\mathbf{Z}_{cal} = \frac{-0.03}{\sqrt{\left(0.9 * \frac{0.1}{400}\right)}} = -2$$

Step4:

α= 1% so, α=0.01

Making decision using P-value

We check the P (Z=-2) from the table we get 0.0228, So P_{value} = 0.0228

Now comparing the P_{value} and level of significant we see that **(**000000287**) $P_{value} < \alpha$ (0.01)** Hence **we reject the Null hypothesis**

Decision: We reject the null hypothesis

Conclusion: At the 1% significance level there is enough evidence to suggest that the proportion of students that own an android phone is lower than the marketers' claim.

Exercise

1. A local maternity center claimed that 98% of children of antenatal women usually survived in their center. To test the claim a researcher take a sample of twelve months from the maternity center that discovered that there are 47 living children of two hundred and fifty antenatal woman. At 5% level of significant what can you say about the local maternity center?
2. A recent study on the safety of airplane drinking water that was conducted by the U.S. Environmental Protection Agency (EPA). A study found that out of a random sample of 316 airplanes tested, 40 had coliform bacteria in the drinking water drawn from restrooms and kitchens. As a benchmark comparison, in 2003 the EPA found that about 3.5% of the U.S. population have coliform bacteria-infected drinking water. The question of interest is whether, based on the results of this study, we can conclude that drinking water on airplanes is more contaminated than drinking water in general
3. Data from the Center for Disease Control estimates that about 30.4% of American teenagers were overweight in 2008. The definition of overweight is a body mass index (BMI) of over 25. The percentage was very similar for boys and girls. A professor in public health at a major university wants to determine whether the proportion has changed since 2008. He samples 800 randomly selected incoming freshman at universities around the country. Using the BMI measurements, he finds that 210, or about 26%, of them are overweight. The professor tests the hypotheses H_0: $p = 0.304$ versus Ha: $p \neq 0.304$. The P-value is about 0.011. If the professor uses a significance level of 0.05, what conclusion can he draw?

F.3 Testing for proportion of a population

Example 1

In a simple random sample of 600 students taken from a university, 400 are found to be smokers. In another simple random sample of 900students from another university 450 are smokers Do the data indicate that there is a significant difference in the habit of smoking in the two universities.

Solution

Here we are dealing with proportion from two different population, It is important to note that whenever two proportion has no significance difference they must be the same i.e. $P_1 = P_2$

Step1.

Null: there is no significant difference in the habit of smoking in the two universities i.e. $P_1 = P_2$

Alternative: there is a significant difference in the habit of smoking in the two universities i.e $P_1 \neq P_2$

Mathematically

H_0: $P_1 = P_2$

Ha: $P_1 \neq P_2$

Step2

α=0.5 (whenever it is not given we use this)

Since the sample size is up to 30 then we will need to use Z-table also it must be one tails test since the alternative test is with less than to sign.

Step3

N_1=600, smoker1=400, P_1=400/600=0.6667 (from the first population)

N_2=900, smoker2=450, P_2=450/900=0.5 (from the second population)

Since we are dealing with two populations given the proportion we need to use the fourth formula stated above.

Firstly we have to calculate the P

$$P = \frac{n_1 * P_1 + n_2 * P_2}{n_1 + n_2} \qquad P = \frac{600 * 0.6667 + 900 * 0.5}{600 + 900}$$

$$P = \frac{600*0.6667+900*0.5}{600+900} \qquad P=0.5667$$

$$Z_{cal} = \frac{P_1 - P_2}{\sqrt{\left(P\ (1-P)\left(\frac{1}{n_1}+\frac{1}{n_2}\right)\right)}}$$

$$Z_{cal} = \frac{0.6667-0.5}{\sqrt{\left((0.5667*(1-0.5667)\left(\frac{1}{600}+\frac{1}{900}\right)\right)}}$$

$$Z_{cal} = \frac{0.1667}{\sqrt{\left((0.5667*0.433)\left(\frac{1}{600}+\frac{1}{900}\right)\right)}}$$

$$Z_{cal} = \frac{0.1667}{\sqrt{\left((0.5667*0.433)(0.0017+0,0011)\right)}}$$

$$Z_{cal} = \frac{0.1667}{\sqrt{\left((0.5667*0.433)(0.0028)\right)}}$$

$$Z_{cal} = \frac{0.1667}{\sqrt{0.2454*0.0028}}$$

$$Z_{cal} = \frac{0.1667}{0.0262}$$

$$Z_{cal} = 6.36$$

Step4

α= 5% = 0.05 (since the level of significant is not given.)

$\frac{\alpha}{2} = \frac{0.05}{2} = 0.025$ (Since we are dealing with two tails test)

We need to look for the value of 0.025 from the normal distribution table and that give 1.96

Step5

Comparing the $Z_{cal(0.8165)}$ and $Z_{table(1.96)}$ we see that Zcal >Ztable which follows rule3 of the Rejection rules therefore :

Decision: we reject the null hypothesis.

Conclusion: at 5% level of significant we can say that there is enough evidence to conclude that there is a significant difference in the habit of smoking between the two universities.

Example 6.3.2

A health magazine claims that marriage status is one of the most telling factors for a person's happiness. Use the table below to test the claim that the percent of married people that are unhappy is lower than the percent of single or divorced people that are unhappy. The data was collected randomly. (Use a 10% significance level.)

	Married	Single or Divorced
Happy	75	90
Unhappy	125	110
Total	200	200

A tattoo magazine claimed that the percent of men that have at least one tattoo is greater than the percent of women with at least one tattoo. Test this claim with the following sample data. A random sample of 794 women found that 137 of them had at least one tattoo. A random sample of 502 men found that 110 of them had at least one tattoo. (Use a 5% significance level.)

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Data from the Center for Disease Control estimates that about 30.4% of American teenagers were overweight in 2008. The definition of overweight is a body mass index (BMI) of over 25. The percentage was very similar for boys and girls.

A professor in public health at a major university wants to determine whether the proportion has changed since 2008. He samples 800 randomly selected incoming freshman at universities around the country. Using the BMI measurements, he finds that 210, or about 26%, of them are overweight.

The professor tests the hypotheses H0: $p = 0.304$ versus Ha: $p \neq 0.304$. The P-value is about 0.011. If the professor uses a significance level of 0.05, what conclusion can he draw?

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