Hash Functions, Proofs of Work, Blockchains and Decentralization

CMSC 23280/ECON 23040, Autumn 2018
Lecture 2

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University of Chicago

Class logistics

1. Website for weeks 1-5:
   https://people.cs.uchicago.edu/~davidcash/23280-winter-19/
   (Or: https://david.cash forwards to my page, then click link)

2. Get on Piazza if you were not added automatically

3. Request access to Canvas if you were not added automatically

4. Assignment 1 will be out tonight and due in one week

Lecture 2 Outline

1. Cryptographic Hash Functions
   - Blockchains
   - Proofs of Work
2. Putting DCash “on the blockchain”, with an authority
3. The idea of decentralization
4. Decentralized DCash with an Angel
5. Decentralized DCash via proofs-of-work

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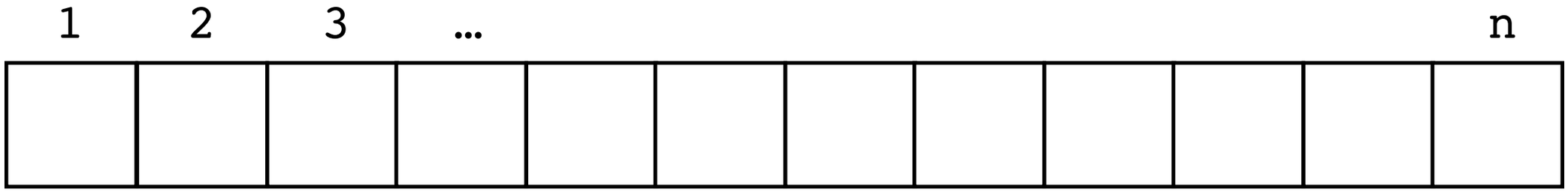

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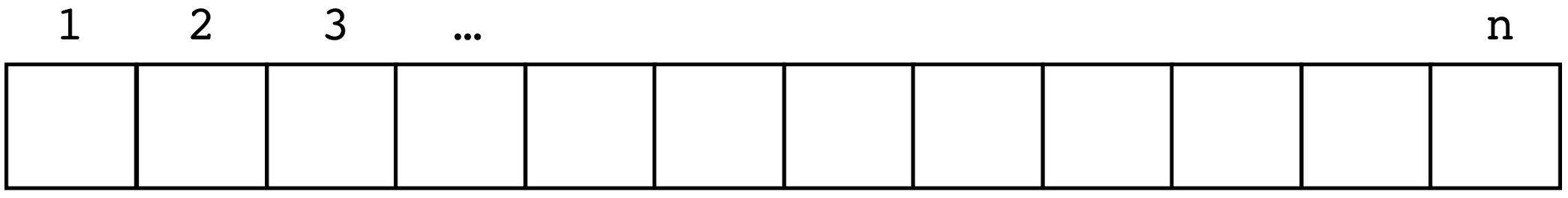

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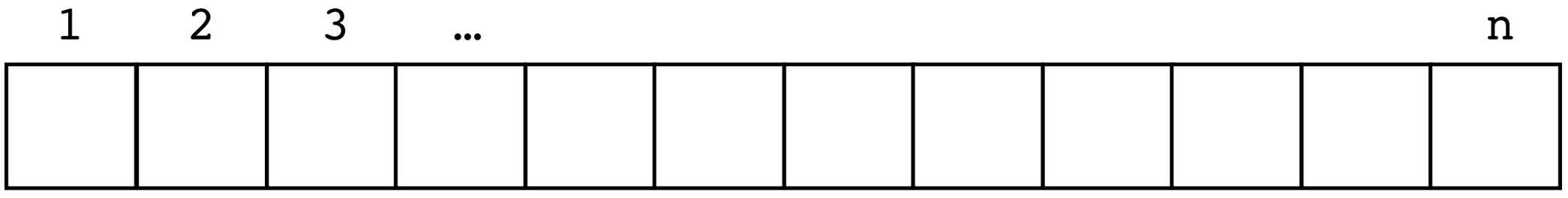

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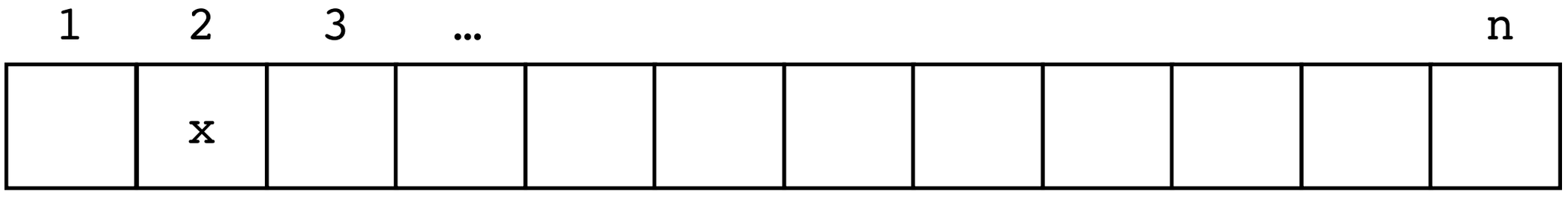

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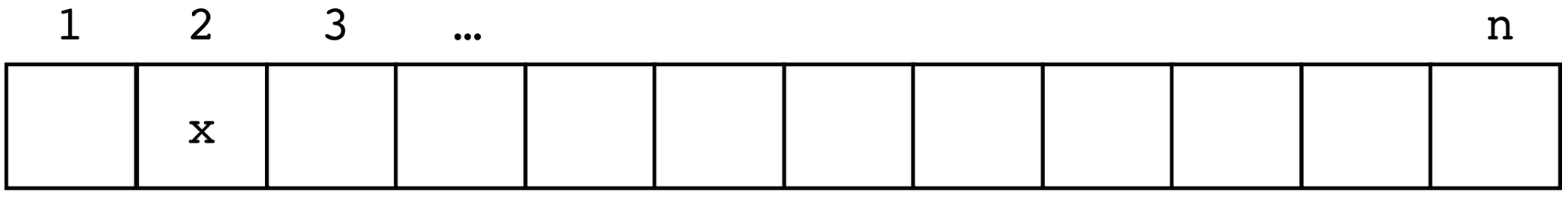

- Simple hash functions work, like `h(x)=x mod n` (or slightly more complicated)
- Collisions happen: `x≠x'` but `h(x)=h(x')`, and must be handled by table.

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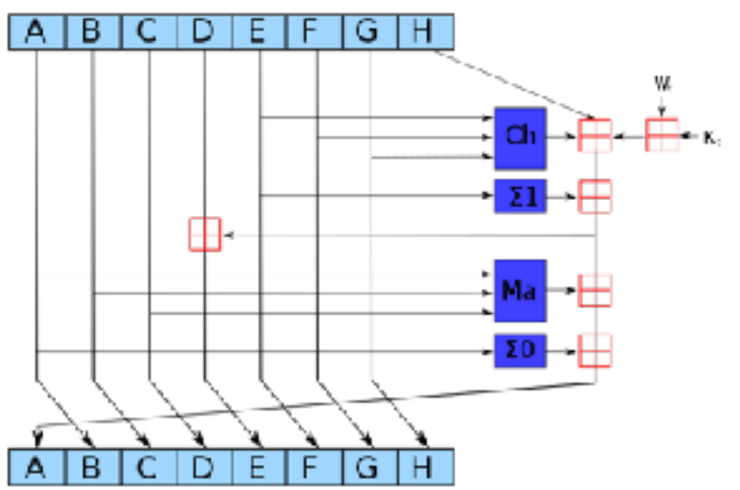

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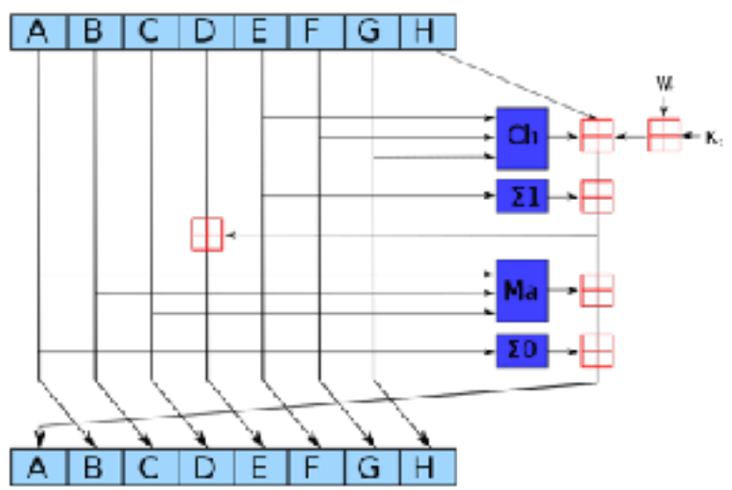
- … are much more resilient to adversarial inputs



Aside: Huge, Astronomical, and Depressingly Large

# Steps	Who can do that many?
2^{30}	Your laptop (one day)
2^{56}	Strong computer with GPUs
2^{80}	All computers on Bitcoin network in a few days
2^{128}	US Gov in ??? years, or very large quantum computer*
2^{256}	Nobody ever?
2^{1024}	Nobody ever?

*Not directly comparable but this is an estimate of equivalent power. Quantum computers are most effective against public-key crypto like digital signatures, but they also speed up attacks on hash functions.

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- After the market closes, you can only reveal the `x` you chose. Finding another `x` to change your prediction amounts to finding a collision.

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You can ignore the alphabet soup of hash function names. Bitcoin uses SHA256.

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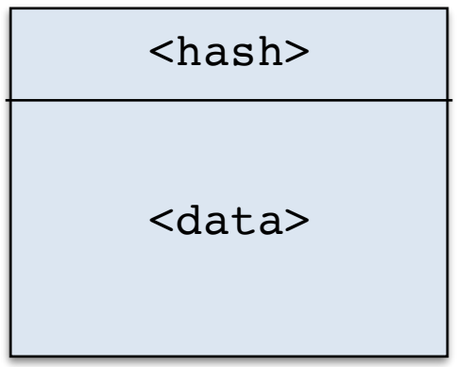

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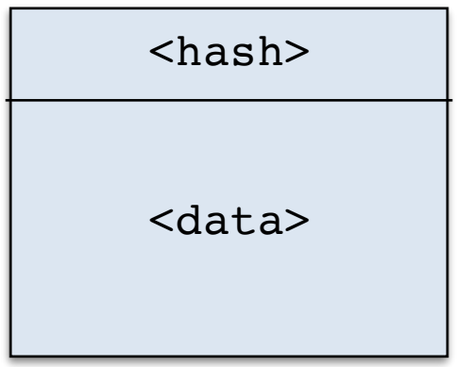

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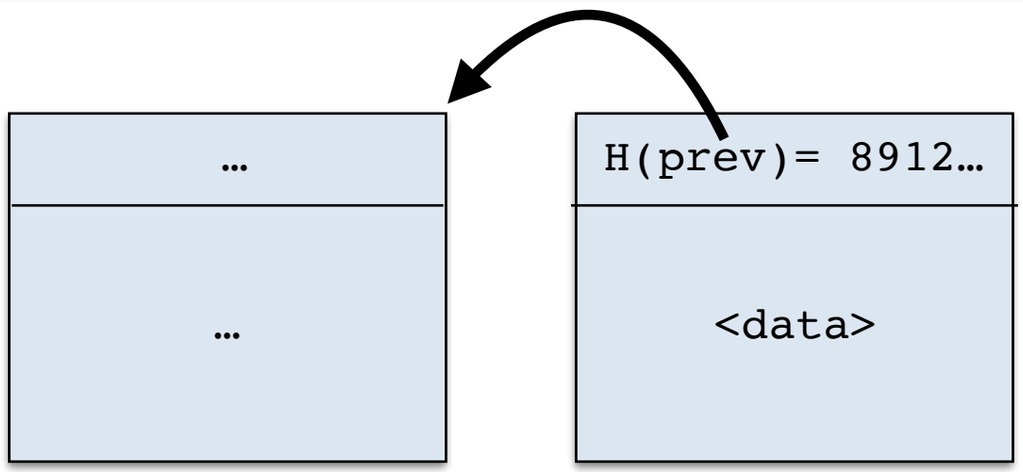

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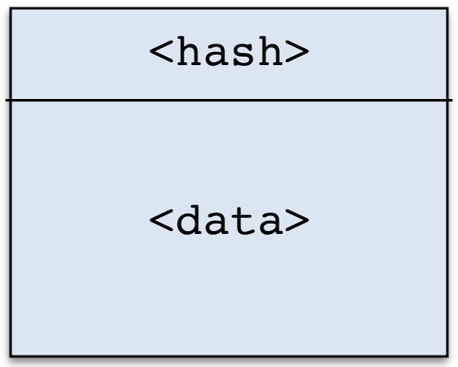

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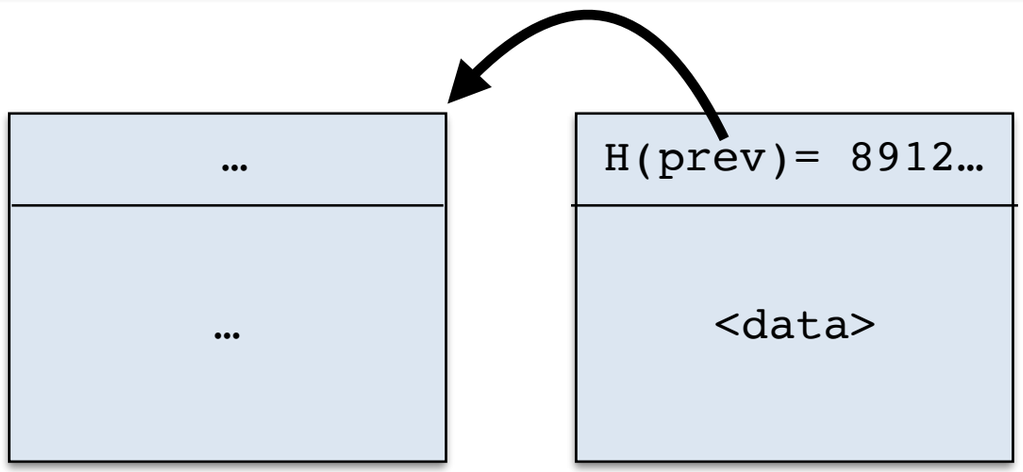

- Note: Input to H should include *both* fields of previous block (hash *and* data)

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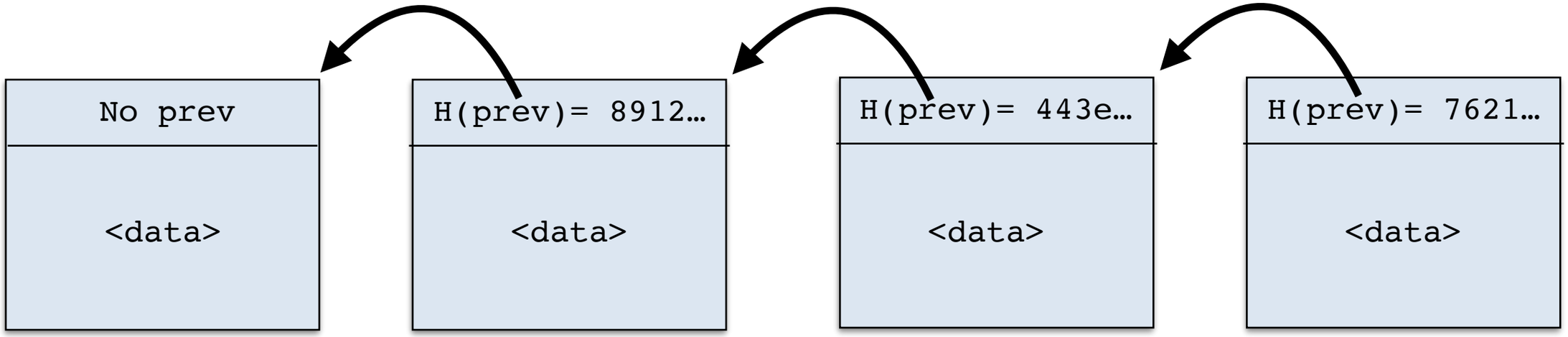

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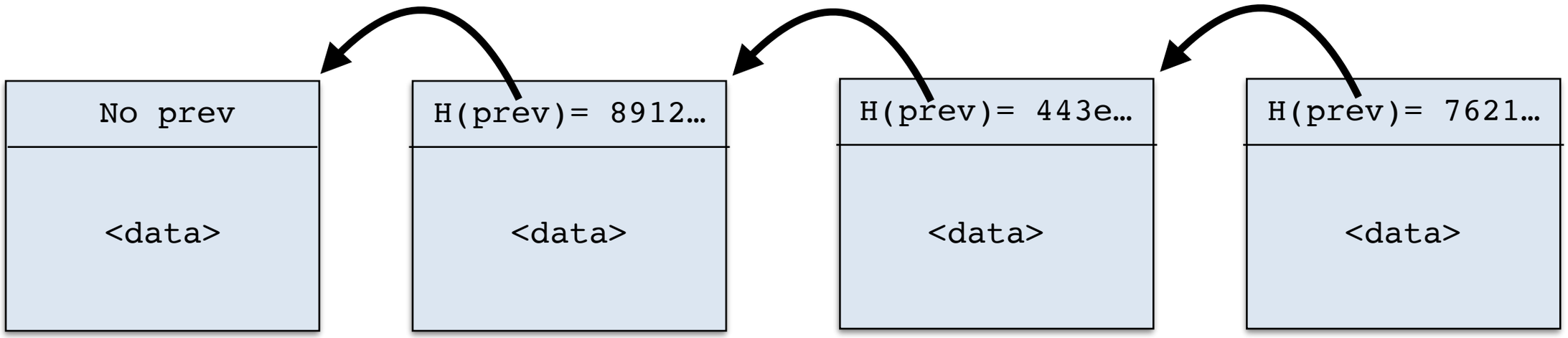
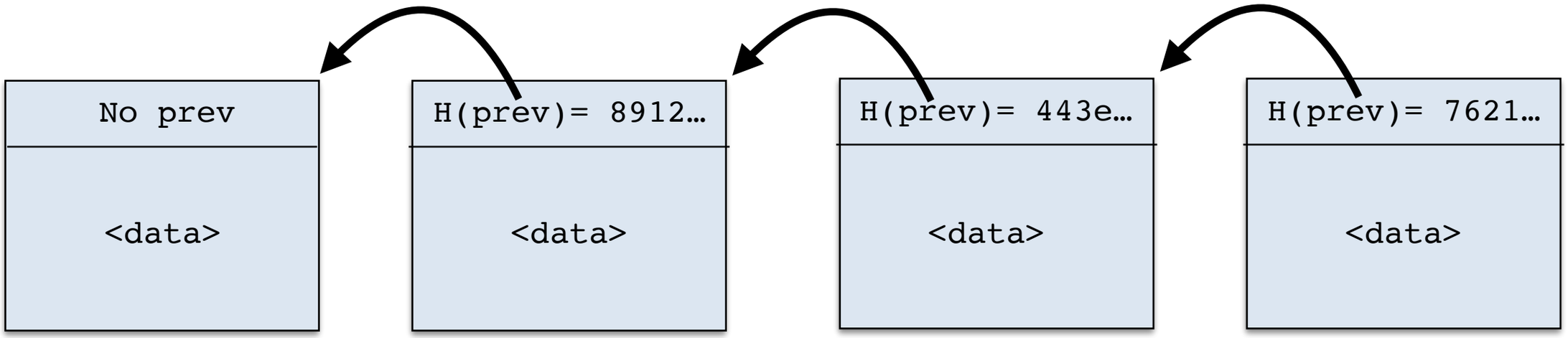


Given the last block, it's easy to add a block to a blockchain. Just evaluating `H` on the last block and put the result in your new block's hash field.

Blockchains Integrity

Blockchain integrity property: If you know the last hash field (`7612…` in example), you can check integrity of data in entire blockchain



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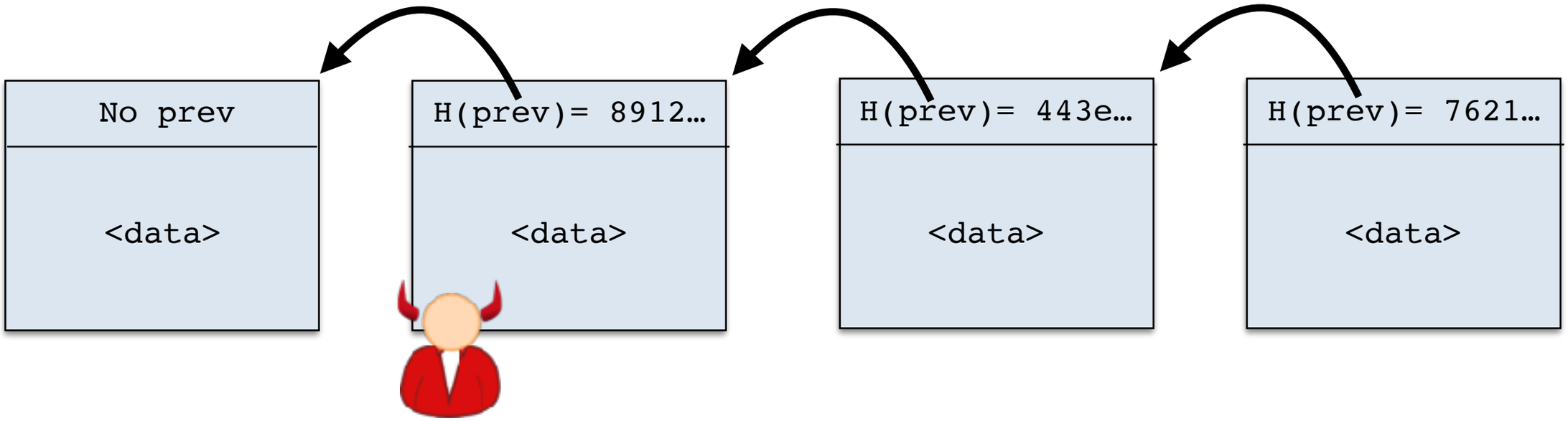

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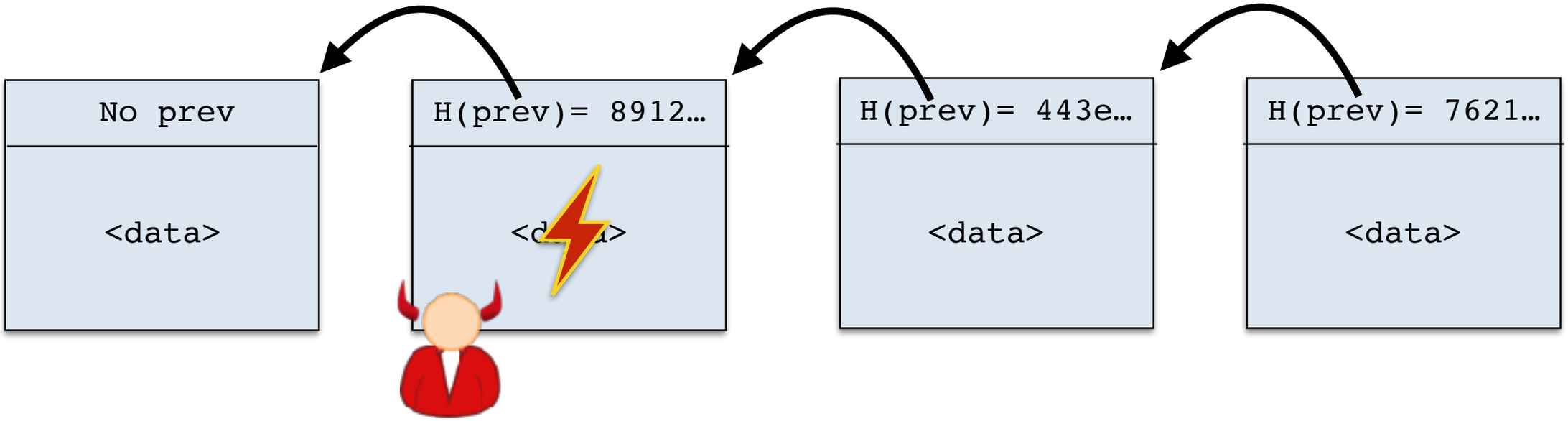

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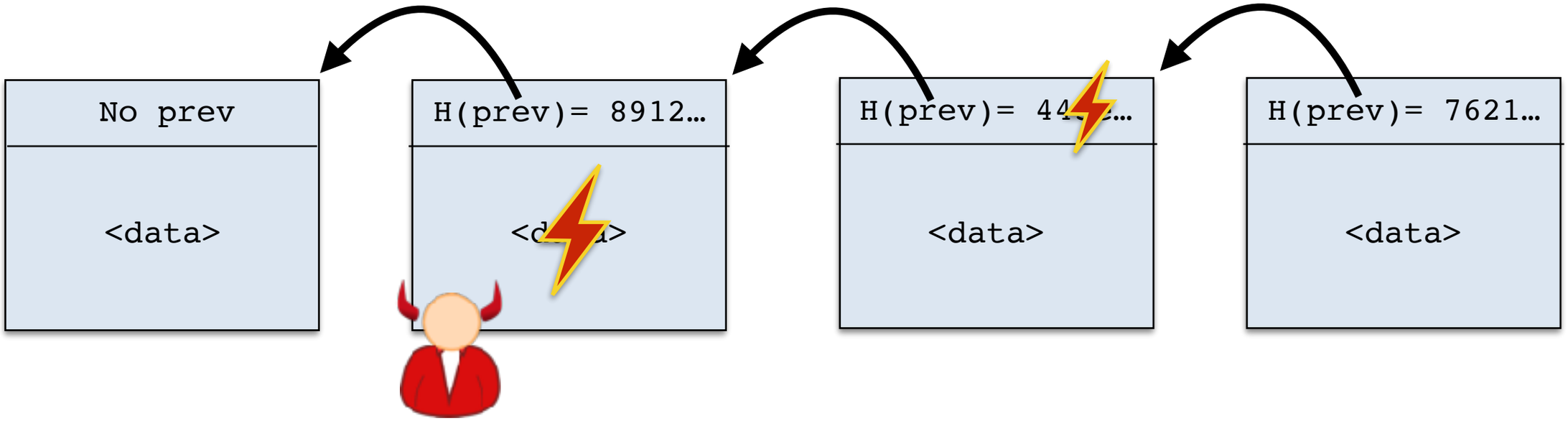

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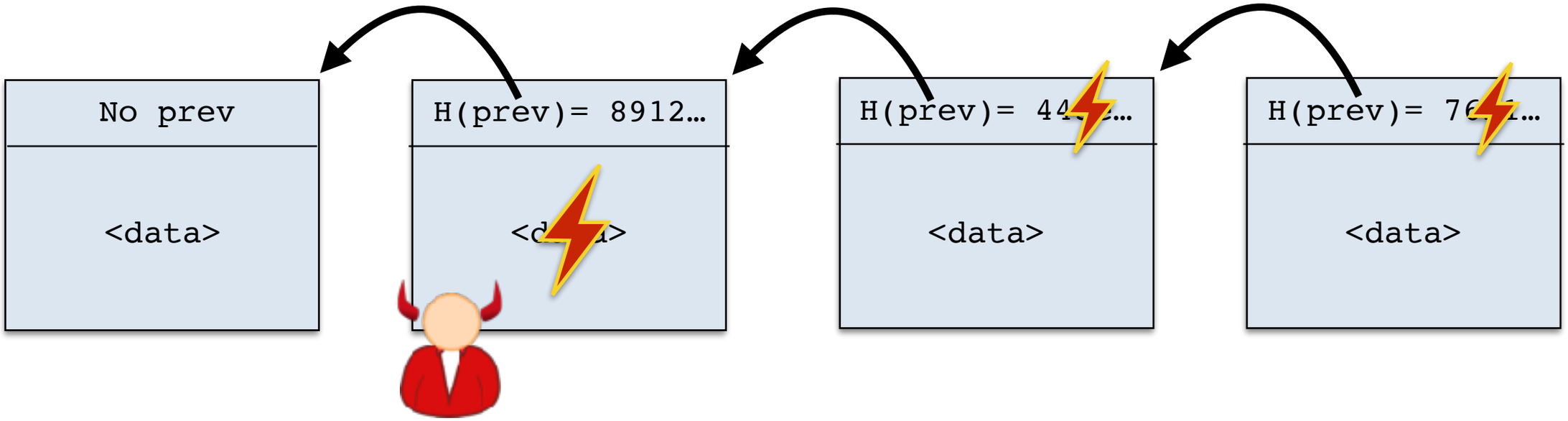

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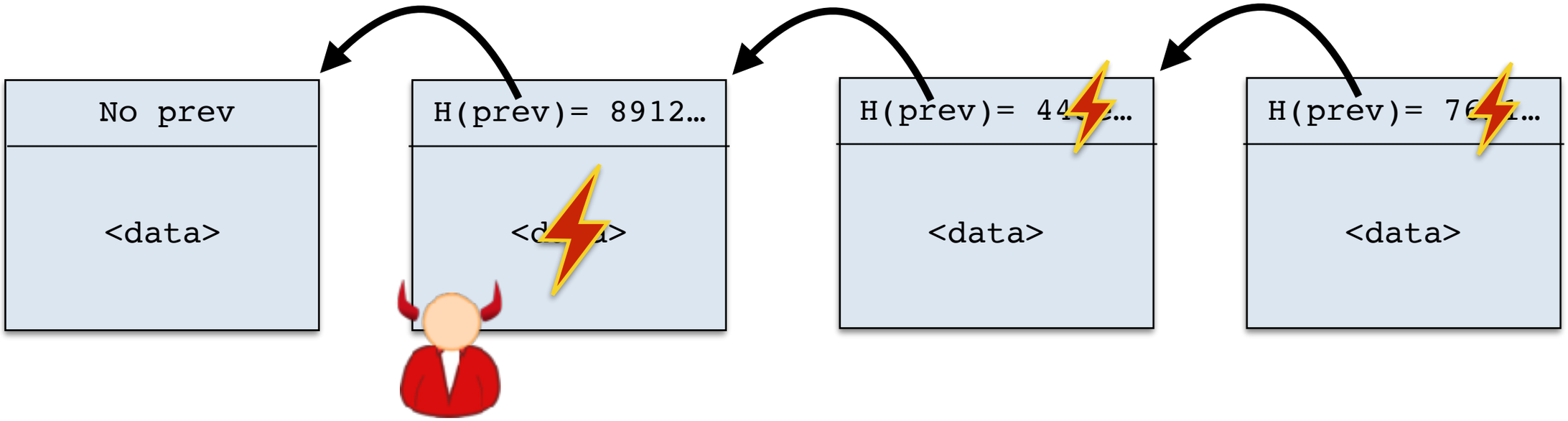

Theorem that can be formalized proved: Changing/deleting/adding a block without changing the final hash requires finding a collision in `H` (which we believe is infeasible).

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   - **Proofs of Work**

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- Probability `1/2` that it starts with at least one zero, `1/4` that it starts with at least two zeros, `1/8` for three, `1/16` for four, etc
- Once `z≈80` this probability very small by computer standards
- In probability jargon: Number of starting zeros is geometric with parameter `1/2`.

Hash Function Application: Proofs of Work Puzzles

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- **z=20** is quick to solve, **z=70** is solvable only by powerful computers

Proofs-of-Work for Anti-Spam (Not used, sadly)

POW = Proof of Work

Sender

Email server

Proofs-of-Work for Anti-Spam (Not used, sadly)

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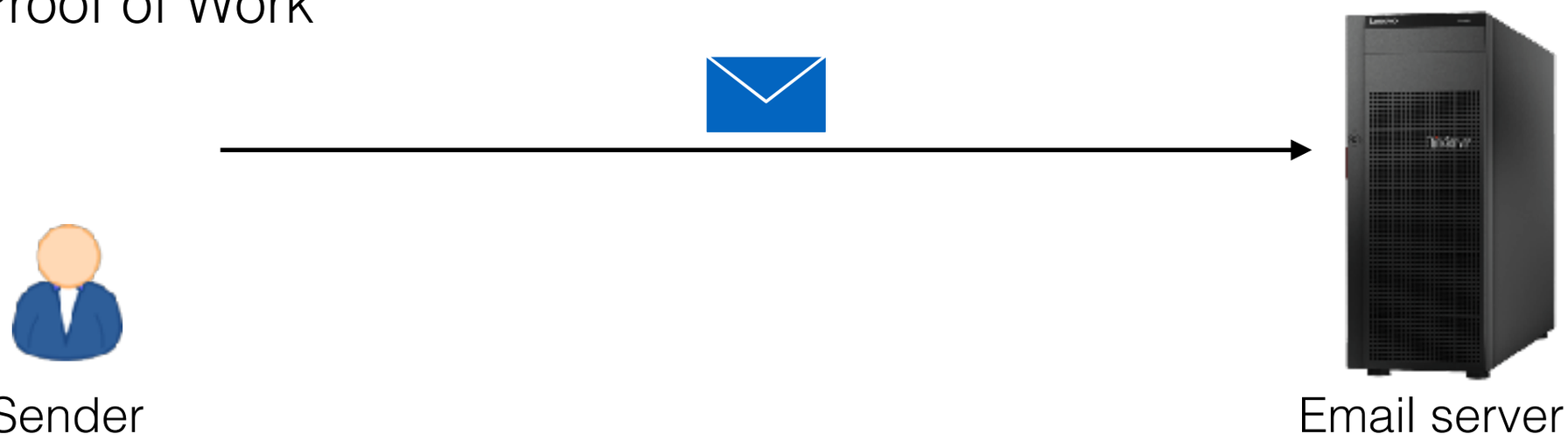

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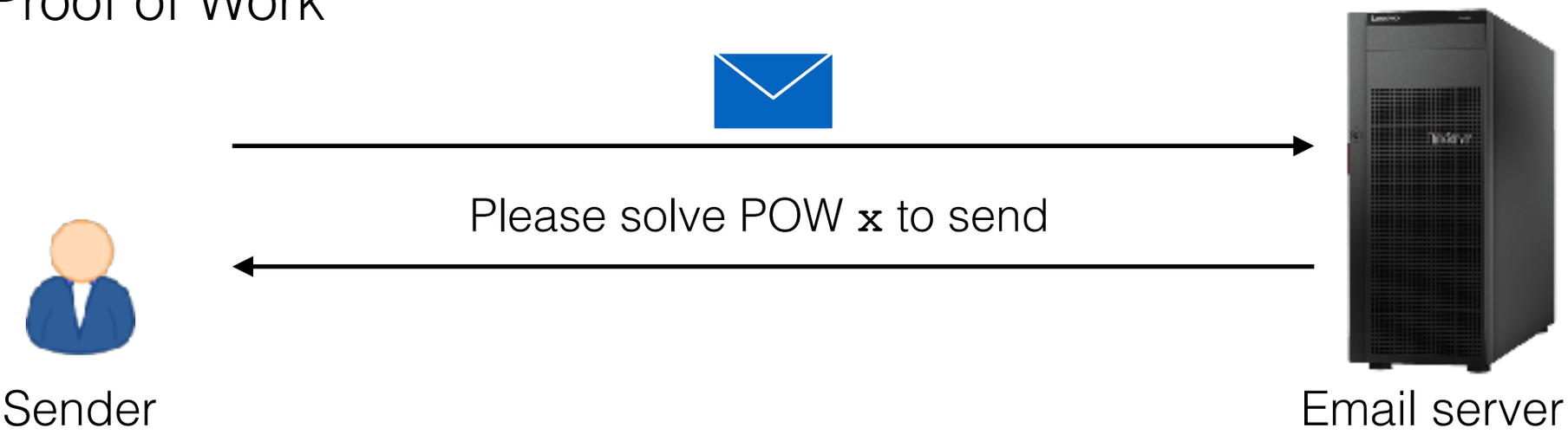

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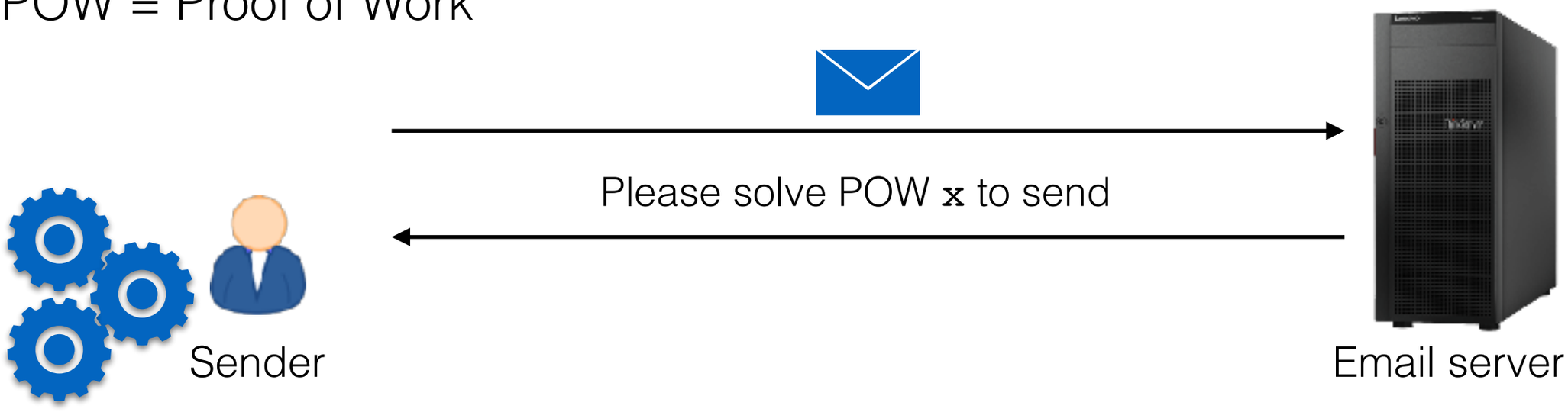

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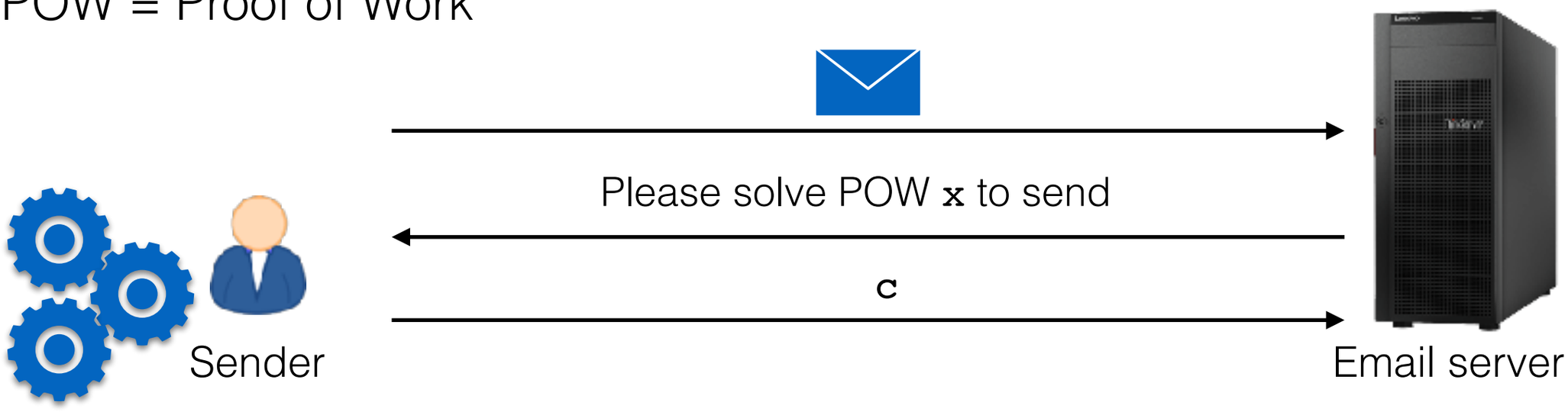

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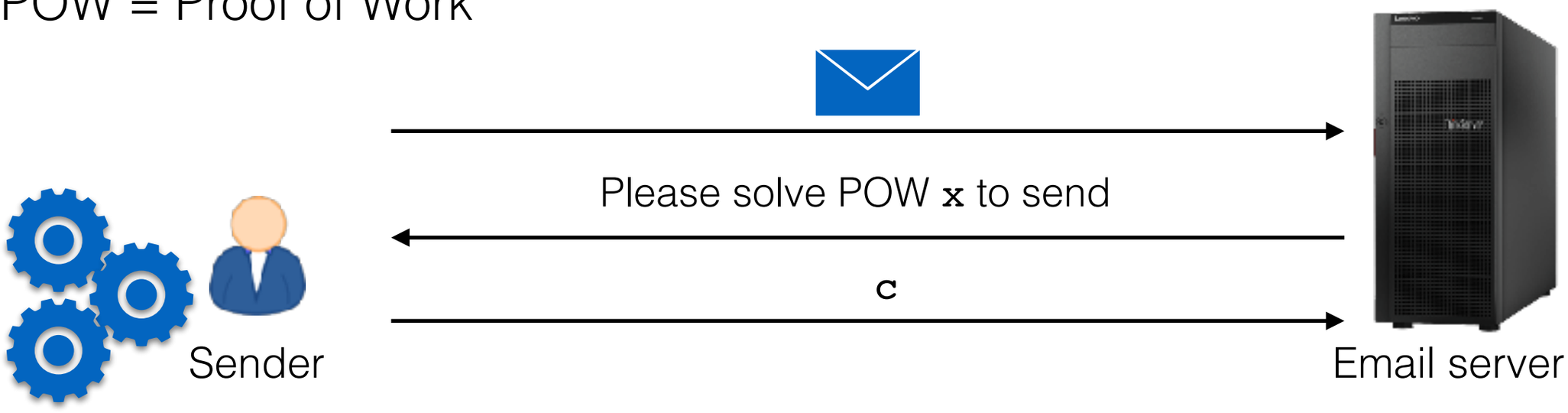

Spammer

Email server

Proofs-of-Work for Anti-Spam (Not used, sadly)

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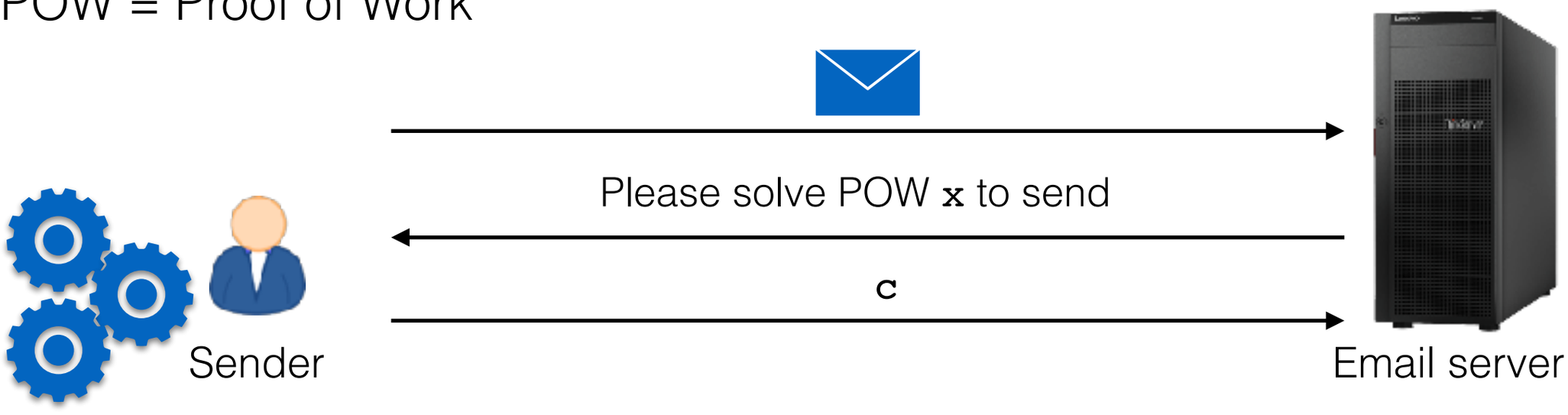

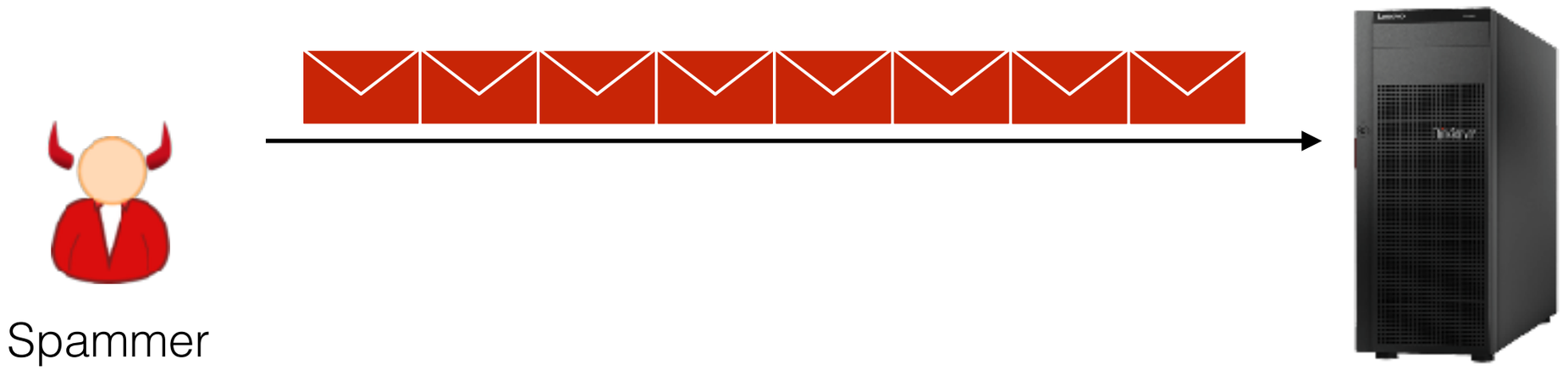

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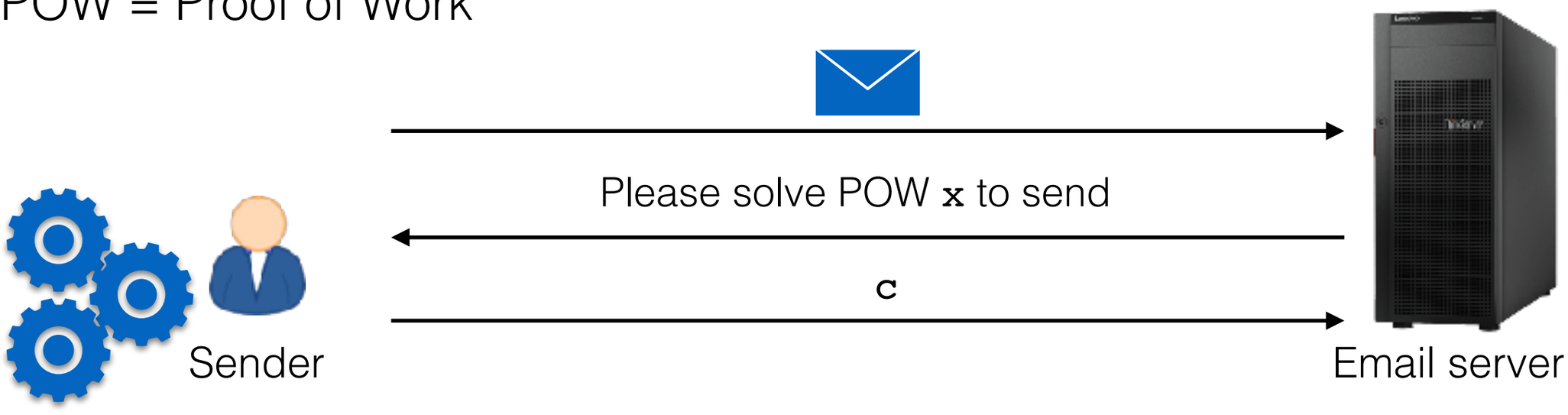

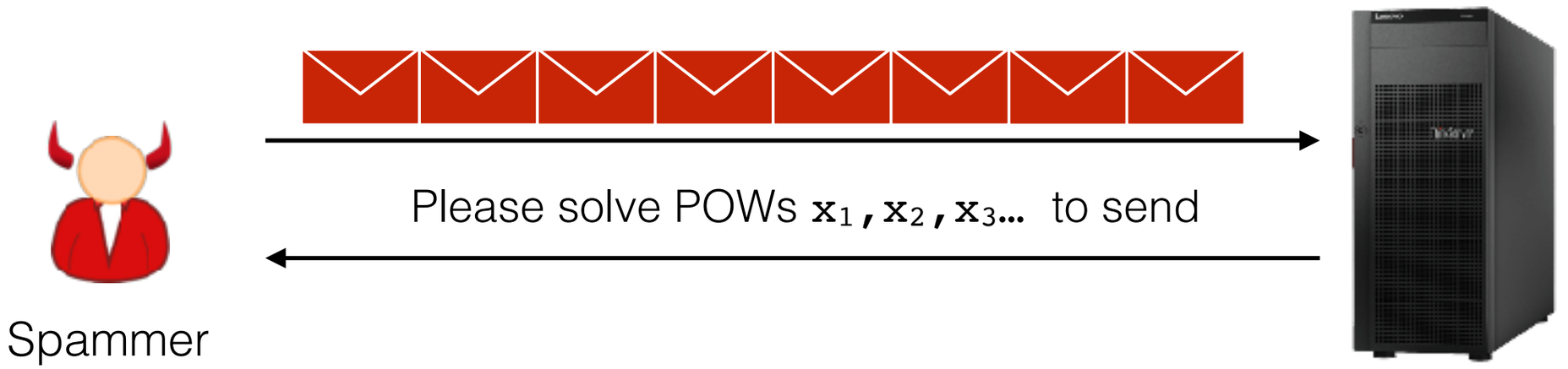

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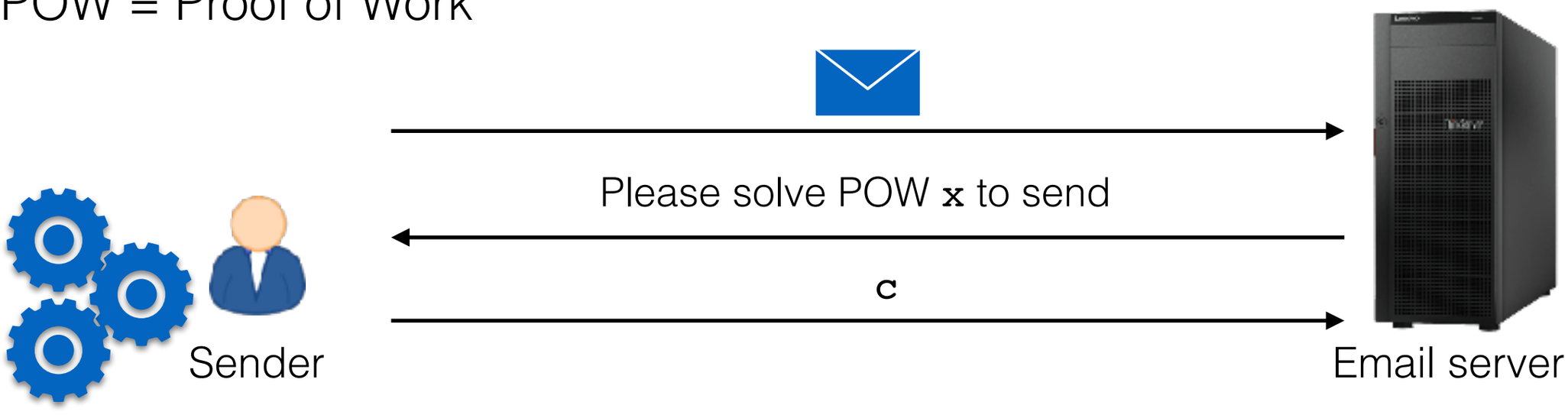

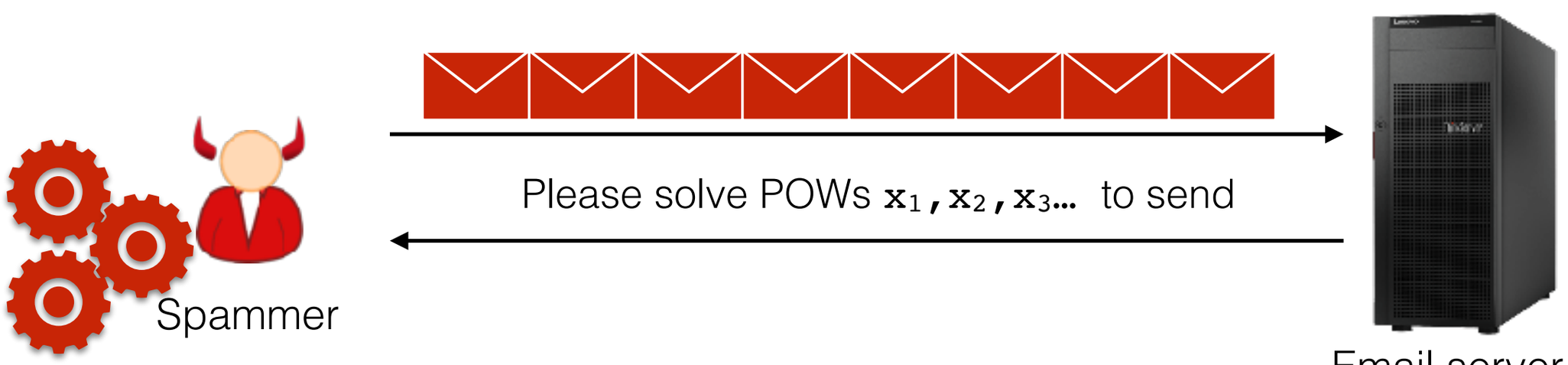

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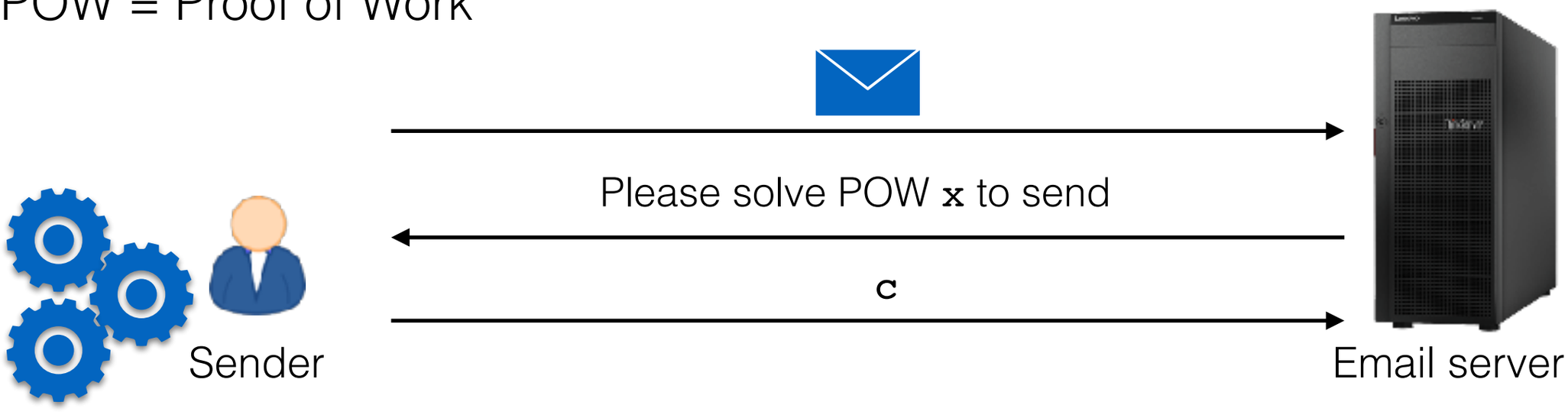

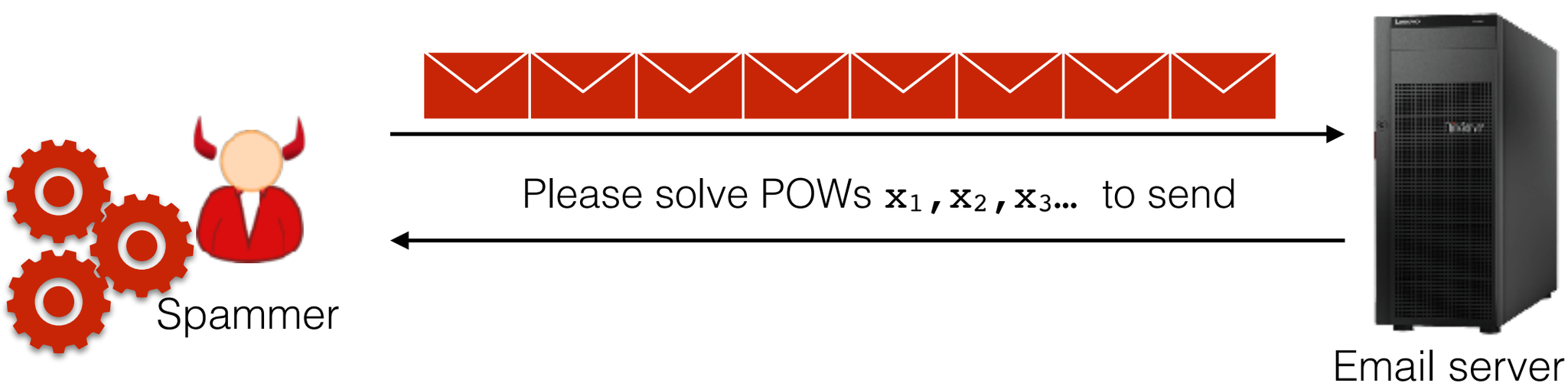

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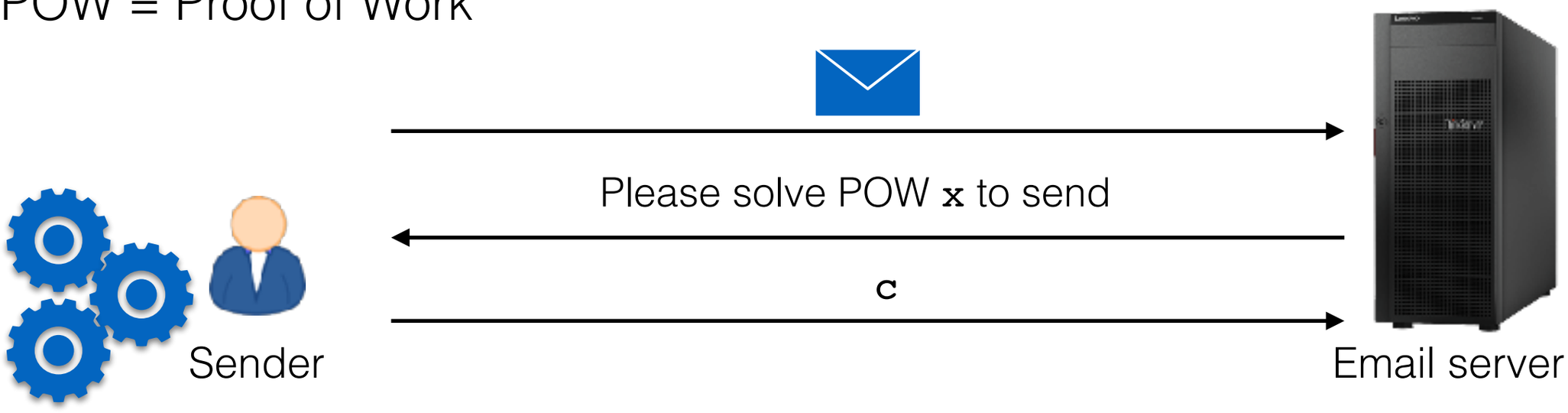

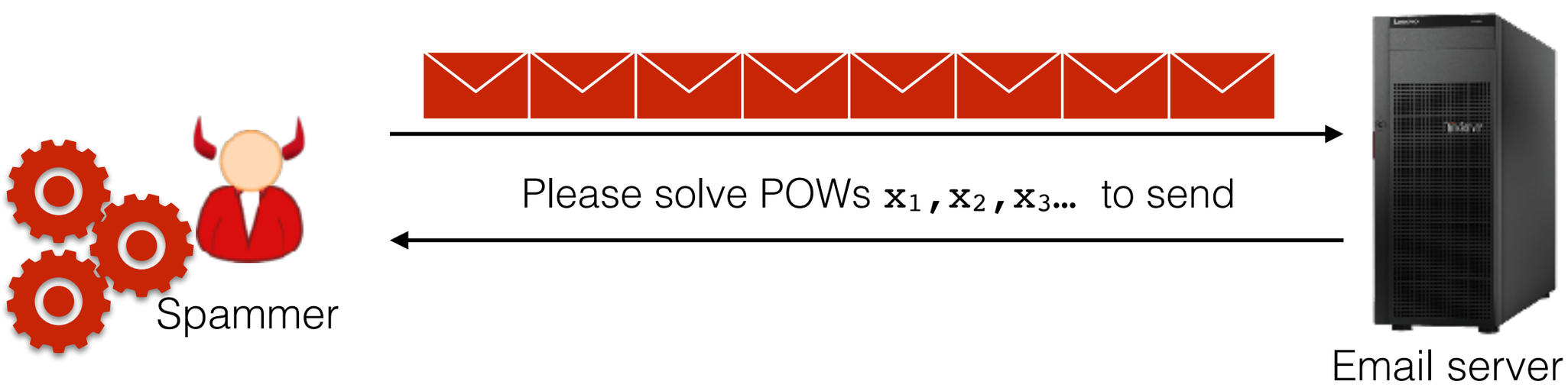

- Evaluating POW takes time/money/compute hardware for good and bad parties
- Set hardness parameter z so that:
  1. Normal senders can solve one puzzle quickly, without noticing the work
  2. It is unprofitable for spammers to solve millions of puzzles

Lecture 2 Outline

1. Cryptographic Hash Functions

   - Blockchains

   - Proofs of Work

2. Putting DCash “on the blockchain”, with an authority

3. The idea of decentralization

4. Decentralized DCash with an Angel

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Recalling DCash

Initialization: Ben, Emily, and David all generate keys for digital signatures

David's verification key: PK_{david} = 5e7843…
Ben's verification key: PK_{ben} = 88f01e…
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- In the terminology of the start of Chapter 3, this is an "account-based ledger"
- To determine if a transaction is valid, we must rerun entire history of ledger

DCash 2.0 (a.k.a. Scroogecoin, Text section 1.5)

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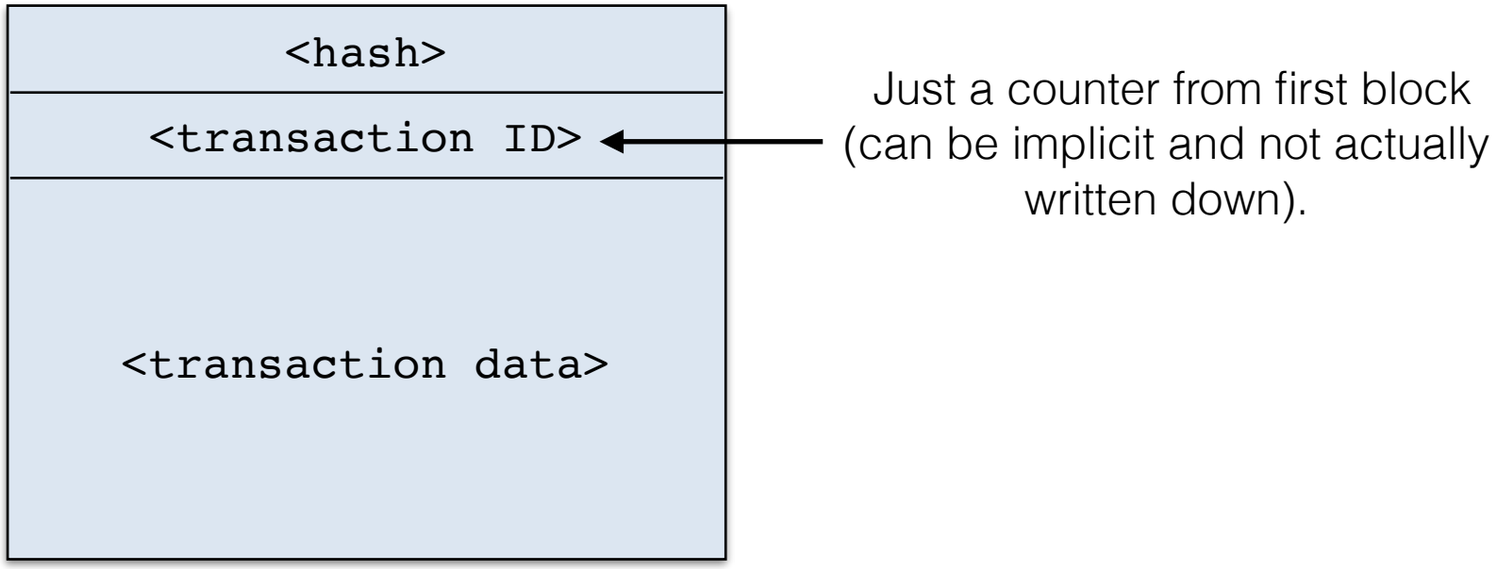
Authority’s responsibilities:

1. Publish blockchain contents publicly.
2. Create coins and assign them to owners (public keys) at will.
3. Receive transactions notifications and commit them to the blockchain.

DCash 2.0 Blocks, Transactions, and Coins

- One transaction per block



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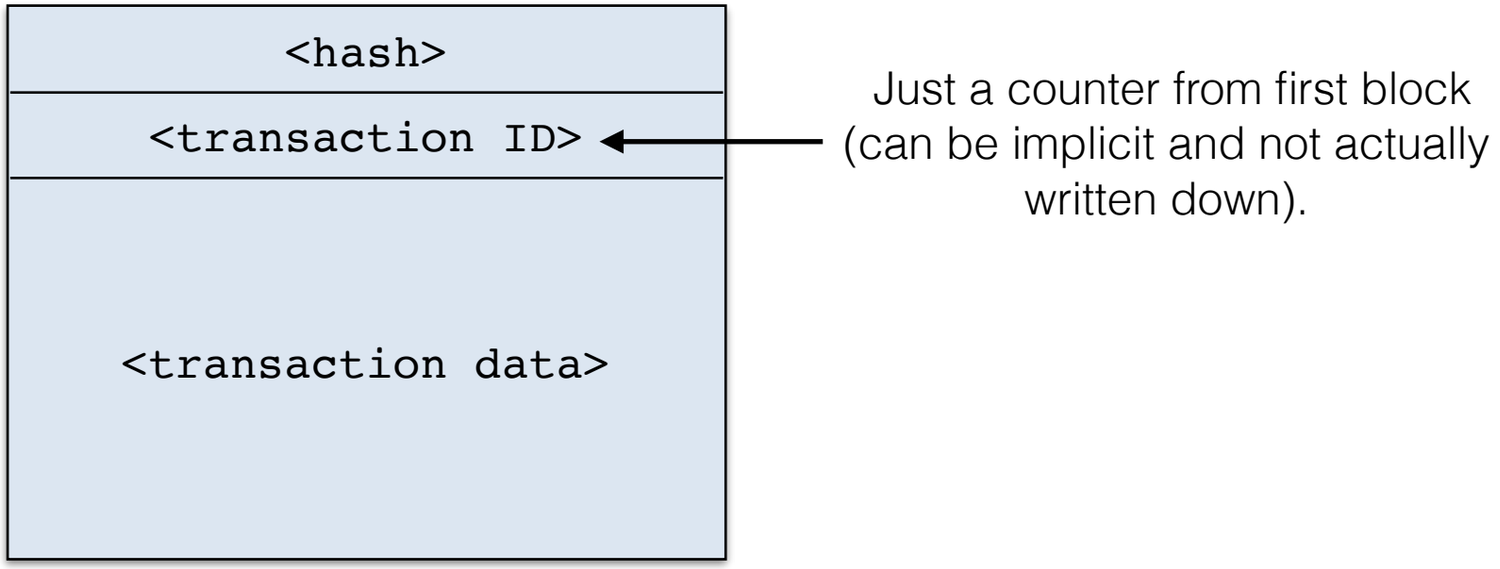

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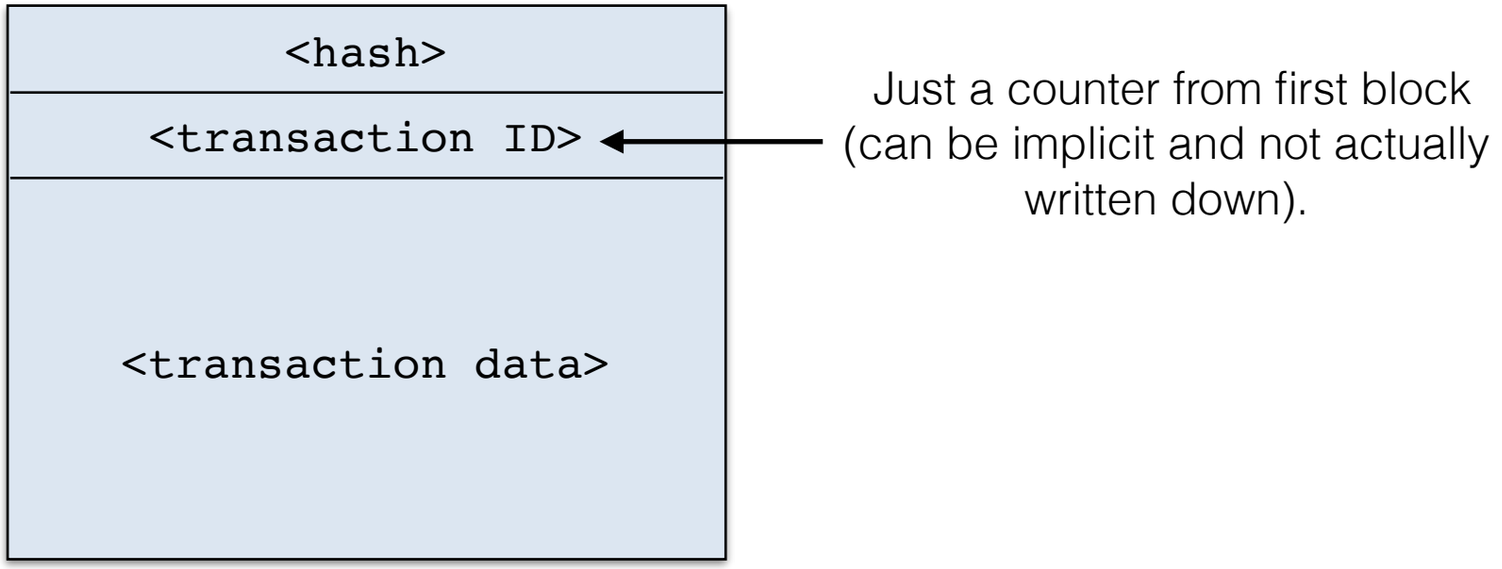

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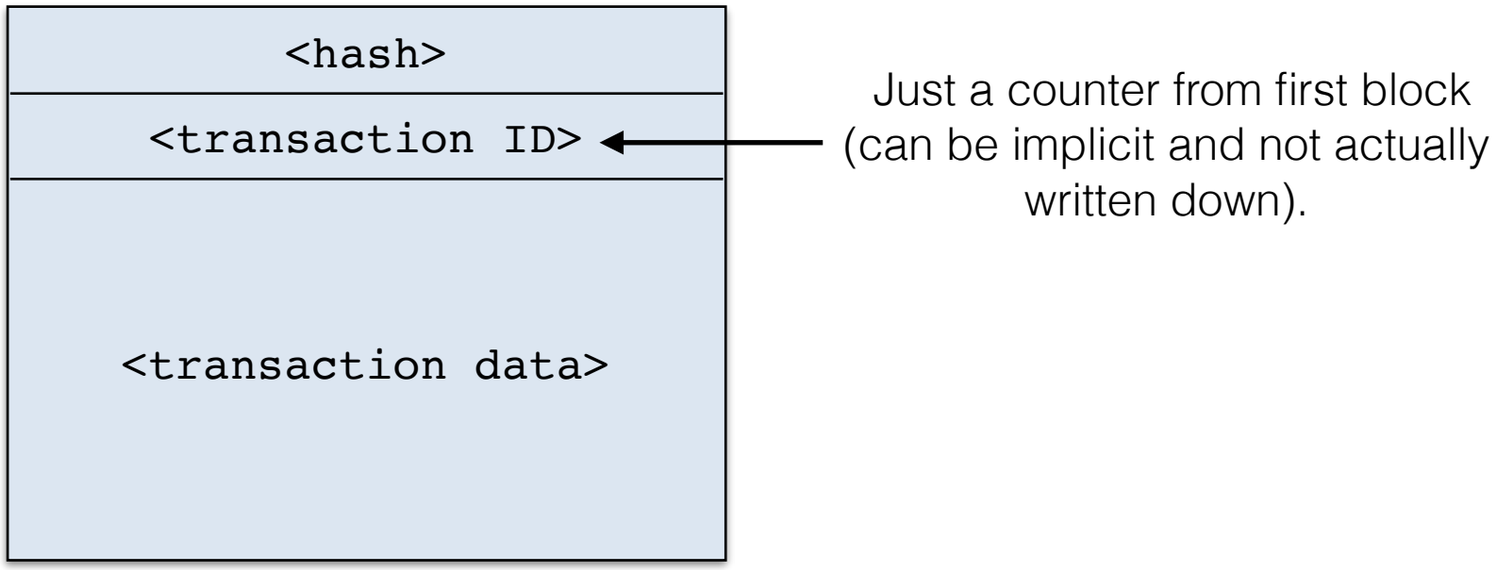
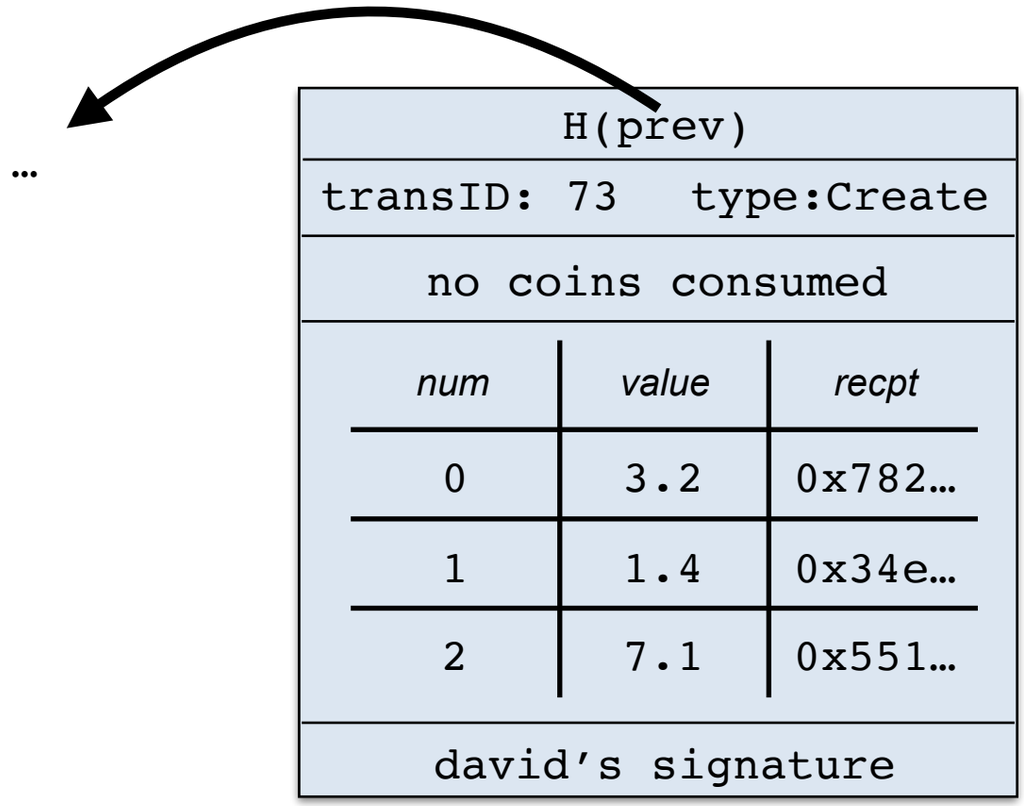


- Two types of transactions: CreateCoins and PayCoins

First Type of Transaction: CreateCoins

- CreateCoins transactions… create coins.
- Authority decides when to create coins and who gets them



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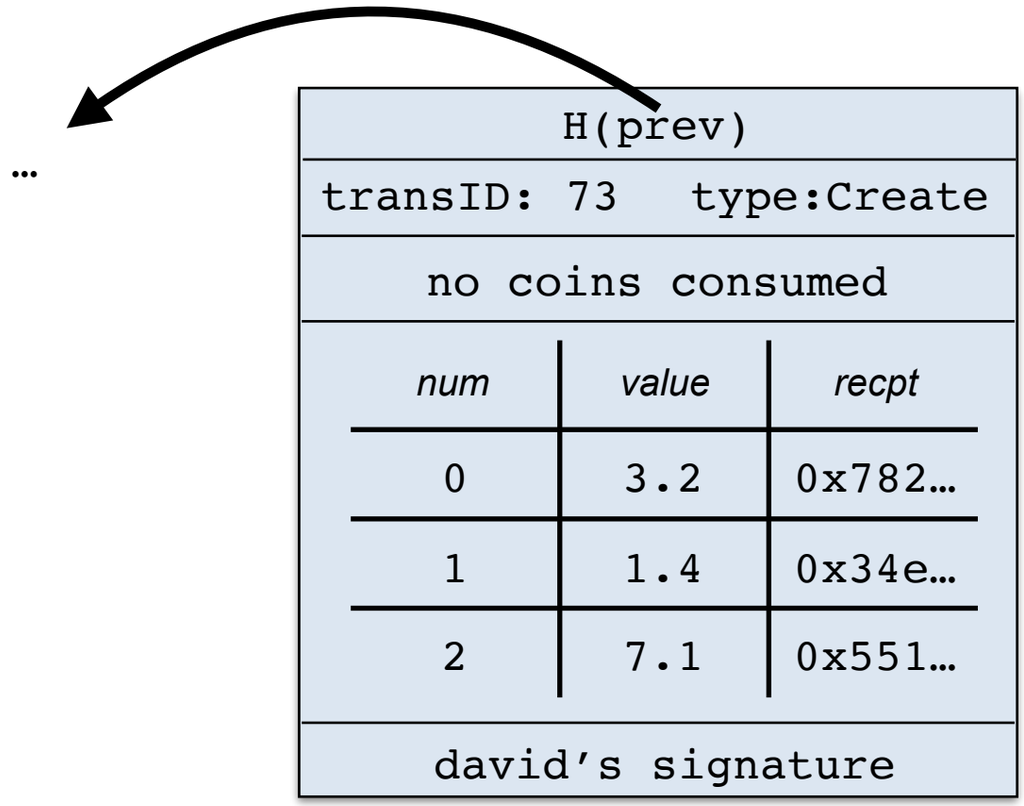

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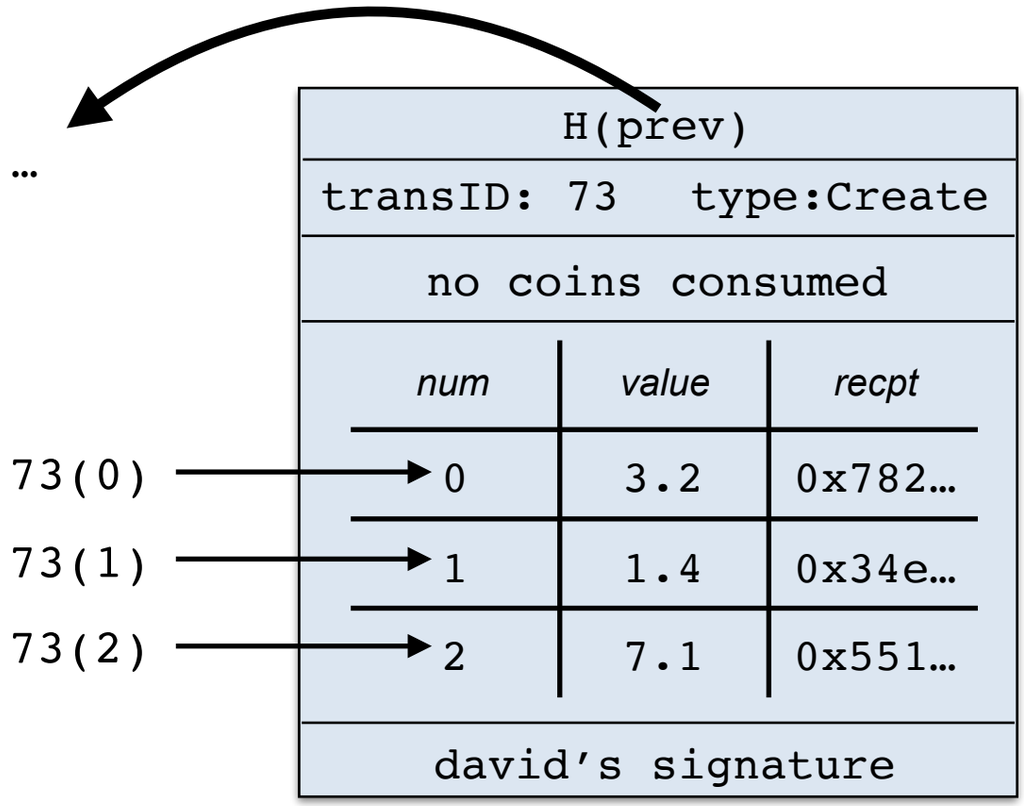

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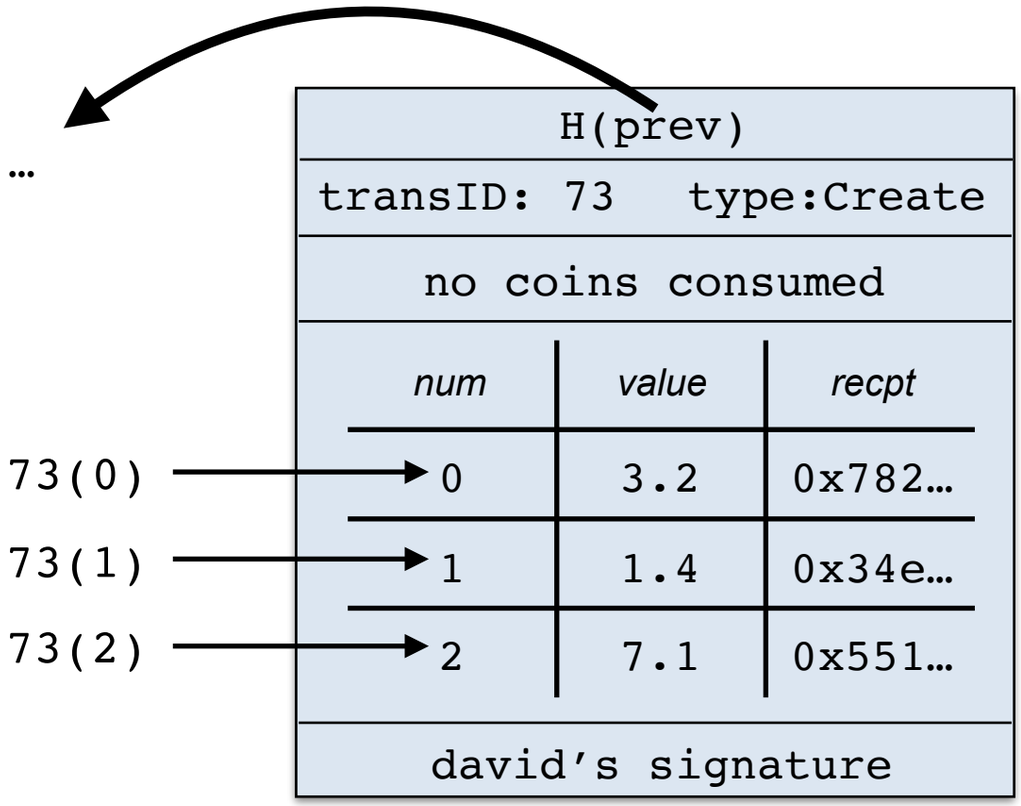

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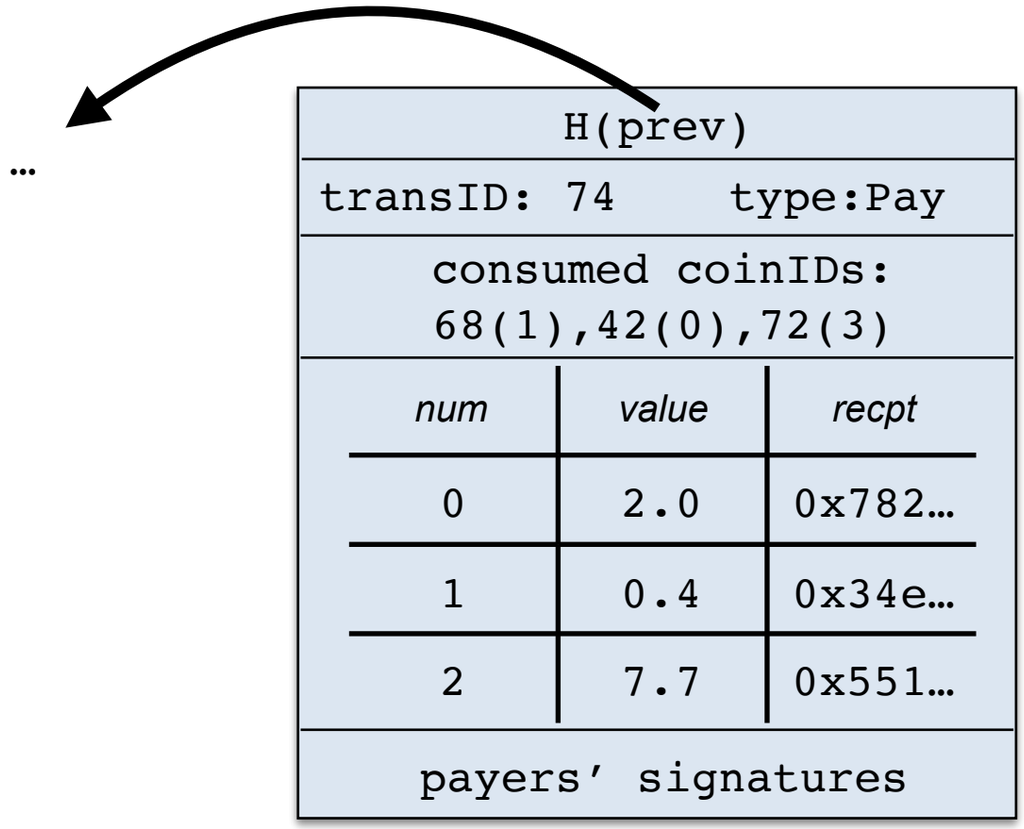
  - **`coinIDs`** are unique and never reused
- Coins can have different values.

Second Type of Transaction: PayCoins

- PayCoins transactions consume some number of coins and create new coins owned by (potentially) different keys
- Payers must sign transaction



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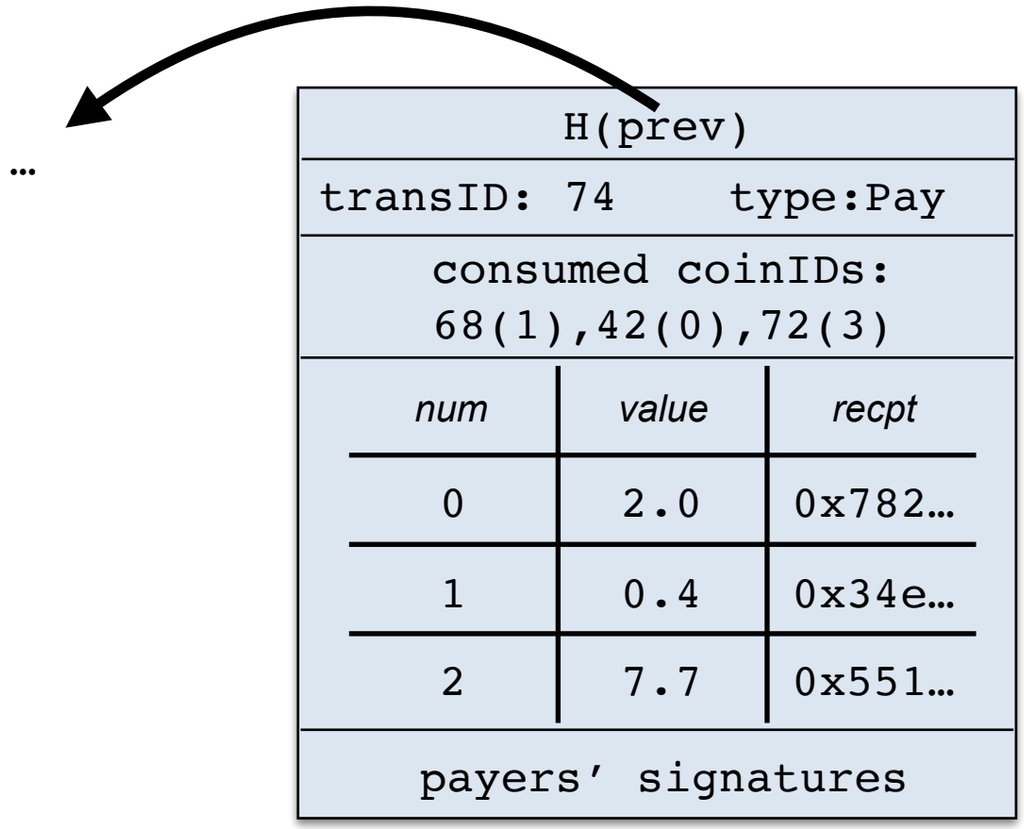

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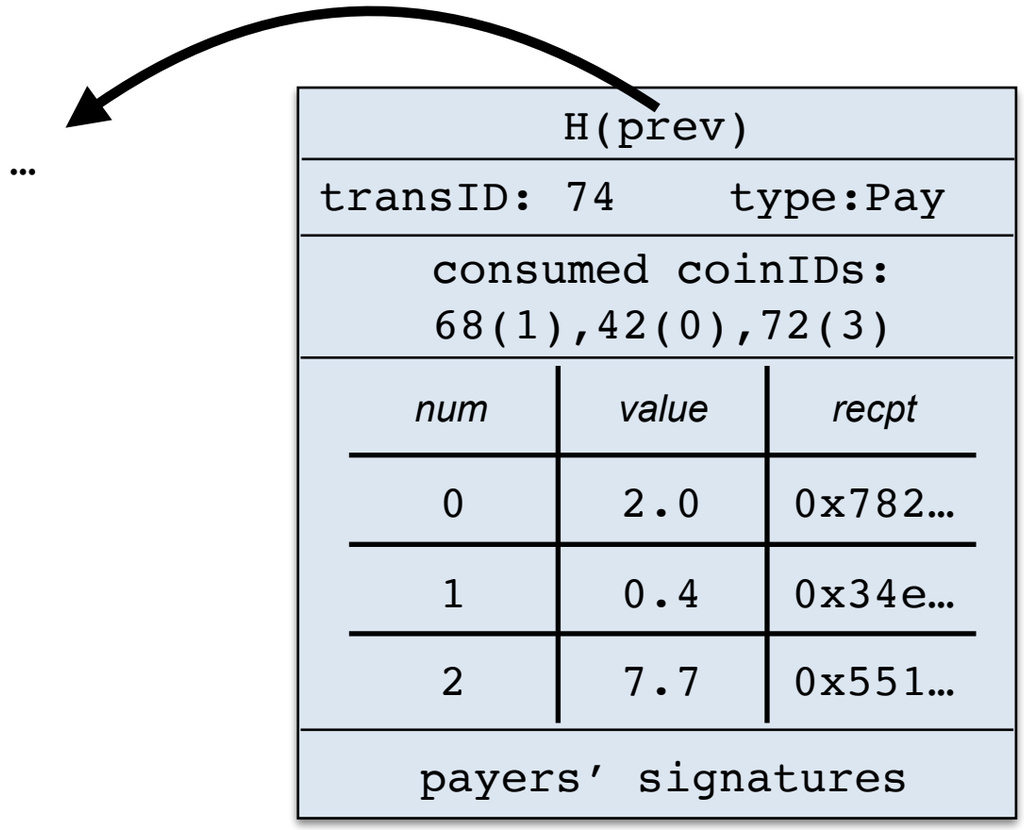

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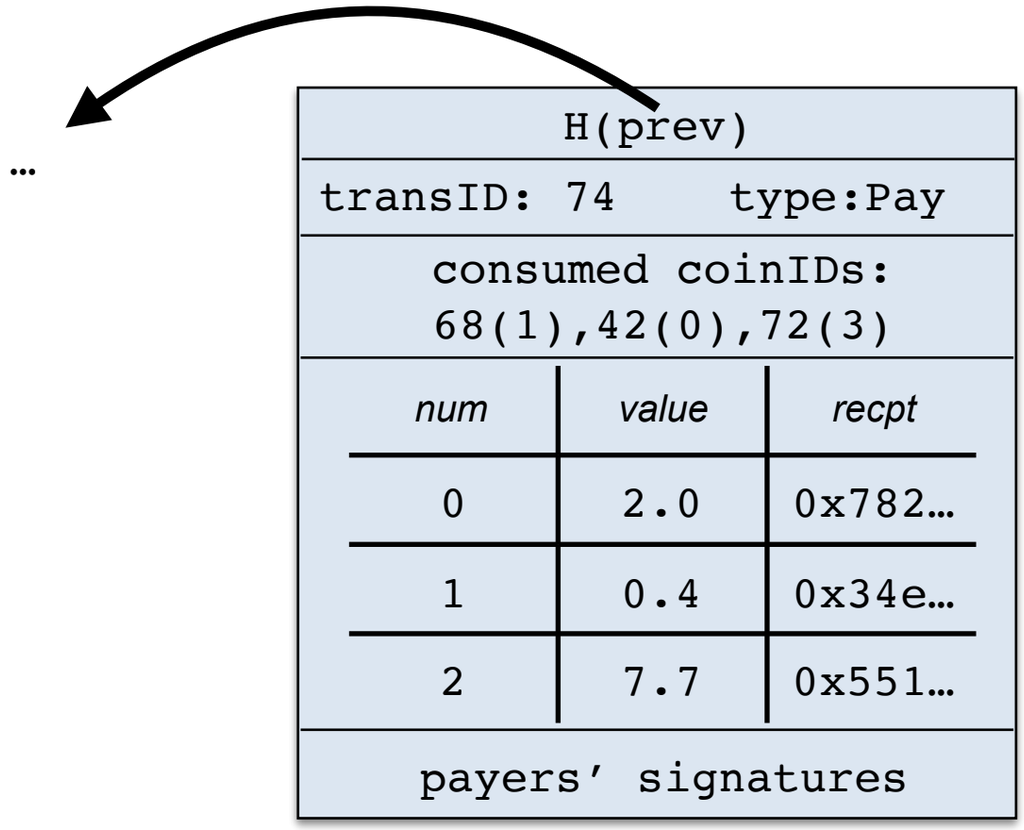

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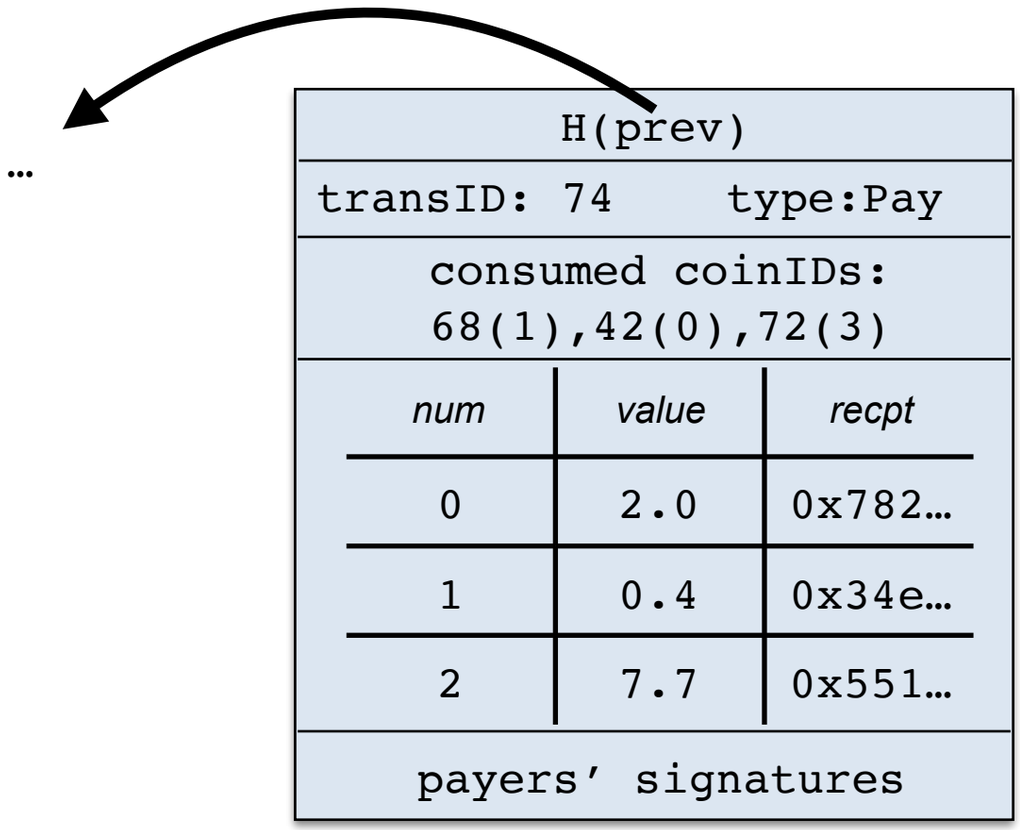
- This is "transaction oriented": Each transaction says where its funds came from.

Valid transactions in DCash 2.0

- CreateCoins transactions are valid if the authority signed them and the hash matches the previous block

- PayCoins transactions are valid if:
  1. Consumed coins were indeed created previously
  2. Consumed coins have not been consumed in a previous block
  3. Consumed coins sum to same value as created created coins
  4. Signatures from payers are all valid
  5. Hash matches previous block

Quickly validating PayCoins Transactions



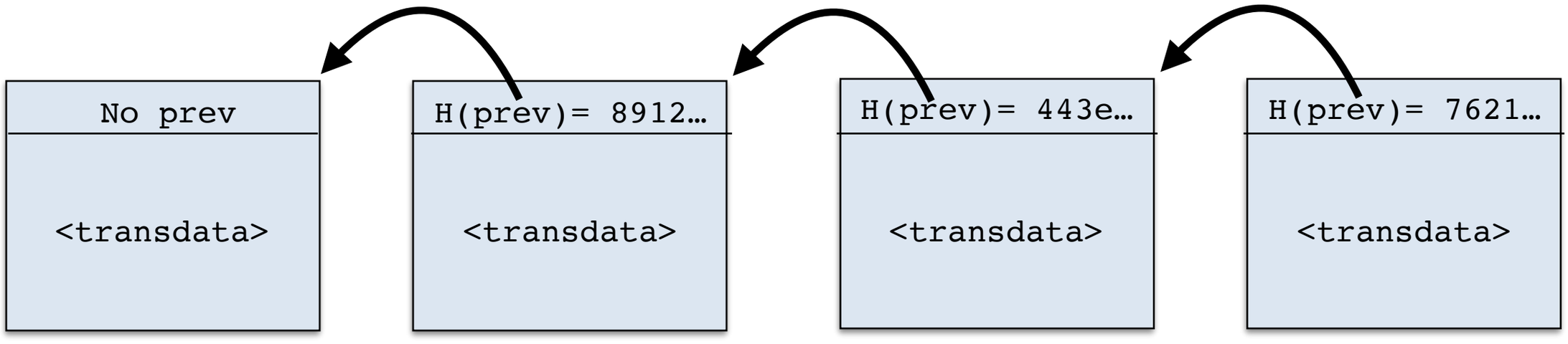
1. Step backwards and check if consumed coins were spent *since transaction 42*
2. Look up consumed coins, sum up their values, compare to output sum
3. Check sigs (using public keys of owners of consumed coin)

- This is easier than with our original ledger: We don't have to compute how much value each account has.

Tricks with PayCoins Transactions

- Can consume a coin, pay someone, and pay yourself the change.
- Can split coins by paying yourself twice in one transaction.
- Easy to extend DCash 2.0 to allow multiple transactions in a block.

(Semi-) Trust in the DCash Authority



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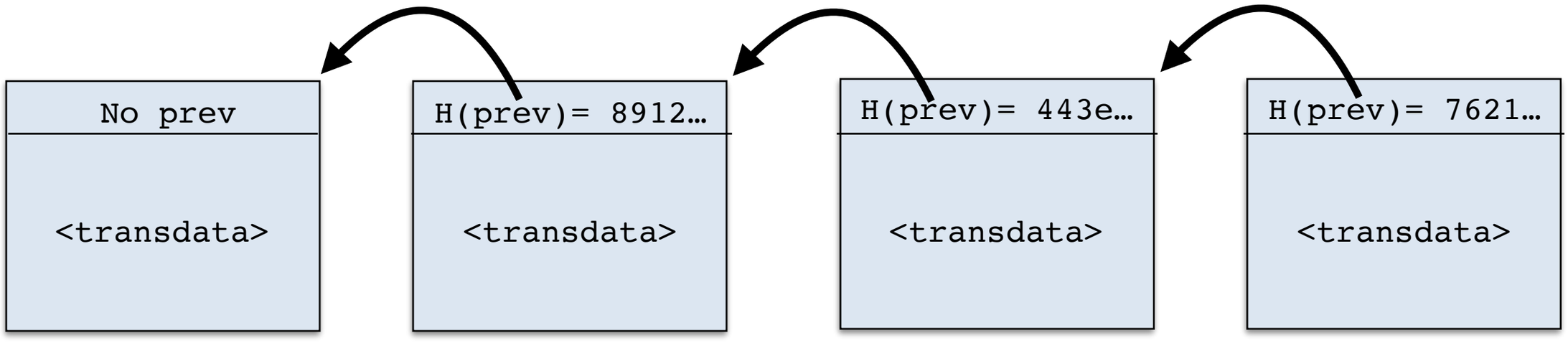

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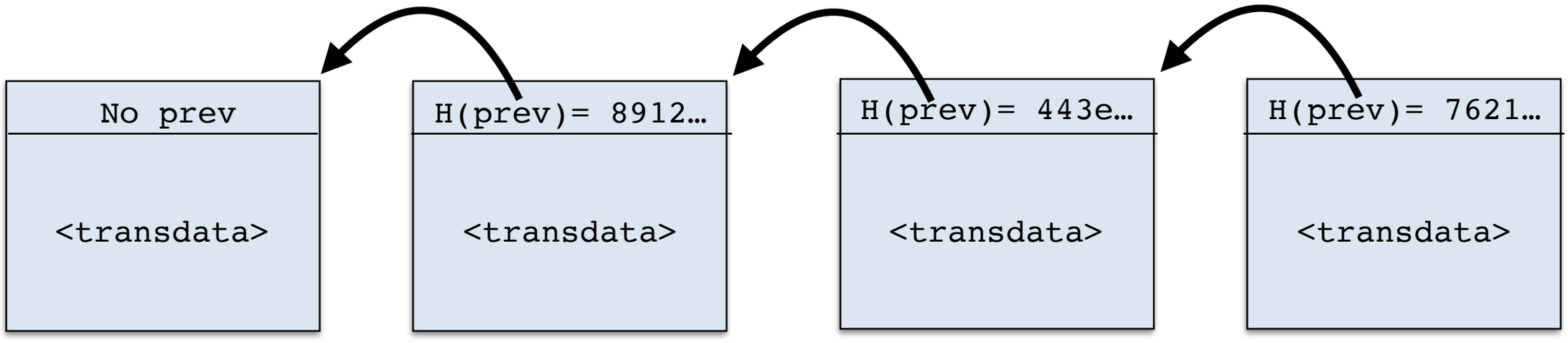

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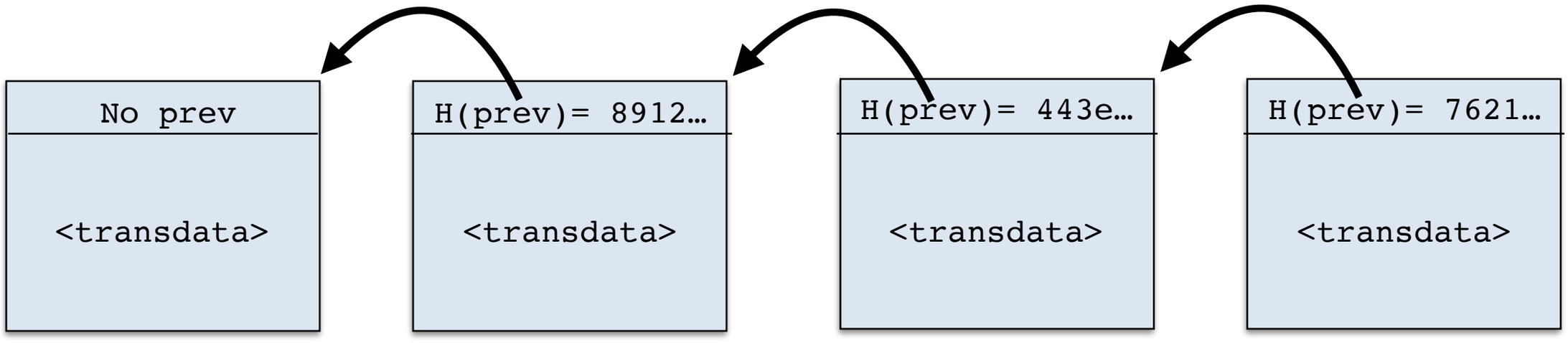

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- Can the authority delete transactions?

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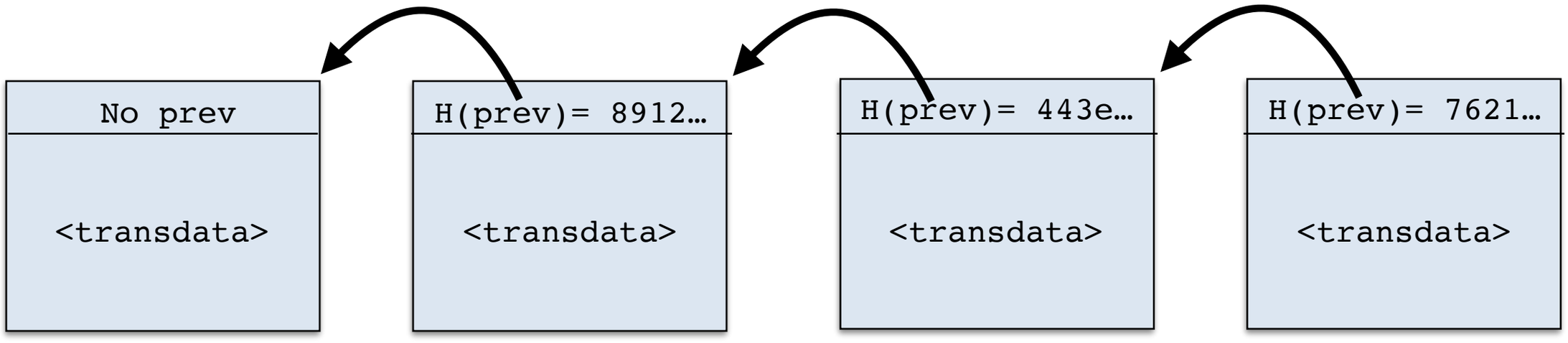

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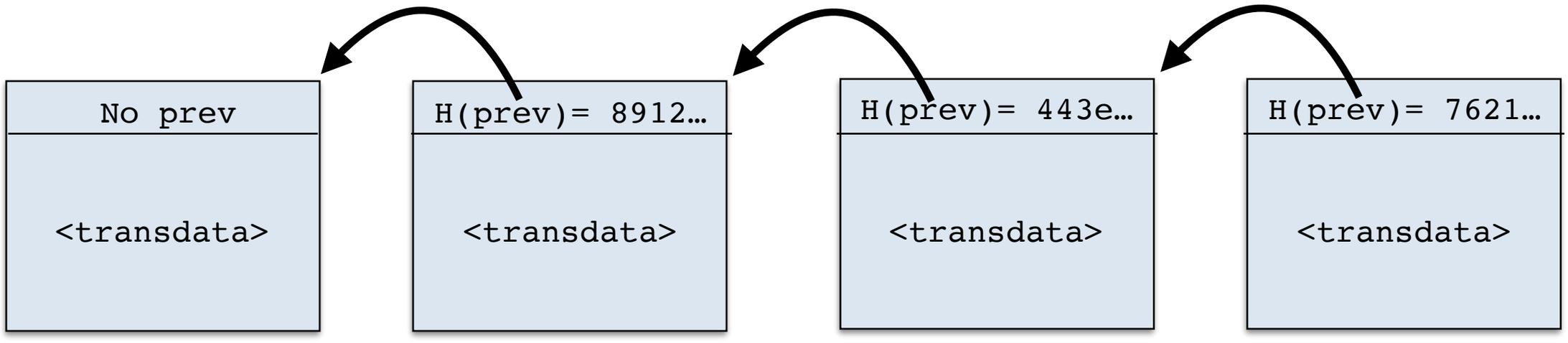

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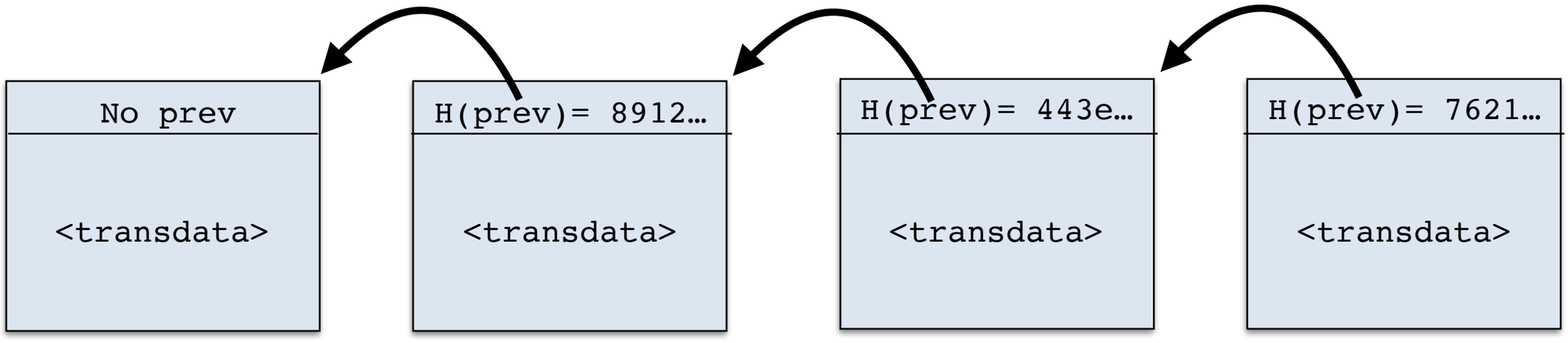

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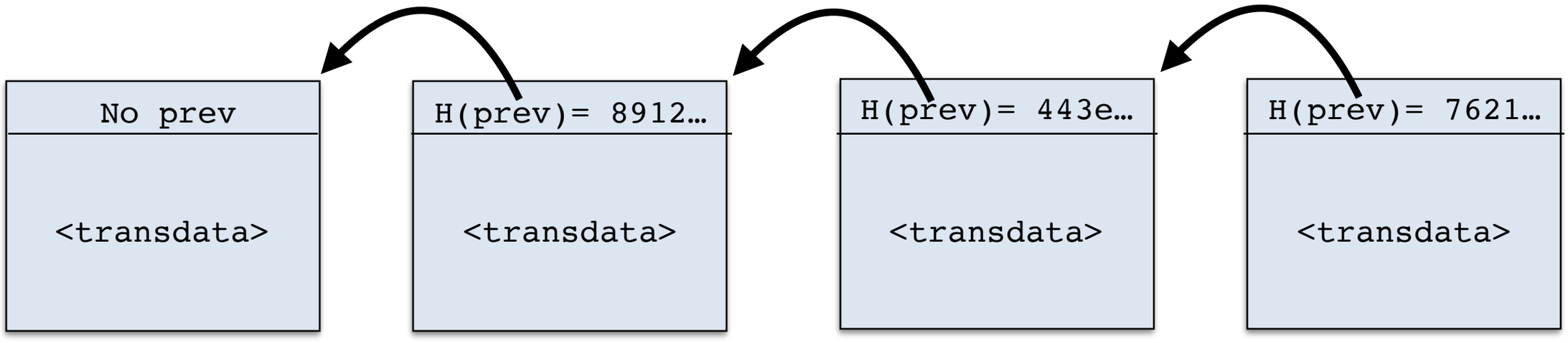

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2. Refuse transactions, locking people out or extorting them.
3. Walk away from the entire affair, rendering all coins worthless.

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Decentralized Technologies vs Centralized

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Decentralized

1. The Internet
2. Email
3. The Web
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Decentralized	Centralized
1. The Internet	1. AOL
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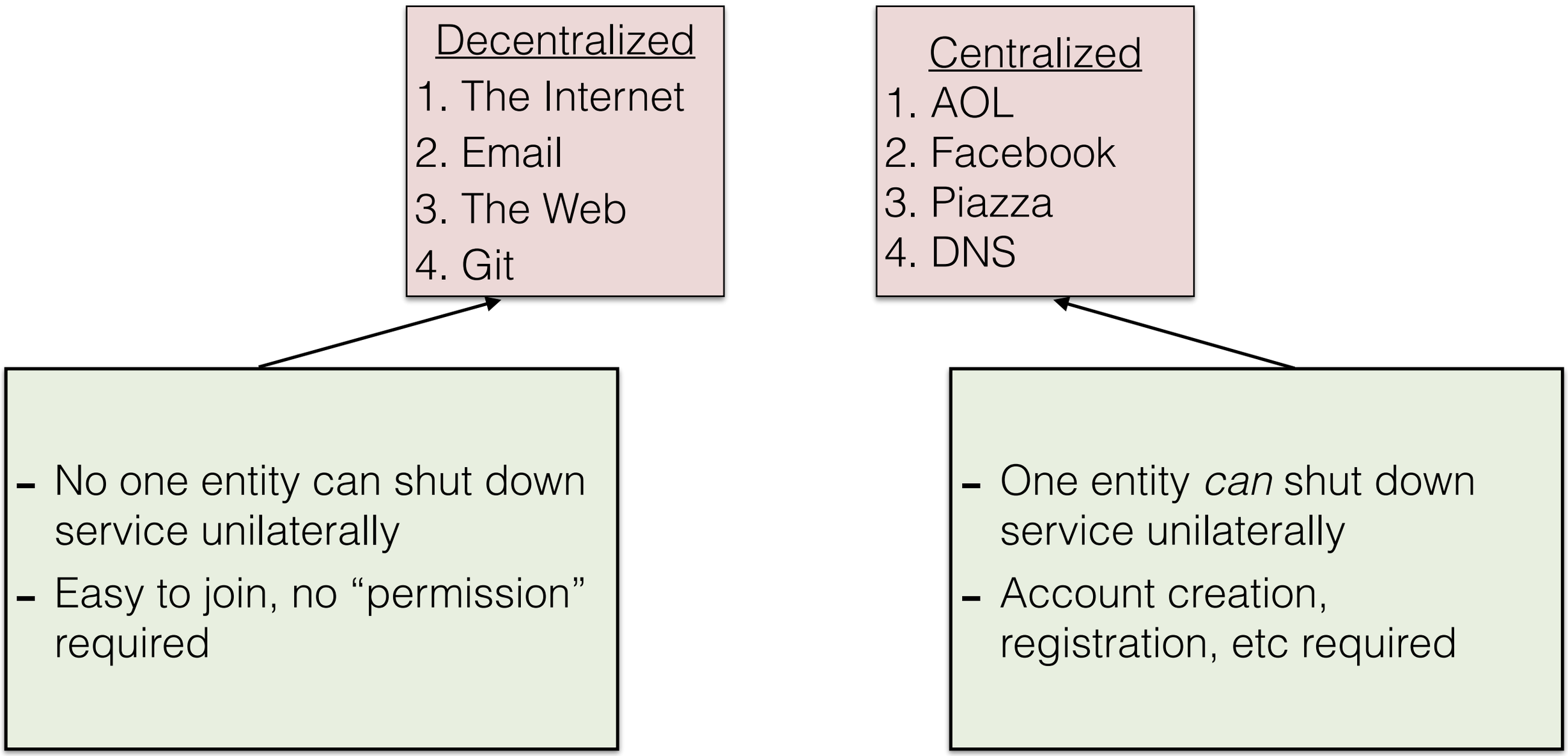
- No one entity can shut down service unilaterally
- Easy to join, no “permission” required

Decentralized Technologies vs Centralized



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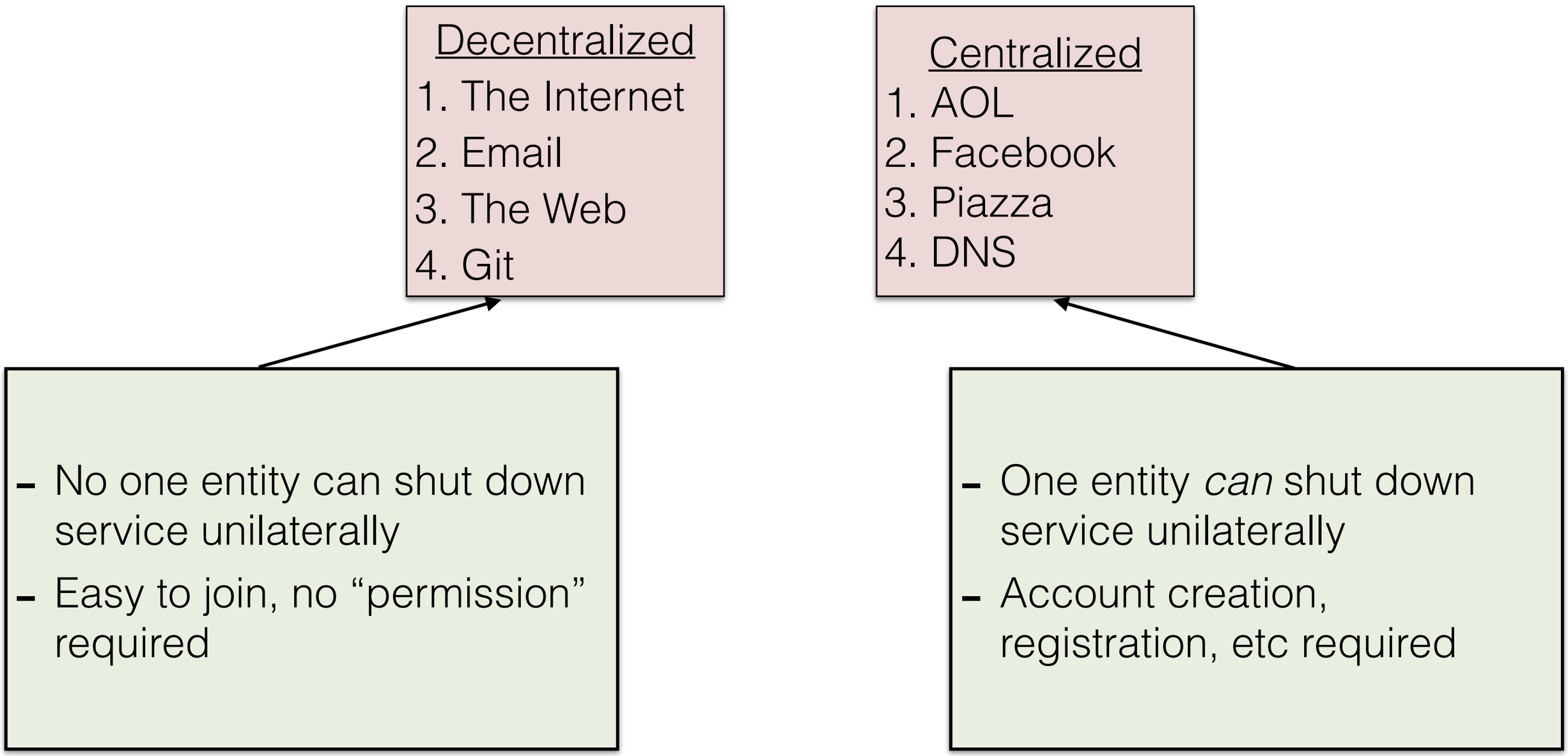


Note: A system may run on thousands of different computers yet still be centralized and controlled by one organization.

Decentralization as Studied in Computer Science

- Computer scientists of studied decentralized systems since the 1970's.
- The news was mostly bad: In many models it is impossible to distribute decisions
- The relevant problem for us called *distributed concensus*

Impossibility of Distributed Consensus with One Faulty Process

MICHAEL J. FISCHER

Yale University, New Haven, Connecticut

NANCY A. LYNCH

Massachusetts Institute of Technology, Cambridge, Massachusetts

AND

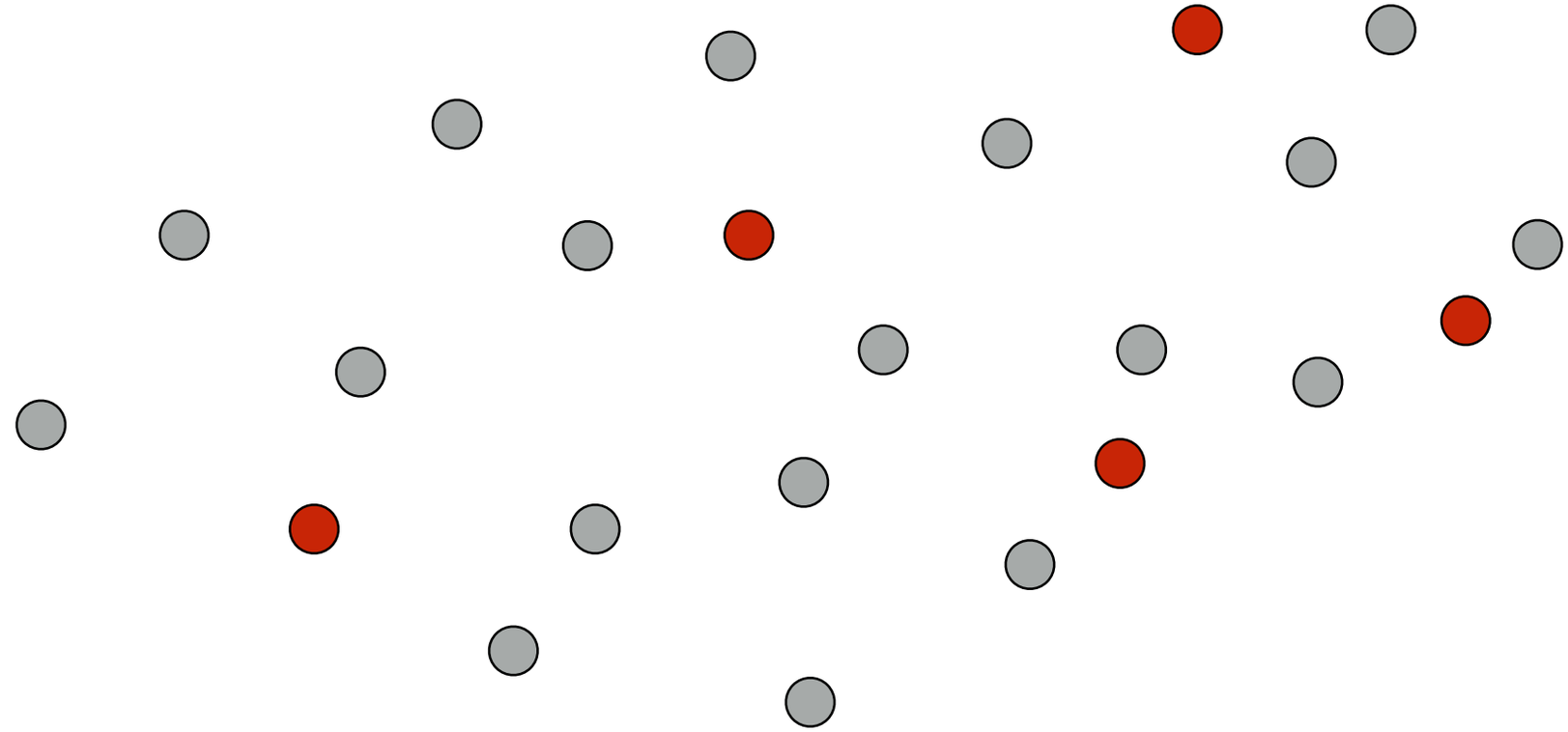
MICHAEL S. PATERSON

University of Warwick, Coventry, England

Abstract. The consensus problem involves an asynchronous system of processes, some of which may be unreliable. The problem is for the reliable processes to agree on a binary value. In this paper, it is shown that every protocol for this problem has the possibility of nontermination, even with only one faulty process. By way of contrast, solutions are known for the synchronous case, the "Byzantine Generals" problem.

Distributed Consensus

- Several nodes communicate asynchronously
- Every node is either *honest* or *malicious*
- Honest nodes supply input values and follow specified protocol



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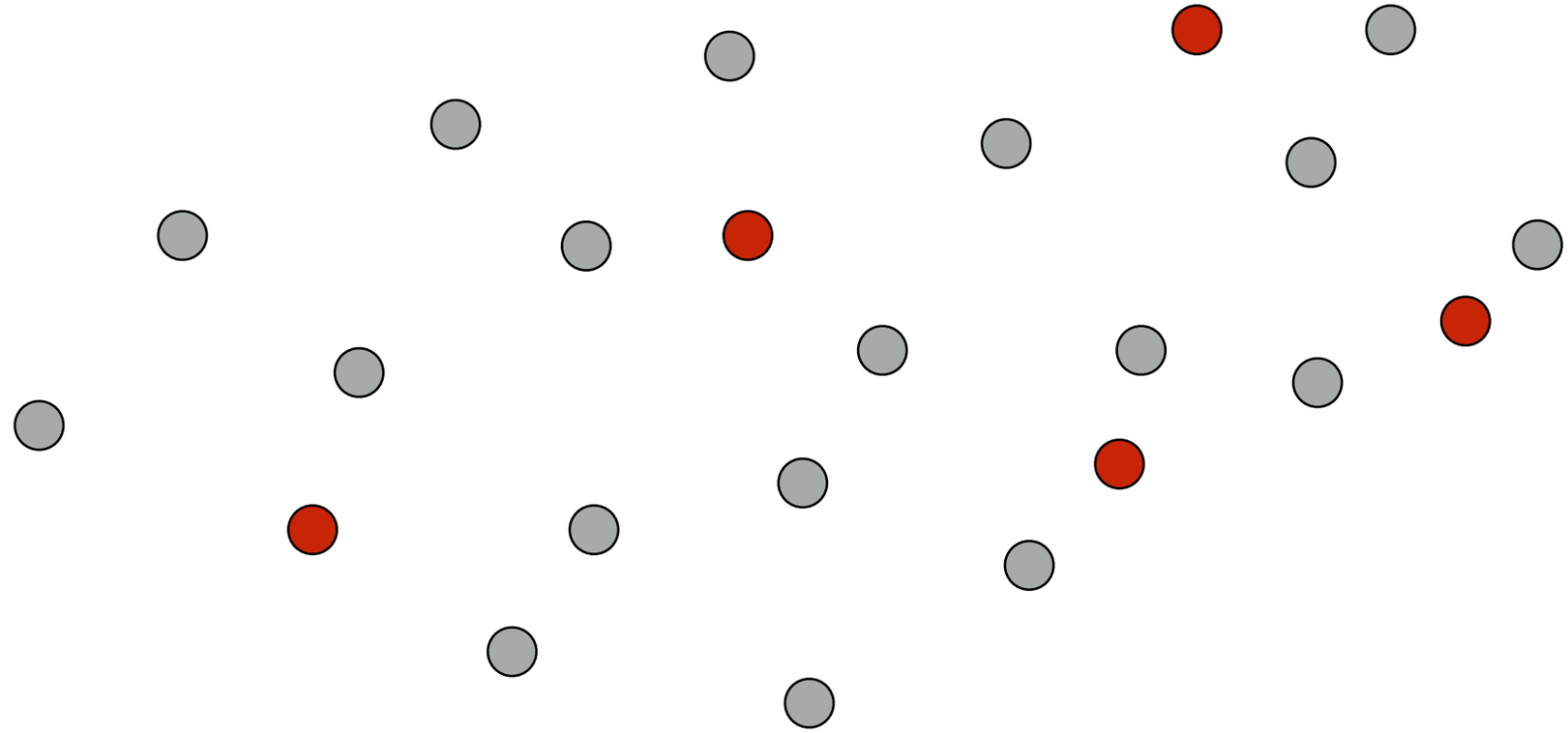
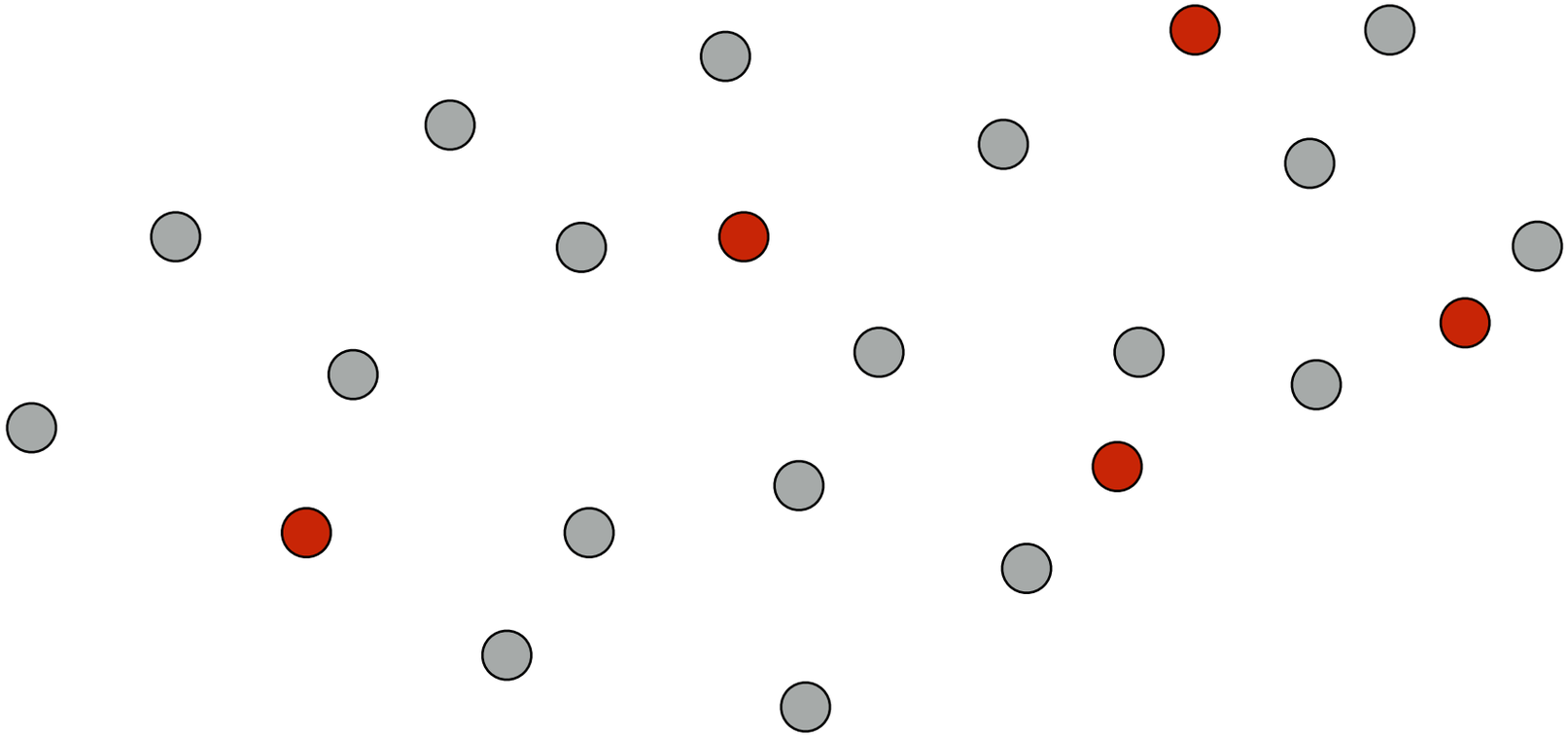


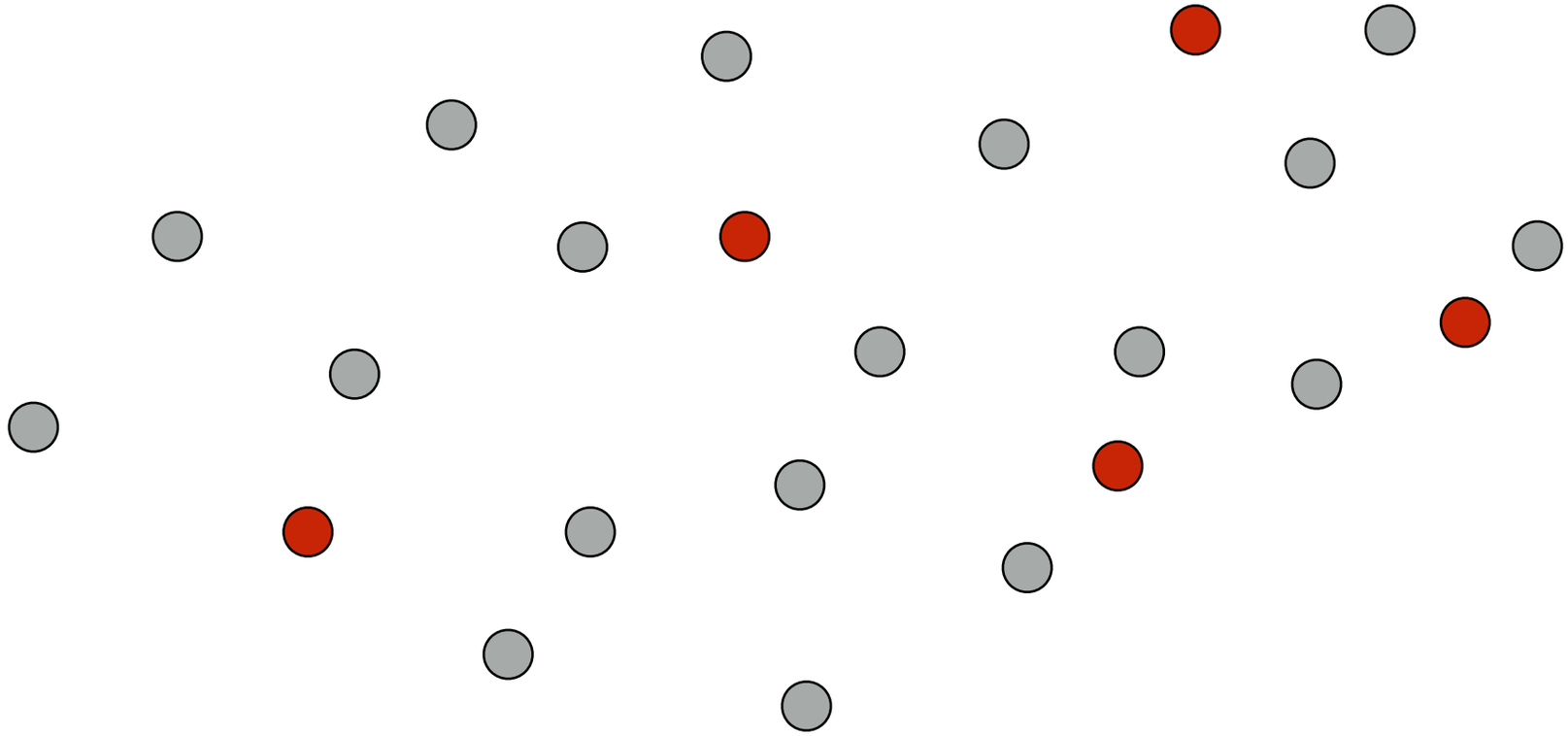
Distibuted consensus problem:
At start: All honest nodes have input value
At end: Protocol terminates and
1. All honest nodes agree on same value
2. The agreed-on value originated from an honest node.

Distributed Consensus of Blockchain State



Distributed Consensus of Blockchain State

- Nodes connected to network broadcast transaction information



Distributed Consensus of Blockchain State

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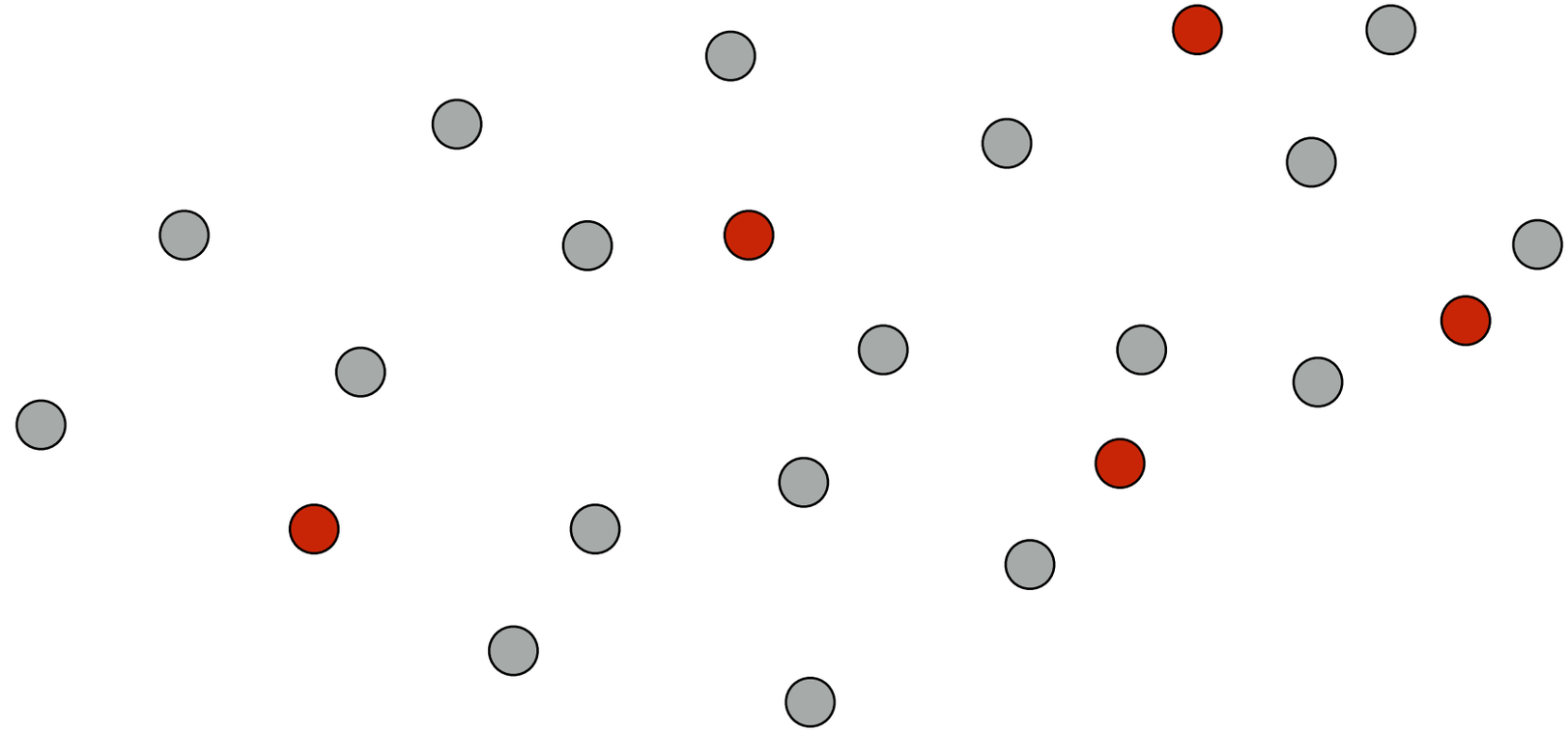
- Specified protocol says how to decide on the state of blockchain, but some nodes are malicious and don't follow protocol.



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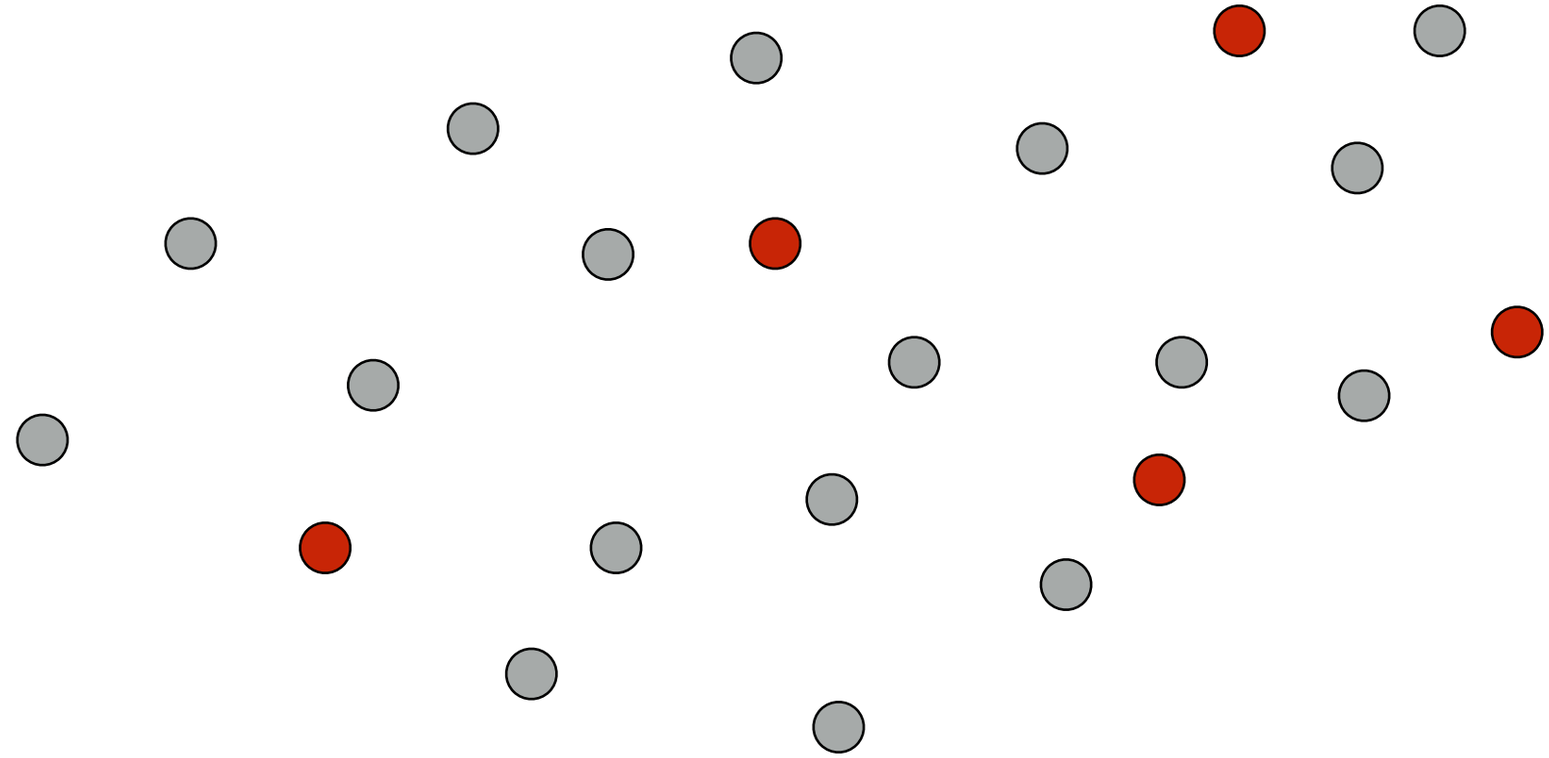

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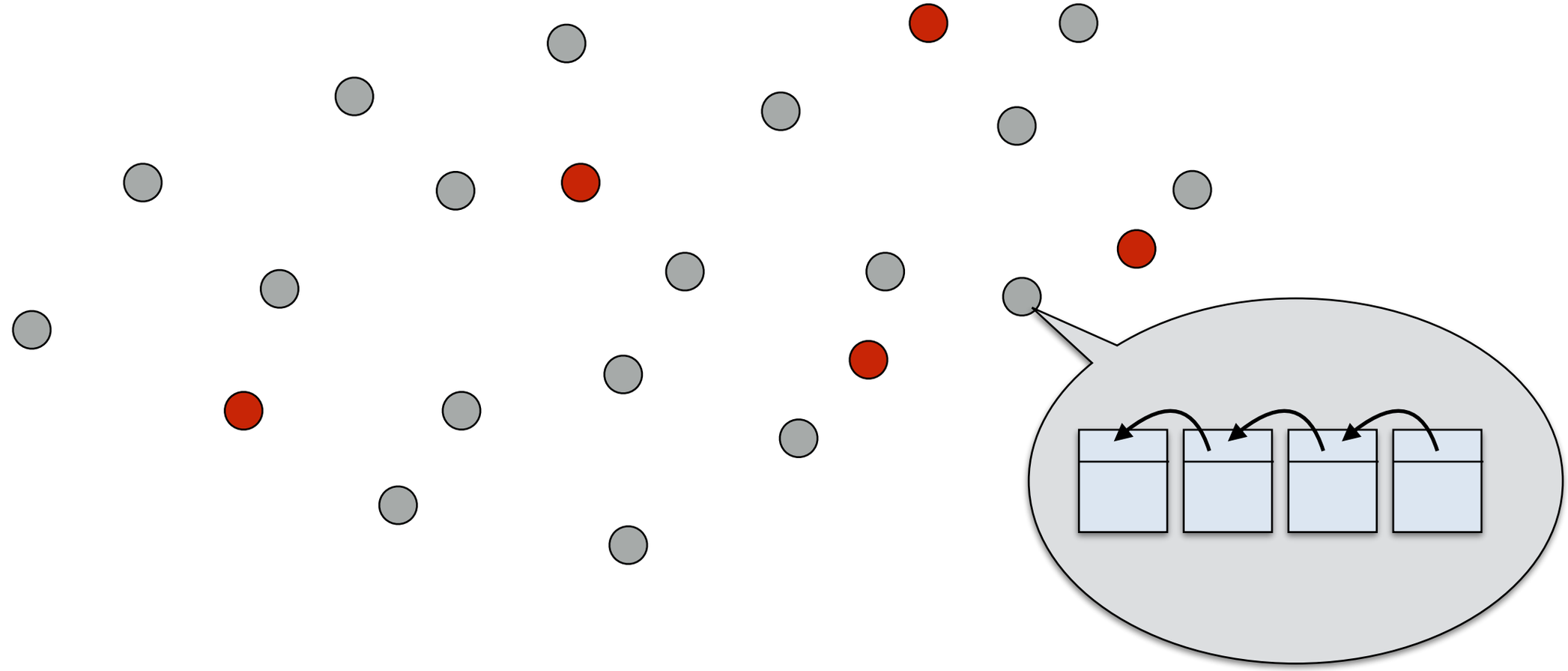

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Identities and Sybil Attacks

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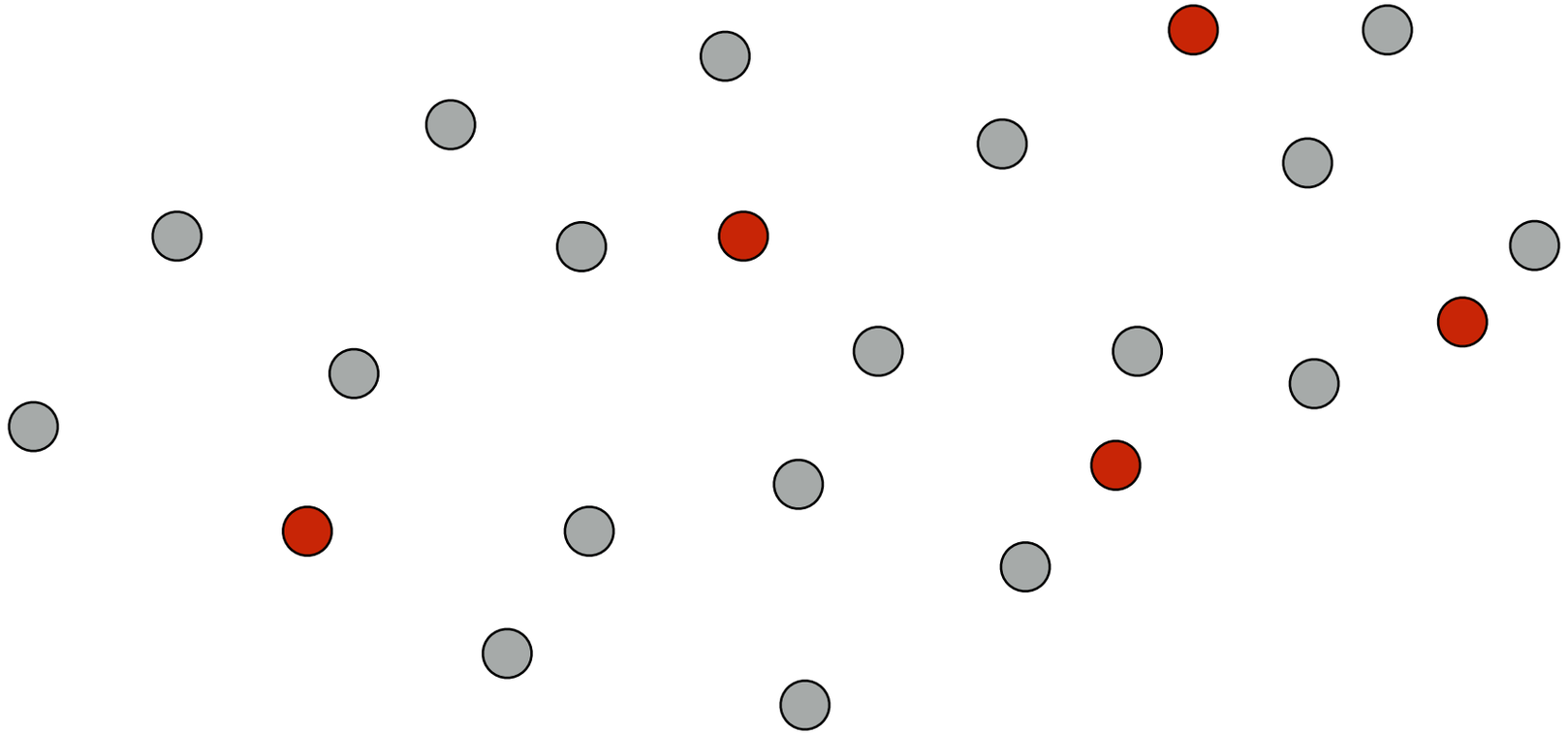

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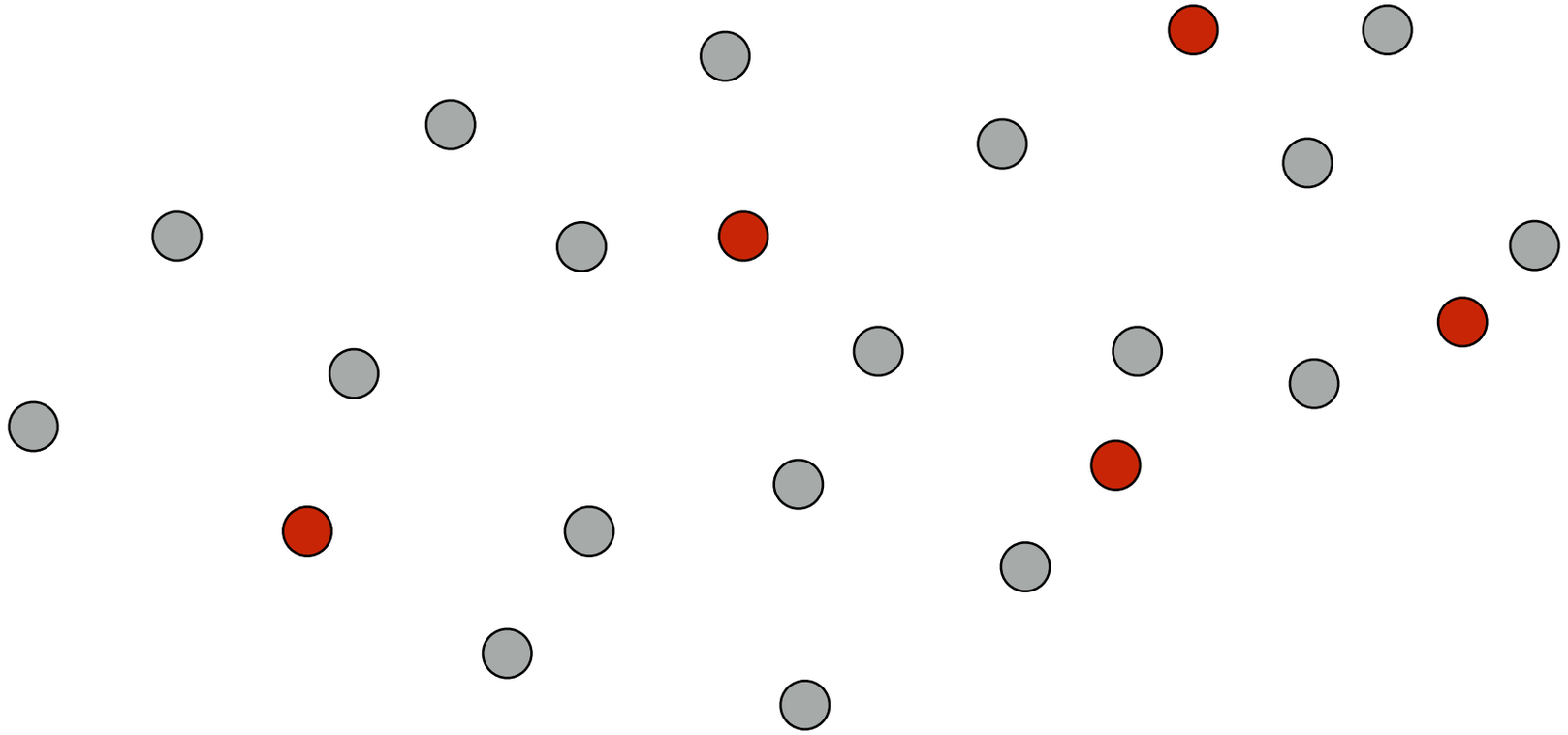

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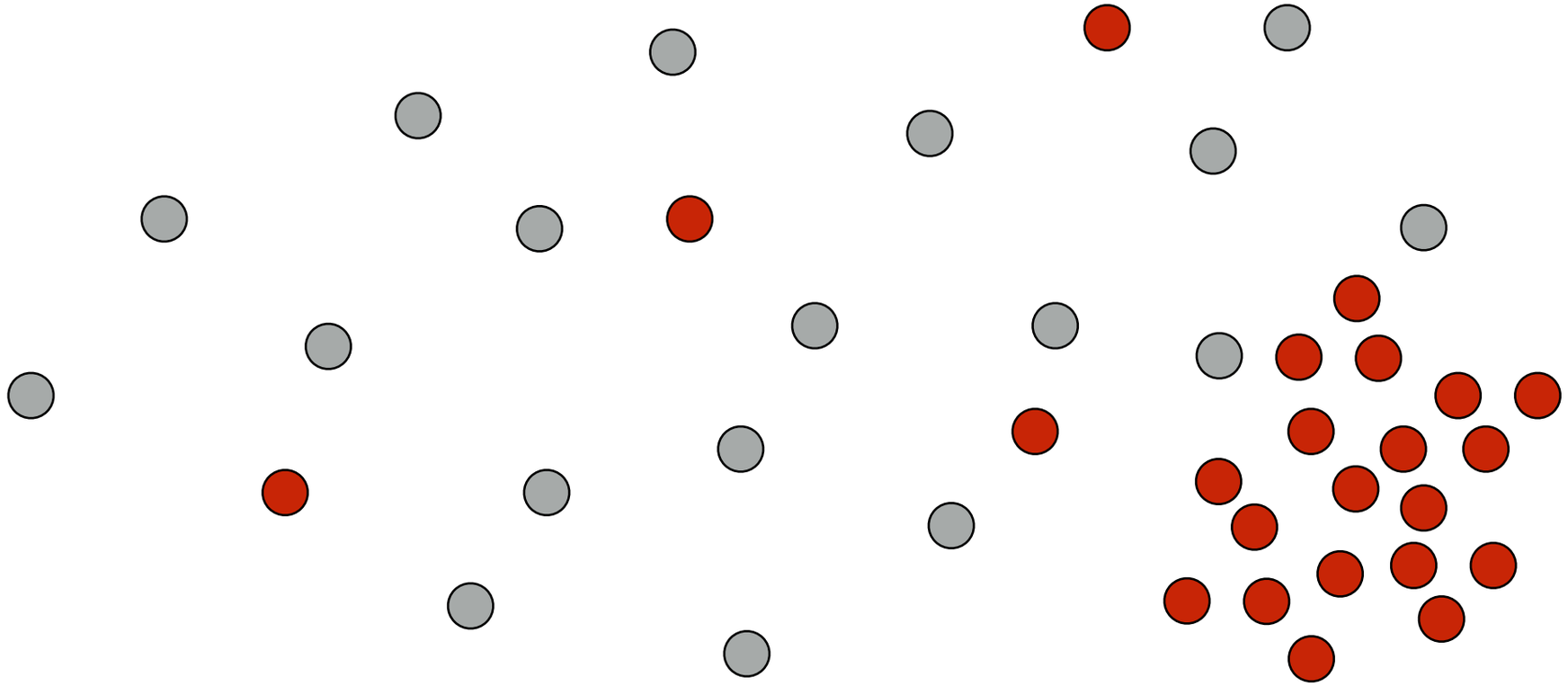

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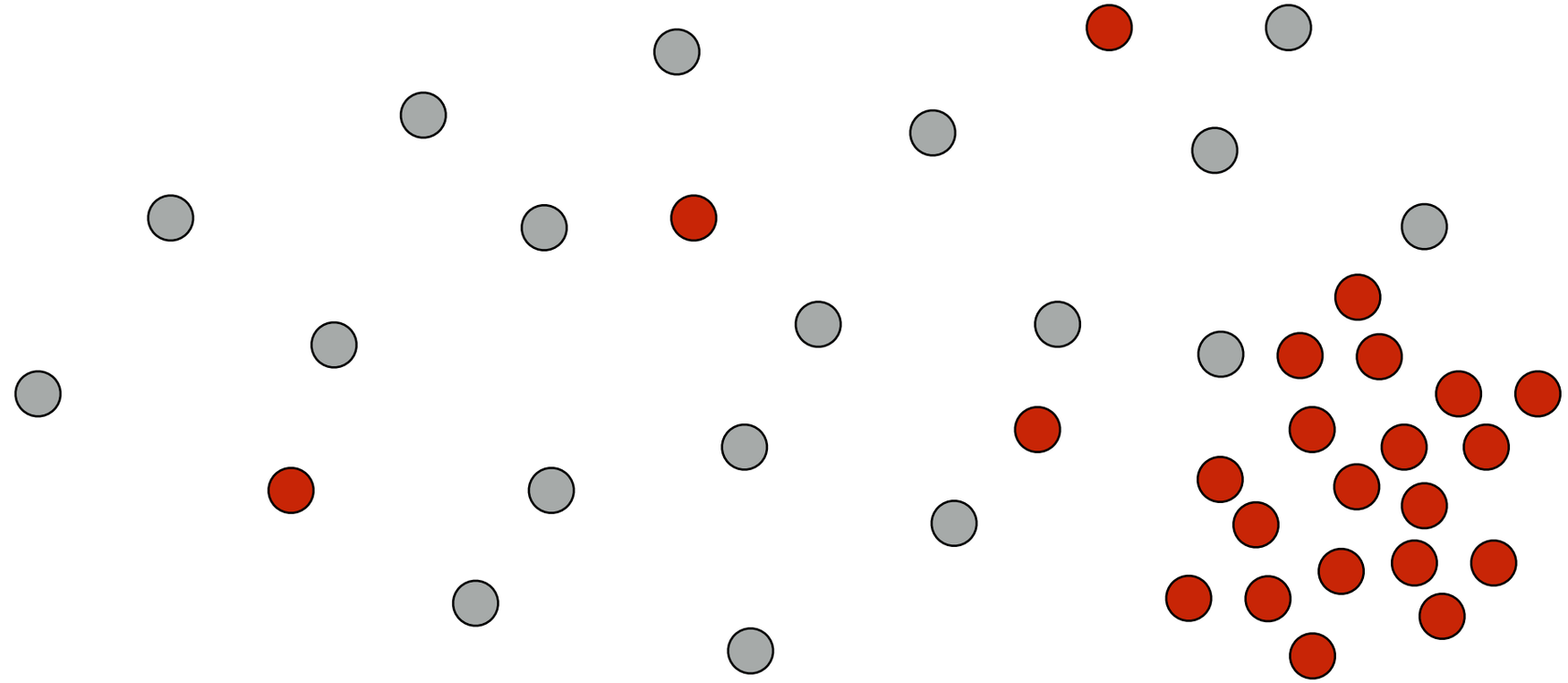

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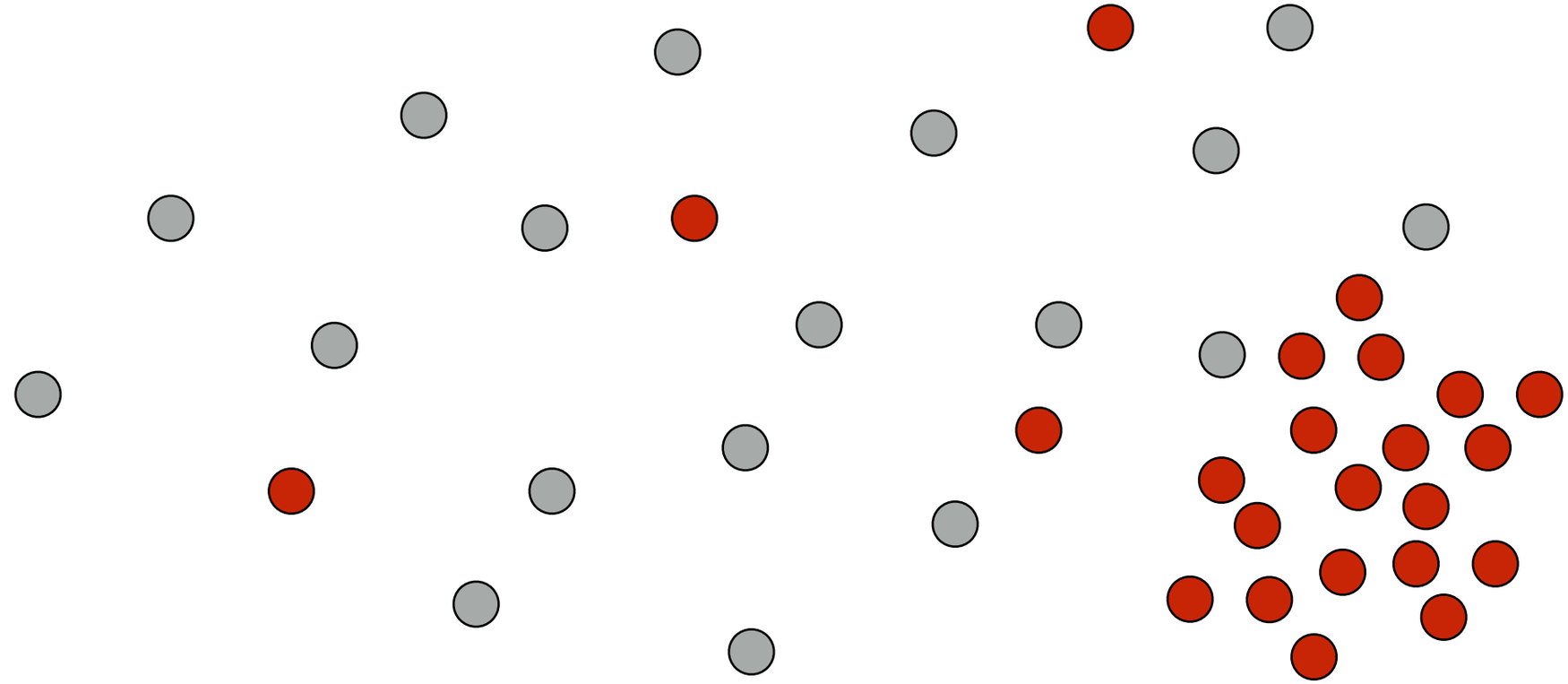

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Goal: Have all honest nodes agree on one of their views of the blockchain.

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DCash 3.0: Consensus with an angel (text section 2.3)

- In this protocol, time is divided into rounds.
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This protocol doesn't exist in reality! We will build the angel and personal views using POWs later.

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Note 2: A node may *only* add a block to its view when the angel chooses it.

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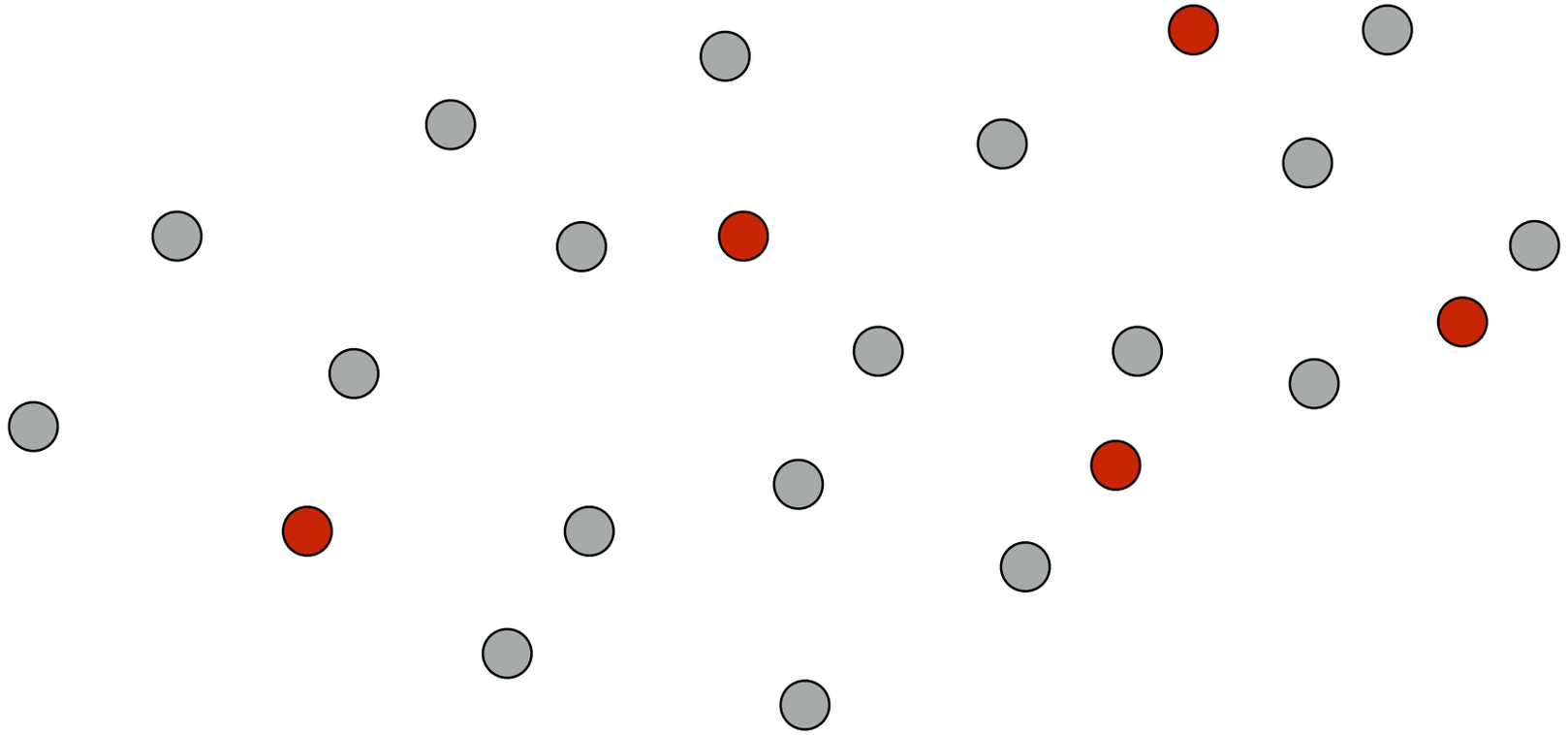

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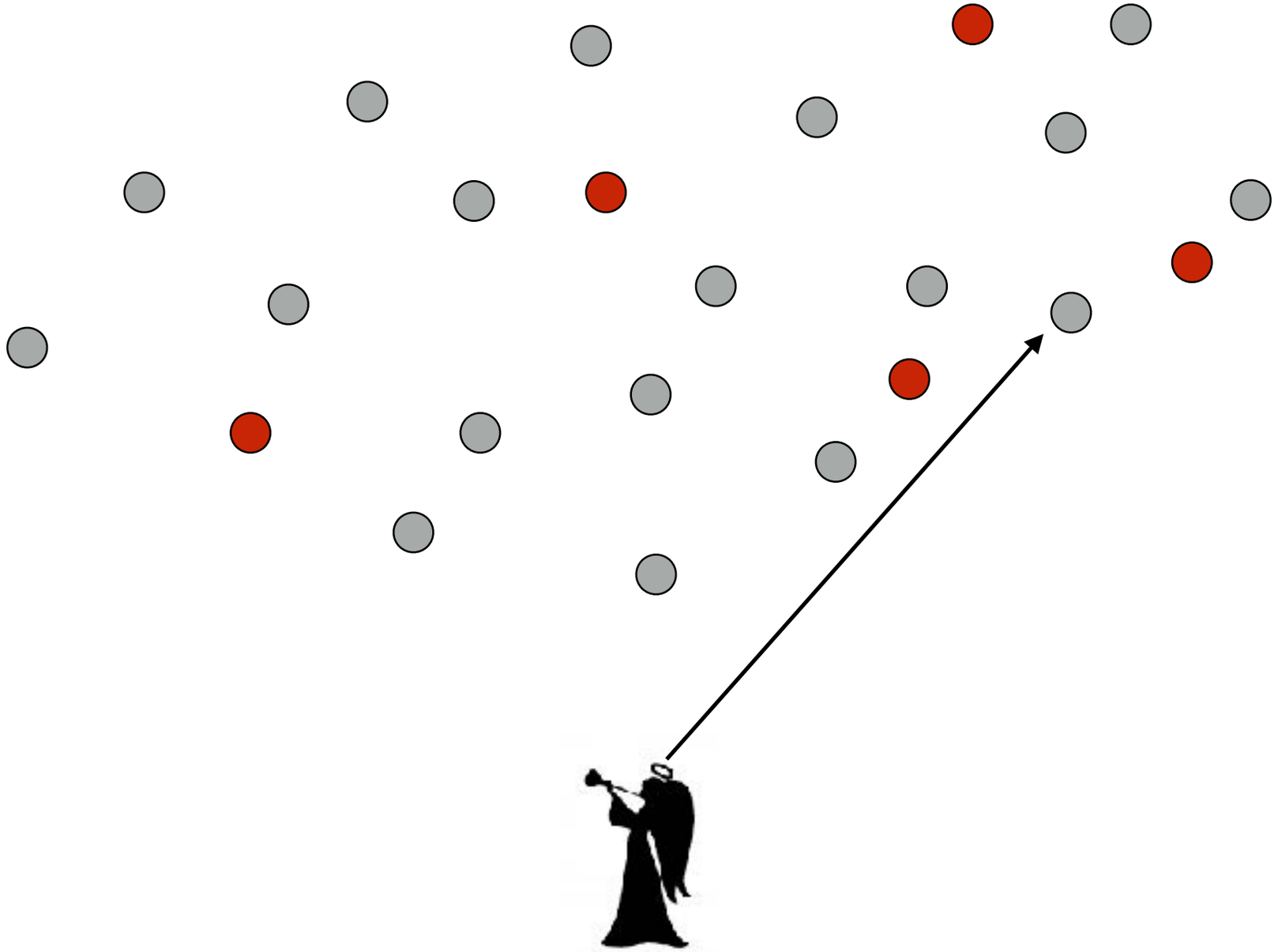

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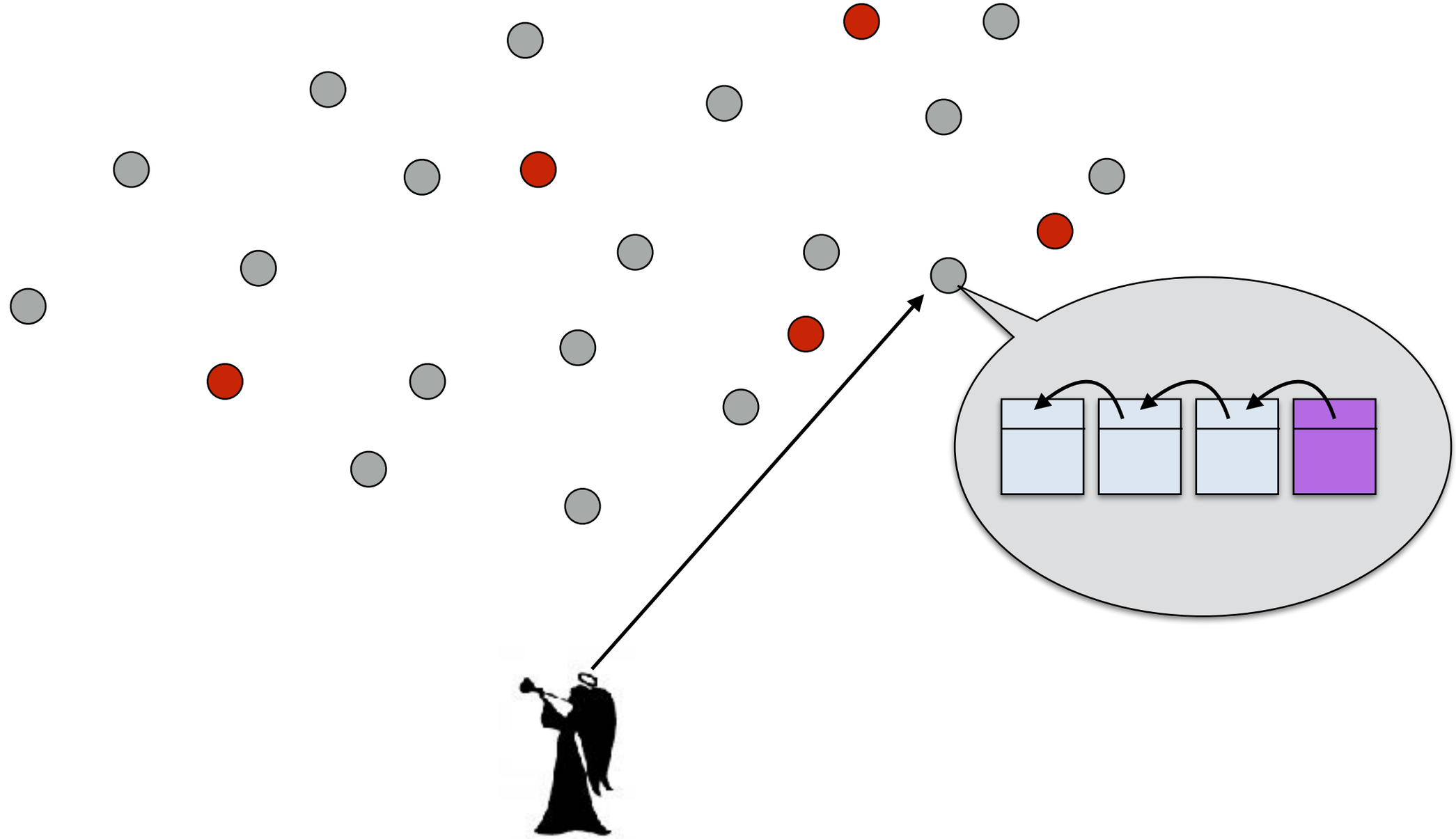

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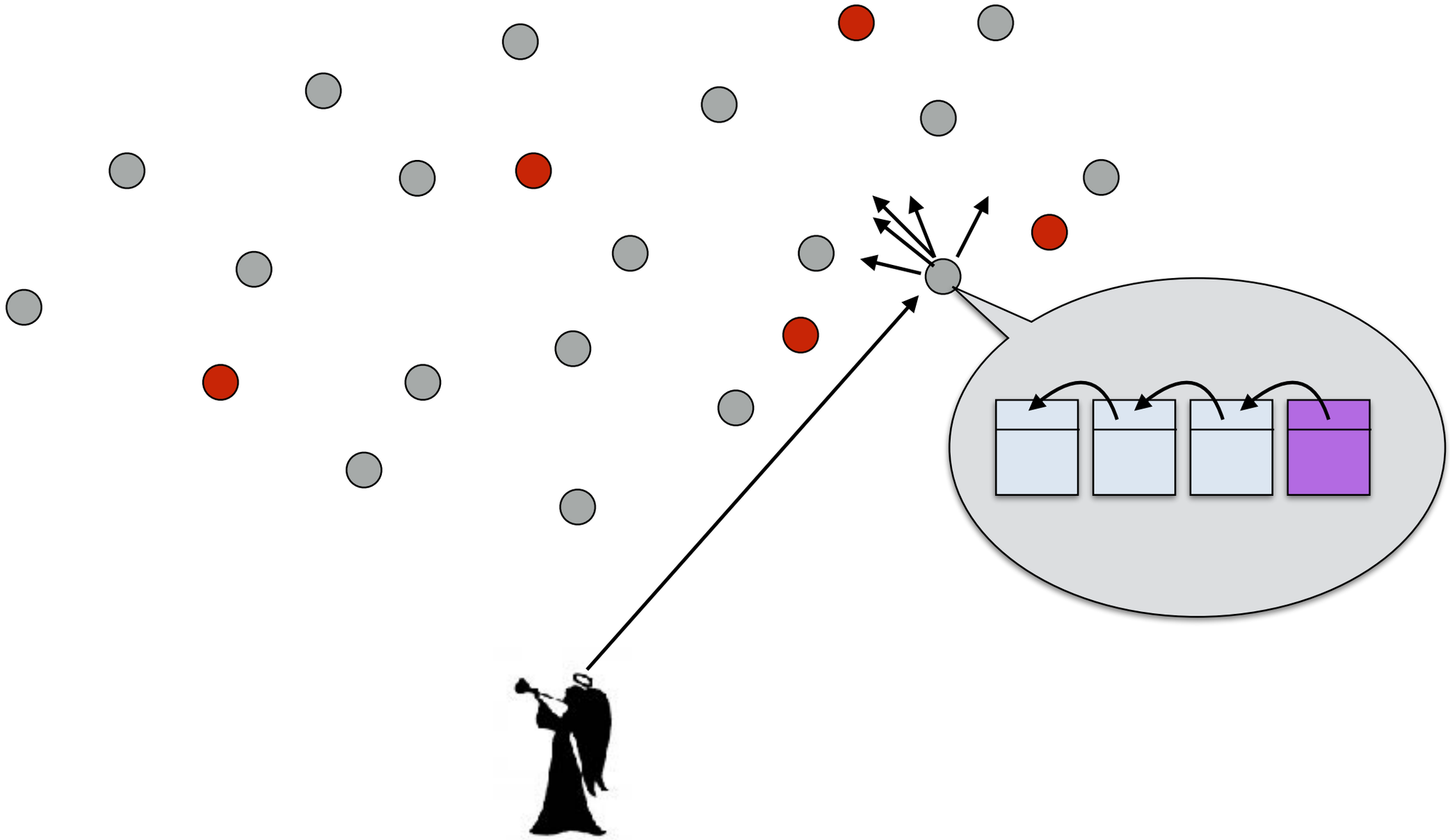

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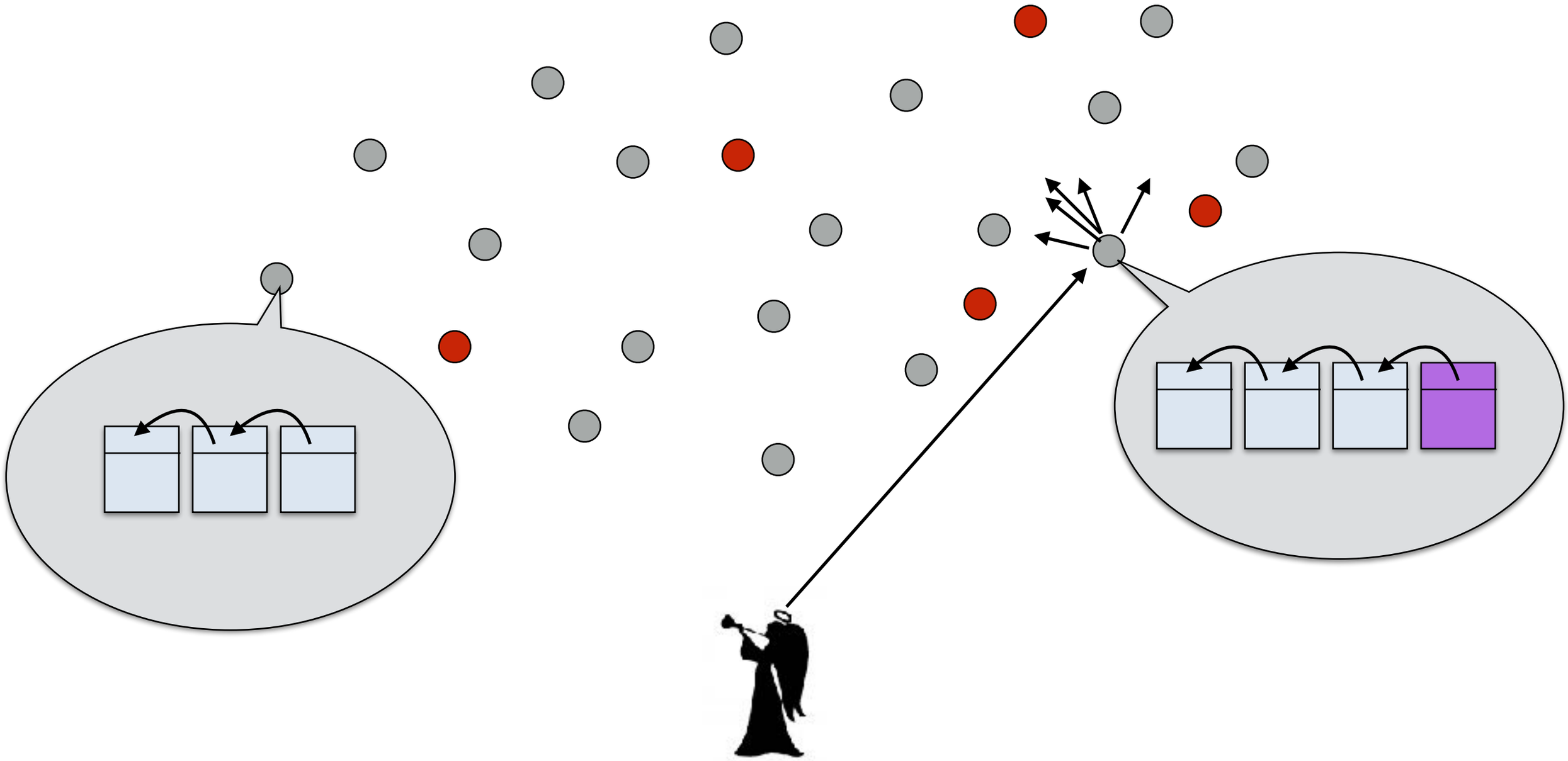

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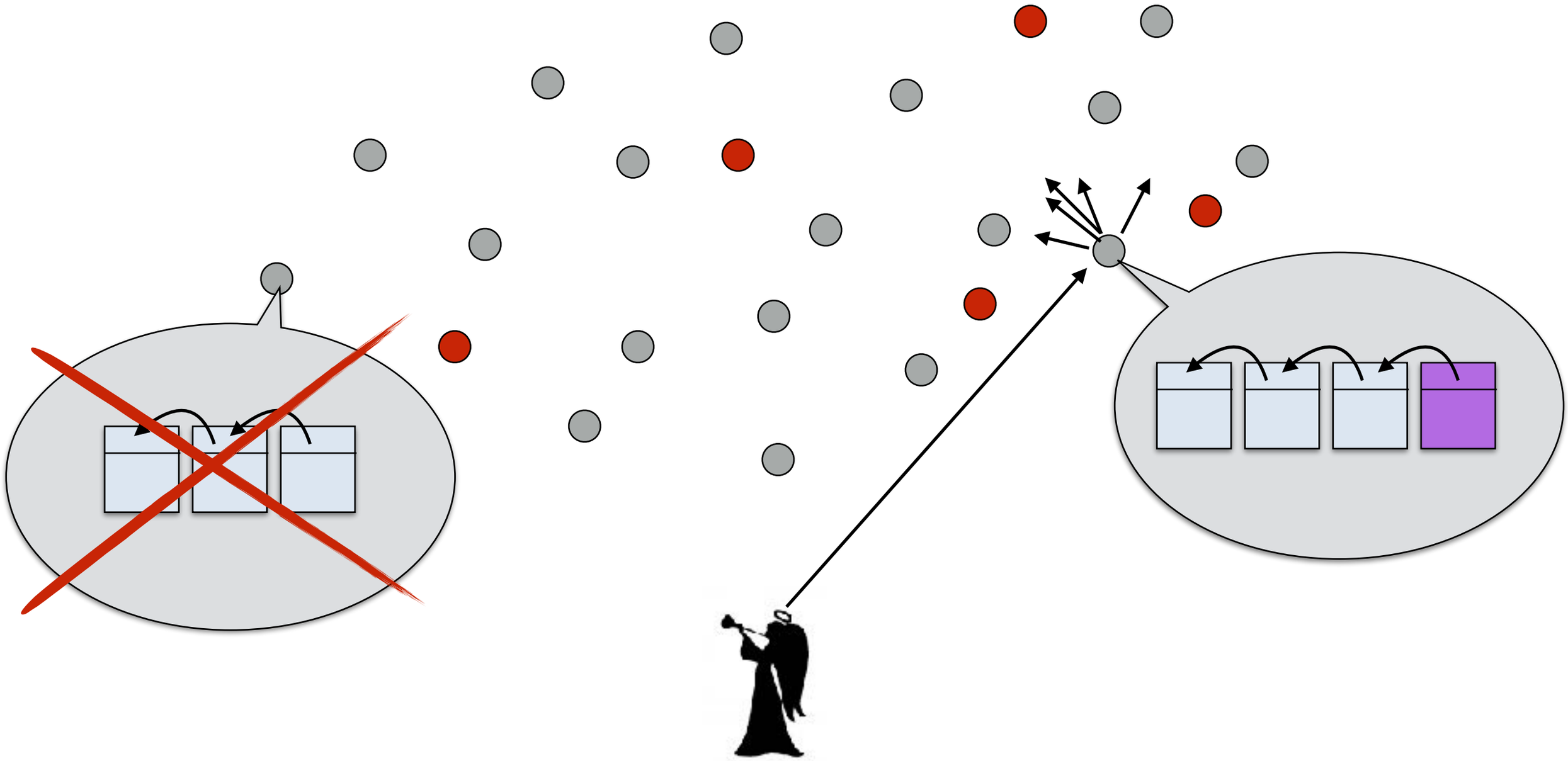

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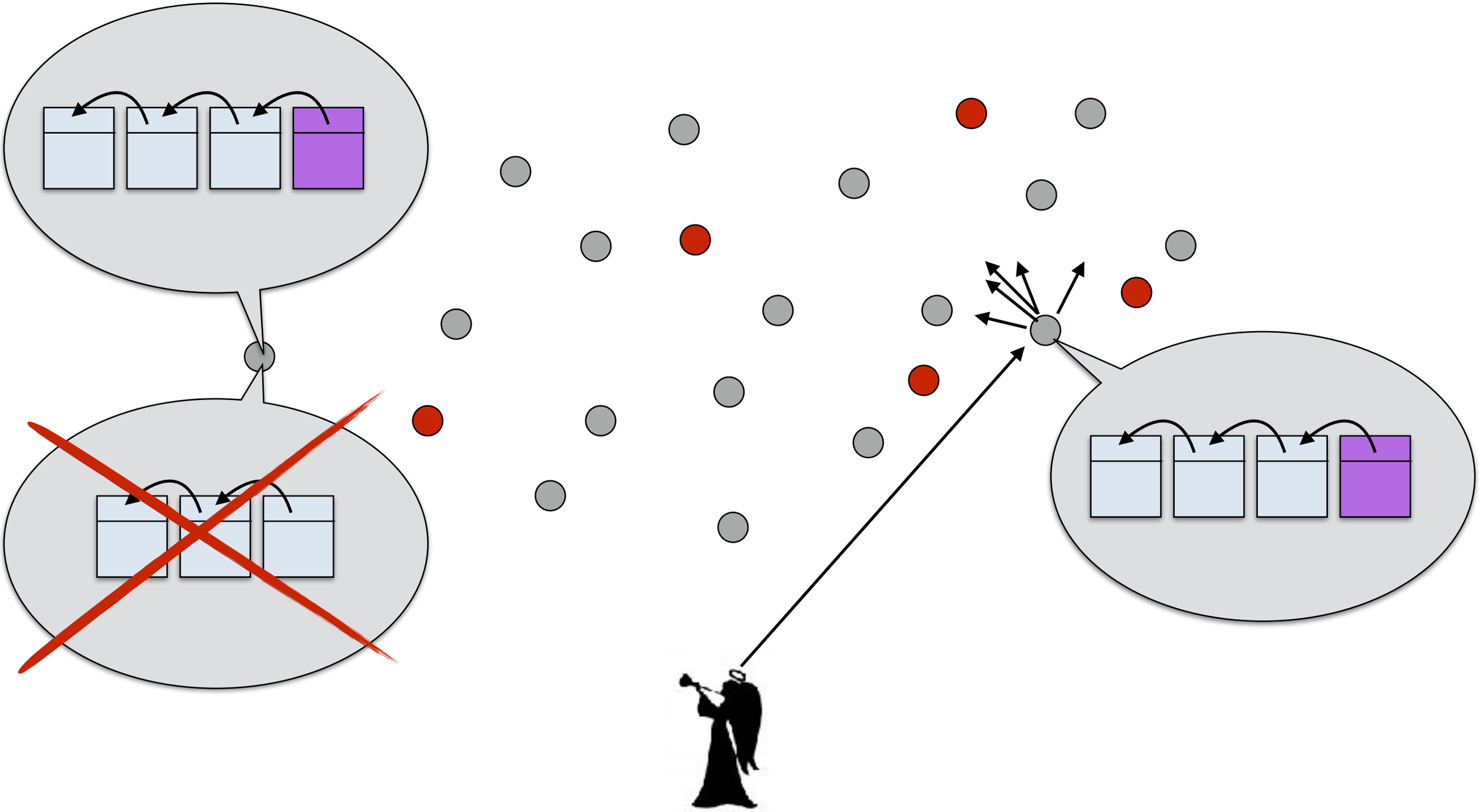

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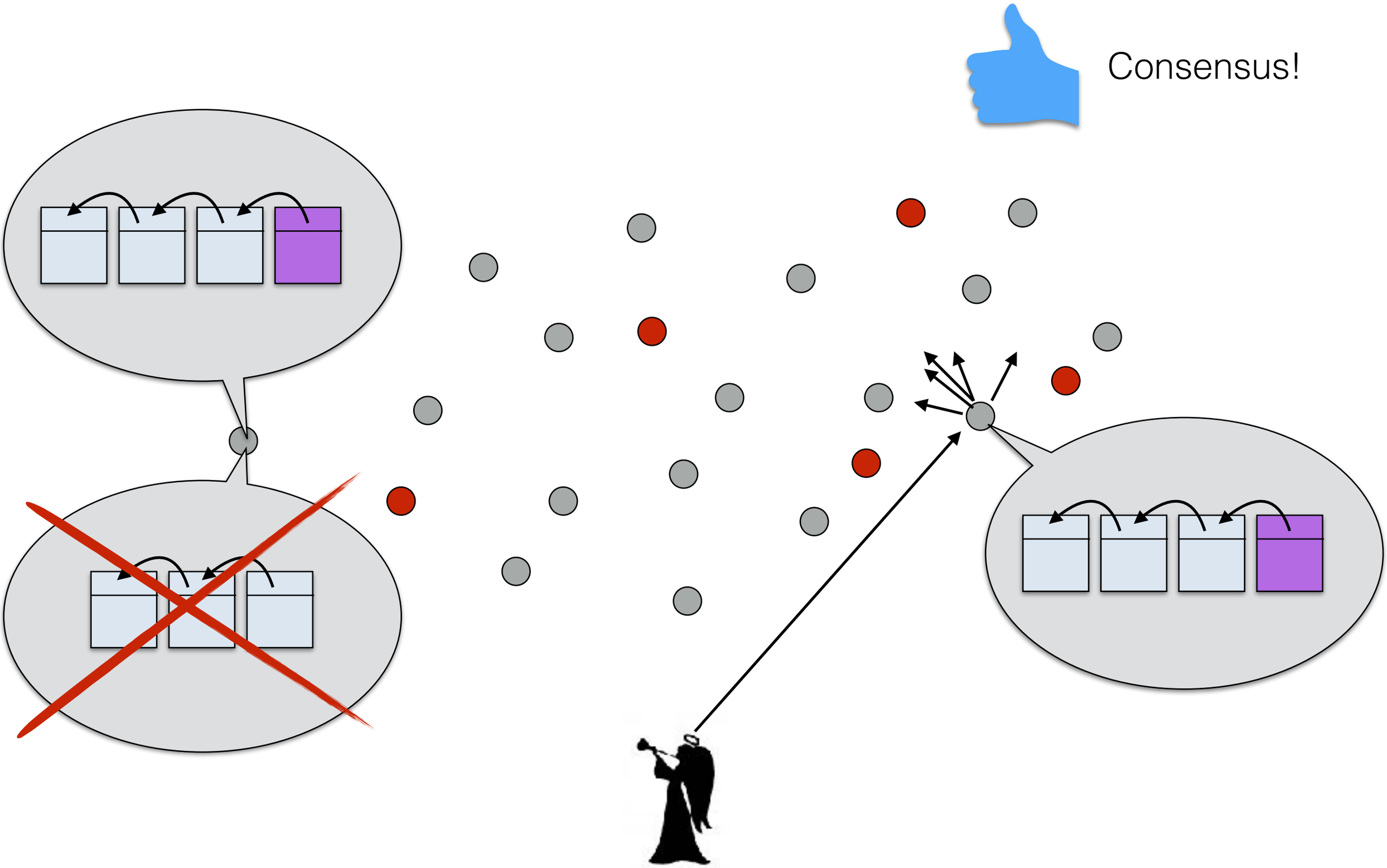

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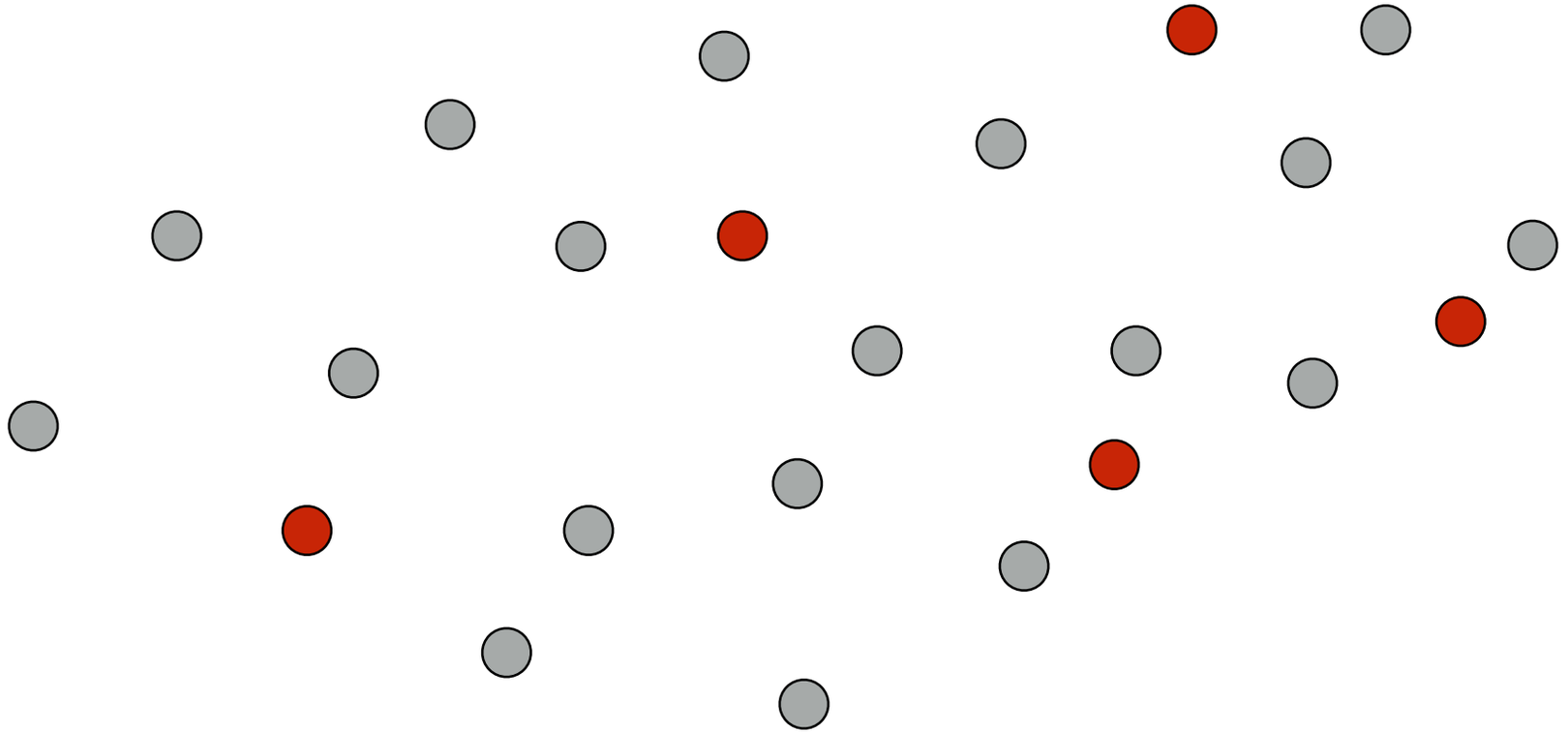

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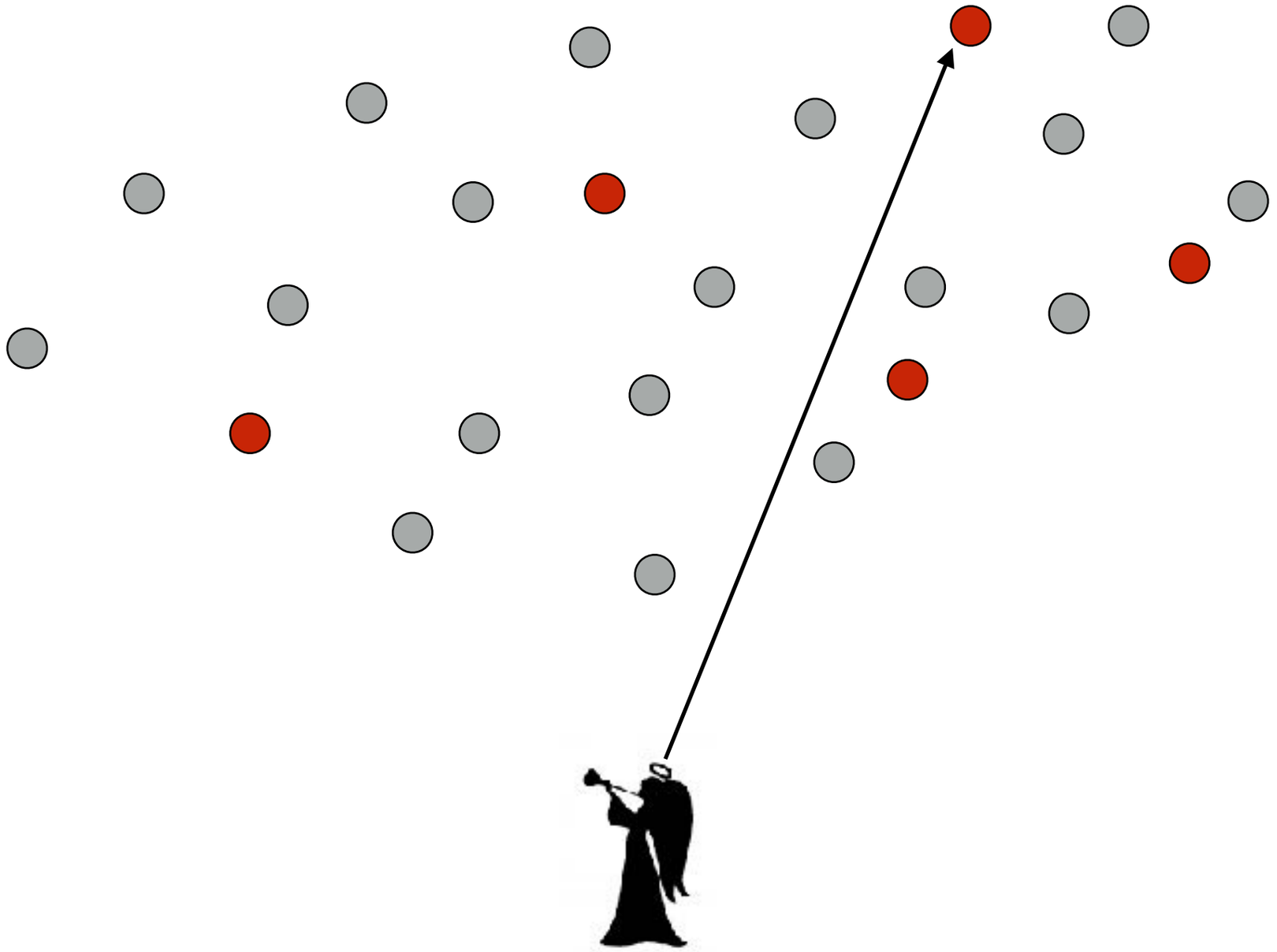

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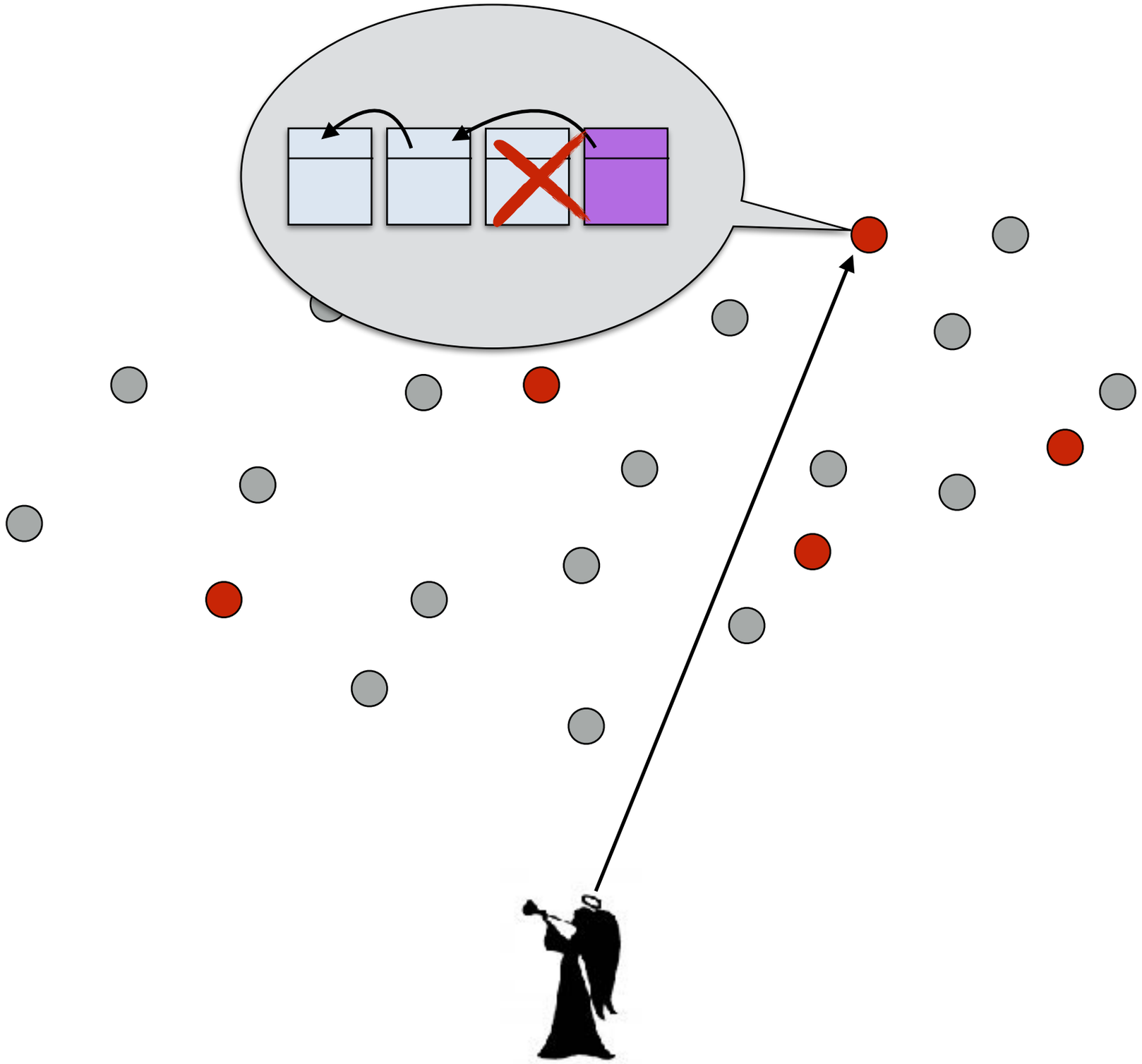

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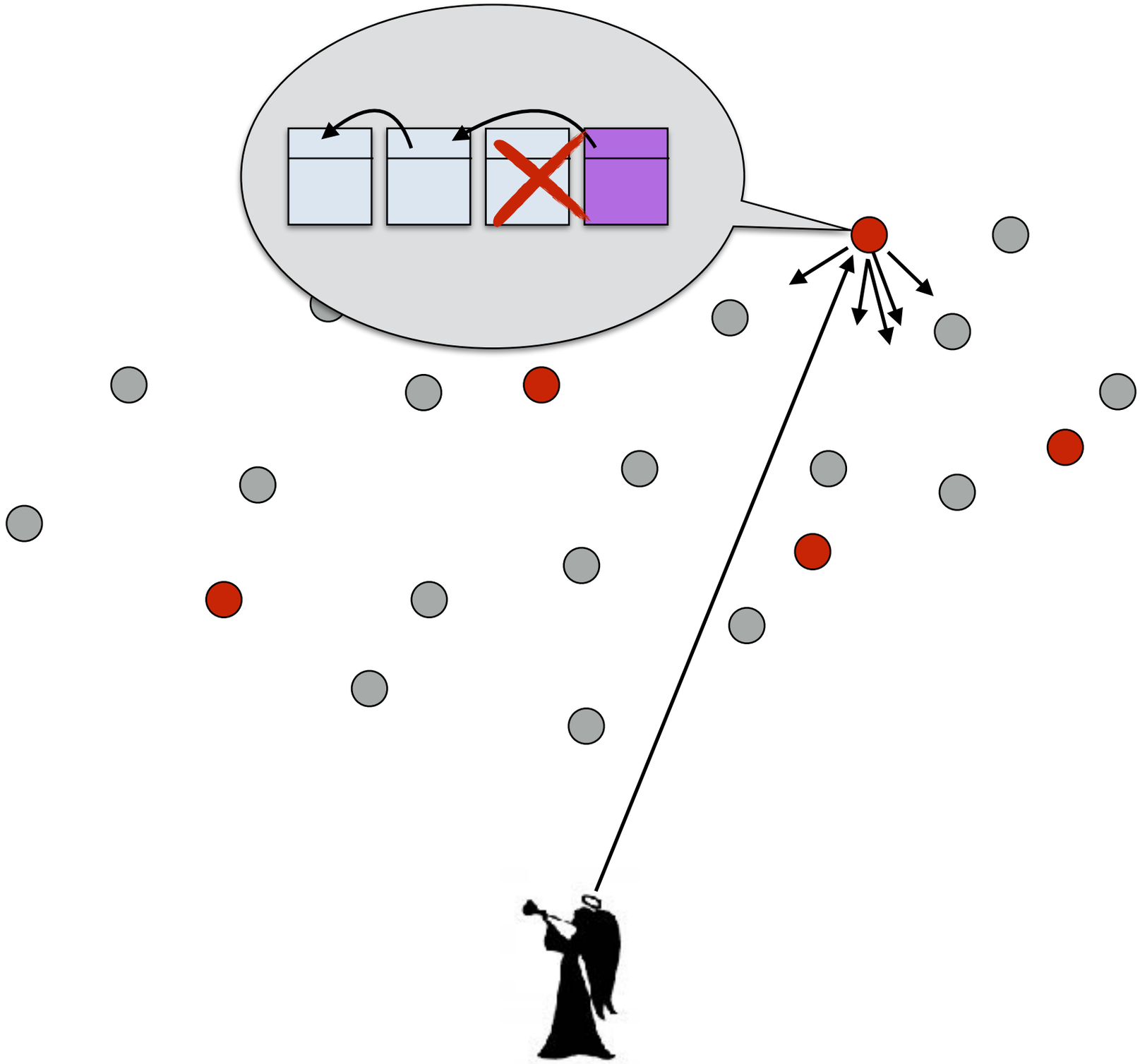

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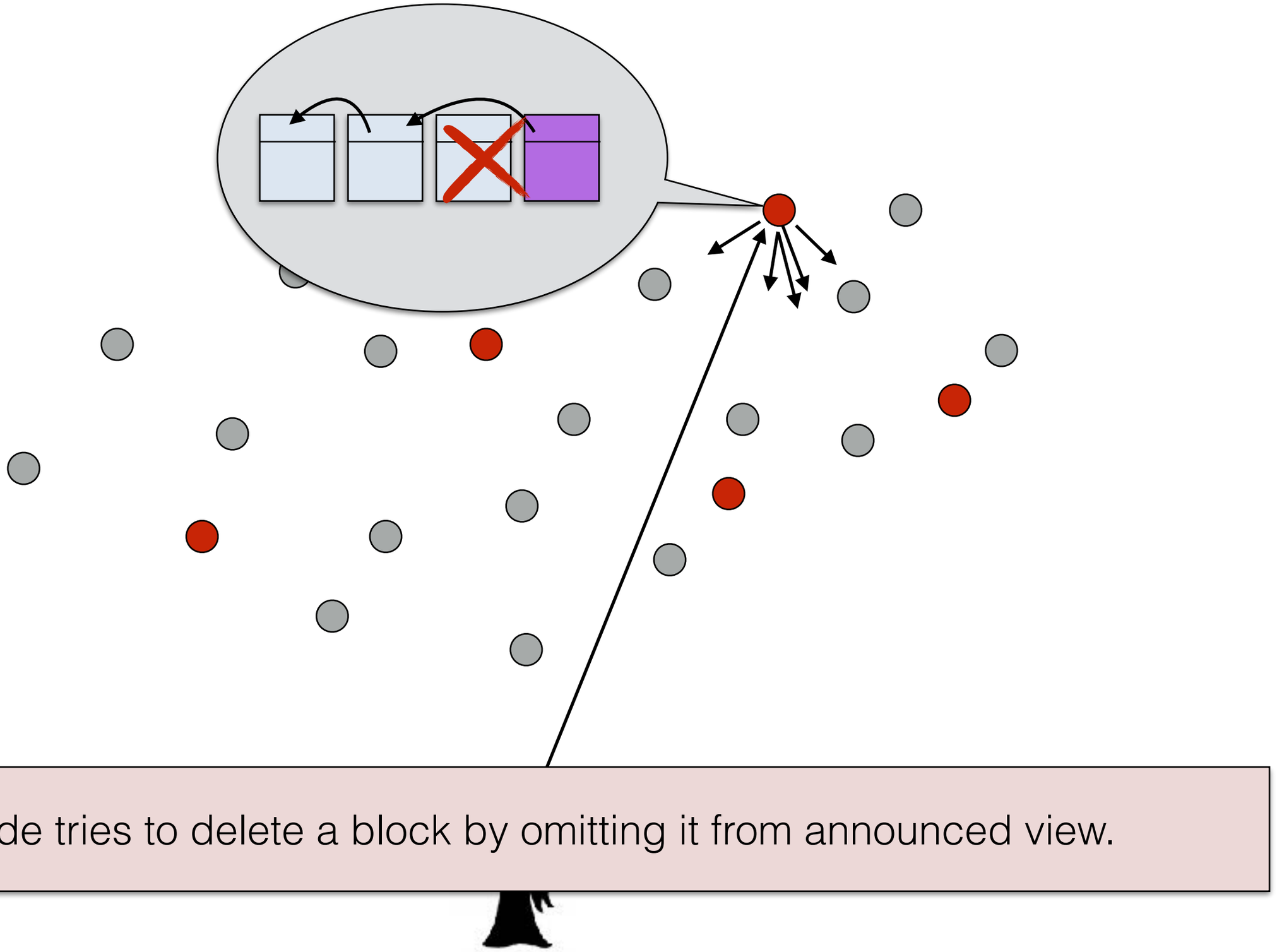

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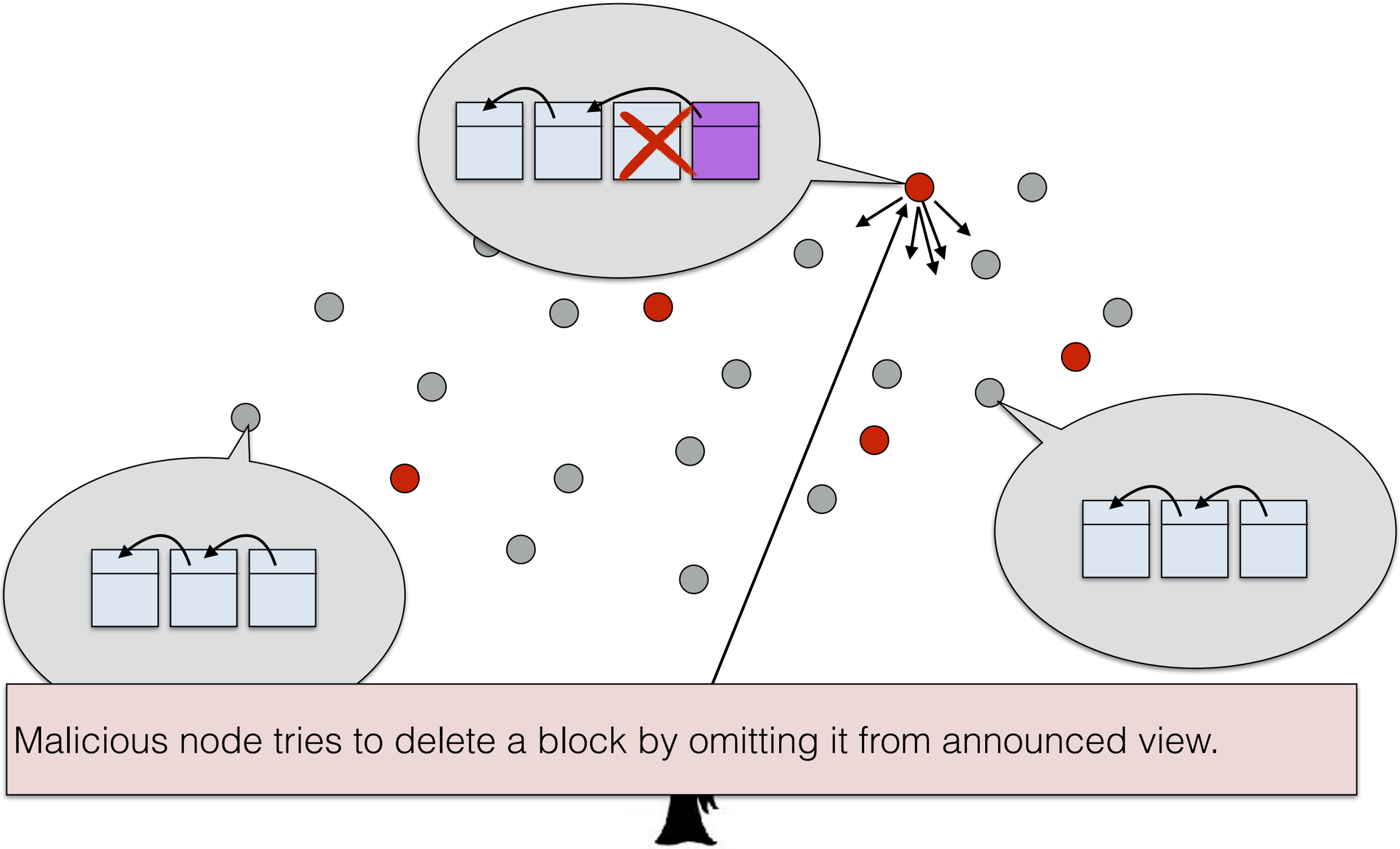

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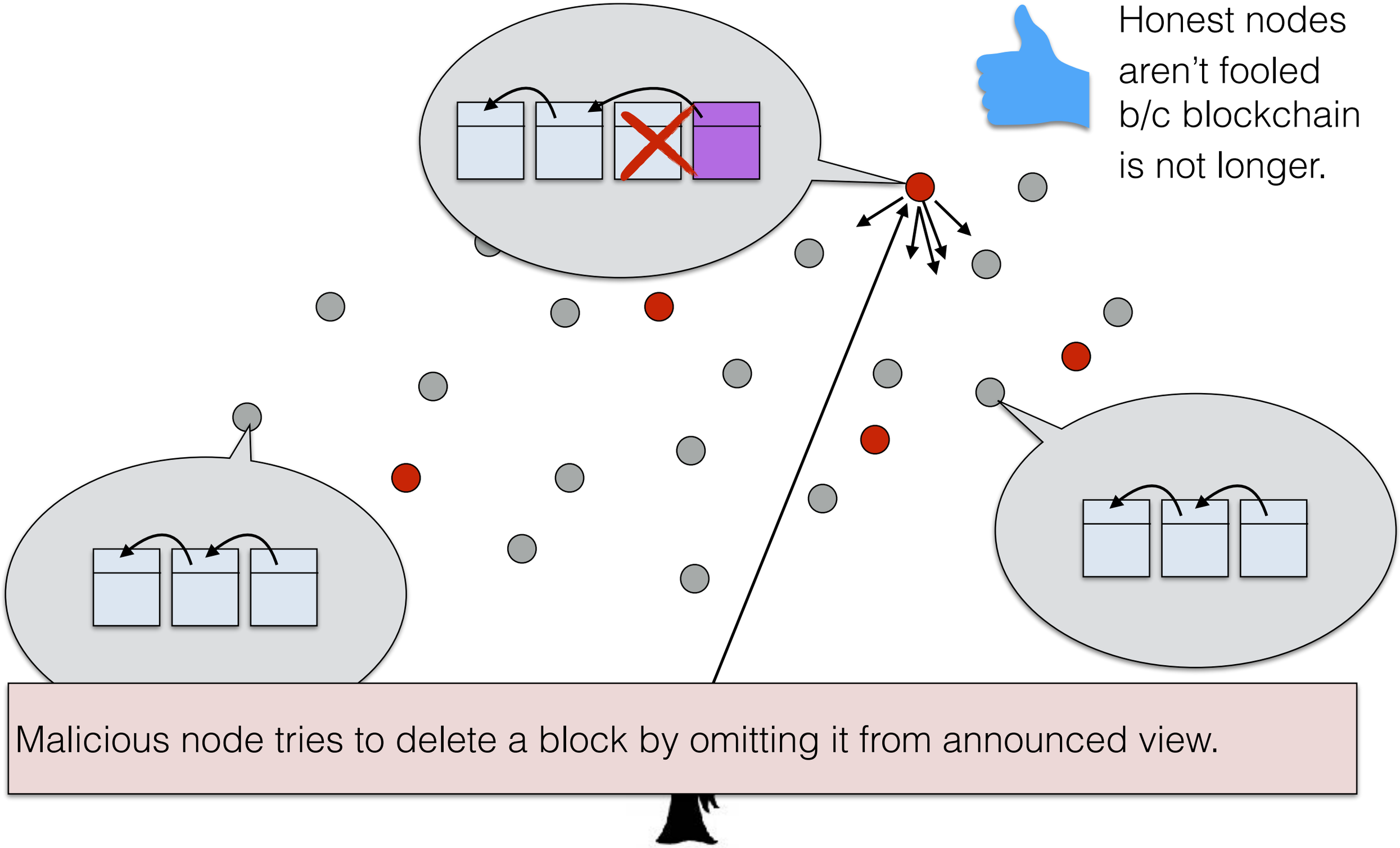

Consensus!
Honest nodes aren't fooled b/c blockchain is not longer.

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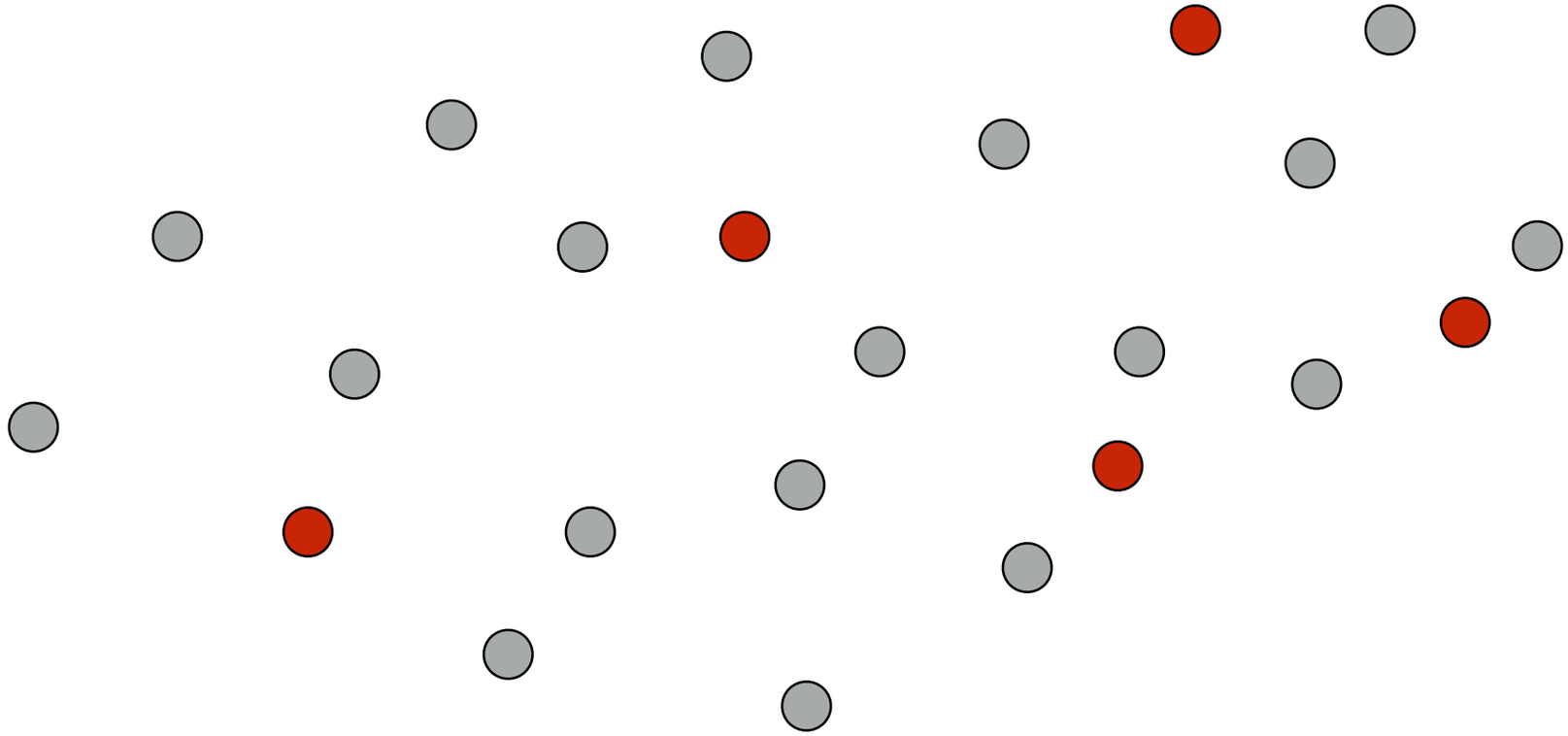
Example: Angel picks an malicious node twice



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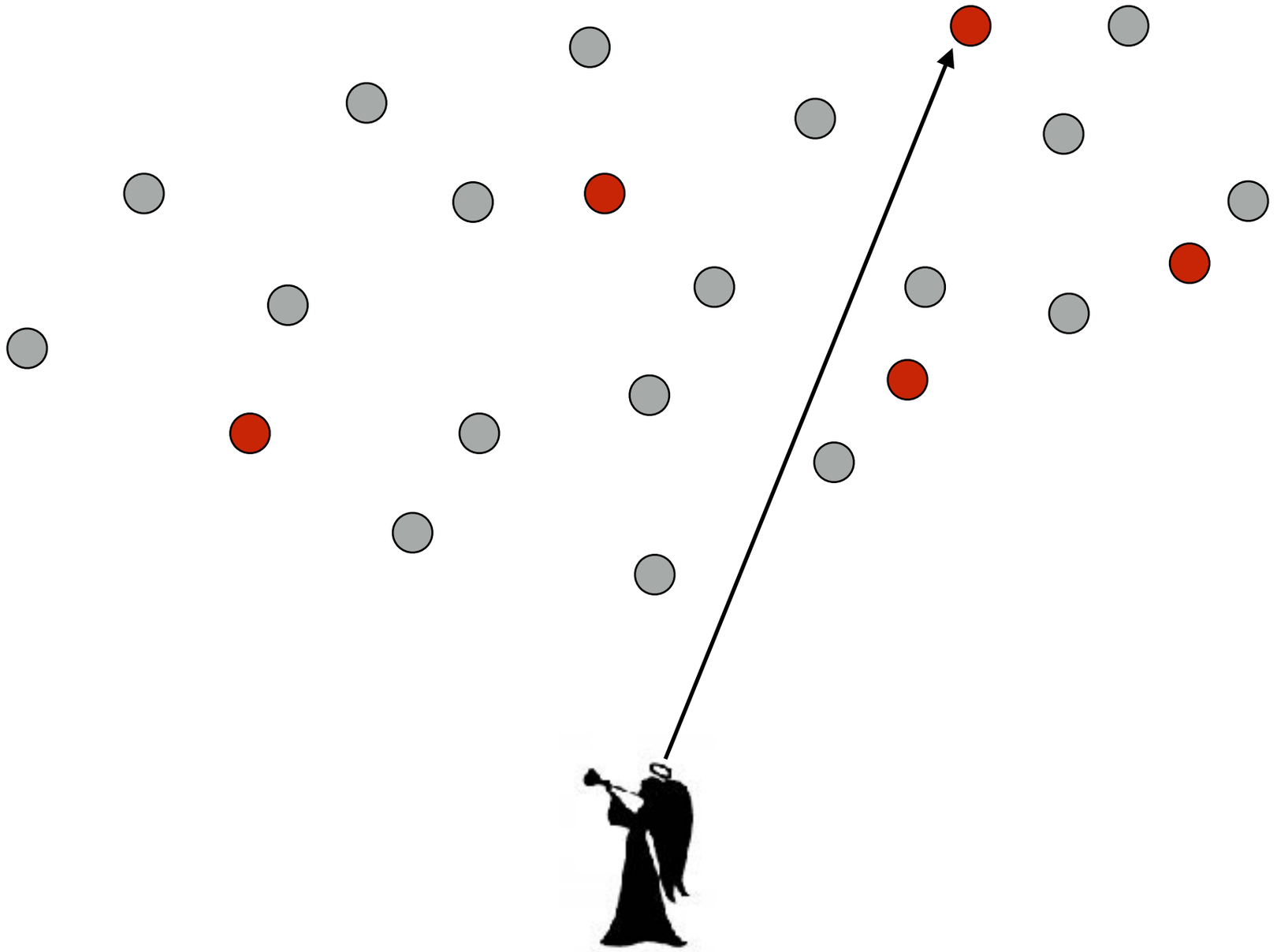

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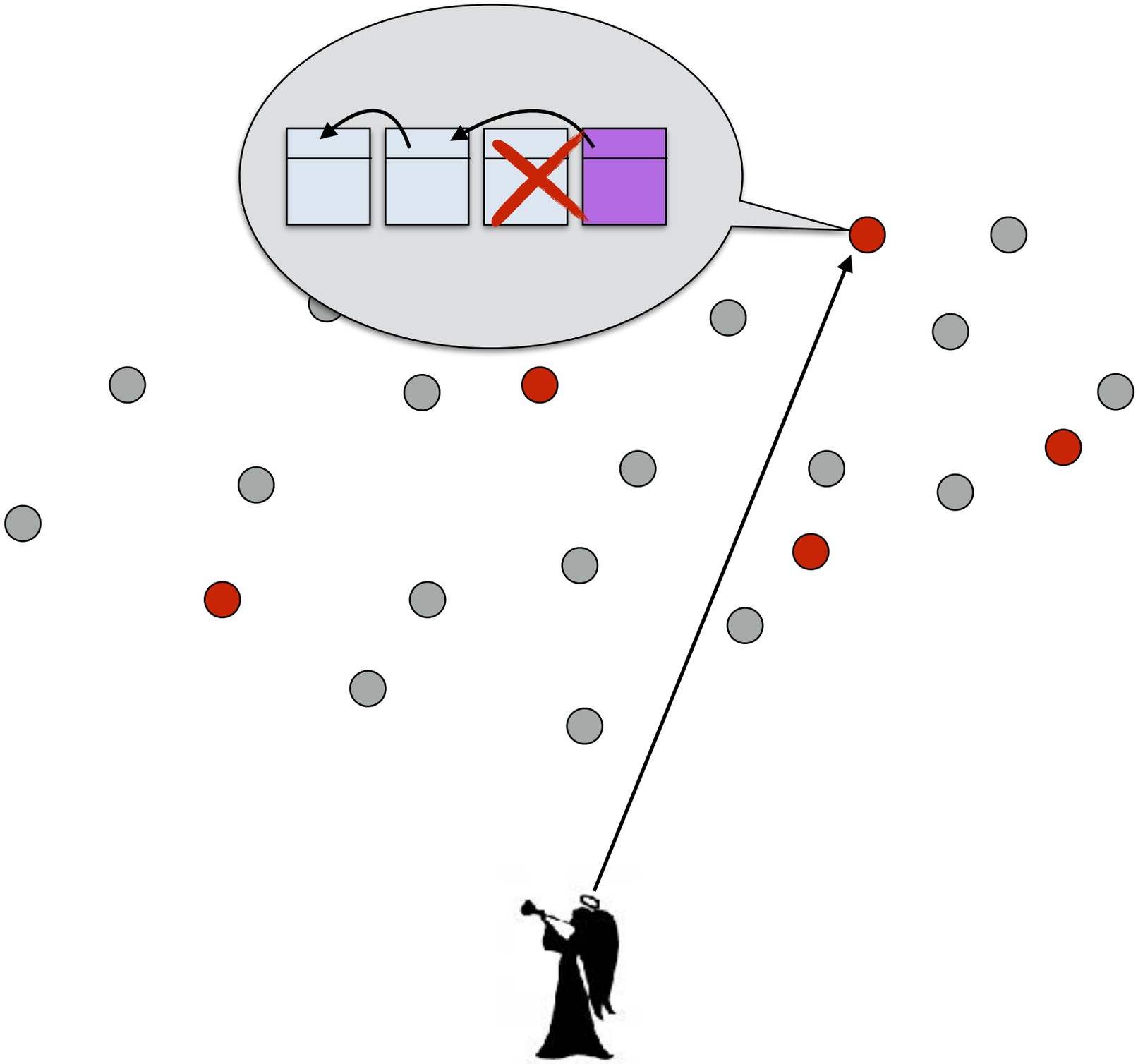

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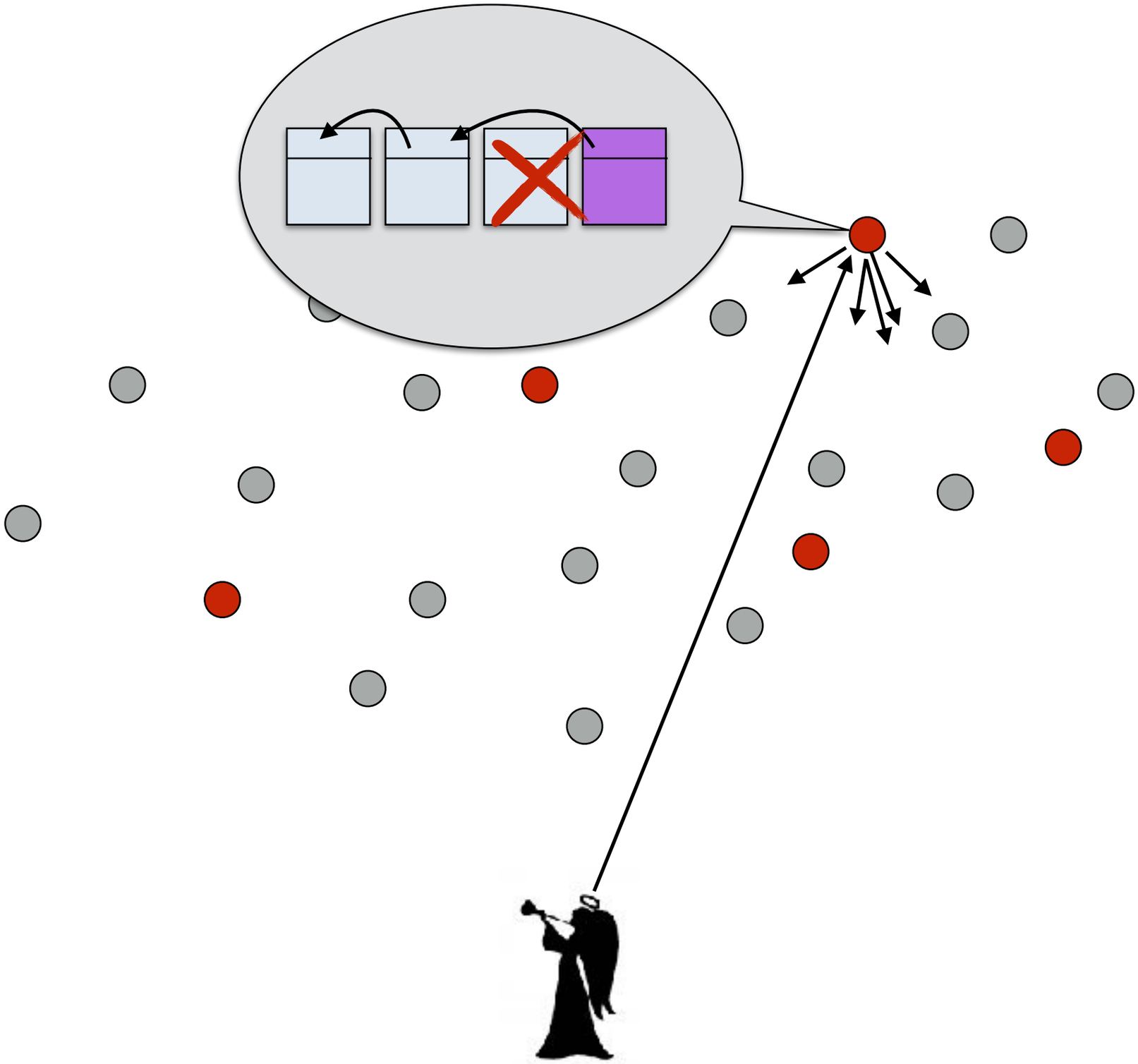

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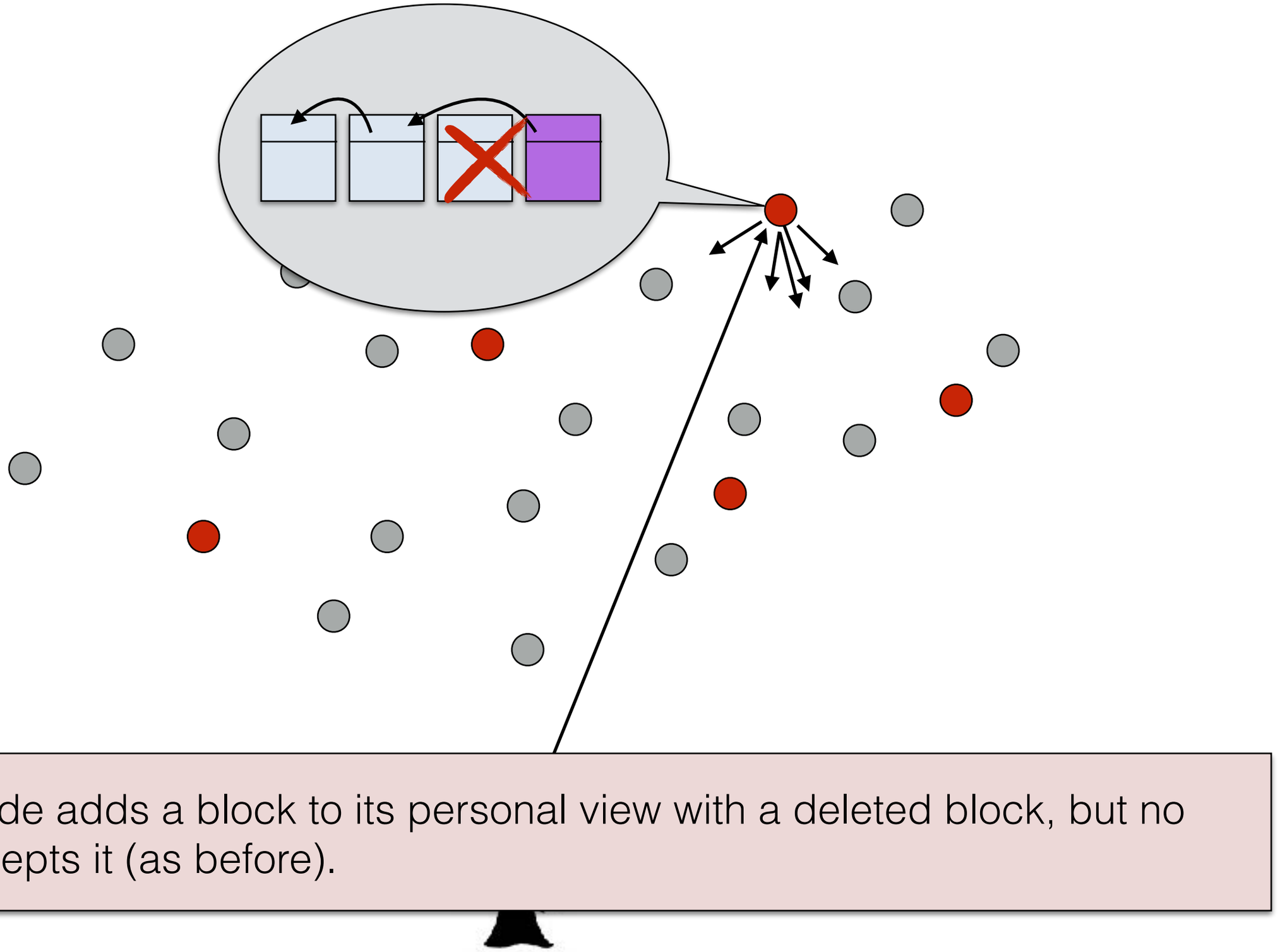

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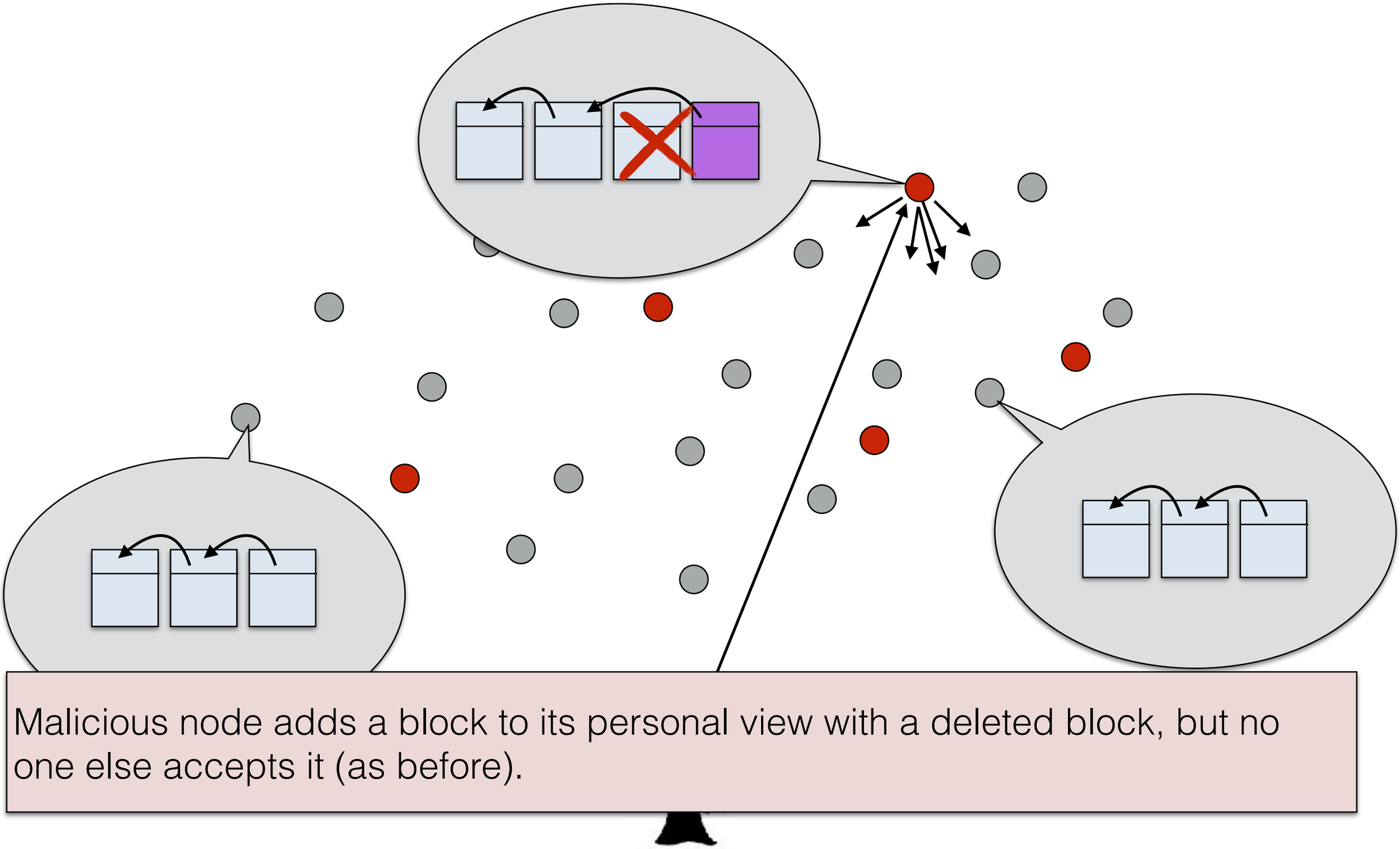

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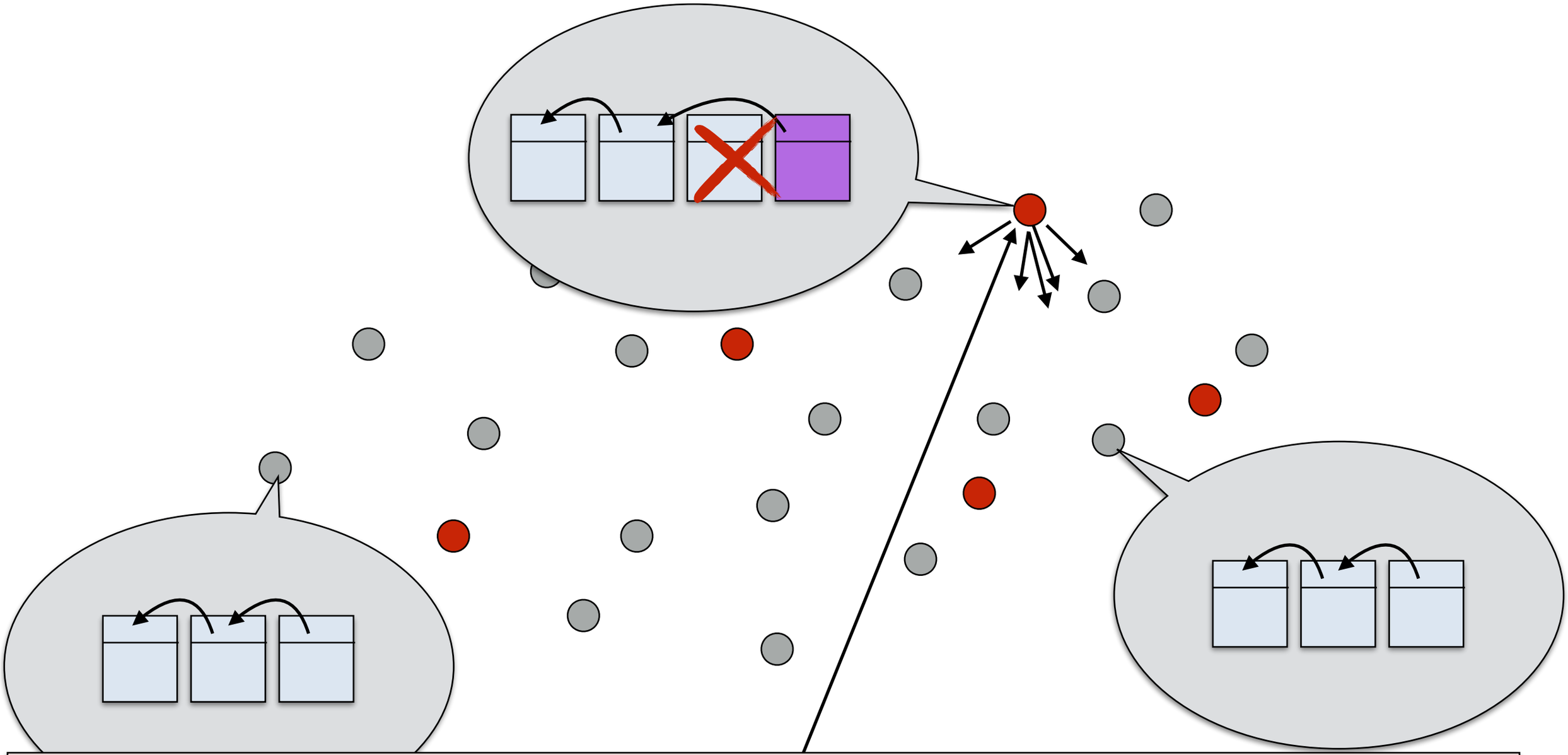

Malicious node adds a block to its personal view with a deleted block, but no one else accepts it (as before).

However, the malicious node adds the block to its view of the chain.

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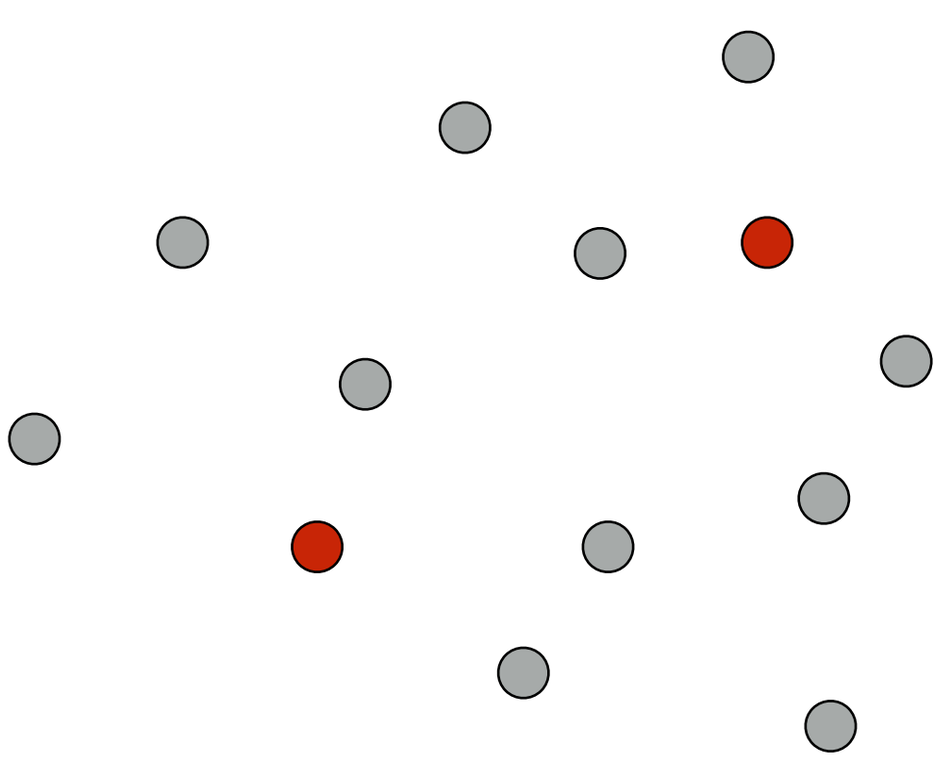

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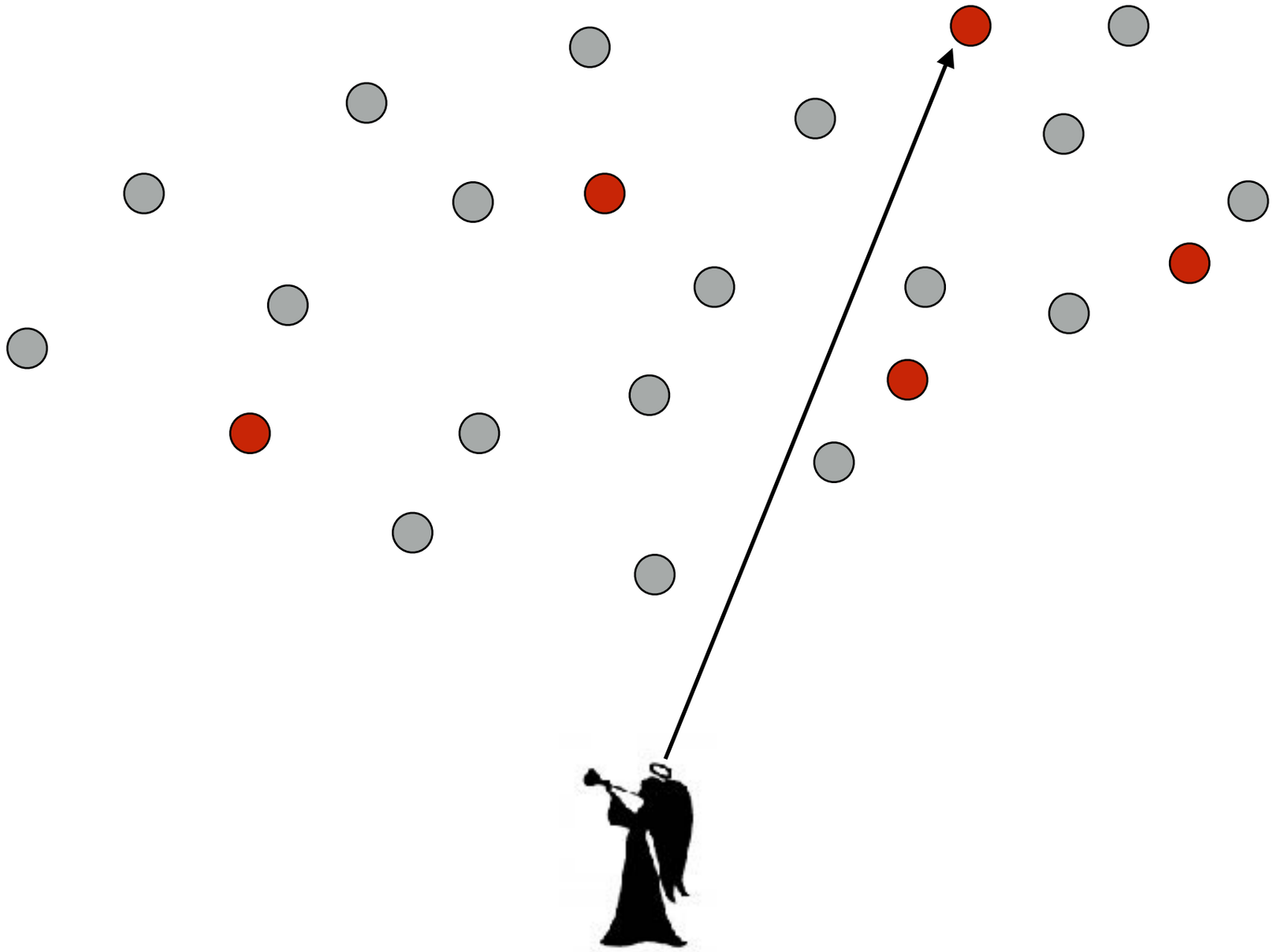

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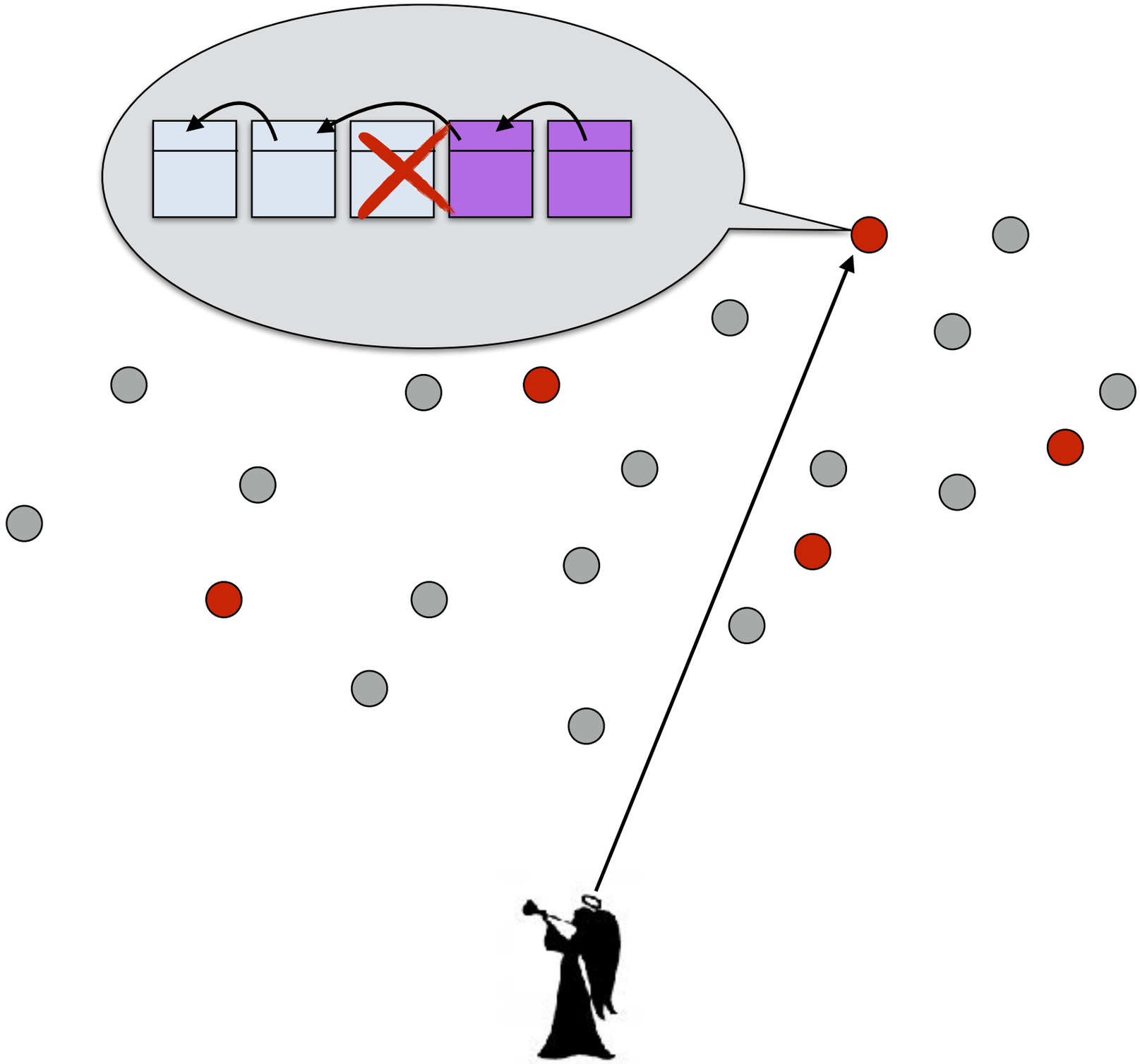

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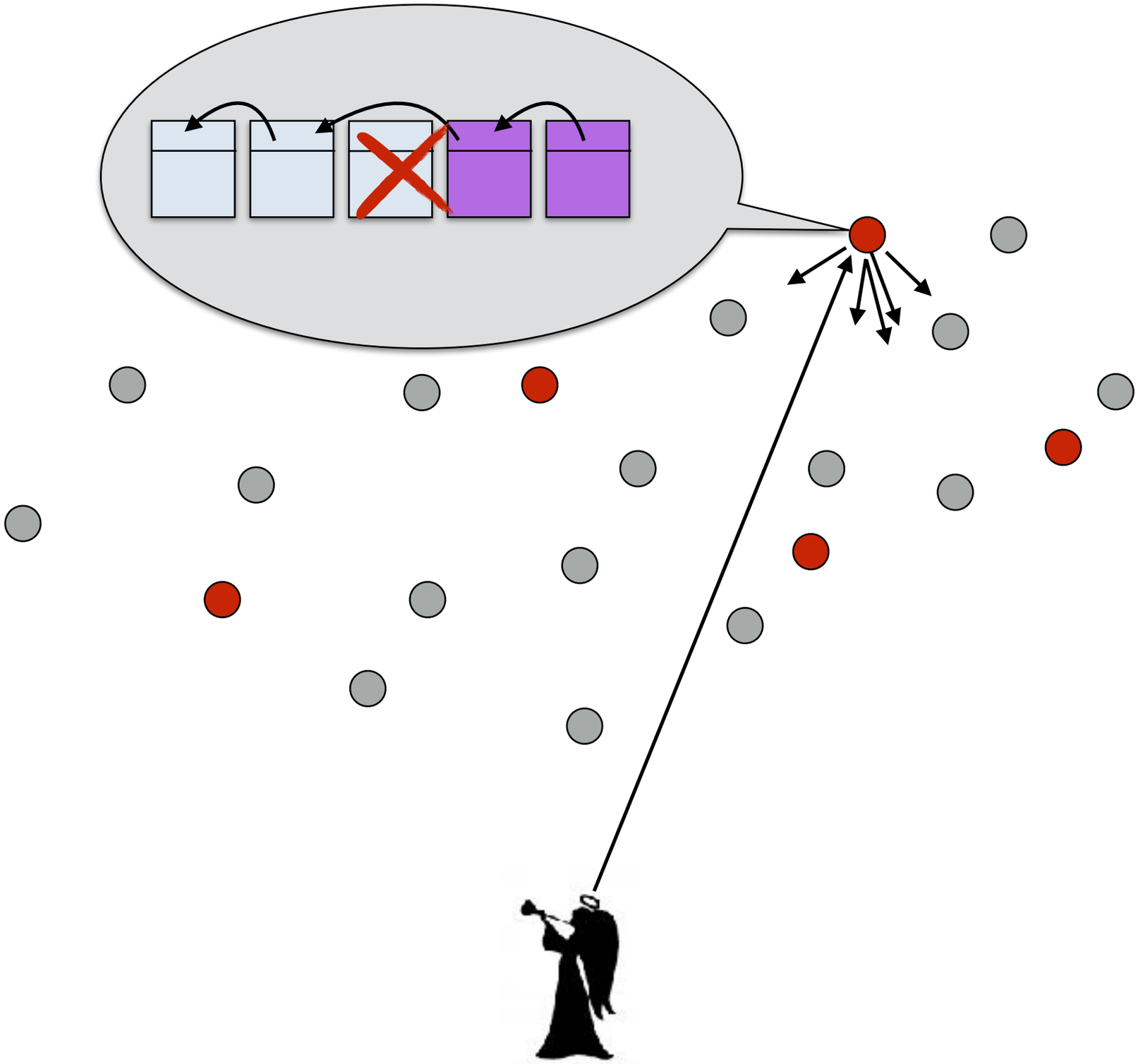

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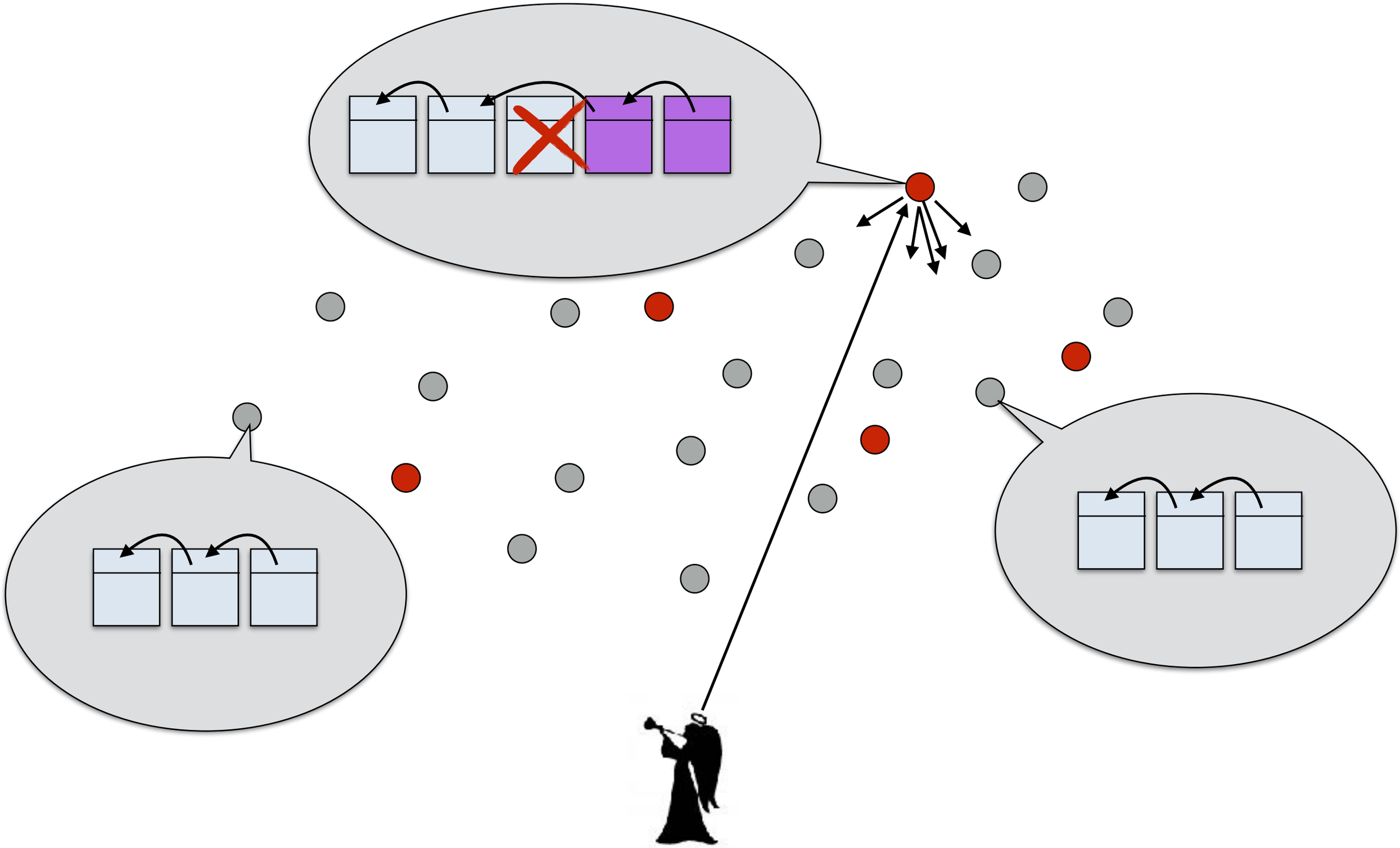

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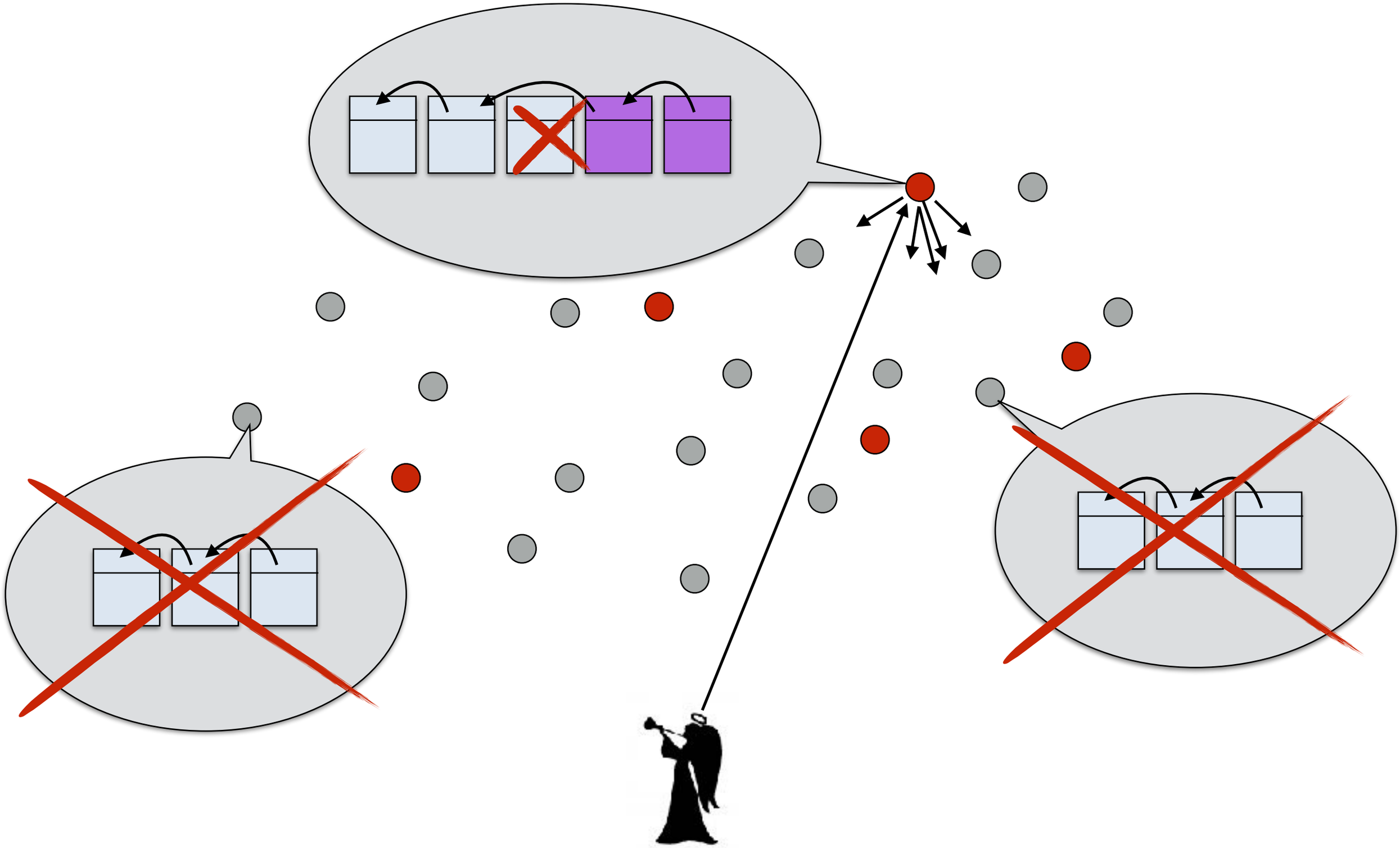

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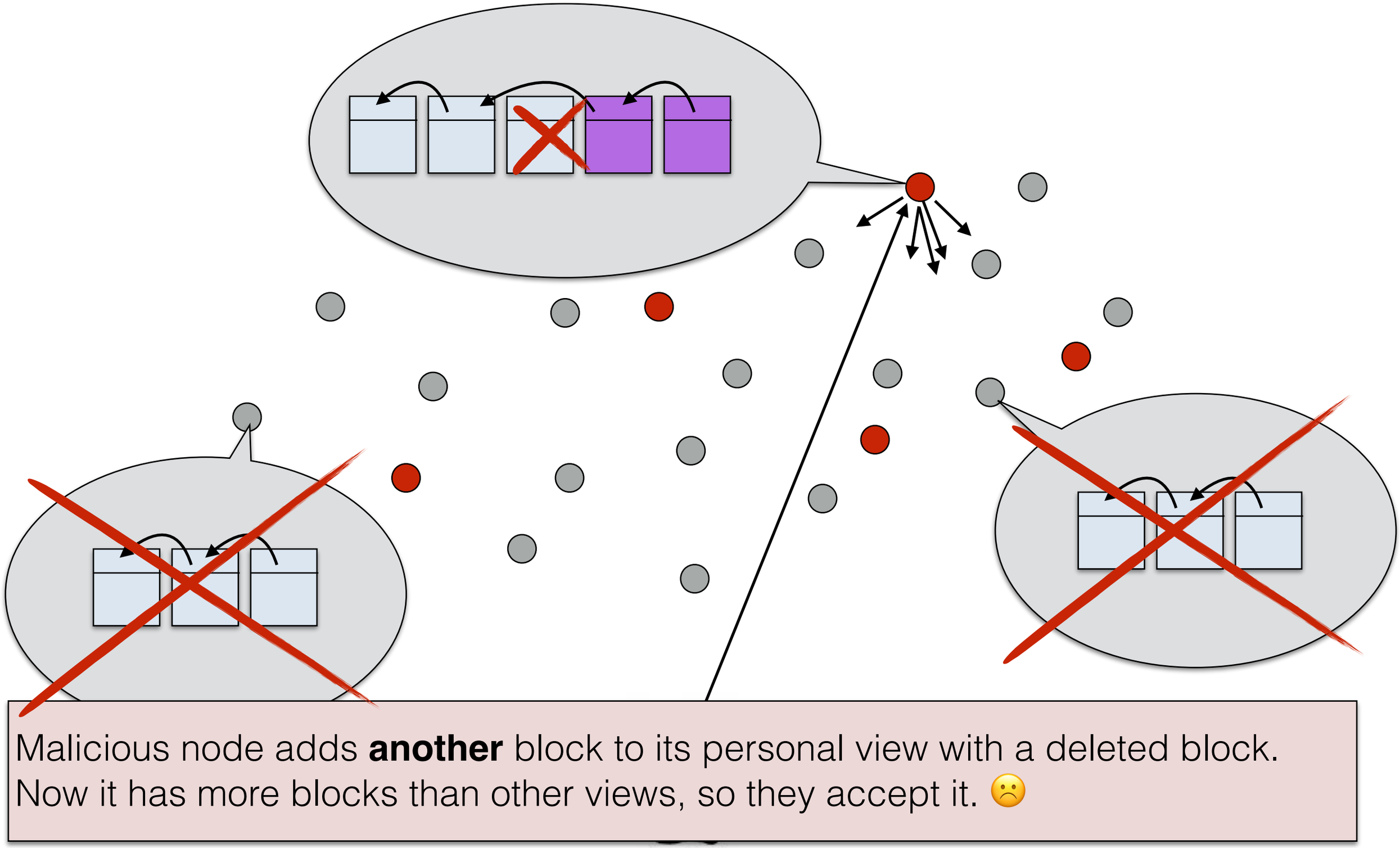

Malicious node adds **another** block to its personal view with a deleted block.
Now it has more blocks than other views, so they accept it. ☹

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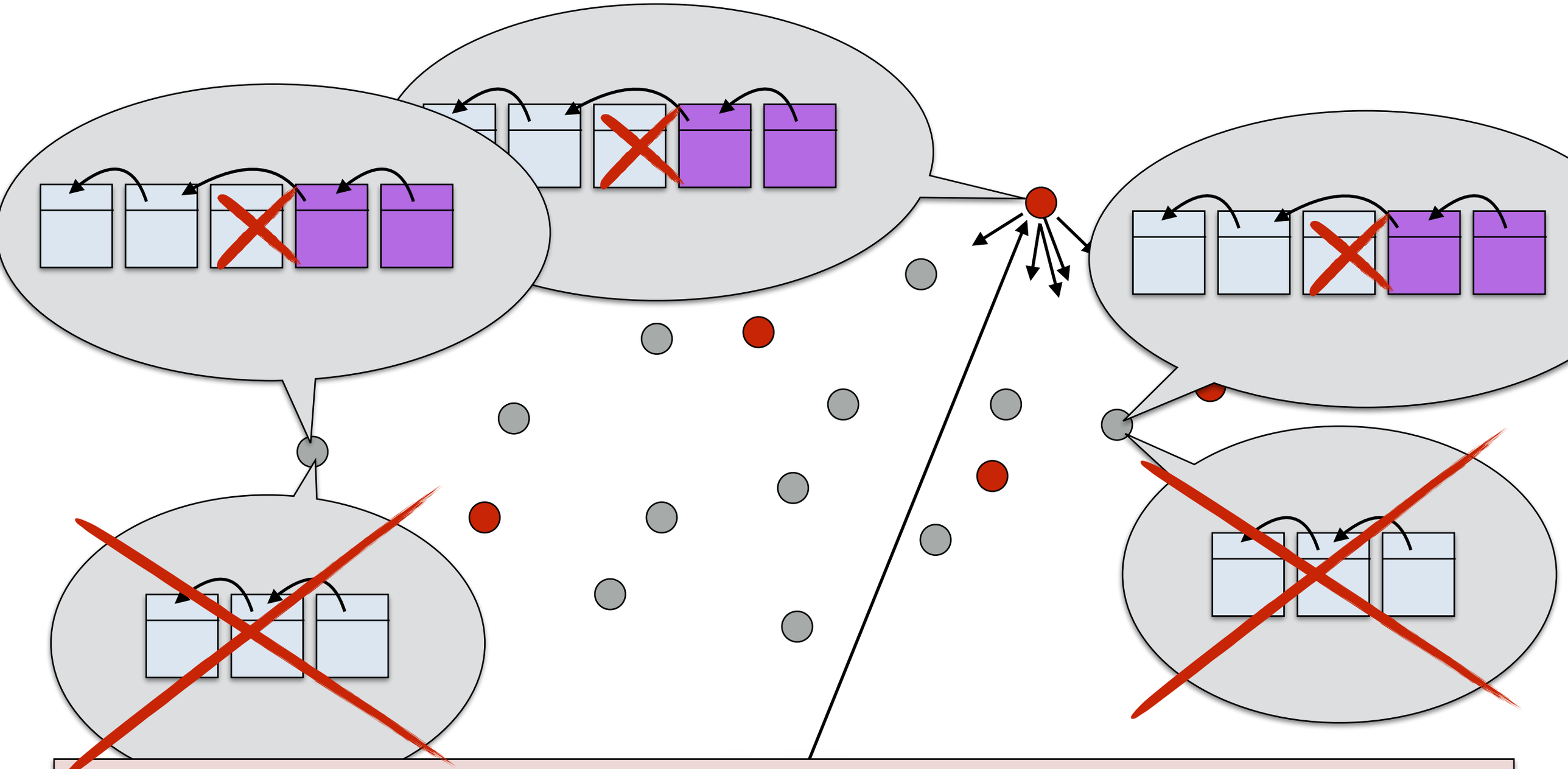

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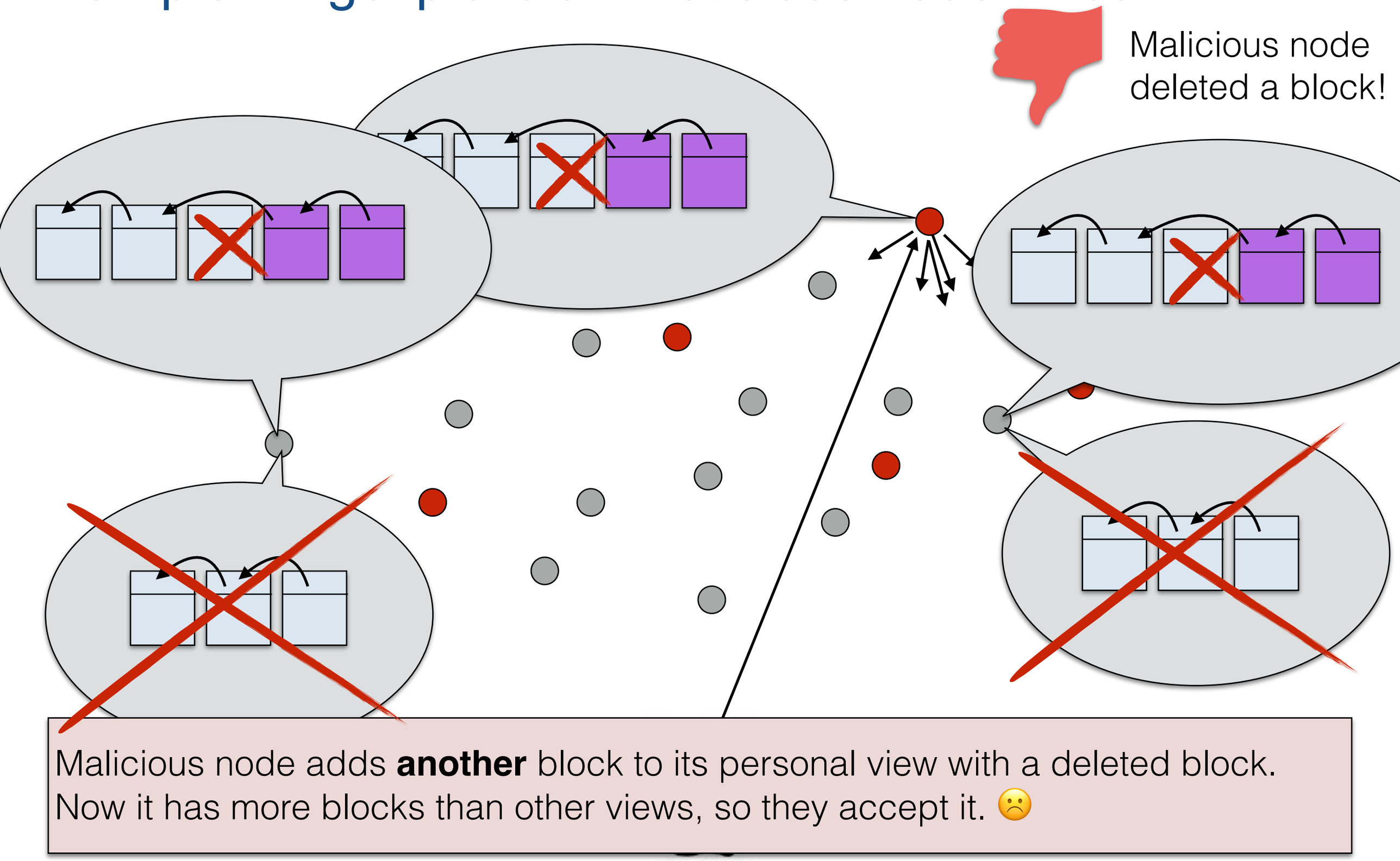

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Going forward: “Confirmations” for a transaction are subsequent blocks that follow the transaction’s block. More confirmations mean the transaction is “more official”.

Problem to solve later: Sybil attacks

Vulnerable to Sybil attacks: If there are a ton of malicious nodes, then the probability a malicious node is chosen is high.

Probability that angel picks honest node = # honest / total

Probability that angel picks malicious node = # malicious / total

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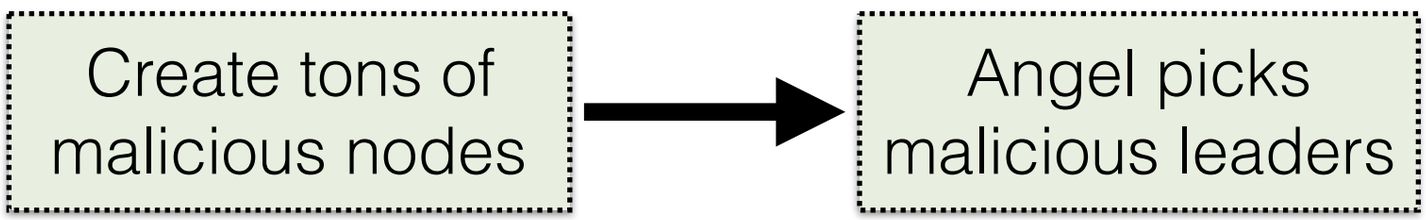

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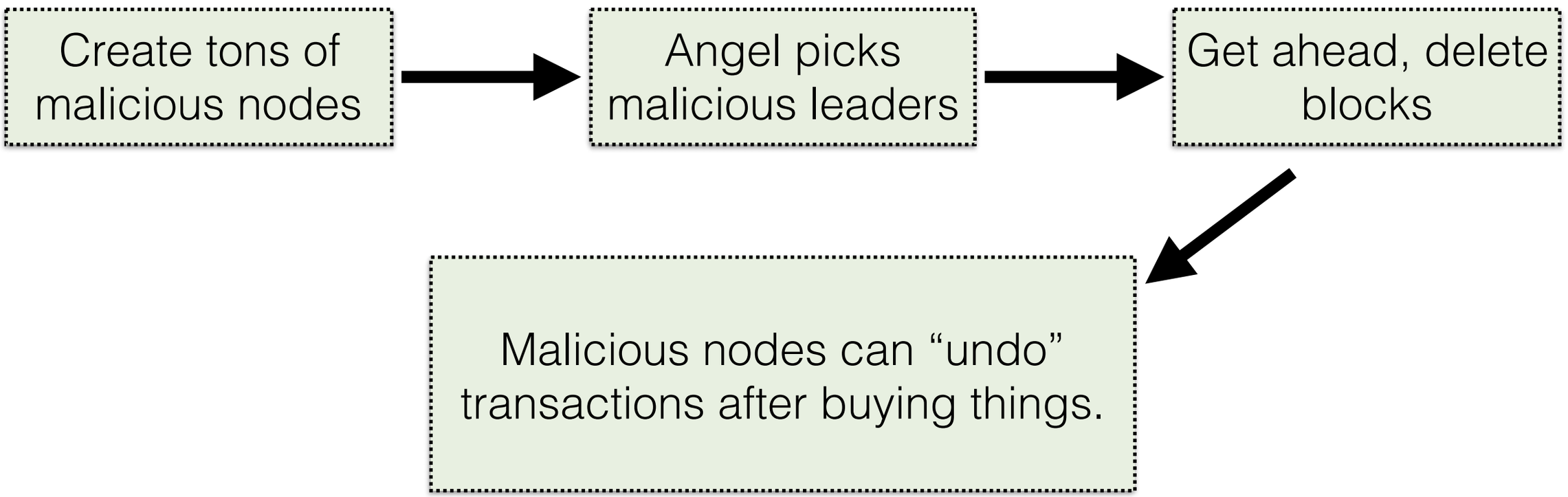

Lecture 2 Outline

1. Cryptographic Hash Functions
   - Blockchains
   - Proofs of Work
2. Putting DCash “on the blockchain”, with an authority
3. The idea of decentralization
4. Decentralized DCash with an Angel
5. **Decentralized DCash via proofs-of-work — Next time.**

The End