University of Nebraska-Lincoln
Digital Signal Processing: Quiz 1

August 29, 2025

Name: ______________________________ **Total Points: 10**

1. **(10 points)** Consider the frequency response

$$H(e^{j\omega}) = \frac{1 + e^{-j\omega}}{3 - 2e^{-j\omega}}$$

Find the **real part** of $H(e^{j\omega})$ as a function of ω. **Note:** don't leave any j's, i's, $\sqrt{-1}$'s, etc.

Solution:

First, express all terms in terms of sines and cosines:

$$e^{-j\omega} = \cos\omega - j\sin\omega$$

$$1 + e^{-j\omega} = 1 + \cos\omega - j\sin\omega$$

$$3 - 2e^{-j\omega} = 3 - 2\cos\omega + j2\sin\omega$$

Let $A = 1 + \cos\omega$, $B = -\sin\omega$, $C = 3 - 2\cos\omega$, $D = 2\sin\omega$, so

$$H(e^{j\omega}) = \frac{A + jB}{C + jD}$$

Multiply numerator and denominator by the complex conjugate of the denominator:

$$(C - jD)$$

$$H(e^{j\omega}) = \frac{(A + jB)(C - jD)}{(C + jD)(C - jD)}$$

$$= \frac{AC + BD + j(BC - AD)}{C^2 + D^2}$$

The real part is:

$$\mathrm{Re}\left\{H(e^{j\omega})\right\} = \frac{AC + BD}{C^2 + D^2}$$

Substitute back:

$$AC = (1 + \cos\omega)(3 - 2\cos\omega) = 3 + 3\cos\omega - 2\cos\omega - 2\cos^2\omega = 3 + \cos\omega - 2\cos^2\omega$$

$$BD = (-\sin\omega)(2\sin\omega) = -2\sin^2\omega$$

$$AC + BD = 3 + \cos\omega - 2\cos^2\omega - 2\sin^2\omega$$

Recall that $\sin^2\omega + \cos^2\omega = 1$, so:

$$-2\cos^2\omega - 2\sin^2\omega = -2(\cos^2\omega + \sin^2\omega) = -2$$

$$AC + BD = 3 + \cos\omega - 2 = 1 + \cos\omega$$

The denominator:

$$C^2 + D^2 = (3 - 2\cos\omega)^2 + [2\sin\omega]^2 = 9 - 12\cos\omega + 4\cos^2\omega + 4\sin^2\omega$$

$$= 9 - 12\cos\omega + 4(\cos^2\omega + \sin^2\omega)$$

$$= 9 - 12\cos\omega + 4(1) = 13 - 12\cos\omega$$

Therefore,

$$\boxed{\mathrm{Re}\left\{H(e^{j\omega})\right\} = \frac{1 + \cos\omega}{13 - 12\cos\omega}}$$