University of Nebraska-Lincoln
Digital Signal Processing: Quiz 5

October 10, 2025

Name: _____ **Total Points: 10**

Given: A discrete-time LTI system has the following impulse response:

$$h[n] = \delta[n] - \delta[n-1]$$

where $\delta[n]$ is the unit impulse function.

1. **(10 points)** Find the frequency response $H(e^{j\omega})$ of this system and determine:
   (a) The magnitude response $|H(e^{j\omega})|$
   (b) The phase response $\angle H(e^{j\omega})$
   (c) Evaluate both the magnitude and phase at $\omega = 0$ and $\omega = \pi$

   Hints:

$$H(e^{j\omega}) = \sum_{n=-\infty}^{\infty} h[n]e^{-j\omega n}$$

$$H(e^{j\omega}) = |H(e^{j\omega})|e^{j\angle H(e^{j\omega})}$$

   Useful Euler's identities:

$$e^{j\theta} = \cos(\theta) + j\sin(\theta) \tag{1}$$

$$e^{j\theta} - e^{-j\theta} = 2j\sin(\theta) \tag{2}$$

$$e^{j\theta} + e^{-j\theta} = 2\cos(\theta) \tag{3}$$

$$j = e^{j\pi/2} \tag{4}$$

   Note: You may express your final answer either in terms of complex exponentials or sinusoids/cosines.

Solution:

The frequency response is the DTFT of the impulse response:

$$H(e^{j\omega}) = \sum_{n=-\infty}^{\infty} h[n]e^{-j\omega n}$$

$$H(e^{j\omega}) = 1 - e^{-j\omega}$$

$$H(e^{j\omega}) = e^{-j\omega/2}(e^{j\omega/2} - e^{-j\omega/2})$$

$$H(e^{j\omega}) = e^{-j\omega/2} \cdot 2j\sin(\omega/2)$$

$$H(e^{j\omega}) = 2\sin(\omega/2)e^{j(\pi/2-\omega/2)}$$

(a) Magnitude response:

$$|H(e^{j\omega})| = 2|\sin(\omega/2)|$$

(b) Phase response:

$$\angle H(e^{j\omega}) = \pi/2 - \omega/2$$

(c) Evaluations:

At $\omega = 0$:

$$|H(e^{j0})| = 2|\sin(0)| = 0$$
$$\angle H(e^{j0}) = \pi/2 - 0 = \pi/2$$

At $\omega = \pi$:

$$|H(e^{j\pi})| = 2|\sin(\pi/2)| = 2$$
$$\angle H(e^{j\pi}) = \pi/2 - \pi/2 = 0$$