# Discrete Time IIR Filtering

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## What is a Filter?

**Key Concept:** Filters are LTI systems that modify frequency components of signals.
**Definition:** Any system that modifies certain frequencies relative to others

**Two Main Classes:**

- **IIR (Infinite Impulse Response):** Rational transfer function $H(z) = \frac{B(z)}{A(z)}$
- IIR Filters typically implemented as difference equations in discrete-time
- **FIR (Finite Impulse Response):** Polynomial transfer function $H(z) = \sum_{n=0}^{M} b[n]z^{-n}$

**Design Focus:**

- Determine parameters to approximate desired frequency response
- Meet design specifications
- Maintain causality and stability

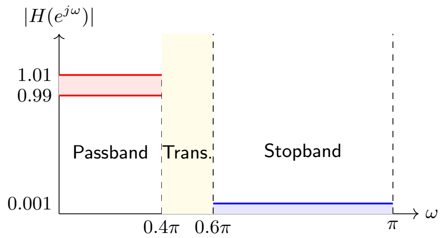




## Example Specifications

**Common specification for all designs:**

Passband: $0.99 \leq |H(e^{j\omega})| \leq 1.01$, $|\omega| \leq 0.4\pi$
Stopband: $|H(e^{j\omega})| \leq 0.001$, $0.6\pi \leq |\omega| \leq \pi$








## Filter Order Comparison

**For specifications:** $\omega_p = 0.4\pi$, $\omega_s = 0.6\pi$, $\delta_p = 0.01$, $\delta_s = 0.001$

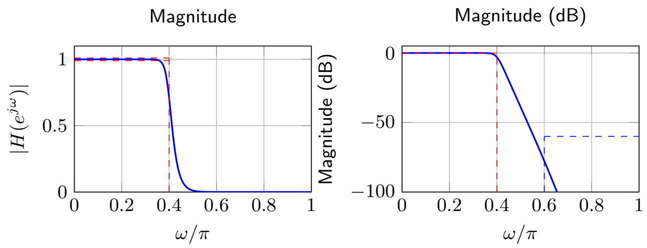
| Filter Type | Minimum Order | Zero Locations |
|---|---|---|
| Butterworth | 14 | 14 zeros at $z = -1$ |
| Chebyshev I | 8 | 8 zeros at $z = -1$ |
| Chebyshev II | 8 | 8 zeros on unit circle |
| Elliptic | 6 | 6 zeros on unit circle |

**Key Observations:**

- Elliptic achieves lowest order (optimal)
- Butterworth requires $\sim 2\times$ the order of Chebyshev
- Chebyshev I simpler than II (all zeros at $z = -1$)
- Order directly impacts computational complexity

## Butterworth: Magnitude Response

**Order 14 Butterworth Filter**



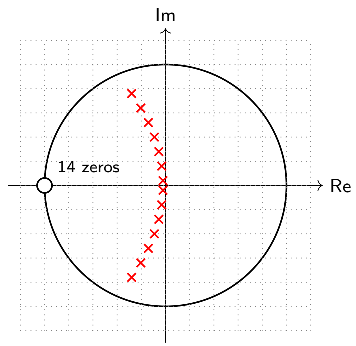

**Characteristics:**

- Smooth, monotonic decrease (no ripple)
- Maximally flat at $\omega = 0$
- Gradual transition band roll-off





## Butterworth: Pole-Zero Plot




**Order 14 Butterworth Filter**

**Key Features:**

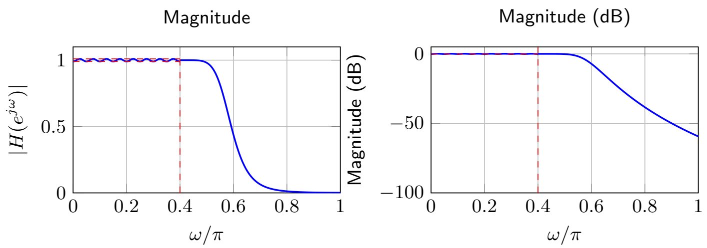
- Leftward bow increases with $|\mathrm{Im}|$ via a quadratic dependence on Im
- Poles placed at small negative Re and in conjugate pairs
- All poles kept inside the unit circle for stability





## Chebyshev Type I: Magnitude Response

**Order 8 Chebyshev Type I Filter**



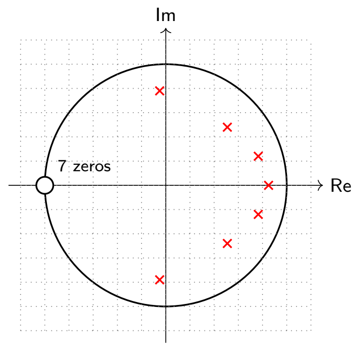

**Characteristics:**

- Equiripple behavior in the passband (visualized between $1 - 0.01$ and $1 + 0.01$)
- Monotonic, smoothly decaying stopband (no ripples)





## Chebyshev Type I: Pole-Zero Plot



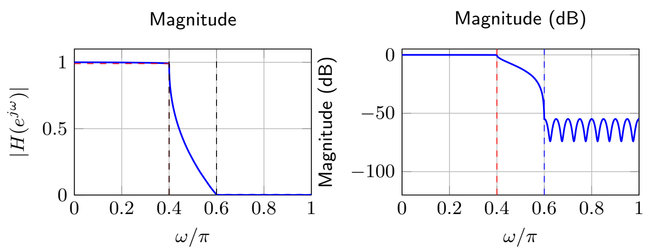

**Key Features:**

- 7 zeros stacked at $z = -1$
- 7 poles lie on a smooth quadratic curve
- All poles remain inside the unit circle

# Chebyshev Type II: Magnitude Response



**Order 8 Chebyshev Type II Filter**



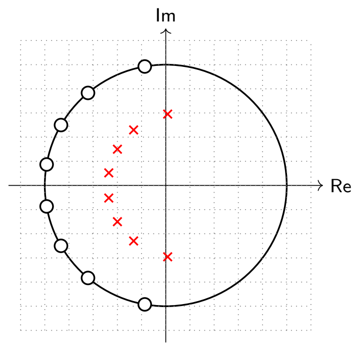

**Characteristics:**

- Monotonic in passband (smooth like Butterworth)
- Equiripple in stopband (oscillates around the stopband baseline)
- Stopband nulls originate from zeros placed on or near the unit circle



# Chebyshev Type II: Pole-Zero Plot






**Order 8 Chebyshev Type II Filter**

**Key Features:**

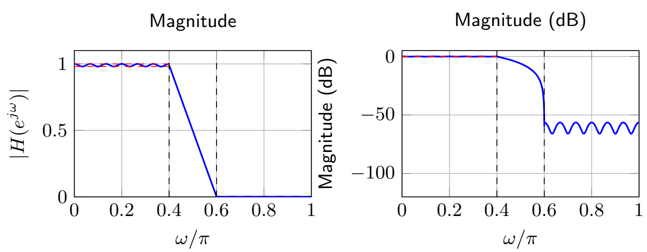
- 8 zeros on the unit circle arranged as conjugate pairs
- 8 poles placed inside the unit circle



# Elliptic: Magnitude Response



**Order 6 Elliptic Filter (Optimal)**



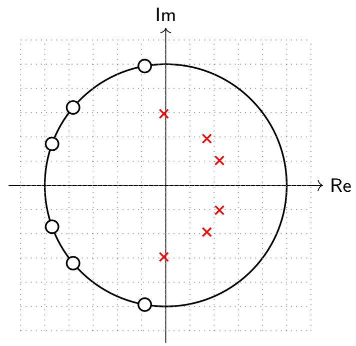

**Characteristics:**

- Equiripple in both passband and stopband
- Smooth finite transition band
- Sharp transition and strong stopband attenuation typical of elliptic filters



# Elliptic: Pole-Zero Plot






**Order 6 Elliptic Filter**

**Key Features:**

- 6 zeros on the unit circle arranged as conjugate pairs (placed similarly to the Chebyshev Type II zeros)
- 6 poles in complex conjugate pairs and within unit circle

# Key Takeaways

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**Visual Summary:**

| Type | Order | Passband | Stopband |
|---|---|---|---|
| Butterworth | 14 | Monotonic | Monotonic |
| Chebyshev I | 8 | Equiripple | Monotonic |
| Chebyshev II | 8 | Monotonic | Equiripple |
| Elliptic | 6 | Equiripple | Equiripple |

**Selection Guide:**

- **Minimize order/cost?** $\rightarrow$ Elliptic
- **Smooth response?** $\rightarrow$ Butterworth
- **Balance efficiency & simplicity?** $\rightarrow$ Chebyshev I
- **Smooth passband, lower order?** $\rightarrow$ Chebyshev II

**All zeros at $z = -1$:** Butterworth, Chebyshev I (simpler implementation)
**Zeros on unit circle:** Chebyshev II, Elliptic (stopband nulls, causes stopband ripples)