University of Nebraska-Lincoln

ECEN 463/863 - Digital Signal Processing

Midterm Exam

October 29, 2025

Question	Points	Score
1	20	
2	20	
3	20	
4	20	
5	20	
Total	**100**	

Name: ____

"On my honor, I have neither given nor received aid on this exam"

Student Signature: ____

1. **(20 points)** For each of the following input/output relations for a discrete time system, please indicate whether the system is:

L or NL: Linear or Not Linear

TI or NTI: Time Invariant or Not Time Invariant

C or NC: Causal or Not Causal

B or NB: BIBO Stable or Not BIBO Stable

If for any property it is not possible to say, then indicate this by writing **CBD** (Cannot Be Determined).

Input/Output Relation	**L/NL**	**TI/NTI**	**C/NC**	**B/NB**
$y[n] = 3x[n-1] + 2x[n-3] - x[n-7]$				
$y[n] = x[n] \cdot x[n-2] + 5$				
$y[n] = \sum_{k=0}^{3} x[n-k] \left(\frac{1}{2}\right)^k$				
$y[n] = x[2n] + \sin(0.1\pi n)$				
$y[n] = \max\{x[n], x[n-1], x[n-4]\}$				

Notes:

(a) In our notation, if the upper limit of a summation is higher than or equal to the lower limit, a summation occurs; otherwise, the summation returns a zero.

(b) $\max\{\}$ returns the maximum of three values – e.g., $\max\{7, -3, 10\} = 10$.

(c) Show all work you did to obtain your answers. You can either show that the relation meets our definition, or show a counter-example that shows the relation does not.

2. **(20 points)** For each of the following sequences, please indicate "yes" it is an eigensequence or "no" it is not an eigensequence for discrete-time LTI systems.

(a) $x_1[n] = \left(\frac{1}{3}\right)^n u[n]$

(b) $x_2[n] = e^{j\frac{2\pi}{3}n}$

(c) $x_3[n] = \cos(\frac{\pi}{5}n)$

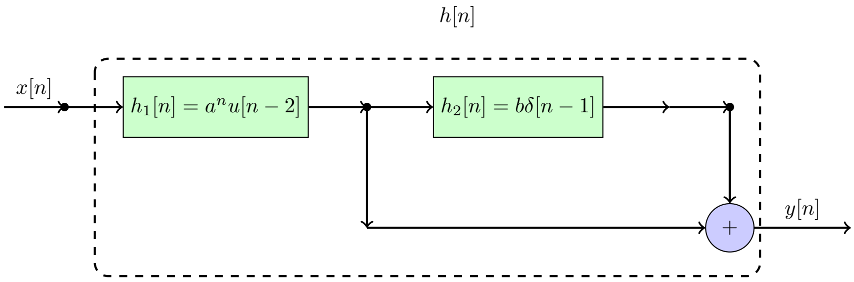
(d) $x_4[n] = \delta[n-3]$

(e) $x_5[n] = 3$

Note:

- $u[n]$ denotes unit step function
- $\delta[n]$ denotes unit impulse (delta) function

3. **(20 points)** Consider the system in the figure below (where $|a| < 1$):



(a) **(8 points)** Find the impulse response $h[n]$ of the overall system.

(b) **(8 points)** Find the frequency response $H(e^{j\omega})$ of the overall system.

(c) **(4 points)** Specify a difference equation that relates the output $y[n]$ to the input $x[n]$.

4. **(20 points)** Compute the DTFT of the following sequence:

$$x[n] = n\left(\frac{1}{3}\right)^n u[n] + (-2)^n\, u[-n-1]$$

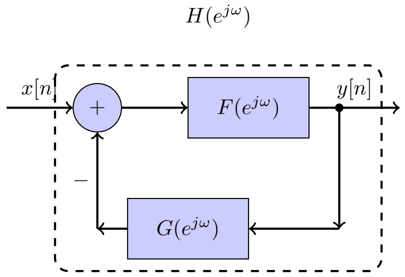
Show all steps in your computation.

5. **(20 points)** A linear time-invariant system $F(e^{j\omega})$ with input $x[n]$ and output $y[n]$ is described by the difference equation:

$$y[n] + 4y[n-2] = x[n] - 0.5x[n-1]$$

(a) **(7 points)** Give the frequency response $F(e^{j\omega})$. Is this system causal? Is this system stable?

(b) **(7 points)** Suppose that $F(e^{j\omega})$ is interconnected in a negative-feedback arrangement with a causal, linear time-invariant system $G(e^{j\omega})$, as shown in the figure below. Derive an expression for the overall system frequency response $H(e^{j\omega})$. Your expression should depend on $F(e^{j\omega})$ and $G(e^{j\omega})$ only.



(c) **(6 points)** Find $G(e^{j\omega})$ such that $H(e^{j\omega}) = 1$. Is the system $G(e^{j\omega})$ stable?

Note: By achieving $H(e^{j\omega}) = 1$, we're effectively controlling the system $F(e^{j\omega})$. That is, we're making $F(e^{j\omega})$ produce whatever output we set it.

Reference Formulas

Euler's Identity:

$$e^{j\omega} = \cos(\omega) + j\sin(\omega)$$

Unit Step Function:

$$u[n] = \begin{cases} 1 & \text{if } n \geq 0 \\ 0 & \text{if } n < 0 \end{cases}$$

Unit Impulse Function:

$$\delta[n] = \begin{cases} 1 & \text{if } n = 0 \\ 0 & \text{if } n \neq 0 \end{cases}$$

Impulse Sampling Property:

$$x[n]\delta[n-k] = x[k]\delta[n-k]$$

DTFT (Analysis):

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\omega n}$$

Inverse DTFT (Synthesis):

$$x[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\omega})e^{j\omega n}d\omega$$

Convolution Sum:

$$y[n] = x[n] * h[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k] = \sum_{k=-\infty}^{\infty} h[k]x[n-k]$$

Convolution Property:

$$x[n] * h[n] \overset{\text{DTFT}}{\longleftrightarrow} X(e^{j\omega})H(e^{j\omega})$$

Geometric Series:

$$\sum_{n=0}^{\infty} ar^n = \frac{a}{1-r} \quad \text{for } |r| < 1$$

$$\sum_{n=0}^{N-1} ar^n = a\frac{1-r^N}{1-r} \quad \text{for } r \neq 1$$

Trigonometric Identities:

$$\cos(\omega) = \frac{e^{j\omega} + e^{-j\omega}}{2}, \quad \sin(\omega) = \frac{e^{j\omega} - e^{-j\omega}}{2j}$$

DTFT Tables

TABLE 2.1 SYMMETRY PROPERTIES OF THE FOURIER TRANSFORM

Sequence $x[n]$	Fourier Transform $X(e^{j\omega})$
1. $x^*[n]$	$X^*(e^{-j\omega})$
2. $x^*[-n]$	$X^*(e^{j\omega})$
3. $\mathcal{R}e\{x[n]\}$	$X_e(e^{j\omega})$ (conjugate-symmetric part of $X(e^{j\omega})$)
4. $j\mathcal{I}m\{x[n]\}$	$X_o(e^{j\omega})$ (conjugate-antisymmetric part of $X(e^{j\omega})$)
5. $x_e[n]$ (conjugate-symmetric part of $x[n]$)	$X_R(e^{j\omega}) = \mathcal{R}e\{X(e^{j\omega})\}$
6. $x_o[n]$ (conjugate-antisymmetric part of $x[n]$)	$jX_I(e^{j\omega}) = j\mathcal{I}m\{X(e^{j\omega})\}$
The following properties apply only when $x[n]$ is real:	
7. Any real $x[n]$	$X(e^{j\omega}) = X^*(e^{-j\omega})$ (Fourier transform is conjugate symmetric)
8. Any real $x[n]$	$X_R(e^{j\omega}) = X_R(e^{-j\omega})$ (real part is even)
9. Any real $x[n]$	$X_I(e^{j\omega}) = -X_I(e^{-j\omega})$ (imaginary part is odd)
10. Any real $x[n]$	$\lvert X(e^{j\omega})\rvert = \lvert X(e^{-j\omega})\rvert$ (magnitude is even)
11. Any real $x[n]$	$\angle X(e^{j\omega}) = -\angle X(e^{-j\omega})$ (phase is odd)
12. $x_e[n]$ (even part of $x[n]$)	$X_R(e^{j\omega})$
13. $x_o[n]$ (odd part of $x[n]$)	$jX_I(e^{j\omega})$

TABLE 2.2 FOURIER TRANSFORM THEOREMS

Sequence $x[n]$ $y[n]$	Fourier Transform $X(e^{j\omega})$ $Y(e^{j\omega})$
1. $ax[n] + by[n]$	$aX(e^{j\omega}) + bY(e^{j\omega})$
2. $x[n - n_d]$ (n_d an integer)	$e^{-j\omega n_d}X(e^{j\omega})$
3. $e^{j\omega_0 n}x[n]$	$X(e^{j(\omega-\omega_0)})$
4. $x[-n]$	$X(e^{-j\omega})$ $X^*(e^{j\omega})$ if $x[n]$ real.
5. $nx[n]$	$j\dfrac{dX(e^{j\omega})}{d\omega}$
6. $x[n] * y[n]$	$X(e^{j\omega})Y(e^{j\omega})$
7. $x[n]y[n]$	$\dfrac{1}{2\pi}\displaystyle\int_{-\pi}^{\pi} X(e^{j\theta})Y(e^{j(\omega-\theta)})d\theta$
Parseval's theorem:	
8. $\displaystyle\sum_{n=-\infty}^{\infty} \lvert x[n]\rvert^2 = \frac{1}{2\pi}\int_{-\pi}^{\pi} \lvert X(e^{j\omega})\rvert^2 d\omega$	
9. $\displaystyle\sum_{n=-\infty}^{\infty} x[n]y^*[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\omega})Y^*(e^{j\omega})d\omega$	

TABLE 2.3 FOURIER TRANSFORM PAIRS

Sequence	Fourier Transform
1. $\delta[n]$	1
2. $\delta[n - n_0]$	$e^{-j\omega n_0}$
3. $1 \quad (-\infty < n < \infty)$	$\displaystyle\sum_{k=-\infty}^{\infty} 2\pi\delta(\omega + 2\pi k)$
4. $a^n u[n] \quad (\lvert a\rvert < 1)$	$\dfrac{1}{1 - ae^{-j\omega}}$
5. $u[n]$	$\dfrac{1}{1 - e^{-j\omega}} + \displaystyle\sum_{k=-\infty}^{\infty} \pi\delta(\omega + 2\pi k)$
6. $(n+1)a^n u[n] \quad (\lvert a\rvert < 1)$	$\dfrac{1}{(1 - ae^{-j\omega})^2}$
7. $\dfrac{r^n \sin\omega_p(n+1)}{\sin\omega_p}u[n] \quad (\lvert r\rvert < 1)$	$\dfrac{1}{1 - 2r\cos\omega_p e^{-j\omega} + r^2 e^{-j2\omega}}$
8. $\dfrac{\sin\omega_c n}{\pi n}$	$X(e^{j\omega}) = \begin{cases} 1, & \lvert\omega\rvert < \omega_c, \\ 0, & \omega_c < \lvert\omega\rvert \le \pi \end{cases}$
9. $x[n] = \begin{cases} 1, & 0 \le n \le M \\ 0, & \text{otherwise} \end{cases}$	$\dfrac{\sin[\omega(M+1)/2]}{\sin(\omega/2)}e^{-j\omega M/2}$
10. $e^{j\omega_0 n}$	$\displaystyle\sum_{k=-\infty}^{\infty} 2\pi\delta(\omega - \omega_0 + 2\pi k)$
11. $\cos(\omega_0 n + \phi)$	$\displaystyle\sum_{k=-\infty}^{\infty} [\pi e^{j\phi}\delta(\omega - \omega_0 + 2\pi k) + \pi e^{-j\phi}\delta(\omega + \omega_0 + 2\pi k)]$