Discrete Time IIR Filtering

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What is a Filter?

Key Concept: Filters are LTI systems that modify frequency components of signals.
Definition: Any system that modifies certain frequencies relative to others

Two Main Classes:

- **IIR (Infinite Impulse Response):** Rational transfer function $H(z) = \frac{B(z)}{A(z)}$
- IIR Filters typically implemented as difference equations in discrete-time
- **FIR (Finite Impulse Response):** Polynomial transfer function $H(z) = \sum_{n=0}^{M} b[n]z^{-n}$

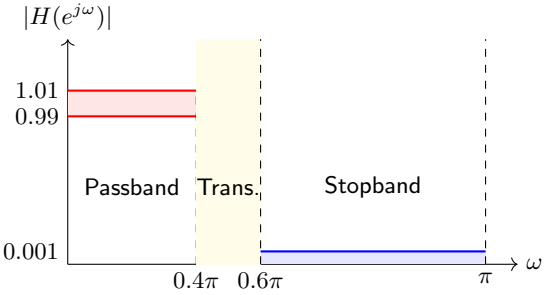
Design Focus:

- Determine parameters to approximate desired frequency response
- Meet design specifications
- Maintain causality and stability

Example Specifications

Common specification for all designs:

Passband: $0.99 \leq |H(e^{j\omega})| \leq 1.01$, $|\omega| \leq 0.4\pi$
Stopband: $|H(e^{j\omega})| \leq 0.001$, $0.6\pi \leq |\omega| \leq \pi$



Filter Order Comparison

For specifications: $\omega_p = 0.4\pi$, $\omega_s = 0.6\pi$, $\delta_p = 0.01$, $\delta_s = 0.001$

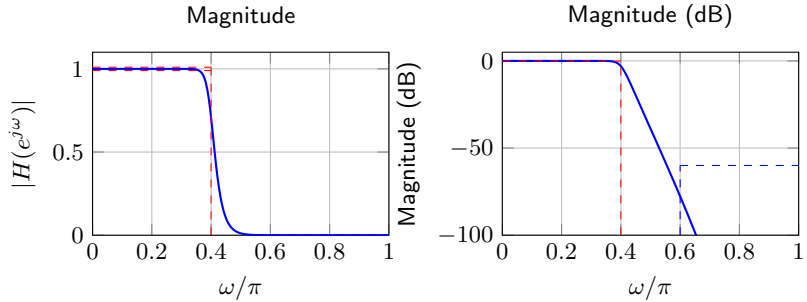
Filter Type	Minimum Order	Zero Locations
Butterworth	14	14 zeros at $z = -1$
Chebyshev I	8	8 zeros at $z = -1$
Chebyshev II	8	8 zeros on unit circle
Elliptic	6	6 zeros on unit circle

Key Observations:

- Elliptic achieves lowest order (optimal)
- Butterworth requires $\sim 2\times$ the order of Chebyshev
- Chebyshev I simpler than II (all zeros at $z = -1$)
- Order directly impacts computational complexity

Butterworth: Magnitude Response

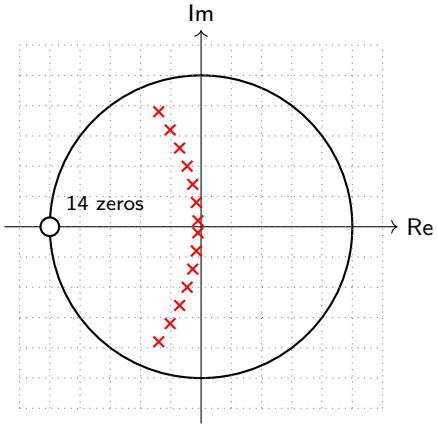
Order 14 Butterworth Filter



Characteristics:

- Smooth, monotonic decrease (no ripple)
- Maximally flat at $\omega = 0$
- Gradual transition band roll-off

Butterworth: Pole-Zero Plot



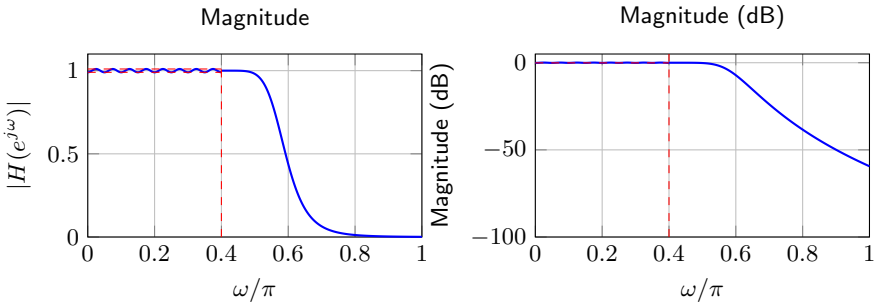
Order 14 Butterworth Filter

Key Features:

- Leftward bow increases with $|\mathrm{Im}|$ via a quadratic dependence on Im
- Poles placed at small negative Re and in conjugate pairs
- All poles kept inside the unit circle for stability

Chebyshev Type I: Magnitude Response

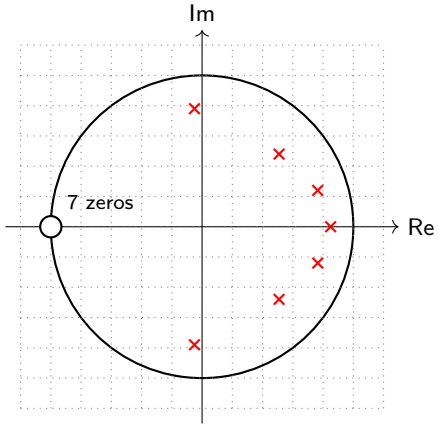
Order 8 Chebyshev Type I Filter



Characteristics:

- Equiripple behavior in the passband (visualized between $1 - 0.01$ and $1 + 0.01$)
- Monotonic, smoothly decaying stopband (no ripples)

Chebyshev Type I: Pole-Zero Plot

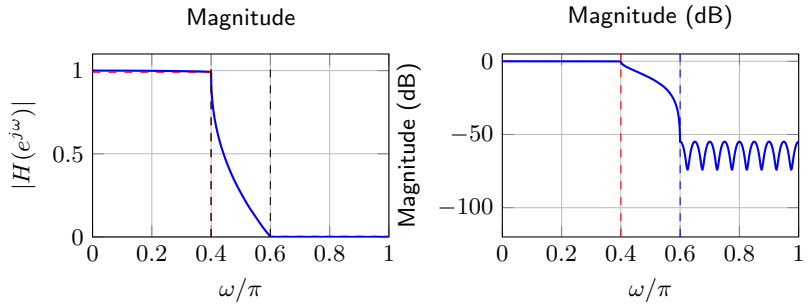


Key Features:

- 7 zeros stacked at $z = -1$
- 7 poles lie on a smooth quadratic curve
- All poles remain inside the unit circle

Chebyshev Type II: Magnitude Response

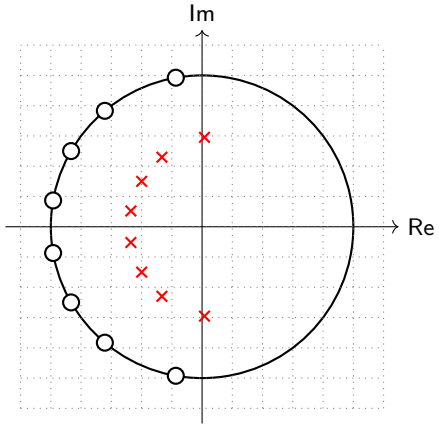
Order 8 Chebyshev Type II Filter



Characteristics:

- Monotonic in passband (smooth like Butterworth)
- Equiripple in stopband (oscillates around the stopband baseline)
- Stopband nulls originate from zeros placed on or near the unit circle

Chebyshev Type II: Pole-Zero Plot



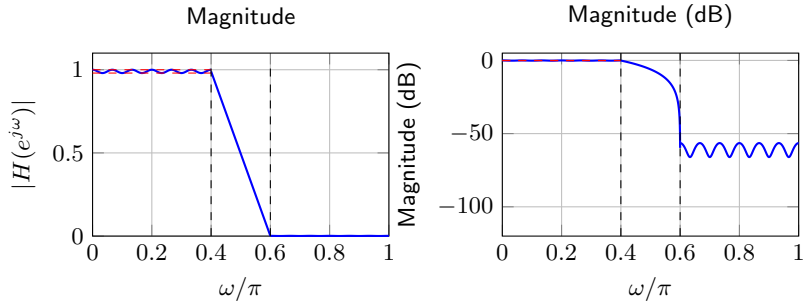
Order 8 Chebyshev Type II Filter

Key Features:

- 8 zeros on the unit circle arranged as conjugate pairs
- 8 poles placed inside the unit circle

Elliptic: Magnitude Response

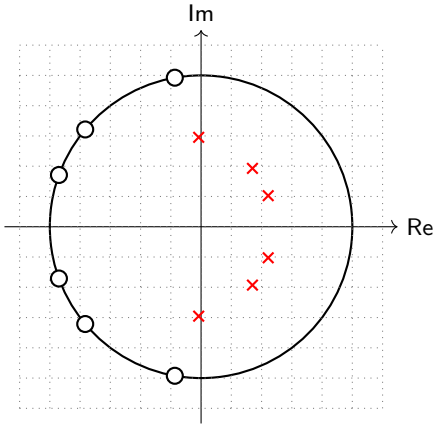
Order 6 Elliptic Filter (Optimal)



Characteristics:

- Equiripple in both passband and stopband
- Smooth finite transition band
- Sharp transition and strong stopband attenuation typical of elliptic filters

Elliptic: Pole-Zero Plot



Order 6 Elliptic Filter

Key Features:

- 6 zeros on the unit circle arranged as conjugate pairs (placed similarly to the Chebyshev Type II zeros)
- 6 poles in complex conjugate pairs and within unit circle

Key Takeaways

Visual Summary:

Type	Order	Passband	Stopband
Butterworth	14	Monotonic	Monotonic
Chebyshev I	8	Equiripple	Monotonic
Chebyshev II	8	Monotonic	Equiripple
Elliptic	6	Equiripple	Equiripple

Selection Guide:

- **Minimize order/cost?** $\rightarrow$ Elliptic
- **Smooth response?** $\rightarrow$ Butterworth
- **Balance efficiency & simplicity?** $\rightarrow$ Chebyshev I
- **Smooth passband, lower order?** $\rightarrow$ Chebyshev II

All zeros at $z = -1$: Butterworth, Chebyshev I (simpler implementation)
Zeros on unit circle: Chebyshev II, Elliptic (stopband nulls, causes stopband ripples)