Assignment 02
Frequency Domain Analysis of Circuits

ECEN 222, Spring 2026

University of Nebraska-Lincoln

Instructions

This assignment focuses on analyzing AC circuits in the frequency domain using phasor techniques. You will convert time-domain signals to phasors, calculate impedances, analyze circuits, and convert results back to the time domain.

Key Formulas

Phasor Conversion:

$$v(t) = V_m \cos(\omega t + \phi) \quad \Leftrightarrow \quad \mathbf{V} = V_m \angle \phi$$

Impedances:

- Resistor: $\mathbf{Z}_R = R$
- Inductor: $\mathbf{Z}_L = j\omega L = \omega L \angle 90°$
- Capacitor: $\mathbf{Z}_C = \frac{1}{j\omega C} = \frac{-j}{\omega C} = \frac{1}{\omega C} \angle -90°$

Complex Number Operations:

- Rectangular to Polar: $a + jb = \sqrt{a^2 + b^2} \angle \tan^{-1}(b/a)$
- Polar to Rectangular: $r\angle\theta = r\cos\theta + jr\sin\theta$

AC Power:

- Real Power: $P = V_{rms} I_{rms} \cos\theta$ (W)
- Reactive Power: $Q = V_{rms} I_{rms} \sin\theta$ (VAR)
- Apparent Power: $S = V_{rms} I_{rms}$ (VA)
- Complex Power: $\mathbf{S} = P + jQ = \mathbf{V}_{rms} \mathbf{I}_{rms}^*$

Problems

1. **[12.5 points]**

Given the following time-domain signals:

$$v_1(t) = 15\cos(5000t + 60^\circ)\ \text{V}$$
$$v_2(t) = 8\cos(5000t - 30^\circ)\ \text{V}$$
$$i(t) = 3\cos(5000t + 15^\circ)\ \text{A}$$

(a) Convert each signal to phasor form.

Solution:
For a sinusoidal signal $v(t) = V_m \cos(\omega t + \phi)$, the phasor is $\mathbf{V} = V_m\angle\phi$.

$$v_1(t) = 15\cos(5000t + 60^\circ)\ \text{V} \quad \Rightarrow \quad \mathbf{V}_1 = 15\angle 60^\circ\ \text{V}$$
$$v_2(t) = 8\cos(5000t - 30^\circ)\ \text{V} \quad \Rightarrow \quad \mathbf{V}_2 = 8\angle -30^\circ\ \text{V}$$
$$i(t) = 3\cos(5000t + 15^\circ)\ \text{A} \quad \Rightarrow \quad \mathbf{I} = 3\angle 15^\circ\ \text{A}$$

(b) Calculate $\mathbf{V}_1 + \mathbf{V}_2$ in both rectangular and polar forms.

Solution:
First, convert each phasor to rectangular form:

$$\mathbf{V}_1 = 15\angle 60^\circ = 15\cos(60^\circ) + j15\sin(60^\circ) = 7.5 + j12.99\ \text{V}$$
$$\mathbf{V}_2 = 8\angle -30^\circ = 8\cos(-30^\circ) + j8\sin(-30^\circ) = 6.93 - j4.0\ \text{V}$$

Now add:

$$\begin{aligned}\mathbf{V}_1 + \mathbf{V}_2 &= (7.5 + j12.99) + (6.93 - j4.0)\\ &= 14.43 + j8.99\ \text{V}\end{aligned}$$

Convert to polar form:

$$|\mathbf{V}_1 + \mathbf{V}_2| = \sqrt{14.43^2 + 8.99^2} = \sqrt{208.22 + 80.82} = 17.00\ \text{V}$$
$$\angle(\mathbf{V}_1 + \mathbf{V}_2) = \tan^{-1}\left(\frac{8.99}{14.43}\right) = 31.9^\circ$$

Answer: $\mathbf{V}_1 + \mathbf{V}_2 = 14.43 + j8.99\ \text{V} = 17.00\angle 31.9^\circ\ \text{V}$

(c) Calculate $\mathbf{V}_1 - \mathbf{V}_2$ in both rectangular and polar forms.

Solution:
Using the rectangular forms from part (b):

$$\begin{aligned}\mathbf{V}_1 - \mathbf{V}_2 &= (7.5 + j12.99) - (6.93 - j4.0) \\ &= 0.57 + j16.99 \text{ V}\end{aligned}$$

Convert to polar form:

$$\begin{aligned}|\mathbf{V}_1 - \mathbf{V}_2| &= \sqrt{0.57^2 + 16.99^2} = \sqrt{0.32 + 288.66} = 17.00 \text{ V} \\ \angle(\mathbf{V}_1 - \mathbf{V}_2) &= \tan^{-1}\left(\frac{16.99}{0.57}\right) = 88.1^\circ\end{aligned}$$

Answer: $\mathbf{V}_1 - \mathbf{V}_2 = 0.57 + j16.99$ V $= 17.00\angle 88.1^\circ$ V

(d) Calculate the impedance $\mathbf{Z} = \mathbf{V}_1/\mathbf{I}$ and express it in both rectangular and polar forms. What type of element(s) does this impedance represent?

Solution:

$$\mathbf{Z} = \frac{\mathbf{V}_1}{\mathbf{I}} = \frac{15\angle 60^\circ}{3\angle 15^\circ} = \frac{15}{3}\angle(60^\circ - 15^\circ) = 5\angle 45^\circ\,\Omega$$

Convert to rectangular form:

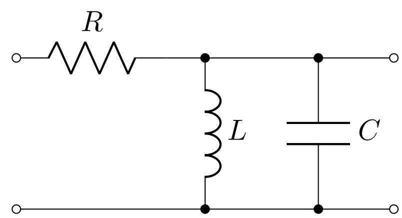
$$\mathbf{Z} = 5\cos(45^\circ) + j5\sin(45^\circ) = 3.54 + j3.54\,\Omega$$

Answer: $\mathbf{Z} = 3.54 + j3.54\,\Omega = 5\angle 45^\circ\,\Omega$

This impedance has both a positive real part (resistive) and a positive imaginary part (inductive). It represents a resistor in series with an inductor. Since $\omega = 5000$ rad/s and $X_L = 3.54\,\Omega$, we have $L = X_L/\omega = 3.54/5000 = 0.708$ mH.

2. **[12.5 points]**

Consider a circuit with a resistor $R = 50\,\Omega$ in series with a parallel combination of $L = 20$ mH and $C = 2\,\mu$F.



(a) Calculate the impedance of the parallel LC combination at $f = 1000$ Hz. Express in both rectangular and polar forms.

Solution:
First, calculate the angular frequency: $\omega = 2\pi f = 2\pi(1000) = 6283.2$ rad/s

Calculate individual impedances:

$$\mathbf{Z}_L = j\omega L = j(6283.2)(0.02) = j125.66\,\Omega$$
$$\mathbf{Z}_C = \frac{1}{j\omega C} = \frac{1}{j(6283.2)(2\times10^{-6})} = \frac{-j}{0.01257} = -j79.58\,\Omega$$

For parallel combination:

$$\begin{aligned}\mathbf{Z}_{LC} &= \frac{\mathbf{Z}_L\mathbf{Z}_C}{\mathbf{Z}_L+\mathbf{Z}_C} = \frac{(j125.66)(-j79.58)}{j125.66 - j79.58} \\ &= \frac{9999.7}{j46.08} = \frac{9999.7}{46.08\angle 90^\circ} = 217.0\angle -90^\circ\,\Omega \\ &= -j217.0\,\Omega\end{aligned}$$

Answer: $\mathbf{Z}_{LC} = -j217.0\,\Omega = 217.0\angle -90^\circ\,\Omega$ (capacitive)

(b) Calculate the total impedance $\mathbf{Z}_{tot}$ at $f = 1000$ Hz. Express your answer in both rectangular and polar forms.

Solution:
The total impedance is:

$$\mathbf{Z}_{tot} = R + \mathbf{Z}_{LC} = 50 - j217.0\,\Omega$$

Convert to polar form:

$$|\mathbf{Z}_{tot}| = \sqrt{50^2 + 217.0^2} = \sqrt{2500 + 47089} = 222.7\,\Omega$$
$$\angle\mathbf{Z}_{tot} = \tan^{-1}\left(\frac{-217.0}{50}\right) = -77.0^\circ$$

Answer: $\mathbf{Z}_{tot} = 50 - j217.0\,\Omega = 222.7\angle -77.0^\circ\,\Omega$

(c) At what frequency does the parallel LC combination have infinite impedance? What is the total impedance at this frequency?

Solution:
Parallel LC has infinite impedance at resonance when $\mathbf{Z}_L = -\mathbf{Z}_C$:

$$j\omega L = -\left(\frac{-j}{\omega C}\right)$$
$$\omega L = \frac{1}{\omega C}$$
$$\omega^2 = \frac{1}{LC}$$
$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(0.02)(2\times 10^{-6})}} = \frac{1}{\sqrt{4\times 10^{-8}}} = \frac{1}{2\times 10^{-4}} = 5000 \text{ rad/s}$$

Corresponding frequency:

$$f_0 = \frac{\omega_0}{2\pi} = \frac{5000}{2\pi} = 795.8 \text{ Hz}$$

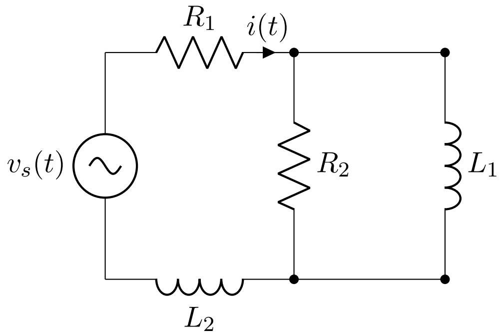
At this frequency, the parallel LC combination has infinite impedance (acts as an open circuit), so:

$$\mathbf{Z}_{tot}(f_0) = R + \infty \approx \infty\,\Omega$$

Answer: $f_0 = 795.8$ Hz, $\mathbf{Z}_{tot} = 50\,\Omega$ (purely resistive)

3. **[12.5 points]**

For the circuit shown below, the voltage source is $v_s(t) = 24\cos(3000t)$ V, $R_1 = 30\,\Omega$, $R_2 = 60\,\Omega$, $L_1 = 20$ mH, and $L_2 = 40$ mH.



(a) Calculate the impedances of L_1 and L_2 at $\omega = 3000$ rad/s.

Solution:

$$\mathbf{Z}_{L_1} = j\omega L_1 = j(3000)(0.02) = j60\,\Omega$$
$$\mathbf{Z}_{L_2} = j\omega L_2 = j(3000)(0.04) = j120\,\Omega$$

(b) Calculate the impedance of the parallel combination of R_2 and L_1.

Solution:

$$\begin{aligned}\mathbf{Z}_{par} &= \frac{R_2 \cdot \mathbf{Z}_{L_1}}{R_2 + \mathbf{Z}_{L_1}} = \frac{(60)(j60)}{60 + j60} = \frac{j3600}{60 + j60} \\ &= \frac{j3600}{84.85\angle 45^\circ} = \frac{3600\angle 90^\circ}{84.85\angle 45^\circ} = 42.43\angle 45^\circ\,\Omega \\ &= 42.43\cos(45^\circ) + j42.43\sin(45^\circ) = 30.0 + j30.0\,\Omega\end{aligned}$$

(c) Calculate the total impedance $\mathbf{Z}_{tot}$ in both rectangular and polar forms. (Hint: The circuit topology is R_1 in series with $(R_2 \parallel L_1)$, all in series with L_2.)

Solution:

$$
\begin{aligned}
\mathbf{Z}_{tot} &= R_1 + \mathbf{Z}_{par} + \mathbf{Z}_{L_2} \\
&= 30 + (30.0 + j30.0) + j120 \\
&= 60.0 + j150.0\,\Omega
\end{aligned}
$$

Convert to polar form:

$$
\begin{aligned}
|\mathbf{Z}_{tot}| &= \sqrt{60.0^2 + 150.0^2} = \sqrt{3600 + 22500} = 161.6\,\Omega \\
\angle\mathbf{Z}_{tot} &= \tan^{-1}\left(\frac{150.0}{60.0}\right) = 68.2^\circ
\end{aligned}
$$

Answer: $\mathbf{Z}_{tot} = 60.0 + j150.0\,\Omega = 161.6\angle 68.2^\circ\,\Omega$

(d) Find the current $i(t)$ in both phasor and time-domain forms.

Solution:
First convert the source to phasor form: $\mathbf{V}_s = 24\angle 0^\circ$ V

$$
\mathbf{I} = \frac{\mathbf{V}_s}{\mathbf{Z}_{tot}} = \frac{24\angle 0^\circ}{161.6\angle 68.2^\circ} = 0.149\angle -68.2^\circ \text{ A}
$$

Convert to time domain:

$$
i(t) = 0.149\cos(3000t - 68.2^\circ) \text{ A} = 149\cos(3000t - 68.2^\circ) \text{ mA}
$$

Answer: $\mathbf{I} = 0.149\angle -68.2^\circ$ A, $i(t) = 0.149\cos(3000t - 68.2^\circ)$ A

(e) Find the voltage across the parallel branch and determine the current through R_2 and L_1 individually.

Solution:
Voltage across parallel branch:

$$
\begin{aligned}
\mathbf{V}_{par} &= \mathbf{I}\cdot\mathbf{Z}_{par} = (0.149\angle -68.2^\circ)(42.43\angle 45^\circ) \\
&= 6.32\angle -23.2^\circ \text{ V}
\end{aligned}
$$

Current through R_2:

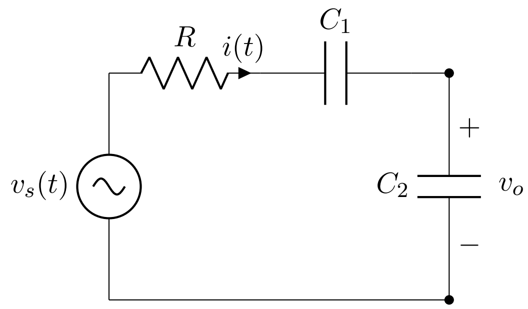
$$\mathbf{I}_{R_2} = \frac{\mathbf{V}_{par}}{R_2} = \frac{6.32\angle -23.2^\circ}{60} = 0.105\angle -23.2^\circ \text{ A}$$
$$i_{R_2}(t) = 0.105\cos(3000t - 23.2^\circ) \text{ A}$$

Current through L_1:

$$\mathbf{I}_{L_1} = \frac{\mathbf{V}_{par}}{\mathbf{Z}_{L_1}} = \frac{6.32\angle -23.2^\circ}{60\angle 90^\circ} = 0.105\angle -113.2^\circ \text{ A}$$
$$i_{L_1}(t) = 0.105\cos(3000t - 113.2^\circ) \text{ A}$$

4. **[12.5 points]**

For the circuit shown below, the voltage source is $v_s(t) = 50\cos(4000t)$ V, $R = 100\,\Omega$, $C_1 = 5\,\mu\text{F}$, and $C_2 = 10\,\mu\text{F}$.



(a) Calculate the impedance of C_1 and C_2 individually at $\omega = 4000$ rad/s.

Solution:

$$\mathbf{Z}_{C_1} = \frac{1}{j\omega C_1} = \frac{1}{j(4000)(5\times 10^{-6})} = \frac{-j}{0.02} = -j50\,\Omega$$
$$\mathbf{Z}_{C_2} = \frac{1}{j\omega C_2} = \frac{1}{j(4000)(10\times 10^{-6})} = \frac{-j}{0.04} = -j25\,\Omega$$

(b) Calculate the total impedance Z_{tot} of the three series components.

Solution:

$$\begin{aligned}\mathbf{Z}_{tot} &= R + \mathbf{Z}_{C_1} + \mathbf{Z}_{C_2} \\ &= 100 - j50 - j25 \\ &= 100 - j75\,\Omega\end{aligned}$$

Convert to polar form:

$$|\mathbf{Z}_{tot}| = \sqrt{100^2 + 75^2} = \sqrt{10000 + 5625} = 125\,\Omega$$
$$\angle\mathbf{Z}_{tot} = \tan^{-1}\left(\frac{-75}{100}\right) = -36.9^\circ$$

Answer: $\mathbf{Z}_{tot} = 100 - j75\,\Omega = 125\angle -36.9^\circ\,\Omega$

(c) Find the current $i(t)$ in both phasor and time-domain forms.

Solution:
Source phasor: $\mathbf{V}_s = 50\angle 0^\circ$ V

$$\mathbf{I} = \frac{\mathbf{V}_s}{\mathbf{Z}_{tot}} = \frac{50\angle 0^\circ}{125\angle -36.9^\circ} = 0.4\angle 36.9^\circ \text{ A}$$

Time domain:

$$i(t) = 0.4\cos(4000t + 36.9^\circ) \text{ A}$$

Answer: $\mathbf{I} = 0.4\angle 36.9^\circ$ A, $i(t) = 0.4\cos(4000t + 36.9^\circ)$ A

(d) Find $v_o(t)$.

Solution:
The output voltage is across C_2:

$$\begin{aligned}\mathbf{V}_o &= \mathbf{I}\cdot\mathbf{Z}_{C_2} = (0.4\angle 36.9^\circ)(-j25)\\ &= (0.4\angle 36.9^\circ)(25\angle -90^\circ)\\ &= 10\angle -53.1^\circ \text{ V}\end{aligned}$$

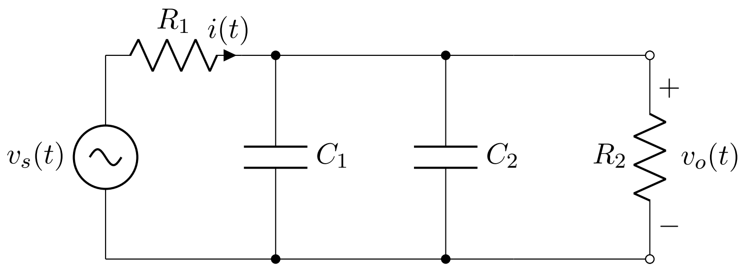
Time domain:

$$v_o(t) = 10\cos(4000t - 53.1^\circ) \text{ V}$$

Answer: $v_o(t) = 10\cos(4000t - 53.1^\circ)$ V

5. **[12.5 points]**

Consider the circuit shown below with $v_s(t) = 20\cos(8000t)$ V, $R_1 = 1$ kΩ, $R_2 = 3$ kΩ, $C_1 = 100$ nF, and $C_2 = 50$ nF.



(a) Calculate the impedances of C_1, C_2, and R_2 at $\omega = 8000$ rad/s, then find the equivalent impedance of the parallel combination.

Solution:

$$\mathbf{Z}_{C_1} = \frac{1}{j\omega C_1} = \frac{1}{j(8000)(100\times10^{-9})} = \frac{-j}{8\times10^{-4}} = -j1250\,\Omega$$
$$\mathbf{Z}_{C_2} = \frac{1}{j\omega C_2} = \frac{1}{j(8000)(50\times10^{-9})} = \frac{-j}{4\times10^{-4}} = -j2500\,\Omega$$
$$\mathbf{Z}_{R_2} = R_2 = 3000\,\Omega$$

For parallel combination:

$$\begin{aligned}
\frac{1}{\mathbf{Z}_{par}} &= \frac{1}{\mathbf{Z}_{C_1}} + \frac{1}{\mathbf{Z}_{C_2}} + \frac{1}{R_2} \\
&= \frac{1}{-j1250} + \frac{1}{-j2500} + \frac{1}{3000} \\
&= j8\times10^{-4} + j4\times10^{-4} + 3.33\times10^{-4} \\
&= 3.33\times10^{-4} + j12\times10^{-4} \\
\mathbf{Z}_{par} &= \frac{1}{3.33\times10^{-4} + j12\times10^{-4}} = \frac{1}{12.45\times10^{-4}\angle 74.5^\circ} \\
&= 803.2\angle -74.5^\circ\,\Omega = 214.7 - j773.8\,\Omega
\end{aligned}$$

Answer: $\mathbf{Z}_{par} = 214.7 - j773.8\,\Omega = 803.2\angle -74.5^\circ\,\Omega$

(b) Calculate the total circuit impedance $\mathbf{Z}_{tot}$ in both rectangular and polar forms.

Solution:

$$\begin{aligned}\mathbf{Z}_{tot} &= R_1 + \mathbf{Z}_{par} = 1000 + (214.7 - j773.8)\\ &= 1214.7 - j773.8\,\Omega\end{aligned}$$

Polar form:

$$\begin{aligned}|\mathbf{Z}_{tot}| &= \sqrt{1214.7^2 + 773.8^2} = \sqrt{1475496 + 598766} = 1441\,\Omega\\ \angle\mathbf{Z}_{tot} &= \tan^{-1}\left(\frac{-773.8}{1214.7}\right) = -32.5^\circ\end{aligned}$$

Answer: $\mathbf{Z}_{tot} = 1214.7 - j773.8\,\Omega = 1441\angle - 32.5^\circ\,\Omega$

(c) Find the input current $i(t)$ in both phasor and time-domain forms.

Solution:
Source phasor: $\mathbf{V}_s = 20\angle 0^\circ$ V

$$\mathbf{I} = \frac{\mathbf{V}_s}{\mathbf{Z}_{tot}} = \frac{20\angle 0^\circ}{1441\angle - 32.5^\circ} = 0.01388\angle 32.5^\circ \text{ A}$$

Time domain:

$$i(t) = 0.01388\cos(8000t + 32.5^\circ) \text{ A} = 13.88\cos(8000t + 32.5^\circ) \text{ mA}$$

Answer: $\mathbf{I} = 13.88\angle 32.5^\circ$ mA, $i(t) = 13.88\cos(8000t + 32.5^\circ)$ mA

(d) Find the output voltage $v_o(t)$.

Solution:
Output voltage across parallel combination:

$$\begin{aligned}\mathbf{V}_o &= \mathbf{I}\cdot\mathbf{Z}_{par} = (0.01388\angle 32.5^\circ)(803.2\angle - 74.5^\circ)\\ &= 11.15\angle - 42.0^\circ \text{ V}\end{aligned}$$

Time domain:

$$v_o(t) = 11.15\cos(8000t - 42.0^\circ) \text{ V}$$

Answer: $v_o(t) = 11.15\cos(8000t - 42.0°)$ V

(e) Calculate the magnitude ratio $|\mathbf{V}_o|/|\mathbf{V}_s|$ and the phase shift.

Solution:

$$\text{Magnitude ratio: } \frac{|\mathbf{V}_o|}{|\mathbf{V}_s|} = \frac{11.15}{20} = 0.5575$$
$$\text{Phase shift: } \Delta\phi = -42.0° - 0° = -42.0°$$

This circuit acts as a voltage divider with gain of 0.56 and introduces a phase lag of 42.0°.

Answer: Magnitude ratio = 0.5575, Phase shift = −42.0°

(f) Find the current through each capacitor (C_1 and C_2) individually.

Solution:

Current through C_1:

$$\begin{aligned}\mathbf{I}_{C_1} &= \frac{\mathbf{V}_o}{\mathbf{Z}_{C_1}} = \frac{11.15\angle-42.0°}{-j1250} = \frac{11.15\angle-42.0°}{1250\angle-90°} \\ &= 0.00892\angle 48.0° \text{ A} = 8.92\angle 48.0° \text{ mA} \\ i_{C_1}(t) &= 8.92\cos(8000t + 48.0°) \text{ mA}\end{aligned}$$

Current through C_2:

$$\begin{aligned}\mathbf{I}_{C_2} &= \frac{\mathbf{V}_o}{\mathbf{Z}_{C_2}} = \frac{11.15\angle-42.0°}{-j2500} = \frac{11.15\angle-42.0°}{2500\angle-90°} \\ &= 0.00446\angle 48.0° \text{ A} = 4.46\angle 48.0° \text{ mA} \\ i_{C_2}(t) &= 4.46\cos(8000t + 48.0°) \text{ mA}\end{aligned}$$

6. **[12.5 points]**

A load is connected to an AC source. The voltage across the load and the current through the load are measured as:

$$v(t) = 120\sqrt{2}\cos(377t) \text{ V}$$
$$i(t) = 6\sqrt{2}\cos(377t - 53.13^\circ) \text{ A}$$

(a) Convert the voltage and current to phasor form, then calculate the RMS phasors $\mathbf{V}_{rms}$ and $\mathbf{I}_{rms}$. (Hint: The $\sqrt{2}$ factor is included to indicate peak values.)

Solution:
The signals are already in peak form. Convert to phasors:

$$v(t) = 120\sqrt{2}\cos(377t) \text{ V} \quad \Rightarrow \quad \mathbf{V} = 120\sqrt{2}\angle 0^\circ \text{ V}$$
$$i(t) = 6\sqrt{2}\cos(377t - 53.13^\circ) \text{ A} \quad \Rightarrow \quad \mathbf{I} = 6\sqrt{2}\angle - 53.13^\circ \text{ A}$$

RMS values:

$$\mathbf{V}_{rms} = \frac{\mathbf{V}}{\sqrt{2}} = \frac{120\sqrt{2}}{\sqrt{2}}\angle 0^\circ = 120\angle 0^\circ \text{ V}$$
$$\mathbf{I}_{rms} = \frac{\mathbf{I}}{\sqrt{2}} = \frac{6\sqrt{2}}{\sqrt{2}}\angle - 53.13^\circ = 6\angle - 53.13^\circ \text{ A}$$

Answer: $\mathbf{V}_{rms} = 120\angle 0^\circ$ V, $\mathbf{I}_{rms} = 6\angle - 53.13^\circ$ A

(b) Find the real power P, reactive power Q, and apparent power $\mathbf{S}$. Include proper units.

Solution:
The phase angle between voltage and current is $\theta = 0^\circ - (-53.13^\circ) = 53.13^\circ$.

$$S = V_{rms}I_{rms} = (120)(6) = 720 \text{ VA}$$
$$P = V_{rms}I_{rms}\cos\theta = (120)(6)\cos(53.13^\circ) = 720(0.6) = 432 \text{ W}$$
$$Q = V_{rms}I_{rms}\sin\theta = (120)(6)\sin(53.13^\circ) = 720(0.8) = 576 \text{ VAR}$$

Alternatively, using complex power:

$$\mathbf{S} = \mathbf{V}_{rms}\mathbf{I}_{rms}^* = (120\angle 0^\circ)(6\angle 53.13^\circ)$$
$$= 720\angle 53.13^\circ = 432 + j576 \text{ VA}$$

Answer: $P = 432$ W, $Q = 576$ VAR (inductive), $S = 720$ VA, $\mathbf{S} = 432 + j576$ VA

(c) Calculate the power factor and state whether the load is inductive or capacitive. Explain your reasoning.

Solution:

$$\text{Power factor} = \cos\theta = \cos(53.13^\circ) = 0.6 \text{ lagging}$$

The load is **inductive** because:

- The current lags the voltage by 53.13° (current phase angle is −53.13° while voltage is at 0°)
- Reactive power $Q > 0$ (positive reactive power indicates inductive load)
- The power factor is "lagging"

Answer: Power factor = 0.6 lagging, Load is inductive

(d) Find the impedance of the load using $\mathbf{Z} = \mathbf{V}_{rms}/\mathbf{I}_{rms}$ in both rectangular and polar forms. What circuit elements could this load represent?

Solution:

$$\mathbf{Z} = \frac{\mathbf{V}_{rms}}{\mathbf{I}_{rms}} = \frac{120\angle 0^\circ}{6\angle -53.13^\circ} = 20\angle 53.13^\circ\,\Omega$$

Convert to rectangular form:

$$\begin{aligned}\mathbf{Z} &= 20\cos(53.13^\circ) + j20\sin(53.13^\circ)\\ &= 20(0.6) + j20(0.8) = 12 + j16\,\Omega\end{aligned}$$

This impedance represents a resistor in series with an inductor:

$$\begin{aligned}&R = 12\,\Omega\\ &X_L = 16\,\Omega \quad \Rightarrow \quad L = \frac{X_L}{\omega} = \frac{16}{377} = 42.4 \text{ mH}\end{aligned}$$

Answer: $\mathbf{Z} = 12 + j16\,\Omega = 20\angle 53.13^\circ\,\Omega$. The load is a $12\,\Omega$ resistor in series with a 42.4 mH inductor.

(e) If you wanted to improve the power factor to unity (1.0), what value of capacitor would you need to place in parallel with this load? (Note: $\omega = 377$ rad/s corresponds to $f = 60$ Hz)

Solution:
To achieve unity power factor, we need to cancel the reactive power:

$$Q_C = -Q_L = -576 \text{ VAR}$$

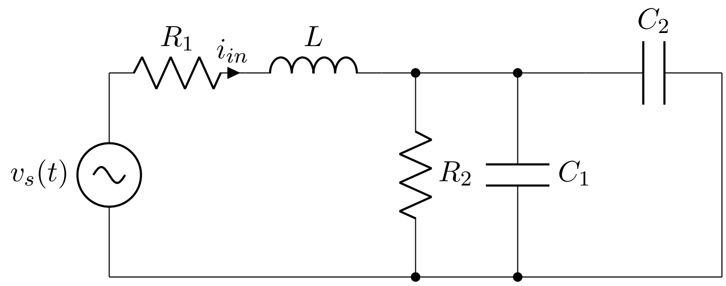
For a capacitor:

$$\begin{aligned} Q_C &= -V_{rms}^2 \omega C \quad \text{(negative because capacitive)} \\ -576 &= -\frac{V_{rms}^2}{X_C} = -V_{rms}^2 \omega C \\ 576 &= (120)^2 \omega C \\ C &= \frac{576}{(120)^2(377)} = \frac{576}{5428800} = 106.1 \times 10^{-6} \text{ F} \end{aligned}$$

Answer: $C = 106.1\,\mu\text{F}$ (placed in parallel with the load)

7. **[12.5 points]**

Consider the circuit with $v_s(t) = 100\cos(5000t)$ V, $R_1 = 20\,\Omega$, $R_2 = 80\,\Omega$, $L = 8$ mH, $C_1 = 10\,\mu$F, and $C_2 = 40\,\mu$F.



(a) Calculate all component impedances at $\omega = 5000$ rad/s.

Solution:

$$\begin{aligned}
\mathbf{Z}_{R_1} &= R_1 = 20\,\Omega \\
\mathbf{Z}_{R_2} &= R_2 = 80\,\Omega \\
\mathbf{Z}_L &= j\omega L = j(5000)(0.008) = j40\,\Omega \\
\mathbf{Z}_{C_1} &= \frac{1}{j\omega C_1} = \frac{1}{j(5000)(10\times10^{-6})} = \frac{-j}{0.05} = -j20\,\Omega \\
\mathbf{Z}_{C_2} &= \frac{1}{j\omega C_2} = \frac{1}{j(5000)(40\times10^{-6})} = \frac{-j}{0.2} = -j5\,\Omega
\end{aligned}$$

(b) Calculate the total impedance $\mathbf{Z}_{tot}$ in both rectangular and polar forms.

Solution:

The circuit topology is: R_1 in series with L, then parallel combination of $(R_2 \parallel C_1 \parallel C_2)$.

First, find the parallel impedance:

$$\begin{aligned}
\frac{1}{\mathbf{Z}_{par}} &= \frac{1}{R_2} + \frac{1}{\mathbf{Z}_{C_1}} + \frac{1}{\mathbf{Z}_{C_2}} \\
&= \frac{1}{80} + \frac{1}{-j20} + \frac{1}{-j5} \\
&= 0.0125 + j0.05 + j0.2 = 0.0125 + j0.25 \\
\mathbf{Z}_{par} &= \frac{1}{0.0125 + j0.25} = \frac{1}{0.2503\angle 87.1^\circ} = 3.996\angle -87.1^\circ\,\Omega \\
&= 0.20 - j3.99\,\Omega
\end{aligned}$$

Total impedance:

$$\begin{aligned}\mathbf{Z}_{tot} &= R_1 + \mathbf{Z}_L + \mathbf{Z}_{par} \\ &= 20 + j40 + (0.20 - j3.99) \\ &= 20.20 + j36.01\,\Omega\end{aligned}$$

Polar form:

$$\begin{aligned}|\mathbf{Z}_{tot}| &= \sqrt{20.20^2 + 36.01^2} = \sqrt{408.04 + 1296.72} = 41.29\,\Omega \\ \angle\mathbf{Z}_{tot} &= \tan^{-1}\left(\frac{36.01}{20.20}\right) = 60.7^\circ\end{aligned}$$

Answer: $\mathbf{Z}_{tot} = 20.20 + j36.01\,\Omega = 41.29\angle 60.7^\circ\,\Omega$

(c) Find the input current $i_{in}(t)$ in both phasor and time-domain forms.

Solution:

Source phasor: $\mathbf{V}_s = 100\angle 0^\circ$ V

$$\mathbf{I}_{in} = \frac{\mathbf{V}_s}{\mathbf{Z}_{tot}} = \frac{100\angle 0^\circ}{41.29\angle 60.7^\circ} = 2.42\angle -60.7^\circ \text{ A}$$

Time domain:

$$i_{in}(t) = 2.42\cos(5000t - 60.7^\circ) \text{ A}$$

Answer: $\mathbf{I}_{in} = 2.42\angle -60.7^\circ$ A, $i_{in}(t) = 2.42\cos(5000t - 60.7^\circ)$ A

(d) Calculate the voltage across the parallel branch ($R_2 \parallel C_1 \parallel C_2$).

Solution:

$$\begin{aligned}\mathbf{V}_{par} &= \mathbf{I}_{in} \cdot \mathbf{Z}_{par} = (2.42\angle -60.7^\circ)(3.996\angle -87.1^\circ) \\ &= 9.67\angle -147.8^\circ \text{ V}\end{aligned}$$

Time domain:

$$v_{par}(t) = 9.67\cos(5000t - 147.8^\circ) \text{ V}$$

Answer: $\mathbf{V}_{par} = 9.67\angle -147.8^\circ$ V

(e) Find the current through R_2, C_1, and C_2 individually.

Solution:
Current through R_2:

$$\mathbf{I}_{R_2} = \frac{\mathbf{V}_{par}}{R_2} = \frac{9.67\angle -147.8^\circ}{80} = 0.121\angle -147.8^\circ \text{ A}$$
$$i_{R_2}(t) = 0.121\cos(5000t - 147.8^\circ) \text{ A} = 121\cos(5000t - 147.8^\circ) \text{ mA}$$

Current through C_1:

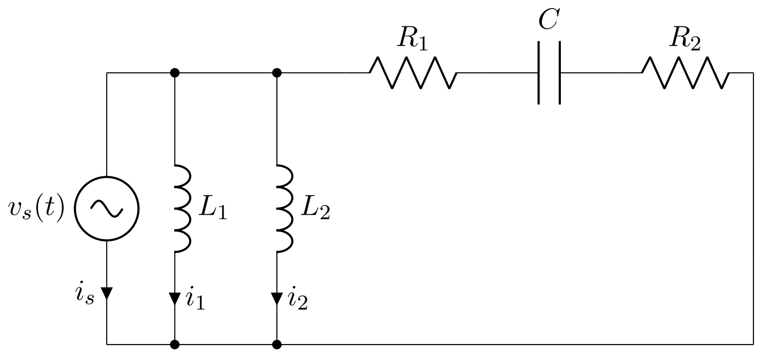
$$\begin{aligned}\mathbf{I}_{C_1} &= \frac{\mathbf{V}_{par}}{\mathbf{Z}_{C_1}} = \frac{9.67\angle -147.8^\circ}{-j20} = \frac{9.67\angle -147.8^\circ}{20\angle -90^\circ} \\ &= 0.484\angle -57.8^\circ \text{ A} \\ i_{C_1}(t) &= 0.484\cos(5000t - 57.8^\circ) \text{ A} = 484\cos(5000t - 57.8^\circ) \text{ mA}\end{aligned}$$

Current through C_2:

$$\begin{aligned}\mathbf{I}_{C_2} &= \frac{\mathbf{V}_{par}}{\mathbf{Z}_{C_2}} = \frac{9.67\angle -147.8^\circ}{-j5} = \frac{9.67\angle -147.8^\circ}{5\angle -90^\circ} \\ &= 1.934\angle -57.8^\circ \text{ A} \\ i_{C_2}(t) &= 1.934\cos(5000t - 57.8^\circ) \text{ A}\end{aligned}$$

8. **[12.5 points]**

Analyze the circuit shown below with $v_s(t) = 60\cos(4000t)$ V, $R_1 = 50\,\Omega$, $R_2 = 100\,\Omega$, $L_1 = 25$ mH, $L_2 = 50$ mH, and $C = 5\,\mu$F.



(a) Calculate the impedance of each inductor and the capacitor at $\omega = 4000$ rad/s.

Solution:

$$\mathbf{Z}_{L_1} = j\omega L_1 = j(4000)(0.025) = j100\,\Omega$$
$$\mathbf{Z}_{L_2} = j\omega L_2 = j(4000)(0.050) = j200\,\Omega$$
$$\mathbf{Z}_C = \frac{1}{j\omega C} = \frac{1}{j(4000)(5\times10^{-6})} = \frac{-j}{0.02} = -j50\,\Omega$$

(b) Calculate the equivalent impedance of the parallel inductor combination ($L_1 \parallel L_2$).

Solution:

$$\begin{aligned}\mathbf{Z}_{L,par} &= \frac{\mathbf{Z}_{L_1}\cdot\mathbf{Z}_{L_2}}{\mathbf{Z}_{L_1}+\mathbf{Z}_{L_2}} = \frac{(j100)(j200)}{j100+j200}\\ &= \frac{-20000}{j300} = \frac{-20000}{300\angle 90^\circ} = \frac{20000}{300}\angle -90^\circ\\ &= 66.67\angle 90^\circ = j66.67\,\Omega\end{aligned}$$

Alternatively, for inductors in parallel:

$$L_{eq} = \frac{L_1 L_2}{L_1+L_2} = \frac{(0.025)(0.050)}{0.025+0.050} = \frac{0.00125}{0.075} = 0.01667\text{ H}$$
$$\mathbf{Z}_{L,par} = j\omega L_{eq} = j(4000)(0.01667) = j66.67\,\Omega$$

Answer: $\mathbf{Z}_{L,par} = j66.67\,\Omega$

(c) Calculate the total impedance $\mathbf{Z}_{tot}$ in both rectangular and polar forms.

Solution:
First, calculate the series impedance:

$$\begin{aligned}\mathbf{Z}_{series} &= R_1 + \mathbf{Z}_C + R_2 \\ &= 50 - j50 + 100 \\ &= 150 - j50\,\Omega\end{aligned}$$

Now find the parallel combination:

$$\begin{aligned}\mathbf{Z}_{tot} &= \frac{\mathbf{Z}_{L,par} \cdot \mathbf{Z}_{series}}{\mathbf{Z}_{L,par} + \mathbf{Z}_{series}} = \frac{(j66.67)(150 - j50)}{j66.67 + 150 - j50} \\ &= \frac{j66.67(150 - j50)}{150 + j16.67} = \frac{j10000.5 + 3333.5}{150 + j16.67} \\ &= \frac{3333.5 + j10000.5}{150 + j16.67}\end{aligned}$$

Convert to polar:

$$\begin{aligned}\text{Numerator: } & |3333.5 + j10000.5| = \sqrt{3333.5^2 + 10000.5^2} = 10543\,\Omega \\ & \angle = \tan^{-1}(10000.5/3333.5) = 71.6^\circ \\ \text{Denominator: } & |150 + j16.67| = \sqrt{150^2 + 16.67^2} = 150.9\,\Omega \\ & \angle = \tan^{-1}(16.67/150) = 6.34^\circ\end{aligned}$$

$$\mathbf{Z}_{tot} = \frac{10543\angle 71.6^\circ}{150.9\angle 6.34^\circ} = 69.87\angle 65.3^\circ\,\Omega$$

Convert to rectangular:

$$\mathbf{Z}_{tot} = 69.87\cos(65.3^\circ) + j69.87\sin(65.3^\circ) = 29.1 + j63.5\,\Omega$$

Answer: $\mathbf{Z}_{tot} = 29.1 + j63.5\,\Omega = 69.87\angle 65.3^\circ\,\Omega$

(d) Find the total source current $i_s(t)$ in both phasor and time-domain forms.

Solution:
Source phasor: $\mathbf{V}_s = 60\angle 0^\circ$ V

$$\mathbf{I}_s = \frac{\mathbf{V}_s}{\mathbf{Z}_{tot}} = \frac{60\angle 0^\circ}{69.87\angle 65.3^\circ} = 0.859\angle -65.3^\circ \text{ A}$$

Time domain:

$$i_s(t) = 0.859\cos(4000t - 65.3^\circ) \text{ A} = 859\cos(4000t - 65.3^\circ) \text{ mA}$$

Answer: $\mathbf{I}_s = 0.859\angle -65.3^\circ$ A, $i_s(t) = 0.859\cos(4000t - 65.3^\circ)$ A

(e) Find the voltage across the parallel inductor combination, then determine the individual currents $i_1(t)$ and $i_2(t)$ through each inductor.

Solution:
Since the inductors are in parallel with the entire circuit and connected directly to the source, the voltage across them is the source voltage:

$$\mathbf{V}_{L,par} = \mathbf{V}_s = 60\angle 0^\circ \text{ V}$$

Current through L_1:

$$\mathbf{I}_1 = \frac{\mathbf{V}_{L,par}}{\mathbf{Z}_{L_1}} = \frac{60\angle 0^\circ}{100\angle 90^\circ} = 0.6\angle -90^\circ \text{ A}$$
$$i_1(t) = 0.6\cos(4000t - 90^\circ) \text{ A} = 600\cos(4000t - 90^\circ) \text{ mA}$$

Current through L_2:

$$\mathbf{I}_2 = \frac{\mathbf{V}_{L,par}}{\mathbf{Z}_{L_2}} = \frac{60\angle 0^\circ}{200\angle 90^\circ} = 0.3\angle -90^\circ \text{ A}$$
$$i_2(t) = 0.3\cos(4000t - 90^\circ) \text{ A} = 300\cos(4000t - 90^\circ) \text{ mA}$$

Current through series branch ($R_1 + C + R_2$):

$$\mathbf{I}_{series} = \frac{\mathbf{V}_s}{\mathbf{Z}_{series}} = \frac{60\angle 0^\circ}{150 - j50} = \frac{60\angle 0^\circ}{158.1\angle -18.4^\circ}$$
$$= 0.379\angle 18.4^\circ \text{ A}$$

Verification: $\mathbf{I}_s = \mathbf{I}_1 + \mathbf{I}_2 + \mathbf{I}_{series}$ (by current division)

(f) Calculate the total real power, reactive power, and complex power delivered by the source. Express complex power in rectangular form. (Hint:You already know the source voltage and current.)

Solution:
Convert to RMS values:

$$\mathbf{V}_{s,rms} = \frac{60}{\sqrt{2}}\angle 0^\circ = 42.43\angle 0^\circ \text{ V}$$
$$\mathbf{I}_{s,rms} = \frac{0.859}{\sqrt{2}}\angle -65.3^\circ = 0.607\angle -65.3^\circ \text{ A}$$

Complex power:

$$\mathbf{S} = \mathbf{V}_{s,rms}\mathbf{I}^*_{s,rms} = (42.43\angle 0^\circ)(0.607\angle 65.3^\circ)$$
$$= 25.76\angle 65.3^\circ \text{ VA}$$

Convert to rectangular form:

$$\mathbf{S} = 25.76\cos(65.3^\circ) + j25.76\sin(65.3^\circ)$$
$$= 10.74 + j23.43 \text{ VA}$$

Therefore:

$$P = 10.74 \text{ W (real power)}$$
$$Q = 23.43 \text{ VAR (reactive power, inductive)}$$
$$S = |\mathbf{S}| = 25.76 \text{ VA (apparent power)}$$

Answer: $P = 10.74$ W, $Q = 23.43$ VAR, $\mathbf{S} = 10.74 + j23.43$ VA