Operational Amplifier Applications

Feedback Configurations and Mathematical Operations

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Outline

Review: The Ideal Op-Amp

Five Ideal OpAmp Assumptions:

1. $Z_{in} = \infty \Rightarrow i_+ = i_- = 0$
2. $Z_{out} = 0$ (ideal voltage source)
3. $A = \infty$ (infinite open-loop gain)
4. Infinite bandwidth
5. Infinite CMRR

Basic Relationship:

$$v_{out} = A(v_+ - v_-)$$

With $A \to \infty$, for bounded output:

$$v_+ - v_- \to 0 \quad \text{(virtual short)}$$

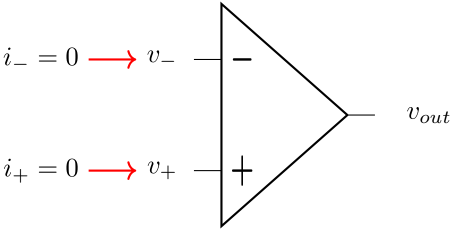


Figure 1: Ideal op-amp terminals

Key Insight

- $v_+ = v_-$ (virtual short)
- $i_+ = i_- = 0$ (no input current)

Negative Feedback Concept

What is Negative Feedback?

- Output is fed back to inverting input
- Opposes changes in output
- ☺ Makes gain predictable

Why Use Feedback?

- ☺ Precise, stable gain
- ☺ Insensitive to A variations
- ☺ Improved linearity
- ☺ Controlled impedances

Key Insight

With negative feedback and ideal op-amp: $v_+ = v_-$

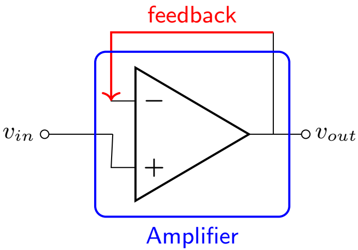


Figure 2: Negative feedback block diagram

Result:

- Op-amp adjusts v_{out} to make $v_- = v_+$
- ☺ Gain determined by external components

The Inverting Amplifier

Circuit Configuration:

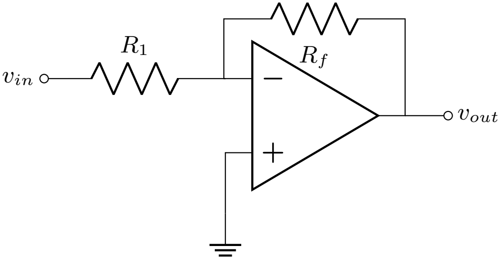


Figure 3: Inverting amplifier circuit

Analysis:

1. Since $v_+ = 0$ (AC Ground, be careful!):

$$v_- = v_+ = 0 \quad \text{(AC ground)}$$

2. Current through R_1:

$$i_1 = \frac{v_{in} - v_-}{R_1} = \frac{v_{in}}{R_1}$$

3. Since $i_- = 0$, all of i_1 flows through R_f:

$$i_f = i_1 = \frac{v_{in}}{R_1}$$

4. Voltage across R_f:

$$v_{out} - v_- = -i_f R_f$$

Inverting Amplifier: Results

Voltage Gain:

$$\boxed{A_v = \frac{v_{out}}{v_{in}} = -\frac{R_f}{R_1}}$$

- 😐 Negative sign: 180° phase shift
- ☺ Magnitude set by resistor ratio
- ☺ Independent of op-amp gain A

Input Impedance:

$$\boxed{R_{in} = \frac{v_{in}}{i_1} = R_1}$$

- 😐 Not infinite!
- ☺ Determined by R_1

Design Examples:

Example 1: Unity gain inverter

- $R_1 = R_f = 10\ \text{k}\Omega$
- $A_v = -1$
- $R_{in} = 10\ \text{k}\Omega$

Example 2: Gain of -10

- $R_1 = 10\ \text{k}\Omega$, $R_f = 100\ \text{k}\Omega$
- $A_v = -10$
- $R_{in} = 10\ \text{k}\Omega$

Example 3: Gain of -0.5

- $R_1 = 20\ \text{k}\Omega$, $R_f = 10\ \text{k}\Omega$
- $A_v = -0.5$ (attenuation!)
- $R_{in} = 20\ \text{k}\Omega$

The Noninverting Amplifier

Circuit Configuration:

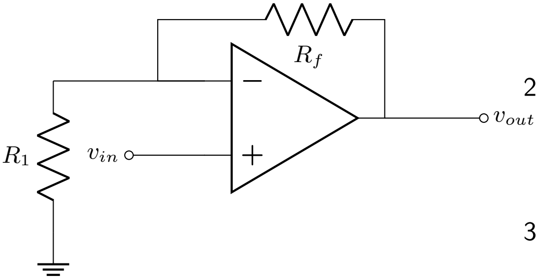


Figure 4: Noninverting amplifier circuit

Analysis:

1. Since $v_+ = v_{in}$:

$$v_- = v_+ = v_{in}$$

2. Voltage divider at inverting input:

$$v_- = v_{out}\frac{R_1}{R_1 + R_f}$$

3. Solve for gain:

$$v_{in} = v_{out}\frac{R_1}{R_1 + R_f}$$

$$\frac{v_{out}}{v_{in}} = \frac{R_1 + R_f}{R_1} = 1 + \frac{R_f}{R_1}$$

Noninverting Amplifier: Key Results

Voltage Gain:

$$\boxed{A_v = 1 + \frac{R_f}{R_1}}$$

- 😐 Always positive (no inversion)
- Minimum gain is 1
- ☺ Set by resistor ratio

Input Impedance:

$$\boxed{R_{in} = \infty}$$

- ☺ Infinite (ideally)
- ☺ No loading of source

Design Examples:

Example 1: Gain of 2

- $R_1 = R_f = 10\ \text{k}\Omega$
- $A_v = 1 + 1 = 2$

Example 2: Gain of 11

- $R_1 = 10\ \text{k}\Omega,\ R_f = 100\ \text{k}\Omega$
- $A_v = 1 + 10 = 11$

Example 3: Gain of 1

- $R_1 = \infty,\ R_f = 0$
- $A_v = 1 + 0 = 1$

Noninverting Amplifier: Voltage Follower (Unity-Gain Buffer)

Special Case - Voltage Follower:

- $R_f = 0$ (short circuit)
- $R_1 = \infty$ (open circuit)
- $A_v = 1$ (unity gain buffer)

Key Features:

- ☺ Output tracks the input: $v_{out} \approx v_{in}$
- ☺ High input impedance, allows it to sense the input without loading it
- ☺ Low output impedance allows it to drive heavy loads
- ☺ Used to isolate stages, prevent loading effects

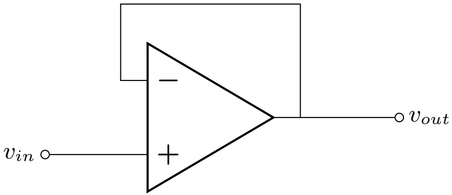


Figure 5: Unity Gain Buffer

The Summing Amplifier

Circuit Configuration:

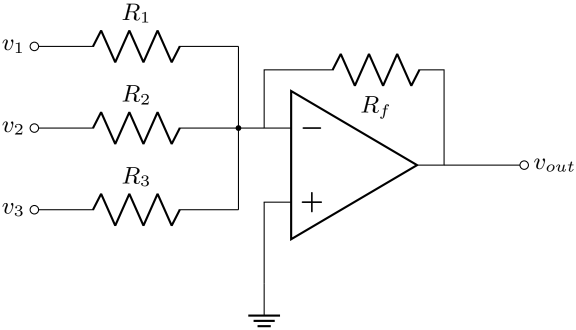


Figure 6: Summing amplifier (3 inputs)

Analysis:

1. Virtual ground: $v_- = 0$
2. Currents from each input:

$$i_1 = \frac{v_1}{R_1}, \quad i_2 = \frac{v_2}{R_2}, \quad i_3 = \frac{v_3}{R_3}$$

3. KCL at inverting node:

$$i_f = i_1 + i_2 + i_3$$

4. Output voltage:

$$v_{out} = -i_f R_f$$

$$v_{out} = -R_f \left(\frac{v_1}{R_1} + \frac{v_2}{R_2} + \frac{v_3}{R_3} \right)$$

Summing Amplifier: Applications

Special Cases:

Case 1: Equal resistors

- $R_1 = R_2 = R_3 = R$, $R_f = R$
- $v_{out} = -(v_1 + v_2 + v_3)$
- Simple inverting summer

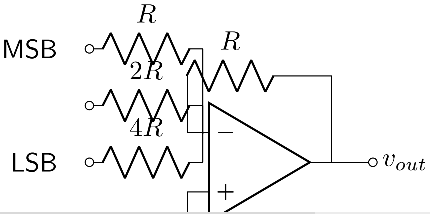
Case 2: Weighted summer

- Different resistor values
- Each input has different weight
- Example: $R_f = 10\ \text{k}\Omega$
  - $R_1 = 10\ \text{k}\Omega \Rightarrow$ weight $= 1$
  - $R_2 = 5\ \text{k}\Omega \Rightarrow$ weight $= 2$
  - $R_3 = 20\ \text{k}\Omega \Rightarrow$ weight $= 0.5$
- $v_{out} = -(v_1 + 2v_2 + 0.5v_3)$

Applications:

- Audio mixing consoles
- Digital-to-analog conversion (DAC)
- Analog computation
- Signal averaging
- Multi-input control systems

Example: 3-bit DAC:



The Difference Amplifier

Circuit Configuration:

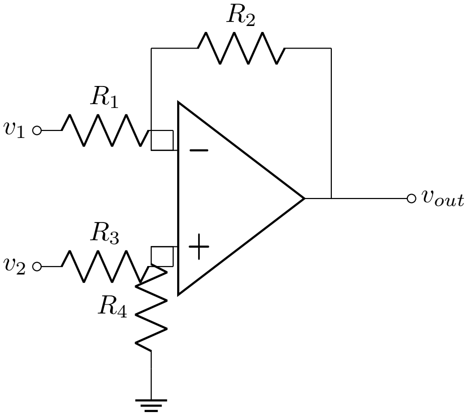


Figure 8: Difference (differential) amplifier

Analysis:

1. Voltage at noninverting input:

$$v_+ = v_2 \frac{R_4}{R_3 + R_4}$$

2. Since $v_- = v_+$:

$$v_- = v_2 \frac{R_4}{R_3 + R_4}$$

3. Current through R_1:

$$i_1 = \frac{v_1 - v_-}{R_1}$$

4. Since $i_- = 0$:

$$v_{out} = v_- - i_1 R_2$$

Difference Amplifier: Key Points

Design Constraint:
For pure differential gain:

$$\boxed{\frac{R_2}{R_1} = \frac{R_4}{R_3}}$$

Common Choice:

- $R_1 = R_3 = R$
- $R_2 = R_4 = kR$
- Differential gain: $A_d = k$

Example:

- $R_1 = R_3 = 10\ \text{k}\Omega$
- $R_2 = R_4 = 100\ \text{k}\Omega$
- $v_{out} = 10(v_2 - v_1)$

Applications:

- Instrumentation
- Sensor signal conditioning
- Noise rejection (common-mode)
- Bridge circuit measurements
- Biomedical amplifiers (ECG, EEG)

Advantages:

- Rejects common-mode signals
- Amplifies differential signal
- Single op-amp solution

Limitations:

- Requires precision resistor matching

The Integrator

Circuit Configuration:

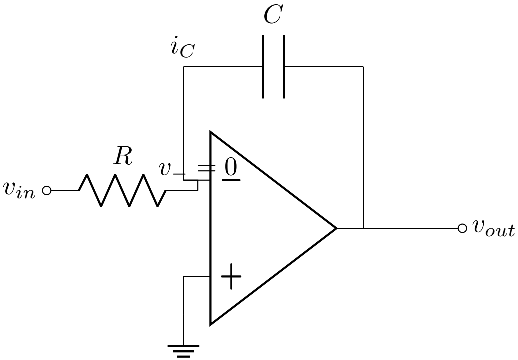


Figure 9: Inverting integrator

Analysis:

1. Virtual ground: $v_- = 0$
2. Current through R:

$$i_R = \frac{v_{in} - 0}{R} = \frac{v_{in}}{R}$$

3. Since $i_- = 0$, all current flows through C:

$$i_C = i_R = \frac{v_{in}}{R}$$

4. Capacitor voltage-current relation:

$$i_C = C\frac{dv_C}{dt}$$

5. Since $v_C = 0 - v_{out} = -v_{out}$:

Integrator: Time-Domain Analysis

Transfer Function:

From:

$$\frac{v_{in}}{R} = -C\frac{dv_{out}}{dt}$$

Rearranging:

$$\frac{dv_{out}}{dt} = -\frac{1}{RC}v_{in}$$

Integrating both sides:

$$\boxed{v_{out}(t) = -\frac{1}{RC}\int_0^t v_{in}(\tau)d\tau + v_{out}(0)}$$

Interpretation:

- Output is (inverted) integral of input
- Time constant: $\tau = RC$

Frequency Response:

In frequency domain (assuming $v_{out}(0) = 0$):

$$\boxed{\frac{V_{out}(j\omega)}{V_{in}(j\omega)} = -\frac{1}{j\omega RC}}$$

Magnitude:

$$|H(j\omega)| = \frac{1}{\omega RC}$$

- Gain decreases with frequency
- -20 dB/decade slope
- Infinite gain at DC (impractical!)

Practical Issue:

Integrator: Waveform Examples

Example 1: Step Input

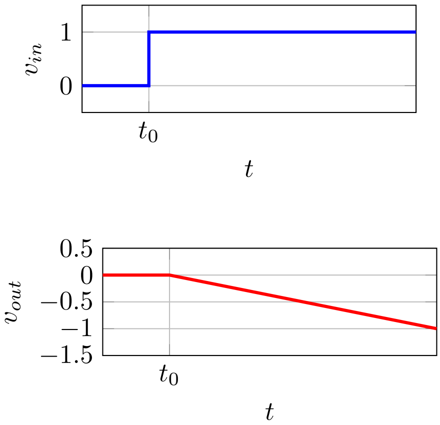


Figure 10: Step input produces ramp output

Example 2: Square Wave Input

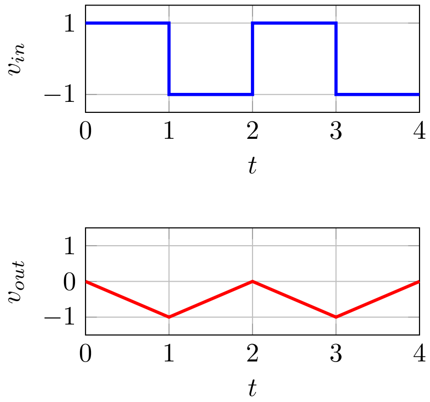


Figure 11: Square wave produces triangle wave

The Differentiator

Circuit Configuration:

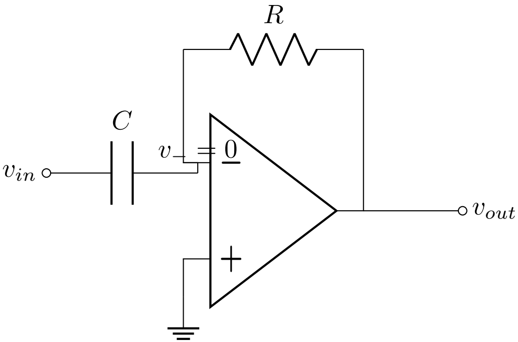


Figure 12: Inverting differentiator

Note: Integrator with R and C swapped!

Analysis:

1. Virtual ground: $v_- = 0$
2. Capacitor current:

$$i_C = C\frac{dv_C}{dt} = C\frac{d(v_{in} - 0)}{dt}$$

$$i_C = C\frac{dv_{in}}{dt}$$

3. Since $i_- = 0$, i_C flows through R:

$$i_R = i_C = C\frac{dv_{in}}{dt}$$

4. Output voltage:

$$v_{out} = 0 - i_R R = -RC\frac{dv_{in}}{dt}$$

Differentiator: Characteristics and Issues

Transfer Function:

Time domain:

$$v_{out} = -RC\frac{dv_{in}}{dt}$$

Frequency domain:

$$\boxed{\frac{V_{out}(j\omega)}{V_{in}(j\omega)} = -j\omega RC}$$

Magnitude:

$$|H(j\omega)| = \omega RC$$

- Gain increases with frequency
- $+20$ dB/decade slope

Practical Problems:

1. **Noise amplification**
   - High-frequency noise magnified
   - Can saturate output
2. **Stability issues**
   - Phase shift can cause oscillation
   - Needs compensation

Practical Solution:

Add small resistor R_s in series with C:

- Limits high-frequency gain
- Improves stability
- $R_s \ll R$ (typically $R_s \approx R/10$)

Warning

Differentiator: Waveform Examples

Example 1: Ramp Input

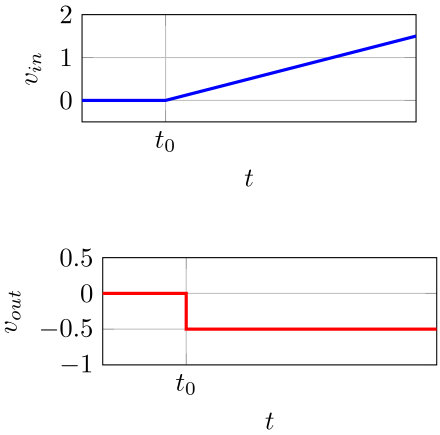


Figure 13: Ramp input produces constant

Example 2: Triangle Wave Input

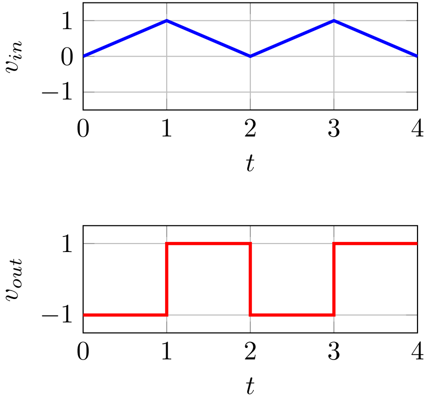


Figure 14: Triangle wave produces square wave

Summary: Op-Amp Applications

Configuration	**Gain**	R_{in}	**Application**
Inverting	$-\frac{R_f}{R_1}$	R_1	Amplification with inversion
Noninverting	$1+\frac{R_f}{R_1}$	∞	Amplification, no inversion
Voltage Follower	1	∞	Buffering, impedance matching
Summing	$-\sum \frac{R_f}{R_i} v_i$	R_i	Audio mixing, DAC
Difference	$\frac{R_2}{R_1}(v_2 - v_1)$	finite	Instrumentation
Integrator	$-\frac{1}{RC}\int v_{in} dt$	∞ (AC)	Analog computation, filters
Differentiator	$-RC\frac{dv_{in}}{dt}$	$\to 0$ (HF)	Rarely used (noise!)

Table 1: Summary of op-amp configurations

Key Analysis Steps

1. Identify configuration
2. Apply virtual short: $v_+ = v_-$ (with negative feedback)

Practice Problem 1

Given: An inverting amplifier with $R_1 = 4.7$ kΩ and $R_f = 47$ kΩ

Find:

(a) The voltage gain A_v

(b) The input impedance R_{in}

(c) If $v_{in} = 0.5$ V, what is v_{out}?

(d) What resistor value should R_f be to achieve $A_v = -15$?

Hint: Use $A_v = -\frac{R_f}{R_1}$ and $R_{in} = R_1$

Practice Problem 1 Solution

Given: $R_1 = 4.7\ \text{k}\Omega$, $R_f = 47\ \text{k}\Omega$

Solutions:

(a) Voltage gain:

$$A_v = -\frac{R_f}{R_1} = -\frac{47\ \text{k}\Omega}{4.7\ \text{k}\Omega} = \boxed{-10\ \text{V/V}}$$

(b) Input impedance:

$$R_{in} = R_1 = \boxed{4.7\ \text{k}\Omega}$$

(c) Output voltage:

$$v_{out} = A_v \cdot v_{in} = (-10)(0.5\ \text{V}) = \boxed{-5\ \text{V}}$$

(d) For $A_v = -15$:

$$-15 = -\frac{R_f}{4.7\ \text{k}\Omega} \Rightarrow R_f = 15 \times 4.7\ \text{k}\Omega = \boxed{70.5\ \text{k}\Omega}$$

(Use standard value 68 kΩ or 75 kΩ in practice)

Practice Problem 2

Given: A noninverting amplifier with the following requirements:

- Voltage gain: $A_v = 5$
- Input voltage: $v_{in} = 0.2$ V
- Choose $R_1 = 10\ \text{k}\Omega$

Find:

(a) The required value of R_f

(b) The output voltage v_{out}

(c) The input impedance

(d) If you need $A_v = 1$ (unity gain buffer), what should the circuit look like?

Hint: Use $A_v = 1 + \frac{R_f}{R_1}$

Practice Problem 2 Solution

Given: $A_v = 5$, $R_1 = 10\ \text{k}\Omega$, $v_{in} = 0.2$ V

Solutions:

(a) Required R_f:

$$5 = 1 + \frac{R_f}{10\ \text{k}\Omega}$$

$$\frac{R_f}{10\ \text{k}\Omega} = 4 \Rightarrow R_f = \boxed{40\ \text{k}\Omega}$$

(b) Output voltage:

$$v_{out} = A_v \cdot v_{in} = 5 \times 0.2\ \text{V} = \boxed{1.0\ \text{V}}$$

(c) Input impedance:

$$R_{in} = \boxed{\infty}\ \text{(ideally)}$$

(d) For unity gain buffer ($A_v = 1$):

- Connect output directly to inverting input (direct feedback)
- Remove R_1 and R_f
- Input to noninverting terminal

Practice Problem 3

Given: A summing amplifier with three inputs

- $R_1 = 10$ kΩ, $R_2 = 20$ kΩ, $R_3 = 5$ kΩ
- $R_f = 20$ kΩ
- Input voltages: $v_1 = 1$ V, $v_2 = 0.5$ V, $v_3 = -0.25$ V

Find:

(a) The weight (coefficient) for each input

(b) The output voltage v_{out}

(c) If you want equal weights for all inputs, what should the resistor values be?

Hint: Use $v_{out} = -\left(\frac{R_f}{R_1}v_1 + \frac{R_f}{R_2}v_2 + \frac{R_f}{R_3}v_3\right)$

Practice Problem 3 Solution

Given: $R_f = 20$ kΩ, $R_1 = 10$ kΩ, $R_2 = 20$ kΩ, $R_3 = 5$ kΩ
Inputs: $v_1 = 1$ V, $v_2 = 0.5$ V, $v_3 = -0.25$ V

Solutions:
(a) Weights:

$$w_1 = \frac{R_f}{R_1} = \frac{20}{10} = \boxed{2}$$

$$w_2 = \frac{R_f}{R_2} = \frac{20}{20} = \boxed{1}$$

$$w_3 = \frac{R_f}{R_3} = \frac{20}{5} = \boxed{4}$$

(b) Output voltage:

$$v_{out} = -(2 \times 1 + 1 \times 0.5 + 4 \times (-0.25))$$

$$v_{out} = -(2 + 0.5 - 1) = \boxed{-1.5\ \text{V}}$$

(c) For equal weights: $R_1 = R_2 = R_3 = R$ (any equal value, e.g., 10 kΩ)

Practice Problem 4

Given: An integrator circuit with $R = 100$ kΩ and $C = 1$ μF

Find:

(a) The time constant $\tau = RC$
(b) If a constant input $v_{in} = 2$ V is applied starting at $t = 0$ (with $v_{out}(0) = 0$), find v_{out} at $t = 0.1$ s
(c) At what time will the output reach -5 V?
(d) What is the magnitude of the transfer function at $f = 10$ Hz?

Hint: $v_{out}(t) = -\frac{1}{RC}\int_0^t v_{in}(\tau)d\tau$, and $|H(f)| = \frac{1}{2\pi f RC}$

Practice Problem 4 Solution

Given: $R = 100$ kΩ, $C = 1$ μF, $v_{in} = 2$ V (constant)

Solutions:

(a) Time constant:

$$\tau = RC = (100 \times 10^3)(1 \times 10^{-6}) = \boxed{0.1\text{ s}}$$

(b) At $t = 0.1$ s:

$$v_{out}(t) = -\frac{1}{RC}\int_0^t 2\,d\tau = -\frac{2}{RC}\cdot t$$

$$v_{out}(0.1) = -\frac{2}{0.1} \times 0.1 = \boxed{-2\text{ V}}$$

(c) For $v_{out} = -5$ V:

$$-5 = -\frac{2}{0.1}\cdot t \Rightarrow t = \frac{5 \times 0.1}{2} = \boxed{0.25\text{ s}}$$

(d) At $f = 10$ Hz:

$$|H(f)| = \frac{1}{2\pi f RC} = \frac{1}{2\pi(10)(0.1)} = \frac{1}{2\pi} \approx \boxed{0.159}$$

Practice Problem 5

Given: A difference amplifier with the following components:

- $R_1 = R_3 = 10\ \text{k}\Omega$
- $R_2 = R_4 = 50\ \text{k}\Omega$
- Input voltages: $v_1 = 2.5$ V, $v_2 = 3.0$ V

Find:

(a) Verify that the resistor matching condition is satisfied

(b) The differential gain A_d

(c) The output voltage v_{out}

(d) If $v_1 = v_2 = 2.5$ V (common-mode), what is v_{out} (ideally)?

Hint: Matching condition: $\frac{R_2}{R_1} = \frac{R_4}{R_3}$, and $v_{out} = A_d(v_2 - v_1)$

Practice Problem 5 Solution

Given: $R_1 = R_3 = 10\ \text{k}\Omega$, $R_2 = R_4 = 50\ \text{k}\Omega$
Inputs: $v_1 = 2.5$ V, $v_2 = 3.0$ V

Solutions:
(a) Check matching condition:

$$\frac{R_2}{R_1} = \frac{50}{10} = 5, \quad \frac{R_4}{R_3} = \frac{50}{10} = 5$$

$$\boxed{\frac{R_2}{R_1} = \frac{R_4}{R_3} \checkmark} \text{ (condition satisfied)}$$

(b) Differential gain:

$$A_d = \frac{R_2}{R_1} = \frac{50}{10} = \boxed{5}$$

(c) Output voltage:

$$v_{out} = A_d(v_2 - v_1) = 5(3.0 - 2.5) = 5(0.5) = \boxed{2.5\ \text{V}}$$

(d) Common mode input ($v_1 = v_2$):

Advanced Application: Instrumentation Amplifier (Preview)

Limitations of Simple Difference Amp:

- Finite input impedance
- Limited CMRR (requires precise matching)
- Gain-impedance tradeoff

Instrumentation Amplifier:

- Three op-amp configuration
- Very high input impedance (both inputs)
- Excellent CMRR (¿100 dB)
- Single resistor sets gain
- Industry standard for precision measurement

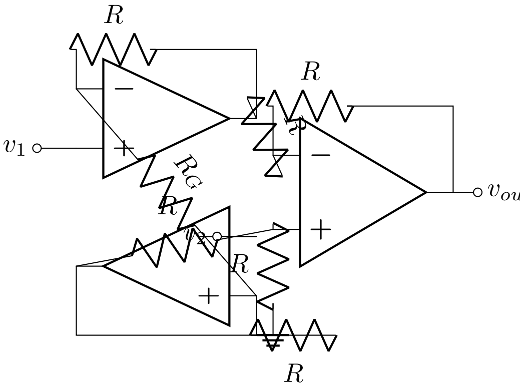


Figure 15: Three op-amp instrumentation

Comparison of Configurations

Feature	**Inverting**	**Noninverting**	**Difference**
Phase shift	180°	0°	0° (for $v_2 - v_1$)
Input impedance	R_1	∞	Finite at both
Minimum gain	0 (can attenuate)	1	0
Gain polarity	Negative	Positive	Positive
Complexity	Simple	Simple	Moderate
Virtual ground	Yes (at v_-)	No	No
CMRR	N/A	N/A	Depends on matching

Table 2: Comparison of basic op-amp configurations

Design Guidelines

- Use **inverting** when: phase inversion acceptable, moderate R_{in} OK
- Use **noninverting** when: high R_{in} needed, no phase inversion
- Use **difference** when: differential measurement needed, CMRR important

Practical Considerations

Resistor Selection:

- Typical range: 1 kΩ to 1 MΩ
- Too low: excessive current, power
- Too high: noise pickup, offset errors
- Use 1% or better for precision
- Match resistors for difference amp

Capacitor Selection:

- Integrator: low-leakage (polypropylene, polyester)
- Avoid electrolytic for precision
- Consider temperature coefficients
- ESR affects high-frequency performance

Common Mistakes:

1. Forgetting DC path (integrator saturation)
2. Exceeding op-amp output current limit
3. Ignoring bandwidth limitations
4. Poor layout causing oscillations
5. Wrong polarity on electrolytic caps

Verification Steps:

- Check DC operating point
- Verify gain with known input
- Test frequency response
- Check for distortion/clipping
- Measure with realistic loads