Discrete Time Filtering

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What is a Filter?

Key Concept: Filters are LTI systems that modify frequency components of signals.
Definition: Any system that modifies certain frequencies relative to others

Two Main Classes:

- **IIR (Infinite Impulse Response):** Rational transfer function $H(z) = \frac{B(z)}{A(z)}$
- **FIR (Finite Impulse Response):** Polynomial transfer function $H(z) = \sum_{n=0}^{M} b[n]z^{-n}$

Design Focus:

- Determine parameters to approximate desired frequency response
- Meet design specifications
- Maintain causality and stability

Elliptic Filters

Key Concept: Equiripple in *both* passband and stopband

Optimal Property:

- Lowest order filter for given specifications
- Minimum approximation error ripples equally in both bands

Characteristics:

- Zeros arrayed on unit circle in stopband (like Chebyshev II)
- Sharpest transition band for given order

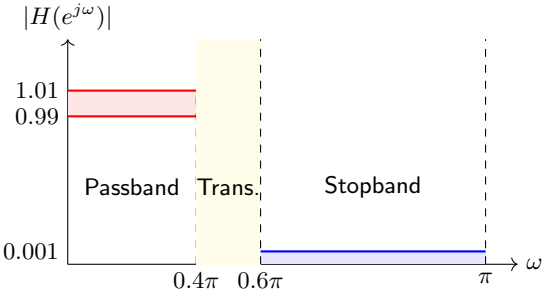
Trade-off:

- Lowest order $\rightarrow$ fewer computations
- More complex implementation than Butterworth
- Group delay typically less uniform than Butterworth

Example Specifications

Common specification for all designs:

Passband: $0.99 \leq |H(e^{j\omega})| \leq 1.01$, $|\omega| \leq 0.4\pi$
Stopband: $|H(e^{j\omega})| \leq 0.001$, $0.6\pi \leq |\omega| \leq \pi$



Filter Order Comparison

For specifications: $\omega_p = 0.4\pi$, $\omega_s = 0.6\pi$, $\delta_p = 0.01$, $\delta_s = 0.001$

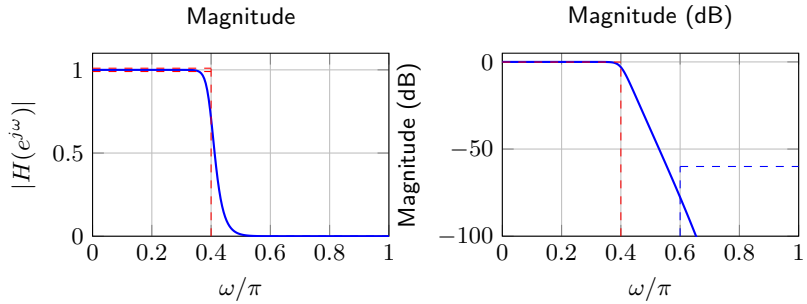
Filter Type	Minimum Order	Zero Locations
Butterworth	14	14 zeros at $z = -1$
Chebyshev I	8	8 zeros at $z = -1$
Chebyshev II	8	8 zeros on unit circle
Elliptic	6	6 zeros on unit circle

Key Observations:

- Elliptic achieves lowest order (optimal)
- Butterworth requires $\sim 2\times$ the order of Chebyshev
- Chebyshev I simpler than II (all zeros at $z = -1$)
- Order directly impacts computational complexity

Butterworth: Magnitude Response

Order 14 Butterworth Filter

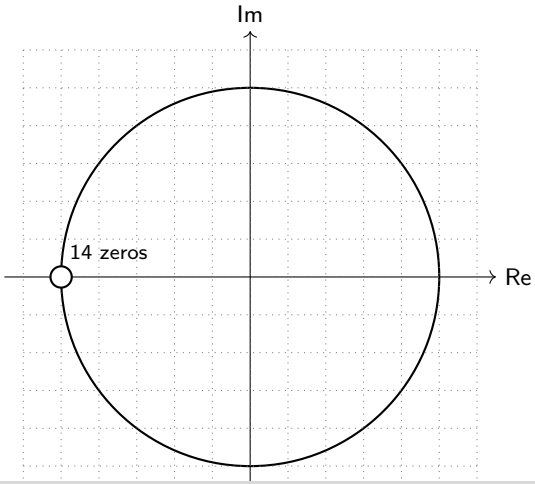


Characteristics:

- Smooth, monotonic decrease (no ripple)
- Maximally flat at $\omega = 0$
- Gradual transition band roll-off

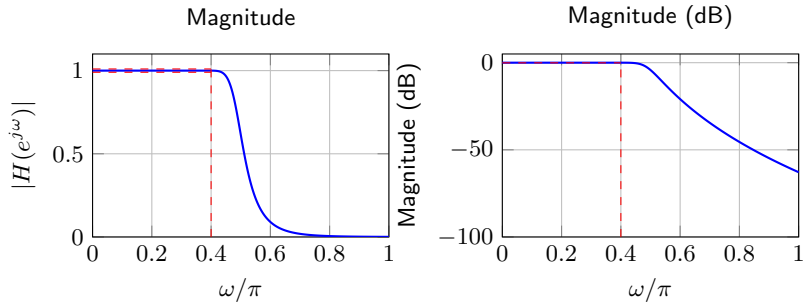
Butterworth: Pole-Zero Plot

Order 14 Butterworth Filter



Chebyshev Type I: Magnitude Response

Order 8 Chebyshev Type I Filter

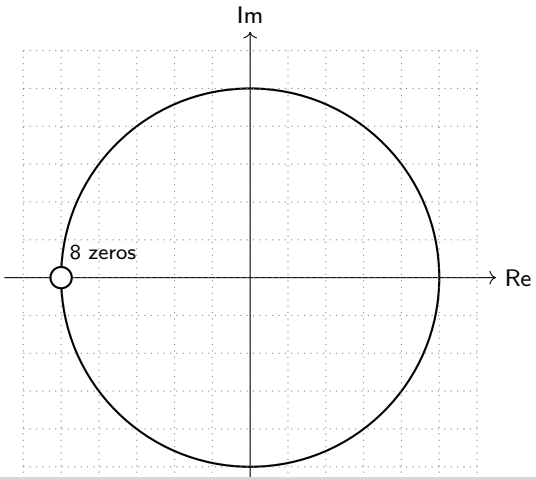


Characteristics:

- Equiripple in passband (oscillates between 0. 99 and 1.01)
- Monotonic in stopband
- Sharper transition than Butterworth

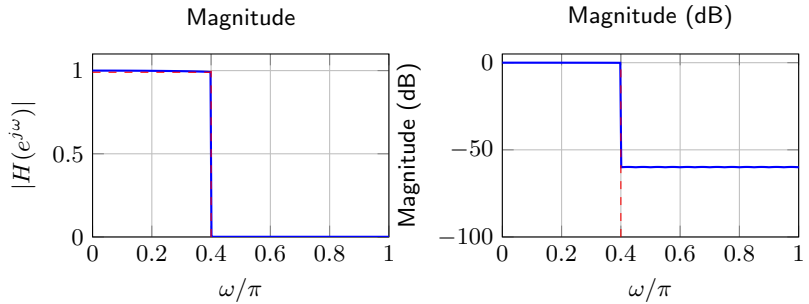
Chebyshev Type I: Pole-Zero Plot

Order 8 Chebyshev Type I Filter



Chebyshev Type II: Magnitude Response

Order 8 Chebyshev Type II Filter

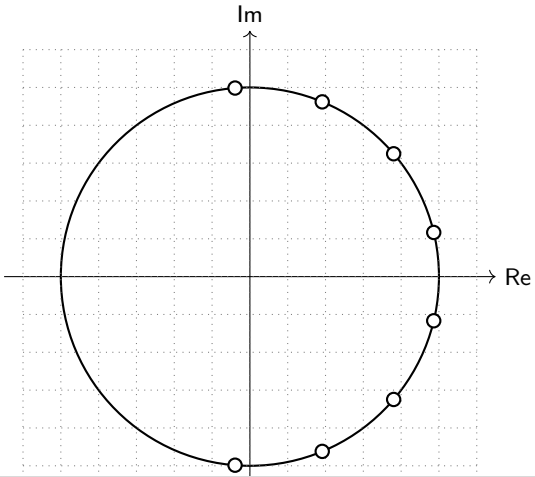


Characteristics:

- Monotonic in passband (smooth like Butterworth)
- Equiripple in stopband (oscillates around δ_s)
- Stopband nulls from zeros on unit circle

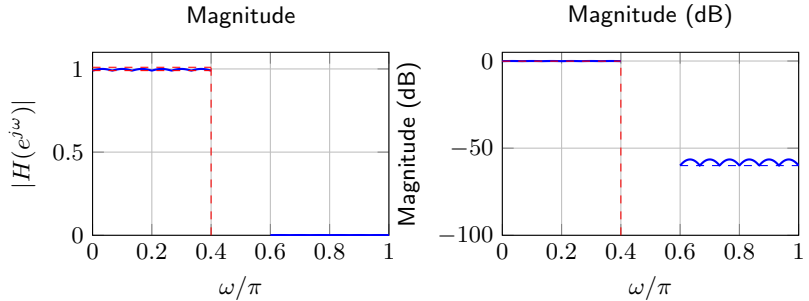
Chebyshev Type II: Pole-Zero Plot

Order 8 Chebyshev Type II Filter



Elliptic: Magnitude Response

Order 6 Elliptic Filter (Optimal)

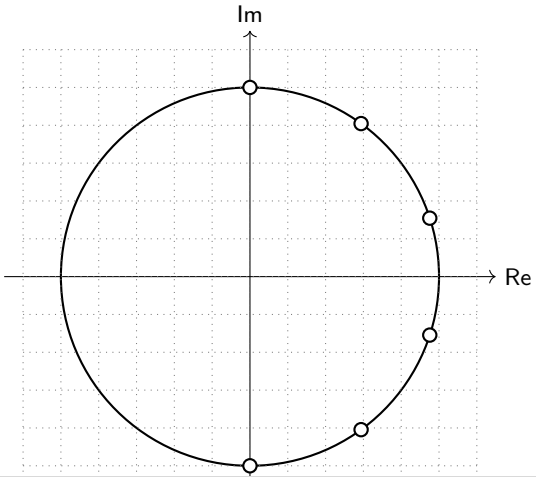


Characteristics:

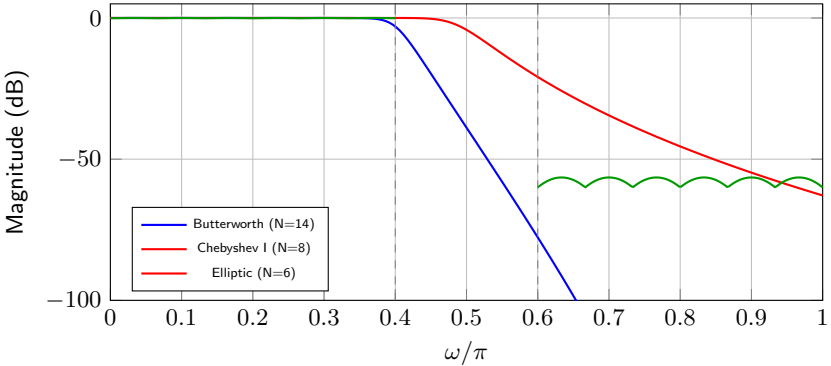
- Equiripple in *both* passband and stopband
- Sharpest transition band (only 6th order!)
- Optimal filter (lowest order for given specs)

Elliptic: Pole-Zero Plot

Order 6 Elliptic Filter



Side-by-Side Comparison: Magnitude



Observations:

- Elliptic has sharpest transition (lowest order)
- Butterworth smoothest, but requires highest order
- All meet specifications, different trade-offs

Key Takeaways

Visual Summary:

Type	Order	Passband	Stopband
Butterworth	14	Monotonic	Monotonic
Chebyshev I	8	Equiripple	Monotonic
Chebyshev II	8	Monotonic	Equiripple
Elliptic	6	Equiripple	Equiripple

Selection Guide:

- **Minimize order/cost?** $\rightarrow$ Elliptic
- **Smooth response?** $\rightarrow$ Butterworth
- **Balance efficiency & simplicity?** $\rightarrow$ Chebyshev I
- **Smooth passband, lower order?** $\rightarrow$ Chebyshev II

All zeros at $z = -1$: Butterworth, Chebyshev I (simpler implementation)
Zeros on unit circle: Chebyshev II, Elliptic (stopband nulls)