ECEN 222

Maxx Seminario

Introduction to
Time Domain
Analysis

First-Order
Circuits

Second-Order
Circuits

Summary

Time Domain Analysis of RLC Circuits

Maxx Seminario

University of Nebraska-Lincoln

Spring 2026

Why Time Domain Analysis?

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Motivation:

- What happens when we switch circuits on/off?
- How do voltages and currents change with time?
- How do R, L, and C interact?

Applications:

- Power supply turn-on behavior
- Pulse and digital circuits
- Signal filtering and shaping
- Oscillators and timing circuits

Mathematical Tool

Differential equations describe how energy storage elements (L, C) respond in time

Goal for this lecture

Review RLC circuits analysis in the time domain using differential equations

Review: I-V Relations for Passive Elements

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Element	I-V Relation	Integrated Form	Key Property
Resistor	$v = iR$	—	Instantaneous
Capacitor	$i = C\dfrac{dv}{dt}$	$v(t) = \dfrac{1}{C}\displaystyle\int_{t_0}^{t} i(\tau)d\tau + v(t_0)$	v cannot change instantly
Inductor	$v = L\dfrac{di}{dt}$	$i(t) = \dfrac{1}{L}\displaystyle\int_{t_0}^{t} v(\tau)d\tau + i(t_0)$	i cannot change instantly

Continuity in Time Domain

- **Capacitor voltage** $v_C(t)$ is continuous (cannot jump)
- **Inductor current** $i_L(t)$ is continuous (cannot jump)
- Used to find **initial conditions** for differential equations

First-Order Circuits: Definition

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What is a First-Order Circuit?

A circuit containing:

- **One** energy storage element (L or C)
- Any number of resistors
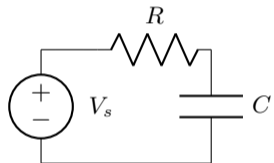- Sources (independent or dependent)

Mathematical Form
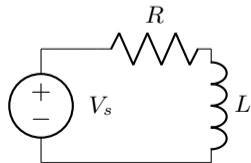
Results in a **first-order** differential equation:

$$f(t) = \frac{dx}{dt} + \frac{1}{\tau}x$$

where x is $v_C(t)$ or $i_L(t)$

RC Circuit:

RL Circuit:

RC Circuit Analysis: Source-Free Response
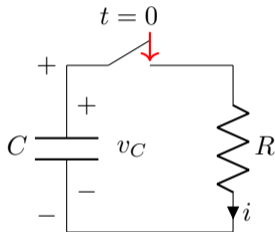
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Circuit:

Initial Condition: $v_C(0) = V_0$
Apply KCL:

$$i_C + i_R = 0$$

$$C\frac{dv_C}{dt} + \frac{v_C}{R} = 0$$

Differential Equation:

$$\frac{dv_C}{dt} + \frac{1}{RC}v_C = 0$$

Solution: Assuming $v_C(t) = Ke^{st}$ and plug into differential equation gives:

$$s + \frac{1}{RC} = 0 \quad \Rightarrow \quad s = -\frac{1}{RC}$$

General Solution

$$v_C(t) = V_0 e^{-t/RC}$$

$$i(t) = \frac{v_C(t)}{R} = \frac{V_0}{R}e^{-t/RC}$$

RC Circuit: Time Constant and Response

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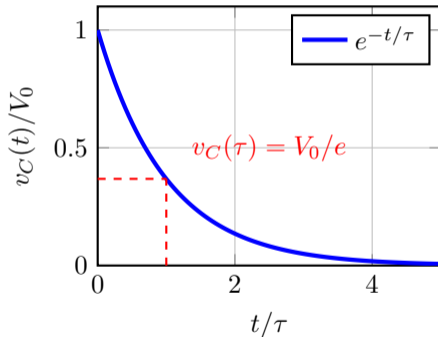Time Domain
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Time Constant: $\tau = RC$

- Time for voltage to decay to 36.8% ($1/e$) of initial value
- After 5τ: essentially zero ($< 1\%$)

Time	Voltage
$t = 0$	V_0
$t = \tau$	$0.368V_0$
$t = 2\tau$	$0.135V_0$
$t = 3\tau$	$0.050V_0$
$t = 5\tau$	$0.007V_0$

Voltage Response:

RC Circuit: Step Response

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Circuit with DC Source:

KVL for $t > 0$ **and** $v_C(0^-) = 0$:

$$V_s = v_R + v_C = iR + v_C$$

$$V_s = RC\frac{dv_C}{dt} + v_C$$

Solution Form:

$$v_C(t) = v_C(\infty) + [v_C(0^+) - v_C(\infty)]e^{-t/\tau}$$

where:

- $v_C(0^+) = 0$ (voltage continuity)
- $v_C(\infty) = V_s$ (steady-state voltage)
- $\tau = RC$

Step Response

$$v_C(t) = V_s(1 - e^{-t/RC})$$

$$i(t) = \frac{V_s}{R}e^{-t/RC}$$

RC Step Response: Charging Behavior
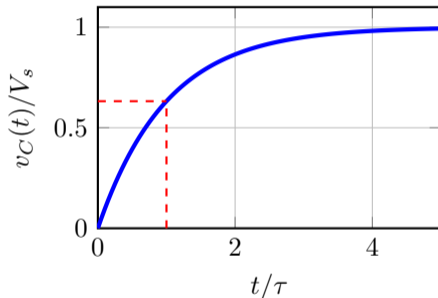
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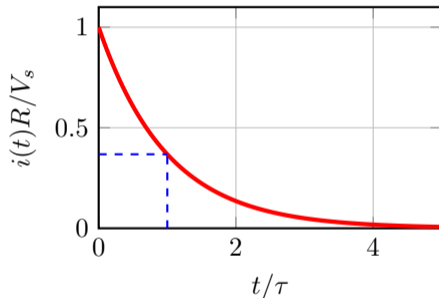Time Domain
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Voltage rises logarithmically:

Current decays exponentially:

Physical Interpretation

Initially: capacitor is a short circuit ($v_C = 0$, i is maximum)
Finally: capacitor is an open circuit ($v_C = V_s$, $i = 0$)

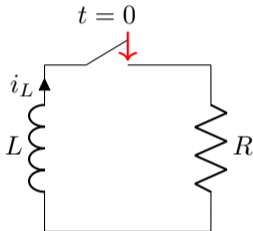# RL Circuit Analysis: Source-Free Response

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Circuit:

Initial Condition: $i_L(0) = I_0$
Apply KVL:

$$v_L + v_R = 0$$

$$L\frac{di_L}{dt} + Ri_L = 0$$

Differential Equation:

$$\frac{di_L}{dt} + \frac{R}{L}i_L = 0$$

Solution: Assume $i_L(t) = Ke^{st}$

$$s + \frac{R}{L} = 0 \quad \Rightarrow \quad s = -\frac{R}{L}$$

General Solution

$$i_L(t) = I_0 e^{-Rt/L}$$

$$v_L(t) = L\frac{di_L}{dt} = -RI_0 e^{-Rt/L}$$

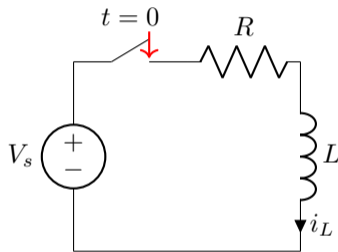# RL Circuit: Step Response

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Circuit with DC Source:

Initial Condition: $i_L(0^-) = 0$

KVL for $t > 0$:

$$V_s = v_R + v_L = Ri_L + L\frac{di_L}{dt}$$

Solution Form:

$$i_L(t) = i_L(\infty) + [i_L(0^+) - i_L(\infty)]e^{-t/\tau}$$

where:

- $i_L(0^+) = 0$ (currrent continuity)
- $i_L(\infty) = V_s/R$ (current steady-state)
- $\tau = L/R$

Step Response

$$i_L(t) = \frac{V_s}{R}(1 - e^{-Rt/L})$$

$$v_L(t) = V_s e^{-Rt/L}$$

General Solution Method for First-Order Circuits

1 **Find Initial Condition**: $x(0^+)$
  - Use continuity: $v_C(0^+) = v_C(0^-)$ or $i_L(0^+) = i_L(0^-)$
  - Analyze circuit just before switching occurs

2 **Find Final (Steady-State) Value**: $x(\infty)$
  - Capacitors become open circuits (DC)
  - Inductors become short circuits (DC)
  - Solve the DC circuit

3 **Find Time Constant**: τ
  - RC circuits: $\tau = R_{eq}C$
  - RL circuits: $\tau = L/R_{eq}$
  - R_{eq} is Thévenin resistance seen by L or C

General Solution

$$x(t) = x(\infty) + [x(0^+) - x(\infty)]e^{-t/\tau}$$

Second-Order Circuits: RLC Combinations

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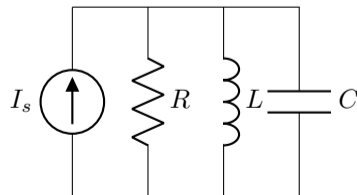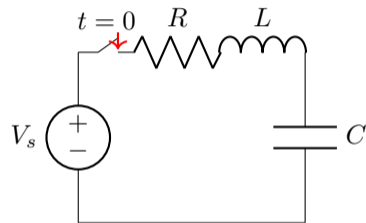**What is a Second-Order Circuit?**

A circuit containing:

- **Two** energy storage elements
- Energy storage devices may be L or C
- Resistors and sources

Mathematical Form

Results in a **second-order** differential equation:

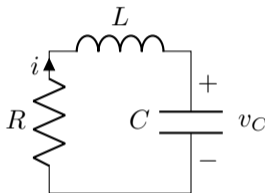$$\frac{d^2x}{dt^2} + 2\alpha\frac{dx}{dt} + \omega_0^2 x = f(t)$$

Series RLC Circuit: Differential Equation

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Source-Free Series RLC:

$$v_R + v_L + v_C = 0$$

$$Ri + L\frac{di}{dt} + v_C = 0$$

$$RC\frac{dv_C}{dt} + LC\frac{d^2v_C}{dt^2} + v_C = 0$$

Standard Form:

$$\frac{d^2v_C}{dt^2} + \frac{R}{L}\frac{dv_C}{dt} + \frac{1}{LC}v_C = 0$$

Natural Frequency: $\omega_0 = \frac{1}{\sqrt{LC}}$

Damping Coefficient: $\alpha = \frac{R}{2L}$

Standard Form

$$\frac{d^2v_C}{dt^2} + 2\alpha\frac{dv_C}{dt} + \omega_0^2 v_C = 0$$

Characteristic Equation and Damping

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Characteristic Equation:
Assume solution: $v_C(t) = Ke^{st}$

$$s^2 + 2\alpha s + \omega_0^2 = 0$$

Roots:

$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2}$$

Damping Ratio:

$$\zeta = \frac{\alpha}{\omega_0} = \frac{R}{2}\sqrt{\frac{C}{L}}$$

Three Cases:

1 **Overdamped**: $\alpha > \omega_0$ ($\zeta > 1$)
  - Two real, distinct roots
  - No oscillation

2 **Critically Damped**: $\alpha = \omega_0$ ($\zeta = 1$)
  - Two real, equal roots
  - Fastest response without overshoot

3 **Underdamped**: $\alpha < \omega_0$ ($\zeta < 1$)
  - Complex conjugate roots
  - Oscillatory response

Overdamped Response ($\alpha > \omega_0$)

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Roots:

$$s_1 = -\alpha + \sqrt{\alpha^2 - \omega_0^2}$$

$$s_2 = -\alpha - \sqrt{\alpha^2 - \omega_0^2}$$

Both roots are **real and negative**.

General Solution

$$v_C(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}$$

where A_1 and A_2 are determined by initial conditions
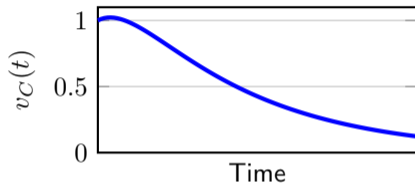
Response Shape:

Figure 3: Slow exponential decay, no oscillation

Characteristics:

- No overshoot
- Slow settling
- High resistance

Critically Damped Response ($\alpha = \omega_0$)

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Roots:

$$s_1 = s_2 = -\alpha$$

Repeated real root.

General Solution

$$v_C(t) = (A_1 + A_2 t)e^{-\alpha t}$$

where A_1 and A_2 are determined by initial conditions

Critical Resistance:

$$R_{\text{crit}} = 2\sqrt{\frac{L}{C}}$$
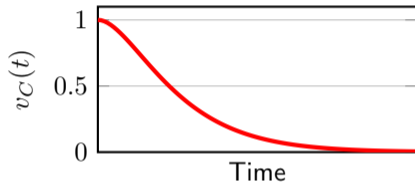
Response Shape:

Figure 4: Fastest settling without overshoot

Characteristics:

- No overshoot
- Fastest response
- Boundary case

Underdamped Response ($\alpha < \omega_0$)

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Roots:

$$s_{1,2} = -\alpha \pm j\omega_d$$

where the **damped frequency** is:

$$\omega_d = \sqrt{\omega_0^2 - \alpha^2}$$

General Solution

$$v_C(t) = e^{-\alpha t}(A_1 \cos(\omega_d t) + A_2 \sin(\omega_d t))$$

or equivalently:

$$v_C(t) = Be^{-\alpha t}\cos(\omega_d t + \phi)$$
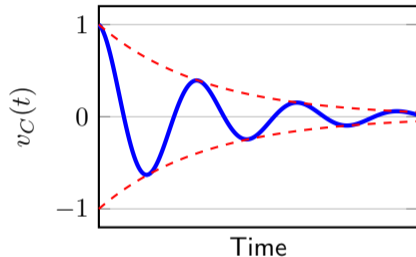
Response Shape:

Figure 5: Oscillatory with exponential decay

Characteristics:

- Oscillates at ω_d
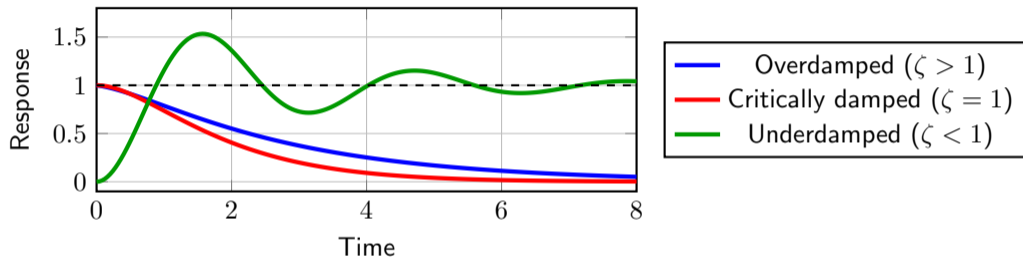- Envelope decays at rate α
- Low resistance

Comparison of Damping Cases

Design Implications

- **Overdamped**: Slow but stable (e.g., door closers)
- **Critically damped**: Optimal speed without overshoot (e.g., measuring instruments)
- **Underdamped**: Fast but oscillatory (e.g., speakers, control systems)
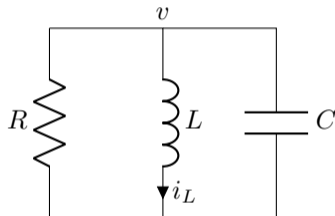
Parallel RLC Circuit

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Source-Free Parallel RLC:

Apply KCL:

$$i_R + i_L + i_C = 0$$

$$\frac{v}{R} + \frac{1}{L}\int v\,dt + C\frac{dv}{dt} = 0$$

Differentiate and rearrange:

$$\frac{d^2v}{dt^2} + \frac{1}{RC}\frac{dv}{dt} + \frac{1}{LC}v = 0$$

Natural Frequency: $\omega_0 = \frac{1}{\sqrt{LC}}$

Damping Coefficient: $\alpha = \frac{1}{2RC}$

Note: Same ω_0 as series RLC, but damping depends on $1/R$ (opposite of series)

Summary: Time Domain Analysis

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First-Order Circuits:

- One energy storage element (L or C)
- First-order differential equation
- Exponential response: $e^{-t/\tau}$
- Time constant: $\tau = RC$ or $\tau = L/R$

Solution Method:

$$x(t) = x(\infty) + [x(0^+) - x(\infty)]e^{-t/\tau}$$

Key Concepts:

- Continuity of v_C and i_L
- Initial conditions
- Steady-state values

Second-Order Circuits:

- Two energy storage elements
- Second-order differential equation
- Response depends on damping

Damping Cases:

1. Overdamped: slow, no oscillation
2. Critically damped: fastest, no overshoot
3. Underdamped: oscillatory

Parameters:

- Natural frequency: $\omega_0 = 1/\sqrt{LC}$
- Damping coefficient: α
- Damping ratio: $\zeta = \alpha/\omega_0$