

Assignment 02

Frequency Domain Analysis of Circuits

ECEN 222, Spring 2026

University of Nebraska-Lincoln

Instructions

This assignment focuses on analyzing AC circuits in the frequency domain using phasor techniques. You will convert time-domain signals to phasors, calculate impedances, analyze circuits, and convert results back to the time domain.

Key Formulas

Phasor Conversion:

$$v(t) = V_m \cos(\omega t + \phi) \Leftrightarrow \mathbf{V} = V_m \angle \phi$$

Impedances:

- Resistor: $\mathbf{Z}_R = R$
- Inductor: $\mathbf{Z}_L = j\omega L = \omega L \angle 90^\circ$
- Capacitor: $\mathbf{Z}_C = \frac{1}{j\omega C} = \frac{-j}{\omega C} = \frac{1}{\omega C} \angle -90^\circ$

Complex Number Operations:

- Rectangular to Polar: $a + jb = \sqrt{a^2 + b^2} \angle \tan^{-1}(b/a)$
- Polar to Rectangular: $r \angle \theta = r \cos \theta + jr \sin \theta$

AC Power:

- Real Power: $P = V_{rms} I_{rms} \cos \theta$ (W)
- Reactive Power: $Q = V_{rms} I_{rms} \sin \theta$ (VAR)
- Apparent Power: $S = V_{rms} I_{rms}$ (VA)
- Complex Power: $\mathbf{S} = P + jQ = \mathbf{V}_{rms} \mathbf{I}_{rms}^*$

Problems

1. [12.5 points]

Given the following time-domain signals:

$$\begin{aligned}v_1(t) &= 15 \cos(5000t + 60^\circ) \text{ V} \\v_2(t) &= 8 \cos(5000t - 30^\circ) \text{ V} \\i(t) &= 3 \cos(5000t + 15^\circ) \text{ A}\end{aligned}$$

(a) Convert each signal to phasor form.

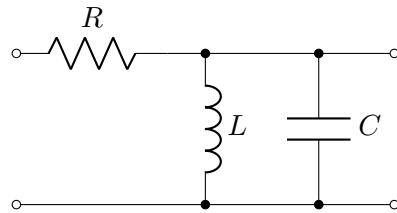
(b) Calculate $\mathbf{V}_1 + \mathbf{V}_2$ in both rectangular and polar forms.

(c) Calculate $\mathbf{V}_1 - \mathbf{V}_2$ in both rectangular and polar forms.

(d) Calculate the impedance $\mathbf{Z} = \mathbf{V}_1/\mathbf{I}$ and express it in both rectangular and polar forms. What type of element(s) does this impedance represent?

2. [12.5 points]

Consider a circuit with a resistor $R = 50\Omega$ in series with a parallel combination of $L = 20\text{ mH}$ and $C = 2\mu\text{F}$.



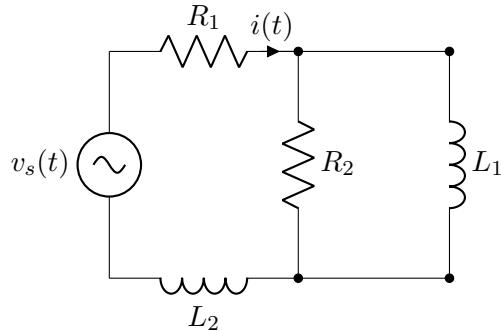
(a) Calculate the impedance of the parallel LC combination at $f = 1000\text{ Hz}$. Express in both rectangular and polar forms.

(b) Calculate the total impedance \mathbf{Z}_{tot} at $f = 1000\text{ Hz}$. Express your answer in both rectangular and polar forms.

(c) At what frequency does the parallel LC combination have infinite impedance? What is the total impedance at this frequency?

3. [12.5 points]

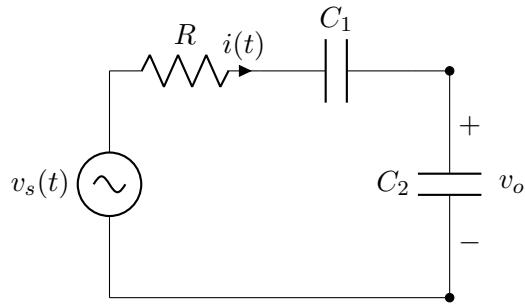
For the circuit shown below, the voltage source is $v_s(t) = 24 \cos(3000t)$ V, $R_1 = 30 \Omega$, $R_2 = 60 \Omega$, $L_1 = 20 \text{ mH}$, and $L_2 = 40 \text{ mH}$.



- (a) Calculate the impedances of L_1 and L_2 at $\omega = 3000$ rad/s.
- (b) Calculate the impedance of the parallel combination of R_2 and L_1 .
- (c) Calculate the total impedance Z_{tot} in both rectangular and polar forms. (Hint: The circuit topology is R_1 in series with $(R_2 \parallel L_1)$, all in series with L_2 .)
- (d) Find the current $i(t)$ in both phasor and time-domain forms.
- (e) Find the voltage across the parallel branch and determine the current through R_2 and L_1 individually.

4. [12.5 points]

For the circuit shown below, the voltage source is $v_s(t) = 50 \cos(4000t)$ V, $R = 100 \Omega$, $C_1 = 5 \mu\text{F}$, and $C_2 = 10 \mu\text{F}$.



- (a) Calculate the impedance of C_1 and C_2 individually at $\omega = 4000$ rad/s.

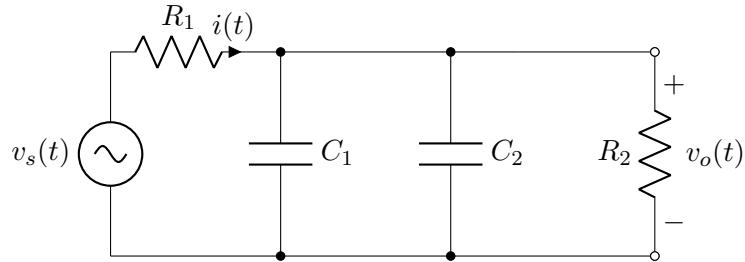
- (b) Calculate the total impedance Z_{tot} of the three series components.

- (c) Find the current $i(t)$ in both phasor and time-domain forms.

- (d) Find $v_o(t)$.

5. [12.5 points]

Consider the circuit shown below with $v_s(t) = 20 \cos(8000t)$ V, $R_1 = 1$ k Ω , $R_2 = 3$ k Ω , $C_1 = 100$ nF, and $C_2 = 50$ nF.



- (a) Calculate the impedances of C_1 , C_2 , and R_2 at $\omega = 8000$ rad/s, then find the equivalent impedance of the parallel combination.
- (b) Calculate the total circuit impedance Z_{tot} in both rectangular and polar forms.
- (c) Find the input current $i(t)$ in both phasor and time-domain forms.
- (d) Find the output voltage $v_o(t)$.
- (e) Calculate the magnitude ratio $|\mathbf{V}_o|/|\mathbf{V}_s|$ and the phase shift.

(f) Find the current through each capacitor (C_1 and C_2) individually.

6. [12.5 points]

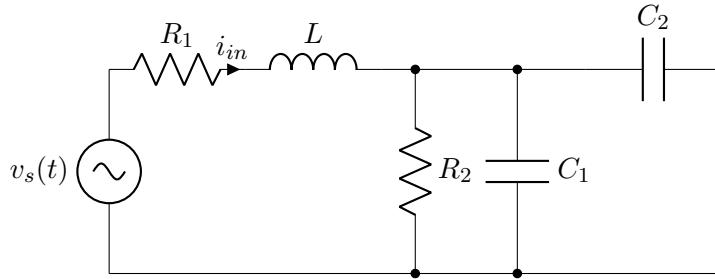
A load is connected to an AC source. The voltage across the load and the current through the load are measured as:

$$v(t) = 120\sqrt{2} \cos(377t) \text{ V}$$
$$i(t) = 6\sqrt{2} \cos(377t - 53.13^\circ) \text{ A}$$

- (a) Convert the voltage and current to phasor form, then calculate the RMS phasors \mathbf{V}_{rms} and \mathbf{I}_{rms} . (Hint: The $\sqrt{2}$ factor is included to indicate peak values.)
- (b) Find the real power P , reactive power Q , and apparent power \mathbf{S} . Include proper units.
- (c) Calculate the power factor and state whether the load is inductive or capacitive. Explain your reasoning.
- (d) Find the impedance of the load using $\mathbf{Z} = \mathbf{V}_{rms}/\mathbf{I}_{rms}$ in both rectangular and polar forms. What circuit elements could this load represent?
- (e) If you wanted to improve the power factor to unity (1.0), what value of capacitor would you need to place in parallel with this load? (Note: $\omega = 377 \text{ rad/s}$ corresponds to $f = 60 \text{ Hz}$)

7. [12.5 points]

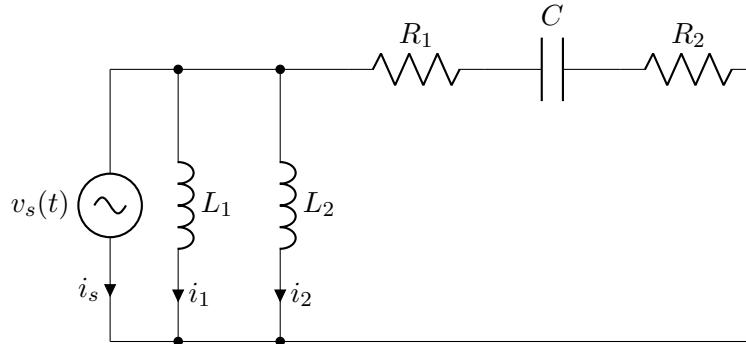
Consider the circuit with $v_s(t) = 100 \cos(5000t)$ V, $R_1 = 20\Omega$, $R_2 = 80\Omega$, $L = 8$ mH, $C_1 = 10\mu\text{F}$, and $C_2 = 40\mu\text{F}$.



- (a) Calculate all component impedances at $\omega = 5000$ rad/s.
- (b) Calculate the total impedance \mathbf{Z}_{tot} in both rectangular and polar forms.
- (c) Find the input current $i_{in}(t)$ in both phasor and time-domain forms.
- (d) Calculate the voltage across the parallel branch ($R_2 \parallel C_1 \parallel C_2$).
- (e) Find the current through R_2 , C_1 , and C_2 individually.

8. [12.5 points]

Analyze the circuit shown below with $v_s(t) = 60 \cos(4000t)$ V, $R_1 = 50 \Omega$, $R_2 = 100 \Omega$, $L_1 = 25 \text{ mH}$, $L_2 = 50 \text{ mH}$, and $C = 5 \mu\text{F}$.



- (a) Calculate the impedance of each inductor and the capacitor at $\omega = 4000$ rad/s.
- (b) Calculate the equivalent impedance of the parallel inductor combination ($L_1 \parallel L_2$).
- (c) Calculate the total impedance \mathbf{Z}_{tot} in both rectangular and polar forms.
- (d) Find the total source current $i_s(t)$ in both phasor and time-domain forms.
- (e) Find the voltage across the parallel inductor combination, then determine the individual currents $i_1(t)$ and $i_2(t)$ through each inductor.

(f) Calculate the total real power, reactive power, and complex power delivered by the source. Express complex power in rectangular form. (Hint: You already know the source voltage and current.)