

# Maxwell's Equations to Passive Circuit Elements

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# Maxwell's Equations: The Foundation

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Maxwell's  
Equations Review

From Maxwell to  
Circuit Elements

Summary

**There are four equations governing all electromagnetics:**

Name	Integral Form	Differential Form
Gauss's Law - Electric	$\oint_S \mathbf{E} \cdot d\mathbf{A} = \frac{Q_{\text{enc}}}{\epsilon_0}$	$\nabla \cdot \mathbf{E} = \frac{\rho}{\epsilon_0}$
No magnetic monopoles	$\oint_S \mathbf{B} \cdot d\mathbf{A} = 0$	$\nabla \cdot \mathbf{B} = 0$
Faraday's Law	$\oint_C \mathbf{E} \cdot d\mathbf{l} = -\frac{d\Phi_B}{dt}$	$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$
Ampère-Maxwell Law	$\oint_C \mathbf{B} \cdot d\mathbf{l} = \mu_0 I_{\text{enc}} + \mu_0 \epsilon_0 \frac{d\Phi_E}{dt}$	$\nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \mu_0 \epsilon_0 \frac{\partial \mathbf{E}}{\partial t}$

# What Do Maxwell's Equations Tell Us?

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## Gauss's Law:

- Electric charges create electric fields
- Electric field lines start/end on charges
- Basis for **capacitance**

## No Magnetic Monopoles:

- Magnetic field lines form closed loops
- No isolated magnetic charges
- Magnetic fields from currents only

## Faraday's Law:

- Changing magnetic flux creates voltage
- Basis for **inductance**
- Transformers, generators, motors

## Ampère-Maxwell Law:

- Currents create magnetic fields
- Changing electric fields create magnetic fields
- Basis for electromagnetic waves

## For Circuit Theory

We focus on Gauss's Law (capacitors) and Faraday's Law (inductors)

# The Resistor: Ohm's Law

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## From Electromagnetics:

Current density related to electric field:

$$\mathbf{J} = \sigma \mathbf{E}$$

where  $\sigma$  is conductivity.

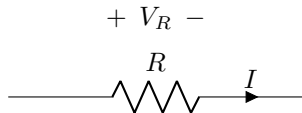
Integrating over a conductor:

$$I = \int_A \mathbf{J} \cdot d\mathbf{A}, \quad V = \int_l \mathbf{E} \cdot d\mathbf{l}$$

Leads to Ohm's Law

$$V = IR, \text{ where } R = \frac{l}{\sigma A}$$

## Circuit Symbol:



## Power Dissipated:

$$P = VI = I^2 R = \frac{V^2}{R}$$

# The Capacitor: Stored Electric Energy

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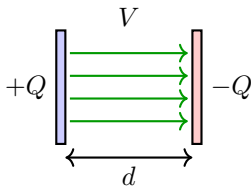
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## From Gauss's Law:

Parallel plate capacitor:

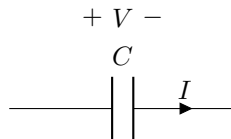


Electric field:  $E = \frac{Q}{\epsilon_0 A}$

Voltage:  $V = Ed = \frac{Qd}{\epsilon_0 A}$

$$Q = CV \quad \text{where} \quad C = \frac{\epsilon_0 A}{d}$$

## Circuit Symbol:



## I-V Relationship

$$I = C \frac{dV}{dt}$$

## Stored Energy:

- $W = \frac{1}{2}CV^2 = \frac{1}{2}QV$
- Stores energy in electric field

# The Inductor: Stored Magnetic Energy

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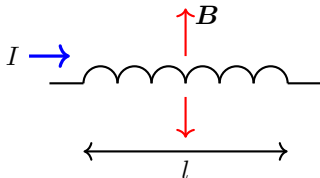
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## From Faraday's Law:

Solenoid inductor:

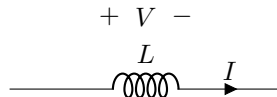


Magnetic flux:  $\Phi_B = NBA$

Faraday's Law:  $V = -\frac{d\Phi_B}{dt}$

For  $B = \frac{\mu_0 NI}{l}$ :

## Circuit Symbol:



## I-V Relationship:

$$V = L \frac{dI}{dt} \text{ where } L = \frac{\mu_0 N^2 A}{l}$$

## Stored Energy:

- $W = \frac{1}{2} LI^2$
- Stores energy in magnetic field

# Passives Summary


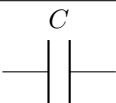
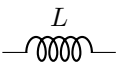
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Element	Symbol	I-V Relation	Energy/Power
Resistor		$V = IR$	$P = I^2 R$ (dissipated)
Capacitor		$I = C \frac{dV}{dt}$	$W = \frac{1}{2} CV^2$ (stored)
Inductor		$V = L \frac{dI}{dt}$	$W = \frac{1}{2} LI^2$ (stored)

## Resistor (R):

- From  $\mathbf{J} = \sigma \mathbf{E}$
- Dissipates energy
- Algebraic relation

## Capacitor (C):

- From Gauss's Law
- Stores electric energy
- Time derivative of  $V$

## Inductor (L):

- From Faraday's Law
- Stores magnetic energy
- Time derivative of  $I$

# Summary

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Summary

## Maxwell's Equations:

- Four fundamental laws
- Gauss's Law  $\rightarrow$  capacitors
- Faraday's Law  $\rightarrow$  inductors
- Unified theory of E&M

## Circuit Elements IV Relations:

- **R**: Dissipates energy ( $V = IR$ )
- **C**: Stores electric energy ( $I = C \frac{dV}{dt}$ )
- **L**: Stores magnetic energy ( $V = L \frac{dI}{dt}$ )

## Frequency Behavior:

- Capacitors: block DC, pass AC
- Inductors: pass DC, block AC
- Resistors: frequency-independent

## Energy Storage:

- Capacitor:  $W_C = \frac{1}{2}CV^2$
- Inductor:  $W_L = \frac{1}{2}LI^2$
- Only R dissipates energy

## Big Picture

Passive circuit theory is the application of Maxwell's equations!