

# Discrete Time Filtering

Maxx Seminario

University of Nebraska-Lincoln

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# What is a Filter?

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#### Introduction

#### Classical Filter Types

#### Design Comparisons

#### Butterworth Filter

#### Chebyshev Type I Filter

#### Chebyshev Type II Filter

#### Elliptic Filter

#### Visual Comparison

#### Conclusion

**Key Concept:** Filters are LTI systems that modify frequency components of signals.

**Definition:** Any system that modifies certain frequencies relative to others

### Two Main Classes:

- **IIR (Infinite Impulse Response):** Rational transfer function  $H(z) = \frac{B(z)}{A(z)}$
- **FIR (Finite Impulse Response):** Polynomial transfer function
$$H(z) = \sum_{n=0}^M b[n]z^{-n}$$

### Design Focus:

- Determine parameters to approximate desired frequency response
- Meet design specifications
- Maintain causality and stability

# Elliptic Filters

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**Key Concept:** Equiripple in *both* passband and stopband

### Optimal Property:

- Lowest order filter for given specifications
- Minimum approximation error ripples equally in both bands

### Characteristics:

- Zeros arrayed on unit circle in stopband (like Chebyshev II)
- Sharpest transition band for given order

### Trade-off:

- Lowest order  $\rightarrow$  fewer computations
- More complex implementation than Butterworth
- Group delay typically less uniform than Butterworth

# Example Specifications

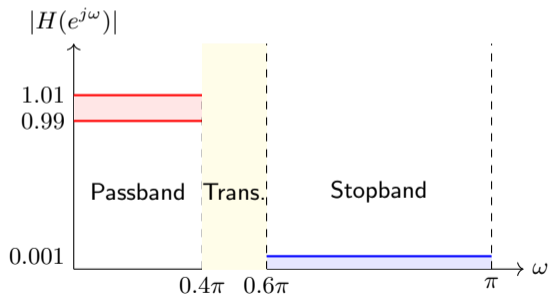
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#### Common specification for all designs:

Passband:  $0.99 \leq |H(e^{j\omega})| \leq 1.01$ ,  $|\omega| \leq 0.4\pi$

Stopband:  $|H(e^{j\omega})| \leq 0.001$ ,  $0.6\pi \leq |\omega| \leq \pi$



# Filter Order Comparison

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**For specifications:**  $\omega_p = 0.4\pi$ ,  $\omega_s = 0.6\pi$ ,  $\delta_p = 0.01$ ,  $\delta_s = 0.001$

Filter Type	Minimum Order	Zero Locations
Butterworth	14	14 zeros at $z = -1$
Chebyshev I	8	8 zeros at $z = -1$
Chebyshev II	8	8 zeros on unit circle
Elliptic	6	6 zeros on unit circle

## Key Observations:

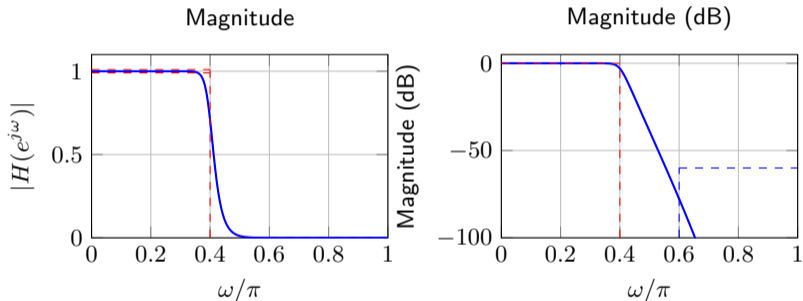
- Elliptic achieves lowest order (optimal)
- Butterworth requires  $\sim 2\times$  the order of Chebyshev
- Chebyshev I simpler than II (all zeros at  $z = -1$ )
- Order directly impacts computational complexity

# Butterworth: Magnitude Response

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## Order 14 Butterworth Filter



### Characteristics:

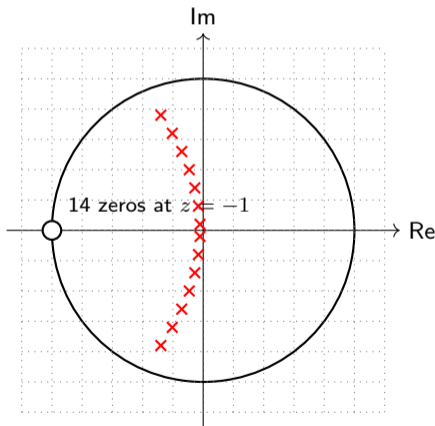
- Smooth, monotonic decrease (no ripple)
- Maximally flat at  $\omega = 0$
- Gradual transition band roll-off

# Butterworth: Pole-Zero Plot (near-vertical poles)

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**Order 14 Butterworth Filter — poles near vertical line at small negative Re and bowed left with increasing Im**

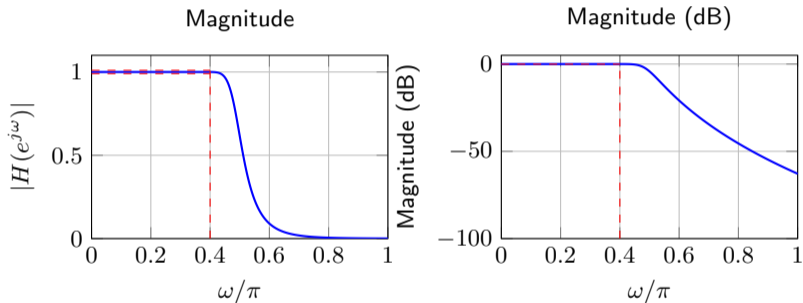


# Chebyshev Type I: Magnitude Response

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## Order 8 Chebyshev Type I Filter



### Characteristics:

- Equiripple in passband (oscillates between 0.99 and 1.01)
- Monotonic in stopband
- Sharper transition than Butterworth

# Chebyshev Type I: Pole-Zero Plot (quadratic bow)

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Butterworth  
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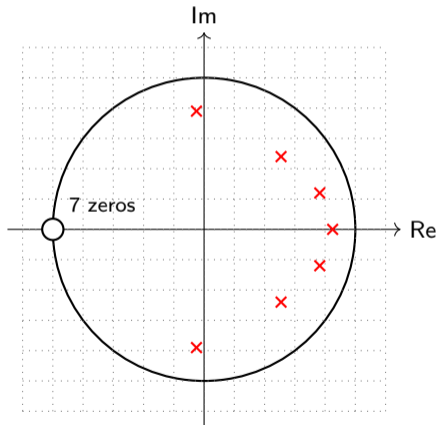
Chebyshev Type I  
Filter

Chebyshev Type II  
Filter

Elliptic Filter

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Conclusion



## Key Features:

- 7 zeros stacked at  $z = -1$
- 7 poles lie on a smooth quadratic curve
- All poles remain inside the unit circle

# Chebyshev Type II: Magnitude Response

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Chebyshev Type I  
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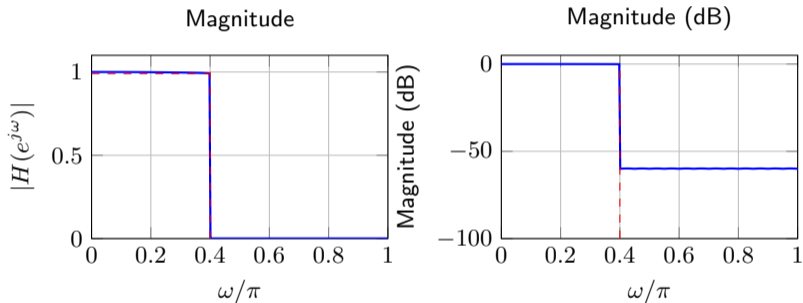
Chebyshev Type II  
Filter

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## Order 8 Chebyshev Type II Filter



### Characteristics:

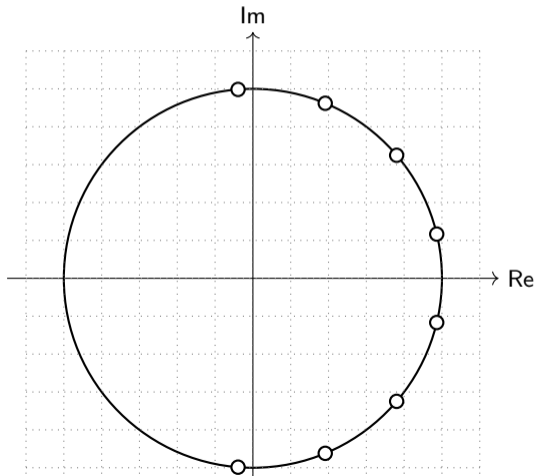
- Monotonic in passband (smooth like Butterworth)
- Equiripple in stopband (oscillates around  $\delta_s$ )
- Stopband nulls from zeros on unit circle

# Chebyshev Type II: Pole-Zero Plot

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## Order 8 Chebyshev Type II Filter



# Elliptic: Magnitude Response

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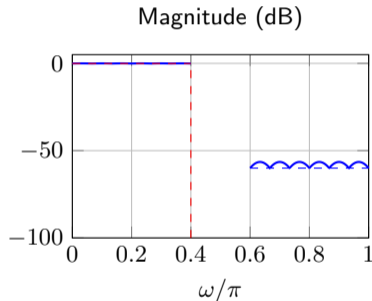
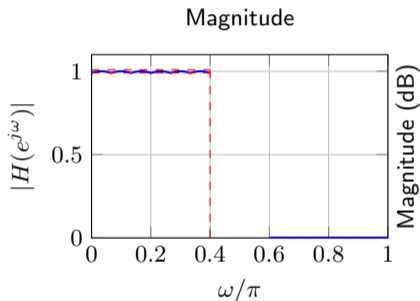
Chebyshev Type II  
Filter

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## Order 6 Elliptic Filter (Optimal)



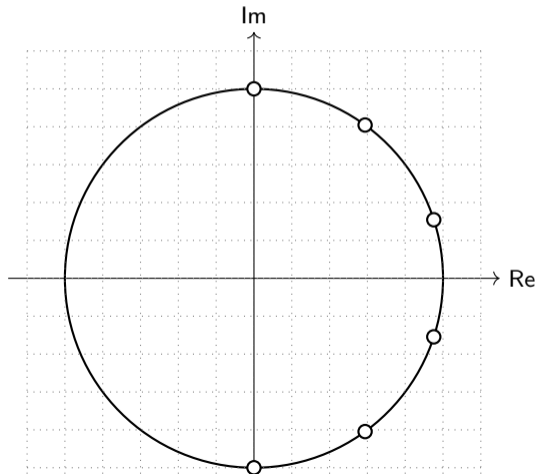
### Characteristics:

- Equiripple in *both* passband and stopband
- Sharpest transition band (only 6th order!)
- Optimal filter (lowest order for given specs)

# Elliptic: Pole-Zero Plot

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## Order 6 Elliptic Filter



# Side-by-Side Comparison: Magnitude

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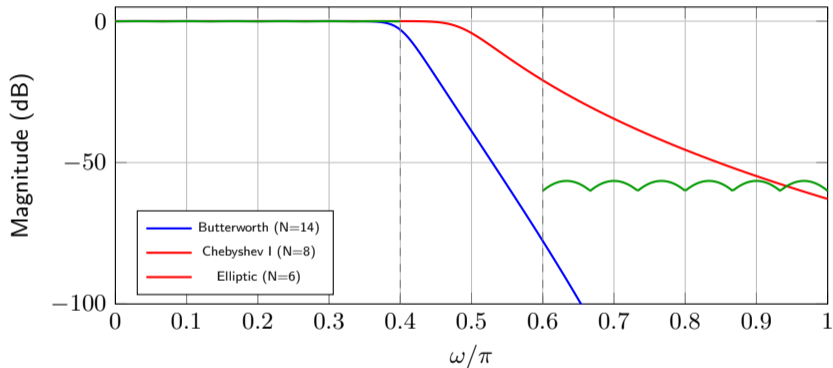
Chebyshev Type I  
Filter

Chebyshev Type II  
Filter

Elliptic Filter

Visual  
Comparison

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## Observations:

- Elliptic has sharpest transition (lowest order)
- Butterworth smoothest, but requires highest order
- All meet specifications, different trade-offs

# Key Takeaways

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## Visual Summary:

Type	Order	Passband	Stopband
Butterworth	14	Monotonic	Monotonic
Chebyshev I	8	Equiripple	Monotonic
Chebyshev II	8	Monotonic	Equiripple
Elliptic	6	Equiripple	Equiripple

## Selection Guide:

- **Minimize order/cost?** → Elliptic
- **Smooth response?** → Butterworth
- **Balance efficiency & simplicity?** → Chebyshev I
- **Smooth passband, lower order?** → Chebyshev II

**All zeros at  $z = -1$ :** Butterworth, Chebyshev I (simpler implementation)

**Zeros on unit circle:** Chebyshev II, Elliptic (stopband nulls)