Assignment 01: Time Domain Response of RLC Circuits

Maxx Seminario

University of Nebraska-Lincoln

Instructions

This assignment focuses on analyzing the time domain response of passive RLC circuits subjected to step inputs. You will sketch output voltage waveforms based on circuit topology and initial/final conditions.

Important Notes:

- You do **not** need to solve differential equations (you may if you wish).
- The key is to determine:
  1. The **initial condition** (at $t = 0$)
  2. The **final steady-state** (as $t \to \infty$)
  3. The **time constant** τ (for first-order) or damping characteristics (for second-order)
- For first-order circuits, sketch the waveform knowing it transitions exponentially or logarithmically between initial and final values.
- Label key features: initial value, final value, approximate time constant on your sketches.

Useful Differential Equation Solutions

For those who wish to derive exact expressions, here are the standard solutions:

First-Order Circuits:
The general form is:

$$x(t) = x(\infty) + [x(0) - x(\infty)]e^{-t/\tau}$$

where $x(t)$ is the voltage or current of interest, and τ is the time constant.

- **RC circuits**: $\tau = RC$
- **RL circuits**: $\tau = L/R$

Second-Order RLC Circuits:
The characteristic equation gives roots based on damping:

$$s^2 + 2\alpha s + \omega_0^2 = 0$$

where $\alpha = \frac{R}{2L}$ (for series RLC) and $\omega_0 = \frac{1}{\sqrt{LC}}$.

- **Overdamped** ($\alpha > \omega_0$):
$$x(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}$$
where $s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2}$ (both real, negative)

- **Critically Damped** ($\alpha = \omega_0$):
$$x(t) = (A_1 + A_2 t)e^{-\alpha t}$$

- **Underdamped** ($\alpha < \omega_0$):
$$x(t) = e^{-\alpha t}(A_1 \cos(\omega_d t) + A_2 \sin(\omega_d t))$$
where $\omega_d = \sqrt{\omega_0^2 - \alpha^2}$ is the damped natural frequency

Problems

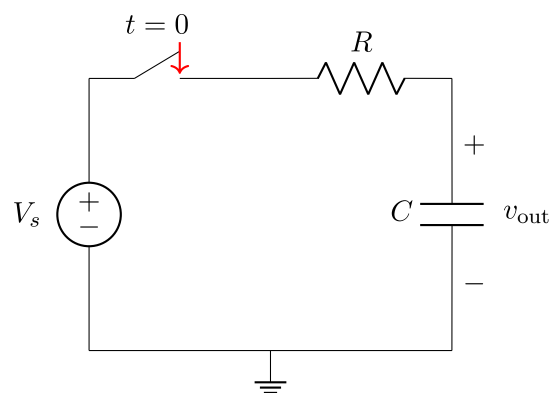
Part A: First-Order Circuits

For each circuit below, assume the circuit has been in steady state for $t < 0$, and the switch changes position at $t = 0$. Sketch $v_{out}(t)$ for $t > 0$. Clearly label:

- Initial voltage $v_{out}(0^+)$
- Final voltage $v_{out}(\infty)$
- Time constant τ in terms of circuit elements
- Shape of the waveform

1. **Simple RC Step Response**

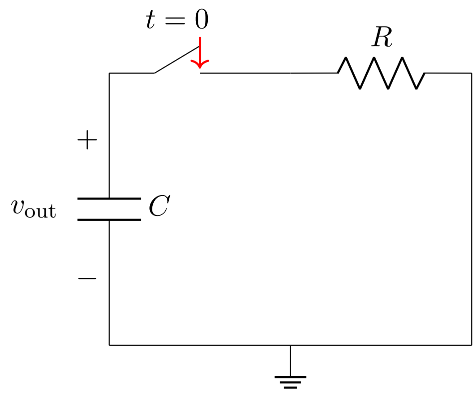
   The switch has been open for a long time. The capacitor is initially uncharged $v_{out}(0^-) = 0$ and $V_s = 1V$. At $t = 0$, the switch closes. Sketch $v_{out}(t)$.



Sketch your answer here:

2. **RC Circuit with Initial Condition**

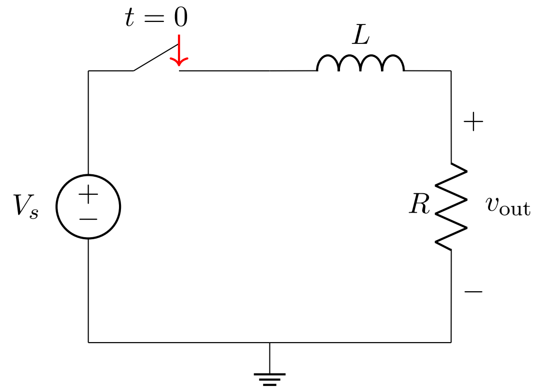
   The capacitor has been charged to V_0 and the switch has been open for a long time. At $t = 0$, the switch closes. Sketch $v_{\text{out}}(t)$.



Sketch your answer here:

3. **RL Step Response**

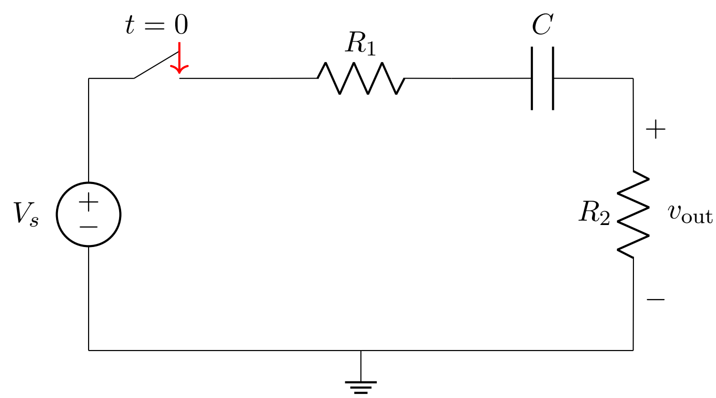
The switch has been open for a long time. At $t = 0$, the switch closes. Sketch $v_{\text{out}}(t)$ across the resistor.



Sketch your answer here:

4. **RC Voltage Divider with Step Input**

   The switch has been open for a long time. At $t = 0$, the switch closes. The capacitor is initially uncharged. Sketch $v_{\text{out}}(t)$.

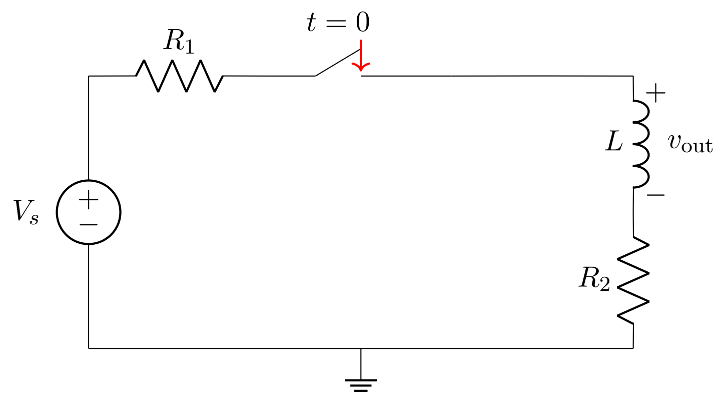


   Hint: Consider the voltage division at $t = 0^+$ and $t = \infty$. The time constant is $\tau = (R_1 + R_2)C$.

   Sketch your answer here:

5. **RL Circuit with Parallel Resistor**

   The switch has been closed for a long time. At $t = 0$, the switch opens. Sketch $v_{\text{out}}(t)$ across the inductor.



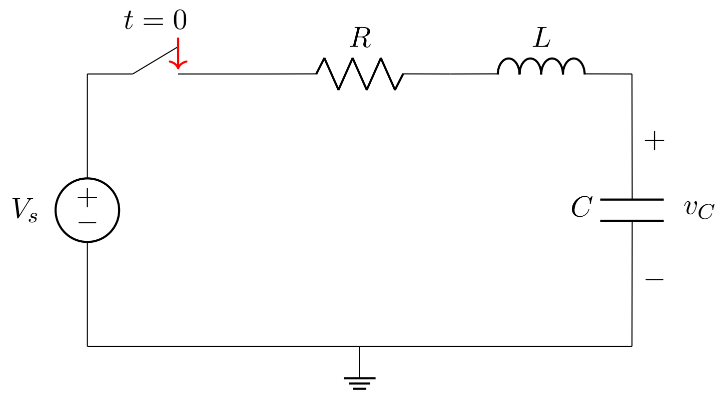
Sketch your answer here:

Part B: Second-Order RLC Circuits

For the following problems, you will analyze second-order RLC circuits.

6. **Series RLC Response Classification**

   Consider the series RLC circuit below. The switch has been open for a long time, and closes at $t = 0$.



(a) Sketch the three possible types of responses for $v_C(t)$:

- Overdamped
- Critically damped
- Underdamped

Clearly label which is which, and indicate key characteristics (oscillation, overshoot, settling behavior).

Sketch your answer here:

(b) For each case below, determine whether the response is overdamped, critically damped, or underdamped. Use the parameters:

$$\alpha = \frac{R}{2L}, \quad \omega_0 = \frac{1}{\sqrt{LC}}$$

Compare α and ω_0 to determine the damping type.

(i) $R = 100\,\Omega$, $L = 10\,\text{mH}$, $C = 10\,\mu\text{F}$

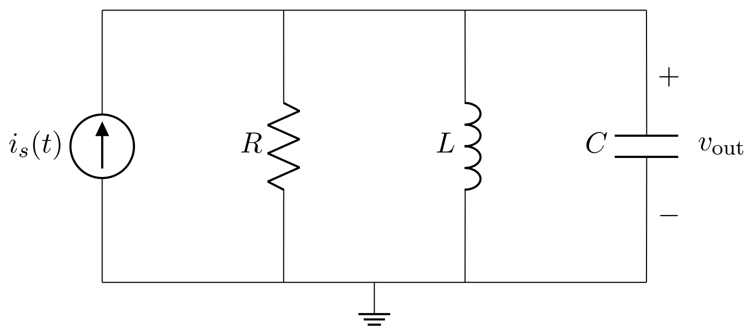
(ii) $R = 20\,\Omega$, $L = 10\,\text{mH}$, $C = 10\,\mu\text{F}$

(iii) $R = 632\,\Omega$, $L = 10\,\text{mH}$, $C = 10\,\mu\text{F}$

For each case, calculate α, ω_0, and state the response type.

7. **Parallel RLC Circuit Analysis (Extra Credit)**

   Consider the parallel RLC circuit shown. The current source is a step function: $i_s(t) = I_0 \cdot u(t)$ where $u(t)$ is the unit step function.



(a) Without specifying values, sketch the general shape of $v_{\text{out}}(t)$ for an underdamped response. Indicate the damped oscillation frequency and the exponential envelope.

Sketch your answer here:

(b) Choose specific values for R, L, and C that would result in a critically damped response. Show your work to verify that $\alpha = \omega_0$.

For a parallel RLC: $\alpha = \frac{1}{2RC}$ and $\omega_0 = \frac{1}{\sqrt{LC}}$.

Submission Guidelines

- Show all work and reasoning clearly.
- Sketches should be neat and properly labeled.
- For sketches, indicate time axis (you may use multiples of τ for first-order).
- Clearly mark initial conditions, final values, and time constants.