MOSFET AC Analysis and Small-Signal Models

Unit 5: Field-Effect Transistors

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AC vs DC Analysis

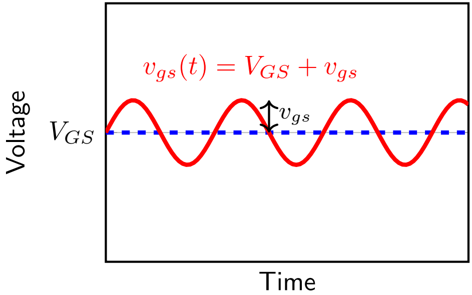
DC Analysis

- Establishes Q-point
- Determines operating region
- Uses large-signal model
- Static voltages and currents

AC Analysis

- Small variations around Q-point
- Uses small-signal model
- Linear approximation
- Frequency-dependent behavior

Total Signal = DC + AC



Key Concept

AC analysis assumes small signals ($v_{gs} \ll V_{GS}$), allowing linear approximation around the Q-point.

Small-Signal Linearization

Nonlinear DC Relationship

$$i_D = k_n(v_{GS} - V_{th})^2(1 + \lambda v_{DS})$$

Total signals:

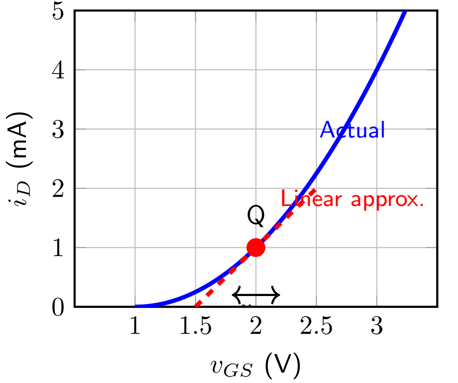
$$v_{GS}(t) = V_{GS} + v_{gs}(t)$$
$$v_{DS}(t) = V_{DS} + v_{ds}(t)$$
$$i_D(t) = I_D + i_d(t)$$

Taylor Series Expansion

$$i_D(t) = I_D + \left.\frac{\partial i_D}{\partial v_{GS}}\right|_Q v_{gs} + \left.\frac{\partial i_D}{\partial v_{DS}}\right|_Q v_{ds} + \ldots$$

For small signals, higher-order terms negligible.

Graphical Interpretation



Slope at Q-point = transconductance g_m

Common Source Amplifier: Voltage Gain Mechanism

Small Changes in V_{GS} Create Large Changes in V_{DS}

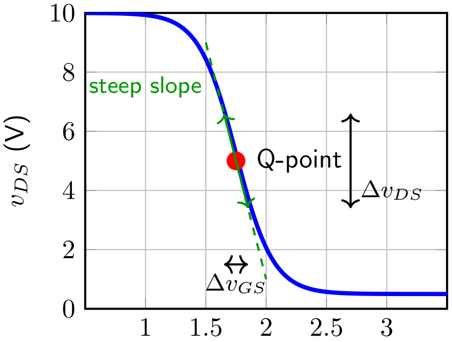
Consider a common source amplifier biased at Q-point:

- Input signal varies v_{GS}
- This modulates drain current i_D
- Current flows through load resistor R_D
- Output voltage v_{DS} changes

Voltage Gain

Small $\Delta v_{GS} \Rightarrow$ Large $\Delta i_D \Rightarrow$ Large Δv_{DS}
This is the fundamental amplification mechanism.

Voltage Transfer Characteristic (VTC)



Voltage Gain:

$$A_v = \frac{\Delta v_{DS}}{\Delta v_{GS}} = \text{slope of VTC}$$

Transconductance g_m

Definition

Transconductance relates small-signal gate voltage to drain current:

$$g_m = \left.\frac{\partial i_D}{\partial v_{GS}}\right|_Q$$

Derivation (Saturation, no CLM)

$$I_D = k_n(V_{GS} - V_{th})^2$$

$$g_m = \frac{\partial}{\partial V_{GS}}\left[k_n(V_{GS} - V_{th})^2\right]$$

$$\boxed{g_m = 2k_n(V_{GS} - V_{th}) = 2k_n V_{OV}}$$

where $V_{OV} = V_{GS} - V_{th}$, overdrive voltage.

Alternative Forms

Since $I_D = k_n V_{OV}^2$, we can write:

$$\boxed{g_m = \sqrt{2k_n I_D} = \frac{2I_D}{V_{OV}}}$$

Units: mA/V or mS (millisiemens)

Physical Interpretation

- ☺ Voltage-to-current gain
- ☺ Larger g_m = better amplification
- ☺ Increases with I_D and device size (W/L)
- ☹ Decreases with V_{OV} (for fixed I_D)

Important

g_m is determined by the DC operating point

Output Resistance r_o - Channel Length Modulation

Channel-Length Modulation (CLM)

In saturation, I_D depends slightly on V_{DS}:

$$I_D = k_n(V_{GS} - V_{th})^2(1 + \lambda V_{DS})$$

where λ is the CLM parameter (V^{-1}).

Output Resistance

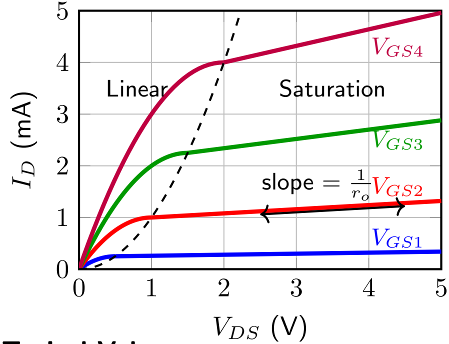
$$r_o = \left.\frac{\partial v_{DS}}{\partial i_D}\right|_Q$$

Derivation:

$$\frac{\partial i_D}{\partial v_{DS}} = k_n(V_{GS} - V_{th})^2\lambda = \lambda I_D$$

$$\boxed{r_o = \frac{1}{\lambda I_D}}$$

I-V Curves with CLM

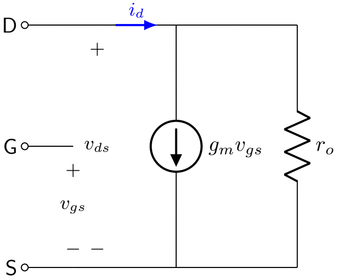


Typical Values

- $\lambda = 0.01$ to 0.1 V^{-1}
- $r_o = 10$ to 100 kΩ
- ☺ Higher r_o = better current source

MOSFET Small-Signal Equivalent Circuit

Small-Signal Model (Saturation)



Key Features:

- Voltage-controlled current source
- Infinite input resistance (DC/low freq)
- Finite output resistance r_o

Small-Signal Parameters

Parameter	Expression
g_m	$2k_n V_{OV} = \sqrt{2k_n I_D}$
r_o	$\frac{1}{\lambda I_D}$
i_d	$g_m v_{gs} + \frac{v_{ds}}{r_o}$

Simplified Model ($r_o \to \infty$ / $\lambda = 0$)

$$\boxed{i_d = g_m v_{gs}}$$

The small-signal model is only valid for:

- Small signal amplitudes
- Frequencies where capacitances can be ignored
- Operation in saturation region

Example: Small-Signal Parameter Calculation

Given DC Operating Point:

- $V_{GS} = 2$ V
- $V_{DS} = 6$ V
- $I_D = 2$ mA
- $V_{th} = 1$ V
- $k_n = 2$ mA/V^2
- $\lambda = 0.02$ V^{-1}

Find: g_m and r_o

Solution:
Step 1: Calculate overdrive voltage

$$V_{OV} = V_{GS} - V_{th} = 2 - 1 = 1 \text{ V}$$

Step 2: Calculate transconductance
Using $g_m = 2k_n V_{OV}$:

$$g_m = 2 \times 2 \times 1 = 4 \text{ mA/V}$$

Step 3: Calculate output impedance

$$r_o = \frac{1}{\lambda I_D} = \frac{1}{0.02 \times 2 \times 10^{-3}} = \frac{1}{4 \times 10^{-5}}$$

$$\boxed{r_o = 25 \text{ k}\Omega}$$

Result: Small-signal model has $g_m = 4$ mA/V and $r_o = 25$ kΩ.

AC Analysis Procedure

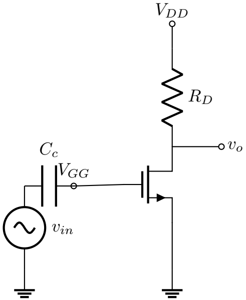
1. **Perform DC Analysis**
   - Find Q-point: V_{GS}, V_{DS}, I_D
   - Verify operating region
2. **Calculate Small-Signal Parameters**
   - $g_m = 2k_n V_{OV}$ or $g_m = \sqrt{2k_n I_D}$
   - $r_o = \frac{1}{\lambda I_D}$ (if CLM included)
3. **Draw AC Equivalent Circuit**
   - Replace MOSFET with small-signal model
   - DC sources $\rightarrow$ ground (short circuit)
   - Capacitors $\rightarrow$ short circuit (at high frequencies)
   - DC biasing resistors remain
4. **Analyze AC Circuit**
   - Apply KVL, KCL, Ohm's law
   - Calculate voltage gain, input/output impedance

Key Point

Always perform DC analysis first to establish the Q-point before conducting AC analysis.

Example: Common Source Amplifier Analysis

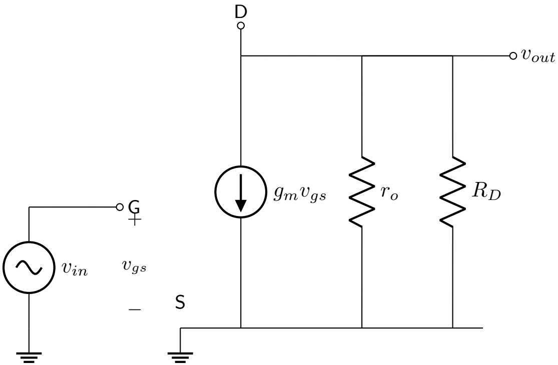
DC Circuit



Given:

- $R_D = 2$ kΩ
- $g_m = 4$ mA/V
- $r_o = 25$ kΩ

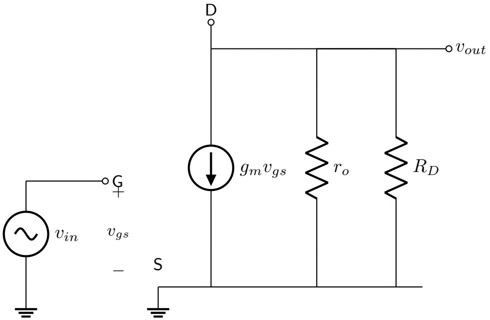
AC Equivalent Circuit



Analysis:

$v_{gs} = v_{in}$ (ideal coupling)

Example: Common Source Amplifier - Small-Signal Analysis



Given:

- $R_D = 2$ kΩ
- $g_m = 4$ mA/V
- $r_o = 25$ kΩ

Small-Signal Analysis:
Current i_d from controlled source

$$i_d = g_m v_{gs} = g_m v_{in}$$

Output voltage (KVL at drain node)
Current flows through $R_D \parallel r_o$:

$$v_{out} = -i_d(R_D \parallel r_o) = -g_m v_{in}(R_D \parallel r_o)$$

$$A_v = \frac{v_{out}}{v_{in}} = -g_m(R_D \parallel r_o)$$

$$R_D \parallel r_o = \frac{2 \times 25}{2 + 25} = 1.85 \text{ k}\Omega$$

$$A_v = -4 \times 1.85 = -7.4$$

Impact of Output Resistance on Gain

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Voltage Gain with CLM

$$A_v = -g_m(R_D \parallel r_o) = -g_m \frac{R_D r_o}{R_D + r_o}$$

Two Limiting Cases:

Case 1: $r_o \gg R_D$ (ideal case)

$$A_v \approx -g_m R_D$$

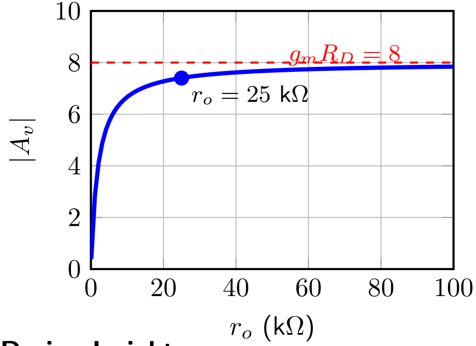
Maximum achievable gain.

Case 2: $r_o \ll R_D$ (high CLM)

$$A_v \approx -g_m r_o = -\frac{g_m}{\lambda I_D}$$

Gain limited by device physics.

Gain vs. r_o



Design Insight:

- ☺ Higher r_o improves gain
- ☺ Long-channel devices have higher r_o

Summary: MOSFET Small-Signal Parameters

Parameter	Definition	Formula	Typical Value
Transconductance	$\frac{\partial i_D}{\partial v_{GS}}$	$g_m = 2k_n V_{OV} = \sqrt{2k_n I_D}$	1-10 mA/V
Output resistance	$\frac{\partial v_{DS}}{\partial i_D}$	$r_o = \frac{1}{\lambda I_D}$	10-100 kΩ
Body transconductance	$\frac{\partial i_D}{\partial v_{BS}}$	$g_{mb} = \eta g_m$	0.1-0.3 g_m

Key Relationships:

- $g_m \propto \sqrt{I_D}$
- $g_m \propto \sqrt{W/L}$
- $r_o \propto 1/I_D$
- $g_m \cdot r_o = \frac{2}{\lambda V_{OV}}$ (intrinsic gain)

Design Trade-offs:

- Large $I_D \Rightarrow$ high g_m, low r_o
- Large $W/L \Rightarrow$ high g_m, more area
- Small $V_{OV} \Rightarrow$ high g_m (for given I_D)
- Long channel $\Rightarrow$ small λ, high r_o