

# Discrete Time Signals

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# What is a Signal?

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Discrete-Time  
Signals

Sequences

Frequencies

- A **signal** conveys information about the state or behavior of a physical system.
- Signals can be synthesized for communication between humans or between humans and machines.
- Mathematically, signals are functions of one or more independent variables (e.g., time, space).
- Example: **Speech signal** is a function of time.  
**Image** is a function of two spatial variables (brightness).

# Signals: Continuous vs. Discrete

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- The independent variable of a signal can be:
  - **Continuous** (Continuous-time or analog signals)
  - **Discrete** (Discrete-time signals)
- Amplitude can also be:
  - **Continuous**
  - **Discrete**
- **Digital signals:** Both time and amplitude are discrete.

# Types of Signal Processing Systems

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- **Continuous-time systems:** Input and output are continuous-time signals.
- **Discrete-time systems:** Input and output are discrete-time signals.
- **Digital systems:** Input and output are digital signals (discrete in both time and amplitude).
- **Digital signal processing (DSP):** Concerned with transforming signals that are discrete in both amplitude and time.

# Discrete-Time Signals as Sequences

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- Discrete-time signals are sequences of numbers:

$$x = \{x[n]\}, \quad -\infty < n < \infty$$

- $n$  is an integer;  $x[n]$  is the  $n$ th sample.
- Often arise from sampling a continuous-time signal  $x_a(t)$ :

$$x[n] = x_a(nT)$$

- $T$  is the sampling period; its reciprocal is the sampling frequency.

# Graphical Representation of a Sequence

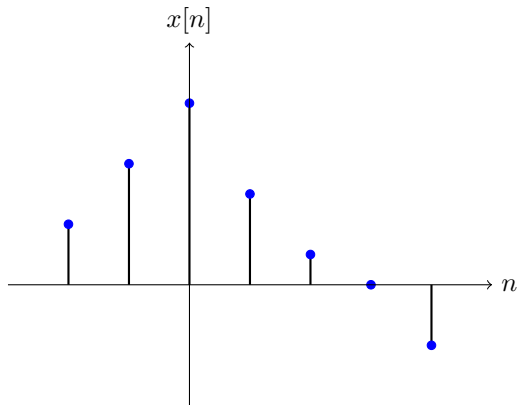
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- $x[n]$  is defined only for integer values of  $n$ .
- Not defined for non-integer values.

# Sampling: From Analog to Discrete

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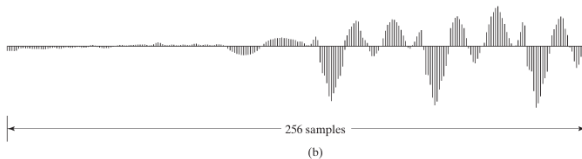
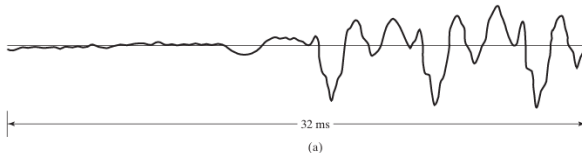
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## Continuous-time (Analog) Signal



- Sampling theorem: original analog signal can be reconstructed if sampled frequently enough.

# Basic Discrete-Time Sequences: Impulse and Step

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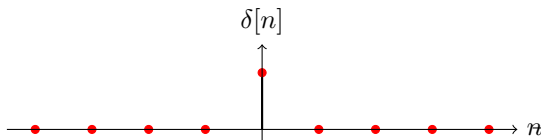
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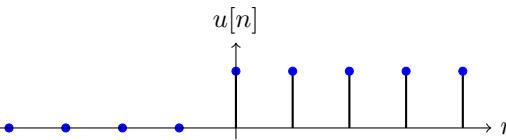
## Unit Sample (Impulse) Sequence

$$\delta[n] = \begin{cases} 1, & n = 0 \\ 0, & n \neq 0 \end{cases}$$



## Unit Step Sequence

$$u[n] = \begin{cases} 1, & n \geq 0 \\ 0, & n < 0 \end{cases}$$





# Basic Discrete-Time Sequences: Exponential and Sinusoidal

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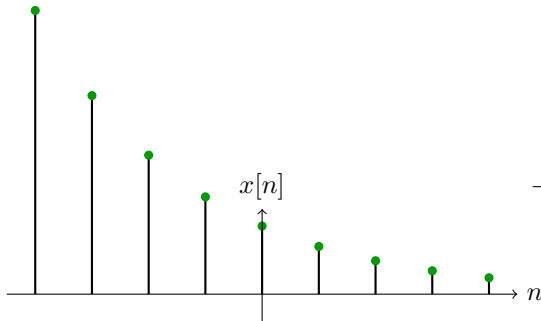
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## Exponential Sequence

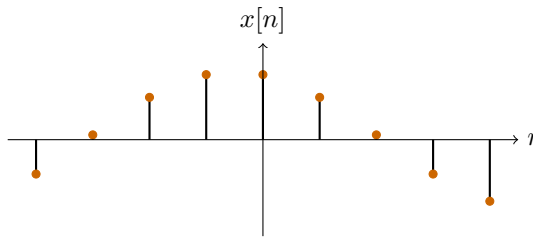
$$x[n] = A\alpha^n$$



Example:  $A = 1.2$ ,  $\alpha = 0.7$

## Sinusoidal Sequence

$$x[n] = A \cos(\omega_0 n + \phi)$$



Example:  $A = 1.2$ ,  $\omega_0 = 0.6$ ,  $\phi = 0.3$

# Impulse Representation of Sequences

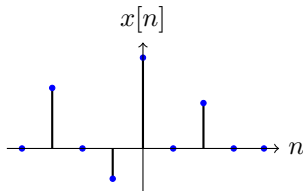
- Any sequence can be written as a sum of scaled, delayed impulses:

$$x[n] = \sum_{k=-\infty}^{\infty} x[k] \delta[n - k]$$

- Example with a random sequence:

$$x[n] = 2\delta[n + 3] - \delta[n + 1] + 3\delta[n] + 1.5\delta[n - 2]$$

$n$	-4	-3	-2	-1	0	1	2	3	4
$x[n]$	0	2	0	-1	3	0	1.5	0	0



# Relations Between Impulse and Step Sequences

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- The unit step is the sum of all impulses up to  $n$ :

$$u[n] = \sum_{k=-\infty}^n \delta[k]$$

- Or, as sum of delayed impulses:

$$u[n] = \sum_{k=0}^{\infty} \delta[n - k]$$

- The impulse is the first backward difference of the unit step:

$$\delta[n] = u[n] - u[n - 1]$$

# Exponential and Sinusoidal Sequences

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- **Real exponential:**  $x[n] = A\alpha^n$  (growth/decay depends on  $|\alpha|$ )

- **Complex exponential:**

$$x[n] = |A||\alpha|^n e^{j(\omega_0 n + \phi)}$$

- **Sinusoidal:**

$$x[n] = |A| \cos(\omega_0 n + \phi)$$

- Key property: In discrete-time,  $e^{j(\omega_0 + 2\pi)n} = e^{j\omega_0 n}$
- Need only consider frequencies in  $-\pi < \omega_0 \leq \pi$  (or  $0 \leq \omega_0 < 2\pi$ )

# Periodicity in Discrete-Time Sinusoids

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- A discrete-time sequence is periodic if  $x[n] = x[n + N]$  for all  $n$ , where  $N$  is a positive integer (**the period**).
- For the sinusoid  $x[n] = A \cos(\omega_0 n + \phi)$ , periodicity requires that:

$$\omega_0 N = 2\pi k, \quad \text{where } k \text{ is an integer}$$

- A similar statement holds for the complex exponential sequence  $x[n] = Ce^{j\omega_0 n}$ :

$$e^{j\omega_0(n+N)} = e^{j\omega_0 n}$$

- **Conclusion:** Not all discrete-time sinusoids or complex exponentials are periodic, and their periodicity depends on whether the frequency  $\omega_0$  satisfies the above condition for some integer  $N$ .

# Example 1: Discrete-Time Sinusoid $x_1[n]$

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- Consider the sequence, what is the period?

$$x_1[n] = \cos\left(\frac{\pi n}{4}\right)$$

# Example 1 Result: Discrete-Time Sinusoid $x_1[n]$

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- $x_1[n]$  is periodic with period  $N = 8$ .
- Proof:

$$x_1[n + 8] = \cos\left(\frac{\pi(n + 8)}{4}\right) = \cos\left(\frac{\pi n}{4} + 2\pi\right) = \cos\left(\frac{\pi n}{4}\right) = x_1[n]$$

## Example 2: Discrete-Time Sinusoid $x_2[n]$

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- Now consider the sequence, what is the period?

$$x_2[n] = \cos\left(\frac{3\pi n}{8}\right)$$



## Example 2 Result: Discrete-Time Sinusoid $x_2[n]$

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- $x_2[n]$  has a higher frequency but is *not* periodic with period 8.

- 

$$x_2[n + 8] = \cos\left(\frac{3\pi(n + 8)}{8}\right) = \cos\left(\frac{3\pi n}{8} + 3\pi\right) = -x_2[n]$$

- $x_2[n]$  has period  $N = 16$ .

# Non-Periodic Discrete-Time Sinusoids

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- Some discrete-time sinusoids are not periodic at all.

- Example:

$$x_3[n] = \cos(n)$$

- For periodicity, we require that for some integer  $N$ :  
 $x_3[n] = x_3[n + N]$  for all  $n$ , which implies

$$\cos(n) = \cos(n + N) \implies N = 2\pi k \text{ for some integer } k$$

- However,  $2\pi$  is not a rational multiple of 1, so there is **no integer**  $N$  that satisfies this condition.
- **Conclusion:**  $x_3[n]$  never exactly repeats itself. This aperiodicity is unique to discrete-time sinusoids with “irrational” frequencies (relative to  $2\pi$ ).
- **Key Point:** In discrete time, only certain frequencies yield periodic sinusoids; others are aperiodic because the period would have to be a non-integer.

# Distinct Frequencies and Periodicity in Discrete-Time

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- $\omega_0$  and  $(\omega_0 + 2\pi r)$  are indistinguishable for integer  $r$ .
- There are exactly  $N$  distinguishable frequencies for sequences periodic with period  $N$ :

$$\omega_k = \frac{2\pi k}{N}, \quad k = 0, 1, \dots, N-1$$

- These properties are fundamental for discrete-time Fourier analysis.

# Indistinguishable Discrete-Time Frequencies

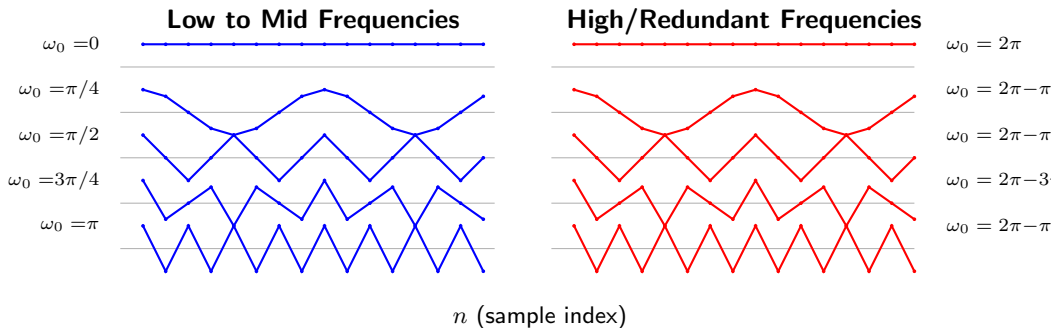
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Each row compares signals with frequencies  $\omega_0$  and  $2\pi - \omega_0$ . Blue and red lines connect the points of each sinusoid for better visibility.

# Oscillation Rate vs. Frequency in Discrete-Time

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- As  $\omega_0$  increases from 0 toward  $\pi$ , oscillations become more rapid.
- As  $\omega_0$  increases from  $\pi$  toward  $2\pi$ , oscillations slow down.
- Due to periodicity,  $\omega_0 = 2\pi$  is identical to  $\omega_0 = 0$ .

# Summary: Discrete-Time Signals & Systems

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- **Signals:** Functions that convey information; can be continuous or discrete in time and amplitude.
- **Discrete-Time Signals:** Sequences  $x[n]$ , typically obtained by sampling a continuous signal.
- **Basic Sequences:**
  - Impulse ( $\delta[n]$ ), step ( $u[n]$ ), exponential, and sinusoidal sequences.
  - Any sequence can be expressed as a sum of scaled, shifted impulses.
- **Impulse and Step Relations:**
  - $u[n]$  is the running sum of  $\delta[n]$ ;  $\delta[n]$  is the difference of  $u[n]$ .
- **Sinusoids in Discrete Time:**
  - Periodicity depends on frequency: only some discrete-time sinusoids are periodic.
  - Frequencies separated by  $2\pi$  are indistinguishable (aliasing effect).
  - Only  $N$  unique frequencies for period- $N$  sequences.
- **Oscillation Rate:** Oscillation increases from 0 to  $\pi$ ; slows from  $\pi$  to  $2\pi$ .