Frequency Domain Analysis of Circuits

Maxx Seminario

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Why Frequency Domain Analysis?

Limitations of Time Domain:

- Differential equations for AC circuits
- Complex trig math
- Difficult for sinusoidal steady-state

Frequency Domain Advantages:

- Converts differential equations to algebra
- Easy handling of sinusoidal signals
- Simplifies AC circuit analysis

Applications:

- AC power systems (60 Hz)
- Audio systems (20 Hz - 20 kHz)
- Radio frequency circuits (MHz - GHz)
- Signal processing and filtering

Domain Transformation Tool

Phasor transform converts time-domain sinusoids to frequency-domain complex numbers

Goal for this lecture

Review frequency domain (phasor) analysis for AC circuits

Sinusoidal Signals: The Foundation

General Sinusoidal Signal:

$$v(t) = V_m \cos(\omega t + \phi)$$

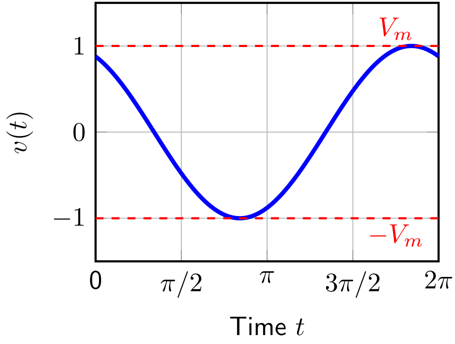
where:

- V_m = amplitude (peak value)
- ω = angular frequency (rad/s)
- ϕ = phase angle (radians or degrees)

Related Parameters:

- Frequency: $f = \omega/(2\pi)$ (Hz)
- Period: $T = 1/f = 2\pi/\omega$ (s)
- RMS value: $V_{rms} = V_m/\sqrt{2}$

Sinusoidal Waveform:



Phasor Concept: From Time to Frequency Domain

Euler's Identity:

$$e^{j\theta} = \cos\theta + j\sin\theta$$

Sinusoid as Complex Exponential:

$$v(t) = V_m \cos(\omega t + \phi)$$

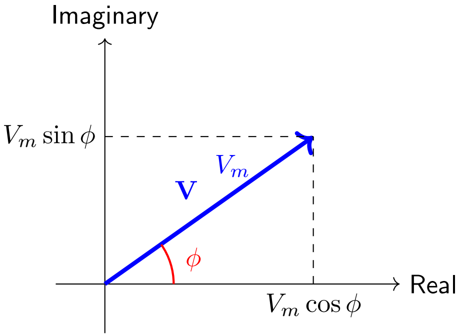
$$v(t) = \mathrm{Re}\{V_m e^{j(\omega t+\phi)}\}$$

$$v(t) = \mathrm{Re}\{V_m e^{j\phi} e^{j\omega t}\}$$

Phasor Definition

$$\mathbf{V} = V_m e^{j\phi} = V_m \angle \phi$$

Phasor Diagram:



Rectangular Form:

$$\mathbf{V} = V_m \cos\phi + jV_m \sin\phi$$

Phasor Transform: Summary

Time Domain	Phasor Domain	Operation
$V_m \cos(\omega t + \phi)$	$\mathbf{V} = V_m \angle \phi$	Domain transformation
$\dfrac{d}{dt}$	$j\omega$	Differentiation $\rightarrow$ multiplication
$\int dt$	$\dfrac{1}{j\omega}$	Integration $\rightarrow$ division
Addition	Addition	Same (LTI Systems)

Key Advantage

- ☺ **Differentiation** in time domain $\rightarrow$ **Multiplication** by $j\omega$ in phasor domain.
- 😐 Phasor analysis only works for **linear circuits** with **sinusoidal sources** at the **same frequency** in **steady-state**

Electrical Impedance

Definition:
Impedance is the ratio of phasor voltage to phasor current:

$$\mathbf{Z} = \frac{\mathbf{V}}{\mathbf{I}}$$

Polar Form:

$$\mathbf{Z} = |\mathbf{Z}|\angle\theta$$

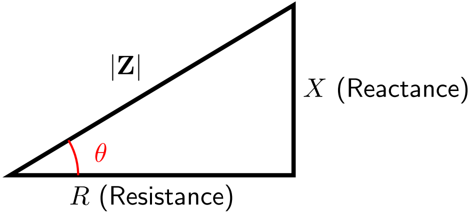
Rectangular Form:

$$\mathbf{Z} = R + jX$$

where:

- R = resistance (real part)
- X = reactance (imaginary part)

Impedance in Complex Plane:



Relationships:

$$|\mathbf{Z}| = \sqrt{R^2 + X^2}$$

$$\theta = \tan^{-1}\left(\frac{X}{R}\right)$$

Impedance of R, L, and C

Element	Time Domain	Impedance	Phase
Resistor	$v = iR$	$\mathbf{Z}_R = R$	0
Inductor	$v = L\dfrac{di}{dt}$	$\mathbf{Z}_L = j\omega L$	$+90$
Capacitor	$i = C\dfrac{dv}{dt}$	$\mathbf{Z}_C = \dfrac{1}{j\omega C} = \dfrac{-j}{\omega C}$	-90

Resistor:

- Real impedance
- V and I in phase
- Frequency independent

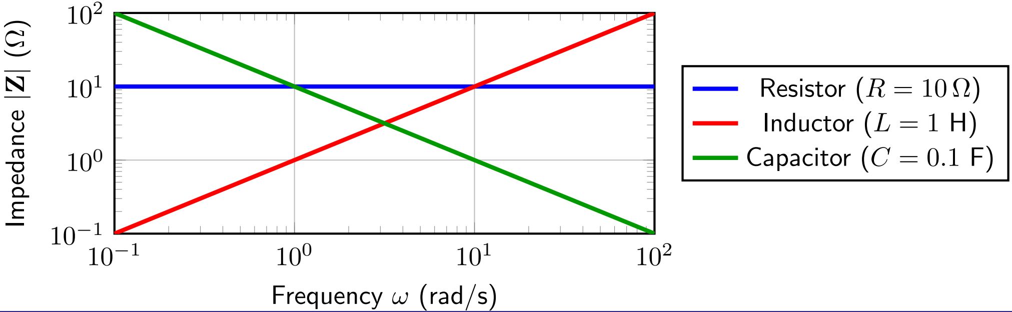
Inductor:

- Imaginary impedance
- V leads I by 90°
- $|\mathbf{Z}_L| = \omega L$ increases with ω

Capacitor:

- Imaginary impedance
- I leads V by 90°
- $|\mathbf{Z}_C| = 1/(\omega C)$ decreases with ω

Frequency Behavior of Impedance



Frequency Behavior

- **Resistor**: Constant impedance (frequency independent)
- **Inductor**: High impedance at high frequencies (blocks AC, passes DC)
- **Capacitor**: Low impedance at high frequencies (blocks DC, passes AC)

Phasor Analysis: Circuit Laws

All DC circuit analysis techniques apply to phasors

Kirchhoff's Voltage Law (KVL):

$$\sum \mathbf{V}_k = 0$$

Kirchhoff's Current Law (KCL):

$$\sum \mathbf{I}_k = 0$$

Ohm's Law:

$$\mathbf{V} = \mathbf{I}\mathbf{Z}$$

Series Impedances:

$$\mathbf{Z}_{eq} = \mathbf{Z}_1 + \mathbf{Z}_2 + \cdots + \mathbf{Z}_n$$

Parallel Impedances:

$$\mathbf{Z}_{eq}^{-1} = \mathbf{Z}_1^{-1} + \mathbf{Z}_2^{-1} + \cdots + \mathbf{Z}_n^{-1}$$

Voltage Divider:

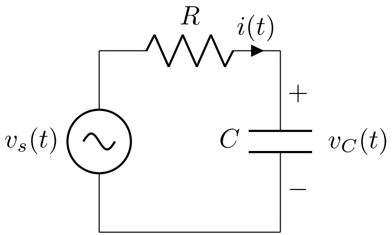
$$\mathbf{V}_k = \mathbf{V}_s \mathbf{Z}_k (\mathbf{Z}_1 + \mathbf{Z}_2)^{-1}$$

Key Point

Replace resistances with impedances, and voltages/currents with phasors. Then use the standard DC techniques

Example: Series RC Circuit

Circuit:



Given:

- $v_s(t) = V_m \cos(\omega t)$
- $R = 100\,\Omega$
- $C = 10\,\mu$F
- $\omega = 1000$ rad/s

Phasor Analysis:

Source phasor: $\mathbf{V}_s = V_m\angle 0$

Impedances:

$$\mathbf{Z}_R = 100\,\Omega$$

$$\mathbf{Z}_C = \frac{-j}{\omega C} = \frac{-j}{0.01} = -j100\,\Omega$$

Total impedance:

$$\mathbf{Z}_{eq} = R - jX_C = 100 - j100$$

$$= 141.4\angle - 45\,\Omega$$

Current:

$$\mathbf{I} = \frac{\mathbf{V}_s}{\mathbf{Z}_{eq}} = \frac{V_m\angle 0}{141.4\angle - 45} = \frac{V_m}{141.4}\angle 45$$

Example: Phasor Diagram

Voltage Divider:
Capacitor voltage:

$$\mathbf{V}_C = \mathbf{V}_s \frac{\mathbf{Z}_C}{\mathbf{Z}_R + \mathbf{Z}_C}$$

$$= \mathbf{V}_s \frac{-j100}{100 - j100}$$

$$= \mathbf{V}_s \frac{100\angle -90}{141.4\angle -45}$$

$$= 0.707 V_m \angle -45$$

Resistor voltage:

$$\mathbf{V}_R = \mathbf{I}R = 0.707 V_m \angle 45$$

Phasor Diagram:

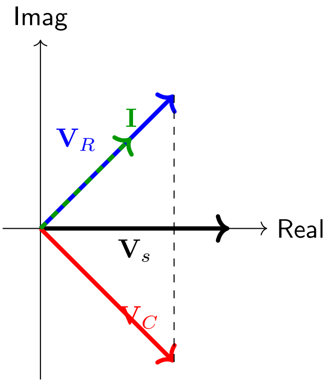
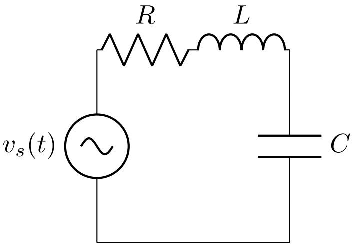


Figure 1: $\mathbf{V}_R + \mathbf{V}_C = \mathbf{V}_s$ (KVL)

Example: Series RLC Circuit

Circuit:



Total Impedance:

$$\mathbf{Z} = R + j\omega L + \frac{1}{j\omega C} = R + j\left(\omega L - \frac{1}{\omega C}\right)$$

$$= R + j(X_L - X_C)$$

Three Cases:

1. **Inductive** $(X_L > X_C)$:
   - Net reactance is positive
   - Voltage leads current
   - Behaves like RL circuit
2. **Capacitive** $(X_L < X_C)$:
   - Net reactance is negative
   - Current leads voltage
   - Behaves like RC circuit
3. **Resonant** $(X_L = X_C)$:
   - Net reactance is zero
   - $\mathbf{Z} = R$ (purely resistive)
   - V and I in phase

Resonance in RLC Circuits

Resonance Condition:
At resonance: $X_L = X_C$

$$\omega_0 L = \frac{1}{\omega_0 C}$$

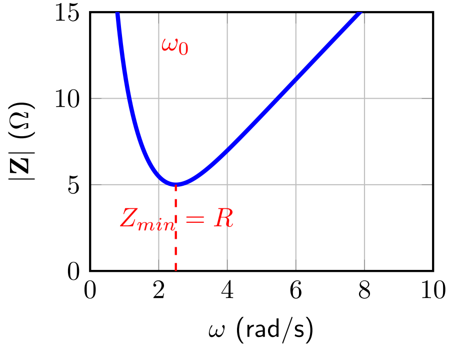
Resonant Frequency

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

At Resonance:

- $\mathbf{Z} = R$ (minimum impedance)
- Maximum current
- Zero phase angle

Impedance vs. Frequency:



AC Power: Instantaneous and Average

Instantaneous Power:
For $v(t) = V_m \cos(\omega t)$ and
$i(t) = I_m \cos(\omega t - \theta)$:

$$p(t) = v(t) \cdot i(t)$$

$$= V_m I_m \cos(\omega t) \cos(\omega t - \theta)$$

Using trig identity:

$$p(t) = \frac{V_m I_m}{2} \cos\theta + \frac{V_m I_m}{2} \cos(2\omega t - \theta)$$

Average Power:

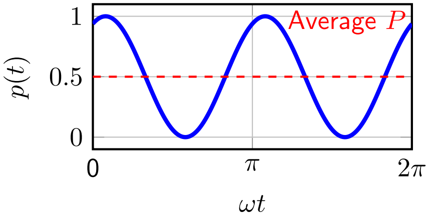
$$P = \frac{1}{T} \int_0^T p(t)\,dt = \frac{V_m I_m}{2} \cos\theta$$

Using RMS Values:

$$V_{rms} = \frac{V_m}{\sqrt{2}}, \quad I_{rms} = \frac{I_m}{\sqrt{2}}$$

Average (Real) Power

$$P = V_{rms} I_{rms} \cos\theta$$



Reactive and Apparent Power

Power Components:

1. **Real (Average) Power**:

$$P = V_{rms} I_{rms} \cos\theta \quad \text{(W)}$$

- Power dissipated (resistors)

2. **Reactive Power**:

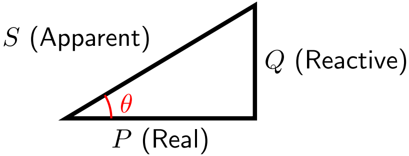
$$Q = V_{rms} I_{rms} \sin\theta \quad \text{(VAR)}$$

- Power stored/returned (L/C)

3. **Apparent Power**:

$$S = V_{rms} I_{rms} \quad \text{(VA)}$$

Power Triangle:



$$S = \sqrt{P^2 + Q^2}$$

$$P = S\cos\theta, \quad Q = S\sin\theta$$

Power Factor:

$$\text{pf} = \cos\theta = \frac{P}{S}$$

Power Factor and Its Importance

Power Factor Definition:

$$\text{pf} = \cos\theta = \frac{P}{S}$$

Range: $0 \leq \text{pf} \leq 1$

Special Cases:

- ☺ **pf = 1** (unity): purely resistive, $\theta = 0$
- ☹ **pf = 0**: purely reactive, $\theta = \pm 90$

Leading vs. Lagging:

- Lagging pf: inductive load (current lags voltage)
- Leading pf: capacitive load (current leads voltage)

Low Power Factor Problems

- ☹ Higher current required
- ☹ Larger conductor sizes needed
- ☹ More I^2R losses in transmission

Power Factor Correction:
Add capacitors in parallel with inductive loads to:

- ☺ Increase power factor
- ☺ Reduce reactive power
- ☺ Lower current draw

Power in Circuit Elements

Element	Phase	Real Power P	Reactive Power Q	pf
Resistor	$\theta = 0$	$I^2 R$	0	1
Inductor	$\theta = 90$	0	$I^2 X_L$ (positive)	0
Capacitor	$\theta = -90$	0	$-I^2 X_C$ (negative)	0

Key Observations

- Only **resistors** dissipate real power (convert to heat · or light if you mess up)
- **Inductors** and **capacitors** store and return energy (reactive power)
- Reactive power from L and C have opposite signs (can cancel to form resonant networks)

Summary: Frequency Domain Analysis

Phasor Analysis:

- Transform: $V_m \cos(\omega t + \phi) \leftrightarrow V_m \angle \phi$
- ☺ Differential equations $\rightarrow$ algebra
- $d/dt \rightarrow j\omega$, $\int dt \rightarrow 1/(j\omega)$

Impedance:

- $\mathbf{Z} = R + jX$
- Resistor: $\mathbf{Z}_R = R$
- Inductor: $\mathbf{Z}_L = j\omega L$
- Capacitor: $\mathbf{Z}_C = 1/(j\omega C)$

Circuit Analysis:

- ☺ All DC techniques apply
- KVL, KCL, voltage/current dividers
- Series/parallel combinations

AC Power:

- Real power: $P = V_{rms} I_{rms} \cos\theta$
- Reactive power: $Q = V_{rms} I_{rms} \sin\theta$
- Apparent power: $S = V_{rms} I_{rms}$

Power Factor:

- pf $= \cos\theta = P/S$
- Lagging pf: inductive
- Leading pf: capacitive
- ☹ Low pf $\rightarrow$ higher losses

Resonance:

- Occurs when $X_L = X_C$
- $\omega_0 = 1/\sqrt{LC}$
- ☺ Minimum Z, maximum I

Comparison: Time vs. Frequency Domain

Aspect	**Time Domain**	**Frequency Domain**
Signals	$v(t)$, $i(t)$ (real functions)	$\mathbf{V}$, $\mathbf{I}$ (complex phasors)
Math	Differential equations	Algebraic equations
Circuit elements	R, L, C (time relations)	$\mathbf{Z}_R$, $\mathbf{Z}_L$, $\mathbf{Z}_C$ (impedances)
Analysis	Initial conditions, transients	Steady-state, magnitude/phase
Advantages	Shows time evolution	Simplifies sinusoidal analysis
Limitations	Complex for AC steady-state	Only sinusoidal steady-state

When to Use Each

Time Domain: Transients, switching, initial conditions, non-sinusoidal signals
Frequency Domain: AC steady-state, sinusoidal sources, impedance analysis

Example 1: Phasor Conversions

Problem: Convert the following time-domain signals to phasors, then perform the operations.

Given:

$$v_1(t) = 10\cos(1000t + 30)\ \text{V}$$
$$v_2(t) = 5\cos(1000t - 45)\ \text{V}$$
$$i(t) = 2\cos(1000t + 60)\ \text{A}$$

Find:

1. Phasor forms of v_1, v_2, and i
2. $\mathbf{V}_1 + \mathbf{V}_2$
3. $\mathbf{V}_1 - \mathbf{V}_2$
4. $\mathbf{V}_1/\mathbf{I}$

Solution:

Part 1: Phasor forms

$$\mathbf{V}_1 = 10\angle 30\ \text{V}$$
$$\mathbf{V}_2 = 5\angle -45\ \text{V}$$
$$\mathbf{I} = 2\angle 60\ \text{A}$$

Part 2: $\mathbf{V}_1 + \mathbf{V}_2$

Convert to rectangular:

$$\mathbf{V}_1 = 10\cos(30) + j10\sin(30)$$
$$= 8.66 + j5.00\ \text{V}$$
$$\mathbf{V}_2 = 5\cos(-45) + j5\sin(-45)$$
$$= 3.54 - j3.54\ \text{V}$$

Example 1: Solution (continued)

Part 2 (continued):

$$\mathbf{V}_1 + \mathbf{V}_2 = (8.66 + j5.00) + (3.54 - j3.54)$$
$$= 12.20 + j1.46 \text{ V}$$

Convert to polar:

$$|\mathbf{V}| = \sqrt{12.20^2 + 1.46^2} = 12.29 \text{ V}$$
$$\angle\mathbf{V} = \tan^{-1}\left(\frac{1.46}{12.20}\right) = 6.83$$

$$\boxed{\mathbf{V}_1 + \mathbf{V}_2 = 12.29\angle 6.83 \text{ V}}$$

Part 3: $\mathbf{V}_1 - \mathbf{V}_2$

$$\mathbf{V}_1 - \mathbf{V}_2 = (8.66 + j5.00) - (3.54 - j3.54)$$
$$= 5.12 + j8.54 \text{ V}$$
$$= 9.96\angle 59.05 \text{ V}$$

$$\boxed{\mathbf{V}_1 - \mathbf{V}_2 = 9.96\angle 59.05 \text{ V}}$$

Part 4: $\mathbf{V}_1/\mathbf{I}$ (This is impedance!)

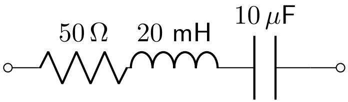
$$\frac{\mathbf{V}_1}{\mathbf{I}} = \frac{10\angle 30}{2\angle 60} = \frac{10}{2}\angle(30 - 60)$$
$$= 5\angle -30\,\Omega$$

$$\boxed{\mathbf{Z} = 5\angle -30\,\Omega = 4.33 - j2.50\,\Omega}$$

Example 2: Impedance at Different Frequencies

Problem: A series circuit contains $R = 50\,\Omega$, $L = 20$ mH, and $C = 10\,\mu$F. Find the total impedance at the following frequencies.

Circuit:



Given:

- $R = 50\,\Omega$
- $L = 20$ mH
- $C = 10\,\mu$F

Find the total impedance at:

- (a) $f = 100$ Hz
- (b) $f = 500$ Hz
- (c) $f = 1000$ Hz

For each frequency, determine:

1. Magnitude $|\mathbf{Z}|$
2. Phase angle θ
3. Whether the circuit is inductive or capacitive

Example 2: Solution - Setup and Part (a)

General Formula for Series RLC:

$$\mathbf{Z} = R + j\omega L + \frac{1}{j\omega C} = R + j\left(\omega L - \frac{1}{\omega C}\right)$$

where $X_L = \omega L$ (inductive reactance) and $X_C = \dfrac{1}{\omega C}$ (capacitive reactance)

Part (a): $f = 100$ **Hz**

$$\omega = 2\pi f = 2\pi(100) = 628.3 \text{ rad/s}$$

$$X_L = \omega L = 628.3 \times 0.02 = 12.57\,\Omega$$

$$X_C = \frac{1}{\omega C} = \frac{1}{628.3 \times 10^{-5}} = 159.2\,\Omega$$

$$X = X_L - X_C = -146.6\,\Omega$$

$$\mathbf{Z} = 50 - j146.6\,\Omega$$

$$\boxed{\mathbf{Z} = 154.9\angle -71.2\,\Omega}$$

Capacitive behavior (negative reactance, current leads)

Example 2: Solution - Parts (b) and (c)

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Part (b): $f = 500$ **Hz**
$\omega = 2\pi(500) = 3141.6$ rad/s

$$X_L = 3141.6 \times 0.02 = 62.83\,\Omega$$
$$X_C = \frac{1}{3141.6 \times 10^{-5}} = 31.83\,\Omega$$
$$X = 62.83 - 31.83 = 31.0\,\Omega$$

$$\mathbf{Z} = 50 + j31.0\,\Omega$$

$$\boxed{\mathbf{Z} = 59.0\angle 31.8\,\Omega}$$

Inductive behavior (positive reactance, current lags)

Part (c): $f = 1000$ **Hz**
$\omega = 2\pi(1000) = 6283.2$ rad/s

$$X_L = 6283.2 \times 0.02 = 125.7\,\Omega$$
$$X_C = \frac{1}{6283.2 \times 10^{-5}} = 15.92\,\Omega$$
$$X = 125.7 - 15.92 = 109.8\,\Omega$$

$$\mathbf{Z} = 50 + j109.8\,\Omega$$

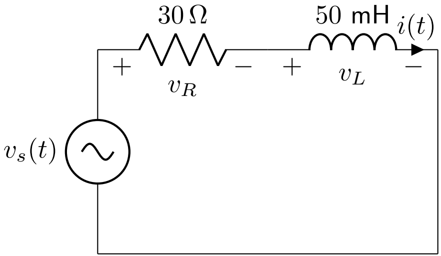
$$\boxed{\mathbf{Z} = 120.7\angle 65.5\,\Omega}$$

Inductive behavior (positive reactance, current lags)

Example 3: Series RL Circuit

Problem: For the circuit shown, find the current, voltage across each element, and draw the phasor diagram.

Circuit:



Given:

- $v_s(t) = 100\cos(2000t)$ V
- $R = 30\,\Omega$
- $L = 50$ mH $= 0.05$ H
- $\omega = 2000$ rad/s

Find:

1. Total impedance $\mathbf{Z}_{tot}$
2. Current $i(t)$ (phasor and time domain)
3. Voltage across resistor $v_R(t)$
4. Voltage across inductor $v_L(t)$
5. Draw the phasor diagram
6. Verify KVL

Example 3: Solution - Impedance and Current

Step 1: Convert to phasor domain
Source phasor:

$$\mathbf{V}_s = 100\angle 0 \text{ V}$$

Step 2: Calculate impedances
Resistor impedance:

$$\mathbf{Z}_R = R = 30\,\Omega$$

Inductor impedance:

$$\begin{aligned}\mathbf{Z}_L &= j\omega L \\ &= j(2000)(0.05) \\ &= j100\,\Omega\end{aligned}$$

Step 3: Total impedance
Rectangular form:

$$\mathbf{Z}_{tot} = \mathbf{Z}_R + \mathbf{Z}_L = 30 + j100\,\Omega$$

Polar form:

$$|\mathbf{Z}_{tot}| = \sqrt{30^2 + 100^2} = 104.4\,\Omega$$

$$\theta_Z = \tan^{-1}\left(\frac{100}{30}\right) = 73.3$$

$$\boxed{\mathbf{Z}_{tot} = 104.4\angle 73.3\,\Omega}$$

Example 3: Solution - Current (continued)

Step 4: Calculate current

Using Ohm's law:

$$\mathbf{I} = \frac{\mathbf{V}_s}{\mathbf{Z}_{tot}} = \frac{100\angle 0}{104.4\angle 73.3} = \frac{100}{104.4}\angle(0 - 73.3) = 0.958\angle -73.3 \text{ A}$$

$$\boxed{\mathbf{I} = 0.958\angle -73.3 \text{ A}}$$

Step 5: Convert to time domain

$$\boxed{i(t) = 0.958\cos(2000t - 73.3) \text{ A}}$$

Interpretation

Current **lags** the voltage by 73.3, which is expected for an inductive circuit (RL circuit).

Example 3: Solution - Element Voltages

Step 6: Voltage across resistor

Using Ohm's law:

$$\begin{aligned}\mathbf{V}_R &= \mathbf{I} \cdot \mathbf{Z}_R \\ &= (0.958\angle-73.3)(30\angle 0) \\ &= 0.958 \times 30\angle(-73.3 + 0) \\ &= 28.7\angle-73.3 \text{ V}\end{aligned}$$

$$\boxed{\mathbf{V}_R = 28.7\angle-73.3 \text{ V}}$$

Time domain:

$$v_R(t) = 28.7\cos(2000t - 73.3) \text{ V}$$

Note: $\mathbf{V}_R$ is in phase with $\mathbf{I}$ (both at -73.3)

Step 7: Voltage across inductor

Using Ohm's law:

$$\begin{aligned}\mathbf{V}_L &= \mathbf{I} \cdot \mathbf{Z}_L \\ &= (0.958\angle-73.3)(100\angle 90) \\ &= 0.958 \times 100\angle(-73.3 + 90) \\ &= 95.8\angle 16.7 \text{ V}\end{aligned}$$

$$\boxed{\mathbf{V}_L = 95.8\angle 16.7 \text{ V}}$$

Time domain:

$$v_L(t) = 95.8\cos(2000t + 16.7) \text{ V}$$

Note: $\mathbf{V}_L$ leads $\mathbf{I}$ by 90 (as expected for an inductor)

Example 3: Solution - Phasor Diagram and Verification

Step 8: Verify KVL

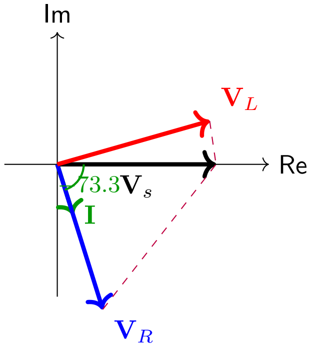
Check: $\mathbf{V}_R + \mathbf{V}_L = \mathbf{V}_s$

Convert to rectangular:

$$\begin{aligned}\mathbf{V}_R &= 28.7\angle -73.3\\ &= 28.7\cos(-73.3) + j28.7\sin(-73.3)\\ &= 8.26 - j27.49 \text{ V}\end{aligned}$$

$$\begin{aligned}\mathbf{V}_L &= 95.8\angle 16.7\\ &= 95.8\cos(16.7) + j95.8\sin(16.7)\\ &= 91.74 + j27.49 \text{ V}\end{aligned}$$

$$\begin{aligned}\mathbf{V}_R + \mathbf{V}_L &= (8.26 - j27.49) + (91.74 + j27.49)\\ &= 100 + j0 = 100\angle 0 \text{ V}\end{aligned}$$



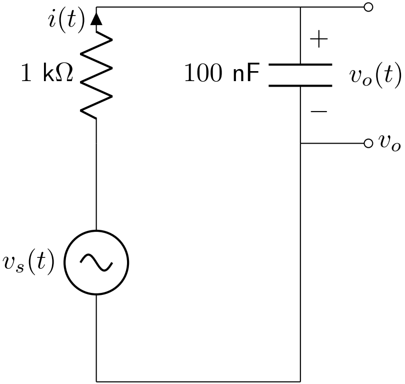
Key Observations

- $\mathbf{V}_R \parallel \mathbf{I}$ (resistor)
- $\mathbf{V}_L \perp \mathbf{I}$, leads by 90 (inductor)
- $\mathbf{V}_R + \mathbf{V}_L = \mathbf{V}_s$ (KVL)

Example 4: RC Voltage Divider

Problem: Analyze the following RC voltage divider circuit.

Circuit:



Given:

- $v_s(t) = 10\cos(10000t)$ V
- $R = 1$ kΩ
- $C = 100$ nF

Find:

1. The impedance of each element
2. The output voltage $\mathbf{V}_o$
3. The output voltage $v_o(t)$
4. The magnitude ratio $|\mathbf{V}_o|/|\mathbf{V}_s|$
5. The phase shift between input and output
6. The current $i(t)$

Example 4: Solution

Solution:
Source phasor: $\mathbf{V}_s = 10\angle 0$ V
$\omega = 10000$ rad/s

1. Impedances:

$$\mathbf{Z}_R = 1000\,\Omega$$

$$\mathbf{Z}_C = \frac{1}{j\omega C} = \frac{1}{j(10^4)(10^{-7})} = -j1000\,\Omega$$

2. Output Voltage (voltage divider)

$$\mathbf{V}_o = \mathbf{V}_s \frac{\mathbf{Z}_C}{\mathbf{Z}_R + \mathbf{Z}_C}$$
$$= 10\angle 0 \cdot \frac{-j1000}{1000 - j1000}$$

$$= 10 \cdot \frac{1000\angle -90}{1414.2\angle -45}$$
$$= 10 \cdot 0.707\angle -45$$
$$= \boxed{7.07\angle -45 \text{ V}}$$

Example 4: Solution (continued)

3. **Time domain**:

$$\boxed{v_o(t) = 7.07\cos(10000t - 45)\ \text{V}}$$

4. **Magnitude ratio**:

$$\boxed{\frac{|\mathbf{V}_o|}{|\mathbf{V}_s|} = \frac{7.07}{10} = 0.707}$$

5. **Phase shift**: $\boxed{-45}$ (output lags input)

6. **Current**:

$$\mathbf{I} = \frac{\mathbf{V}_s}{\mathbf{Z}_R + \mathbf{Z}_C} = \frac{10\angle 0}{1414.2\angle -45}$$
$$= 7.07\angle 45\ \text{mA}$$

$$\boxed{i(t) = 7.07\cos(10000t + 45)\ \text{mA}}$$

Example 5: AC Power Calculation

Problem: Calculate the real, reactive, and apparent power for a load with the following voltage and current.

Given:

$$v(t) = 120\sqrt{2}\cos(377t) \text{ V}$$
$$i(t) = 10\sqrt{2}\cos(377t - 36.87) \text{ A}$$

Note: The coefficients include $\sqrt{2}$ to indicate peak values

Find:

1. RMS voltage and current
2. Real power P
3. Reactive power Q
4. Apparent power S
5. Power factor (and type)
6. Load impedance $\mathbf{Z}$

Example 5: Solution - RMS Values and Powers

Step 1: RMS values

Convert from peak to RMS:

$$V_{rms} = \frac{V_m}{\sqrt{2}} = \frac{120\sqrt{2}}{\sqrt{2}} = \boxed{120 \text{ V}}$$

$$I_{rms} = \frac{I_m}{\sqrt{2}} = \frac{10\sqrt{2}}{\sqrt{2}} = \boxed{10 \text{ A}}$$

Step 2: Determine phase angle

From the time-domain expressions:

- Voltage phase: $\phi_v = 0$
- Current phase: $\phi_i = -36.87$

Phase difference:

$$\theta = \phi_v - \phi_i = 0 - (-36.87) = 36.87$$

Current lags voltage $\Rightarrow$ *inductive load*

Step 3: Real power

$$P = V_{rms} I_{rms} \cos\theta$$
$$= (120)(10)\cos(36.87) = \boxed{960 \text{ W}}$$

Step 4: Reactive power

$$Q = V_{rms} I_{rms} \sin\theta$$
$$= (120)(10)\sin(36.87) = \boxed{720 \text{ VAR}}$$

Step 5: Apparent power

$$S = V_{rms} I_{rms} = \boxed{1200 \text{ VA}}$$

Verify:

$$S = \sqrt{P^2 + Q^2} = \sqrt{960^2 + 720^2} = 1200$$

Example 5: Solution - Power Factor and Impedance

Step 6: Power factor

Method 1 - From angle:

$$\text{pf} = \cos\theta = \cos(36.87)$$
$$= \boxed{0.8 \text{ lagging}}$$

Method 2 - From powers:

$$\text{pf} = \frac{P}{S} = \frac{960}{1200}$$
$$= 0.8 \text{ lagging}$$

"Lagging" because current lags voltage (inductive)

Step 7: Load impedance

Convert to phasor form:

$$\mathbf{V} = 120\angle 0 \text{ V}$$
$$\mathbf{I} = 10\angle -36.87 \text{ A}$$

Calculate impedance:

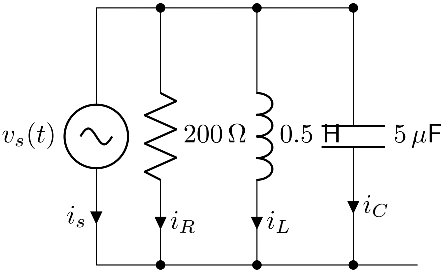
$$\mathbf{Z} = \frac{\mathbf{V}}{\mathbf{I}} = \frac{120\angle 0}{10\angle -36.87}$$
$$= 12\angle 36.87\,\Omega$$
$$\mathbf{Z} = 12(\cos 36.87 + j\sin 36.87)$$
$$= \boxed{9.6 + j7.2\,\Omega}$$

This represents $R = 9.6\,\Omega$ in series with $X_L = 7.2\,\Omega$

Example 6: Parallel RLC Circuit

Problem: Analyze the parallel RLC circuit shown below.

Circuit:



Given:

- $v_s(t) = 50\cos(1000t)$ V

Find:

1. Impedance of each branch
2. Total impedance
3. Source current $i_s(t)$
4. Current through each branch
5. Total real power
6. Total reactive power
7. Is the circuit inductive or capacitive?

Example 6: Solution

Solution:
$\mathbf{V}_s = 50\angle 0$ V, $\omega = 1000$ rad/s

1. **Branch impedances**:

$$\mathbf{Z}_R = 200\,\Omega$$
$$\mathbf{Z}_L = j\omega L = j(1000)(0.5) = j500\,\Omega$$
$$\mathbf{Z}_C = \frac{1}{j\omega C} = \frac{1}{j(1000)(5\times 10^{-6})} = -j200\,\Omega$$

2. **Total impedance**:

$$\frac{1}{\mathbf{Z}_{tot}} = \frac{1}{200} + \frac{1}{j500} + \frac{1}{-j200} = 0.005 + j0.003$$

$$\mathbf{Z}_{tot} = \frac{1}{0.005 + j0.003} = \frac{1}{0.00583\angle 30.96} = \boxed{171.5\angle -30.96\,\Omega} = 147.1 - j88.0\,\Omega$$

3. **Source current**:

$$\mathbf{I}_s = \frac{\mathbf{V}_s}{\mathbf{Z}_{tot}} = \frac{50\angle 0}{171.5\angle -30.96} = 0.292\angle 30.96 \text{ A}$$

$$\boxed{i_s(t) = 0.292\cos(1000t + 30.96) \text{ A}}$$

Example 6: Solution (continued)

4. Branch currents:

$$\mathbf{I}_R = \frac{\mathbf{V}_s}{\mathbf{Z}_R} = \frac{50}{200} = 0.25\angle 0 \text{ A}$$

$$\mathbf{I}_L = \frac{50\angle 0}{500\angle 90} = 0.1\angle -90 \text{ A}$$

$$\mathbf{I}_C = \frac{50\angle 0}{200\angle -90} = 0.25\angle 90 \text{ A}$$

5. Total real power:
Only resistor dissipates real power:

$$P = I_R^2 R = \boxed{12.5 \text{ W}}$$

or using $P = V_{rms} I_{s,rms} \cos\theta$:

$$P = 50 \times 0.292 \times \cos(30.96) = 12.5 \text{ W}$$

6. Total reactive power:

$$Q_L = I_L^2 X_L = (0.1)^2 \times 500 = 5 \text{ VAR}$$

$$Q_C = -I_C^2 X_C = -(0.25)^2 \times 200 = -12.5 \text{ VAR}$$

$$Q_{tot} = Q_L + Q_C = \boxed{-7.5 \text{ VAR}}$$

or:

$$Q = V_{rms} I_{s,rms} \sin\theta$$
$$= 50 \times 0.292 \times \sin(30.96) = -7.5 \text{ VAR}$$

7. Circuit behavior:
$Q_{tot} < 0$ and current leads voltage ($\theta > 0$)
Capacitor reactive power dominates