

# Assignment 02

## Frequency Domain Analysis of Circuits

ECEN 222, Spring 2026

University of Nebraska-Lincoln

### Instructions

This assignment focuses on analyzing AC circuits in the frequency domain using phasor techniques. You will convert time-domain signals to phasors, calculate impedances, analyze circuits, and convert results back to the time domain.

### Key Formulas

#### Phasor Conversion:

$$v(t) = V_m \cos(\omega t + \phi) \Leftrightarrow \mathbf{V} = V_m \angle \phi$$

#### Impedances:

- Resistor:  $\mathbf{Z}_R = R$
- Inductor:  $\mathbf{Z}_L = j\omega L = \omega L \angle 90^\circ$
- Capacitor:  $\mathbf{Z}_C = \frac{1}{j\omega C} = \frac{-j}{\omega C} = \frac{1}{\omega C} \angle -90^\circ$

#### Complex Number Operations:

- Rectangular to Polar:  $a + jb = \sqrt{a^2 + b^2} \angle \tan^{-1}(b/a)$
- Polar to Rectangular:  $r \angle \theta = r \cos \theta + jr \sin \theta$

#### AC Power:

- Real Power:  $P = V_{rms} I_{rms} \cos \theta$  (W)
- Reactive Power:  $Q = V_{rms} I_{rms} \sin \theta$  (VAR)
- Apparent Power:  $S = V_{rms} I_{rms}$  (VA)
- Complex Power:  $\mathbf{S} = P + jQ = \mathbf{V}_{rms} \mathbf{I}_{rms}^*$

## Problems

### 1. [12.5 points] Phasor Conversions and Operations

Given the following time-domain signals:

$$v_1(t) = 15 \cos(5000t + 60^\circ) \text{ V}$$

$$v_2(t) = 8 \cos(5000t - 30^\circ) \text{ V}$$

$$i(t) = 3 \cos(5000t + 15^\circ) \text{ A}$$

(a) Convert each signal to phasor form.

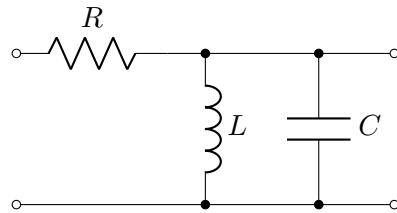
(b) Calculate  $\mathbf{V}_1 + \mathbf{V}_2$  in both rectangular and polar forms.

(c) Calculate  $\mathbf{V}_1 - \mathbf{V}_2$  in both rectangular and polar forms.

(d) Calculate the impedance  $\mathbf{Z} = \mathbf{V}_1/\mathbf{I}$  and express it in both rectangular and polar forms. What type of element(s) does this impedance represent?

**2. [12.5 points] Series-Parallel LC Combination**

Consider a circuit with a resistor  $R = 50\Omega$  in series with a parallel combination of  $L = 20\text{ mH}$  and  $C = 2\mu\text{F}$ .



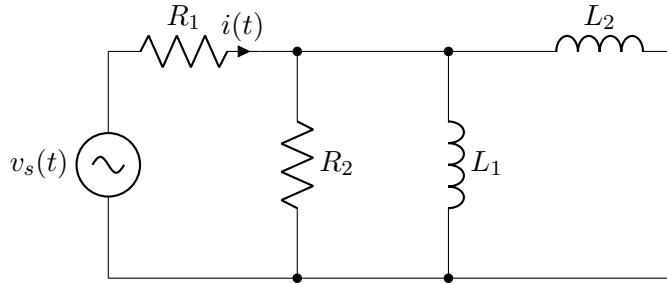
(a) Calculate the impedance of the parallel LC combination at  $f = 796\text{ Hz}$ . Express in both rectangular and polar forms.

(b) Calculate the total impedance  $\mathbf{Z}_{tot}$  at  $f = 796\text{ Hz}$ . Express your answer in both rectangular and polar forms.

(c) At what frequency does the parallel LC combination have infinite impedance? What is the total impedance at this frequency?

**3. [12.5 points] Series-Parallel RL Circuit Analysis**

For the circuit shown below, the voltage source is  $v_s(t) = 24 \cos(3000t)$  V,  $R_1 = 30 \Omega$ ,  $R_2 = 60 \Omega$ ,  $L_1 = 20 \text{ mH}$ , and  $L_2 = 40 \text{ mH}$ .

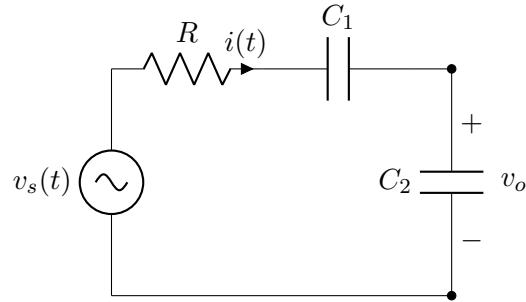


The circuit consists of  $R_1$  in series with a parallel combination of  $R_2$  and  $L_1$ , all in series with  $L_2$ .

- (a) Calculate the impedances of  $L_1$  and  $L_2$  at  $\omega = 3000$  rad/s.
  
  
  
  
  
- (b) Calculate the impedance of the parallel combination of  $R_2$  and  $L_1$ .
  
  
  
  
  
- (c) Calculate the total impedance  $Z_{tot}$  in both rectangular and polar forms.
  
  
  
  
  
- (d) Find the current  $i(t)$  in both phasor and time-domain forms.
  
  
  
  
  
- (e) Find the voltage across the parallel branch and determine the current through  $R_2$  and  $L_1$  individually.

**4. [12.5 points] Multiple Capacitor Circuit**

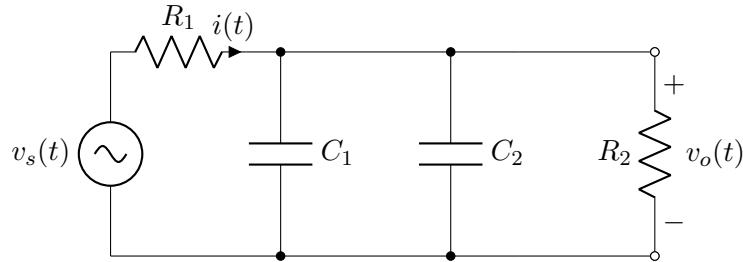
For the circuit shown below, the voltage source is  $v_s(t) = 50 \cos(4000t)$  V,  $R = 100 \Omega$ ,  $C_1 = 5 \mu\text{F}$ , and  $C_2 = 10 \mu\text{F}$ .



- (a) Calculate the impedance of  $C_1$  and  $C_2$  individually at  $\omega = 4000$  rad/s.
  
  
  
  
  
- (b) If the parallel branch includes only  $C_2$  (treating it as the load), calculate the total impedance  $\mathbf{Z}_{tot}$  in both rectangular and polar forms.
  
  
  
  
  
- (c) Find the current  $i(t)$  in both phasor and time-domain forms.
  
  
  
  
  
- (d) Find  $v_o(t)$ .

**5. [12.5 points] Series-Parallel RC Network with Load**

Consider the circuit shown below with  $v_s(t) = 20 \cos(8000t)$  V,  $R_1 = 1$  k $\Omega$ ,  $R_2 = 3$  k $\Omega$ ,  $C_1 = 100$  nF, and  $C_2 = 50$  nF.



(a) Calculate the impedances of  $C_1$  and  $C_2$  at  $\omega = 8000$  rad/s, then find the equivalent impedance of the parallel capacitor combination.

(b) Calculate the total circuit impedance  $\mathbf{Z}_{tot}$  in both rectangular and polar forms.

(c) Find the input current  $i(t)$  in both phasor and time-domain forms.

(d) Find the output voltage  $v_o(t)$ .

(e) Calculate the magnitude ratio  $|\mathbf{V}_o|/|\mathbf{V}_s|$  and the phase shift.

(f) Find the current through each capacitor ( $C_1$  and  $C_2$ ) individually.

**6. [12.5 points] Complex Power Analysis**

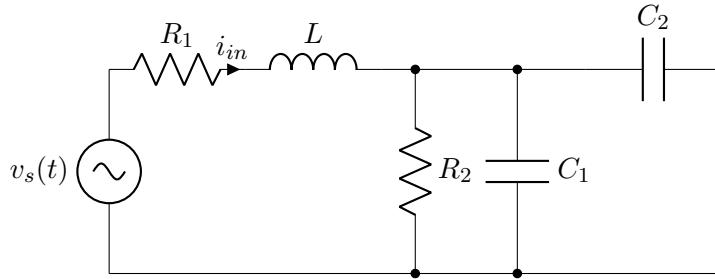
A load is connected to an AC source. The voltage across and current through the load are measured as:

$$v(t) = 120\sqrt{2} \cos(377t) \text{ V}$$
$$i(t) = 6\sqrt{2} \cos(377t - 53.13^\circ) \text{ A}$$

- (a) Convert the voltage and current to phasor form, then calculate the RMS phasors  $\mathbf{V}_{rms}$  and  $\mathbf{I}_{rms}$ . (Hint: The  $\sqrt{2}$  factor is included to indicate peak values.)
- (b) Find the real power  $P$ , reactive power  $Q$ , and apparent power  $\mathbf{S}$ . Include proper units.
- (c) Calculate the power factor and state whether the load is inductive or capacitive. Explain your reasoning.
- (d) Find the impedance of the load using  $\mathbf{Z} = \mathbf{V}_{rms}/\mathbf{I}_{rms}$  in both rectangular and polar forms. What circuit elements could this load represent?
- (e) If you wanted to improve the power factor to unity (1.0), what value of capacitor would you need to place in parallel with this load? (Note:  $\omega = 377 \text{ rad/s}$  corresponds to  $f = 60 \text{ Hz}$ )

**7. [12.5 points] Complex RLC Network with Series-Parallel Combination**

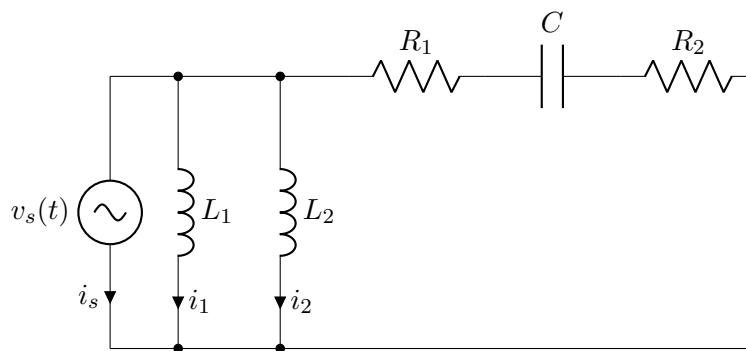
Consider the circuit with  $v_s(t) = 100 \cos(5000t)$  V,  $R_1 = 20 \Omega$ ,  $R_2 = 80 \Omega$ ,  $L = 8 \text{ mH}$ ,  $C_1 = 10 \mu\text{F}$ , and  $C_2 = 40 \mu\text{F}$ .



- (a) Calculate all component impedances at  $\omega = 5000$  rad/s.
- (b) Calculate the total impedance  $\mathbf{Z}_{tot}$  in both rectangular and polar forms.
- (c) Find the input current  $i_{in}(t)$  in both phasor and time-domain forms.
- (d) Calculate the voltage across the parallel branch ( $R_2 \parallel C_1$ ).
- (e) Find the current through  $R_2$  and through  $C_1$  individually.

**8. [12.5 points] Multi-Inductor Parallel-Series Circuit**

Analyze the circuit shown below with  $v_s(t) = 60 \cos(4000t)$  V,  $R_1 = 50 \Omega$ ,  $R_2 = 100 \Omega$ ,  $L_1 = 25 \text{ mH}$ ,  $L_2 = 50 \text{ mH}$ , and  $C = 5 \mu\text{F}$ .



- (a) Calculate the impedance of each inductor and the capacitor at  $\omega = 4000$  rad/s.
- (b) Calculate the equivalent impedance of the parallel inductor combination ( $L_1 \parallel L_2$ ).
- (c) Calculate the total impedance  $\mathbf{Z}_{tot}$  in both rectangular and polar forms.
- (d) Find the total source current  $i_s(t)$  in both phasor and time-domain forms.
- (e) Find the voltage across the parallel inductor combination, then determine the individual currents  $i_1(t)$  and  $i_2(t)$  through each inductor.

(f) Calculate the total real power, reactive power, and complex power delivered by the source. Express complex power in rectangular form. (Hint: You already know the source voltage and current.)