# Maxwell's Equations to Passive Circuit Elements

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# Maxwell's Equations: The Foundation

**There are four equations governing all electromagnetics**:

| Name | Integral Form | Differential Form |
|---|---|---|
| Gauss's Law - Electric | $\oint_S \mathbf{E} \cdot d\mathbf{A} = \frac{Q_{\text{enc}}}{\epsilon_0}$ | $\nabla \cdot \mathbf{E} = \frac{\rho}{\epsilon_0}$ |
| No magnetic monopoles | $\oint_S \mathbf{B} \cdot d\mathbf{A} = 0$ | $\nabla \cdot \mathbf{B} = 0$ |
| Faraday's Law | $\oint_C \mathbf{E} \cdot d\mathbf{l} = -\frac{d\Phi_B}{dt}$ | $\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$ |
| Ampère-Maxwell Law | $\oint_C \mathbf{B} \cdot d\mathbf{l} = \mu_0 I_{\text{enc}} + \mu_0 \epsilon_0 \frac{d\Phi_E}{dt}$ | $\nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \mu_0 \epsilon_0 \frac{\partial \mathbf{E}}{\partial t}$ |

# What Do Maxwell's Equations Tell Us?



**Gauss's Law**:

- Electric charges create electric fields
- Electric field lines start/end on charges
- Basis for **capacitance**

**No Magnetic Monopoles**:

- Magnetic field lines form closed loops
- No isolated magnetic charges
- Magnetic fields from currents only

**Faraday's Law**:

- Changing magnetic flux creates voltage
- Basis for **inductance**
- Transformers, generators, motors

**Ampère-Maxwell Law**:

- Currents create magnetic fields
- Changing electric fields create magnetic fields
- Basis for electromagnetic waves

**For Circuit Theory**

We focus on Gauss's Law (capacitors) and Faraday's Law (inductors)

# The Resistor: Ohm's Law

**From Electromagnetics**:

Current density related to electric field:

$$\boldsymbol{J} = \sigma \boldsymbol{E}$$

where $\sigma$ is conductivity.

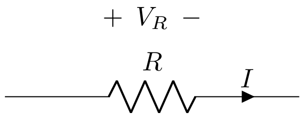
Integrating over a conductor:

$$I = \int_A \boldsymbol{J} \cdot d\boldsymbol{A}, \quad V = \int_l \boldsymbol{E} \cdot d\boldsymbol{l}$$

**Leads to Ohm's Law**

$V = IR$, where $R = \dfrac{l}{\sigma A}$

**Circuit Symbol**:



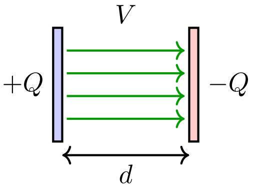

**Power Dissipated:**

$$P = VI = I^2 R = \frac{V^2}{R}$$

# The Capacitor: Stored Electric Energy

**From Gauss's Law**:

Parallel plate capacitor:

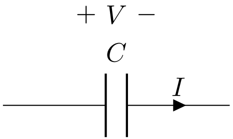



Electric field: $E = \frac{Q}{\epsilon_0 A}$

Voltage: $V = Ed = \frac{Qd}{\epsilon_0 A}$

$$Q = CV \quad \text{where} \quad C = \frac{\epsilon_0 A}{d}$$

**Circuit Symbol**:




### I-V Relationship

$I = C\frac{dV}{dt}$

### Stored Energy:

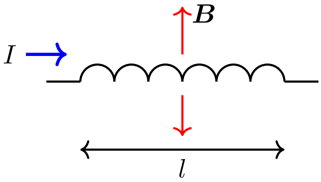
- $W = \frac{1}{2}CV^2 = \frac{1}{2}QV$
- Stores energy in electric field

# The Inductor: Stored Magnetic Energy



**From Faraday's Law**:

Solenoid inductor:

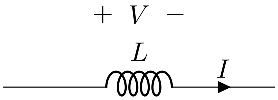



Magnetic flux: $\Phi_B = NBA$

Faraday's Law: $V = -\dfrac{d\Phi_B}{dt}$

For $B = \dfrac{\mu_0 NI}{l}$:

**Circuit Symbol**:



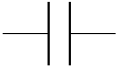

**I-V Relationship:**

$V = L\frac{dI}{dt}$ where $L = \dfrac{\mu_0 N^2 A}{l}$

**Stored Energy:**

- $W = \frac{1}{2}LI^2$
- Stores energy in magnetic field

# Passives Summary

| Element | Symbol | I-V Relation | Energy/Power |
|---|---|---|---|
| Resistor | $R$  | $V = IR$ | $P = I^2R$ (dissipated) |
| Capacitor | $C$  | $I = C\frac{dV}{dt}$ | $W = \frac{1}{2}CV^2$ (stored) |
| Inductor | $L$  | $V = L\frac{dI}{dt}$ | $W = \frac{1}{2}LI^2$ (stored) |

**Resistor (R)**:

- From $\boldsymbol{J} = \sigma\boldsymbol{E}$
- Dissipates energy
- Algebraic relation

**Capacitor (C)**:

- From Gauss's Law
- Stores electric energy
- Time derivative of $V$

**Inductor (L)**:

- From Faraday's Law
- Stores magnetic energy
- Time derivative of $I$

# Summary

**Maxwell's Equations**:

- Four fundamental laws
- Gauss's Law $\rightarrow$ capacitors
- Faraday's Law $\rightarrow$ inductors
- Unified theory of E&M

**Circuit Elements IV Relations**:

- **R**: Dissipates energy ($V = IR$)
- **C**: Stores electric energy ($I = C\frac{dV}{dt}$)
- **L**: Stores magnetic energy ($V = L\frac{dI}{dt}$)

**Frequency Behavior**:

- Capacitors: block DC, pass AC
- Inductors: pass DC, block AC
- Resistors: frequency-independent

**Energy Storage**:

- Capacitor: $W_C = \frac{1}{2}CV^2$
- Inductor: $W_L = \frac{1}{2}LI^2$
- Only R dissipates energy

**Big Picture**

Passive circuit theory is the application of Maxwell's equations!