Assignment 02
Frequency Domain Analysis of Circuits

ECEN 222, Spring 2026

University of Nebraska-Lincoln

Instructions

This assignment focuses on analyzing AC circuits in the frequency domain using phasor techniques. You will convert time-domain signals to phasors, calculate impedances, analyze circuits, and convert results back to the time domain.

Key Formulas

Phasor Conversion:

$$v(t) = V_m \cos(\omega t + \phi) \quad \Leftrightarrow \quad \mathbf{V} = V_m \angle \phi$$

Impedances:

- Resistor: $\mathbf{Z}_R = R$
- Inductor: $\mathbf{Z}_L = j\omega L = \omega L \angle 90^\circ$
- Capacitor: $\mathbf{Z}_C = \frac{1}{j\omega C} = \frac{-j}{\omega C} = \frac{1}{\omega C} \angle -90^\circ$

Complex Number Operations:

- Rectangular to Polar: $a + jb = \sqrt{a^2 + b^2} \angle \tan^{-1}(b/a)$
- Polar to Rectangular: $r\angle\theta = r\cos\theta + jr\sin\theta$

AC Power:

- Real Power: $P = V_{rms} I_{rms} \cos\theta$ (W)
- Reactive Power: $Q = V_{rms} I_{rms} \sin\theta$ (VAR)
- Apparent Power: $S = V_{rms} I_{rms}$ (VA)
- Complex Power: $\mathbf{S} = P + jQ = \mathbf{V}_{rms} \mathbf{I}^*_{rms}$

Problems

1. **[12.5 points]**

   Given the following time-domain signals:

$$
\begin{aligned}
v_1(t) &= 15\cos(5000t + 60^\circ)\ \text{V} \\
v_2(t) &= 8\cos(5000t - 30^\circ)\ \text{V} \\
i(t) &= 3\cos(5000t + 15^\circ)\ \text{A}
\end{aligned}
$$

   (a) Convert each signal to phasor form.

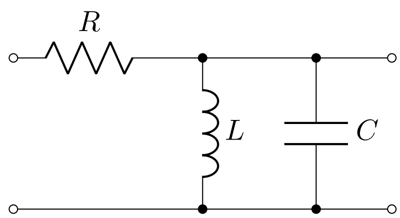
   (b) Calculate $\mathbf{V}_1 + \mathbf{V}_2$ in both rectangular and polar forms.

   (c) Calculate $\mathbf{V}_1 - \mathbf{V}_2$ in both rectangular and polar forms.

   (d) Calculate the impedance $\mathbf{Z} = \mathbf{V}_1/\mathbf{I}$ and express it in both rectangular and polar forms. What type of element(s) does this impedance represent?

2. **[12.5 points]**

Consider a circuit with a resistor $R = 50\,\Omega$ in series with a parallel combination of $L = 20$ mH and $C = 2\,\mu$F.



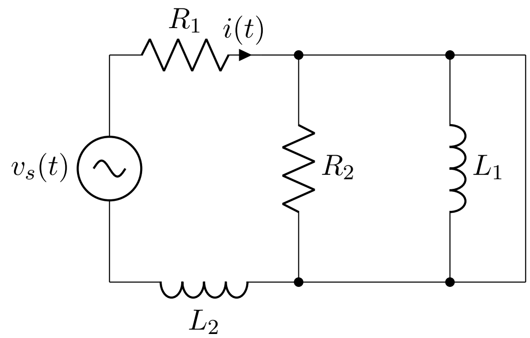
(a) Calculate the impedance of the parallel LC combination at $f = 1000$ Hz. Express in both rectangular and polar forms.

(b) Calculate the total impedance $\mathbf{Z}_{tot}$ at $f = 1000$ Hz. Express your answer in both rectangular and polar forms.

(c) At what frequency does the parallel LC combination have infinite impedance? What is the total impedance at this frequency?

3. **[12.5 points]**

For the circuit shown below, the voltage source is $v_s(t) = 24\cos(3000t)$ V, $R_1 = 30\,\Omega$, $R_2 = 60\,\Omega$, $L_1 = 20$ mH, and $L_2 = 40$ mH.



(a) Calculate the impedances of L_1 and L_2 at $\omega = 3000$ rad/s.

(b) Calculate the impedance of the parallel combination of R_2 and L_1.

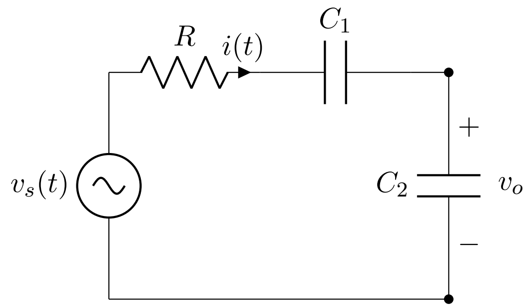
(c) Calculate the total impedance $\mathbf{Z}_{tot}$ in both rectangular and polar forms. (Hint: The circuit topology is R_1 in series with $(R_2 \parallel L_1)$, all in series with L_2.)

(d) Find the current $i(t)$ in both phasor and time-domain forms.

(e) Find the voltage across the parallel branch and determine the current through R_2 and L_1 individually.

4. **[12.5 points]**

For the circuit shown below, the voltage source is $v_s(t) = 50\cos(4000t)$ V, $R = 100\,\Omega$, $C_1 = 5\,\mu$F, and $C_2 = 10\,\mu$F.



(a) Calculate the impedance of C_1 and C_2 individually at $\omega = 4000$ rad/s.

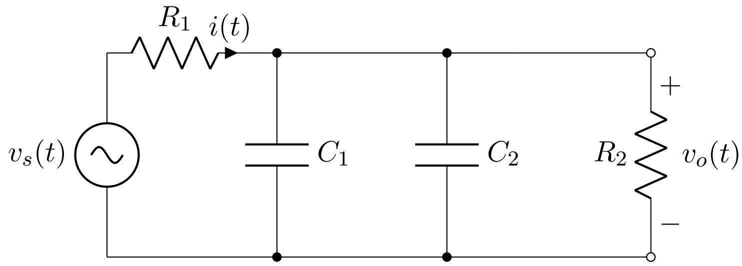
(b) If the parallel branch includes only C_2 (treating it as the load), calculate the total impedance $\mathbf{Z}_{tot}$ in both rectangular and polar forms.

(c) Find the current $i(t)$ in both phasor and time-domain forms.

(d) Find $v_o(t)$.

5. **[12.5 points]**

Consider the circuit shown below with $v_s(t) = 20\cos(8000t)$ V, $R_1 = 1$ kΩ, $R_2 = 3$ kΩ, $C_1 = 100$ nF, and $C_2 = 50$ nF.



(a) Calculate the impedances of C_1 and C_2 at $\omega = 8000$ rad/s, then find the equivalent impedance of the parallel capacitor combination.

(b) Calculate the total circuit impedance $\mathbf{Z}_{tot}$ in both rectangular and polar forms.

(c) Find the input current $i(t)$ in both phasor and time-domain forms.

(d) Find the output voltage $v_o(t)$.

(e) Calculate the magnitude ratio $|\mathbf{V}_o|/|\mathbf{V}_s|$ and the phase shift.

(f) Find the current through each capacitor (C_1 and C_2) individually.

6. **[12.5 points]**

A load is connected to an AC source. The voltage across and current through the load are measured as:

$$v(t) = 120\sqrt{2}\cos(377t) \text{ V}$$
$$i(t) = 6\sqrt{2}\cos(377t - 53.13°) \text{ A}$$

(a) Convert the voltage and current to phasor form, then calculate the RMS phasors $\mathbf{V}_{rms}$ and $\mathbf{I}_{rms}$. (Hint: The $\sqrt{2}$ factor is included to indicate peak values.)

(b) Find the real power P, reactive power Q, and apparent power $\mathbf{S}$. Include proper units.

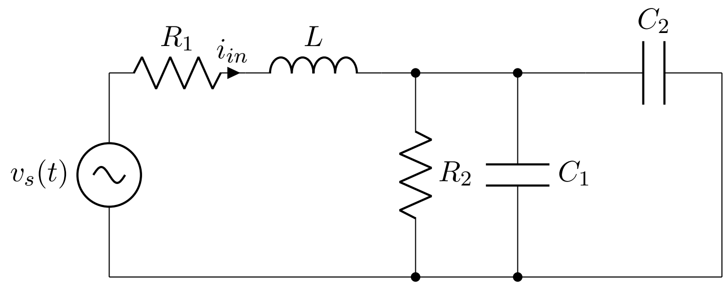
(c) Calculate the power factor and state whether the load is inductive or capacitive. Explain your reasoning.

(d) Find the impedance of the load using $\mathbf{Z} = \mathbf{V}_{rms}/\mathbf{I}_{rms}$ in both rectangular and polar forms. What circuit elements could this load represent?

(e) If you wanted to improve the power factor to unity (1.0), what value of capacitor would you need to place in parallel with this load? (Note: $\omega = 377$ rad/s corresponds to $f = 60$ Hz)

7. **[12.5 points]**

Consider the circuit with $v_s(t) = 100\cos(5000t)$ V, $R_1 = 20\,\Omega$, $R_2 = 80\,\Omega$, $L = 8$ mH, $C_1 = 10\,\mu$F, and $C_2 = 40\,\mu$F.



(a) Calculate all component impedances at $\omega = 5000$ rad/s.

(b) Calculate the total impedance $\mathbf{Z}_{tot}$ in both rectangular and polar forms.

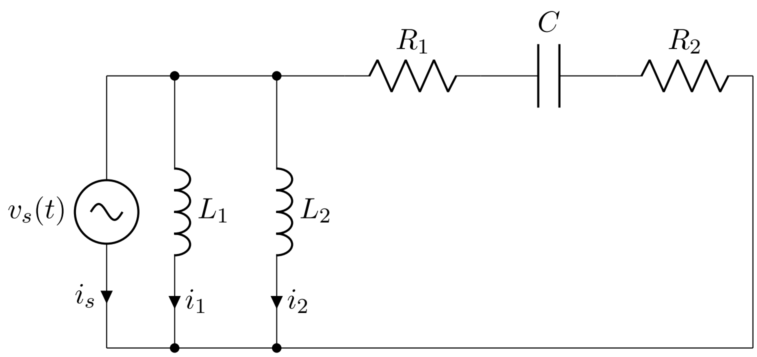
(c) Find the input current $i_{in}(t)$ in both phasor and time-domain forms.

(d) Calculate the voltage across the parallel branch ($R_2 \parallel C_1$).

(e) Find the current through R_2 and through C_1 individually.

8. **[12.5 points]**

Analyze the circuit shown below with $v_s(t) = 60\cos(4000t)$ V, $R_1 = 50\,\Omega$, $R_2 = 100\,\Omega$, $L_1 = 25$ mH, $L_2 = 50$ mH, and $C = 5\,\mu$F.



(a) Calculate the impedance of each inductor and the capacitor at $\omega = 4000$ rad/s.

(b) Calculate the equivalent impedance of the parallel inductor combination ($L_1 \parallel L_2$).

(c) Calculate the total impedance $\mathbf{Z}_{tot}$ in both rectangular and polar forms.

(d) Find the total source current $i_s(t)$ in both phasor and time-domain forms.

(e) Find the voltage across the parallel inductor combination, then determine the individual currents $i_1(t)$ and $i_2(t)$ through each inductor.

(f) Calculate the total real power, reactive power, and complex power delivered by the source. Express complex power in rectangular form. (Hint:You already know the source voltage and current.)