Complex Numbers Review

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Spring 2026

Why Complex Numbers in Circuits?

The Challenge:

- AC circuits involve sinusoids
- Trigonometry gets messy
- Need a better mathematical tool

The Solution:

- Complex numbers simplify AC analysis
- Turn trig into algebra
- Enable frequency domain methods

Key Idea:

- Represent sinusoids as rotating phasors
- Use complex exponentials
- Mathematics becomes elegant

Goal

Build intuition with complex numbers to prepare for frequency domain circuit analysis

What is a Complex Number?

Definition: A complex number z has a real part and an imaginary part

Rectangular Form:

$$z = a + jb$$

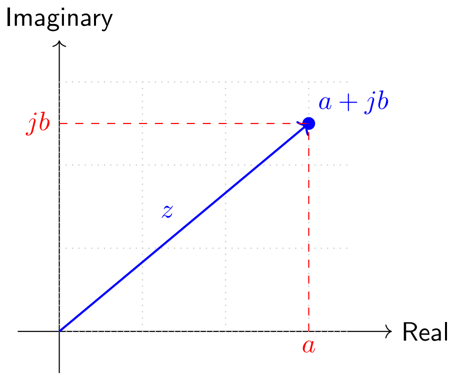
where:

- $a =$ real part, Re$\{z\}$
- $b =$ imaginary part, Im$\{z\}$
- $j = \sqrt{-1}$

Examples:

- 7 (purely real, $b = 0$)
- $j6$ (purely imaginary, $a = 0$)
- $3 + j4$

Complex Plane Visualization:



Polar Form of Complex Numbers

Complex numbers can also be expressed in **polar form**:

Polar Representation:

$$z = r\angle\theta = re^{j\theta}$$

- r = magnitude (length of vector)
- θ = angle (phase)

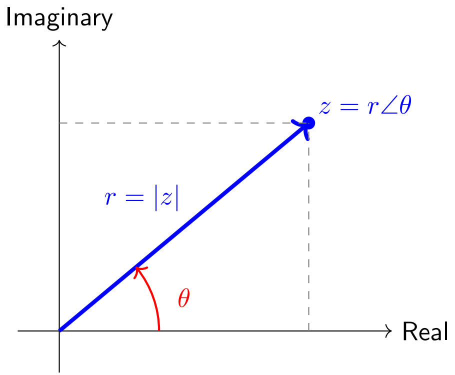
Rectangular $\rightarrow$ Polar:

$$r = \sqrt{a^2 + b^2} = |z|$$
$$\theta = \arctan(b/a)$$

Polar $\rightarrow$ Rectangular:

$$a = r\cos(\theta)$$
$$b = r\sin(\theta)$$

Polar Visualization:



Euler's Formula

Euler's Formula connects complex exponentials to trigonometry:

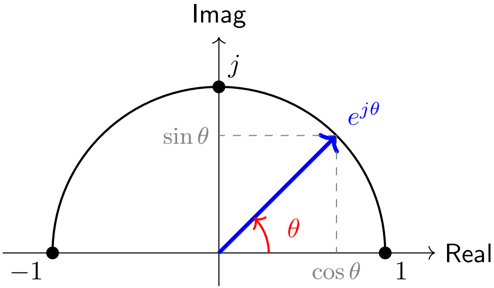
$$\boxed{e^{j\theta} = \cos(\theta) + j\sin(\theta)}$$

Key Insights:

- $e^{j\theta}$ represents rotation
- Real part: $\cos(\theta)$
- Imaginary part: $\sin(\theta)$
- Magnitude is always 1

$$e^{j0} = 1$$
$$e^{j\pi/2} = j$$
$$e^{j\pi} = -1$$
$$e^{j3\pi/2} = -j$$

The Unit Circle:



General Polar Form with Euler's Formula

Any complex number can be written using Euler's formula:

$$\boxed{z = re^{j\theta} = r[\cos(\theta) + j\sin(\theta)] = r\cos(\theta) + jr\sin(\theta)}$$

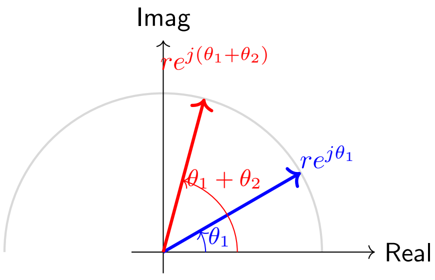
Why This Matters:

- Multiplication becomes addition of angles
- Division becomes subtraction of angles
- Powers become angle multiplication
- Rotation is just adding to θ

Example: Multiply $2e^{j30} \times 3e^{j45}$

$$= (2 \times 3) \cdot e^{j(30+45)}$$
$$= 6e^{j75}$$

Visualizing Rotation:



Multiplying by $e^{j\theta_2}$ rotates by angle θ_2

Addition and Subtraction

Rule: Add/subtract complex numbers in *rectangular form*

Addition:

$$z_1 + z_2 = (a_1 + jb_1) + (a_2 + jb_2)$$

$$= (a_1 + a_2) + j(b_1 + b_2)$$

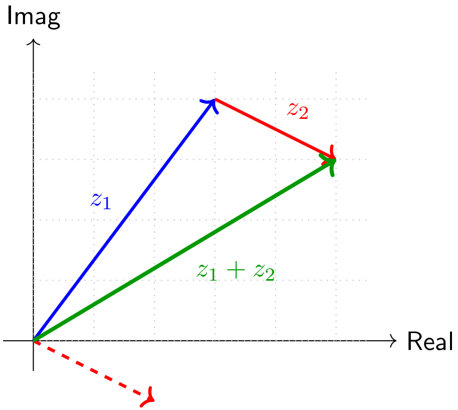
Example:

$$(3 + j4) + (2 - j1)$$
$$= (3 + 2) + j(4 - 1)$$
$$= 5 + j3$$

Geometric Interpretation:

- Add like vectors
- Tip-to-tail method

Vector Addition Visualization:



Multiplication and Division

Rule: Multiply/divide complex numbers in *polar form*

Multiplication:

$$z_1 \cdot z_2 = r_1 e^{j\theta_1} \cdot r_2 e^{j\theta_2}$$

$$= (r_1 r_2) e^{j(\theta_1 + \theta_2)}$$

- Multiply magnitudes: $r_1 \times r_2$
- Add angles: $\theta_1 + \theta_2$

Example:

$$(2\angle 30) \times (3\angle 45)$$
$$= 6\angle 75$$

Division:

$$\frac{z_1}{z_2} = \frac{r_1 e^{j\theta_1}}{r_2 e^{j\theta_2}}$$

$$= \frac{r_1}{r_2} e^{j(\theta_1 - \theta_2)}$$

- Divide magnitudes: r_1/r_2
- Subtract angles: $\theta_1 - \theta_2$

Example:

$$\frac{10\angle 60}{2\angle 20}$$
$$= 5\angle 40$$

Complex Conjugate

The **complex conjugate** z^* flips the sign of the imaginary part:

Definitions:

If $z = a + jb$, then:

$$z^* = a - jb$$

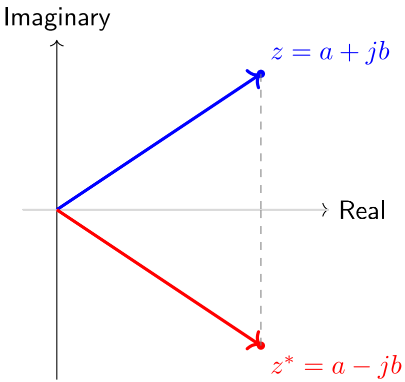
If $z = re^{j\theta}$, then:

$$z^* = re^{-j\theta}$$

Properties:

- $z \cdot z^* = |z|^2 = r^2$
- $(z^*)^* = z$
- $(z_1 + z_2)^* = z_1^* + z_2^*$
- $(z_1 \cdot z_2)^* = z_1^* \cdot z_2^*$

Geometric Visualization:



Sinusoids and Complex Exponentials

Key Connection: Sinusoids can be represented as complex exponentials

From Euler's Formula:

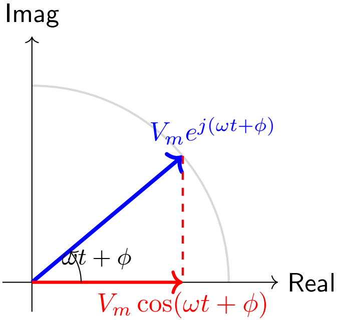
$$e^{j\omega t} = \cos(\omega t) + j\sin(\omega t)$$
$$e^{-j\omega t} = \cos(\omega t) - j\sin(\omega t)$$
$$\cos(\omega t) = \frac{e^{j\omega t} + e^{-j\omega t}}{2}$$
$$\sin(\omega t) = \frac{e^{j\omega t} - e^{-j\omega t}}{2j}$$

General Sinusoid:

$$v(t) = V_m \cos(\omega t + \phi)$$

$$v(t) = \text{Re}\{V_m e^{j(\omega t+\phi)}\}$$

Rotating Phasor Interpretation:



The real part of the rotating phasor gives us the sinusoid in the time domain

Phasor Representation

Phasor: A complex number representing amplitude and phase of a sinusoid

Time Domain → Phasor Domain:

$$v(t) = V_m \cos(\omega t + \phi)$$

$$\boxed{\mathbf{V} = V_m e^{j\phi} = V_m \angle \phi}$$

- Drop the $e^{j\omega t}$ time dependence
- Keep magnitude V_m and phase ϕ

Examples:

$$\begin{aligned} 10\cos(\omega t + 30) &\to 10\angle 30 \\ 5\sin(\omega t) &\to 5\angle -90 \\ -3\cos(\omega t) &\to 3\angle 180 \end{aligned}$$

Why Phasors?

- ☺ Easy to add sinusoids
- ☺ Simplifies circuit analysis
- ☺ Natural for AC steady-state

Adding Sinusoids:

- ☹ Time domain (difficult):

$$v_1(t) + v_2(t) = 10\cos(\omega t + 30) + 5\cos(\omega t - 45)$$

- ☺ Phasor domain (easy):

$$\begin{aligned} \mathbf{V} &= 10\angle 30 + 5\angle -45 \\ &= (8.66 + j5) + (3.54 - j3.54) = 12.2 + j1.46 \\ &= 12.3\angle 6.8 \end{aligned}$$

Summary: Complex Numbers for Circuit Analysis

Concepts:

- Complex numbers: $z = a + jb$
- Polar form: $z = re^{j\theta} = r\angle\theta$
- Euler's formula: $e^{j\theta} = \cos\theta + j\sin\theta$
- Unit circle representation

Operations:

- Add/subtract in rectangular form
- Multiply/divide in polar form
- Complex conjugate: $z^* = a - jb$

For Circuit Analysis:

- Sinusoids $\leftrightarrow$ Rotating phasors
- Time domain $\leftrightarrow$ Frequency domain
- Phasors capture magnitude & phase
- Simplify AC circuit analysis

Next Lecture:

- Apply to impedance (Z)
- Analyze AC circuits with phasors
- Frequency domain methods

Remember

Complex numbers are mathematical tools to make computation easier when dealing with sinusoidal signals in circuits. You will get the same result if you compute in time domain.

Practice Problems

Try these to check your understanding:

1. Convert $4 + j3$ to polar form.
2. Convert $10\angle 135$ to rectangular form.
3. Compute: $(2 + j3) + (1 - j5)$
4. Compute: $(5\angle 60) \times (2\angle 30)$
5. Find the complex conjugate of $3 - j4$.
6. Express $v(t) = 15\cos(\omega t + 45)$ as a phasor.
7. Add the phasors: $8\angle 0 + 6\angle 90$