

Frequency Domain Analysis of Circuits

Maxx Seminario

University of Nebraska-Lincoln

Spring 2026

Why Frequency Domain Analysis?

ECEN 222

Maxx Seminario

Introduction to
Frequency
Domain

Phasor
Representation

Impedance

Phasor Circuit
Analysis

AC Power
Analysis

Summary

Phasor Basics
Problems

Impedance
Calculation
Problems

Circuit Analysis

AC Power
Problems

Limitations of Time Domain:

- Differential equations for AC circuits
- Complex trig math
- Difficult for sinusoidal steady-state

Frequency Domain Advantages:

- Converts differential equations to algebra
- Easy handling of sinusoidal signals
- Simplifies AC circuit analysis

Applications:

- AC power systems (60 Hz)
- Audio systems (20 Hz - 20 kHz)
- Radio frequency circuits (MHz - GHz)
- Signal processing and filtering

Domain Transformation Tool

Phasor transform converts time-domain sinusoids to frequency-domain complex numbers

Goal for this lecture

Review frequency domain (phasor) analysis for AC circuits

Sinusoidal Signals: The Foundation

ECEN 222

Maxx Seminario

Introduction to
Frequency
Domain

Phasor
Representation

Impedance

Phasor Circuit
Analysis

AC Power
Analysis

Summary

Phasor Basics
Problems

Impedance
Calculation
Problems

Circuit Analysis

AC Power
Problems

General Sinusoidal Signal:

$$v(t) = V_m \cos(\omega t + \phi)$$

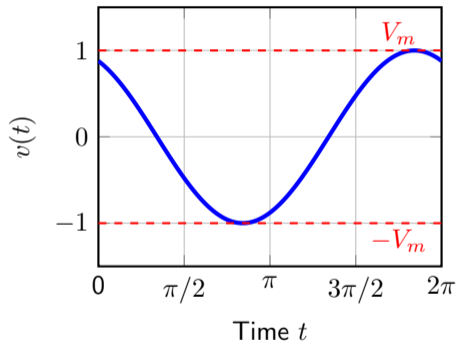
where:

- V_m = amplitude (peak value)
- ω = angular frequency (rad/s)
- ϕ = phase angle (radians or degrees)

Related Parameters:

- Frequency: $f = \omega/(2\pi)$ (Hz)
- Period: $T = 1/f = 2\pi/\omega$ (s)
- RMS value: $V_{rms} = V_m/\sqrt{2}$

Sinusoidal Waveform:



Phasor Concept: From Time to Frequency Domain

ECEN 222

Maxx Seminario

Introduction to
Frequency
Domain

Phasor
Representation

Impedance

Phasor Circuit
Analysis

AC Power
Analysis

Summary

Phasor Basics
Problems

Impedance
Calculation
Problems

Circuit Analysis

AC Power
Problems

Euler's Identity:

$$e^{j\theta} = \cos \theta + j \sin \theta$$

Sinusoid as Complex Exponential:

$$v(t) = V_m \cos(\omega t + \phi)$$

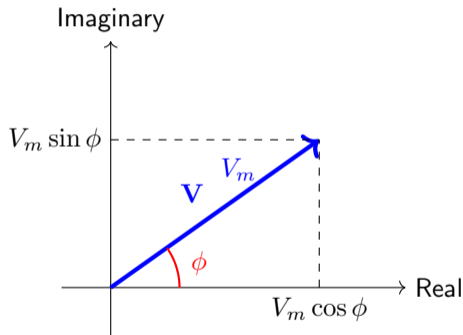
$$v(t) = \operatorname{Re}\{V_m e^{j(\omega t + \phi)}\}$$

$$v(t) = \operatorname{Re}\{V_m e^{j\phi} e^{j\omega t}\}$$

Phasor Definition

$$\mathbf{V} = V_m e^{j\phi} = V_m \angle \phi$$

Phasor Diagram:



Rectangular Form:

$$\mathbf{V} = V_m \cos \phi + j V_m \sin \phi$$

Phasor Transform: Summary

ECEN 222

Maxx Seminario

Introduction to
Frequency
Domain

Phasor
Representation

Impedance

Phasor Circuit
Analysis

AC Power
Analysis

Summary

Phasor Basics
Problems

Impedance
Calculation
Problems

Circuit Analysis

AC Power
Problems

Time Domain	Phasor Domain	Operation
$V_m \cos(\omega t + \phi)$	$\mathbf{V} = V_m \angle \phi$	Domain transformation
$\frac{d}{dt}$	$j\omega$	Differentiation \rightarrow multiplication
$\int dt$	$\frac{1}{j\omega}$	Integration \rightarrow division
Addition	Addition	Same (LTI Systems)

Key Advantage

- 😊 **Differentiation** in time domain \rightarrow **Multiplication** by $j\omega$ in phasor domain.
- 😞 Phasor analysis only works for **linear circuits** with **sinusoidal sources** at the **same frequency** in **steady-state**

Electrical Impedance

ECEN 222

Maxx Seminario

Introduction to
Frequency
Domain

Phasor
Representation

Impedance

Phasor Circuit
Analysis

AC Power
Analysis

Summary

Phasor Basics
Problems

Impedance
Calculation
Problems

Circuit Analysis

AC Power
Problems

Definition:

Impedance is the ratio of phasor voltage to phasor current:

$$\mathbf{Z} = \frac{\mathbf{V}}{\mathbf{I}}$$

Polar Form:

$$\mathbf{Z} = |\mathbf{Z}| \angle \theta$$

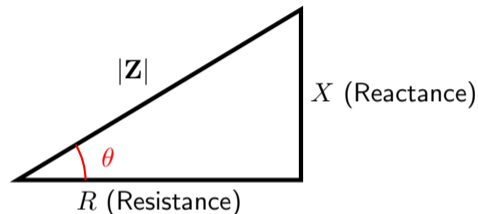
Rectangular Form:

$$\mathbf{Z} = R + jX$$

where:

- R = resistance (real part)
- X = reactance (imaginary part)

Impedance in Complex Plane:



Relationships:

$$|\mathbf{Z}| = \sqrt{R^2 + X^2}$$

$$\theta = \tan^{-1} \left(\frac{X}{R} \right)$$

Impedance of R, L, and C

ECEN 222

Maxx Seminario

Introduction to
Frequency
Domain

Phasor
Representation

Impedance

Phasor Circuit
Analysis

AC Power
Analysis

Summary

Phasor Basics
Problems

Impedance
Calculation
Problems

Circuit Analysis

AC Power
Problems

Element	Time Domain	Impedance	Phase
Resistor	$v = iR$	$\mathbf{Z}_R = R$	0
Inductor	$v = L \frac{di}{dt}$	$\mathbf{Z}_L = j\omega L$	+90
Capacitor	$i = C \frac{dv}{dt}$	$\mathbf{Z}_C = \frac{1}{j\omega C} = \frac{-j}{\omega C}$	-90

Resistor:

- Real impedance
- V and I in phase
- Frequency independent

Inductor:

- Imaginary impedance
- V leads I by 90°
- $|\mathbf{Z}_L| = \omega L$ increases with ω

Capacitor:

- Imaginary impedance
- I leads V by 90°
- $|\mathbf{Z}_C| = 1/(\omega C)$ decreases with ω

Frequency Behavior of Impedance

ECEN 222

Maxx Seminario

Introduction to
Frequency
Domain

Phasor
Representation

Impedance

Phasor Circuit
Analysis

AC Power
Analysis

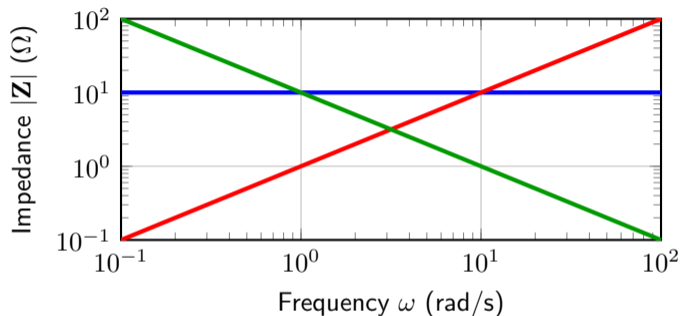
Summary

Phasor Basics
Problems

Impedance
Calculation
Problems

Circuit Analysis

AC Power
Problems



- Resistor ($R = 10 \Omega$)
- Inductor ($L = 1 \text{ H}$)
- Capacitor ($C = 0.1 \text{ F}$)

Frequency Behavior

- **Resistor:** Constant impedance (frequency independent)
- **Inductor:** High impedance at high frequencies (blocks AC, passes DC)
- **Capacitor:** Low impedance at high frequencies (blocks DC, passes AC)

Phasor Analysis: Circuit Laws

ECEN 222

Maxx Seminario

All DC circuit analysis techniques apply to phasors

Kirchhoff's Voltage Law (KVL):

$$\sum \mathbf{V}_k = 0$$

Kirchhoff's Current Law (KCL):

$$\sum \mathbf{I}_k = 0$$

Ohm's Law:

$$\mathbf{V} = \mathbf{I}\mathbf{Z}$$

Series Impedances:

$$\mathbf{Z}_{eq} = \mathbf{Z}_1 + \mathbf{Z}_2 + \cdots + \mathbf{Z}_n$$

Parallel Impedances:

$$\mathbf{Z}_{eq}^{-1} = \mathbf{Z}_1^{-1} + \mathbf{Z}_2^{-1} + \cdots + \mathbf{Z}_n^{-1}$$

Voltage Divider:

$$\mathbf{V}_k = \mathbf{V}_s \mathbf{Z}_k (\mathbf{Z}_1 + \mathbf{Z}_2)^{-1}$$

Key Point

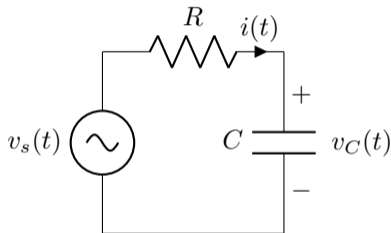
Replace resistances with impedances, and voltages/currents with phasors. Then use the standard DC techniques

Example: Series RC Circuit

ECEN 222

Maxx Seminario

Circuit:



Given:

- $v_s(t) = V_m \cos(\omega t)$
- $R = 100 \Omega$
- $C = 10 \mu\text{F}$
- $\omega = 1000 \text{ rad/s}$

Phasor Analysis:

Source phasor: $\mathbf{V}_s = V_m \angle 0$

Impedances:

$$\mathbf{Z}_R = 100 \Omega$$

$$\mathbf{Z}_C = \frac{-j}{\omega C} = \frac{-j}{0.01} = -j100 \Omega$$

Total impedance:

$$\begin{aligned}\mathbf{Z}_{eq} &= R - jX_C = 100 - j100 \\ &= 141.4 \angle -45^\circ \Omega\end{aligned}$$

Current:

$$\mathbf{I} = \frac{\mathbf{V}_s}{\mathbf{Z}_{eq}} = \frac{V_m \angle 0}{141.4 \angle -45^\circ} = \frac{V_m}{141.4} \angle 45^\circ$$

Example: Phasor Diagram

ECEN 222

Maxx Seminario

Voltage Divider:

Capacitor voltage:

$$\begin{aligned}\mathbf{V}_C &= \mathbf{V}_s \frac{\mathbf{Z}_C}{\mathbf{Z}_R + \mathbf{Z}_C} \\ &= \mathbf{V}_s \frac{-j100}{100 - j100} \\ &= \mathbf{V}_s \frac{100 \angle -90}{141.4 \angle -45} \\ &= 0.707V_m \angle -45\end{aligned}$$

Resistor voltage:

$$\mathbf{V}_R = \mathbf{I}R = 0.707V_m \angle 45$$

Phasor Diagram:

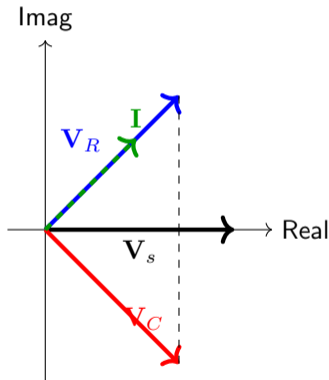


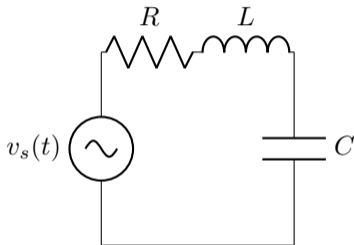
Figure 1: $\mathbf{V}_R + \mathbf{V}_C = \mathbf{V}_s$ (KVL)

Example: Series RLC Circuit

ECEN 222

Maxx Seminario

Circuit:



Total Impedance:

$$\begin{aligned}\mathbf{Z} &= R + j\omega L + \frac{1}{j\omega C} = R + j\left(\omega L - \frac{1}{\omega C}\right) \\ &= R + j(X_L - X_C)\end{aligned}$$

Three Cases:

1. Inductive ($X_L > X_C$):

- Net reactance is positive
- Voltage leads current
- Behaves like RL circuit

2. Capacitive ($X_L < X_C$):

- Net reactance is negative
- Current leads voltage
- Behaves like RC circuit

3. Resonant ($X_L = X_C$):

- Net reactance is zero
- $\mathbf{Z} = R$ (purely resistive)
- \mathbf{V} and \mathbf{I} in phase

Resonance in RLC Circuits

ECEN 222

Maxx Seminario

Resonance Condition:

At resonance: $X_L = X_C$

$$\omega_0 L = \frac{1}{\omega_0 C}$$

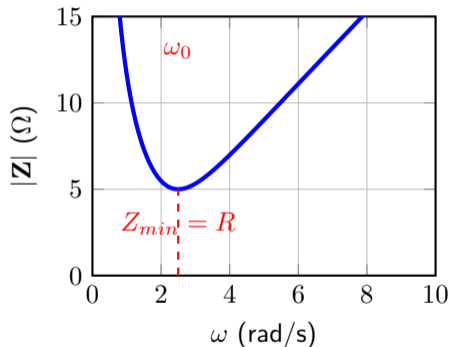
Resonant Frequency

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

At Resonance:

- $Z = R$ (minimum impedance)
- Maximum current
- Zero phase angle

Impedance vs. Frequency:



AC Power: Instantaneous and Average

ECEN 222

Maxx Seminario

Introduction to
Frequency
Domain

Phasor
Representation

Impedance

Phasor Circuit
Analysis

AC Power
Analysis

Summary

Phasor Basics
Problems

Impedance
Calculation
Problems

Circuit Analysis

AC Power
Problems

Instantaneous Power:

For $v(t) = V_m \cos(\omega t)$ and
 $i(t) = I_m \cos(\omega t - \theta)$:

$$p(t) = v(t) \cdot i(t)$$

$$= V_m I_m \cos(\omega t) \cos(\omega t - \theta)$$

Using trig identity:

$$p(t) = \frac{V_m I_m}{2} \cos \theta + \frac{V_m I_m}{2} \cos(2\omega t - \theta)$$

Average Power:

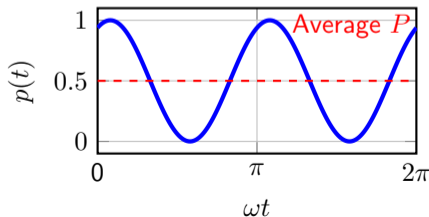
$$P = \frac{1}{T} \int_0^T p(t) dt = \frac{V_m I_m}{2} \cos \theta$$

Using RMS Values:

$$V_{rms} = \frac{V_m}{\sqrt{2}}, \quad I_{rms} = \frac{I_m}{\sqrt{2}}$$

Average (Real) Power

$$P = V_{rms} I_{rms} \cos \theta$$



Reactive and Apparent Power

ECEN 222

Maxx Seminario

Power Components:

1. Real (Average) Power:

$$P = V_{rms} I_{rms} \cos \theta \quad (\text{W})$$

- Power dissipated (resistors)

2. Reactive Power:

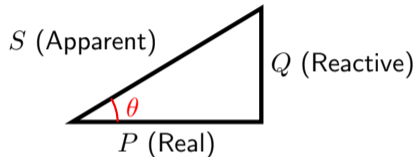
$$Q = V_{rms} I_{rms} \sin \theta \quad (\text{VAR})$$

- Power stored/returned (L/C)

3. Apparent Power:

$$S = V_{rms} I_{rms} \quad (\text{VA})$$

Power Triangle:



$$S = \sqrt{P^2 + Q^2}$$

$$P = S \cos \theta, \quad Q = S \sin \theta$$

Power Factor:

$$\text{pf} = \cos \theta = \frac{P}{S}$$

Power Factor and Its Importance

ECEN 222

Maxx Seminario

Power Factor Definition:

$$\text{pf} = \cos \theta = \frac{P}{S}$$

Range: $0 \leq \text{pf} \leq 1$

Special Cases:

😊 **pf = 1** (unity): purely resistive, $\theta = 0$

☹️ **pf = 0**: purely reactive, $\theta = \pm 90$

Leading vs. Lagging:

- Lagging pf: inductive load (current lags voltage)
- Leading pf: capacitive load (current leads voltage)

Low Power Factor Problems

- ☹️ Higher current required
- ☹️ Larger conductor sizes needed
- ☹️ More I^2R losses in transmission

Power Factor Correction:

Add capacitors in parallel with inductive loads to:

- 😊 Increase power factor
- 😊 Reduce reactive power
- 😊 Lower current draw

Power in Circuit Elements

ECEN 222

Maxx Seminario

Introduction to
Frequency
Domain

Phasor
Representation

Impedance

Phasor Circuit
Analysis

AC Power
Analysis

Summary

Phasor Basics
Problems

Impedance
Calculation
Problems

Circuit Analysis

AC Power
Problems

Element	Phase	Real Power P	Reactive Power Q	pf
Resistor	$\theta = 0$	$I^2 R$	0	1
Inductor	$\theta = 90$	0	$I^2 X_L$ (positive)	0
Capacitor	$\theta = -90$	0	$-I^2 X_C$ (negative)	0

Key Observations

- Only **resistors** dissipate real power (convert to heat · or light if you mess up)
- **Inductors** and **capacitors** store and return energy (reactive power)
- Reactive power from L and C have opposite signs (can cancel to form resonant networks)

Summary: Frequency Domain Analysis

ECEN 222

Maxx Seminario

Introduction to
Frequency
Domain

Phasor
Representation

Impedance

Phasor Circuit
Analysis

AC Power
Analysis

Summary

Phasor Basics
Problems

Impedance
Calculation
Problems

Circuit Analysis

AC Power
Problems

Phasor Analysis:

- Transform: $V_m \cos(\omega t + \phi) \leftrightarrow V_m \angle \phi$
- ☺ Differential equations \rightarrow algebra
- $d/dt \rightarrow j\omega$, $\int dt \rightarrow 1/(j\omega)$

Impedance:

- $\mathbf{Z} = R + jX$
- Resistor: $\mathbf{Z}_R = R$
- Inductor: $\mathbf{Z}_L = j\omega L$
- Capacitor: $\mathbf{Z}_C = 1/(j\omega C)$

Circuit Analysis:

- ☺ All DC techniques apply
- KVL, KCL, voltage/current dividers
- Series/parallel combinations

AC Power:

- Real power: $P = V_{rms} I_{rms} \cos \theta$
- Reactive power: $Q = V_{rms} I_{rms} \sin \theta$
- Apparent power: $S = V_{rms} I_{rms}$

Power Factor:

- $\text{pf} = \cos \theta = P/S$
- Lagging pf: inductive
- Leading pf: capacitive
- ☹ Low pf \rightarrow higher losses

Resonance:

- Occurs when $X_L = X_C$
- $\omega_0 = 1/\sqrt{LC}$
- ☺ Minimum Z , maximum I

Comparison: Time vs. Frequency Domain

ECEN 222

Maxx Seminario

Introduction to
Frequency
Domain

Phasor
Representation

Impedance

Phasor Circuit
Analysis

AC Power
Analysis

Summary

Phasor Basics
Problems

Impedance
Calculation
Problems

Circuit Analysis

AC Power
Problems

Aspect	Time Domain	Frequency Domain
Signals	$v(t)$, $i(t)$ (real functions)	\mathbf{V} , \mathbf{I} (complex phasors)
Math	Differential equations	Algebraic equations
Circuit elements	R, L, C (time relations)	\mathbf{Z}_R , \mathbf{Z}_L , \mathbf{Z}_C (impedances)
Analysis	Initial conditions, transients	Steady-state, magnitude/phase
Advantages	Shows time evolution	Simplifies sinusoidal analysis
Limitations	Complex for AC steady-state	Only sinusoidal steady-state

When to Use Each

Time Domain: Transients, switching, initial conditions, non-sinusoidal signals

Frequency Domain: AC steady-state, sinusoidal sources, impedance analysis

Example 1: Phasor Conversions

ECEN 222

Maxx Seminario

Introduction to
Frequency
Domain

Phasor
Representation

Impedance

Phasor Circuit
Analysis

AC Power
Analysis

Summary

Phasor Basics
Problems

Impedance
Calculation
Problems

Circuit Analysis

AC Power
Problems

Problem: Convert the following time-domain signals to phasors, then perform the operations.

Given:

$$v_1(t) = 10 \cos(1000t + 30) \text{ V}$$

$$v_2(t) = 5 \cos(1000t - 45) \text{ V}$$

$$i(t) = 2 \cos(1000t + 60) \text{ A}$$

Find:

- 1 Phasor forms of v_1 , v_2 , and i
- 2 $\mathbf{V}_1 + \mathbf{V}_2$
- 3 $\mathbf{V}_1 - \mathbf{V}_2$
- 4 \mathbf{V}_1/\mathbf{I}

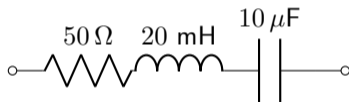
Example 2: Impedance at Different Frequencies

ECEN 222

Maxx Seminario

Problem: A series circuit contains $R = 50\ \Omega$, $L = 20\ \text{mH}$, and $C = 10\ \mu\text{F}$. Find the total impedance at the following frequencies.

Circuit:



Given:

- $R = 50\ \Omega$
- $L = 20\ \text{mH}$
- $C = 10\ \mu\text{F}$

Find the total impedance at:

- (a) $f = 100\ \text{Hz}$
- (b) $f = 500\ \text{Hz}$
- (c) $f = 1000\ \text{Hz}$

For each frequency, determine:

- 1 Magnitude $|\mathbf{Z}|$
- 2 Phase angle θ
- 3 Whether the circuit is inductive or capacitive

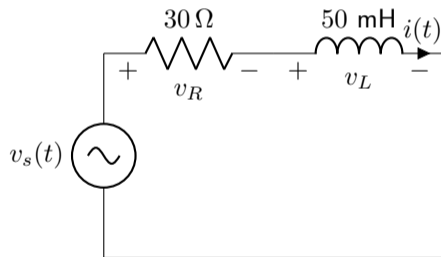
Example 3: Series RL Circuit

ECEN 222

Maxx Seminario

Problem: For the circuit shown, find the current, voltage across each element, and draw the phasor diagram.

Circuit:



Given:

- $v_s(t) = 100 \cos(2000t)\text{ V}$
- $R = 30\ \Omega$
- $L = 50\text{ mH} = 0.05\text{ H}$
- $\omega = 2000\text{ rad/s}$

Find:

- 1 Total impedance \mathbf{Z}_{tot}
- 2 Current $i(t)$ (phasor and time domain)
- 3 Voltage across resistor $v_R(t)$
- 4 Voltage across inductor $v_L(t)$
- 5 Draw the phasor diagram
- 6 Verify KVL

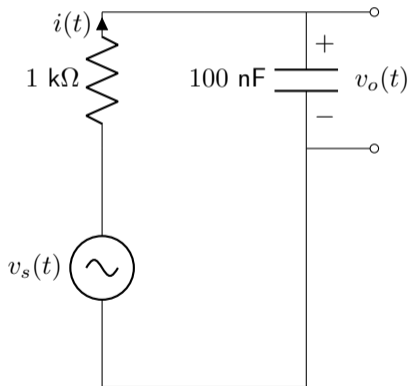
Example 4: RC Voltage Divider

ECEN 222

Maxx Seminario

Problem: Analyze the following RC voltage divider circuit.

Circuit:



Given:

- $v_s(t) = 10 \cos(10000t) \text{ V}$
- $R = 1 \text{ k}\Omega$
- $C = 100 \text{ nF}$

Find:

- 1 The impedance of each element
- 2 The output voltage \mathbf{V}_o
- 3 The output voltage $v_o(t)$
- 4 The magnitude ratio $|\mathbf{V}_o|/|\mathbf{V}_s|$
- 5 The phase shift between input and output
- 6 The current $i(t)$

Example 5: AC Power Calculation

ECEN 222

Maxx Seminario

Introduction to
Frequency
Domain

Phasor
Representation

Impedance

Phasor Circuit
Analysis

AC Power
Analysis

Summary

Phasor Basics
Problems

Impedance
Calculation
Problems

Circuit Analysis

AC Power
Problems

Problem: Calculate the real, reactive, and apparent power for a load with the following voltage and current.

Given:

$$v(t) = 120\sqrt{2} \cos(377t) \text{ V}$$

$$i(t) = 10\sqrt{2} \cos(377t - 36.87) \text{ A}$$

Note: The coefficients include $\sqrt{2}$ to indicate peak values

Find:

- 1 RMS voltage and current
- 2 Real power P
- 3 Reactive power Q
- 4 Apparent power S
- 5 Power factor (and type)
- 6 Load impedance \mathbf{Z}

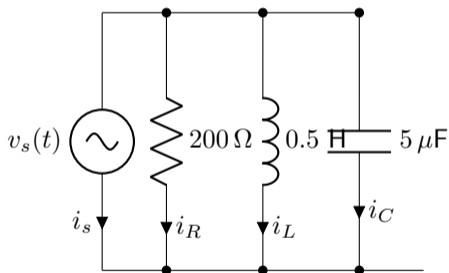
Example 6: Parallel RLC Circuit

ECEN 222

Maxx Seminario

Problem: Analyze the parallel RLC circuit shown below.

Circuit:



Given:

■ $v_s(t) = 50 \cos(1000t)\ \text{V}$

Find:

- 1 Impedance of each branch
- 2 Total impedance
- 3 Source current $i_s(t)$
- 4 Current through each branch
- 5 Total real power
- 6 Total reactive power
- 7 Is the circuit inductive or capacitive?