

## Continuous-Time Processing of Discrete-Time Signals

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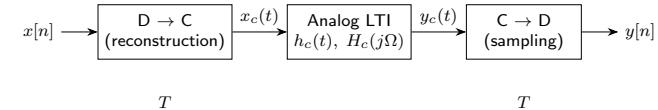
Fall 2025

## Overview: System Architecture

### Complementary to Previous Topic:

- Previously: Discrete-time processing of continuous-time signals
- Now: Continuous-time processing of discrete-time signals

### General System Configuration:



$T$

$T$

### Key Characteristics:

- Input and output: discrete-time sequences
- Intermediate processing: continuous-time domain
- Provides useful interpretation of certain discrete-time systems
- Not typically implemented

## Bandlimited Signal Constraint

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Introduction  
Sampling  
Nyquist  
Sampling  
Reconstruction  
Aliasing  
Windowing  
Summary

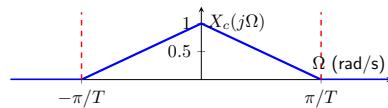
### Fundamental Property:

The ideal D/C converter produces a bandlimited signal:

$$X_c(j\Omega) = 0, \quad |\Omega| \geq \pi/T$$

**Consequence:** No aliasing in C/D conversion

- $y_c(t)$  is also bandlimited:  $Y_c(j\Omega) = 0$  for  $|\Omega| \geq \pi/T$
- Sampling  $y_c(t)$  at rate  $1/T$  satisfies Nyquist criterion
- Perfect reconstruction of  $y[n]$  from  $y_c(nT)$



## Time-Domain Representations

### Input Signal - Bandlimited Interpolation:

$$x_c(t) = \sum_{n=-\infty}^{\infty} x[n] \frac{\sin[\pi(t - nT)/T]}{\pi(t - nT)/T}$$

### Output Signal - After Continuous-Time Processing:

$$y_c(t) = \sum_{n=-\infty}^{\infty} y[n] \frac{\sin[\pi(t - nT)/T]}{\pi(t - nT)/T}$$

where  $y[n] = y_c(nT)$

### Key Relationships:

- $x[n] = x_c(nT)$  - samples of reconstructed signal
- $y[n] = y_c(nT)$  - samples of processed signal
- Both sequences connected through continuous-time system

## Frequency Domain Representations

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Frequency  
Domain Analysis  
Example  
Continuous-DT  
Discrete-DT  
Summary

### Three Key Equations:

#### 1. D/C Conversion:

$$X_c(j\Omega) = TX(e^{j\Omega T}), \quad |\Omega| < \pi/T$$

#### 2. Continuous-Time Processing:

$$Y_c(j\Omega) = H_c(j\Omega)X_c(j\Omega)$$

#### 3. C/D Conversion:

$$Y(e^{j\omega}) = \frac{1}{T}Y_c\left(j\frac{\omega}{T}\right), \quad |\omega| < \pi$$

## Overall Discrete-Time System Response

### Combining All Relationships:

Substitute Eq. (1) and (2) into Eq. (3):

$$\begin{aligned} Y(e^{j\omega}) &= \frac{1}{T}Y_c\left(j\frac{\omega}{T}\right) \\ &= \frac{1}{T}H_c\left(j\frac{\omega}{T}\right)X_c\left(j\frac{\omega}{T}\right) \\ &= \frac{1}{T}H_c\left(j\frac{\omega}{T}\right) \cdot TX(e^{j\omega}) \end{aligned}$$

### Result - Effective Discrete-Time Frequency Response:

$$H(e^{j\omega}) = H_c\left(j\frac{\omega}{T}\right), \quad |\omega| < \pi$$

Therefore the overall system behaves as a discrete-time system with frequency response  $H(e^{j\omega})$

## Design Relationship

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Frequency Domain Analysis  
Example: Windowed DFT  
Example: Windowed DCT  
Example: Windowed DCT  
Summary

**Forward Design:** Given desired  $H(e^{j\omega})$ , find  $H_c(j\Omega)$

**Solution:**

$$H_c(j\Omega) = H(e^{j\Omega T}), \quad |\Omega| < \pi/T$$

**Arbitrary Extension:**

- Since  $X_c(j\Omega) = 0$  for  $|\Omega| \geq \pi/T$ , we can choose  $H_c(j\Omega)$  arbitrarily above  $\pi/T$
- Typically (out of convenience):  $H_c(j\Omega) = 0$  for  $|\Omega| \geq \pi/T$
- Makes  $H_c(j\Omega)$  bandlimited

**Key Notes:**

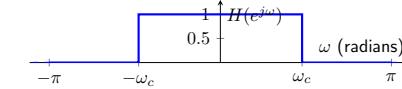
- This is the **inverse** of impulse invariance
- Impulse invariance:  $H(e^{j\omega}) = H_c(j\omega/T)$
- This method:  $H_c(j\Omega) = H(e^{j\Omega T})$

## Frequency Domain Illustration

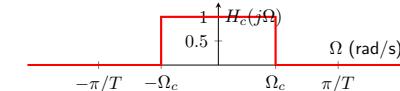
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Frequency Domain Analysis  
Example: Windowed DFT  
Example: Windowed DCT  
Example: Windowed DCT  
Summary

**Discrete-Time Frequency Response  $H(e^{j\omega})$ :**



**Continuous-Time Frequency Response  $H_c(j\Omega)$ :**



**Relationship:**  $H_c(j\Omega) = H(e^{j\Omega T})$  for  $|\Omega| < \pi/T$

## Example: Noninteger Delay System

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Discrete-Time Frequency Response:  
Example: Noninteger Delay  
Continuous-Time Analysis  
Example: Noninteger Delay  
Continuous-Time Analysis  
Summary

### Discrete-Time Frequency Response:

$$H(e^{j\omega}) = e^{-j\omega\Delta}, \quad |\omega| < \pi$$

### Case 1 - Integer Delay ( $\Delta = n_0$ , integer):

$$y[n] = x[n - n_0]$$

Straightforward interpretation: shift sequence by  $n_0$  samples

### Case 2 - Noninteger Delay ( $\Delta$ not integer):

- Expression  $y[n] = x[n - \Delta]$  has no direct meaning
- Cannot shift discrete sequence by fractional samples
- Need continuous-time interpretation

## Continuous-Time Interpretation

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Discrete-Time Frequency Response:  
Example: Noninteger Delay  
Continuous-Time Analysis  
Example: Noninteger Delay  
Continuous-Time Analysis  
Summary

### Apply Design Relationship:

$$H_c(j\Omega) = H(e^{j\Omega T}) = e^{-j\Omega\Delta T}$$

We recognize this is an ideal time delay

$$y_c(t) = x_c(t - \Delta T)$$

### Physical Interpretation:

- 1 Start with discrete sequence  $x[n]$
- 2 Reconstruct bandlimited  $x_c(t)$  via D/C converter
- 3 Delay  $x_c(t)$  by  $\Delta T$  seconds
- 4 Sample delayed signal to get  $y[n] = y_c(nT)$

Therefore the noninteger delay operates on the **interpolated** continuous signal

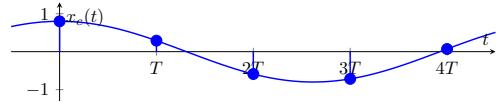
## Noninteger Delay: Time Domain Visualization

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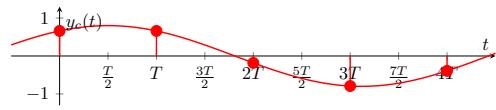
Frequency Domain Analysis  
Example:  
Noninteger Delay  
Example: Moving Average  
Summary

**Example:**  $\Delta = 0.5$  (half-sample delay)

Input: Interpolated Signal with Samples  $x[n]$



Output: Delayed by  $T/2$ , Sampled at  $y[n]$



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Frequency Domain Analysis  
Example:  
Moving Average  
Summary

## Example: Moving-Average System

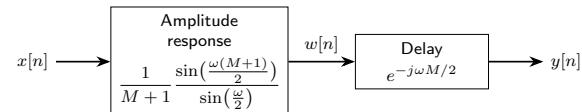
**General  $(M + 1)$ -Point Moving Average:**

$$y[n] = \frac{1}{M+1} \sum_{k=0}^M x[n-k]$$

**Frequency Response** (from DTFT Lecture):

$$H(e^{j\omega}) = \frac{1}{M+1} \frac{\sin[\omega(M+1)/2]}{\sin(\omega/2)} e^{-j\omega M/2}$$

**Decomposition:**



## Moving Average: Integer vs. Noninteger Delay

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Frequency Domain Analysis  
Example: Moving Average  
Summary

### Case 1 - Odd Number of Points ( $M$ even):

Example:  $M = 4$  (5-point average)

$$\text{Delay} = \frac{M}{2} = 2 \text{ samples (integer)}$$
$$y[n] = w[n-2]$$

Simple interpretation: 2-sample shift

### Case 2 - Even Number of Points ( $M$ odd):

Example:  $M = 5$  (6-point average)

$$\text{Delay} = \frac{M}{2} = 2.5 \text{ samples (noninteger)}$$

Must use continuous-time interpretation:

- Bandlimited interpolation of  $w[n]$
- Continuous delay of  $MT/2 = 2.5T$  seconds
- Resampling to get  $y[n]$

## Moving Average: Numerical Example

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Frequency Domain Analysis  
Example: Moving Average  
Summary

**Input:**  $x[n] = \cos(0.25\pi n)$

**System:** 6-point moving average ( $M = 5$ )

### Frequency Response at Input Frequency:

$$H(e^{j0.25\pi}) = \frac{1}{6} \frac{\sin[3(0.25\pi)]}{\sin(0.125\pi)} e^{-j(0.25\pi)(2.5)}$$

Calculate magnitude:

$$|H(e^{j0.25\pi})| = \frac{1}{6} \frac{\sin(0.75\pi)}{\sin(0.125\pi)} \approx 0.308$$

Calculate phase:

$$\angle H(e^{j0.25\pi}) = -0.25\pi \times 2.5 = -0.625\pi$$

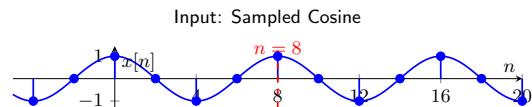
## Moving Average: Visualization

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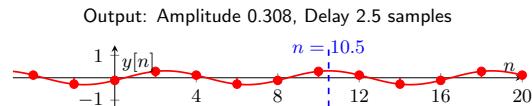
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Frequency Domain Analysis  
Example: Windowed Data  
Example: Moving Average  
Summary

**Input Signal:**  $x[n] = \cos(0.25\pi n)$



**Output Signal:**  $y[n] = 0.308 \cos[0.25\pi(n - 2.5)]$



## Summary: Concepts

### System Architecture:

$$x[n] \xrightarrow{\text{D/C}} x_c(t) \xrightarrow{H_c(j\Omega)} y_c(t) \xrightarrow{\text{C/D}} y[n]$$

### Fundamental Relationships:

$$H(e^{j\omega}) = H_c\left(j\frac{\omega}{T}\right) \Leftrightarrow H_c(j\Omega) = H(e^{j\Omega T})$$

### Limitations:

- Not typically used for actual implementation
- Requires bandlimited signals for exact analysis
- Mainly a conceptual/analytical tool

## Summary: Mathematical Framework

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Introduction

Convolution

Correlation

Sampling

Interpolation

Windowing

Summary

### Time Domain:

$$x_c(t) = \sum_{n=-\infty}^{\infty} x[n] \frac{\sin[\pi(t - nT)/T]}{\pi(t - nT)/T}$$

$$h[n] = \frac{\sin[\pi(n - \Delta)]}{\pi(n - \Delta)} \text{ for delay } \Delta$$

### Frequency Domain:

$$X_c(j\Omega) = TX(e^{j\Omega T}), \quad Y_c(j\Omega) = H_c(j\Omega)X_c(j\Omega)$$

$$Y(e^{j\omega}) = \frac{1}{T} Y_c\left(j\frac{\omega}{T}\right) = H_c\left(j\frac{\omega}{T}\right) X(e^{j\omega})$$

### Design Equation:

$$\boxed{H_c(j\Omega) = H(e^{j\Omega T}), \quad |\Omega| < \pi/T}$$