

Assignment3_1

September 26, 2020

1 Assignment 3.1

Your task is to use the Euler method to compare the difference between a numerical approximation and the ‘real’ solution to an ODE. This assignment is worth 20 marks and 2 of those marks will be awarded for good comments in your code.

1.1 Part 1 :

(7 marks out 20). Choose an ODE with a well known solution or one that you know how to solve. Some examples include : $y' = xy$, $y' = \frac{x}{y}$, $y' = x^4y^{-2}$. Define the initial conditions and check it makes sense, so for example if you chose $y' = \frac{3x^2}{y}$, $y(0) = 1$, this is a separable differential equation and so rearranging this gives :

$$\int y dy = \int 3x^2 dx$$
$$\frac{y^2}{2} = x^3 + c$$

hence, for $y(0) = 1$

$$c = \frac{1}{2}$$

therefore the ‘real’ solution is

$$y^2 = 2x^3 + 1$$

and

$$y = \sqrt{2x^3 + 1}$$

(taking positive solutions only)

Once you have chosen your ODE, write a code to work out a numerical solution to this differential equation. The user should provide as inputs the step-size h and the value of x for which the solution is sought. You may use the examples in the notes as the basis for your code.

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1.2 Part 2 :

(7 marks out of 20). Add to your code so that it works out the difference between the approximated value and the true value at each step and store this difference as a list. Examine some of the values in this list in your own way, to help you understand what is happening in your approximation. Or you could plot it as a function of x (see above example). Make a few comments in the markdown cell on your observations.

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1.3 Part 3 :

(6 marks out of 20). Choose a cut off value in difference that will interrupt the program (i.e. break out of the loop) and inform you that the solutions are diverging with an appropriate message. HINT: the deviation from the true solution can be positive or negative and so you will probably want to look at the absolute value of this!!

Make a few observations, in the cells below, on how the cut-off point changes with the value of h .

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OBSERVATIONS: MAKE SOME OBSERVATIONS HERE