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4.7

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5.1 $\backslash subsuperscript$

$\binom{n}{m}$ - number of m element subsets of a set of n elements. 'n choose m'. A binomial number

$\binom{n}{m} = \frac{n^{\underline{m}}}{m!} \quad if\ m > 0,\ else\ 0$

$\binom{n}{m} = \binom{n-1}{m} + \binom{n-1}{m-1} \quad for\ int\ m$

$\binom{r}{0} = 1$

$\binom{r}{1} = 1$

$\binom{r}{2} = \frac{r(r-1)}{2}$

$\binom{n}{k} = \binom{n}{n-k} \quad for\ int\ n \geq 0,\ int\ k$

$\binom{r}{k} = \frac{r}{k}\binom{r-1}{k-1} \quad for\ int\ k \neq 0$

$k\binom{r}{k} = r\binom{r-1}{k-1} \quad for\ int\ k$

$(r - k)\binom{r}{k} = r\binom{r-1}{k} \quad for\ int\ k$

$\binom{n}{n} = 1 \quad if\ n \geq 0,\ else\ 0$

$\binom{n}{k} = \frac{n!}{k!(n-k)!} \quad for\ int\ n \geq k \geq 0$

$\sum_{k \leq n} \left(\frac{r+k}{k}\right) = \binom{r+n+1}{n} \quad for\ int\ n$

$\sum_{0 \leq k \leq n} \binom{k}{m} = \binom{n+1}{m+1} \quad for\ m, n \geq 0$

$(x + y)^r = \sum_k \binom{r}{k} x^k y^{r-k} \quad for\ int\ r \geq 0 \,||\, \left|\frac{x}{y}\right| < 1$

$2^n{}_{(for\ n=3)} := \binom{3}{0} + \binom{3}{1} + \binom{3}{2} + \binom{3}{3}$ - number of subsets of an n element set

$\binom{r}{k} = (-1)^k \binom{k-r-1}{k} \quad for\ int\ k$

$(-1)^m \binom{-n-1}{m} = (-1)^m \binom{-m-1}{n} \quad for\ int\ m, m \geq 0$

$$\sum_{k\le m} \binom{r}{k}(-1)^k = (-1)^m \binom{r-1}{m}$$

$$\sum_{k\le m} \binom{m+r}{k} x^k y^{m-k} = \sum_{k\le m} \binom{-r}{k}(-x)^k (x+y)^{m-k} \quad for\ int\ m$$

$$2^m = \sum_{k\le m} \binom{m+k}{k} 2^{-k}$$

$$\binom{r}{m}\binom{m}{k} = \binom{r}{k}\binom{r-k}{m-k} \quad for\ int\ m, k$$

Trinomial Coefficients:

$\binom{a+b+c}{a,b,c} = \frac{(a+b+c)!}{a!b!c!}$ // same for 4, …, n

$$(x+y+z)^n = \sum_{\substack{a+b+c=n \\ 0\le a,b,c\le n}} \binom{a+b+c}{a,b,c} x^a y^b z^c = \sum_{\substack{a+b+c=n \\ 0\le a,b,c\le n}} \left(\frac{(a+b+c)!}{a!b!c!}\right) x^a y^b z^c$$

$\sum_k \binom{r}{m+k}\binom{s}{n-k} = \binom{r+s}{m+n} \quad for\ int\ m, n$ // Vandermonde convolution

$$\sum_k \binom{l}{m+k}\binom{s}{n+k} = \binom{l+s}{l-m+n} \quad for\ int\ l \ge 0,\ int\ m, n$$

$$\sum_k \binom{l}{m+k}\binom{s+k}{n}(-1)^k = (-1)^{l+m}\binom{s-m}{n-l} \quad for\ int\ l \ge 0,\ int\ m, n$$

$$\sum_{k\le 1} \binom{l-k}{m}\binom{s}{k-n}(-1)^k = (-1)^{l+m}\binom{s-m-1}{l-m-n} \quad for\ int\ l, m, n \ge 0$$

$$\sum_{0\le k\le l} \binom{l-k}{m}\binom{q+k}{n} = \binom{l+q+1}{m+n+1} \quad for\ int\ l, m \ge 0,\ \ int\ n \ge q \ge 0$$

$$\sum_k \binom{a+b}{a+k}\binom{b+c}{b+k}\binom{c+a}{c+k}(-1)^k = \frac{(a+b+c)!}{a!b!c!} \quad for\ a, b, c \ge 0$$

5.2