2

Transformation is linear if:

$T(x + y) = T(x) + T(y)$

$T(ax) = aT(x)$

$T(0) = (0)$

$T(- x) =- T(x)$

Rotation / Reflection / Projection

Elementary matrices

Markov Chains

3.

$det\ A_{2x2} = ad - bc$

$det\ A = a_{11}c_{11}(A) + a_{12}c_{12}(A) +...+ a_{1n}c_{1n}(A)$

$c_{ij} = (- 1)^{i+j}det(A_{ij})$

+-+...

-+-...

+-+...

Can calc det from any col or row

det = 0 if A is not invertible, or has row/col of all zeros or has duplicate row/col

If rows are swapped, negate the det

If row is multiplied by c, the det of new matrix is u(det A)

Adding/subtracting different rows doesn't change det

For lower/upper triangular or diagonal, det = product of the diagonally

$det\ AB = det\ A\ det\ B$

$det(A^k) = (det\ A)^k\ \ for\ k \geq 1$

$det(A^{-1}) = \frac{1}{det\ A}$

$A\ is\ orthogonal\ if\ A^{-1} = A^T$

$adj\ A = [c_{ij}(A)]^T$

$A(adj\ A) = det(A) * I = (adj\ A)A$

$A^{-1} = (\frac{adj}{det})A$

$det(cA_{nxn}) = c^n det(A_{nxn})$

$det(\lambda I - A ?? A - I\lambda) = 0$ >> solve for λ

$(\lambda I - A)x = 0$ >> solve for x - eigenvector, and write as $\vec{x} = [\ {}^{-1}_{1}]t,\ for\ t \neq 0$

A is diagonalizable if $P^{-1}AP$ is invertible $= diag(\lambda_1,...)$, where P is a concat of eigenvectors.

5.
solve matrix - https://matrix.reshish.com/gauss-jordanElimination.php
REF, RREF, null, solution - http://www.math.odu.edu/~bogacki/cgi-bin/lat.cgi
im - https://www.mathdetail.com/col.php
Eigenspace - https://www.dcode.fr/matrix-eigenspaces
Last hope - https://www.symbolab.com/solver/matrix-calculator/

Subspace if:
$0\ in\ U$
$x \in U,\ y \in U\ >\ x + y \in U$
$x \in U\ >\ cx \in U\ for\ c \in R$

Eigenspace
$\lambda I - A$ or $A - \lambda I$
$E_\lambda = \{Null(\lambda I - A)\} = span(\lambda I - A) = span(eigenvector)$

Null
$x \in null(A)\ \Leftrightarrow\ Ax = 0$ (has the size of a col)

Span
$x \in span(A)\ \Leftrightarrow\ Ax = b$ (has the size of a col)

Dependent
if exists $t_1x_1 + ... + t_nx_n = 0$ where not all $t = 0$
$dependent = more\ cols\ than\ rows\ ||\ !\ isIdentity(RREF(matrix(set)))$

Col space (im)
$im(A) = filter(A, RREF\ col\ has\ leading\ 1)$ (has the size of a col)

Row space
$im(A) = filter(RREF, RREF\ row\ not\ empty)$ (has the size of a row)

$rank\ A = rank\ A^T$
$dim(null\ A) = n - r$
$dim(row\ A) = dim(col\ A) = r$
Where dim is the number of bases
Bases are independent (not repeating) components of span/im/row/null

TODO:
eigenvectors and eigenspaces
crank up the lamp
prepare the programs
review the notes