$!x_1 !x_2 + !x_1 x_2 + x_1 x_2$

$!x_1 (!x_2 + x_2)$

$!x_1 + x_1 x_2$

$!x_1 (1 + x_2)$

$!x_1 + x_2$

$x_2 + !x_1 !x_2$

$x_2 + (x_1 + x_2)$

$x_1 + x_2$

$!x_1 !x_2 + !x_1 x_2 + x_1 x_2$

$!x_1 + x_1 x_2$

$!x_1 + x_2$

Nr	x_1	x_2	x_3	Minterm
0	0	0	0	$m_0 = \bar{x}_1 \bar{x}_2 \bar{x}_3$
1	0	0	1	$m_1 = \bar{x}_1 \bar{x}_2 x_3$
2	0	1	0	$m_2 = \bar{x}_1 x_2 \bar{x}_3$
3	0	1	1	$m_3 = \bar{x}_1 x_2 x_3$
4	1	0	0	$m_4 = x_1 \bar{x}_2 \bar{x}_3$
5	1	0	1	$m_5 = x_1 \bar{x}_2 x_3$
6	1	1	0	$m_6 = x_1 x_2 \bar{x}_3$

6 1 1 0 $m_6 = x_1 x_2 \bar{x}_3$

7 1 1 1 $m_7 = x_1 x_2 x_3$

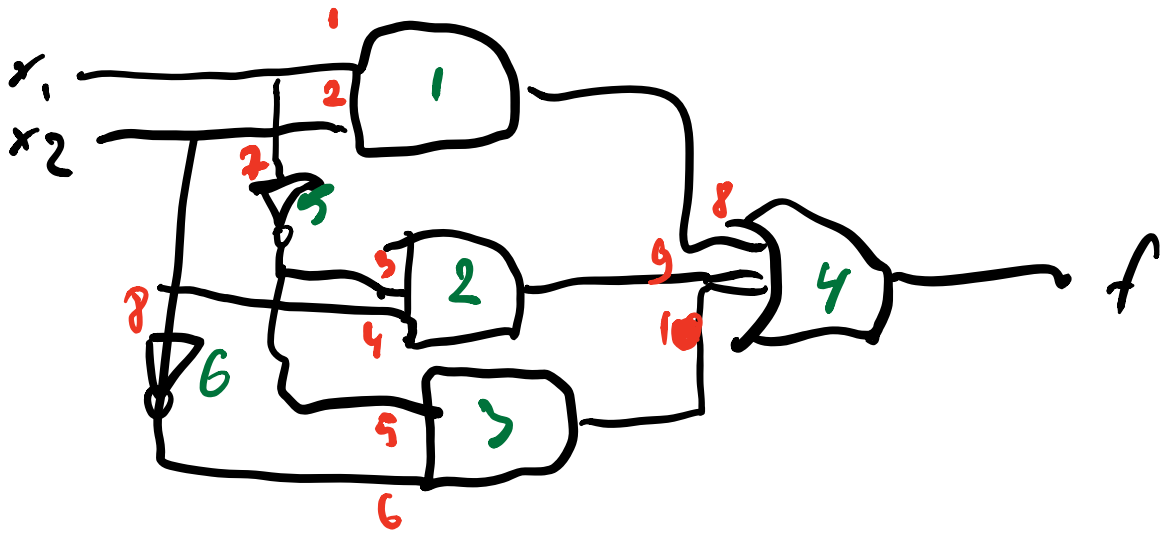
0	0	1	$\bar{x}_1 \bar{x}_2 \cdot 1$
0	1	1	$\bar{x}_1 x_2 \cdot 1$
1	0	0	$x_1 \bar{x}_2 \cdot 0$
1	1	1	$x_1 x_2 \cdot 1$

$\bar{x}_1 \bar{x}_2 + \bar{x}_1 x_2 + x_1 x_2$

a) Canonical sum-of-products
(regular solution)

b) Minimal-cost realization
(most efficient solution)

Cost = #gates + #inputs



6 + 11 = 17

2 + 3 = 5

$$f(x_1, x_2, x_3) = \sum(m_1, m_4, m_5, m_6) =$$

$$= \sum m(1, 4, 5, 6)$$

rows that are not 0

= Canonical SOP expression

Sum-of-products

m - minterm $0 = \bar{x}$ $1 = x$ (all = 1)

M - Maxterm $0 = x$ $1 = \bar{x}$ (all = 0)

Product-of-sums (POS) - inverse of SOP

$(x+y)(\bar{x}+y)(\bar{x}+\bar{y})$

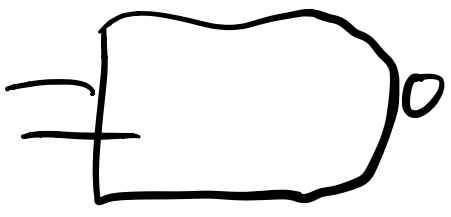
$\sum$ - sum

$\prod$ - product

$f(x_1, x_2, x_3) = \prod(M_0, M_2, M_3, M_7) =$

$= \prod M(0, 2, 3, 7)$

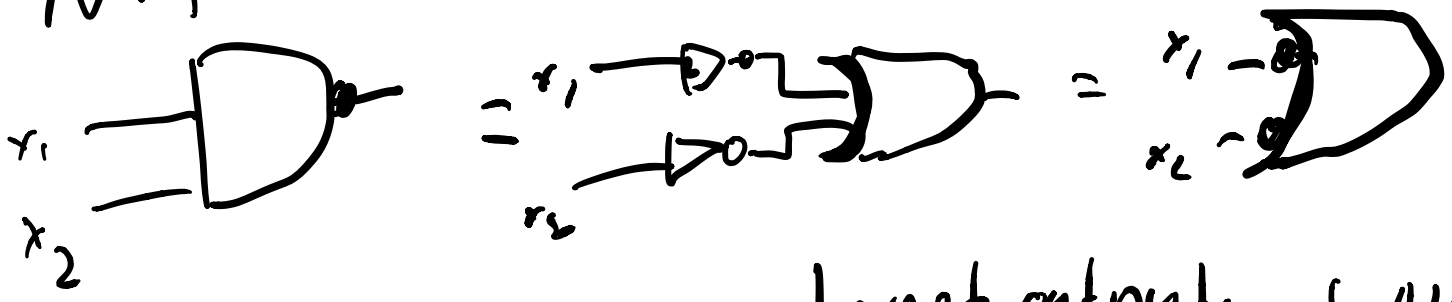
in N $f = \prod M(\varepsilon) = \sum m(not\ \varepsilon)$

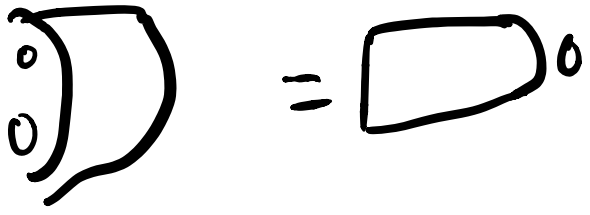


NAND

NOR

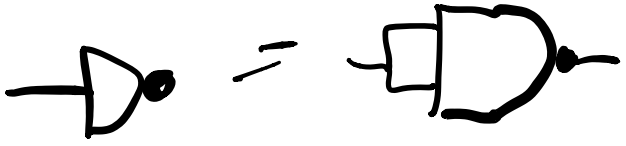
NAND



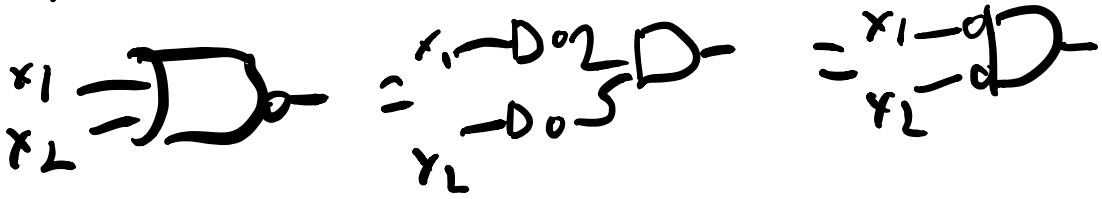


Invert outputs of AND
Invert inputs of OR
Use 15th to convert OR to NAND
Use NAND for inversions

x	y	or	and	nand	nor
0	0	0	0	1	1
0	1	1	0	1	0
1	0	1	0	1	0
1	1	1	1	0	0



NOR



$\overline{(x_1 + x_2)} = \bar{x}_1 \bar{x}_2$

invert outputs of OR

invert inputs of AND

Use 15th to convert AND to NOR

Use NOR for inversions