OPNS 523 - Assignment: Aguirregabiria (1999) - PART I and II

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PART I

1. The intuition behind using K-convex functions for the (s, S) inventory policies is that the objective functions where neither convex nor concave. That meant that it was complicated to prove that some policy was the optimal. We add some K that denotes the setup cost, so when we are trying to optimize our objective function if we our inventory level x is below out threshold s we need to order because the savings exceed K. In other words, if we have an objective function with some K-convex form, we can still achieve an optimal policy since we are dealing with a simple generalization of convex functions. (Porteus 2002)

2. The decision rules shown in Aguirregabiria (1999, p. 293-4) are a combination of marginal conditions of optimality (markup decision) and optimal discrete choice (inventory decision). With this, they estimate structural parameters that exploit moment conditions based on optimal discrete choice but no with the markup conditions, meaning that they don't exploit Euler equations insights. Recall that an Euler equation shows the evolution of some economic variable in a dynamic choice problem along an optimal path (Parker 2007). This means that as Aguirregabiria doesn't exploit the economic intuition behind the Euler equation. Moreover, the author might be loosing some economic implications of the observed behavior.
   Is doing this sacrificing much information? Partially, even though only considering the discrete choices won't get us to see the inter-temporal rate of marginal conditions, Aguirregaribia claims that because of the endogeneity nature of markup, the optimality of an (S, s) rule does not get affected. Moreover, as the author claims, estimating the Euler equations provide some imprecise estimates of the structural parameters.

3. We can express Aguirregaribia's (1999) estimation procedure in Aguirregabiria and Mira's (2002) terms as follows: The basic idea is that instead of solving for a full DP in each likelihood evaluation, we just implement one policy iteration step for each likelihood evaluation by incorporating some prior CCP information. Instead of jump-

ing directly to Aguirregaribia's (1999) stage 1 where he makes a nonparametric kernel estimation and an estimation of the transition probabilities of the state variables, we instead use Aguirregabiria and Mira's (2002) and create a suitable approximation of the initial policy function that point us toward the true parameter.

REFERENCES

Parker, J. A. (2007). *Euler equations.* New Palgrave Dictionary of Economics, available online at http://www.princeton.edu/ jparker, 10, 2016.

Porteus, E L. (2002). *Foundations of Stochastic Inventory Theory.* Stanford Business Books, Stanford, CA.

PART II

1. We can express the double integrated value function $\tilde{V}_i = \mathrm{E}_s[\bar{V_{i,s}}]$ as a function of $f_i = \mathrm{E}_s[U_{i,s}(1) - log(P_{i,s}(1))]$ and $\tilde{V}_Q$ as follows. First, let $\tilde{V}_Q = \beta V(p_w, i)$:

$$
\begin{aligned}
\tilde{V}_i &= \mathrm{E}_s[\bar{V_{i,s}}] \\
&= \mathrm{E}_s[\mathrm{E}_\epsilon[V_{i,s,\epsilon}]] \\
&= \mathrm{E}_s[\mathrm{E}_\epsilon[\max_q U_{i,s}(q) + \epsilon_q + \tilde{V}_Q]] \\
&= \mathrm{E}_s[\mathrm{E}_\epsilon[U_{i,s}(q) + \epsilon_q + \tilde{V}_Q - log(P(s|i))]] \\
&= \mathrm{E}_s[U_{i,s}(q) + \tilde{V}_Q - log(P(s|i))] \\
&= \mathrm{E}_s[U_{i,s}(1) + \tilde{V}_Q - \log(P_{i,s}(1))] \qquad \text{(assuming max utility comes from ordering)} \\
&= \mathrm{E}_s[U_{i,s}(1) - \log(P_{i,s}(1))] + \tilde{V}_Q \\
&= f_i + \tilde{V}_Q
\end{aligned}
$$

2. To express $P_{i,s}$ in terms of U and f note that we can apply Gumbel properties and divide into two parts as follows:

$$
\begin{aligned}
\tilde{v}_{i,s}(1) &= U_{i,s}(1) + \tilde{V}_Q \\
\tilde{v}_{i,s}(0) &= U_{i,s}(0) + \tilde{V}_{i-s}
\end{aligned}
$$

And then,

$$\begin{aligned}\mathbf{P}_{i,s}(1) &= \frac{\exp(v_{i,s}(1))}{\exp(v_{i,s}(0)) + v_{i,s}(1))} \\ &= \frac{1}{1+\exp(v_{i,s}(0) - v_{i,s}(1))} \\ &= \frac{1}{1+\exp(U_{i,s}(0) + \tilde{V}_{i,s} - U_{i,s}(1) + \tilde{V}_Q)}\end{aligned}$$

3. First, we would estimate the distribution function of the demand based on the demand data, i.e. we obtain the empirical demand distribution function. Second, we would obtain the parameters by maximizing the log-likelihood function. Then, we would estimate the critical fractile solution by using the empirical demand distribution function and the estimated parameters. As for the shipping costs, given that we are observing the inventory level, the sales and the quantity ordered (implied by the inventory level), we can roughly estimate the fixed costs η_q by assuming the agent used optimal q^* when ordering inventory.