OPNS 523 - Assignment: Rosenbaum (2013)

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Question 1

The expression $\mathrm{E}[N_r(x,y)]$ can be interpreted as the minimum value, over the set of possible firms that can fulfill contract r by producing part p, of firm f's expected per-unit cost to supply a given contract r given it's location over (x,y). Moreover, we can derive a closed-form expression for $\mathrm{E}[N_r(x,y)]$ in terms of $\overline{C}_{rf}$ as follows:

$$
\begin{aligned}
\mathrm{E}[N_r(x,y)] &= \mathrm{E}[\min_{f' \in \mathcal{F}_{p(r)}} C_{rf'}(x_f, y_f)] \\
&= \mathrm{E}[\min_{f' \in \mathcal{F}_{p(r)}} \overline{C}_r + \overline{C}_{rf} + \eta_{rf}] \\
&= \overline{C}_r + \mathrm{E}[\min_{f' \in \mathcal{F}_{p(r)}} \overline{C}_{rf} + \eta_{rf}] \\
&= \overline{C}_r - \mathrm{E}[\max_{f' \in \mathcal{F}_{p(r)}} -\overline{C}_{rf} - \eta_{rf}] \\
&= \overline{C}_r - LSE(-\overline{C}_{rf}) - \gamma
\end{aligned}
$$

Where on the forth equality we converted the minimization into a maximization and we assumed that η_{rf} followed a Gumbel distribution (since it is an independent extreme value distribution type-I). And the last equality was possible because of the Gumbel distribution preservation across maximization resulting in another Gumbel with parameter $LSE(-\overline{C}_{rf})$.

Question 2

The author uses a nested logit structure with two error terms because of the structure estimation that he is trying to model. His model is based on the need of inter-dependencies among different nests of choices in two different stages. In the first stage, each firm (supplier) choose a location for its plants ($\mathcal{I}$ nests of choices). On the second stage, now those suppliers compete for contracts to produce some p part in one (or many) of their plants (k^{th} element of the ith nest) with the lowest cost for the assembler. The two error terms are also solved at two different stages of the game: first we have η_{rf}, where η_{rf} is a contract/supplier form cost that resolves up front, meaning that it is unknown to all firms by the time the firm is choosing a plant location. Then we have ϵ_{ri} that represents the contract/supplier plant specific

cost that resolves after the firm has chosen a plant and before competing for a contract to produce some part p.