1. (a) (15 pts) Prove that if n/2 or more chips are bad, Snape cannot necessarily deter- mine which chips are good using any strategy based on this kind of pairwise test. Assume that the bad chips can conspire to fool Snape.

   (b) (15 pts) Consider the problem of finding a single good chip from among the n chips, assuming that more than n/2 of the chips are good. Prove that $\lfloor n/2 \rfloor$ pairwise tests are sufficient to reduce the problem to one of nearly half the size.

   Starting with n chips, sort the chips into pairs. Since we can guarantee that at least one chip is bad, discard any pairs that both report bad, or report conflicting status' about one another. The only pairs that are left are pairs that report (good, good). With the pairs that are remaining, allow 1 chip in each pair to be the "left" chip and the other to be the "right" chip. Since each pair either contains 2 good or 2 bad, discard the "left" chips and repeat the process above on the right chips.

   Each time we discard a (good, bad) or (bad, bad) pair we remove either 2 bad chips or 1 bad chip and 1 good chip. Either way, we never remove more good chips than bad chips so we may repeat this process until there are 1, 2, or 3 chips left. If there are 1 or 2 chips left, we can conclude that they are good. If there are 3 chips left, at most 1 may be bad in which case the 2 good chips will announce that when compared to the bad chip.

   $\lfloor n/2 \rfloor$ pairwise tests are sufficient to reduce the problem to one half the size because even if there are only (good, good) reports, (i.e. good chips being compared with good chips or bad chips being compared with bad chips), we may discard the "left" chips, which effectively divides the problem in half.

   (c) (15 pts) Prove that the good chips can be identified with $\Theta(n)$ pairwise tests, assuming that more than $n/2$ of the chips are good. Give and solve the recurrence that describes the number of tests.

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In (b) we showed that we can find at least one good chip when more than $n/2$ chips are good. After finding this good chip, we can compare it with every other chip exactly once and it will report whether the compared chip is good or bad.