Кузишин Максим

1) Біматричні ігри. Знайти
   - рівновагу по строгому домінуванні
   - ускладнену рівновагу по слабкому домінуванні
   - рівновагу в обмежених стратегіях
   - рівновагу Неша
   - рівновагу Штакельберга для першого та другого гравця

   Для ігор, які задані матрицями

$$\begin{pmatrix} (3,4) & (4,1) & (3,2) \\ (2,3) & (3,7) & (5,3) \\ (1,8) & (2,3) & (4,3) \end{pmatrix}$$

1. No strong domination
2. No weak domination
3. minmax
   $u_1(S_1=1)=3 \mid \alpha_1=0$ and $u_1(S_1=2)=2 \mid \alpha_1=1$ and $u_1(S_1=3)=1 \mid \alpha_1=2$
   $u_2(S_2=1)=3 \mid \alpha_1=0$ and $u_2(S_2=2)=1 \mid \alpha_1=1$ and $u_2(S_2=3)=2 \mid \alpha_1=2$

   maxmin
   $u_1(S_2=1)=3 \mid \alpha_1=0$ and $u_1(S_2=2)=4 \mid \alpha_1=1$ and $u_1(S_2=3)=5 \mid \alpha_1=2$
   $u_2(S_1=1)=4 \mid \alpha_1=0$ and $u_2(S_1=2)=7 \mid \alpha_1=1$ and $u_2(S_3=3)=8 \mid \alpha_1=2$
   $S_* = (1, 1)$
4. $b_2(S_1) = \{(1:1);\ (2:2);\ (3:1)\}$ $b_1(S_2) = \{(1:1);\ (1:2);\ (2:3)\}$
   Nash equilibrium: $S^* = NE(G) = \{(1:1)\}$
5. First player:
   a. $b_2(S_1) = \{(1:1);\ (2:2);\ (3:1)\}\, with\ u_1 = 3,\ u_1 = 3,\ u_1 = 1$

      Result: $S_1 = \{(1:1), (2:2)\}$

   Second player:

   b. $b_1(S_2) = \{(1:1);\ (1:2);\ (2:3)\}$ with $u_2 = 4, u_2 = 1, u_2 = 3$
      Result: $S_2 = \{(1:1)\}$

$$\begin{pmatrix} (0,4) & (1,2) & (1,4) & (0,2) & (2,1) & (3,4) \\ (1,0) & (2,1) & (4,2) & (4,3) & (0,1) & (3,1) \\ (4,0) & (0,2) & (4,3) & (2,3) & (4,3) & (0,1) \\ (3,2) & (0,1) & (4,0) & (4,0) & (1,0) & (0,4) \\ (1,3) & (2,3) & (1,1) & (3,0) & (4,0) & (1,3) \end{pmatrix}$$

1. No strong domination
2. No weak domination
3. Minmax:

   $u_1(S_1=1)=0 \mid \alpha_1=0$ and $u_1(S_1=2)=0 \mid \alpha_1=1$ and $u_1(S_1=3)=0 \mid \alpha_1=2$ and $u_1(S_1=4)=0 \mid \alpha_1=3$ and $u_1(\mathbf{S_1=5})=1 \mid \alpha_1=4$

   $u_2(S_2=1)=0 \mid \alpha_1=0$ and $u_2(\mathbf{S_2=1})=1 \mid \alpha_1=1$ and $u_2(S_2=3)=0 \mid \alpha_1=2$ and $u_2(S_2=4)=0 \mid \alpha_1=3$ and $u_2(S_2=5)=0 \mid \alpha_1=4$ and $u_2(\mathbf{S_2=6})=1 \mid \alpha_1=5$

   maxmin:

   $u_1(S_2=1)=3 \mid \alpha_1=0$ and $u_1(S_2=2)=2 \mid \alpha_1=1$ and $u_1(\mathbf{S_2=3})=4 \mid \alpha_1=2$ and $u_1(\mathbf{S_2=4})=4 \mid \alpha_1=3$ and $u_1(\mathbf{S_2=5})=4 \mid \alpha_1=4$ and $u_1(S_2=6)=3 \mid \alpha_1=5$

   $u_2(\mathbf{S_1=1})=4 \mid \alpha_1=0$ and $u_2(S_1=2)=3 \mid \alpha_1=1$ and $u_2(S_3=3)=3 \mid \alpha_1=2$ and $u_2(\mathbf{S_3=4})=4 \mid \alpha_1=3$ and $u_2(S_3=5)=3 \mid \alpha_1=4$

   $S_* = (3, 3)$
4. $b_1(S_2) = \{(3{:}1); (2{:}2); (5{:}2); (2{:}3); (3{:}3); (4{:}3); (2{:}4); (4{:}4); (3{:}5); (5{:}5); (1{:}6); (2{,}6)\}$
   $b_2(S_1) = \{(1{:}1); (1{:}3); (1{:}6); (2{:}4); (3{:}3); (3{:}4); (3{:}5); (4{:}6); (5{:}1); (5{:}2); (5{:}6)\}$
   Nash equilibrium: $S^* = NE(G) = \{(5{:}2); (3{:}3); (2{:}4); (3{:}5); (1{:}6)\}$
5. Stackelberg

   Second player:

   a. $b_2(S_1) = \{(1{:}1); (1{:}3); (1{:}6); (2{:}4); (3{:}3); (3{:}4); (3{:}5); (4{:}6); (5{:}1); (5{:}2); (5{:}6)\}$

   $$with\ u_2 = 4, u_2 = 3, u_2 = 3, u_2 = 4, u_2 = 3$$

   Result: $S_1 = \{(1{:}1); (1{:}3); (1{:}6); (4{:}6)\}$

   First player:

   b. $b_1(S_2) = \{(3{:}1); (2{:}2); (5{:}2); (2{:}3); (3{:}3); (4{:}3); (2{:}4); (4{:}4); (3{:}5); (5{:}5); (1{:}6); (2{,}6)\}$
   with $u_1 = 4, u_1 = 2, u_1 = 4, u_1 = 4, u_1 = 4, u_1 = 3$

   Result: $S_2 = \{(3{:}1); (2{:}3); (3{:}3); (4{:}3); (2{:}4); (4{:}4); (5{:}5)\}$

Знайти рівновагу Неша в мішаних стратегіях для гри

$$\begin{pmatrix} (4,3) & (5,1) & (6,2) \\ (2,1) & (8,4) & (3,6) \\ (3,0) & (9,6) & (2,8) \end{pmatrix}$$

$$P = (p_1; \ \ p_2; \ 1 - \ p_2 - p_2)$$

$$\begin{cases} x = 4p_1 + 5p_2 + \ 6 - 6p_1 - 6p_2 = 6 - 2p_1 - p_2 \\ x \ = 2p_1 + 8p_2 + \ 3 - 3p_1 - 3p_2 = 3 - p_1 + 5p_2 \\ x \ = 3p_1 + 9p_2 + \ 2 - 2p_1 - 2p_2 = 2 + p_1 + 7p_2 \end{cases}$$

$$3 - p_1 + 5p_2 = 2 + p_1 + 7p_2$$

$$p_2 = 0.5 - \ p_1$$

$$6 - 2p_1 - p_2 = 3 - p_1 + 5p_2$$
$$3 = p_1 + 6p_2 = p_1 + 3 - 6p_1$$

$$p_1 = -0.2; p_2 = 0.7; 1 - p_1 - p_2 = 0.5$$

Same for second player:

$$P = (p_1; \ \ p_2; \ 1 - \ p_2 - p_2)$$

$$\begin{cases} x = 3p_1 + p_2 + \ 2 - 2p_1 - 2p_2 = 2 + p_1 - p_2 \\ x \ = p_1 + 4p_2 + \ 6 - 6p_1 - 6p_2 = 6 - 5p_1 - 2p_2 \\ x \ = 6p_2 + \ 8 - 8p_1 - 8p_2 = 8 - 8p_1 - 2p_2 \end{cases}$$

$$2 + p_1 - p_2 = 6 - 5p_1 - 2p_2$$

$$p_2 = 4 - \ 6p_1$$

$$2 + p_1 - p_2 = 8 - 8p_1 - 2p_2$$
$$p_1 = \frac{2}{3}, p_2 = 0$$

$$p_1 = \frac{2}{3}; p_2 = 0; 1 - p_1 - p_2 = \frac{1}{3}$$

Дуополія Курно

Statement:

Demand function: $X = 16 - x_1 - x_2$

$$C_1 = 4x_1$$
$$C_2 = 6x_2$$

Solution:

$$Profit\ for\ player\ 2: P_2 = X * x_2 - C_2 \quad -> max$$
$$P_2 = x_2(10 - x_2 - x_1) \quad -> max$$
$$10 - 2x_2 - x_1 = 0$$
$$b_2(x_1) = x_2 = \frac{10 - x_1}{2}$$
$$Profit\ for\ player\ 1: P_1 = X * x_1 - C_1 \quad -> max$$
$$P_1 = x_1(12 - x_1 - x_2) \quad -> max$$
$$12 - 2x_1 - x_2 = 0$$
$$b_1(x_2) = x_1 = \frac{12 - x_2}{2}$$

$$\begin{cases} x_1 = \frac{12 - x_2}{2} \\ x_2 = \frac{10 - x_1}{2} \end{cases} \qquad \begin{cases} 2x_2 = 24 - 4x_1 \\ 2x_2 = 10 - x_1 \end{cases}$$
$$\boldsymbol{x_1 = \frac{14}{3};\ x_2 = \frac{8}{3}}$$

$$P_2^* = x_2(10 - x_2 - x_1) = \frac{64}{9}$$

$$P_1^* = x_1(12 - x_1 - x_2) = \frac{196}{9}$$

$$\boldsymbol{NE(G)} = \{\frac{64}{9}, \frac{196}{9}\}$$

Stackelberg equilibrium for the first player:

$$P_1 = x_1(12 - x_1 - x_2) \ -> max$$

$$b_2(x_1) = x_2 = \frac{10 - x_1}{2}$$

$$P_1 = x_1\left(12 - x_1 - \frac{10 - x_1}{2}\right) \ -> max$$

$$7 - x_1 = 0$$

$$x_1 = 7;\ x_2 = 1.5$$

$$P_1^* = 7.5 * 4 - 4 * 7 = 24.5$$

$$P_2^* = 7.5 * 1.5 - 6 * 1.5 = 5.25$$

Stackelberg equilibrium for the second player:

$$P_2 = x_2(10 - x_1 - x_2) \ -> max$$

$$b_1(x_2) = x_1 = \frac{12 - x_2}{2}$$

$$P_2 = x_2\left(10 - x_2 - \frac{12 - x_2}{2}\right) -> max$$

$$4 - x_2 = 0$$

$$x_1 = 4; \ x_2 = 4$$

$$P_1^* = 8*4 - 4*4 = 16$$

$$P_2^* = 8*4 - 6*4 = 8$$

Дуополія Бертрана

Statement:

$$x_1 = 8 - p_1 + p_2$$

$$x_2 = 5 - p_2 + p_1$$

$$C_1 = 4x_1$$

$$C_2 = 2x_2$$

Nash equilibrium

For the second player:

$$P_2 = p_2 x_2 - C_2 = (p_2 - 2)(5 - p_2 + p_1) -> max$$

$$5 - 2p_2 + p_1 + 2 = 0$$

$$p_2 = \frac{7 + p_1}{2}$$

For the first player:

$$P_1 = p_1 x_1 - C_1 = (p_1 - 4)(8 - p_1 + p_2) -> max$$

$$8 - 2p_1 + p_2 + 4 = 0$$

$$p_1 = \frac{12 + p_2}{2}$$

$$\begin{cases} p_1 = \frac{12 + p_2}{2} \\ p_2 = \frac{7 + p_1}{2} \end{cases}$$

$$4p_1 = 24 + 7 + p_1$$

$$p_1 = \frac{31}{3}$$

$$p_2 = \frac{26}{3}$$

$$\boldsymbol{NE(G)} = \{\frac{31}{3}, \frac{26}{3}\}$$

$$x_1^* = \frac{19}{3}$$

$$x_2^* = \frac{20}{3}$$

Stackelberg equilibrium:

$$\begin{cases} p_1 = \frac{12 + p_2}{2} \\ p_2 = \frac{7 + p_1}{2} \end{cases}$$

$$P_1 = p_1 x_1 - C_1 = 0.5(p_1 - 4)(23 - p_1) \ -> \ max$$

$$23 - 2p_1 + 4 = 0$$

$$p_1 = 13.5$$

$$p_2 = 10.25$$

$$x_1^* = 4.75; \ x_2^* = 8.25$$

$$P_1^* = 45.125; \ P_2^* = 68.0625$$

Stackelberg equilibrium for the other player:

$$P_2 = p_2 x_2 - C_2 = 0.5(p_2 - 2)(22 - p_1) \ -> \ max$$

$$22 - 2p_2 + 4 = 0$$

$$p_1 = 12.5$$

$$p_2 = 13$$

$$P_1^* = 72.25; \ P_2^* = 49.5$$