

Gravity Within Earth

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What is this?

A little bit of working for finding the gravity within Earth

The problem

I want to find the hydrostatic pressure within a self gravitation sphere of fluid. We give the sphere a density ρ_0 and radius R . Taking the navier stokes equation and ignoring all velocity related terms we get the hydrostatic balence:

$$\nabla p = \vec{g}\rho_0 \quad (1)$$

Due to azimuthal and polar symetries, p and \vec{g} have only radial dependence, with \vec{g} also only having a radial component. Thus,

$$\frac{\partial p}{\partial r} = g(r)\rho_0 \quad (2)$$

By applying gauss's law of gravitation we get that:

$$g(r) = G\frac{m(r)}{r^2}, \quad (3)$$

where $m(r)$ is the mass contained within a sphere of radius r and is given by $m(r) = \rho_0\frac{4}{3}\pi r^3$. Therefore,

$$\frac{\partial p}{\partial r} = \frac{2}{3}\pi G\rho_0^2(R^2 - r^2) \quad (4)$$

which we integrate to a depth of r to get:

$$P(r) = \frac{2}{3}\pi G\rho_0^2(R^2 - r^2) \quad (5)$$