Godanov Scheme

Maximilian Williams

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What is this?

A little discussion on the Godanov scheme

Problem

I was trying to find an explicit scheme for solving the advection-diffusion equation. The diffusion part is not an issue, so lets just look at the advection equation.

$$\frac{\partial T}{\partial t} + v\frac{\partial T}{\partial x} = 0 \tag{1}$$

We will employ the goodanove scheme to numerically solve this equation. We discritize $\frac{\partial T}{\partial t}$ as:

$$\frac{\partial T}{\partial t} = \frac{T_i^{n+1} - T_i^n}{\Delta t}. \tag{2}$$

Now here comes the trick with the Godanov scheme. If $v > 0$, then we use a upstream scheme:

$$\frac{\partial T}{\partial x} = \frac{T_{i-1}^n - T_i^n}{\Delta x}. \tag{3}$$

while if $v < 0$ we use a downwind scheme:

$$\frac{\partial T}{\partial x} = \frac{T_{i+1}^n - T_i^n}{\Delta x}. \tag{4}$$

In doing this, we never use information that travelled against the advection velocity v. The MEPDE for this scheme looks like:

$$\frac{\partial T}{\partial t} + v\frac{\partial T}{\partial x} =\mid v \mid \frac{\Delta x}{2}(1- \mid v \mid \frac{\Delta t}{\Delta x})\frac{\partial^2 T}{\partial x^2} + \mathcal{O}(\Delta x^2, \Delta t^2). \tag{5}$$

So, this scheme has some numerical diffusion, with coeffecient $\kappa' =\mid v_c \mid \frac{\Delta x}{2}(1- \mid v_c \mid)$, where we will use v_c to denote the maximum velocity in our problem. Lets overeastimate this a little so that $\kappa' =\mid v_c \mid \frac{\Delta x}{2}$. We have neglected the diffusion term in our advection diffusion equation, but lets add it in to see how this numerical diffusion affects our numerical solution:

$$\frac{\partial T}{\partial t} + v\frac{\partial T}{\partial x} = (\kappa + \kappa')\frac{\partial^2 T}{\partial x^2}, \tag{6}$$

So long as $\kappa >> \kappa'$ this numerical diffusivity is negligable and this scheme is accurate. However, this may require a very fine grid and so take lots of memory.