Numerically Solving Poissons Equation

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September 2021

Description of the problem

We wish to solve for the streamfunction ψ given the voricity ω (equation **??**) in a discritized domain $\mathcal{D}$ in equation **??**.

$$\Delta^2 \psi = -\omega \tag{1}$$

$$\omega = \nabla \times \psi \tag{2}$$

Finite Difference Approximation of derivatives

Given a function $f : R \to R$, the taylor series about the point $x \pm \delta x$ is given by equation **??**:

$$f(x \pm \delta x) = f(x) \pm f^{(1)}(x)\delta x + \frac{1}{2!} f^{(2)}(x)\delta x^2 + \mathcal{O}(\delta x^3), \tag{3}$$

where $f^{(n)}(x)$ is the n^{th} derivative of f evaluated at x. By rearanging for $f^{(1)}(x)$ and $f^{(2)}(x)$ in the $x + \delta$ and $x - \delta$ variations of equation **??**, the first and second order central finite difference is obtained:

$$f^{(1)}(x) = \frac{f(x + \delta x) - f(x - \delta x)}{2\delta x} + \mathcal{O}\delta x \tag{4}$$

$$f^{(2)}(x) = \frac{f(x + \delta x) - 2f(x) + f(x - \delta x)}{\delta x^2} + \mathcal{O}\delta x^2 \tag{5}$$

The Jacobi Method

We suppose that we know the vorticity ω in the domain $\mathcal{D}$ and have a guess ψ for the streamfunction.