

# CS 8803 LCS

## Project 1

Maya Iyer  
Email: mayaiyer@gatech.edu  
GTID: 903572496

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Honor Code: I commit to uphold the ideals of honor and integrity by refusing to betray the trust bestowed upon me as a member of the Georgia Tech community.

## 1 CNF Encoding of Einstein's Puzzle

### 1.0.1 Definition of Propositions

The following table illustrates the meaning of the propositions in the CNF.

1: color(1,red):	26: nation(1,british)	51: drink(1,tea)	76: cigar(1,prince)	101: pet(1,dog)
2: color(2,red):	27: nation(2,british)	52: drink(2,tea)	77: cigar(2,prince)	102: pet(2,dog)
3: color(3,red):	28: nation(3,british)	53: drink(3,tea)	78: cigar(3,prince)	103: pet(3,dog)
4: color(4,red):	29: nation(4,british)	54: drink(4,tea)	79: cigar(4,prince)	104: pet(4,dog)
5: color(5,red):	30: nation(5,british)	55: drink(5,tea)	80: cigar(5,prince)	105: pet(5,dog)
6: color(1,green):	31: nation(1,swedish)	56: drink(1,coffee)	81: cigar(1,blends)	106: pet(1,cat)
7: color(2,green):	32: nation(2,swedish)	57: drink(2,coffee)	82: cigar(2,blends)	107: pet(2,cat)
8: color(3,green):	33: nation(3,swedish)	58: drink(3,coffee)	83: cigar(3,blends)	108: pet(3,cat)
9: color(4,green):	34: nation(4,swedish)	59: drink(4,coffee)	84: cigar(4,blends)	109: pet(4,cat)
10: color(5,green):	35: nation(5,swedish)	60: drink(5,coffee)	85: cigar(5,blends)	110: pet(5,cat)
11: color(1,white):	36: nation(1,danish)	61: drink(1,water)	86: cigar(1,pallmall)	111: pet(1,bird)
12: color(2,white):	37: nation(2,danish)	62: drink(2,water)	87: cigar(2,pallmall)	112: pet(2,bird)
13: color(3,white):	38: nation(3,danish)	63: drink(3,water)	88: cigar(3,pallmall)	113: pet(3,bird)
14: color(4,white):	39: nation(4,danish)	64: drink(4,water)	89: cigar(4,pallmall)	114: pet(4,bird)
15: color(5,white):	40: nation(5,danish)	65: drink(5,water)	90: cigar(5,pallmall)	115: pet(5,bird)
16: color(1,blue)	41: nation(1,norwegian)	66: drink(1,beer)	91: cigar(1,bluemasters)	116: pet(1,horse)
17: color(2,blue)	42: nation(2,norwegian)	67: drink(2,beer)	92: cigar(2,bluemasters)	117: pet(2,horse)
18: color(3,blue)	43: nation(3,norwegian)	68: drink(3,beer)	93: cigar(3,bluemasters)	118: pet(3,horse)
19: color(4,blue)	44: nation(4,norwegian)	69: drink(4,beer)	94: cigar(4,bluemasters)	119: pet(4,horse)
20: color(5,blue)	45: nation(5,norwegian)	70: drink(5,beer)	95: cigar(5,bluemasters)	120: pet(5,horse)
21: color(1,yellow)	46: nation(1,german)	71: drink(1,milk)	96: cigar(1,dunhill)	121: pet(1,fish)
22: color(2,yellow)	47: nation(2,german)	72: drink(2,milk)	97: cigar(2,dunhill)	122: pet(2,fish)
23: color(3,yellow)	48: nation(3,german)	73: drink(3,milk)	98: cigar(3,dunhill)	123: pet(3,fish)
24: color(4,yellow)	49: nation(4,german)	74: drink(4,milk)	99: cigar(4,dunhill)	124: pet(4,fish)
25: color(5,yellow)	50: nation(5,german)	75: drink(5,milk)	100: cigar(5,dunhill)	125: pet(5,fish)

For example, proposition 1 describes if house 1 is red. Proposition 107 describes if house 2 has a cat. Each proposition just describes if a property belongs to a

certain house.

There are 25 propositions that describe color, one for each combination of color and house. Since there are 5 colors and 5 houses, we arrive at 25 propositions that are sufficient to describe every possible combination. We follow the same procedure for nationalities, drinks, cigars, and pets.

### 1.0.2 Initial Constraints

We must ensure every house has only one color, nationality, drink, cigar, and pet. Similarly, every color/nationality/drink/cigar/pet is unique, meaning, for example, only one house can be red. This is how we define our initial constraints.

We define 5 groups: color, nationality, drink, cigar, and pet. In each group, there are 5 categories. For example, in color, there are red, green, white, blue, and yellow. Each category belongs to exactly one house. We define this with multiple clauses. Let's use the group color as an example.

#### Formulae for Initial Constraints

At least one house is red:

$$color(1, red) \vee color(2, red) \vee color(3, red) \vee color(4, red) \vee color(5, red)$$

Two houses cannot be red:

$$\begin{aligned} &(\neg color(1, red) \vee \neg color(2, red)) \wedge (\neg color(1, red) \vee \neg color(3, red)) \wedge \\ &(\neg color(2, red) \vee \neg color(3, red)) \wedge (\neg color(1, red) \vee \neg color(4, red)) \wedge \\ &(\neg color(2, red) \vee \neg color(4, red)) \wedge (\neg color(3, red) \vee \neg color(4, red)) \wedge \\ &(\neg color(1, red) \vee \neg color(5, red)) \wedge (\neg color(2, red) \vee \neg color(5, red)) \wedge \\ &(\neg color(3, red) \vee \neg color(5, red)) \wedge (\neg color(4, red) \vee \neg color(5, red)) \wedge \end{aligned}$$

A house cannot be red and another color:

$$\begin{aligned} &(\neg color(1, red) \vee \neg color(1, green)) \wedge (\neg color(1, red) \vee \neg color(1, white)) \wedge \\ &(\neg color(1, red) \vee \neg color(1, blue)) \wedge (\neg color(1, red) \vee \neg color(1, yellow)) \wedge \\ &(\neg color(2, red) \vee \neg color(2, green)) \wedge (\neg color(2, red) \vee \neg color(2, white)) \wedge \\ &(\neg color(2, red) \vee \neg color(2, blue)) \wedge (\neg color(2, red) \vee \neg color(2, yellow)) \wedge \\ &(\neg color(3, red) \vee \neg color(3, green)) \wedge (\neg color(3, red) \vee \neg color(3, white)) \wedge \\ &(\neg color(3, red) \vee \neg color(3, blue)) \wedge (\neg color(3, red) \vee \neg color(3, yellow)) \wedge \\ &(\neg color(4, red) \vee \neg color(4, green)) \wedge (\neg color(4, red) \vee \neg color(4, white)) \wedge \\ &(\neg color(4, red) \vee \neg color(4, blue)) \wedge (\neg color(4, red) \vee \neg color(4, yellow)) \wedge \\ &(\neg color(5, red) \vee \neg color(5, green)) \wedge (\neg color(5, red) \vee \neg color(5, white)) \wedge \\ &(\neg color(5, red) \vee \neg color(5, blue)) \wedge (\neg color(5, red) \vee \neg color(5, yellow)) \end{aligned}$$

### 1.0.3 Constraints from Hints

There are also a series of hints in the puzzle from which we can derive more constraints. They fall into 4 categories: direct assignments, pair relationships, neighbors, and relative locations.

### Direct Assignments

There are three clues that fall under this category:

- The man living in the center house drinks milk.
- The Norwegian lives in the first house.
- The Norwegian lives next to the blue house.

For these, we just directly assign the propositions to true. For example, with the first clue, *The man living in the center house drinks milk.*, the milk property belongs to the 3rd house. This means we add a clause (73) to the formula.

### Pair Relationships

There are eight clues that fall under this category:

- The Brit lives in the red house.
- The Swede keeps dogs as pets.
- The Dane drinks tea.
- The green house's owner drinks coffee.
- The person who smokes Pall Mall rears birds.
- The owner of the yellow house smokes Dunhill.
- The owner who smokes Bluemasters drinks beer.
- The German smokes Prince.

To set up these relationships, we create a pairing. For example with *The Brit lives in the red house.* we create 10 clauses. Essentially we are saying either the British person is in the house, or it isn't red:

$$(-26 \vee 1) \wedge (26 \vee -1) \wedge (-27 \vee 2) \wedge (27 \vee -2) \wedge (-28 \vee 3) \wedge (28 \vee -3) \wedge (-29 \vee 4) \wedge (29 \vee -4) \wedge (-30 \vee 5) \wedge (30 \vee -5)$$

### Neighbors

There are three clues that fall under this category:

- The man who smokes Blends lives next to the one who keeps cats.
- The man who keeps the horse lives next to the man who smokes Dunhill.
- The man who smokes Blends has a neighbor who drinks water.

To set up these relationships, we create a pairing with every possible combination of neighbors. For example with *The man who smokes Blends has a neighbor who drinks water.*, we create 5 clauses. Essentially we pair up every combination of neighbors possible:

$$(-81 \vee 62) \wedge (-85 \vee 64) \wedge (-82 \vee 61 \vee 63) \wedge (-83 \vee 62 \vee 64) \wedge (-84 \vee 63 \vee 65)$$

### Relative Locations

There is only one clue that falls under this category:

- The green house is on the left of the white house.

For this clue, we do a similar thing with the neighbors. We create groupings to determine which pairs of houses allow the clue to be true. We create 16 clauses:

$$\begin{aligned}
& (-11 \vee -10) \wedge (-11 \vee -9) \wedge (-11 \vee -8) \wedge (-11 \vee -7) \wedge \\
& (-12 \vee -10) \wedge (-12 \vee -9) \wedge (-12 \vee -8) \wedge (-13 \vee -10) \wedge \\
& (-13 \vee -9) \wedge (-13 \vee -6) \wedge (-14 \vee -10) \wedge (-14 \vee -7) \wedge \\
& (-14 \vee -6) \wedge (-15 \vee -8) \wedge (-15 \vee -7) \wedge (-15 \vee -6)
\end{aligned}$$

## 2 Satisfying Assignment

-125	-93	-63	-31	3
-123	-92	-62	-30	9
-122	-91	-60	-29	15
-121	-90	-58	-27	17
-120	-89	-57	-26	21
-119	-87	-56	-25	28
-118	-86	-55	-24	35
-116	-85	-54	-23	37
-115	-84	-53	-22	41
-114	-83	-51	-20	49
-112	-81	-50	-19	52
-111	-80	-48	-18	59
-110	-78	-47	-16	61
-109	-77	-46	-14	70
-108	-76	-45	-13	73
-107	-75	-44	-12	79
-104	-74	-43	-11	82
-103	-72	-42	-10	88
-102	-71	-40	-8	95
-101	-69	-39	-7	96
-100	-68	-38	-6	105
-99	-67	-36	-5	106
-98	-66	-34	-4	113
-97	-65	-33	-2	117
-94	-64	-32	-1	124

### 3 Solution

house number	color	nationality	drink	cigar	pet
1	yellow	norwegian	water	dunhill	cat
2	blue	danish	tea	blends	horse
3	red	british	milk	pallmall	bird
4	green	german	coffee	prince	fish
5	white	swedish	beer	bluemasters	dog

The German person who lives in the green house, who drinks coffee, and who smokes Prince, owns the fish.