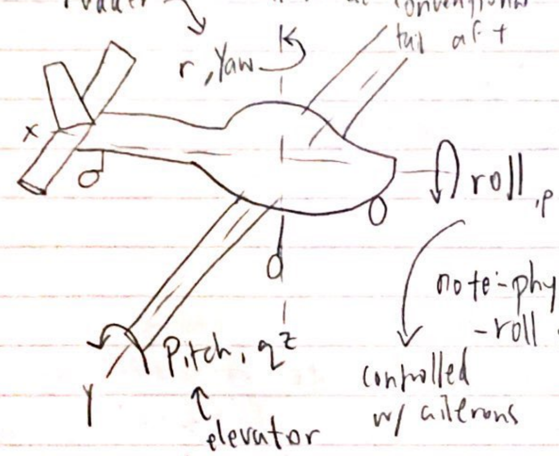
Anatomy of the Tail

Purpose: stability & control
Keep in mind: size, location, configuration, types of controls

all moveable ← conventional tail aft ← canard → three surface → flap

rudder



HT - controls pitch
VT - controls yaw (sometimes roll)

note - physics wise pitch can sometimes be indipendant
-roll & yaw always coupled

Note: control systems are in Miscellaneous design notes

Right hand rule: + roll angle = right wing down, left up
+ pitch = nose up
+ yaw = nose to the right

Physics things: aircraft must be staticly & dynamically stable
- Keep in mind: spin recovery

Tail Sizing
- Step 1: determine tail config ✓ $\boxed{\text{Conventional}}$
- Step 2: Estimate the geometry based on historical data
  - estimate HT & VT volumes
- Step 3: After aircraft designed: more sophisticated sizing:
  - reshaping will depend on

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CS: Control Surface

AOA Angle of Attack

AOY Angle of Yaw

AR: Aspect Ratio

KTAS knots true air speed

cruise + 5-10 KTAS

$1.2V_s$

Tail must be capable of:

- trimming @ low airspeeds @ a forward CG location
- trimming at high speeds @ aft CG location
  - be able to trim for cruise let-down
    - to begin & maintain decent @ cruise + 5-10 knots
- HT & VT must be of low Aspect Ratio, - reduces risk of tail stalls but must be responsive to AOA & AOY
  - HT: $3 < AR_{HT} < 5$
  - VT: $0.9 < AR_{VT} < 2$
- Rudder must not lock - usually + dorsal fin
- HT & VT must be strong enough to control plane during:
  - balking landing, crosswind landing etc. w/o excessive CS deflections
- HT must allow stalling bc if not: minimum speed > Stalling speed so will need: higher approach speeds, longer runways
- HT & VT must result in Stability derivatives
  - such as: ↓ to ensure aircraft is statistically & dynamically stable

C_{LW}	Wing lift coefficient	
$C_{L\alpha}$	Change in lift coefficient due to *AOA*	/deg or /rad
$C_{L\alpha HT}$	Change in horizontal tail lift coefficient due to *AOA*	/deg or /rad
$C_{l\beta}$	Change in coefficient of rolling moment due to sideslip angle	/deg or /rad
$C_{L\beta}$	Change in lift coefficient due to sideslip angle	/deg or /rad
$C_{L\delta e}$	Change in lift coefficient due to elevator deflection	/deg or /rad
$C_{L\delta f}$	Change in lift coefficient due to flap deflection	/deg or /rad
$C_{L\delta spoiler}$	Change in lift coefficient due to spoiler deflection	/deg or /rad
C_m	2D coefficient of moment	
C_{MGC}	Chord length of MGC	ft or m
C_{mo}	Coefficient of moment at zero *AOA*	/deg or /rad
C_{mq}	Change in coefficient of pitching moment due to pitch rate	
$C_{m\alpha}$	Change in coefficient of pitching moment due to *AOA*	/deg or /rad
$C_{m\beta}$	Change in coefficient of pitching moment due to sideslip angle	/deg or /rad
$C_{m\delta e}$	Change in coefficient of pitching moment due to elevator deflection	/deg or /rad
$C_{m\delta f}$	Change in drag coefficient of pitching moment due to flap deflection	/deg or /rad
C_{nr}	Change in coefficient of yawing moment due to yaw rate	
$C_{n\beta}$	Change in coefficient of yawing moment due to sideslip angle	/deg or /rad
$C_{n\delta r}$	Change in coefficient of yawing moment due to rudder deflection	
C_{REF}	Length of wing MGC	ft or m
C_{VT}	Chord length of vertical tail	ft or m
$C_{y\beta}$	Change in coefficient of side force due to sideslip angle	/deg or /rad

Scanned with CamScanner

- The HT or VT should not have detrimental impact on non-linear behavior of the ← stability derivatives @ high AOAs & AOYs
  - should be recoverable in spins or deep-stalls
- HT & VT must be designed for minimum structural weight
- Simple, reliable, & not require excessive CS deflection to manuever plane - excessiveness will cause stalling in surface
- not require excessive strength to move CS - CFR part 23.155 & .397
- cognisant of limitations: rudder doesn't interphere w/ tail, shorter tail = more torque to move it, etc.

↑ ↑ ↑

many of the avove are discussed in 23.3 : Preliminary aircraft design checklist

Fundamentals of Static Stability & Control

Mechanics

Dynamics ↙ / ↳ Statics

- equilibrium of matter
  - linear & angular acceleration $\neq 0$

- equilibrium of matter
  - linear & angular acceleration = 0

Aircraft Stability & Control: 6 deg of freedom motion

3 rotational ↲ ↳ 3 linear

- sum of individual components

$$C_m = \frac{M}{qS_c}$$

M ← pitching moment ⇒ torque produced by the aerodynamic force on the airfoil if that force is considered to be aplied not at center of pressure but @ aerodynamic center of airfoil

q ← dynamic pressure

S ← wing area

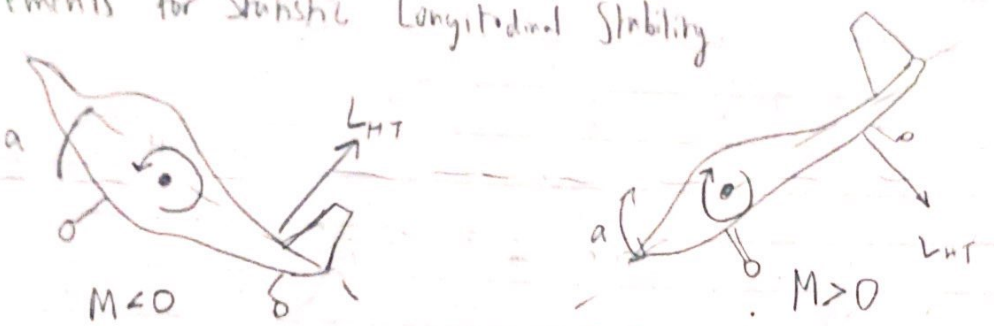
c ← length of chord

C_{mo} = where the pitching moment graph intersects the y axis

M = moment
a = AOA

C_{ma} = slope of pitching moment curve ← must be negative to be stable

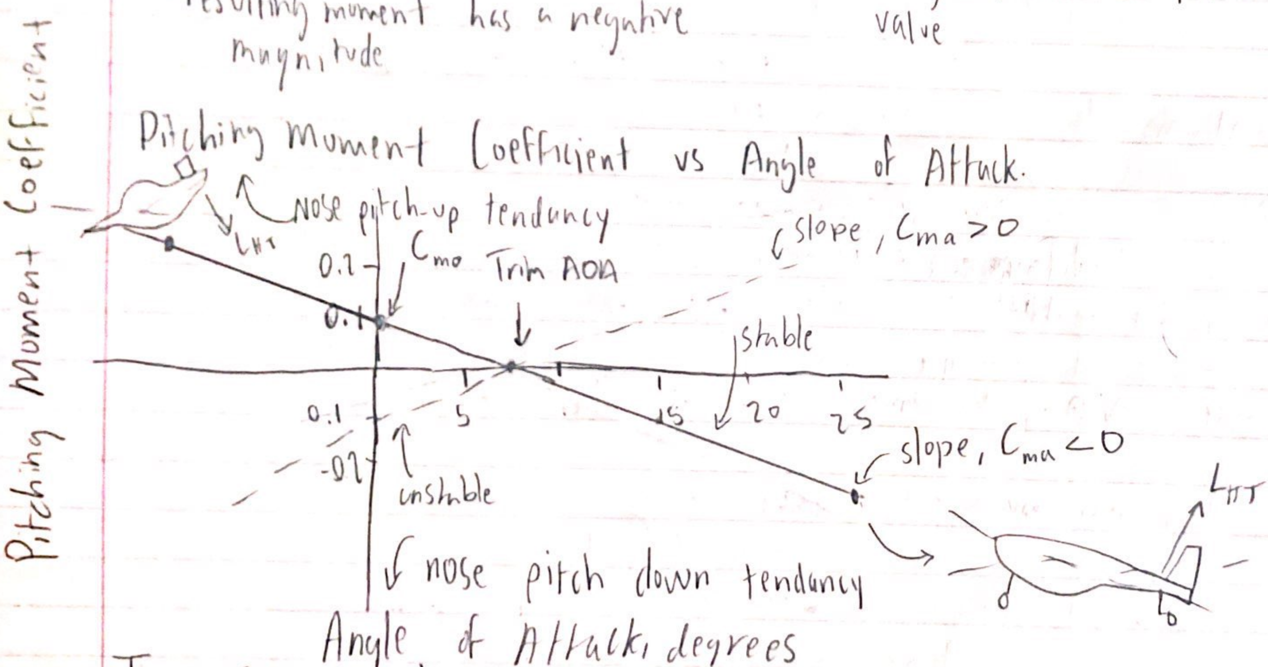
Requirements for Statistic Longitudinal Stability



High AOA
- HT generates a lift force L_{HT}
  - reduces AOA by lowering the nose
- resulting moment has a negative magnitude

LOW AOA
- HT generates lift in downwards direction
  - tendancy to increase AOA
- resulting moment has positive value

Pitching Moment Coefficient vs Angle of Attack.



Trim AOA: no tendancy for the HT to increase or decrease the AOA
Stable Aircraft: an airplane who's HT generates enough lift force to force the aircraft to a specific AOA

Note: the requirement for trimmability requires a negative camber:
$C_{mo} > 0$; positive: $C_{mo} < 0$

- Google Calculators

∂ = derivative

Trimmability:

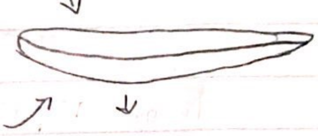
- the intersection to the y axis (C_m-axis), denoted by C_{m0}, must be larger than zero:

Requirement for static Stability:

where the slope intersects the y axis

$C_{m_\alpha} = \dfrac{\partial C_m}{\partial \alpha} < 0$ — pitching moment coefficient; angle of attack; their slope

$C_{m0} > 0$



Requirement for Static Trimability → Negative Camber

When $C_m = 0$: AoA > 0 ← meaning it can generate lift upwards & simultaniusly be staticly stable

$C_{m\alpha}$ is short hand for: $C_{m\alpha} = \dfrac{\partial C_m}{\partial \alpha} = \dfrac{\partial C_m}{\partial C_L}\dfrac{\partial C_L}{\partial \alpha} = \dfrac{\partial C_m}{\partial C_L} C_{L\alpha}$ — derivative of lift coefficient

Note: moment coefficients ≠ force coefficients

- C_m can refer to pitching moment of 2D aerofoil or 3D moment of a wing
  - distinction made by context

Notes: Pitching moment Coefficient: $\dfrac{\text{moment (M)}}{\text{dynamic pressure}}$

- dynamic pressure: $\dfrac{KE}{\text{volume of a fluid}}$
  - lift is proportional to ↑ & wing area

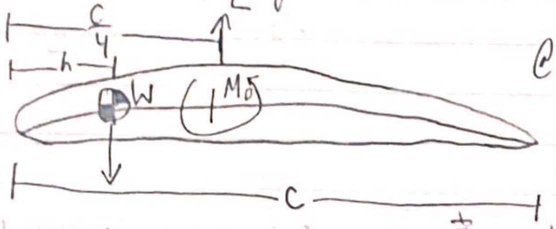
Lift Coefficient: geometric stuff + angle of attack

Lift = constant · (geometric stuff) · $\underbrace{\text{constant} \cdot \text{density} \cdot (\text{velocity})^2 \cdot \text{area}}_{\text{flight stuff}}$ · AOA

⇓

Lift = constant

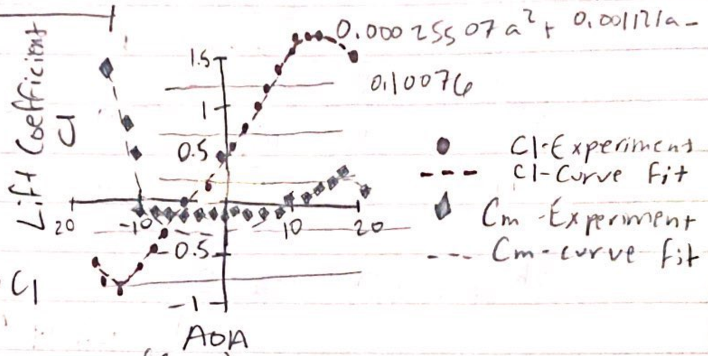
Example: Determine the pitching moment curve for the NACA 4412 airfoil:



@ c/4: $C_l(a) = 0.40575 + 0.11885a + 0.00028215a^2 - 0.00017205a^3$

$C_m(a) = -0.000012469a^3 + 0.00025507a^2 + 0.001121a - 0.10076$

q = dynamic pressure



Solution:

Lift of an Airfoil: $L = qC \cdot C_l$

Moment is given by: $M_0 = qC^2 \cdot C_l$

Sum of moment about the CG: $M = M_0 - L \cdot (\frac{c}{4} - h)$

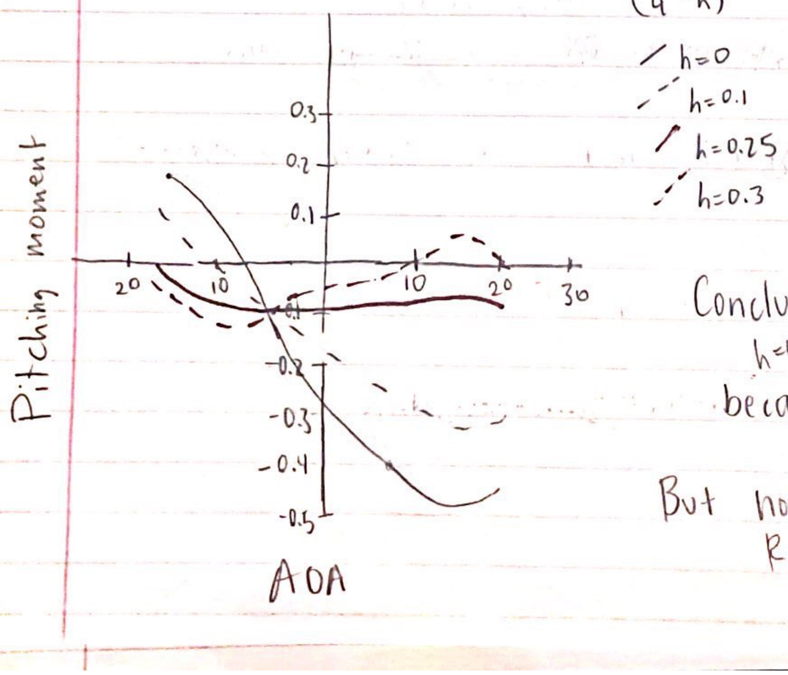
e-x-p-a-n-s-i-o-n

$$M = \frac{qC^2 \cdot Cm(a) - qC \cdot Cl(a) \cdot (\frac{c}{4} - h)}{qC}$$

$$Cm = C \cdot Cm(a) - Cl(a) \cdot (c/4 - h)$$

after cubic splines: $Cm = C \cdot [-0.000012469a^3 + 0.00255507a^2 + 0.001121a - 0.10076] +$

$\vec{C}$ $- [-0.00017205a^3 + 0.00028215a^2 + 0.11885a + 0.40575] \cdot (\frac{c}{4} - h)$

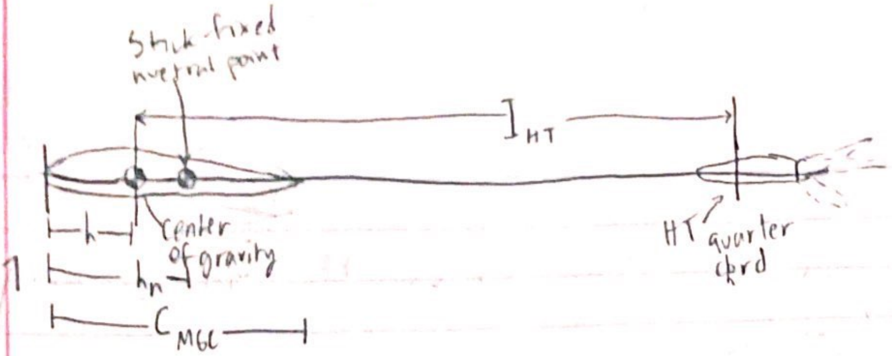


Conclusion:

h=0 is the greatest stability because the slope is the steepest

But none have a good trim ability?? Right??

δ_e = elevator deflection



Forces & Moments for Longitudinal Equilibrium:
- longitudinal equilibrium = lift = weight, drag = thrust, & pitching moment (C_m) = 0

Along x-axis:

$$D_W + D_{HT} + D_{VT} + D_{FUS} + D_{LDG} + \dots - T_{\cos\varepsilon} = 0$$

* subscripts refer to: wing, HT, VT, fuselage, landing gear, etc. *

Along Z axis:

$$L_W + L_{HT} + L_{FUS} + L_{LDG} + \dots + T_{\sin\varepsilon} = W$$

Moment about y-axis (pitch)

$$M_W + M_{HT} + M_{VT} + M_{FUS} + M_{LDG} + M_T + \dots = 0$$

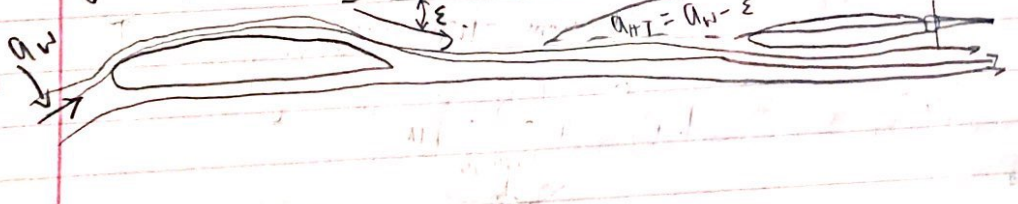
Common Expressions for Aerodynamic Coefficients.

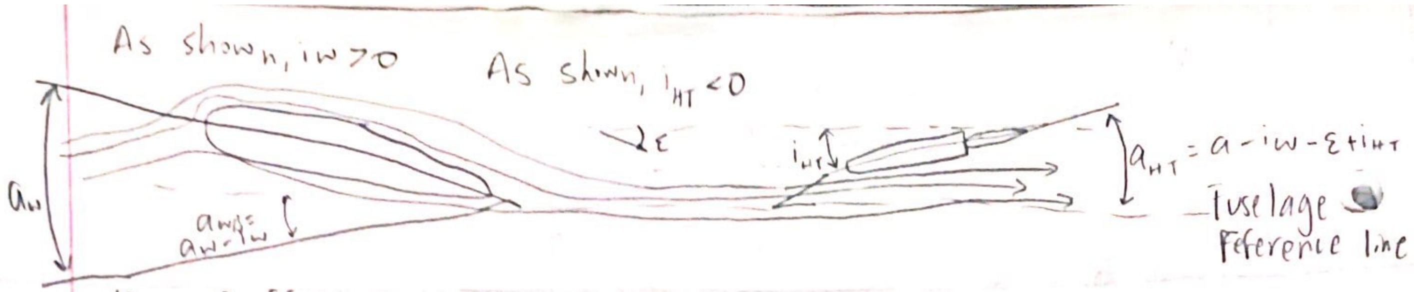
Note: - moments are always taken about the CG
- in equasions that follow: the lift, drag, & moment for the entire aircraft are expressed in terms of coefficients that are linear combinations of varius contributions

$$C_L = C_{L0} + C_{L\alpha}\alpha + C_{L\beta}\beta + C_{L\delta_e}\delta_e + C_{L\delta_f}\delta_f + C_{L\delta spoiler}\delta_{spoiler} + \dots$$

Lift Coefficient of the tail:

$$C_{LHT} = C_{L0_{HT}} + C_{L\alpha_{HT}} \cdot \alpha_{HT} + C_{L\delta e} \cdot \delta e + \dots$$



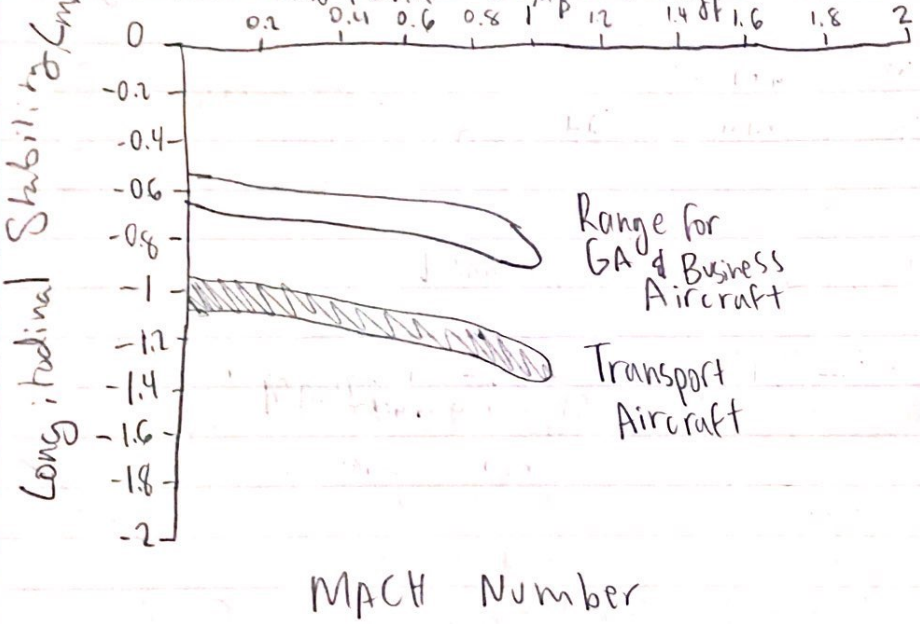


Drag Coefficient:

$$C_D = C_{D_{min}} + C_{D_a} a + C_{D_\beta} \beta + C_{D_{\delta_e}} \delta_e + C_{D_{\delta_f}} \delta_f + C_{D_{\delta_{spoiler}}} \delta_{spoiler} + \ldots$$

Pitching Moment Coefficient:

$$C_m = C_{m_0} + C_{m_a} \cdot a + C_{m_\beta} \cdot \beta + C_{m_{\delta_f}} \delta_f + \ldots$$



Modeling the Pitching Moment for a Simple Wing-HT system

- pitching moment curve: C_m ; function of the AoA & location of CG
- mean geometric chord: MGC ↓

physical position of CG: h
physical position of stick fixed nuetral point (defined later): h_n

$$C_m = C_{m_0} + C_{m_a} a = C_{m_0} + C_{L_a} \underbrace{\left(\frac{h - h_n}{C_{MGC}}\right)}_{\text{static margin}} a$$

List of consideration

Did take into account:
- volume coefficient
- aspect ratio

-37.4"

If CG moves behind neutral point : $h-h_n$ will be positive
= aircraft is unstable @ high AOA
= Cm slope will be positive
= you will die

Horizontal Tail Downwash Angle
- wing downwash will create AOA of the tail
- must be accounted for
- 10° AOA for wing is NOT 10° AOA for HT

Downwash Per Momentum Theory
- Simplest method to predict downwash behind the wing
- BUT assumes an elliptical planform & returns an average value
- in real flow : the magnitude varies w/ position in space
- ease of estimation & ok for conceptual & preliminary design
- NOT to position the height of the HT w/ respect to the wing

$$\varepsilon = \varepsilon_0 + \frac{d\varepsilon}{d\alpha}\alpha$$

derivitive - the change in downwash w/ AOA

residual downwash (when $\alpha = 0$) - only present if the wing features cambered airfoils

Also can be estimated using momentum theory, in terms of C_{L}:

units in radians $\rightarrow$ $\varepsilon = \frac{2C_{LW}}{\pi AR}$ ← lift coefficient of the wing

also

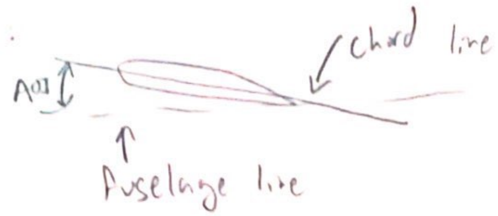
lift curve slope of the wing

$$\varepsilon = \varepsilon_0 + \frac{d\varepsilon}{d\alpha}\alpha = \frac{2C_{LW}}{\pi AR} = \frac{2}{\pi AR}\left(C_{L0W} + C_{L\alpha W}\,\alpha\right)$$

we can define the residual & AOA-dependant downwash:

$$\varepsilon_0 = \frac{2C_{L0W}}{\pi AR} \quad \& \quad \frac{d\varepsilon}{d\alpha} = \frac{2C_{L\alpha W}}{\pi AR}$$

AOI: angle of incidence.



However as seen from the derivative - the expression is an average of the entire column of air being deflected downward

-Real Flow: size & position of HT in space will cause downwash angle to vary along span

combining the equasions:

$$a_{HT} = a_W - \varepsilon = a_W\left(1 - \frac{2C_{L\alpha W}}{\pi AR}\right) - \frac{2C_{L0W}}{\pi AR}$$

but none of these conside AOI

most aircrafts: positive i_W & negative i_{HT}

$\uparrow$ AOI $\rightarrow \uparrow$ AOA $\downarrow$ AOA HT

$$a_{HT} = a_W - i_W - \varepsilon + i_{HT} \approx a_W\left(1 - \frac{2C_{L\alpha W}}{\pi AR}\right) - \frac{2C_{L0W}}{\pi AR} - i_W + i_{HT}$$

$$a_{HT} = a_{WB} - \varepsilon + i_{HT}$$

Note: cessna 182 Skylane's $C_{m\alpha}$ is -0.9

Longitudinal Equilibrium for any configuration:

note: these equasions determine AOA, elevator deflection & Thrust of steady flight (δe, T)

Along x-axis

$$C_{D_{min}} + C_{D\alpha}\alpha + C_{D\delta e}\delta e + C_{D\delta f}\delta f - \frac{T\cos\varepsilon_T}{qS} = 0$$

dynamic pressure

Thrust force angle

Along z-axis

$$C_{L0} + C_{L\alpha}\alpha + C_{L\delta e}\delta e + C_{L\delta f}\delta f + \frac{T\sin\varepsilon_T}{qS} = \frac{W}{qS}$$

Moment about y-axis (pitch)

$$C_{m0} + C_{m\alpha}\alpha + C_{m\delta e}\delta e + C_{m\delta f}\delta f + C_{mT} C_{mTN} = 0$$

Note: all coefficients are sums of their individual coefficients

Must be rearanged bc coefficients are sums:

along x-axis:

$$C_{D_\alpha}\alpha + C_{D_{\delta e}}\delta e - \frac{\cos \varepsilon T}{qS} T = -C_{D_{min}} - C_{D_{\delta f}}\delta f$$

Along z-axis:

$$C_{L_\alpha}\alpha + C_{L_{\delta e}}\delta e + \frac{\sin \varepsilon T}{qS} T = \frac{W}{qS} - C_{L0} - C_{L_{\delta f}}\delta f$$

However: propeller normal force serves as a problem for moment equations $\rightarrow C_{m_{TN}}$

Moment about y axis (pitch)

$$C_{m_\alpha}\alpha + C_{m_{\delta e}}\delta e + C_{m_T} + C_{m_{TN}} = -C_{m0} - C_{m_{\delta f}}\delta f$$

Can just treat $C_{m_{TN}}$ as a constant of average

Other option: solve for thrust use that to calculate normal force

In either case → can be rearanged:

$$C_{m_\alpha}\alpha + C_{m_{\delta e}}\delta e + \frac{z_T}{qSC_{MGC}} T = \frac{-T_N \cdot X_T}{qSC_{MGC}} - C_{m0} - C_{m_{\delta f}}\delta f$$

Matrix form:

$$\begin{bmatrix} C_{D_\alpha} & C_{D_{\delta e}} & \frac{-\cos \varepsilon T}{qS} \\ C_{L_\alpha} & C_{L_{\delta e}} & \frac{\sin \varepsilon T}{qS} \\ C_{m_\alpha} & C_{m_{\delta e}} & \frac{z_T}{qSC_{MGC}} \end{bmatrix} \begin{bmatrix} \alpha \\ \delta e \\ T \end{bmatrix} = \begin{bmatrix} -C_{D_{min}} - C_{D_{\delta f}}\delta f \\ \frac{W}{qS} - C_{L0} - C_{L_{\delta f}}\delta f \\ \frac{-T_N \cdot X_T}{qSC_{MGC}} - C_{m_0} - C_{m_{\delta f}}\delta f \end{bmatrix}$$

When solved: get the AOA, elevator deflection, & thrust required for a longitudinally stable flight

S = wing area

- Solution can be implamented w/ simple 3x3 Cramers' Rule
  - can be used to estimate suitable elevator deflection range & power requirements @ varius conditions

Typical values of the derivates for small planes.
note: all angles are in radians

$0.015 < C_{D_{min}} < 0.06$ $\quad$ $0 < C_{L_0} < 0.6$ $\quad$ $-0.2 < C_{m_0} < 0.2$

$C_{D_\alpha} = 2kC_{L_\alpha}C_L$ $\quad$ $3.0 < C_{L_\alpha} < 6.0$ $\quad$ $C_{m_\alpha} = C_{L_\alpha}(h_{CG} - h_{AC})$

$C_{D_{\delta e}} \approx \frac{0.001}{\delta e_{max}}$ $\quad$ $C_{L_{\delta e}} \approx \frac{0.2}{\delta e_{max}}$ $\quad$ $-0.5 < C_{m_{\delta e}} < -1$

$C_{D_{\delta f}} \approx \frac{0.01}{\delta f_{max}}$ $\quad$ $C_{L_{\delta f}} \approx \frac{0.4}{\delta f_{max}}$ $\quad$ $-0.7 < C_{m_{\delta f}} < -0.2$

Typical $\begin{cases} \delta e_{max}: 0.349 - 0.436 \text{ (radians)} \\ \delta f_{max}: 0.524 - 0.785 \text{ (radians)} \end{cases}$

Example 2:
Determine the α, δe, & T required for a stable level flight w/ flaps retracted @ S-L & (a) 180 KCAS ($q = 109.8\ lb_f/ft^2$) & (b) 75 KCAS ($q = 19.06\ lb/ft^2$)
The aircraft weighs $3400\ lb_f$, $S = 144.9\ ft^2$, $CMGC = 3.78\ ft$, $x_T = 13.06\ ft$, $z_T = -0.5\ ft$, $\varepsilon = 0°$, $k = 0.04207$ &

$C_{D_\alpha} = 0.0863$ $\quad$ $C_{D_{\delta e}} = 0.000175$ $\quad$ $C_{D_{min}} = 0.02541$ $\quad$ $T_N = 50\ lbf$

$C_{L_\alpha} = 4.8$ $\quad$ $C_{L_{\delta e}} = 0.355$ $\quad$ $C_{L_0} = 0.2$

$C_{m_\alpha} = -0.72$ $\quad$ $C_{m_{\delta e}} = -0.923$ $\quad$ $C_{m_0} = 0$

Note: all stability derivates are in radian
- Propelar efficiency η_p: 0.85 (high speed), 0.6 (low speed)

$\frac{\sin \varepsilon_T}{qS} \quad \underline{0}$

$-\frac{\cos \varepsilon_T}{qS} = \frac{-\cos(0)}{144.9\,ft^2 \cdot 109.8\,\frac{lbf}{ft^2}}$

$\frac{\overset{-0.5}{z_T}}{qSc_{MGC}}$; $q \to 109.8$, $S \to 144.9$, $c_{MGC} \to 3.78$

Solution:

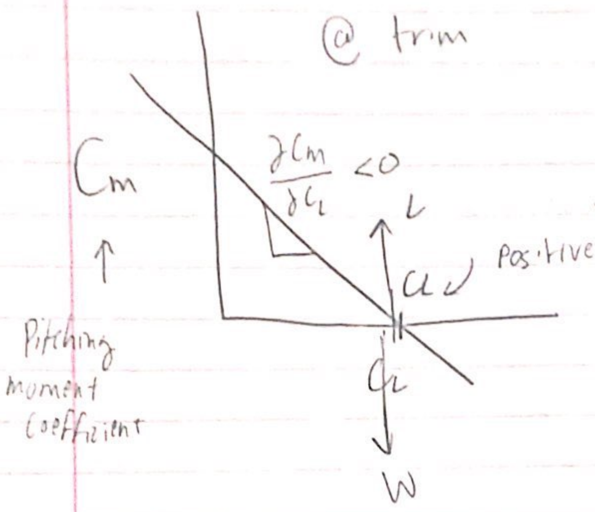
$$\begin{bmatrix} 0.0863 & 0.000175 & -\frac{1}{(144.9)(109.8)} \\ 4.8 & 0.355 & 0 \\ -0.72 & -0.923 & \frac{-0.5}{109.8(144.9)(3.78)} \end{bmatrix} \begin{Bmatrix} \alpha \\ \delta_e \\ T \end{Bmatrix} = \begin{bmatrix} -C_{D_{min}} - C_{D_{\delta f}}\delta f \\ \frac{W}{qS} - C_{L_0} - C_{L_{\delta f}}\delta f \\ \frac{T_N \cdot x_T}{qSc_{MGC}} - C_{m_0} - C_{m_{\delta f}}\delta f \end{bmatrix}$$

$-\underset{0.02541}{C_{D_{min}}} - \underset{0.000175}{C_{D_{\delta f}}}\,\overset{?}{\delta f}$

∂ = derivative

Static Stability

@ trim



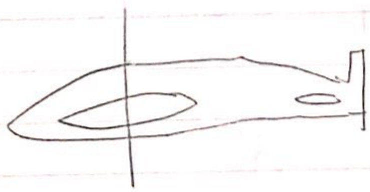
$\frac{\delta C_m}{\delta \alpha} < 0$ $\qquad$ $\frac{\partial C_m}{\partial C_L} = \frac{\partial C_m}{\partial \alpha} \cdot \frac{1}{\frac{\partial C_L}{\partial \alpha}}$

$C_{m_0} > 0$

to be Statically Stable.
$C_{m_0} > 0$

$\frac{\partial C_m}{\partial C_L}$ = Static Margin

Static Margin: distance between nuetral point & CG



Nuetral point: the position of center of gravity at which stability is nuetral
- a good reference point for CG

Nuetral point → N_p

$$h_n = h_0 + n_s V_s \left(\frac{a_s}{a_w}\right)\left(1 - \left(\frac{d\varepsilon}{d\alpha}\right)\right)$$ Buddy Gone

h_n = nuetral point

h_0 = aerodynamic center of the wing (typically 25% chord)

n_s = Stabalizer efficiency (typically 0.6 but 0.9 for T-tail)

V_s = Stabiliser volume coefficient

a_s = lift curve slope of Stabaliser

a_w = lift curve slope of wing

$\frac{d\varepsilon}{d\alpha}$ = change in stabiliser downwash angle vs change in wing AOA (typically 0.5-0.33)

$V_s = \frac{S_s L_s}{S_w c}$

S_s = stabaliser area

L_s = tail arm length

S_w = wing area

c = average chord of the wing

$$a = \frac{A \cdot a_0}{A + 18.25 a_0}$$ A = aspect ration

a_0 = lift slope for the infinite wing (0.11/degree)

Center of gravity <u>should</u> be right above or below center of lift

-should be infront of or at nuetral point

h_0 = aerodynamic center of the wing = 0.25

$n_s = 0.6$

$V_s = \frac{(3412)(76.85)}{(27648)\,6} = 1.58$ Stabalizer area = 3412.5 in^2

$\frac{48}{96} = 2$

$$a_s = \frac{(5.44)(0.11)}{(5.44 + 18.25 \cdot 0.11)} = 0.0803$$

$$a_w = \frac{2 \cdot 0.095}{(2 + 18.25 \cdot 0.095)} = 0.051$$

$$\frac{d\varepsilon}{d\alpha} = \frac{2}{\pi AR} \cdot \frac{d}{d\alpha}\left(C_{L_0} + C_{L_\alpha} \alpha \right) = \frac{2 C_{L_{\alpha w}}}{\pi AR}$$

0.4

Cl

wing lift curve slope

aspect ration (of wing?) (yes!)

Aircraft Lift curve slope

0.4

$Cl = Cl_0 + Cl_\alpha \alpha$

$-0.0024\alpha^2 + 0.102\alpha + 0.47 - 0.4$

$-0.0024\alpha^2 + 0.102\alpha + 0.07$

(0.5-0.33)

$(60+36)(0.971) - 30$ + shipping

$\rightarrow 33.6(.0971)\cdot 10$

$[(60)(0.971)-10]$ Gusset stuff $36(0.971)-10)$ shipping

hood T-shirts

hoodies

we lost 11.0744 / 2

Tail Gusset: Most in stress

Compression during landing $S_c = \frac{F}{A}$ ← Surface Area

Compressive Strength

- doesn't take into account thickness

Aft Tension on Strut: 1107.1193 lb

Fore: 889.85 lb/ft

$E = 9{,}990$ ksi

$S_c = 36000$ for 6061 t-6

Compression: 159.5 $\frac{lb}{ft}$

$\frac{159.5}{0.08} = \cancel{36000}$

~~Tear Out:~~ Compression

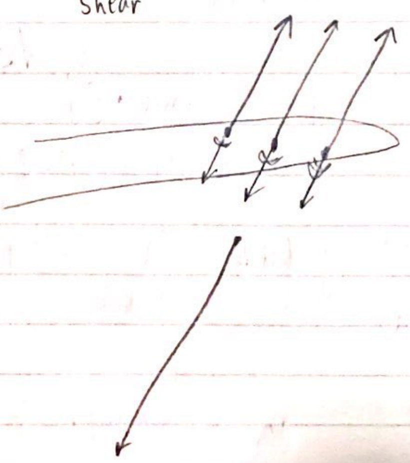
Bearing: $\frac{1107.12 \cdot 1.8}{(2)0.25(0.058)(x)} = 56000$ ~~min fs~~

safety

double shear

1/8"

bolts minimum



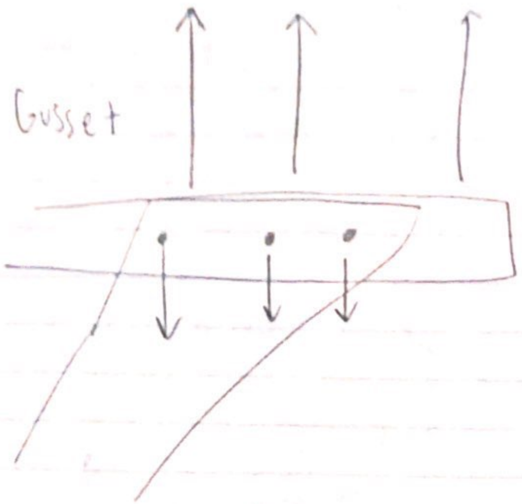
$\boxed{\text{Gusset: 1/8"} \quad \text{3 bolts}}$

$\frac{1107.12}{(0.058)\cdot 3\cdot x} = 24000$

1/3" away from edge ✓

$\frac{x\,4176 = 1107.12}{4176}$

Our Gusset



$\downarrow + \downarrow + \downarrow = 1107$

Empannage Tail:

$$\frac{1.8 \cdot 1185.3}{3/32 \cdot 4 \cdot x} = 56000$$

$$\frac{x \cdot 21000 = 2133.54}{21000}$$

$$\frac{1.8 \cdot 1185.3}{1/4 \cdot 2 \cdot x} = 56000$$

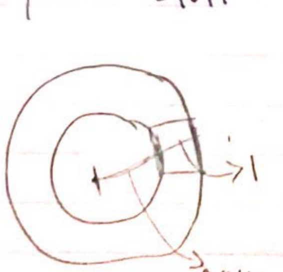
$$\frac{1.8 \cdot 1185.3}{\frac{5}{32} \cdot 2 \cdot x} = 56000$$

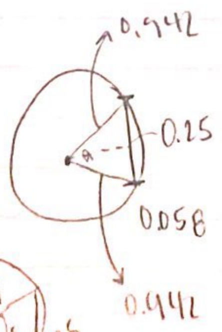
area: 0.0013846

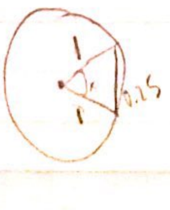
$(0.25 \cdot 0.058 - .0013846) + 0.001295$

0.1167

precalc stuff







$x = 14.361$

law of cosines:

$0.25^2 = 0.942^2 + 0.942^2 - 2(0.942)^2 \cos(a)$

$$\cos^{-1}\left(\frac{0.25^2 - 2(0.942^2)}{-2(0.942)^2}\right)$$

$a = 15.25$

area: $0.942 \cdot 0.942 \cdot \sin(15.25) \cdot \frac{1}{2}$

0.1

$56000 \cdot 1/4 \cdot$ 1185.3

43.48 46.52

A

$\cos(46.52)\frac{a}{3.5}$

Stress on bolt A: 2844.72

all w/ a Safety factor of 1.8

$\frac{3674.43 \cdot 1.8}{1/8 \cdot 1/4 \cdot 2}$

→ 601.8 + 2844.72

$\frac{1185.3 \cdot 1.8}{\frac{5}{32} \cdot 1/8}$

Max bearing stress:

1/16"

For 3/32 Rivet $\frac{x \cdot 1.8}{1/16 \cdot 3/32} = 56000$ $\frac{328.125}{1.8}$

182.3 lb

⇓

double shear: 364.6 lb

check:

↳ but tubing can only take 204.16 lb

For 1/8 Rivet : 243 ⇒ double shear: 486

↳ but tubing can only take 272.22

For 5/32 Rivet 303.82 ⇒ double shear: 607.6

↳ tubing 340.2

1/4" Bolt : 486.11 → double shear 972

↳ tubing 544.4

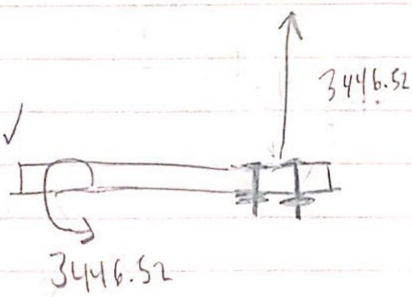
Rivet / Bolt Size	Gusset wall	Single Shear	tubing max
3/32 Rivet	1/16"	182.3 lb	204.16 lb
1/8 Rivet	1/16"	243 lb	272.2 lb
5/32 Rivet	1/16"	303.82 lb	340.2 lb
1/4" Bolt	1/16"	486.11 lb	544.4 lb
3/32 Rivet	1/8"		204.16
1/8 Rivet	1/8"		272.2
5/32 Rivet	1/8"		340.2
1/4" Bolt	1/8"		544.4

6061-t6

$\tau_{pt} = 24000$

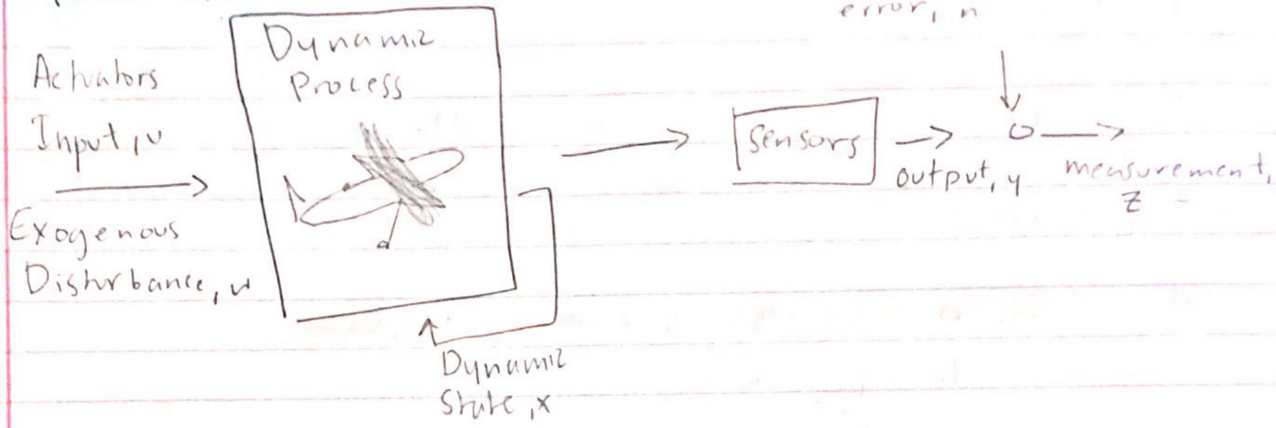
Bearing Stress Consideration:

$$\frac{3446.52}{\frac{\pi}{4}(0.75^2 - 0.25^2)} = 877.65 \checkmark$$



$$\frac{3446.52 \cdot 1.8}{\pi(0.5) \cdot 4 \cdot 0.032}$$

Dynamic Systems



Dynamic Process: current state depends on prior state.

- x = dynamic state
- u = input
- w = exogenous disturbance
- p = parameter
- t or k = time / event index

Observation Process: Measurement may contain error or be incomplete

- y = output (error-free)
- z = measurement
- n = measurement error

All these ( ) can be expressed as **vectors**

Notation for scalar: lower case

Vector: underlined ($\underline{a}$)

- ordered set
- column of scalars
- Dimension = n×1

ex. $\underline{y} = \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}$

Matrices & Transpose

Matrix: capital: $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

Transpose: interchange rows & columns: $A^T = \begin{bmatrix} a & c \\ b & d \end{bmatrix}$

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Point-Mass Dynamics & Aerodynamical/Thrust Forces

The Atmosphere

- Density & pressure decay exponentially w/ altitude
- Temp & speed of sound are piecewise-linear functions of altitude

Air Density, Dynamic Pressure, & Mach Number

ρ = air density, function of height

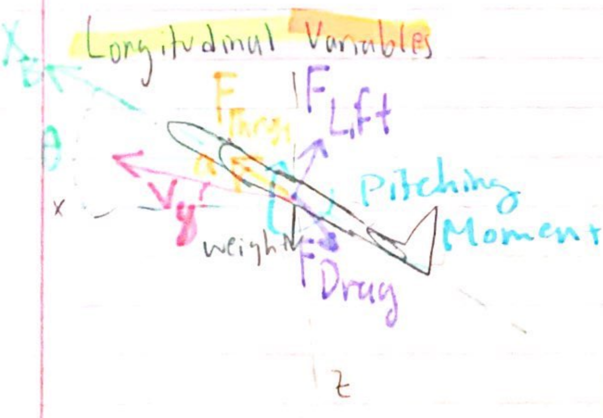
$= \rho_{\text{sea level}} \, e^{-\beta h}$

Dynamic Pressure: $\bar{q} = \frac{1}{2} \rho h V_{air}^2$

True airspeed must increase w/ altitute increase to maintain constant dynamic pressure

- not relevant to our ultralight

Longitudinal Variables



$V(t)$ = axial velocity - along plane centerline

$w(t)$ = normal velocity - perpendicular to cen

$V(t)$ = velocity magnitude - along net direction of flight

$\alpha(t)$ = AOA; angle btwn centerline & direction of flight

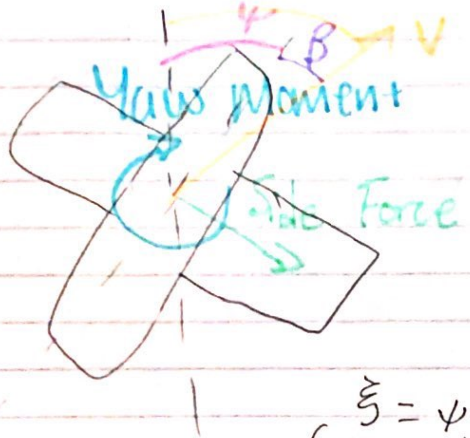
$\gamma(t)$ = flight path angle = angle between direction of flight & local horizontal

$\theta(t)$ = pitch angle = angle between centerline & local horizontal

$\gamma = \theta - \alpha$

(w/ wingtips level)

Lateral Direction & Variables



$$\xi = \psi + \beta$$
(w/ wingtips level)

$\beta(t)$: sideslip angle = angle between centerline & direction of flight
$\psi(t)$: yaw angle = angle between centerline & local horizontal
$\xi(t)$: heading angle = angle between direction of flight & compass reference (ex: north)
$\phi(t)$: roll angle : angle between true vertical & body z axis

Lift & Drag

Lift coefficient: $Lift = \frac{C_L \rho V_{air}^2 S}{2} \approx \frac{\left[C_{L0} + \frac{\partial C_L}{\partial \alpha} \alpha\right] \rho V_{air}^2 S}{2}$
- perpendicular to velocity vector

Drag: $\frac{C_D \rho V_{air}^2 S}{2} \approx \frac{\left[C_{D_0} + \varepsilon C_L^2\right] \rho V_{air}^2 S}{2}$

- drag components sum to produce total drag
  - parallel & opposed to velocity vector
    - skin friction, pressure differentials

Components:
- Parasite drag (friction)
- Wave Drag (shock-induced pressure differential)
- Induced Drag (due to lift generation)

Aerodynamic Center
- 25% chord

Lift Due to Elevator Deflection:

$$C_{L_{\delta E}} \triangleq \frac{\partial C_L}{\partial \delta E} = \tau_{ht}\, \eta_{ht} (C_{L\alpha}) \frac{S_{ht}}{S}$$

$$\Delta C_L = C_{L_{\delta E}}\, \delta E$$

τ_{ht} = Carryover effect
η_{ht} = tail efficiency factor
$(C_{L\alpha})_{ht}$ = Horizontal tail lift-coefficient slope

S_{ht} = Horizontal tail reference area

Lift variation due to elevator deflection

$$\Delta L = C_{L_{\delta E}}\, \bar{q} S\, \delta E$$

Induced Angle of Attack:

$$C_{D_i} = C_L \sin(\alpha_i)$$

induced drag

$$\alpha_i = \frac{C_L}{\pi e AR} \leftarrow \text{induced AOA}$$

$$\varepsilon = \frac{1}{\pi e AR} = \frac{(1+\delta)}{\pi AR}$$

Induced Drag:

$$C_{D_i} = \frac{C_L^2}{\pi e AR} \triangleq \frac{C_L^2 (1+\delta)}{\pi AR} \triangleq \varepsilon C_L^2$$

e = oswalt efficiency factor
= 1 for elliptical distribution
δ = departure from ideal elliptical lift distribution

Pitching Moment of Plane (moment about y axis)

Pressure & shear stress differential · moment arms
Integrate over the airplane surface to produce a net pitching moment

Body-Axis Pitching Moment - M_B

$$= -\iint_{\text{surface}} \left[\Delta P_z(x,y) + \Delta S_z(x,y)\right](x - x_{cm})\, dx\, dy$$

$$+ \iint \left[\Delta P_x(y,z) + \Delta S_x(y,z)\right] \Delta p_x (z - z_{cm})\, dy\, dz$$

Pitching Moment (moment about y-axis)
i = local centers of pressure

$$M_B = -\sum_{i=1}^{I} Z_i (x_i - x_{cm}) + \sum_{i=1}^{I} X_i (z_i - z_{cm})$$

+ interference effects + pure couples
Net effect expressed as

$$M_B = C_m \bar{q} S \bar{c}$$

$\bar{q}$ dynamic pressure, S wing surface area

Net Center of Pressure
nuetral point (net center of pressure) - local centers of pressure can be aggregated at this point
- along body x-axis

$$x_{cp_{net}} = \frac{\left[(x_{cp} C_N)_{wing} + (x_{cp} C_N)_{fuselage} + (x_{cp} C_N)_{tail} + \dots\right]}{C_{N_{total}}}$$

S = reference area

$\boxed{C_N = -C_Z}$ $\boxed{C_A = -C_X}$

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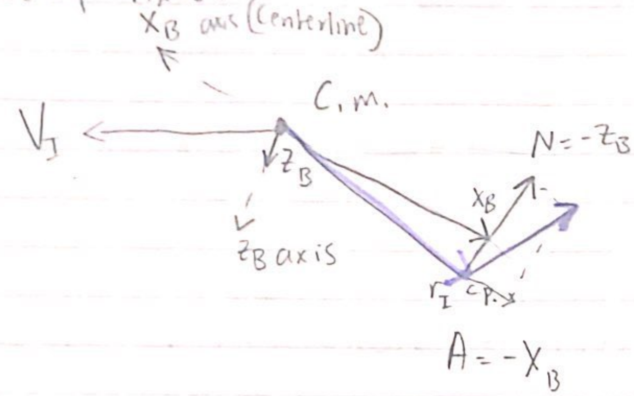
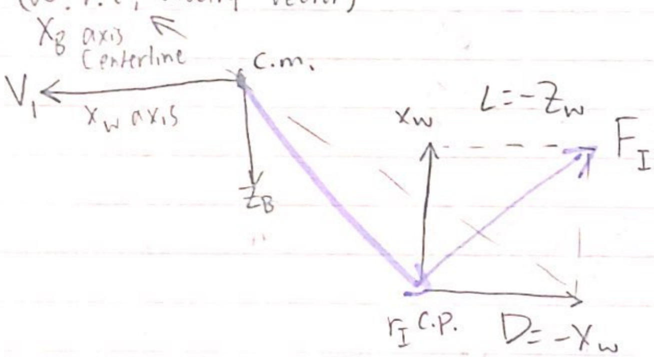
Body Axes:



Wind Axes
(w.r.t. velocity vector)



Static Margin: distance between center of mass (cm) & net center of pressure (cp)

- Body axes
- Normalized by mean aerodynamic chord
- Does not reflect z position of center of pressure
- Positive SM if cp is behind cm

$$\text{Static Margin} \triangleq SM = \frac{100\left(X_{cm} - X_{cp_{net}}\right)_B}{\bar{c}} \%$$

$$= 100\left(h_{cm} - h_{cp_{net}}\right) \%$$

$$h_{cm} \triangleq \frac{X_{cm}}{\bar{c}}$$

Effect of Static Margin on Pitching moment

- Trim angle of attack: zero crossing
  - i.e. Σ moments $= 0$
- Negative Slope required for static stability
- Slope $\frac{\partial C_m}{\partial \alpha}$ varies w/ static margin

$$M_B = (C_{m_0} + C_{m_\alpha} \alpha)\, \bar{q}\, S \bar{c} \qquad \boxed{\alpha_{Trim} = -\frac{C_{m_0}}{C_{m_\alpha}}}$$

Pitch-Moment Coefficient Sensitivity to angle of attack

For small AOA & no control deflection:

$$C_{m_\alpha} \approx -C_{N_{\alpha\, net}} (h_{cm} - h_{cp\, net}) \approx -C_{L_{\alpha\, net}} (h_{cm} - h_{cp\, net})$$

$$\approx -C_{L_{\alpha\, wing}} \left(\frac{x_{cm} - x_{cp\, wing}}{\bar{c}}\right) - C_{L_{\alpha_{ht}}} \left(\frac{x_{cm} - x_{cp\, ht}}{\bar{c}}\right)$$

Horizontal Tail lift Sensitivity

$$\left[\left(C_{L_{\alpha_{ht}}}\right)_{\substack{\text{horizontal} \\ \text{tail}}}\right]_{ref = S} = \left(C_{L_{\alpha_{ht}}}\right)_{ref = S_{ht}} \left(1 - \frac{\partial \varepsilon}{\partial \alpha}\right) \eta_{elas} \left(\frac{S_{ht}}{S}\right) \left(\frac{V_{ht}}{V_N}\right)^2$$

V_{ht} = airspeed at horizontal tail

ε = Down wash angle due to wing @ tail

$\frac{\partial \varepsilon}{\partial \alpha}$ = Sensitivity of downwash to angle of attack

η_{elas} : Aeroelasticity effect

Aerodynamic Center & Center of Pressure of a Wing

$x_{ac} = x$ for which $\frac{\partial C_m}{\partial a} = 0$

$\neq x_{cp}$ for asymmetrical airfoil

$= x_{cp}$ for symmetrical

Effect of Static margin on pitching moment

$$M_B = C_m \bar{q} S\bar{c} \approx [C_{m0} - C_{Na}(h_{cm} - h_{cp\,net})a]\bar{q} S \bar{c}$$

$$\approx [C_{m0} - C_{La}(h_{cm} - h_{cp\,net})a]\bar{q} S \bar{c}$$

$$M_B = (C_{m0} + C_{ma} a) q S \bar{c}$$

$= 0$ in trim

Typically: - Static margin is positive } for static
- $\frac{\partial C_m}{\partial a}$ is negative } pitch stability

Effect of Elevator Deflection of Pitching Coefficient

Control Deflection - shifts curve up & down, affecting trim angle of attack

$$M_B = (C_{m0} + C_{ma} a + C_{m_{\delta E}} \delta E)\bar{q} S \bar{c}$$

$$a_{Trim} = -\frac{1}{C_{ma}}(C_{m0} + C_{m_{\delta E}} \delta E)$$

Elevator deflection changes C_{m_0}

Tail moment sensitivity to AOA

$$C_{m_{\alpha_{ht}}} = -(C_{L_{\alpha_{ht}}})_{ht} \left(\frac{V_{ht}}{V_N}\right)^2 \left(1 - \frac{\partial \varepsilon}{\partial \alpha}\right) n_{elas} \left[\left(\frac{S_{ht}}{S}\right)\left(\frac{l_{ht}}{\bar{c}}\right)\right]$$

$$= -(C_{L_{\alpha_{ht}}})_{ht} \left(\frac{V_{ht}}{V_N}\right)^2 \left(1 - \frac{\partial \varepsilon}{\partial \alpha}\right) n_{elas} \mathbb{V}_{HT}$$

$$\mathbb{V}_{HT} = \frac{S_{ht} l_{ht}}{S \bar{c}} = \text{Horizontal Tail volume Ratio}$$

Normal Force coefficient variation due to elevator deflection

$$C_{L_{\delta_E}} \triangleq \frac{\partial C_L}{\partial \delta_E} = \tau_{ht} n_{ht} (C_{L_\alpha})_{ht} \frac{S_{ht}}{S} \approx C_{N_{\delta_E}}$$

$$\Delta C_N = C_{N_{\delta_E}} \delta_E$$

τ_{ht} = Carryover effect
n_{ht} = tail efficiency factor

$(C_{L_\alpha})_{ht}$ = Horizontal tail lift-coefficient Slope
S_{ht} = horizontal tail reference area

Pitching moment coefficient variation due to elevator deflection

$$C_{m_{\delta_E}} = C_{N_{\delta_E}} \frac{l_{ht}}{\bar{c}} \approx -\tau_{ht} n_{ht} (C_{L_\alpha})_{ht} \left(\frac{S_{ht}}{S} \cdot \frac{l_{ht}}{\bar{c}}\right)$$

$$= -\tau_{ht} n_{ht} (C_{L_\alpha})_{ht} \mathbb{V}_{HT}$$

Downwash & Elastizity also effect elevator sensitivity

$$\left[\left(\frac{\partial C_L}{\partial \delta_E}\right)_{ht}\right]_{ref=S} = \left(C_{L\delta_E}\right)_{ref=S} = \left(C_{L\delta_E}\right)_{ref=S_{ht}}\left(\frac{V_{tail}}{V_N}\right)^2\left(1-\frac{\partial \varepsilon}{\partial a}\right)\eta_{elas}\left(\frac{S_{ht}}{S}\right)$$

2.05×10^{-3} ; $84.39\ ft/s^2$

$$L = C_l \frac{\rho V^2}{2} A$$

37.4 ; $2\pi \sin(a - 0.03852)$; 23.69

$$37.4 = \cancel{2}\pi \sin(a-0.03852) \cdot \frac{(2.05\times10^{-3})(84.39)^2}{\cancel{2}} \cdot 23.69$$

$$\sin^{-1}\left(\frac{37}{\pi \cdot 23.69 \cdot (2.05 \cdot 10^{-3}) \cdot (84.39)^2}\right) = a - 0.03852$$

$$\sin^{-1}\left(\frac{37}{\pi \cdot 23.69 \cdot (2.05 \cdot 10 \times 10^{-3}) \cdot (84.39)^2}\right) + 0.03852 = a$$

$$1.99 = a$$

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