

# Assignment 2

Due by 12.5 noon

In this assignment, you are asked to implement a C program simulating the Nim game. The program simulates a game between a player and a computer, where the computer uses its own strategy, as explained below.

## Game play and illustration

The ordinary version of the game is between two players and played with  $N$  heaps of  $h_1, h_2, \dots, h_N$  objects. The two players alternately take any number of objects from the top of one of the heaps.

The winner is the one who empties all the heaps.

The following is an example of such a match. It has three heaps which initially contain three, four, and five tokens. Using its strategy, the computer wins.

Turn	Sizes of heaps			Moves
	$h_1$	$h_2$	$h_3$	
1	3	4	5	Computer takes 2 objects from heap 1
2	1	4	5	User takes 3 objects from heap 3
3	1	4	2	Computer takes 1 objects from heap 2
4	1	3	2	User takes 1 objects from heap 2
5	1	2	2	Computer takes 1 objects from heap 1
6	0	2	2	User takes 1 objects from heap 2
7	0	1	2	Computer takes 1 objects from heap 3
8	0	1	1	User takes 1 objects from heap 2
9	0	0	1	Computer takes 1 objects from heap 3
10	0	0	0	Computer wins

## Program implementation

The program asks for the number of heaps  $N$  by the printing:

```
"Enter the number of heaps:\n".
```

If the number of heaps is not between 1 and 32 the program prints `"Error: the number of heaps must be between 1 and 32.\n"` and terminates.

The program continues and asks for heaps sizes by printing

```
"Enter the heap sizes:\n"
```

If the  $i^{th}$  heap is a non-positive number ( $h_i \leq 0$ ) the program prints `"Error: the size of heap X should be positive.\n"` and terminates, where  $X$  is the corresponding value.

You can assume that the user enters only integers from  $-2^{31}$  to  $2^{31} - 1$ . Also, it is assumed that the computer begins the game.

Before a turn is being played, the program checks if the heaps are empty in which case it announces the winner with `"You win!\n"` or `"Computer wins!\n"` and terminates.

If the heaps are still not empty, the program prints the status of the heaps with the text

```
"In turn X the heap sizes are: h1=Y h2=Z ... hn=W.\n", where X, Y, Z and W are the actual corresponding values. Then either the user or the computer plays.
```

When the program reaches a user turn, it prints the following representation of the heaps:

$$h_1 = \begin{Bmatrix} * \\ * \\ \cdot \\ \cdot \\ \cdot \\ * \end{Bmatrix} \quad h_2 = \begin{Bmatrix} * \\ * \\ \cdot \\ \cdot \\ \cdot \\ * \end{Bmatrix} \quad \dots \quad h_N = \begin{Bmatrix} * \\ * \\ \cdot \\ \cdot \\ \cdot \\ * \end{Bmatrix}$$

For example, if  $N = 4$  and  $h_1 = 2, h_2 = 4, h_3 = 2, h_4 = 1$  the program prints:

```

      *
      *
    *  *  *
    *  *  *  *
```

The heaps are separated by the spaces obtained with the tab character (`'\t'`).

After printing the above representation, the user enters the heap index and the number of tokens to remove from it. The program prints the text "Your turn: please enter the heap index and the number of removed objects.\n". If the Input is valid, the program prints "You take X objects from heap Y.\n". If the user input is illegal, the program prints "Error: Invalid input.\nPlease enter again the heap index and the number of removed objects.\n". The program continues to the next turn until a valid input is provided.

In a computer turn, the program computes the heap index and the number of objects to remove according the computer's strategy (see below), and prints "Computer takes X objects from heap Y.\n".

## The computer's strategy

### Definitions

We denote the bitwise exclusive or (XOR) between two integers  $x, y$  as  $x \oplus y$  (see below a detailed definition of XOR). Let  $h_k^t$  be the size of heap  $k$  in turn  $t$ . The **Nim-sum** is defined to be the XOR of all heaps, that is:

$$s_{nim}^t = h_1^t \oplus h_2^t \oplus \dots \oplus h_n^t$$

The **Nim-sum of heap  $i$**  is the XOR of heap  $i$  with the Nim-sum:

$$h_i^t \oplus s_{nim}^t$$

Heap  $i$  is called a **winning heap** if its Nim-sum is less than its size, that is:

$$h_i^t \oplus s_{nim}^t < h_i^t$$

Otherwise the heap is called a **losing heap**.

### Computer strategy

The computer strategy at turn  $t$  is:

1. If  $s_{nim}^t \neq 0$  then find a winner heap  $i' = \min\{i \mid h_i^t \oplus s_{nim}^t < h_i^t\}$ 
  - a. Remove  $h_{i'}^t - h_i^t \oplus s_{nim}^t$  objects from heap  $i'$ . The heap size will be equal to its Nim-sum, i.e:  $h_{i'}^t \oplus s_{nim}^t$ .
2. Otherwise (i.e  $s_{nim}^t = 0$ ) then find  $i' = \min\{i \mid h_i^t > 0\}$  and take one object from heap  $i'$ .

An explanation of the algorithm's correctness can be found at the appendix at the end of the assignment.

## Implementation hints and guidelines

### XOR operation

In the C programming language, operations can be performed on a bit-level using bitwise manipulation. An example of using the bitwise XOR operation is given below:

```
int x=10, y=12, w=100, z=0;
z= x^y;
z^=w;
```

In the above example, four integer variables were defined, x,y,w and z. In line 2 the value of z is set to  $z = x \oplus y$  (now  $z=6$ ) and in line-3 the value of z is set to  $z = x \oplus y \oplus w$ , i.e 98.

### Using arrays in your code

Use constant size arrays to represent the heaps (whose size is defined using #define).

### Eclipse bug fix

As some of you have noticed, when working with eclipse on windows the console is not interacting properly. In this assignment, a file named "SPBufferSet.h" is provided to you. Please include this file from main.c and add the line "SP\_BUFF\_SET();" at the beginning of your main function as shown below:

```
#include <stdio.h>
#include "SPBufferSet.h"

int main() {
    SP_BUFF_SET();
    int x=0;
    printf("Please enter a number:\n");
    scanf("%d", &x);
    printf("x = %d", x);
    return 0;
}
```

## Files partitioning

Your code should be clear and well documented (i.e. contains comments and has reasonable variable names and function names). In this assignment you need to implement three source files:

- 1- `main.c` - This file must only contain the main function. Your main function should be as short as possible, - no more than 60 lines of code.
- 2- `sp_nim.c` - This file contains functions that are directly related to the Nim game. For instance you may want to implement the following functions in `sp_snim`:
  - A function that checks if there's a winner.
  - A function that helps you determines the computer's next move
- 3- `main_aux.c` - This file contains all auxiliary functions needed by the main function, such as:
  - A function that prints the game board
  - A function that checks the validity of an input

Notice that all functions should be as short as possible. A function that has more than 60 lines should be shrunk by cascading the work wisely to other functions. Furthermore you should define constants using macros (`#define`) and add comments in order to make your code more readable.

## Running Examples

In addition to the running examples, we add speech bubbles to clarify the status.

```
nova:~> nim
Enter the number of heaps:
4
Enter the heap sizes:
1 3 2 5
In turn 1 heap sizes are: h1=1 h2=3 h3=2 h4=5.
Computer takes 5 objects from heap 4.
In turn 2 heap sizes are: h1=1 h2=3 h3=2 h4=0.
*
*      *
*      *
Your turn: please enter the heap index and the number of removed objects.
1 1
You take 1 objects from heap 1.
In turn 3 heap sizes are: h1=0 h2=3 h3=2 h4=0.
Computer takes 1 objects from heap 2.
In turn 4 heap sizes are: h1=0 h2=2 h3=2 h4=0.
*      *
*      *
Your turn: please enter the heap index and the number of removed objects.
1 2
Error: Invalid input.
Please enter again the heap index and the number of removed objects.
4 2
Error: Invalid input.
Please enter again the heap index and the number of removed objects.
2 1
You take 1 objects from heap 2.
In turn 5 heap sizes are: h1=0 h2=1 h3=2 h4=0.
Computer takes 1 objects from heap 3.
In turn 6 heap sizes are: h1=0 h2=1 h3=1 h4=0.
*      *
Your turn: please enter the heap index and the number of removed objects.
2 1
You take 1 objects from heap 2.
In turn 7 heap sizes are: h1=0 h2=0 h3=1 h4=0.
Computer takes 1 objects from heap 3.
Computer wins!
```

$$\begin{aligned}
 s_{nim}^t &= 5 \\
 h_1^t \oplus s_{nim}^t &= 4 \\
 h_2^t \oplus s_{nim}^t &= 6 \\
 h_3^t \oplus s_{nim}^t &= 7 \\
 h_4^t \oplus s_{nim}^t &= 0
 \end{aligned}$$

$$\begin{aligned}
 s_{nim}^t &= 1 \\
 h_1^t \oplus s_{nim}^t &= 1 \\
 h_2^t \oplus s_{nim}^t &= 2 \\
 h_3^t \oplus s_{nim}^t &= 3 \\
 h_4^t \oplus s_{nim}^t &= 1
 \end{aligned}$$

$$\begin{aligned}
 s_{nim}^t &= 3 \\
 h_1^t \oplus s_{nim}^t &= 3 \\
 h_2^t \oplus s_{nim}^t &= 2 \\
 h_3^t \oplus s_{nim}^t &= 1 \\
 h_4^t \oplus s_{nim}^t &= 3
 \end{aligned}$$

```

nova:~> nim
Enter the number of heaps:
3
Enter the heap sizes:
1 2 3
In turn 1 heap sizes are: h1=1 h2=2 h3=3.
Computer takes 1 objects from heap 1.
In turn 2 heap sizes are: h1=0 h2=2 h3=3.
      *
    *   *
    *   *

Your turn: please enter the heap index and the number of removed objects.
3 1
You take 1 objects from heap 3.
In turn 3 heap sizes are: h1=0 h2=2 h3=2.
Computer takes 1 objects from heap 2.
In turn 4 heap sizes are: h1=0 h2=1 h3=2.
      *
    *   *

Your turn: please enter the heap index and the number of removed objects.
3 1
You take 1 objects from heap 3.
In turn 5 heap sizes are: h1=0 h2=1 h3=1.
Computer takes 1 objects from heap 2.
In turn 6 heap sizes are: h1=0 h2=0 h3=1.
      *

Your turn: please enter the heap index and the number of removed objects.
3 1
You take 1 objects from heap 3.
You win!

```

$$s_{nim}^t = 0$$

$$s_{nim}^t = 0$$

$$s_{nim}^t = 0$$

## Submission guidelines

Please submit a zip file named **id1\_id2\_assignment2.zip** where id1 and id2 are the ids of the partners. The zipped file must contain the following files:

- Header files - main\_aux.h and sp\_nim.h
- Source files - main.c main\_aux.c and sp\_nim.c
- partners.txt – This file contains the full names, ids and moodle usernames of both partners. Please follow the pattern in the assignment files (**do not change the pattern**)
- makefile – the makefile provided in the assignment files. (Don't change it!)

## Compilation

In order to compile your code on NOVA you need to use the makefile provided to you in the assignment zip file. Please notice that after invoking the 'make' command the resulting executable name is nim. Please refer to assignment 1 for more information.

## Check your code

Input/output examples are provided in the zip file. You should check your code by using the "diff" function as was shown in assignment 1.

## Appendix

The logic behind the definition of winner heaps is that reducing a winner heap from  $h_i^t$  to  $s_{nim}^t \oplus h_i^t$  objects reduces the Nim-sum to zero i.e.  $s_{nim}^{t+1} = 0$ . If on the other hand a player chooses a losing heap, the Nim-sum in the next turn will be different than zero (i.e.  $s_{nim}^{t+1} \neq 0$ ).

There might be cases where a player might be forced to choose a losing heap: if the Nim-sum equals  $s_{nim}^t = 0$  then  $(s_{nim}^t \oplus h_i^t) = h_i^t$  and therefore every heap is a losing heap. The following theorem proves that this is the only scenario.

**Theorem:** There exists a winner heap in turn  $t$  iff the Nim-sum is different from zero, i.e.  $s_{nim}^t \neq 0$ .

Assume that turn  $t$  is a computer turn with a Nim-sum of  $s_{nim}^t \neq 0$ . The computer strategy is to find a winning heap, and to reduce its size from  $h_i^t$  to  $s_{nim}^t \oplus h_i^t$ . By doing so, the computer ensures that the user chooses a losing heap and the Nim-sum in the next computer turn is different from zero  $s_{nim}^{t+2} \neq 0$ .

The winner chooses the last set of tokens and must have a nim sum different than zero. Thus, if there exists a computer turn where the Nim-sum is different than zero, the computer wins.

For the automatic testing, you can assume the following assumptions:

1. If  $s_{nim}^t \neq 0$ , the computer must remove objects from the winner heap of minimal index i.e.  $\min\{i | h_i^t \oplus s_{nim}^t < h_i^t\}$
2. If  $s_{nim}^t = 0$  then the computer finds the non-zero heap of minimal index,  $\min\{i | h_i^t \neq 0\}$ , and removes from it a single object.

Good Luck!