

# Mathematics II

Functions

Conducted by : Chamith Jayasinghe



**SLIIT**  
**ACADEMY**

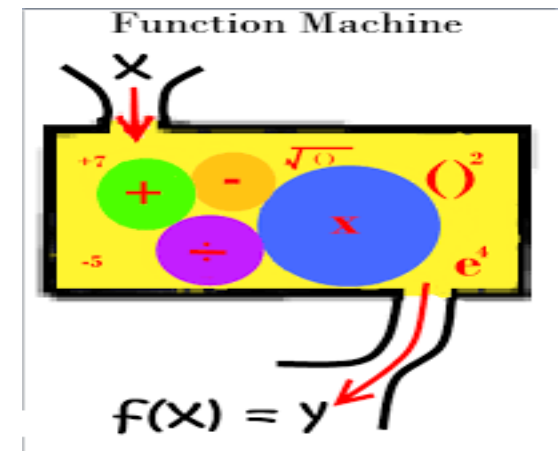
# What is a function?

A function relates an input to an output. It is like a machine that has an input and an output. And the output is related somehow to the input.

$F(x)$  is the classic way of writing a function

There are always three main parts of a function

- The input
- The relationship
- The output



# Input Relationship Output

$f(x)$  shows us the function is called " $f$ ", and " $x$ " goes in

## Example

$f(x) = x^2$  is the function " $f$ " takes " $x$ " and squares it.

Input ( $x$ )  
Relationship (squaring)  
Output ( $y$ )

A function *relates* an input to an output.

$$f(4) = 16$$
$$4 \rightarrow 16$$

This tree grows 20 cm every year, so the height of the tree is *related* to its age using the function ***h***



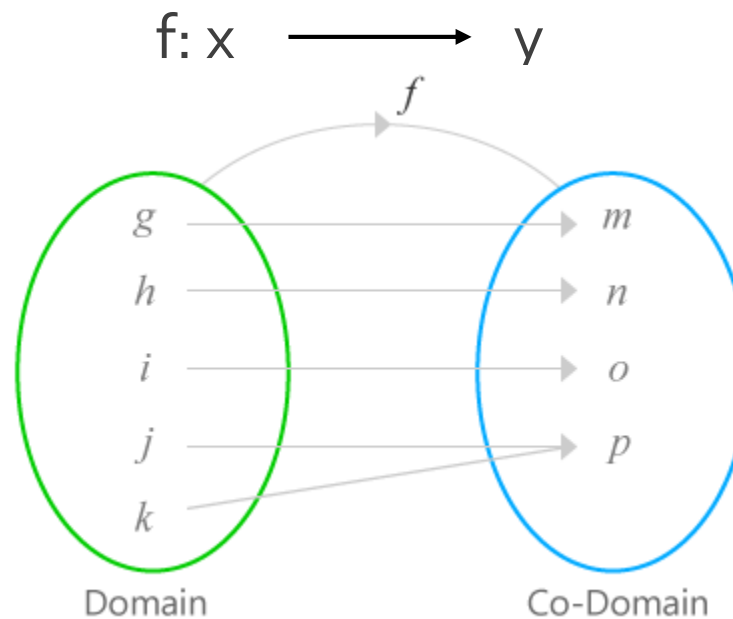
- **$h(\text{age}) = \text{age} \times 20$**

If the age is 10 years, the height is:

**$$h(10) = 10 \times 20 = 200 \text{ cm}$$**

# Definition

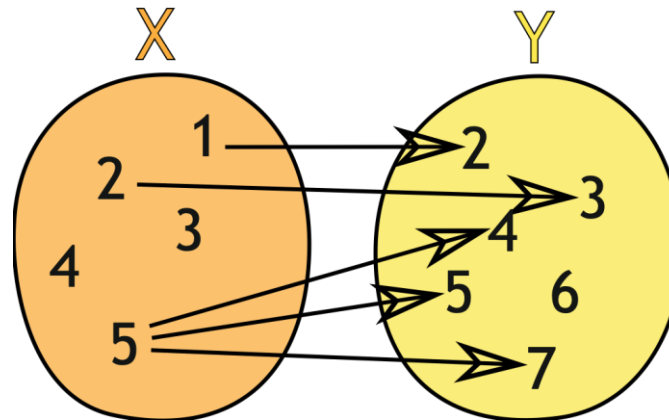
A function  $f$  from a set  $X$  to a set  $Y$  is a relationship between elements of  $X$  and elements of  $Y$  with the property that each element of  $X$  is related to a unique element of  $Y$ .



## Key Point 1

(1)

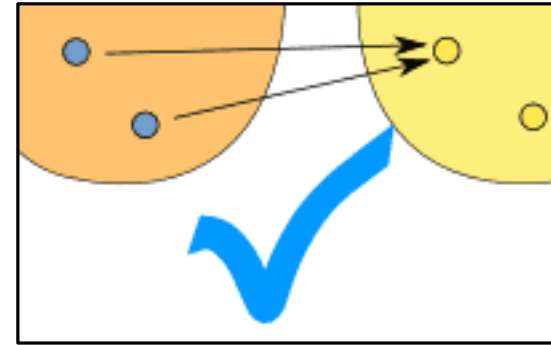
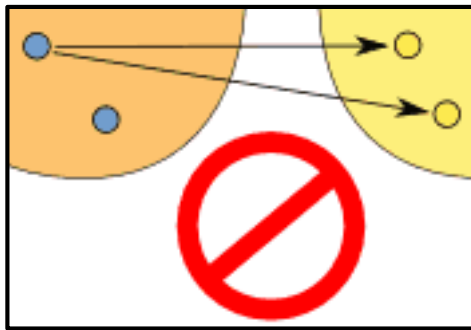
"...each element..." means that every element in  $X$  is related to some element in  $Y$ .



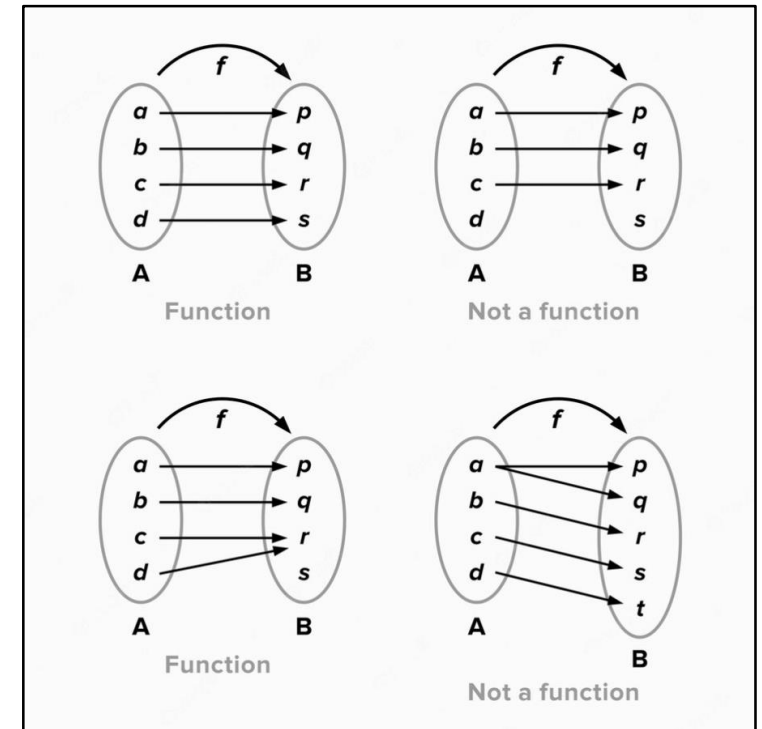
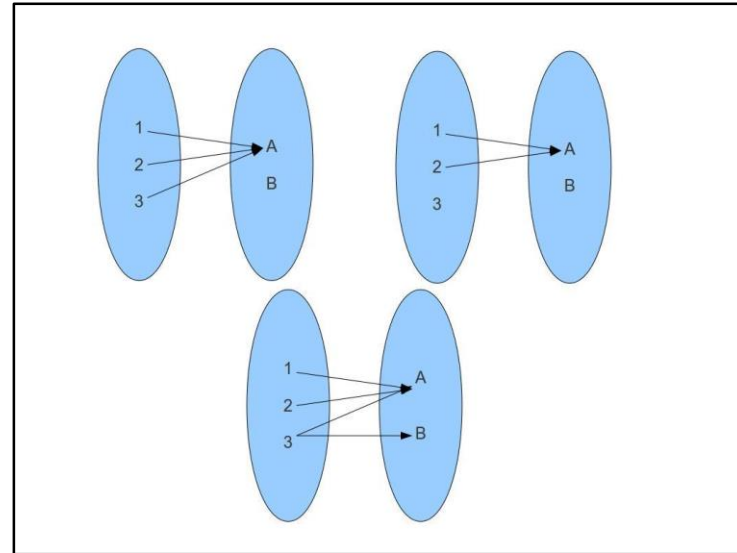
## Key Point 2

- "...exactly one..." means that a function is *single-valued*. It will not give back 2 or more results for the same input.

One-to-many" is **not** allowed, but "many-to-one" **is** allowed

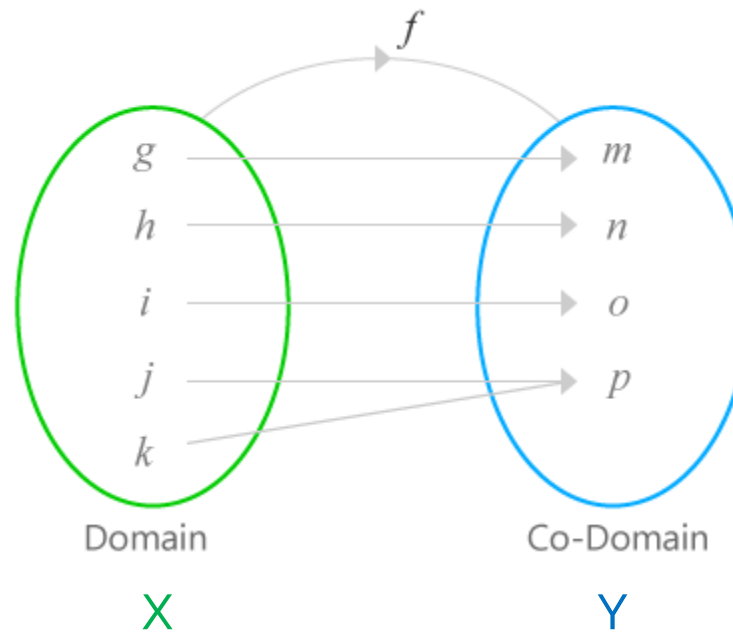


# Name functions





# Domain, Co-domain, Range

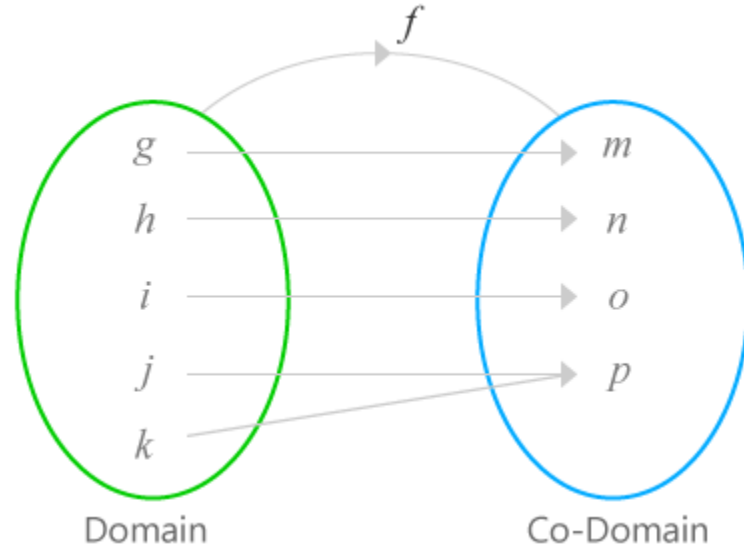


the set "X" is called the **Domain**

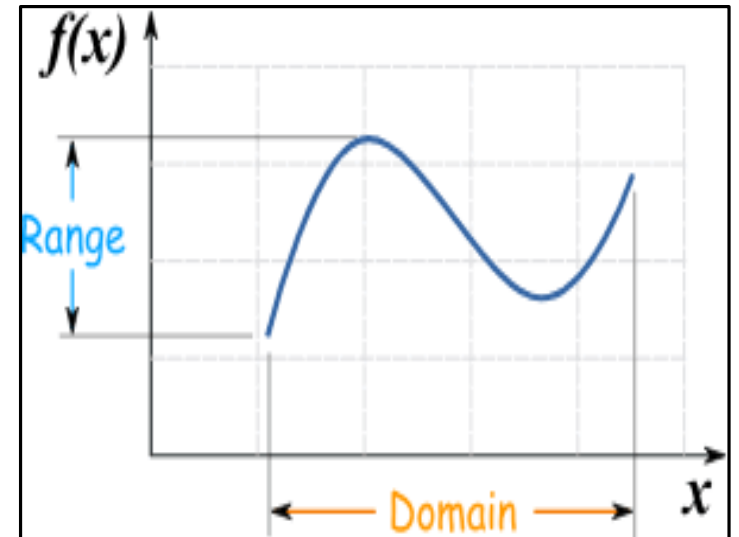
the set "Y" is called the **Codomain**

the set of elements that get pointed to in Y (the actual values produced by the function) is called the **Range**.

# Domain, Co-domain, Range



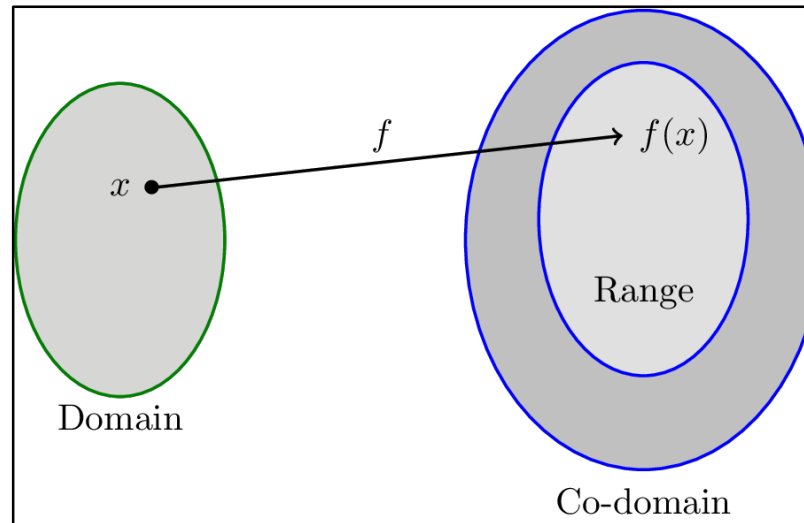
Domain =  $\{g, h, i, j, k\}$   
Co-domain =  $\{m, n, o, p\}$   
Range =  $\{m, n, o, p\}$



# Range/ Image

- The unique element  $y$  to which  $f$  sends  $x$  is denoted by  $f(x)$  and is called  $f$  of  $x$ , or the value of  $f$  at  $x$ , or the image of  $x$  under  $f$ .
- The set of all values of  $f$  taken together is called the range of  $f$  or the image of  $X$  under  $f$ .

$$\text{range of } f = \{y \in Y \mid y=f(x), \text{ for some } x \text{ in } X\}$$



# Domains and Ranges

## Example functions

Function	Domain	Range
$y = x + 2$	$\mathbb{R}$	$\mathbb{R}$
$y = 3x^2 - 7$	$\mathbb{R}$	$\{y : y \geq -7\}$
$y = \sin x$	$\mathbb{R}$	$\{y : -1 \leq y \leq 1\}$
$y = 2^x$	$\mathbb{R}$	$\{y : y > 0\}$
$y = \frac{1}{x}$	$\{x : x \neq 0\}$	$\{y : y \neq 0\}$
$y = \log_2 x$	$\{x : x > 0\}$	$\mathbb{R}$

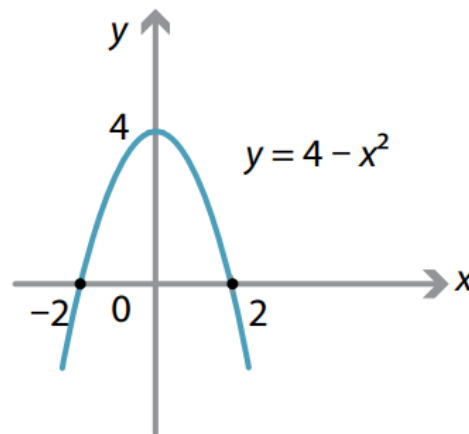
# Domains and Ranges

## Example

What is the domain and range of the function  $f(x) = 4 - x^2$ ?

## Solution

Here a graph of the function helps.



Since  $f(x)$  is defined for all real numbers, we have  $\text{domain}(f) = \mathbb{R}$ .

We can see from the graph that  $\text{range}(f) = \{y : y \leq 4\} = (-\infty, 4]$ .

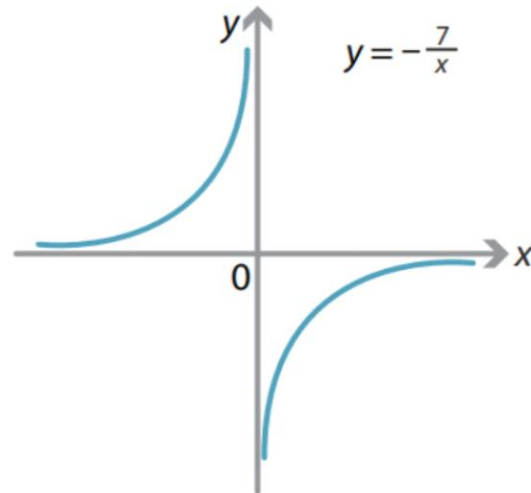
# Domains and Ranges

## Example

What is the domain and range of  $f(x) = -\frac{7}{x}$ ?

## Solution

The graph of  $y = f(x)$  is a rectangular hyperbola.



From the graph, we can see that

$$\text{domain}(f) = \{x \in \mathbb{R} \mid x \neq 0\} = \mathbb{R} \setminus \{0\}$$

$$\text{range}(f) = \{y \in \mathbb{R} \mid y \neq 0\} = \mathbb{R} \setminus \{0\}.$$

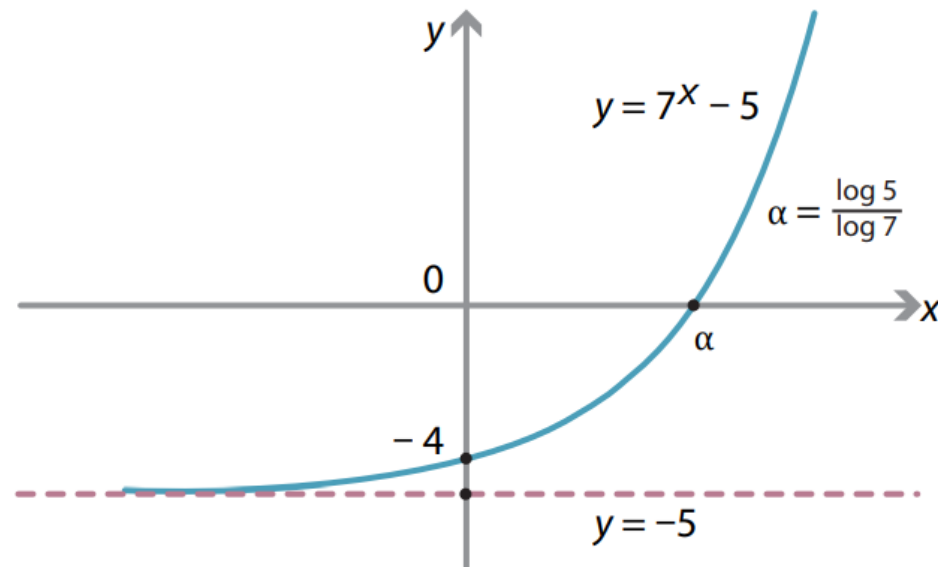
# Domains and Ranges

## Example

What is the domain and range of  $f(x) = 7^x - 5$ ?

## Solution

This is a vertical translation of an exponential function.



Hence,  $\text{domain}(f) = \mathbb{R}$  and  $\text{range}(f) = (-5, \infty)$ .

# Domains and Ranges

## Example

What is the domain and range of  $f(x) = \sqrt{16 - x^2}$ ?

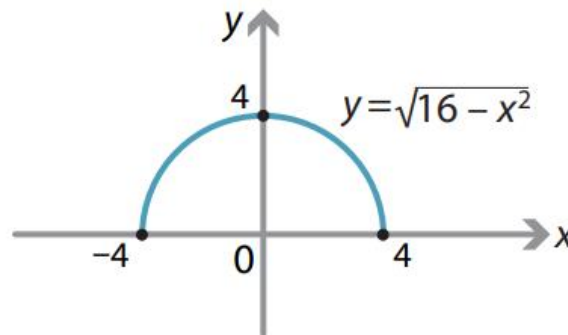
## Solution

We have

$$y = \sqrt{16 - x^2} \iff y^2 = 16 - x^2 \text{ and } y \geq 0$$

$$\iff x^2 + y^2 = 16 \text{ and } y \geq 0.$$

So the graph of  $f(x)$  is the top half of the circle with centre the origin and radius 4.



Hence,  $\text{domain}(f) = [-4, 4]$  and  $\text{range}(f) = [0, 4]$ .



# Domains and Ranges

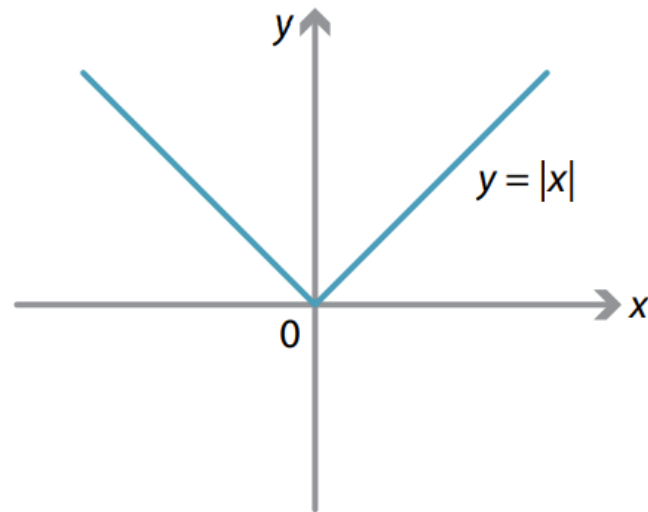
## Example

Find the domain and range of  $f(x) = |x|$ , where as usual

$$|x| = \begin{cases} -x & \text{if } x < 0, \\ x & \text{if } x \geq 0. \end{cases}$$

## Solution

By drawing the graph of  $y = |x|$ , we see that  $\text{domain}(f) = \mathbb{R}$  and  $\text{range}(f) = [0, \infty)$ .



Thank you