SLIIT ACADEMY

Mathematics II
Fractions and Rationalization

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The Number System

- **The Natural Numbers**
  - These are constructed from the first natural number 1 by successively adding 1 each time. (also known as counting numbers)

$$N = \{1,2,3,....\}$$

- We can add or multiply two natural numbers and obtain another natural number.
- However the difference or the ratio of two natural numbers is not always a natural number.

- **Integers**
  - Integers include the natural numbers, the number zero and the negative of the natural number.

$$Z = \{....., -3, -2, -1, 0, 1, 2, 3, ...\}$$

– The set of Integers has been obtained by expanding the set of natural numbers,to make <u>subtraction</u> possible.

– The <u>sum</u>, <u>difference</u> and <u>product</u> of two Integers is always an Integer.

– However, <u>division</u> has limitations.

- **Rational Numbers**
  - Rational numbers are the ratios of Integers, where the divisor is non zero. (Includes decimal values and fractions)

  – The rational numbers can be expressed as a fraction in which the numerator and denominator are Integers.

  – Rational numbers have either <u>terminating</u> or <u>infinitely repeating</u> decimal representations.

Real Numbers

- There is one big problem with rational numbers. It turns out that the rational numbers are not enough to describe the world.

- It can be shown that square root of 2 ($\sqrt{2}$) has no rational solution. There is a rational number arbitrarily close to the solution, but <u>no exact answer</u>.

- A **Real number is a decimal expression** whose digits <u>may or may not terminate or repeat</u>. The real numbers include both rational and irrational numbers. It can be given by an infinite decimal representation such as 2.33482131….. Where digits continue in some way.

- **Irrational Numbers**
  - An irrational number is any real number that is not rational. i.e. it cannot be written as a fraction in which the numerator and denominator are integers.

  - eg: π , $\sqrt{2}$, $\sqrt{3}$

- **Complex Numbers**
  - The square root of -1 is not a real number. It is given a special name, ***the imaginary unit (i).*** Numbers that include the imaginary unit are called Complex numbers.

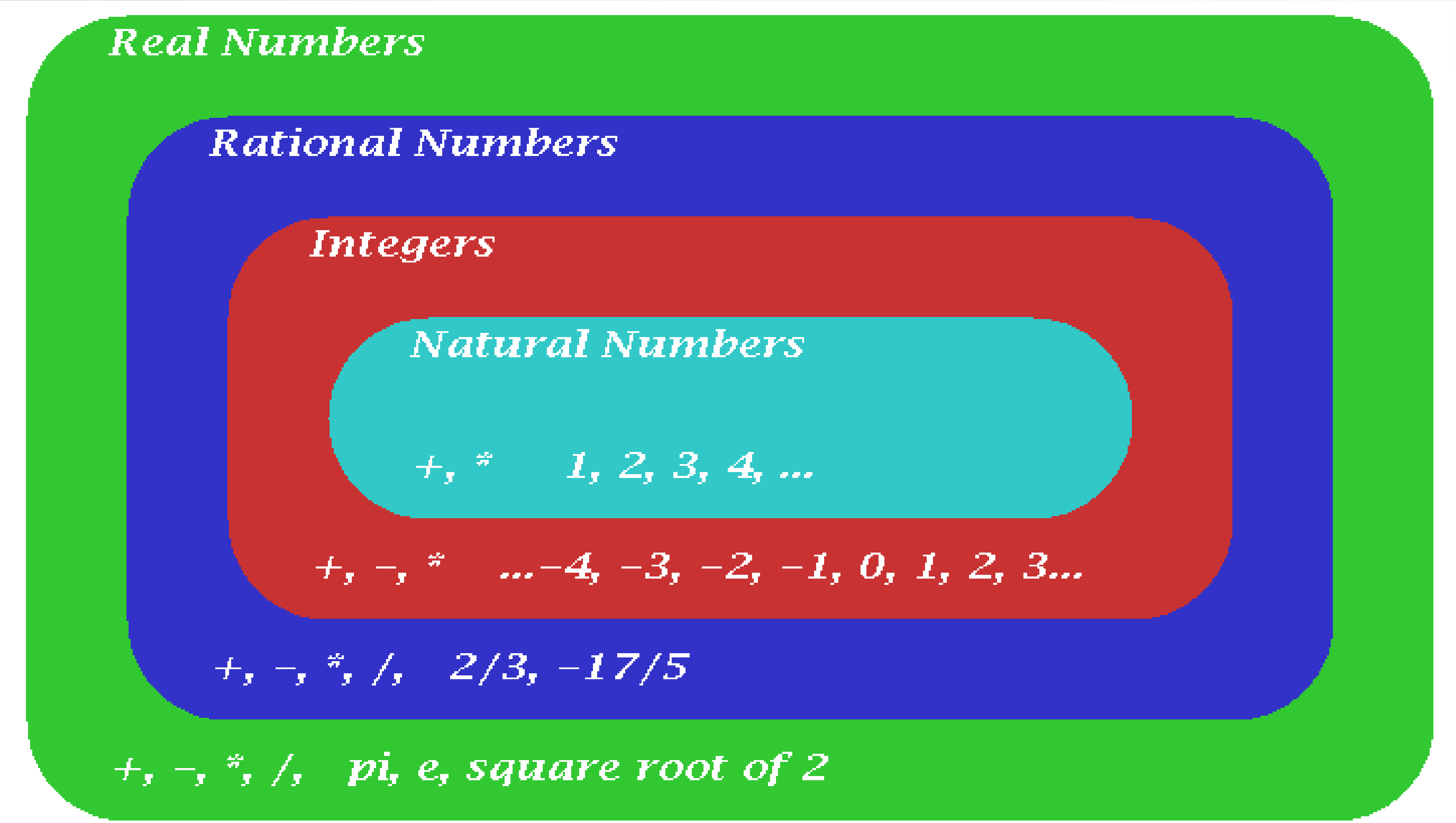
$$z = a + bi$$

  - A generic complex number has the form;

    where ***a*** is called the real part of the complex number and ***b*** is called the imaginary part. When the imaginary part *b* is zero, the complex number is just the real number *a*.

  - With the complex numbers, we finally have an algebraically complete set.

Hierarchy of the Number System



What is a Fraction?

- It is a *value* of the form

$$\frac{a}{b}$$

numerator

denominator

Where *a* and *b* are Integers and $b \neq 0$

Operations with fractions

- ***Add fractions*** (find a common denominator):

$$\frac{a}{b}+\frac{c}{d} = \frac{a}{b}\left(\frac{d}{d}\right)+\frac{c}{d}\left(\frac{b}{b}\right)$$

$$= \frac{ad}{bd}+\frac{bc}{bd}$$

$$= \frac{ad+bc}{bd}$$

- ***Subtract fractions*** (find a common denominator):

$$\frac{a}{b}-\frac{c}{d}=\frac{a}{b}\left(\frac{d}{d}\right)+-\frac{c}{d}\left(\frac{b}{b}\right)$$

$$=\frac{ad}{bd}-\frac{bc}{bd}$$

$$=\frac{ad-bc}{bd}$$

- ***Multiply fractions***

$$\left(\frac{a}{b}\right)\left(\frac{c}{d}\right)=\frac{ac}{bd}$$

- ***Divide fractions***

$$\frac{a/b}{c/d}=\left(\frac{a}{b}\right)\left(\frac{d}{c}\right)=\frac{ad}{bc}$$

Exercises

- Simplify

$$\frac{x}{4}+\frac{x}{2}-\frac{x}{6}$$

$$\frac{x}{4}+\frac{x}{2}-\frac{x}{6}$$

$$=\frac{3x+6x-2x}{12}$$

$$=\frac{7x}{12}$$

Simplify

1. $$\frac{3}{a}+\frac{2}{3a}-\frac{3}{2a}=\frac{18+4-9}{6a}=\frac{13}{6a}$$

2. $$\frac{x+5}{8}+\frac{x-3}{12}=\frac{3(x+5)+2(x-3)}{24}=\frac{3x+15+2x-6}{24}$$
$$=\frac{5x+9}{24}$$

3. $$\frac{3a-4}{7}-\frac{6-a}{14}=\frac{2(3a-4)-(6-a)}{14}=\frac{6a-8-6+a}{14}$$
$$=\frac{7a-14}{14}=\frac{7(a-2)}{14}=\frac{a-2}{2}$$

Simplify

4. $$\frac{2x}{2y} \times \frac{7xy}{28x}$$

5. $$\frac{3a}{7b} \div \frac{7b}{35}$$

Polynomials

- *A* polynomial *p* (in variable *x*) is a function or an expression that can be evaluated by combining the variable and possibly some constants by a finite number of additions, subtractions and multiplications. Note that it <u>excludes</u> divisions and radicals.

- *A* polynomial can always written in standard form as;

$$p(x) = a_n x^n + a_{n-1} x^{n-1} + a_{n-2} x^{n-2} + \dots + a_2 x^2 + a_1 x^1 + a_0 x^0$$

- The integer $n \geq 0$ is called the **degree** of the polynomial.

Examples

$$p(x) = x^5 + 2x^2 - x + 5 = x^5 + 0.x^4 + 0.x^3 + 2x^2 - x + 5$$

$$p(x) = x^3 - 1 = x^3 + 0.x^2 + 0.x - 1$$

Polynomials of the lower degree are been given special names as listed below.

Degree	Name	Example
0	Constant	2
1	Linear	$x+2$
2	Quadratic	x^2-x-2
3	Cubic	x^3-1
4	Quartic	x^4+4x+2
5	Quintic	x^5+3x^2-1
6	Sextic	x^6+x^4+x-1
7	Septic	x^7-c-1
8	Octic	x^8+x+3
9	Nonic	$x^9-x^8+x^6+4$

Rational Expressions

- Expressions that have polynomials as both numerator and denominator and are called rational expressions
  - *eg:* $\dfrac{2}{x}, \dfrac{x^2+2x-4}{x+6}$
- *Proper rational expression*: if the degree of the numerator is less than the degree of the denominator

Exercises

1. $$\frac{5}{8x}+\frac{7}{20x}=\frac{25+14}{40x}=\frac{39}{40x}$$

2. $$\frac{4}{x+6}+\frac{9}{x+4}=\frac{4(x+4)+9(x+6)}{(x+6)(x+4)}=\frac{4x+16+9x+54}{(x+6)(x+4)}$$
$$=\frac{13x+70}{(x+6)(x+4)}$$

3. $$2-\frac{1}{a}+\frac{1}{3a^2}=\frac{(2\times 3a^2)-(3a)+1}{3a^2}=\frac{6a^2-3a+1}{3a^2}$$

4.

$$\frac{1}{x}-\frac{3}{x+4}+\frac{5}{x-2}$$

$$=\frac{(x+4)(x-2)-3(x)(x-2)+5(x)(x+4)}{x(x+4)(x-2)}$$

$$=\frac{(x^2+4x-2x-8)-3(x^2-2x)+5(x^2+4x)}{x(x+4)(x-2)}$$

$$=\frac{3x^2+28x-8}{x(x+4)(x-2)}$$

5.

$$\frac{5}{(3x-2y)(6x+7y)}-\frac{2}{(5x+3y)(2y-3x)}$$

$$=\frac{5}{(3x-2y)(6x+7y)}+\frac{2}{(5x+3y)(3x-2y)}$$

$$=\frac{5(5x+3y)+2(6x+7y)}{(3x-2y)(6x+7y)(5x+3y)}$$

$$=\frac{25x+15y+12x+14y}{(3x-2y)(6x+7y)(5x+3y)}$$

$$=\frac{37x+29y}{(3x-2y)(6x+7y)(5x+3y)}$$

6. $$\frac{x+2}{x+3}-\frac{5-x}{x-3}=\frac{(x+2)(x-3)-(5-x)(x+3)}{(x+3)(x-3)}$$

$$=\frac{(x^2-x-6)-(5x+15-x^2-3x)}{(x+3)(x-3)}$$

$$=\frac{x^2-x-6-5x-15+x^2+3x}{(x+3)(x-3)}$$

$$=\frac{x^2-x-6-5x-15+x^2+3x}{(x+3)(x-3)}$$

$$=\frac{2x^2-3x-21}{(x+3)(x-3)}$$

Expressions involving Radicals

- Expressions involving radical are important to be able to simplify to obtain manageable forms.

$$\frac{\sqrt{x+1}-\dfrac{x}{2\sqrt{x+1}}}{x+1}=\frac{\dfrac{2(x+1)}{2\sqrt{x+1}}-\dfrac{x}{2\sqrt{x+1}}}{x+1}=\frac{\dfrac{2x+2-x}{2\sqrt{x+1}}}{x+1}$$

$$=\frac{x+2}{2\sqrt{x+1}(x+1)}=\frac{x+2}{2(x+1)^{3/2}}$$

Rationalization Techniques

- Rationalization is the process of moving the radical expression from the denominator to the numerator or vice versa.
  - If denominator/numerators is $\frac{\sqrt{a}}{\sqrt{a}}$ multiply by $\sqrt{a}$
  - If denominator/numerators is $\frac{\sqrt{a}+\sqrt{b}}{\sqrt{a}+\sqrt{b}}$ multiply by $\sqrt{a}-\sqrt{b}$
  - If denominator/numerators is $\frac{\sqrt{a}-\sqrt{b}}{\sqrt{a}-\sqrt{b}}$ multiply by $\sqrt{a}+\sqrt{b}$

Example 1

- Rationalize the denominator of $\frac{x}{\sqrt{5}}$.

$$\frac{x}{\sqrt{5}} = \frac{x(\sqrt{5})}{(\sqrt{5})(\sqrt{5})}$$

$$= \frac{\sqrt{5}x}{5}$$

Example 2

- Rationalize the denominator of $\frac{5}{2-\sqrt{3}}$.

$$\frac{5}{2-\sqrt{3}} = \frac{5(2+\sqrt{3})}{(2-\sqrt{3})(2+\sqrt{3})}$$

$$= \frac{10+5\sqrt{3}}{(2)^2-(\sqrt{3})^2}$$

$$= \frac{10+5\sqrt{3}}{4-3}$$

$$= 10+5\sqrt{3}$$

Exercises (Rationalize)

1. $$\frac{1}{\sqrt{x}-\sqrt{x+1}} = \frac{1}{\sqrt{x}-\sqrt{x+1}}\left(\frac{\sqrt{x}+\sqrt{x+1}}{\sqrt{x}+\sqrt{x+1}}\right)$$

$$= \frac{\sqrt{x}+\sqrt{x+1}}{x-(x+1)}$$

$$= -\sqrt{x}-\sqrt{x+1}$$

2. $$\frac{\sqrt{x+1}}{2} = \frac{\sqrt{x+1}}{2}\left(\frac{\sqrt{x+1}}{\sqrt{x+1}}\right)$$

$$= \frac{x+1}{2\sqrt{x+1}}$$

Thank You