SLIIT ACADEMY

Mathematics II
Set Theory

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Set

- A set is a collection of objects considered as a whole.
- The objects of a set are called as elements or members.
- The elements can be anything.
  i.e. numbers, letters of the alphabet, etc.

- Sets are conventionally denoted by capital letters. (A, B, C, etc.)

- Some sets may be described in words.

  eg: B is a set whose members are the first four positive whole numbers.

- Sets can be defined explicitly listing its elements, between curly braces.

  eg: B = {1,2,3,4}

- More complicated sets are sometimes described by a different notation.

  eg: $F = \{ n^2-4 : n \text{ is an Integer and } 0 \leq n \leq 19 \}$

- Above is interpreted as "F equals n^2-4 such that n is a whole number in the range from zero to 19 inclusive."

- The colon ":" indicates "such that". Sometimes the pipe "|" is used instead of the colon.

Set Membership

- If something is an element of a particular set then it is symbolized by "∈".

  If not by "∉".

  eg: F = {-4,-3,0,1,2}

  $-4 \in F$ (-4 is **an element of** F)

  $5 \notin F$ (5 is **not an element of** F)

Subsets

- If every member of the set A is also a member of the set B, then A is said to be a subset of B. It is also said as A is contained in B or B contains A.

- Notation: $A \subseteq B$ or $B \supseteq A$

- eg: If $A = \{2,4,5,7\}$

  $B = \{1,2,3,4,5,6,7\}$

  Then $A \subseteq B$

Note: Any set is considered as a subset of itself

Proper Subset

- If A and B are sets, A is a proper subset of B, if and only if every element of A is in B but, there at least one element of B that is not in A.

- Notation: $A \subset B$

Example

I. $A = \{1,2,3,4,5\}$

$B = \{1,2,3\}$

$B \subset A$

B is a proper subset of A.

II. $C = \{a,b,c\}$

$D = \{a,b,c\}$

$C \subseteq D$

But C is not a proper subset of D.

Set Equality

$$A = B \leftrightarrow A \subseteq B \text{ and } B \subseteq A$$

Given sets A and B, A equal B, if and only if, every element of A is in B and every element of B is in A

Special Sets

- P denotes the set of all primes
- N denotes the set of all natural numbers
- Z denotes the set of all integers
- Q denotes the set of all rational numbers

$$Q = \left\{ \frac{a}{b} \mid a, b \in Z \text{ and } b \neq 0 \right\}$$

- R denotes the set of all real numbers

  [This includes both rational and irrational numbers]

$$P \subset N \subset Z \subset Q \subset R$$

Set Operations

1) **Union**

$$A \cup B = \{x \mid x \in A \text{ or } x \in B\}$$

$A \cup B$ is the set of all elements x such that x is in A or x is in B.

2) **Intersection**

$$A \cap B = \{x \mid x \in A \text{ and } x \in B\}$$

$A \cap B$ is the set of all elements x such that x is in A and x is in B.

3) Difference

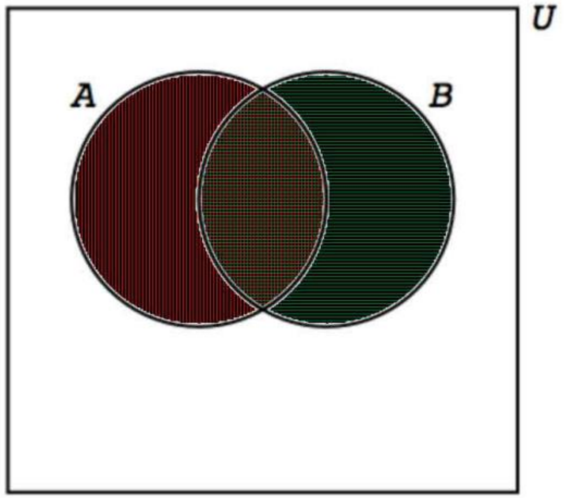
$$A - B = \{x \mid x \in A \text{ and } x \notin B\}$$

4) Complement

$$A^c = A' = \{x \mid x \notin A\}$$

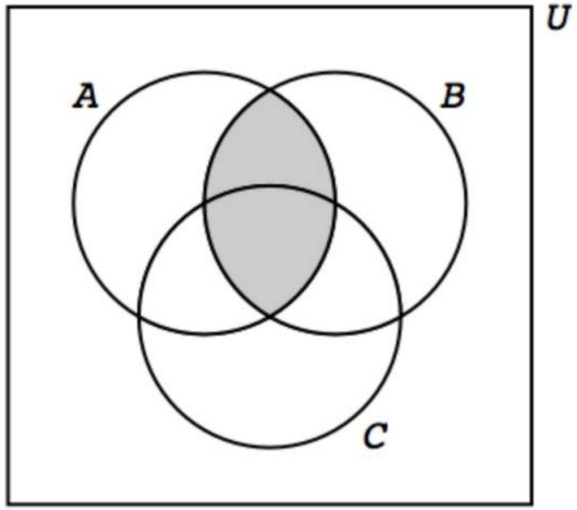
Venn Diagrams

- We can visual subsets of a universal set, and how they interact/overlap, using Venn diagrams



$A \cap B$

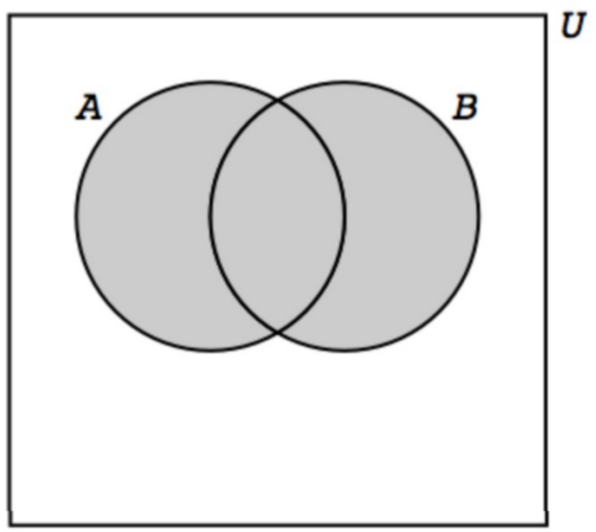
$(A' \cup B')'$



$A \cap B$

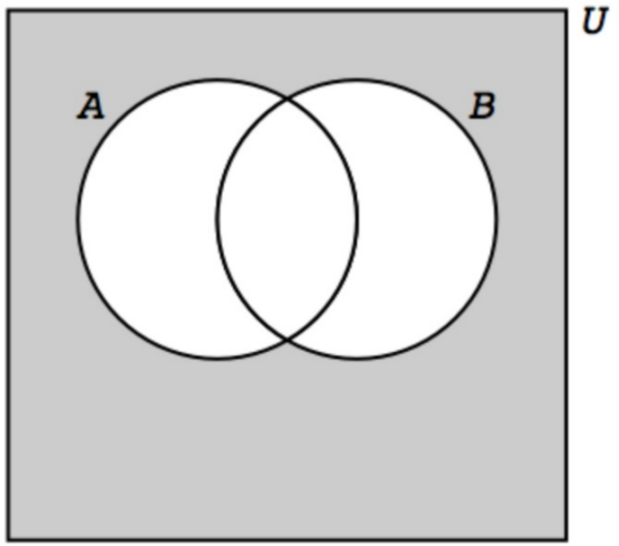
$(A' \cup B')'$

Venn Diagrams



$A \cup B$

$(A' \cap B')'$

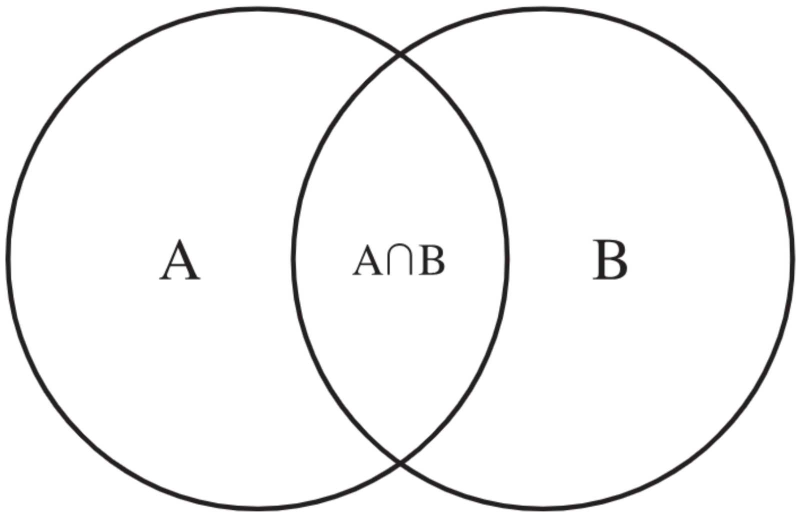


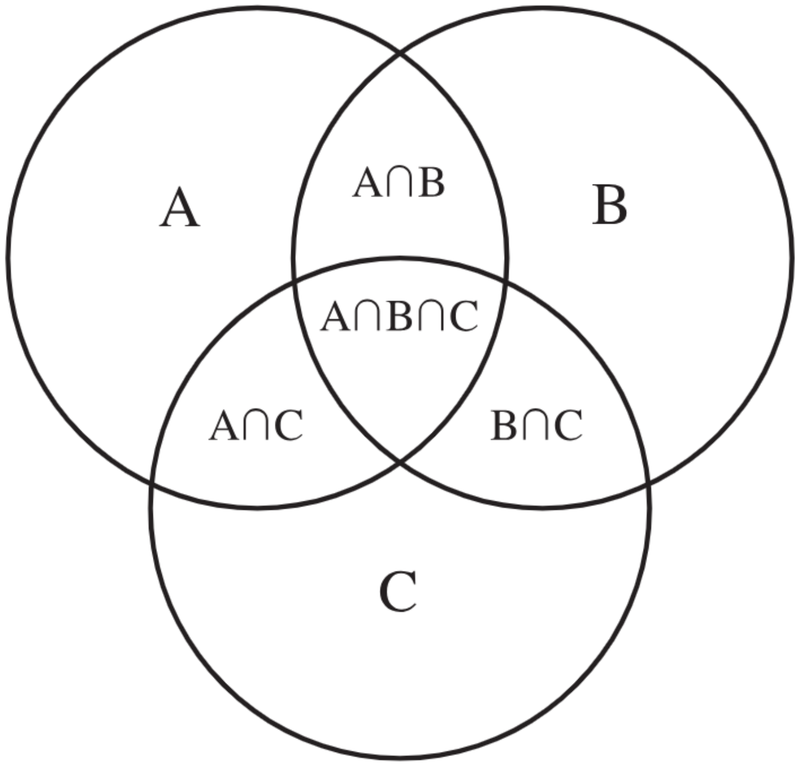
$A' \cap B'$

$(A \cup B)'$

Venn Diagrams

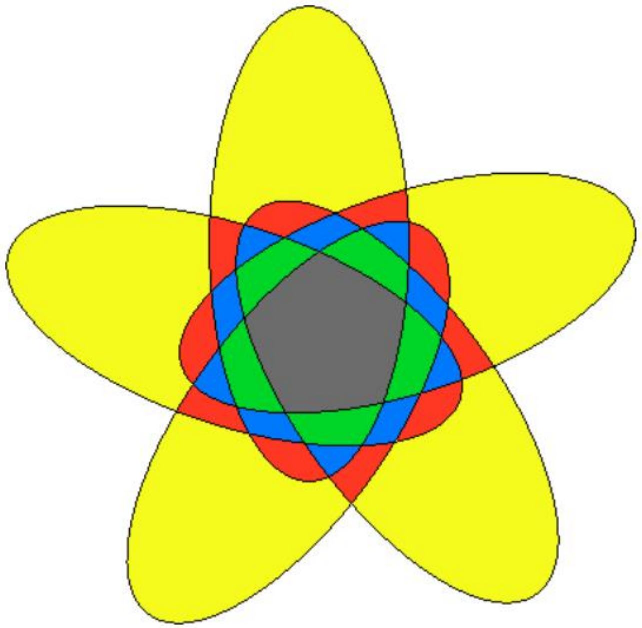
- Venn diagrams using two or three sets are often used in presentations.

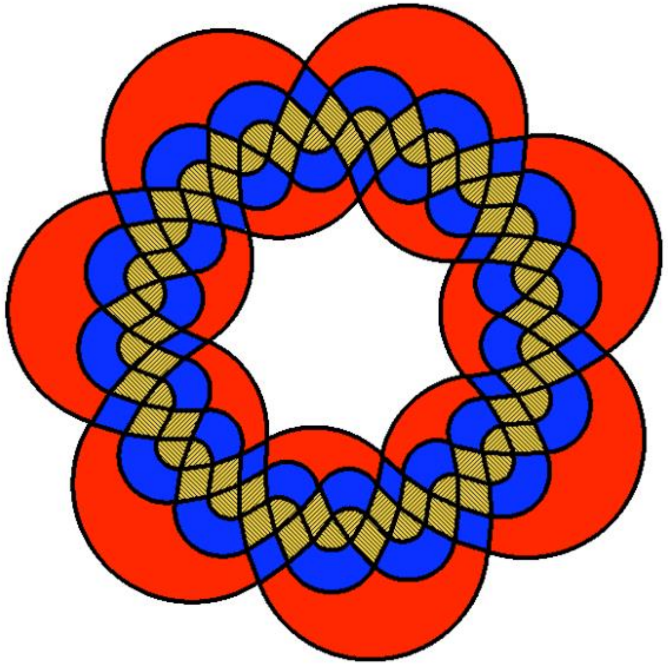




Venn diagrams for presentations

Venn diagrams of more sets are possible, but tend to be confusing as a presentation tool because of the number of possible interactions. The following diagrams show Venn diagrams for five sets on the left and for 7 sets on the right.





The Inclusion-Exclusion Principle

For any finite set, S, we let $n(S)$ denote the number of objects in S.

Example: If $A = \{1, 2, 3, 4, 5, 6, 7\}$ and $B = \{5, 6, 7, 8, 9, 10\}$ then
$n(A) = 7$ and $n(B) = 6$

$A \cup B = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\} \Rightarrow n(A \cup B) = 10.$

$A \cap B = \{5, 6, 7\} \Rightarrow n(A \cap B) = 3.$

The Inclusion-Exclusion Principle

For any finite set, S, we let n (S) denote the number of objects in S.

$$\boxed{n(A \cup B) = n(A) + n(B) - n(A \cap B)}$$

Note :

If two sets A and B do not intersect, then $n(A \cap B) = 0$ and hence $n(A \cup B) = n(A) + n(B)$.

The Inclusion-Exclusion Principle

Formula 1

Set and its complement

$$\boxed{\text{n}\ (A') = \text{n}\ (A^c\ ) = \text{n (U)} - \text{n (A)}}$$

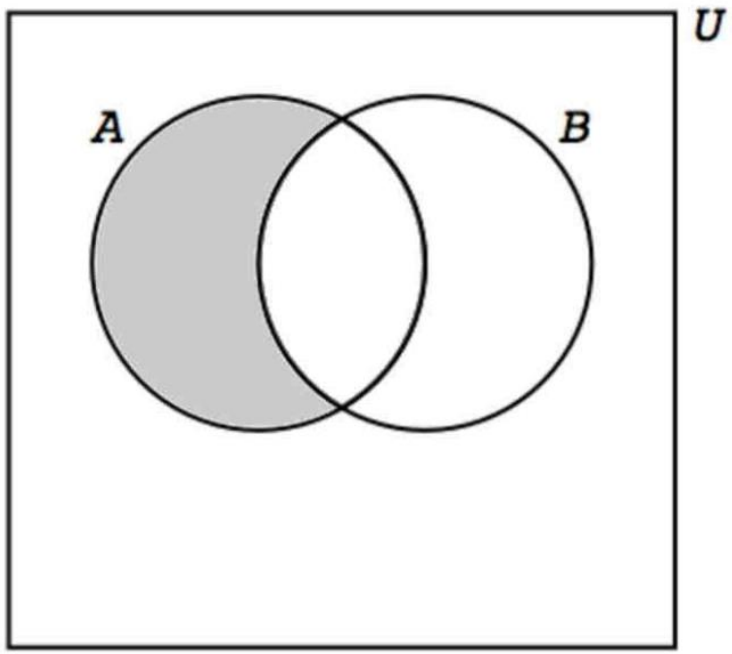
where U is the universal set.

The Inclusion-Exclusion Principle

Formula 2

The shaded region below is A ∩ B^c and (A ∩ B^c) ∩ (A ∩ B) = ∅ so

$$\boxed{n\,(A \cap B^c) = n\,(A) - n\,(A \cap B)}$$



Exercise

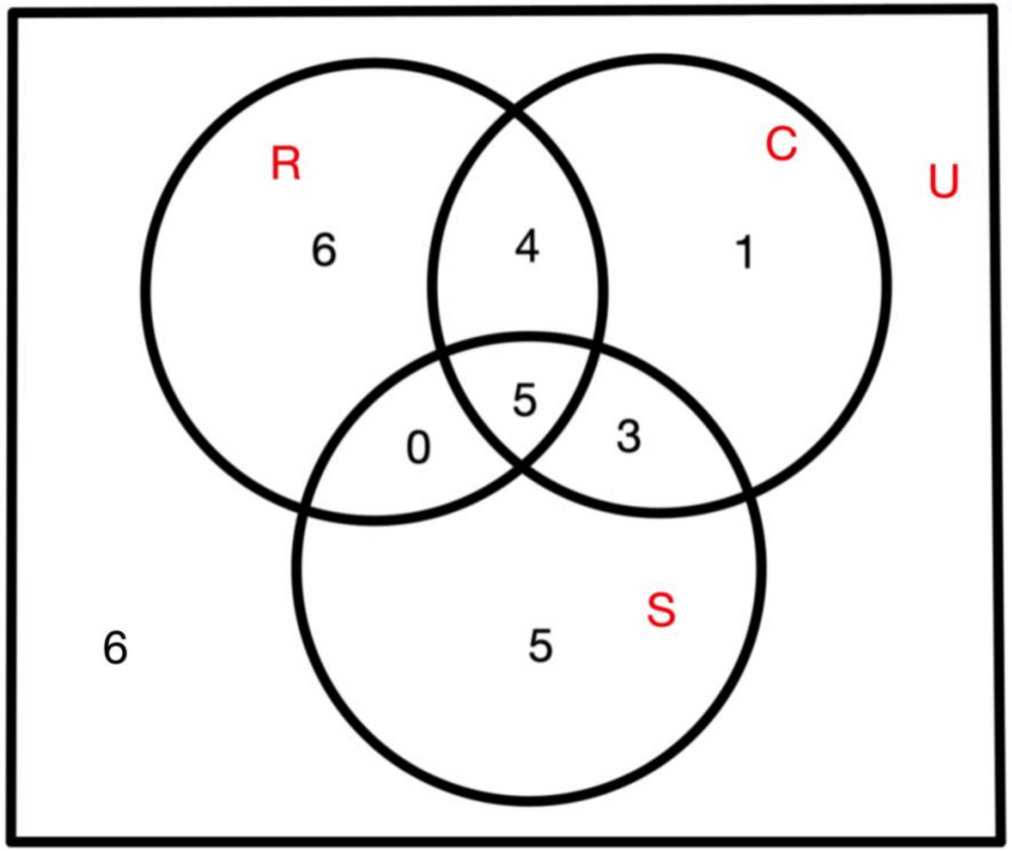
In a group of 30 people, 15 run, 13 swim, 13 cycle, 5 run and swim, 8 cycle and swim, 9 run and cycle, and 5 do all three activities.

(a) Draw a Venn diagram showing the results of the survey.
(b) How many of the 30 people neither run nor cycle?
(c) How many people like run but did not like swim or cycle?

Exercise

(a) Draw a Venn diagram showing the results of the survey.

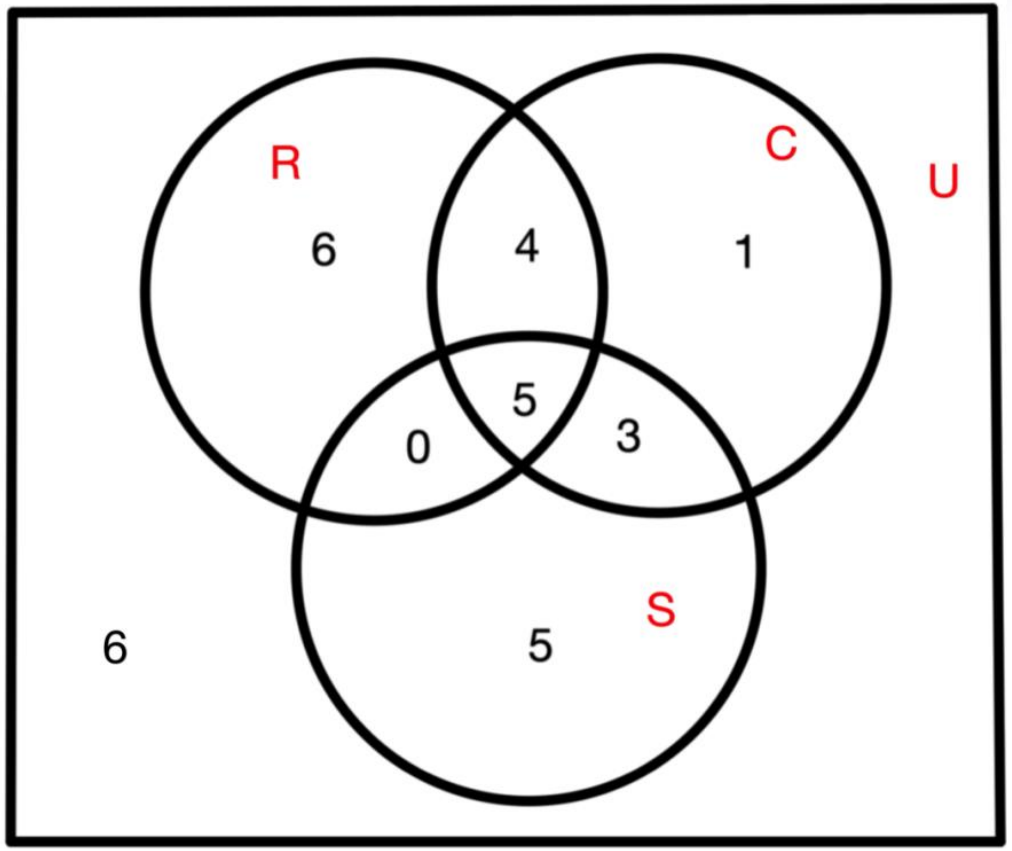
n (U) = 30
n (R) = 15
n (S) = 13
n (C) = 13
n (R ∩ S) = 5
n (C ∩ S) = 8
n (R ∩ C) = 9
R ∩ C ∩ S = 5.



Exercise

(b) How many of the 30 people neither run nor cycle?

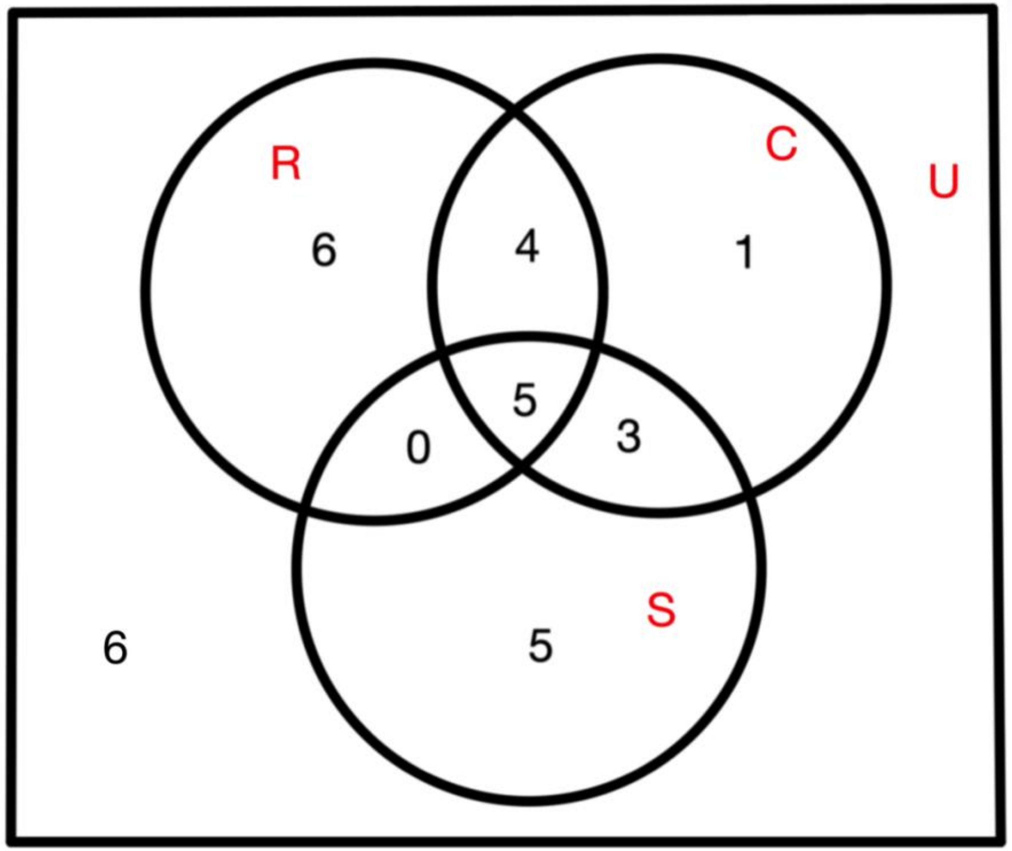
Answer is 6 + 5 = 11



Exercise

(c) How many people like run but did not like swim or cycle?

Answer is 6



Disjoint Sets

- Two sets are called disjoint if and only if they have no elements in common.

- A and B are disjoint $\Leftrightarrow A \cap B = \Phi$

  Eg: A={1,3,5} and B={2,4,6}

  $A \cap B = \Phi$ so A and B sets are disjoint.

Universal Set

- A set which has all the elements in the universe is called an Universal set.

- A Universal set denotes by U.

Empty Set

- A set with no elements is called an empty set or null set.

- Notation: { } or Φ

Set Properties involving Φ

$$A \cup \Phi = A$$

$$A \cap A^c = \Phi$$

$$A \cup A^c = U$$

$$A \cap \Phi = \Phi$$

$$U^c = \Phi$$

$$\Phi^c = U$$

Power Set

- The set of all subsets of A is called as the power set of A.

- Notation: P(A)

eg: A = {a, b}

P(A) = {Φ, {a}, {b}, {a,b}}

Number of elements in A = n

Number of elements in power set of A = 2^n

Ordered Pair

- An ordered pair is a pair of objects (elements) with an order associated with them.

- If the objects are represented by x and y then we can write the ordered pair as (x, y).

Cartesian Product of sets

- The set of all ordered pairs (a,b) where ***a*** is an element of A and ***b*** is an element of B is called the Cartesian product of A and B.

- Notation: A X B

- $A \times B = \{(a,b) \mid a \in A \text{ and } b \in B\}$

Set Identities

1. Commutative laws:

$$A \cup B = B \cup A$$
$$A \cap B = B \cap A$$

2. Associative laws:

$$(A \cup B) \cup C = A \cup (B \cup C)$$
$$(A \cap B) \cap C = A \cap (B \cap C)$$

3. Distributive laws:

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$
$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

4. Intersection with Universal set:

$$A \cap U = A$$

5. Double complement law:

$$(A^c)^c = A$$

6. Idempotent laws:

$$A \cup A = A$$

$$A \cap A = A$$

7. De Morgan's law:

$$(A \cup B)^c = A^c \cap B^c$$

$$(A \cap B)^c = A^c \cup B^c$$

8. Union with Universal set:

$$A \cup U = U$$

9. Absorption laws:

$$A \cup (A \cap B) = A$$
$$A \cap (A \cup B) = A$$

10. Set difference alternative representation:

$$A - B = A \cap B^c$$

Thank You