SLIIT ACADEMY

Mathematics II

Functions

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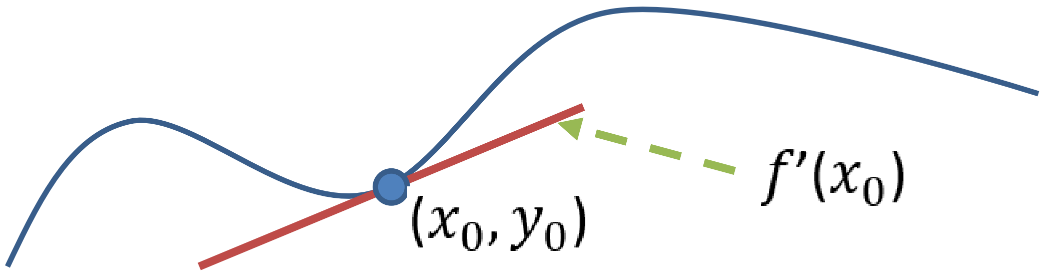
Gradient of a function

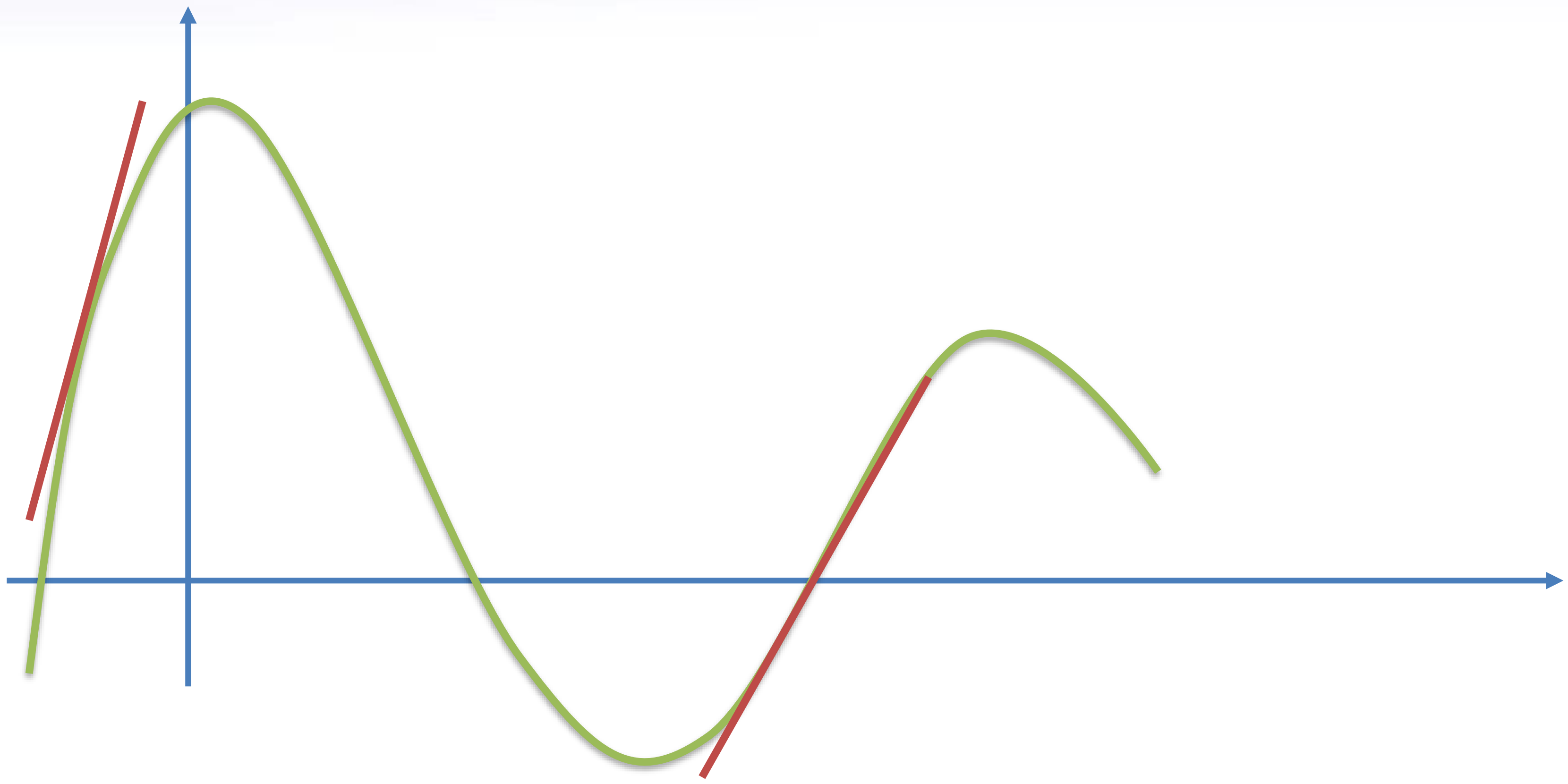
- Gradient of any function at a given coordinate can be found by substituting the x coordinate to $f'(x)$

- Example:
- If the 1^{st} derivative of function $f(x)$ is $f'(x)$
- Then, The gradient of $f(x)$ at coordinate (x_0, y_0) is given by $f'(x_0)$



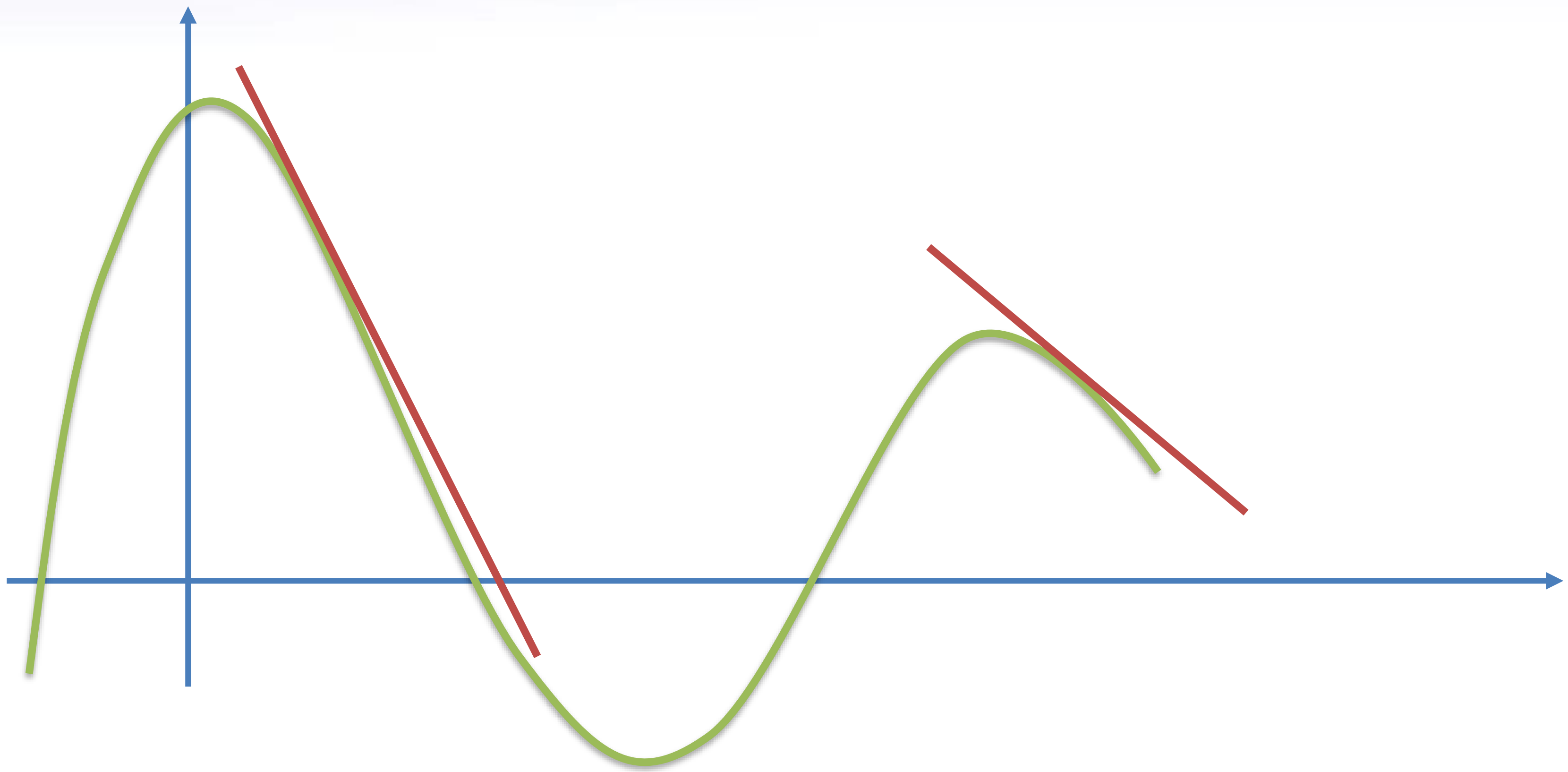
Gradient of a function

- If $f'(x) > 0$ then, the function is increasing



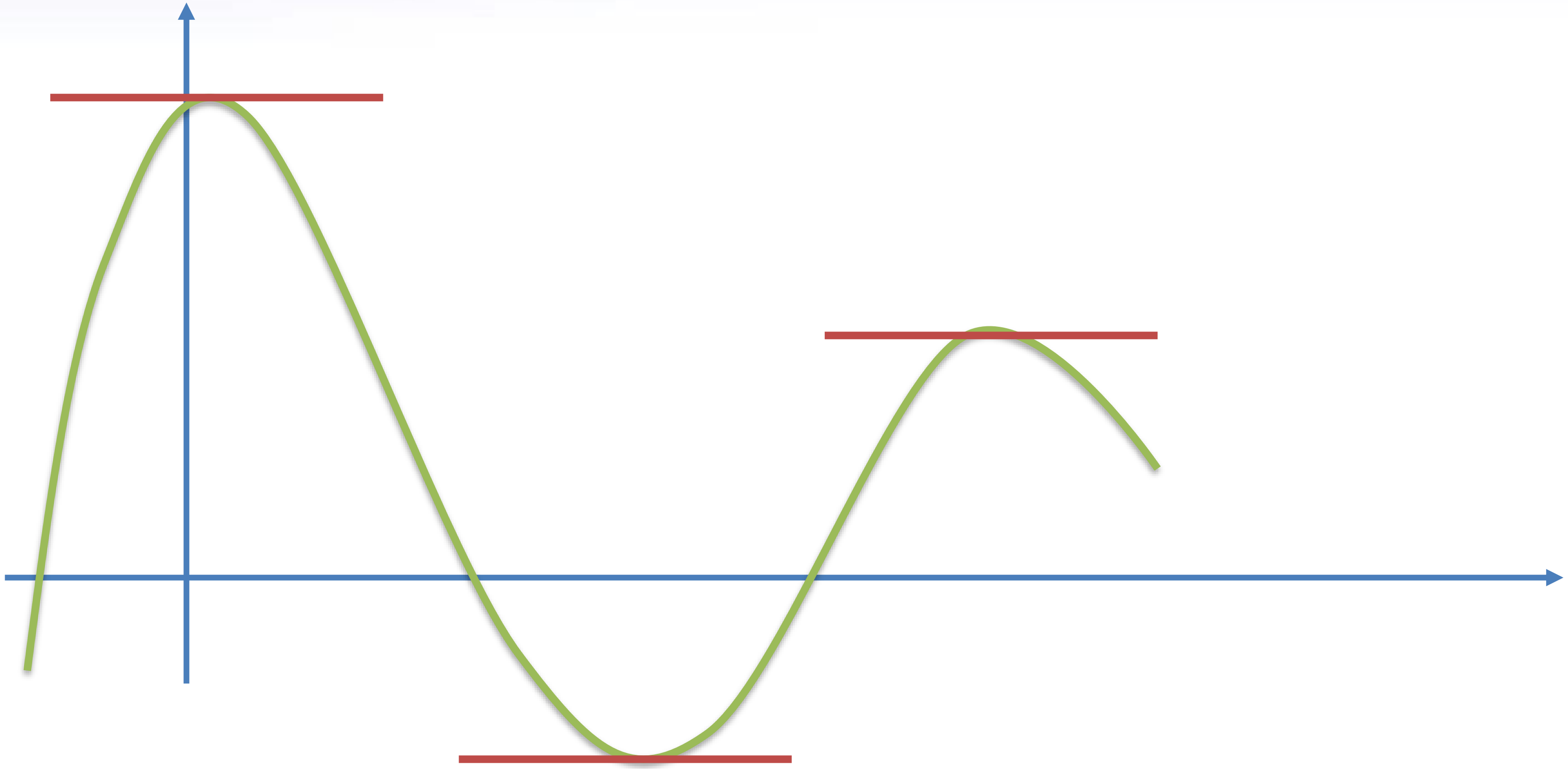
Gradient of a function

- If $f'(x) < 0$ then, the function is decreasing



Gradient of a function

- If $f'(x) = 0$ then, it's a stationary point



Stationary points on a curve

When the 1st derivative $\frac{dy}{dx} = 0$,

The tangent to the curve at the point is parallel to the x axis and called stationary points.

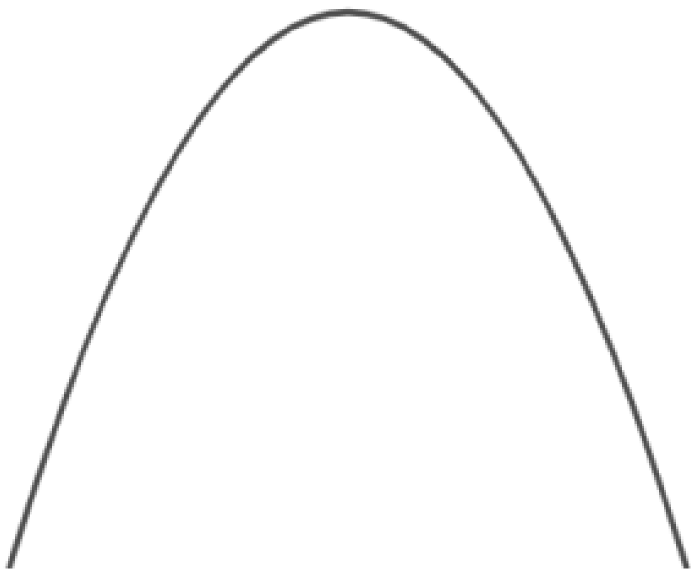
x- coordinates of the stationary points are given by the solutions to the equation, $\frac{dy}{dx} = 0$.

❖ There are three types of stationary points

Stationary points on a curve

There are three types of stationary points

Minimum



Maximum

Stationary points on a curve

There are three types of stationary points

Inflection

The nature of stationary points

After the stationary points are found, the nature of the stationary points can be determined using $\mathbf{1}\boldsymbol{st}$ Derivative .

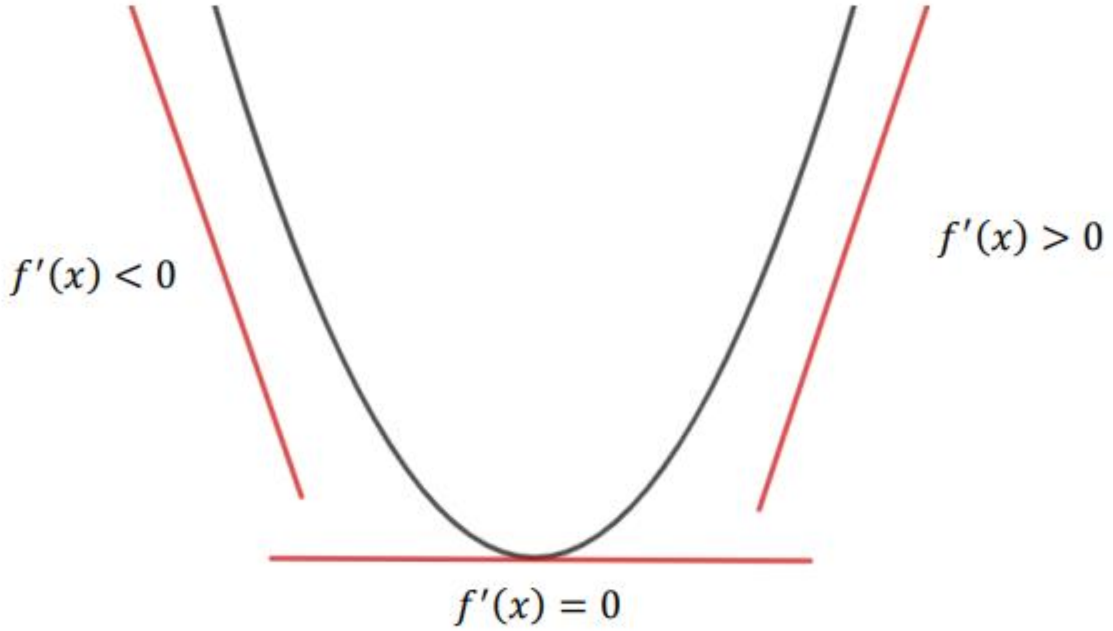
Using 1^{st} Derivative

In order to find the nature of the stationary points the gradient of the functions at both the sides of the stationary point should be considered.

The nature of stationary points

1.) Minimum

Minimum:



If $f'(x_0) = 0$ and,

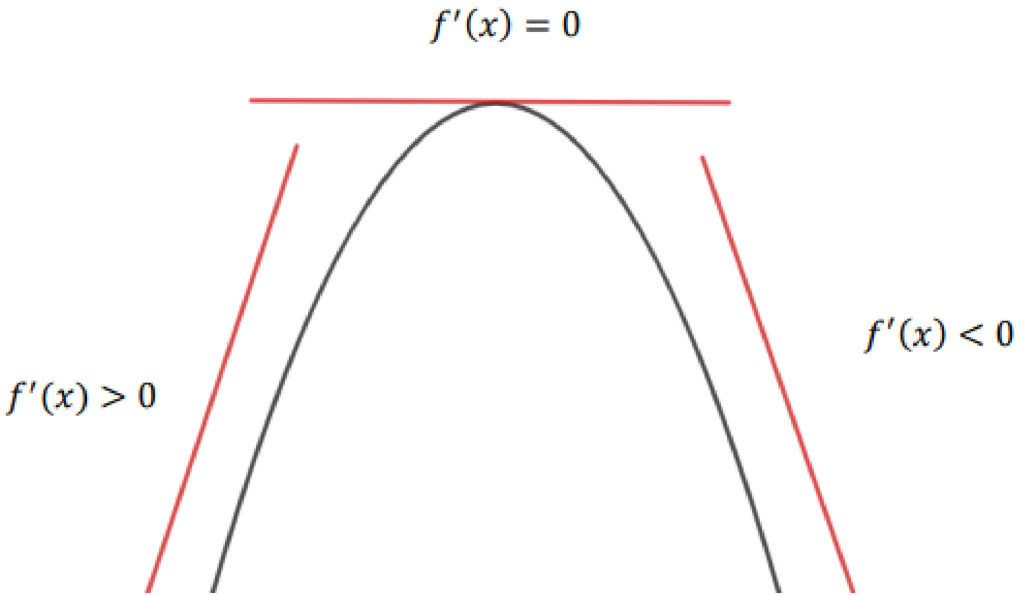
$x < x_0$	and	$x > x_0$
$f'(x) < 0$		$f'(x) > 0$

Then the stationary point at $x = x_0$ is a minimum

The nature of stationary points

2.) Maximum

Maximum:



If $f'(x_0) = 0$ and,

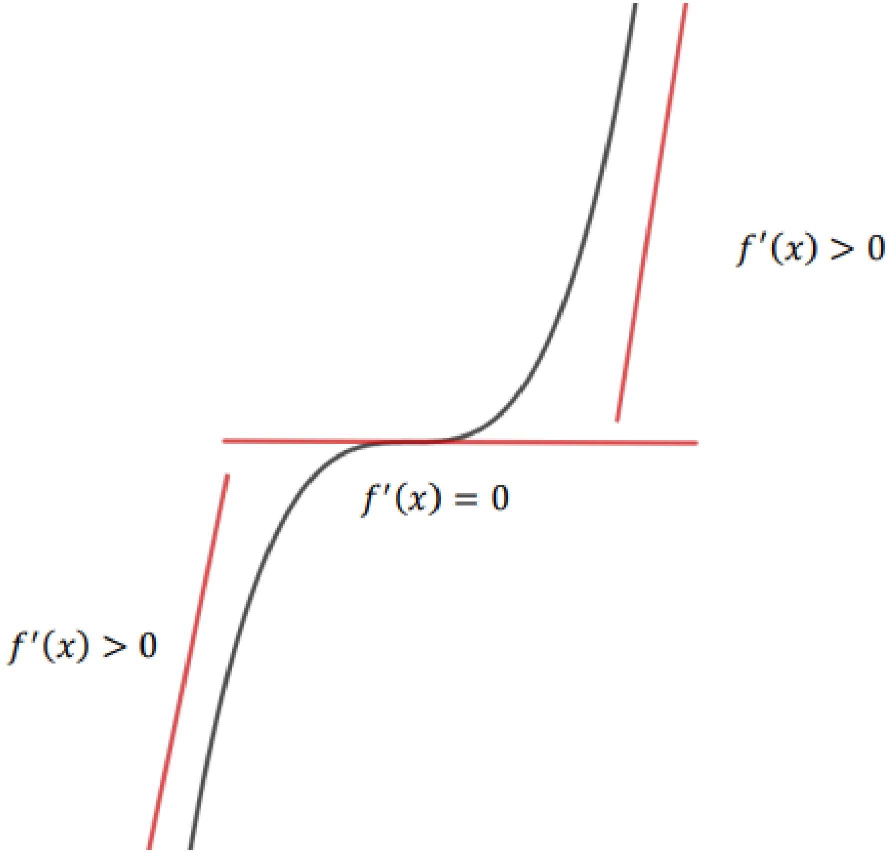
$x < x_0$	and	$x > x_0$
$f'(x) > 0$		$f'(x) < 0$

Then the stationary point at $x = x_0$ is a maximum

The nature of stationary points

3.) Inflection

Inflection:



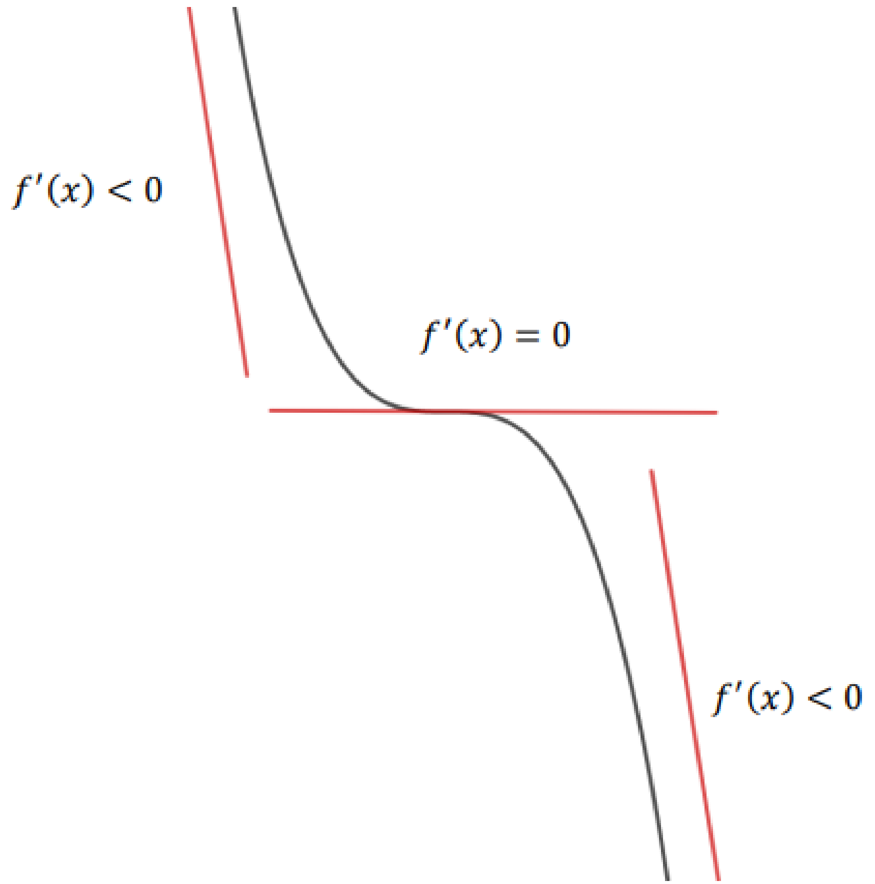
If $f'(x_0) = 0$ and,

$x < x_0$	and	$x > x_0$
$f'(x) > 0$		$f'(x) > 0$

Then the stationary point at $x = x_0$ is a inflation

The nature of stationary points

3.) Inflection



If $f'(x_0) = 0$ and,

$x < x_0$ and $x > x_0$

$f'(x) < 0$ $f'(x) < 0$

Then the stationary point at $x = x_0$ is a inflation

Example 1

- Use differentiation to get the nature of stationary points the graph of $f(x) = x^3 - \frac{3}{2}x^2$

1. $f'(x) = 3x^2 - 3x$

2. Stationary values:

$$f'(x) = 0$$
$$3x^2 - 3x = 0$$
$$3(x)(x-1) = 0$$
$$x = 0, x = 1$$

Stationary *numbers* $x = 0$, and $x = 1$

intervals $(-\infty,0), (0,1), (1,\infty)$

SLIIT COMPUTING

Example 1

- Use differentiation to get the nature of stationary points the graph of $f(x) = x^3 - \frac{3}{2}x^2$

$f'(x) = 3x^2 - 3x$

X = 0 X = 1

Interval	-∝< x < 0	0< x < 1	1 < x < ∝
Test value	x = -1	x = 1/2	x= 2
Sign of f'(x)	+	-	+
Conclusion	Increasing	Decreasing	Increasing

SLIIT COMPUTING

Example 1

- Use differentiation to get the nature of stationary points the graph of $f(x) = x^3 - \frac{3}{2}x^2$

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X = 0 X = 1

Interval	$-\propto < x < 0$	$0 < x < 1$	$1 < x < \propto$
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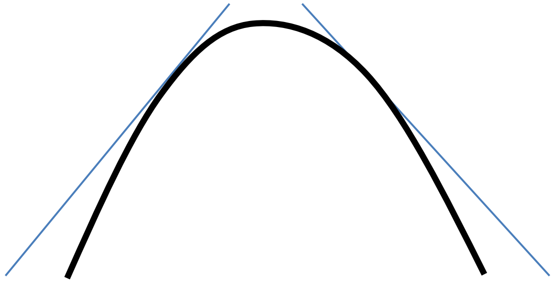

Maximum

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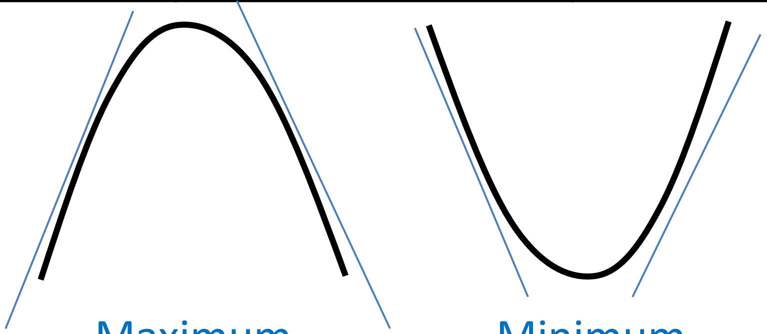
Test value	x = -1	x = 1/2	x= 2
Sign of f′(x)	+	-	+
Conclusion	Increasing	Decreasing	Increasing



Maximum Minimum

Example 2

- Use differentiation to get the nature of stationary points the graph of $f(x) = x^4 - 8x^2$

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- Use differentiation to get the nature of stationary points the graph of $f(x) = x^4 - 8x^2$

1. $f'(x) = 4x^3 - 16x$

2. Stationary values:

$$f'(x) = 0$$
$$4x^3 - 16x = 0$$
$$4(x)(x^2 - 4) = 0$$
$$4(x)(x-2)(x+2) = 0$$
$$x = 0,\ x = 2,\ x = -2$$

Stationary *numbers* $x = -2$, $x = 0$ and $x = 2$

intervals $(-\infty,-2)\,,(-2,0)\,,(0,2)\,,(2,\infty)$

SLIIT COMPUTING

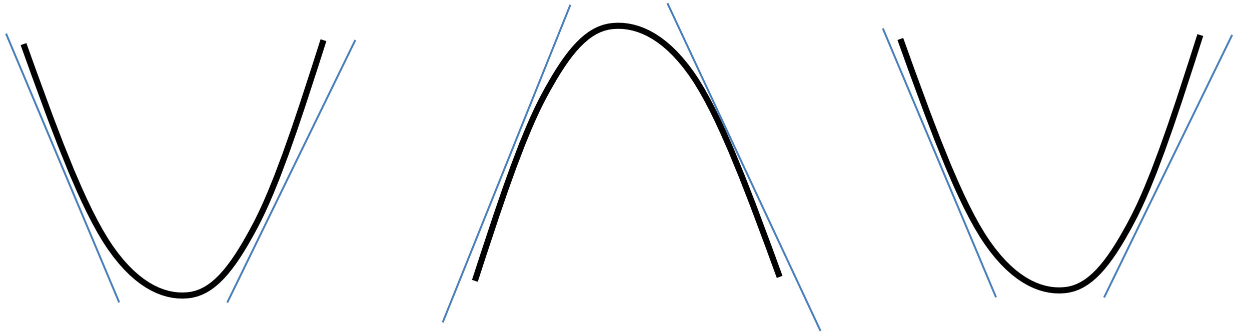
Example 2

- Use differentiation to get the nature of stationary points the graph of $f(x) = x^4 - 8x^2$

$$f'(x) = 4x^3 - 16x$$
$$f'(x) = 4x(x-2)(x+2)$$

X = -2 X = 0 X = 2

Interval	$-\propto < x < -2$	$-2 < x < 0$	$0 < x < 2$	$2 < x < \propto$
Test value	x = -3	x = -1	x= 1	x= 3
Sign of f′(x)	-	+	-	+
Conclusion	Decreasing	Increasing	Decreasing	Increasing



Minimum Maximum Minimum

Exercise

Find stationary points and determine the nature of the following graphs.

1.) $f(x) = 5x^2 - 2x$

2.) $f(x) = x^4 + 8x^3 + 2$

3.) $f(x) = x^3 - 9x^2 + 24x - 10$

4.) $f(x) = \frac{x^5}{5} + \frac{x^4}{4} + \frac{x^3}{12}$

Thank You