

Part a

1st iteration $\rightarrow i = 2 \rightarrow 2^1 \rightarrow 2^{\overline{1}}$
 2nd iteration $\rightarrow i = 4 \rightarrow 2^2 \rightarrow 2^{\overline{2}}$
 3rd iteration $\rightarrow i = 16 \rightarrow 2^4 \rightarrow 2^{\overline{3}}$
 4th iteration $\rightarrow i = 256 \rightarrow 2^8 \rightarrow 2^{\overline{4}}$
 5th iteration $\rightarrow i = 65,536 \rightarrow 2^{16} \rightarrow 2^{\overline{5}}$

$x = \text{iteration index}$

$$i = 2^{2^x} < n \rightarrow \log 2^{2^x} < \log n$$

$$2^x < \log n \rightarrow \log 2^x < \log(\log n)$$

$$x < \log(\log n)$$

$$\Theta(1) + \Theta \log(\log n) = \boxed{\Theta \log(\log n)}$$

Part b

$n = 9$

1st iteration $\rightarrow i = 1$, inner loop ~~executed~~ not executed
 2nd iteration $\rightarrow i = 2$, inner loop ~~executed~~ not executed
 3rd iteration $\rightarrow i = 3$, inner loop executes 3^3 times
 4th iteration $\rightarrow i = 4$, inner loop not executed
 5th iteration $\rightarrow i = 5$, inner loop not executed
 6th iteration $\rightarrow i = 6$, inner loop executes 6^3 times
 7th iteration $\rightarrow i = 7$, inner loop not executed
 8th iteration $\rightarrow i = 8$, inner loop not executed
 9th iteration $\rightarrow i = 9$, inner loop executes 9^3 times

When $n = 9$, inner loop executes

$$3^3 + 6^3 + 9^3 \text{ times}$$

$$\sum_{x=1}^{\sqrt{n}} \Theta(x \sqrt{n})^3 \rightarrow \sum_{x=1}^{\sqrt{n}} \Theta(x^3 \sqrt{n}^3)$$

$$\sqrt{n}^3 \cdot \sum_{x=1}^{\sqrt{n}} \Theta(x^3) \rightarrow \sqrt{n}^3 \cdot \Theta(\sqrt{n}^4)$$

via general form of the arithmetic series

$$n^{3/2} \cdot \Theta(n^{4/2}) \rightarrow \boxed{\Theta(n^{7/2})}$$

Part c

$$\sum_{i=1}^n \sum_{k=1}^n \left(\Theta(1) + \sum_{m=1}^n \Theta(1) \right) \rightarrow \sum_{i=1}^n \sum_{k=1}^n \Theta(1) = \Theta(n^2) + n \sum_{m=1}^n \log n \rightarrow \boxed{\Theta(n^2) + O(n \log n)}$$

$$m \leq n \rightarrow x \log 2 = \log n$$

$$2^x = m \rightarrow x \leq \log n$$

$$2^x \leq n$$

Part d

10th iteration $\rightarrow i = 10, \text{size} = 10, \text{new size} = 15$, executed 10 times
 15th iteration $\rightarrow i = 15, \text{size} = 15, \text{new size} = 22$, executed 15 times
 22nd iteration $\rightarrow i = 22, \text{size} = 22, \text{new size} = 33$, executed 22 times
 33rd iteration $\rightarrow i = 33, \text{size} = 33, \text{new size} = 49$, executed 33 times

When $n = 45$, inner loop executes

$$10 + 15 + 22 + 33 \text{ times}$$

$$10(1.5^0) + 10(1.5^1) + 10(1.5^2) + 10(1.5^3)$$

$$10(1.5^x) < n$$

$$\Theta \frac{n}{10}$$

$$\boxed{\Theta n}$$

$$\sum_{x=0}^{\log_{1.5} \frac{n}{10}} \left(\Theta(10(1.5^x)) \right)$$

$$1.5^x < \frac{n}{10}$$

$$x \log_{1.5} 1.5 < \log_{1.5} \frac{n}{10}$$

$$x < \log_{1.5} \frac{n}{10}$$