CSCI 104 - Counting

1a. $\{v, n, s, a, l\}$

case 1 - v _ _ _ _ $= \binom{4}{4} = 1$
case 2 - v v _ _ _ $= \binom{4}{3} = 4$
case 3 - v v v _ _ $= \binom{4}{2} = 6$

→ 11 subsets

1b.

case 1 - 1v $= \frac{5!}{1!} \cdot \binom{4}{4} = 120$
case 2 - 2vs $= 5!/2! \cdot \binom{4}{3} = 240$
case 3 - 3vs $= 5!/3! \cdot \binom{4}{2} = 120$

→ 480 strings

2. $\binom{13}{2}\binom{4}{2}\binom{4}{2}\binom{11}{1}\binom{4}{1} = 78 \cdot 6 \cdot 6 \cdot 11 \cdot 4 =$ 123,552 ways

3. case 1 - fighting couple has no songs $= \binom{21}{5}$
case 2 - fighting couple has 1 song $= \binom{20}{5}$

$\binom{21}{5} + \binom{20}{5}$ = 35 853 ways

4.

2 nodes → 2 cases

3 nodes → 5 cases

4 nodes → 14 cases
(5 cases, 5 cases, 2 cases, 2 cases)

5 nodes → 42 cases
(14 cases, 14 cases, 5 cases, 5 cases, 2 cases)

$2 \cdot 5 \cdot 42 =$ 420 cases

5.

case 1 - nurse on break

Nurse	Patients								
1	8	7	6	6	5	5	4	4	
2	1	2	2	3	3	4	4	3	= 8 ways
3	1	1	2	1	2	1	2	3	

case 2 - all nurses working

Nurse	Patients								
1	7	6	5	5	4	4	4	3	3
2	1	2	3	2	2	3	4	3	3
3	1	1	1	2	2	2	1	2	3
4	1	1	1	1	2	1	1	2	1

= 9 ways

$8 + 9 =$ **17 ways**