Probability

1. $\dfrac{\overset{\text{1st question}}{15} \cdot \overset{\text{2nd question}}{14} \cdot \overset{\text{3rd question}}{13} \cdot 12 \cdot 11 \cdot 10 \cdot 9 \cdot \overset{\text{8th question}}{8}}{15^8} = \boxed{0.101}$

2. $\dfrac{\underset{\text{odd}}{5} \cdot \underset{\text{odd}}{4} \cdot \underset{\substack{\text{eve 9} \\ -\,2\text{odd} \\ -\,1\text{even} \\ \text{at end}}}{7} \cdot \underset{\overset{7}{-1}}{6} \cdot \underset{\text{even}}{5} = 4200 \qquad \dfrac{4200}{100,000} = 0.042$

$0.00000643 \qquad = \dbinom{8}{5} \cdot 0.042^5 (1 - 0.042)^3$

$$\boxed{6.43 \cdot 10^{-6}}$$

3.

$P(A) = \dbinom{3}{2}\left(\dfrac{1}{2}\right)^2\left(\dfrac{1}{2}\right)^1 + \dbinom{3}{3}\left(\dfrac{1}{2}\right)^3\left(\dfrac{1}{2}\right)^0 = 1/2$

$P(B) = 1 \cdot \dfrac{1}{6} \cdot \dfrac{1}{6} = 1/36$

$P(A \cap B) = P(A) \cdot P(B)$

$P(A \cap B) = \dfrac{1}{72}, \quad P(A) \cdot P(B) = 1/2 \cdot 1/36 = \dfrac{1}{72} \quad \rightarrow P(A \cap B) = P(A) \cdot P(B)$ ✓

$$\boxed{\text{independent}}$$

4.

Geometric:

$P(f) = \dbinom{4}{1}\dbinom{13}{5}, \quad P(tot) = \dbinom{52}{5} \rightarrow \dfrac{\dbinom{4}{1}\dbinom{13}{5}}{\dbinom{52}{5}} = 0.0019808$

expected: $\dfrac{1}{p} \rightarrow \dfrac{1}{0.00198} = \boxed{505 \text{ hands}}$

5.

superstar $\rightarrow$ P(4/5 wins) = $\binom{5}{4} \cdot 0.7^4 \cdot 0.3 = 0.360$

no superstar $\rightarrow$ P(4/5 wins) = $\binom{5}{4} \cdot 0.5^4 \cdot 0.5 = 0.156$

P(4/5 wins) $\rightarrow$ $0.360 \cdot 0.75 + 0.156 \cdot 0.25 = 0.307$

$$\frac{3.60 \cdot 0.75}{0.309} = \boxed{0.874}$$