

CSC1 104 - Counting

1a. $\{u, n, s, a, l\}$

Case 1 - \underline{u} --- = $\binom{4}{4} = 1$

Case 2 - $\underline{u} \underline{u}$ --- = $\binom{4}{3} = 4$

Case 3 - $\underline{u} \underline{u} \underline{u}$ --- = $\binom{4}{2} = 6$

11 subsets

1b.

Case 1 - $1u = \frac{5!}{1!} \cdot \binom{4}{4} = 120$

Case 2 - $2us = \frac{5!}{2!} \cdot \binom{4}{3} = 240$

Case 3 - $3us = \frac{5!}{3!} \cdot \binom{4}{2} = 120$

480 strings

2. ~~123456789101112131415161718192021222324252627282930313233343536373839404142434445464748495051525354555657585960616263646566676869707172737475767778798081828384858687888990919293949596979899100~~ $\binom{15}{2} \binom{4}{2} \binom{4}{2} \binom{11}{1} \binom{4}{1} = 78 \cdot 6 \cdot 6 \cdot 11 \cdot 4 = 125,552$ ways

3. Case 1 - fighting couple has no songs = $\binom{21}{5} \binom{21}{5} + \binom{20}{5}$

Case 2 - fighting couple has 1 song = $\binom{20}{5} = 35,853$ ways

4.

2 nodes $\begin{matrix} (n) \\ | \\ (1) \end{matrix}, \begin{matrix} (n) \\ | \\ (1) \end{matrix} \rightarrow 2 \text{ cases}$

3 nodes $\begin{matrix} (n) \\ | \\ (1) \\ | \\ (2) \end{matrix}, \begin{matrix} (n) \\ | \\ (1) \\ | \\ (2) \end{matrix}, \begin{matrix} (n) \\ | \\ (1) \\ | \\ (2) \end{matrix}, \begin{matrix} (n) \\ | \\ (1) \\ | \\ (2) \end{matrix}, \begin{matrix} (n) \\ | \\ (1) \\ | \\ (2) \end{matrix} \rightarrow 5 \text{ cases}$

4 nodes $\begin{matrix} (n) \\ | \\ (3) \end{matrix}, \begin{matrix} (n) \\ | \\ (3) \end{matrix}, \begin{matrix} (n) \\ | \\ (1) \\ | \\ (2) \end{matrix}, \begin{matrix} (n) \\ | \\ (2) \\ | \\ (1) \end{matrix} \rightarrow 14 \text{ cases}$
 5 cases 5 cases 2 cases 2 cases

5 nodes $\begin{matrix} (n) \\ | \\ (4) \end{matrix}, \begin{matrix} (n) \\ | \\ (4) \end{matrix}, \begin{matrix} (n) \\ | \\ (1) \\ | \\ (3) \end{matrix}, \begin{matrix} (n) \\ | \\ (3) \\ | \\ (1) \end{matrix}, \begin{matrix} (n) \\ | \\ (2) \\ | \\ (2) \end{matrix} \rightarrow 42 \text{ cases}$
 14 cases 14 cases 5 cases 5 cases 2 cases

$2 \cdot 5 \cdot 42 = 420 \text{ cases}$

5.

case 1 - nurse on break

Nurse	Patients									
1	8	7	6	6	5	5	4	4		
2	1	2	2	3	3	4	4	3	= 8 ways	
3	1	1	2	1	2	1	2	3		

case 2 - all nurses working

Nurse	Patients									
1	7	6	5	5	4	4	4	3	3	
2	1	2	3	2	2	3	4	3	3	
3	1	1	1	2	2	2	1	2	3	= 9 ways
4	1	1	1	1	2	1	1	2	1	

$$8 + 9 = \boxed{17 \text{ ways}}$$