

$$\begin{aligned}
d(p, g) &= \int_{-1}^1 (p(x) - g(x))^2 dx \\
&= \int_c^1 (p(x) - b_1 x - b_0)^2 dx + \int_{-1}^c (p(x) - a_1 x - a_0)^2 dx \\
&= \int_c^1 (p(x)^2 + b_1^2 x^2 + b_0^2 - 2b_1 x p(x) - 2b_0 p(x) + 2b_1 b_0 x) dx \\
&\quad + \int_{-1}^c (p(x)^2 + a_1^2 x^2 + a_0^2 - 2a_1 x p(x) - 2a_0 p(x) + 2a_1 a_0 x) dx \\
&= \left\{ \int_c^1 x^2 dx \right\} b_1^2 + \left\{ 2 \int_c^1 x dx \right\} b_1 b_0 + \left\{ \int_c^1 1 dx \right\} b_0^2 \quad (1) \\
&\quad - \left\{ 2 \int_c^1 x p(x) dx \right\} b_1 - \left\{ 2 \int_c^1 p(x) dx \right\} b_0 \\
&\quad + \left\{ \int_{-1}^c x^2 dx \right\} a_1^2 + \left\{ 2 \int_{-1}^c x dx \right\} a_1 a_0 + \left\{ \int_{-1}^c 1 dx \right\} a_0^2 \\
&\quad - \left\{ 2 \int_{-1}^c x p(x) dx \right\} a_1 - \left\{ 2 \int_{-1}^c p(x) dx \right\} a_0 + \int_{-1}^1 p(x)^2 dx \\
&= c_0 b_1^2 + c_1 b_1 b_0 + c_2 b_0^2 + c_3 b_1 + c_4 b_0 \\
&\quad + c_5 a_1^2 + c_6 a_1 a_0 + c_7 a_0^2 + c_8 a_1 + c_9 a_0 + c_{const}
\end{aligned}$$

$$\begin{aligned}
c_0 &= \int_c^1 x^2 dx \\
c_1 &= 2 \int_c^1 x dx \\
c_2 &= \int_c^1 1 dx \\
c_3 &= -2 \int_c^1 xp(x) dx \\
c_4 &= -2 \int_c^1 p(x) dx \\
c_5 &= \int_{-1}^c x^2 dx \\
c_6 &= 2 \int_{-1}^c x dx \\
c_7 &= \int_{-1}^c 1 dx \\
c_8 &= -2 \int_{-1}^c xp(x) dx \\
c_9 &= -2 \int_{-1}^c p(x) dx \\
c_{const} &= \int_{-1}^1 p(x)^2 dx
\end{aligned} \tag{2}$$

$$\begin{aligned}
b_1 c + b_0 &= a_1 c + a_0 \\
a_0 &= b_1 c + b_0 - a_1 c
\end{aligned} \tag{3}$$

$$\begin{aligned}
d(p, g) &= c_0 b_1^2 + c_1 b_1 b_0 + c_2 b_0^2 + c_3 b_1 + c_4 b_0 \\
&+ c_5 a_1^2 + c_6 a_1 (b_1 c + b_0 - a_1 c) + c_7 (b_1 c + b_0 - a_1 c)^2 + c_8 a_1 + c_9 (b_1 c + b_0 - a_1 c) + c_{const} \\
&= c_0 b_1^2 + c_1 b_1 b_0 + c_2 b_0^2 + c_3 b_1 + c_4 b_0 \\
&+ c_5 a_1^2 + c_6 a_1 b_1 + c_6 b_0 a_1 - c_6 a_1^2 \\
&+ c_7 c^2 b_1^2 + c_7 b_0^2 + c_7 c^2 a_1^2 + 2c_7 c b_1 b_0 - 2c_7 c b_0 a_1 - 2c_7 c^2 a_1 b_1 \\
&+ c_8 a_1 + c_9 c b_1 + c_9 b_0 - c_9 c a_1 + c_{const} \\
&= (c_0 + c_7 c^2) b_1^2 + (c_2 + c_7) b_0^2 + (c_5 - c_6 c + c_7 c^2) a_1^2 \\
&+ (c_1 + 2c_7 c) b_1 b_0 + (c_6 - 2c_7 c) b_0 a_1 + (c_6 c - 2c_7 c^2) a_1 b_1 \\
&+ (c_3 + c_9 c) b_1 + (c_4 + c_9) b_0 + (c_8 - c_9 c) a_1 + c_{const} \\
&= d_1 b_1^2 + d_2 b_0^2 + d_3 a_1^2 + d_4 b_1 b_0 + d_5 b_0 a_1 + d_6 a_1 b_1 + d_7 b_1 + d_8 b_0 + d_9 a_1 + c_{const}
\end{aligned} \tag{4}$$

$$\begin{aligned}
d_1 &= c_0 + c_7 c^2 \\
d_2 &= c_2 + c_7 \\
d_3 &= c_5 - c_6 c + c_7 c^2 \\
d_4 &= c_1 + 2c_7 c \\
d_5 &= c_6 - 2c_7 c \\
d_6 &= c_6 c - 2c_7 c^2 \\
d_7 &= c_3 + c_9 c \\
d_8 &= c_4 + c_9 \\
d_9 &= c_8 - c_9 c
\end{aligned} \tag{5}$$

$$\begin{aligned}
d(p, g) &= d_1 b_1^2 + d_2 b_0^2 + d_3 a_1^2 + d_4 b_1 b_0 + d_5 b_0 a_1 + d_6 a_1 b_1 + d_7 b_1 + d_8 b_0 + d_9 a_1 + c_{const} \\
\frac{\partial d(p, g)}{\partial b_1} &= 2d_1 b_1 + d_4 b_0 + d_6 a_1 + d_7 \\
\frac{\partial d(p, g)}{\partial b_0} &= 2d_2 b_0 + d_4 b_1 + d_5 a_1 + d_8 \\
\frac{\partial d(p, g)}{\partial a_1} &= 2d_3 a_1 + d_5 b_0 + d_6 b_1 + d_9
\end{aligned} \tag{6}$$

$$\begin{pmatrix} 2d_1 & d_4 & d_6 \\ d_4 & 2d_2 & d_5 \\ d_6 & d_5 & 2d_3 \end{pmatrix} \begin{pmatrix} b_1 \\ b_0 \\ a_1 \end{pmatrix} = \begin{pmatrix} -d_7 \\ -d_8 \\ -d_9 \end{pmatrix} \tag{7}$$

$$\det A = 8d_1 d_2 d_3 + 2d_4 d_5 d_6 - 2d_1 d_5^2 - 2d_2 d_6^2 - 2d_3 d_4^2 \tag{8}$$

$$\begin{aligned}
\frac{1}{\det A} &\begin{pmatrix} 4d_2 d_3 - d_5^2 & d_5 d_6 - 2d_3 d_4 & d_4 d_5 - 2d_2 d_6 \\ d_5 d_6 - 2d_3 d_4 & 4d_1 d_3 - d_6^2 & d_4 d_6 - 2d_1 d_5 \\ d_4 d_5 - 2d_2 d_6 & d_4 d_6 - 2d_1 d_5 & 4d_1 d_2 - d_4^2 \end{pmatrix} \begin{pmatrix} 2d_1 & d_4 & d_6 \\ d_4 & 2d_2 & d_5 \\ d_6 & d_5 & 2d_3 \end{pmatrix} \begin{pmatrix} b_1 \\ b_0 \\ a_1 \end{pmatrix} \\
&= \frac{1}{\det A} \begin{pmatrix} 4d_2 d_3 - d_5^2 & d_5 d_6 - 2d_3 d_4 & d_4 d_5 - 2d_2 d_6 \\ d_5 d_6 - 2d_3 d_4 & 4d_1 d_3 - d_6^2 & d_4 d_6 - 2d_1 d_5 \\ d_4 d_5 - 2d_2 d_6 & d_4 d_6 - 2d_1 d_5 & 4d_1 d_2 - d_4^2 \end{pmatrix} \begin{pmatrix} -d_7 \\ -d_8 \\ -d_9 \end{pmatrix}
\end{aligned} \tag{9}$$

$$\begin{pmatrix} b_1 \\ b_0 \\ a_1 \end{pmatrix} = \frac{1}{\det A} \begin{pmatrix} 4d_2 d_3 - d_5^2 & d_5 d_6 - 2d_3 d_4 & d_4 d_5 - 2d_2 d_6 \\ d_5 d_6 - 2d_3 d_4 & 4d_1 d_3 - d_6^2 & d_4 d_6 - 2d_1 d_5 \\ d_4 d_5 - 2d_2 d_6 & d_4 d_6 - 2d_1 d_5 & 4d_1 d_2 - d_4^2 \end{pmatrix} \begin{pmatrix} -d_7 \\ -d_8 \\ -d_9 \end{pmatrix} \tag{10}$$