# Math 214 Winter 2019 Written Homework 5

**Instructions:**

- Please write your solutions clearly and neatly. Justify them completely.
- Fill out your name and Section below.
- Upload your solutions (including this cover page) to Gradescope before the deadline. (Friaday, February 22, 11:59pm)

NAME:

SECTION NUMBER:

INSTRUCTOR:

1. (11 points) You are given that $L : \mathbb{R}^4 \to \mathbb{R}^4$ is a linear map. You are also given that $\mathfrak{B} = (\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4)$ is a basis for $\mathbb{R}^4$ and the matrix of $L$ with respect to the basis $\mathfrak{B}$ is

$$B = \begin{bmatrix} 1 & -1 & 1 & -1 \\ -1 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

Your answers to this question may involve the vectors $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4$.

(a) (2 points) Find a basis for the kernel of $L$.

(b) (3 points) Find a basis for the image of $L$.

(c) (4 points) Which of the vectors $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4$ lies in the image of $L$? In each case, explain why or why not.

(d) (2 points) Find the vector $L^{100}(\mathbf{v}_1 - \mathbf{v}_2)$ (as a linear combination of the $\mathbf{v}_i$).

2. (8 points) Let $V_1$ and $V_2$ be the subspaces of $\mathbb{R}^3$ given respectively by the equations:

$$V_1 : \quad x_1 + x_2 + x_3 = 0$$

$$V_2 : \quad 2x_1 - x_2 - x_3 = 0$$

(a) (2 points) Find a basis for $V_1^{\perp}$.

(b) (2 points) Find a basis for $V_2^{\perp}$.

(c) (4 points) Find an orthonormal basis $(\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3)$ of $\mathbb{R}^3$ such that $(\mathbf{u}_1, \mathbf{u}_2)$ is a basis of $V_1$, while $(\mathbf{u}_2, \mathbf{u}_3)$ is a basis of $V_2$.

3. (11 points) In this question, $f : \mathbb{R} \to \mathbb{R}$ is a differentiable function. We will use linear algebra try to find a good linear approximation to $f$ near a point $x = a$. Our linear approximation will be of the form $y = c + mx$.

   (a) (3 points) Use the values of $f$ at the point $x = a$ and the nearby point $x = a + h$ to find a good approximation. (You will need to set up a linear system involving $c, m, a, h, f(a), f(a + h)$, where $c$ and $m$ are the variables. You should also be able to solve this linear system exactly for the vector $\begin{bmatrix} c \\ m \end{bmatrix}$.)

   (b) (2 points) Explain what happens to your solution as $h \mapsto 0$. Why is this reasonable?

(c) (4 points) Now we will use three points $x = a$, $x = a + h$ and $x = a - h$. The linear system you get now is overdetermined, so it is not possible in general to solve exactly. Nevertheless you can find a *least-squares* solution. What is the least squares solution for $\begin{bmatrix} c \\ m \end{bmatrix}$? (The answer you get should be quite a bit more complicated than that of part (a). You may continue your work on the next page.)

Part (c) contd ...

(d) (2 points) What happens when you let $h \mapsto 0$ in the solution of part (c)? How does this compare with your answer to part (b)?