Cloud computing [40 pts]

A cloud computing provider needs to process 1000 jobs, indexed by $i = 1, \ldots, 1000$ with 20 machines, indexed by $j = 1, \ldots, 20$. We denote by c_{ij} the energy consumption resulting from performing job i on machine j. The objective of the problem is to allocate jobs to machines, while minimizing the total energy consumption and ensuring that each job gets fully processed.

a. Assume that each machine j has finite capacity, and can process up to C_j jobs. Formulate the problem as a linear optimization model. Is it a network flow? If so, propose a network representation, including the supply and/or demand at each node, the cost of each arc, and the capacity on each arc. If not, explain why not. Is the solution guaranteed to be integral?

b. You have access to the following data files:
   - energy.csv: A matrix of size 1000×20 that indicates the energy consumption c_{ij} for each job i (rows) and each machine j (columns).
   - capacity.csv: A vector of size 20 that indicates the capacity C_j of machine j.

   Implement the model, and report (i) the optimal energy consumption; (ii) the energy consumption that would be achieved if each job could be assigned to any machine without any consideration for machine capacities; and (iii) the number of jobs that are not (fully) assigned to the machine with the lowest energy consumption.

c. Write the dual of your linear optimization formulation. Propose a metric β_{ij} such that each job $i = 1, \ldots, 1000$ is served by a machine $j = 1, \ldots, 20$ that minimizes β_{ij}. Explain the underlying intuition in simple terms.

d. Actually, all jobs are not created equal. Accordingly, we replace the machine capacity constraint by a machine utilization constraint. Let r_{ij} be the utilization of machine j when performing job i, and let U_j be the maximal utilization of machine j. Formulate the problem as a linear optimization model. Is it a network flow? If so, propose a network representation, including the supply and/or demand at each node, the cost of each arc, and the capacity on each arc. If not, explain why not. Is the solution guaranteed to be integral?

e. You have access to the following data files:
   - utilization.csv: A matrix of size 1000×20 that indicates the utilization r_{ij} for each job i (rows) and each machine j (columns).
   - maxutil.csv: A vector of size 20 that indicates the maximal utilization U_j of machine j.

   Implement the model computationally, and report (i) the optimal energy consumption; (ii) the energy consumption that would be achieved if each job could be assigned to any machine without any consideration for machine utilization; and (iii) the number of jobs that are not (fully) assigned to the machine with the lowest energy consumption.

f. Provide a scatter plot showing the maximal utilization of each machine on the horizontal axis and the dual variable of the corresponding utilization constraint on the vertical axis. Comment briefly on the values and the trends that you observe.