```
Proof summary for theory top_group
    Theory top_group totals: 0 formulas, 0 attempted, 0 succeeded (0.00
s)

Proof summary for theory commutative_semigroup
    IMP_commutative_groupoid_TCC1.........proved - complete
[shostak](0.22 s)
    IMP_semigroup_TCC1....................proved - complete
[shostak](0.22 s)
    commutative_semigroup_TCC1............proved - complete
[shostak](0.20 s)
    commutative_semigroup_is_semigroup....proved - complete
[shostak](0.26 s)
    commutative_semigroup_is_commutative_groupoid...proved -
complete   [shostak](0.25 s)
    Theory commutative_semigroup totals: 5 formulas, 5 attempted, 5
succeeded (1.15 s)

Proof summary for theory commutative_groupoid
    commutative_groupoid_TCC1.............proved - complete
[shostak](0.23 s)
    commutative...........................proved - complete
[shostak](0.23 s)
    commutative_groupoid_is_groupoid......proved - complete
[shostak](0.27 s)
    Theory commutative_groupoid totals: 3 formulas, 3 attempted, 3
succeeded (0.73 s)

Proof summary for theory cyclic_monoid
    IMP_monoid_TCC1.......................proved - complete
[shostak](0.21 s)
    cyclic_monoid_TCC1....................proved - complete
[shostak](0.22 s)
    cyclic_monoid_is......................proved - complete
[shostak](0.21 s)
    cyclic_monoid_is_monoid...............proved - complete
[shostak](0.27 s)
    cyclic_monoid_is_commutative_monoid...proved - complete
[shostak](0.30 s)
    Theory cyclic_monoid totals: 5 formulas, 5 attempted, 5 succeeded
(1.21 s)

Proof summary for theory cyclic_monoid_def
    Theory cyclic_monoid_def totals: 0 formulas, 0 attempted, 0
succeeded (0.00 s)

Proof summary for theory subgroups
    G_TCC1................................proved - complete
```

```
[shostak](0.23 s)
    pg64_1.............................proved - complete
[shostak](0.84 s)
    center_normal_TCC1.................proved - complete
[shostak](0.25 s)
    center_normal......................proved - complete
[shostak](0.38 s)
    Theory subgroups totals: 4 formulas, 4 attempted, 4 succeeded (1.70
s)

Proof summary for theory symmetric_groups
    op_TCC1............................proved - complete
[shostak](0.25 s)
    Sym_is_group.......................proved - complete
[shostak](0.94 s)
    Theory symmetric_groups totals: 2 formulas, 2 attempted, 2
succeeded (1.19 s)

Proof summary for theory group_test
    integer_plus_TCC1..................proved - complete
[shostak](0.33 s)
    nz_rational_mult_TCC1..............proved - complete
[shostak](0.45 s)
    pos_rational_mult_TCC1.............proved - complete
[shostak](0.47 s)
    Theory group_test totals: 3 formulas, 3 attempted, 3 succeeded
(1.26 s)

Proof summary for theory infinite_cyclic_groups
    Z_TCC1.............................proved - complete
[shostak](0.38 s)
    F_TCC1.............................proved - complete
[shostak](0.22 s)
    Z_gen..............................proved - complete
[shostak](0.89 s)
    inf_cyclic_is_Z....................proved - complete
[shostak](0.96 s)
    Theory infinite_cyclic_groups totals: 4 formulas, 4 attempted, 4
succeeded (2.45 s)

Proof summary for theory cayleys
    S_TCC1.............................proved - complete
[shostak](0.24 s)
    cayley_prep_TCC1...................proved - complete
[shostak](0.21 s)
    cayley_prep_TCC2...................proved - complete
[shostak](0.27 s)
    cayley_prep........................proved - complete
```

```
[shostak](0.54 s)
    trans_is_group_TCC1..................proved - complete
[shostak](0.22 s)
    trans_is_group_TCC2..................proved - complete
[shostak](0.32 s)
    trans_is_group.......................proved - complete
[shostak](1.72 s)
    Cayleys_TCC1.........................proved - complete
[shostak](0.22 s)
    Cayleys_TCC2.........................proved - complete
[shostak](0.28 s)
    Cayleys..............................proved - complete
[shostak](0.85 s)
    Theory cayleys totals: 10 formulas, 10 attempted, 10 succeeded
(4.87 s)

Proof summary for theory A_group
    op_TCC1..............................proved - complete
[shostak](0.23 s)
    A_is_group_TCC1......................proved - complete
[shostak](0.21 s)
    A_is_group...........................proved - complete
[shostak](0.79 s)
    Theory A_group totals: 3 formulas, 3 attempted, 3 succeeded (1.23
s)

Proof summary for theory zn
    floor_help...........................proved - complete
[shostak](0.32 s)
    Z_group..............................proved - complete
[shostak](0.36 s)
    Z_TCC1...............................proved - complete
[shostak](0.22 s)
    Z_prep_TCC1..........................proved - complete
[shostak](0.22 s)
    Z_prep_TCC2..........................proved - complete
[shostak](0.23 s)
    Z_prep...............................proved - complete
[shostak](1.13 s)
    Z__TCC1..............................proved - complete
[shostak](0.22 s)
    nZ_plus_TCC1.........................proved - complete
[shostak](0.38 s)
    nZ_prep_TCC1.........................proved - complete
[shostak](0.26 s)
    nZ_prep_TCC2.........................proved - complete
[shostak](0.24 s)
    nZ_prep..............................proved - complete
```

```
[shostak](0.47 s)
    nZ_TCC1.............................proved - complete
[shostak](0.44 s)
    nZ_TCC2.............................proved - complete
[shostak](0.47 s)
    nZ_normal_TCC1......................proved - complete
[shostak](0.60 s)
    nZ_normal...........................proved - complete
[shostak](0.33 s)
    Z_fact_test_TCC1....................proved - complete
[shostak](1.00 s)
    Z_fact_test_TCC2....................proved - complete
[shostak](0.48 s)
    Z_fact_test.........................proved - complete
[shostak](0.23 s)
    Theory zn totals: 18 formulas, 18 attempted, 18 succeeded (7.58 s)

Proof summary for theory group_rew
    IMP_group_TCC1......................proved - complete
[shostak](0.21 s)
    inv_left............................proved - complete
[shostak](0.24 s)
    inv_right...........................proved - complete
[shostak](0.22 s)
    inv_inv.............................proved - complete
[shostak](0.22 s)
    inv_one.............................proved - complete
[shostak](0.23 s)
    inv_in..............................proved - complete
[shostak](0.22 s)
    expt_0..............................proved - complete
[shostak](0.23 s)
    expt_1..............................proved - complete
[shostak](0.21 s)
    expt_m1.............................proved - complete
[shostak](0.22 s)
    one_expt............................proved - complete
[shostak](0.23 s)
    one_left............................proved - complete
[shostak](0.22 s)
    one_right...........................proved - complete
[shostak](0.22 s)
    Theory group_rew totals: 12 formulas, 12 attempted, 12 succeeded
(2.66 s)

Proof summary for theory top_field
    Theory top_field totals: 0 formulas, 0 attempted, 0 succeeded (0.00
s)
```

```
Proof summary for theory commutative_ring_with_one
    IMP_ring_with_one_TCC1..............proved - complete
[shostak](0.22 s)
    IMP_commutative_ring_TCC1...........proved - complete
[shostak](0.30 s)
    commutative_ring_with_one_TCC1........proved - complete
[shostak](0.23 s)
    commutative_ring_with_one_is.........proved - complete
[shostak](0.23 s)
    commutative_ring_with_one_is_commutative_ring...proved -
complete   [shostak](0.39 s)
    commutative_ring_with_one_is_ring_with_one...proved - complete
[shostak](0.39 s)
    commutative_ring_with_one_is_commutative_monoid...proved -
complete   [shostak](0.36 s)
    Theory commutative_ring_with_one totals: 7 formulas, 7 attempted, 7
succeeded (2.13 s)

Proof summary for theory field
    IMP_division_ring_TCC1..............proved - complete
[shostak](0.24 s)
    IMP_integral_domain_TCC1............proved - complete
[shostak](0.36 s)
    field_TCC1..........................proved - complete
[shostak](0.26 s)
    nz_star_TCC1........................proved - complete
[shostak](0.26 s)
    field_is_division_ring..............proved - complete
[shostak](0.51 s)
    field_is_integral_domain............proved - complete
[shostak](0.28 s)
    field_is_abelian_group_TCC1.........proved - complete
[shostak](0.25 s)
    field_is_abelian_group_TCC2.........proved - complete
[shostak](0.25 s)
    field_is_abelian_group..............proved - complete
[shostak](1.33 s)
    mult_div_TCC1.......................proved - complete
[shostak](0.26 s)
    mult_div............................proved - complete
[shostak](0.74 s)
    times_div_right.....................proved - complete
[shostak](0.73 s)
    div_times_TCC1......................proved - complete
[shostak](0.26 s)
    div_times...........................proved - complete
[shostak](1.24 s)
```

```
    cross_mult.............................proved - complete
[shostak](1.43 s)
    add_div................................proved - complete
[shostak](1.50 s)
    minus_div1.............................proved - complete
[shostak](0.34 s)
    sq_div.................................proved - complete
[shostak](0.32 s)
    Theory field totals: 18 formulas, 18 attempted, 18 succeeded (10.55
s)

Proof summary for theory division_ring
    IMP_ring_with_one_TCC1.................proved - complete
[shostak](0.24 s)
    IMP_ring_nz_closed_TCC1................proved - complete
[shostak](0.30 s)
    IMP_group_TCC1.........................proved - complete
[shostak](0.29 s)
    IMP_group_TCC2.........................proved - complete
[shostak](0.31 s)
    IMP_group_TCC3.........................proved - complete
[shostak](0.26 s)
    IMP_group_TCC4.........................proved - complete
[shostak](0.54 s)
    division_ring_TCC1.....................proved - complete
[shostak](0.28 s)
    division_ring_is.......................proved - complete
[shostak](0.25 s)
    division_ring_is_ring_with_one........proved - complete
[shostak](0.47 s)
    division_ring_is_ring_nz_closed.......proved - complete
[shostak](0.28 s)
    division_ring_is_group.................proved - complete
[shostak](0.70 s)
    one_ne_zero............................proved - complete
[shostak](0.30 s)
    cancel_times_right.....................proved - complete
[shostak](0.33 s)
    cancel_times_left......................proved - complete
[shostak](0.34 s)
    idempotent_times.......................proved - complete
[shostak](0.28 s)
    recip_ne_zero..........................proved - complete
[shostak](0.46 s)
    nz_T_div_nz_T_is_nz_T..................proved - complete
[shostak](0.48 s)
    div_simplify...........................proved - complete
[shostak](0.31 s)
```

```
    cancel_div_right.....................proved - complete
[shostak](0.76 s)
    cancel_div_left......................proved - complete
[shostak](0.00 s)
    times_div_left.......................proved - complete
[shostak](0.27 s)
    div_eq_zero..........................proved - complete
[shostak](0.49 s)
    div_mult.............................proved - complete
[shostak](0.29 s)
    div_mult_left........................proved - complete
[shostak](0.29 s)
    div_mult_right.......................proved - complete
[shostak](0.27 s)
    div_distributes......................proved - complete
[shostak](0.46 s)
    div_distributes_minus................proved - complete
[shostak](0.31 s)
    div_div1.............................proved - complete
[shostak](0.31 s)
    div_div2_TCC1........................proved - complete
[shostak](0.26 s)
    div_div2.............................proved - complete
[shostak](0.70 s)
    Theory division_ring totals: 30 formulas, 30 attempted, 30
succeeded (10.84 s)

Proof summary for theory top_sylow
    Theory top_sylow totals: 0 formulas, 0 attempted, 0 succeeded (0.00
s)

Proof summary for theory sylow_theorems
    p_subgroup_sylow?_TCC1...............proved - complete
[shostak](0.32 s)
    p_subgroup_sylow?_TCC2...............proved - complete
[shostak](0.31 s)
    subgroup_is_factor_TCC1..............proved - complete
[shostak](0.37 s)
    subgroup_is_factor_TCC2..............proved - complete
[shostak](0.37 s)
    subgroup_is_factor_TCC3..............proved - complete
[shostak](0.35 s)
    subgroup_is_factor...................proved - complete
[shostak]( n/a s)
    First_Sylow_Theorem_TCC1.............proved - complete
[shostak](0.36 s)
    First_Sylow_Theorem_TCC2.............proved - incomplete
[shostak](0.37 s)
```

```
    First_Sylow_Theorem_TCC3.............proved - incomplete
[shostak](0.36 s)
    First_Sylow_Theorem_TCC4.............proved - incomplete
[shostak](0.38 s)
    First_Sylow_Theorem..................proved - incomplete
[shostak]( n/a s)
    p_group_is_subgroup_TCC1.............proved - complete
[shostak](0.32 s)
    p_group_is_subgroup_TCC2.............proved - incomplete
[shostak](0.58 s)
    p_group_is_subgroup_TCC3.............proved - incomplete
[shostak](0.37 s)
    p_group_is_subgroup..................proved - incomplete
[shostak]( n/a s)
    p_subgroup_sylow_order_TCC1..........proved - incomplete
[shostak](0.38 s)
    p_subgroup_sylow_order...............proved - incomplete
[shostak]( n/a s)
    conjugate_is_p_subgroup_sylow_TCC1....proved - incomplete
[shostak](0.44 s)
    conjugate_is_p_subgroup_sylow.........proved - incomplete
[shostak]( n/a s)
    unique_is_normal.....................proved - incomplete
[shostak]( n/a s)
    Second_Sylow_Theorem_TCC1............proved - incomplete
[shostak](0.32 s)
    Second_Sylow_Theorem_TCC2............proved - incomplete
[shostak](0.43 s)
    Second_Sylow_Theorem.................proved - incomplete
[shostak]( n/a s)
    Third_Sylow_Theorem_TCC1.............proved - incomplete
[shostak](0.42 s)
    Third_Sylow_Theorem_TCC2.............proved - incomplete
[shostak](0.43 s)
    Third_Sylow_Theorem..................proved - incomplete
[shostak]( n/a s)
    Theory sylow_theorems totals: 26 formulas, 26 attempted, 26
succeeded (6.88 s)

Proof summary for theory isomorphism_theorems
    G_TCC1...............................proved - complete
[shostak](0.30 s)
    GP_TCC1..............................proved - complete
[shostak](0.30 s)
    quotient_subgroup_TCC1...............proved - complete
[shostak](0.34 s)
    quotient_subgroup_TCC2...............proved - complete
[shostak](0.39 s)
```

```
    quotient_subgroup_TCC3..............proved - complete
[shostak](0.37 s)
    quotient_subgroup_TCC4..............proved - complete
[shostak](0.35 s)
    quotient_subgroup...................proved - complete
[shostak]( n/a s)
    second_isomorphism_th_aux_TCC1........proved - incomplete
[shostak](0.49 s)
    second_isomorphism_th_aux_TCC2........proved - incomplete
[shostak](0.64 s)
    second_isomorphism_th_aux_TCC3........proved - incomplete
[shostak](0.62 s)
    second_isomorphism_th_aux_TCC4........proved - incomplete
[shostak](0.75 s)
    second_isomorphism_th_aux.............proved - incomplete
[shostak]( n/a s)
    second_isomorphism_th_TCC1............proved - incomplete
[shostak](0.42 s)
    second_isomorphism_th_TCC2............proved - incomplete
[shostak](0.37 s)
    second_isomorphism_th_TCC3............proved - incomplete
[shostak](0.41 s)
    second_isomorphism_th_TCC4............proved - incomplete
[shostak](0.39 s)
    second_isomorphism_th_TCC5............proved - incomplete
[shostak](0.61 s)
    second_isomorphism_th.................proved - incomplete
[shostak]( n/a s)
    third_isomorphism_th_aux_TCC1.........proved - incomplete
[shostak](0.33 s)
    third_isomorphism_th_aux_TCC2.........proved - incomplete
[shostak](0.36 s)
    third_isomorphism_th_aux_TCC3.........proved - incomplete
[shostak](0.36 s)
    third_isomorphism_th_aux..............proved - incomplete
[shostak]( n/a s)
    third_isomorphism_th_TCC1............proved - incomplete
[shostak](0.39 s)
    third_isomorphism_th_TCC2............proved - incomplete
[shostak](0.34 s)
    third_isomorphism_th_TCC3............proved - incomplete
[shostak](0.92 s)
    third_isomorphism_th_TCC4............proved - incomplete
[shostak](0.35 s)
    third_isomorphism_th_TCC5............proved - incomplete
[shostak](0.80 s)
    third_isomorphism_th_TCC6............proved - incomplete
[shostak](0.36 s)
```

```
    third_isomorphism_th.................proved - incomplete
[shostak]( n/a s)
    correspondence_theorem...............proved - incomplete
[shostak]( n/a s)
    Theory isomorphism_theorems totals: 30 formulas, 30 attempted, 30
succeeded (10.94 s)

Proof summary for theory homomorphism_lemmas
    G_TCC1...............................proved - complete
[shostak](0.29 s)
    GP_TCC1..............................proved - complete
[shostak](0.30 s)
    natural_homo_TCC1....................proved - complete
[shostak](0.34 s)
    natural_homo_TCC2....................proved - complete
[shostak](0.36 s)
    natural_homo.........................proved - complete
[shostak]( n/a s)
    homo_inv_TCC1........................proved - complete
[shostak](0.34 s)
    homo_inv.............................proved - complete
[shostak]( n/a s)
    kernel_normal........................proved - complete
[shostak]( n/a s)
    homo_image...........................proved - complete
[shostak]( n/a s)
    homo_image_normal_TCC1...............proved - complete
[shostak](0.37 s)
    homo_image_normal....................proved - complete
[shostak]( n/a s)
    homo_inv_image.......................proved - complete
[shostak]( n/a s)
    homo_inv_image_normal_TCC1...........proved - complete
[shostak](0.37 s)
    homo_inv_image_normal................proved - complete
[shostak]( n/a s)
    kernel_in_inv_image..................proved - complete
[shostak]( n/a s)
    homo_inv_image_image.................proved - complete
[shostak]( n/a s)
    homo_inv_image_image_cor.............proved - complete
[shostak]( n/a s)
    first_isomorphism_th_TCC1............proved - complete
[shostak](0.37 s)
    first_isomorphism_th_TCC2............proved - complete
[shostak](0.44 s)
    first_isomorphism_th_TCC3............proved - complete
[shostak](0.30 s)
```

```
    first_isomorphism_th_TCC4.............proved - complete
[shostak](0.37 s)
    first_isomorphism_th..................proved - complete
[shostak]( n/a s)
    Theory homomorphism_lemmas totals: 22 formulas, 22 attempted, 22
succeeded (3.86 s)

Proof summary for theory products_subgroups
    HK_subgroup...........................proved - incomplete
[shostak]( n/a s)
    HK_subgroup_permute...................proved - incomplete
[shostak]( n/a s)
    H_K_are_subgroups.....................proved - incomplete
[shostak]( n/a s)
    Theory products_subgroups totals: 3 formulas, 3 attempted, 3
succeeded (0.00 s)

Proof summary for theory homomorphisms
    IMP_group_TCC1........................proved - complete
[shostak](0.22 s)
    IMP_group_TCC2........................proved - complete
[shostak](0.24 s)
    homomorphism?_TCC1....................proved - complete
[shostak](0.32 s)
    homo_one_TCC1.........................proved - complete
[shostak](0.23 s)
    homo_one..............................proved - complete
[shostak](0.26 s)
    kernel_TCC1...........................proved - complete
[shostak](0.39 s)
    Theory homomorphisms totals: 6 formulas, 6 attempted, 6 succeeded
(1.67 s)

Proof summary for theory p_groups
    alt_is_action_TCC1....................proved - complete
[shostak](0.39 s)
    alt_is_action.........................proved - complete
[shostak]( n/a s)
    Fix_iff_subset........................proved - complete
[shostak]( n/a s)
    Fix_iff_subset_cor_TCC1...............proved - complete
[shostak](0.43 s)
    Fix_iff_subset_cor....................proved - incomplete
[shostak]( n/a s)
    subgroup_is_p_group_TCC1..............proved - complete
[shostak](0.36 s)
    subgroup_is_p_group...................proved - complete
[shostak]( n/a s)
```

```
    p_group_iff_power.....................proved - incomplete
[shostak]( n/a s)
    p_divides_index.......................proved - incomplete
[shostak]( n/a s)
    factor_cyclic_TCC1....................proved - complete
[shostak](0.42 s)
    factor_cyclic_TCC2....................proved - complete
[shostak](0.32 s)
    factor_cyclic_TCC3....................proved - complete
[shostak](0.50 s)
    factor_cyclic.........................proved - complete
[shostak]( n/a s)
    normalizer_index_TCC1.................proved - complete
[shostak](0.31 s)
    normalizer_index_TCC2.................proved - complete
[shostak](0.33 s)
    normalizer_index_TCC3.................proved - complete
[shostak](0.32 s)
    normalizer_index......................proved - incomplete
[shostak]( n/a s)
    subgroup_proper.......................proved - incomplete
[shostak]( n/a s)
    burside_theorem_TCC1..................proved - complete
[shostak](0.39 s)
    burside_theorem.......................proved - incomplete
[shostak]( n/a s)
    p_square_is_abelian...................proved - incomplete
[shostak]( n/a s)
    Theory p_groups totals: 21 formulas, 21 attempted, 21 succeeded
(3.77 s)

Proof summary for theory normalizer_centralizer
    normalizer_TCC1.......................proved - complete
[shostak](0.48 s)
    centralizer_TCC1......................proved - complete
[shostak](0.41 s)
    a_by_c_TCC1...........................proved - complete
[shostak](0.37 s)
    CL_TCC1...............................proved - complete
[shostak](0.47 s)
    normalizer_is_subgroup................proved - complete
[shostak]( n/a s)
    subset_of_normalizer..................proved - complete
[shostak]( n/a s)
    normal_in_normalizer_TCC1.............proved - complete
[shostak](0.35 s)
    normal_in_normalizer..................proved - complete
[shostak]( n/a s)
```

```
    centralizer_is_subgroup.............proved - complete
[shostak]( n/a s)
    singleton_iff_center................proved - complete
[shostak]( n/a s)
    a_by_c_is_action....................proved - complete
[shostak]( n/a s)
    Fix_is_center_TCC1..................proved - complete
[shostak](0.33 s)
    Fix_is_center.......................proved - complete
[shostak]( n/a s)
    stabilizer_is_centralizer...........proved - complete
[shostak]( n/a s)
    orbit_is_CL.........................proved - complete
[shostak]( n/a s)
    orbits_is_CLs.......................proved - complete
[shostak]( n/a s)
    orbits_nFix_is_CLs_nc...............proved - complete
[shostak]( n/a s)
    CLs_eq_index_TCC1...................proved - complete
[shostak](0.33 s)
    CLs_eq_index_TCC2...................proved - complete
[shostak](0.35 s)
    CLs_eq_index........................proved - incomplete
[shostak]( n/a s)
    class_equation_2_TCC1...............proved - complete
[shostak](0.33 s)
    class_equation_2_TCC2...............proved - incomplete
[shostak](0.54 s)
    class_equation_2....................proved - incomplete
[shostak]( n/a s)
    Theory normalizer_centralizer totals: 23 formulas, 23 attempted, 23
succeeded (3.97 s)

Proof summary for theory cauchy
    fseq_product_TCC1...................proved - complete
[shostak](0.36 s)
    fseq_product_TCC2...................proved - incomplete
[shostak](0.56 s)
    S_TCC1..............................proved - complete
[shostak](0.39 s)
    fseq_product_in.....................proved - incomplete
[shostak]( n/a s)
    fseq_product_o......................proved - incomplete
[shostak]( n/a s)
    fseq_product_one....................proved - incomplete
[shostak]( n/a s)
    fseq_product_power..................proved - incomplete
[shostak]( n/a s)
```

```
    one_in_SE.............................proved - incomplete
[shostak]( n/a s)
    order_SE..............................proved - incomplete
[shostak]( n/a s)
    S_bij_set_seq_TCC1....................proved - complete
[shostak](0.37 s)
    S_bij_set_seq.........................proved - incomplete
[shostak]( n/a s)
    S_is_finite...........................proved - incomplete
[shostak]( n/a s)
    S_card_TCC1...........................proved - incomplete
[shostak](0.35 s)
    S_card................................proved - incomplete
[shostak]( n/a s)
    F_TCC1................................proved - incomplete
[shostak](0.81 s)
    F_1_TCC1..............................proved - complete
[shostak](0.33 s)
    F_1_TCC2..............................proved - complete
[shostak](0.33 s)
    F_2_TCC1..............................proved - complete
[shostak](0.38 s)
    F_o_F12_TCC1..........................proved - incomplete
[shostak](0.64 s)
    F_o_F12...............................proved - incomplete
[shostak]( n/a s)
    fs_o_F21..............................proved - incomplete
[shostak]( n/a s)
    F_in_S................................proved - incomplete
[shostak]( n/a s)
    F_is_action_TCC1......................proved - incomplete
[shostak](0.40 s)
    F_is_action_TCC2......................proved - complete
[shostak](0.32 s)
    F_is_action_TCC3......................proved - complete
[shostak](0.32 s)
    F_is_action_TCC4......................proved - complete
[shostak](0.36 s)
    F_is_action...........................proved - incomplete
[shostak]( n/a s)
    Fixed_subset_TCC1.....................proved - incomplete
[shostak](0.40 s)
    Fixed_subset_TCC2.....................proved - incomplete
[shostak](0.36 s)
    Fixed_subset..........................proved - incomplete
[shostak]( n/a s)
    cauchy................................proved - incomplete
[shostak]( n/a s)
```

```
    cauchy_cor_TCC1.....................proved - complete
[shostak](0.30 s)
    cauchy_cor..........................proved - incomplete
[shostak]( n/a s)
    Theory cauchy totals: 33 formulas, 33 attempted, 33 succeeded (6.98
s)

Proof summary for theory finite_cyclic_groups
    IMP_finite_groups_TCC1..............proved - complete
[shostak](0.22 s)
    prime_order_cycle...................proved - complete
[shostak](0.41 s)
    Theory finite_cyclic_groups totals: 2 formulas, 2 attempted, 2
succeeded (0.63 s)

Proof summary for theory group_action
    group_action?_TCC1..................proved - complete
[shostak](0.34 s)
    group_action?_TCC2..................proved - complete
[shostak](0.33 s)
    stabilizer_TCC1.....................proved - complete
[shostak](0.47 s)
    orbit_TCC1..........................proved - complete
[shostak](0.44 s)
    Fix_TCC1............................proved - complete
[shostak](0.45 s)
    stabilizer_is_subgroup..............proved - complete
[shostak]( n/a s)
    singleton_iff_Fix...................proved - complete
[shostak]( n/a s)
    empty_iff_eq_Fix....................proved - complete
[shostak]( n/a s)
    orbits_nFix_disj_Fix................proved - complete
[shostak]( n/a s)
    orbits_is_union.....................proved - complete
[shostak]( n/a s)
    orbit_nonempty......................proved - complete
[shostak]( n/a s)
    orbits_nonempty.....................proved - complete
[shostak]( n/a s)
    set_orbits_is.......................proved - complete
[shostak]( n/a s)
    orbit_is_finite.....................proved - complete
[shostak]( n/a s)
    orbits_disjoint.....................proved - complete
[shostak]( n/a s)
    orbits_partition....................proved - complete
[shostak]( n/a s)
```

```
    orbits_nFix_partition.................proved - complete
[shostak]( n/a s)
    orbits_eq_index_aux_TCC1..............proved - complete
[shostak](0.38 s)
    orbits_eq_index_aux...................proved - complete
[shostak]( n/a s)
    orbits_eq_index_TCC1..................proved - complete
[shostak](0.39 s)
    orbits_eq_index_TCC2..................proved - complete
[shostak](0.37 s)
    orbits_eq_index.......................proved - incomplete
[shostak]( n/a s)
    counting_formula_TCC1.................proved - incomplete
[shostak](0.45 s)
    counting_formula......................proved - incomplete
[shostak]( n/a s)
    class_equation_TCC1...................proved - complete
[shostak](0.36 s)
    class_equation_TCC2...................proved - incomplete
[shostak](0.51 s)
    class_equation........................proved - incomplete
[shostak]( n/a s)
    Fix_congruence_TCC1...................proved - complete
[shostak](0.35 s)
    Fix_congruence........................proved - incomplete
[shostak]( n/a s)
    Theory group_action totals: 29 formulas, 29 attempted, 29 succeeded
(4.84 s)

Proof summary for theory lagrange_index
    IMP_right_left_cosets_TCC1............proved - complete
[shostak](0.26 s)
    Lagrange_index........................proved - incomplete
[shostak](0.57 s)
    index_divides.........................proved - incomplete
[shostak](0.27 s)
    order_factor_TCC1.....................proved - complete
[shostak](0.29 s)
    order_factor..........................proved - incomplete
[shostak](0.39 s)
    Theory lagrange_index totals: 5 formulas, 5 attempted, 5 succeeded
(1.79 s)

Proof summary for theory class_equation_scaf
    card_rest_aux_TCC1....................proved - complete   [shostak]
( 0.25 s)
    card_rest_aux_TCC2....................proved - complete   [shostak]
( 0.28 s)
```

```
    card_rest_aux_TCC3..................proved - complete   [shostak]
( 0.31 s)
    card_rest_aux.......................proved - complete   [shostak]
( 0.62 s)
    card_partition_TCC1.................proved - complete   [shostak]
( 0.28 s)
    card_partition_TCC2.................proved - incomplete [shostak]
( 1.23 s)
    card_partition......................proved - incomplete [shostak]
(15.49 s)
    divide_sigma_TCC1...................proved - incomplete [shostak]
( 0.28 s)
    divide_sigma_TCC2...................proved - incomplete [shostak]
(40.02 s)
    divide_sigma........................proved - incomplete [shostak]
(23.50 s)
    Theory class_equation_scaf totals: 10 formulas, 10 attempted, 10
succeeded (82.27 s)

Proof summary for theory groups_scaf
    divby_r.............................proved - complete
[shostak]( n/a s)
    subgroup_transitive.................proved - complete
[shostak]( n/a s)
    normal_subgroup_tran................proved - complete
[shostak]( n/a s)
    subgroup_intersection...............proved - complete
[shostak]( n/a s)
    conjugate_is_subgroup...............proved - complete
[shostak]( n/a s)
    center_is_normal_TCC1...............proved - complete
[shostak](0.31 s)
    center_is_normal....................proved - complete
[shostak]( n/a s)
    abelian_eq_center...................proved - complete
[shostak]( n/a s)
    order_gt_1..........................proved - incomplete
[shostak]( n/a s)
    order_gt_p..........................proved - incomplete
[shostak]( n/a s)
    exists_diff_one.....................proved - complete
[shostak]( n/a s)
    one_iff_divides.....................proved - complete
[shostak]( n/a s)
    order_power_TCC1....................proved - complete
[shostak](0.31 s)
    order_power_TCC2....................proved - complete
[shostak](0.35 s)
```

```
    order_power.........................proved - incomplete
[shostak]( n/a s)
    coset_power_nat_TCC1................proved - complete
[shostak](0.32 s)
    coset_power_nat_TCC2................proved - complete
[shostak](0.36 s)
    coset_power_nat_TCC3................proved - complete
[shostak](0.44 s)
    coset_power_nat_TCC4................proved - complete
[shostak](0.49 s)
    coset_power_nat.....................proved - complete
[shostak]( n/a s)
    coset_power_int.....................proved - complete
[shostak]( n/a s)
    factor_of_cyclic_is_cyclic_TCC1.....proved - complete
[shostak](0.54 s)
    factor_of_cyclic_is_cyclic_TCC2.....proved - complete
[shostak](0.35 s)
    factor_of_cyclic_is_cyclic_TCC3.....proved - complete
[shostak](0.53 s)
    factor_of_cyclic_is_cyclic..........proved - complete
[shostak]( n/a s)
    Theory groups_scaf totals: 25 formulas, 25 attempted, 25 succeeded
(4.00 s)

Proof summary for theory finite_groups
    IMP_group_TCC1......................proved - complete
[shostak](0.22 s)
    finite_generated_by.................proved - complete
[shostak](0.27 s)
    finite_generated_by_def_TCC1.......proved - complete
[shostak](0.21 s)
    finite_generated_by_def.............proved - complete
[shostak](1.62 s)
    finite_generated_by_one.............proved - complete
[shostak](0.98 s)
    generated_by_card_1_TCC1............proved - complete
[shostak](0.22 s)
    generated_by_card_1.................proved - complete
[shostak](0.28 s)
    finite_group_elements...............proved - complete
[shostak](0.29 s)
    period_TCC1.........................proved - complete
[shostak](0.28 s)
    a_hat_period_TCC1...................proved - complete
[shostak](0.23 s)
    a_hat_period........................proved - complete
[shostak](0.25 s)
```

```
    finite_subgroup_def.................proved - complete
[shostak](0.47 s)
    orders_equal........................proved - complete
[shostak](0.24 s)
    period_is_generated_order_TCC1.....proved - complete
[shostak](0.25 s)
    period_is_generated_order..........proved - complete
[shostak](0.00 s)
    period_element_divides_group.......proved - complete
[shostak](0.29 s)
    Theory finite_groups totals: 16 formulas, 16 attempted, 16
succeeded (6.11 s)

Proof summary for theory general_properties
    seq_power_TCC1.....................proved - incomplete
[shostak](0.28 s)
    only_power_p_TCC1..................proved - incomplete
[shostak](0.26 s)
    divides_element....................proved - complete
[shostak](0.35 s)
    divides_rel_primes_TCC1............proved - complete
[shostak](0.23 s)
    divides_rel_primes.................proved - incomplete
[shostak](0.64 s)
    divides_product....................proved - incomplete
[shostak](0.55 s)
    product_power_TCC1.................proved - complete
[shostak](0.25 s)
    product_power......................proved - incomplete
[shostak](1.32 s)
    product_only_power_TCC1............proved - incomplete
[shostak](0.25 s)
    product_only_power.................proved - incomplete
[shostak](2.72 s)
    divides_power......................proved - complete
[shostak](0.62 s)
    divides_prime_power_TCC1...........proved - complete
[shostak](0.25 s)
    divides_prime_power_TCC2...........proved - complete
[shostak](0.25 s)
    divides_prime_power................proved - incomplete
[shostak](1.71 s)
    gcd_1_TCC1.........................proved - complete
[shostak](0.24 s)
    gcd_1..............................proved - incomplete
[shostak](0.37 s)
    gcd_1_nd_TCC1......................proved - complete
[shostak](0.24 s)
```

```
    gcd_1_nd..............................proved - incomplete
[shostak](0.30 s)
    gcd_1_ndp.............................proved - incomplete
[shostak](0.47 s)
    gcd_1_gcd_1_TCC1......................proved - incomplete
[shostak](0.26 s)
    gcd_1_gcd_1_TCC2......................proved - incomplete
[shostak](0.27 s)
    gcd_1_gcd_1...........................proved - incomplete
[shostak](0.31 s)
    Theory general_properties totals: 22 formulas, 22 attempted, 22
succeeded (12.14 s)

Proof summary for theory right_left_cosets
    IMP_lagrange_TCC1.....................proved - complete
[shostak](0.23 s)
    nonempty_left_coset_TCC1..............proved - complete
[shostak](0.29 s)
    nonempty_left_coset...................proved - complete
[shostak](0.26 s)
    left_coset_finite_TCC1................proved - complete
[shostak](0.27 s)
    left_coset_finite.....................proved - complete
[shostak](0.31 s)
    left_coset_correspondence.............proved - complete
[shostak](0.32 s)
    left_coset_correspondence_inv.........proved - complete
[shostak](0.36 s)
    finite_left_coset_correspondence_TCC1...proved - complete
[shostak](0.28 s)
    finite_left_coset_correspondence_TCC2...proved - complete
[shostak](0.28 s)
    finite_left_coset_correspondence_TCC3...proved - complete
[shostak](0.29 s)
    finite_left_coset_correspondence......proved - incomplete
[shostak](0.55 s)
    set_left_cosets_full..................proved - complete
[shostak](0.35 s)
    left_cosets_disjoint..................proved - complete
[shostak](0.55 s)
    left_cosets_partition.................proved - complete
[shostak](0.43 s)
    set_right_cosets_full_1...............proved - complete
[shostak](0.47 s)
    right_left_correspondence.............proved - complete
[shostak](1.16 s)
    finite_right_left_correspondence_TCC1...proved - complete
[shostak](0.29 s)
```

```
    finite_right_left_correspondence_TCC2...proved - complete
[shostak](0.28 s)
    finite_right_left_correspondence......proved - incomplete
[shostak](0.33 s)
    index_TCC1............................proved - complete
[shostak](0.26 s)
    index_gt1.............................proved - complete
[shostak](0.28 s)
    divide_TCC1...........................proved - complete
[shostak](0.27 s)
    divide_TCC2...........................proved - complete
[shostak](0.27 s)
    divide_TCC3...........................proved - complete
[shostak](0.99 s)
    card_factor_TCC1......................proved - complete
[shostak](0.60 s)
    card_factor_TCC2......................proved - complete
[shostak](0.24 s)
    card_factor...........................proved - complete
[shostak](1.58 s)
    Theory right_left_cosets totals: 27 formulas, 27 attempted, 27
succeeded (11.79 s)

Proof summary for theory lagrange
    IMP_group_TCC1........................proved - complete
[shostak](0.21 s)
    right_coset_finite_TCC1...............proved - complete
[shostak](0.29 s)
    right_coset_finite....................proved - complete
[shostak](0.34 s)
    finite_right_coset_correspondence_TCC1...proved - complete
[shostak](0.22 s)
    finite_right_coset_correspondence_TCC2...proved - complete
[shostak](0.21 s)
    finite_right_coset_correspondence_TCC3...proved - complete
[shostak](0.23 s)
    finite_right_coset_correspondence.....proved - complete
[shostak](0.39 s)
    set_right_cosets_full.................proved - complete
[shostak](0.31 s)
    right_cosets_disjoint.................proved - complete
[shostak](0.42 s)
    right_cosets_partition................proved - complete
[shostak](0.32 s)
    Lagrange..............................proved - complete
[shostak](0.63 s)
    Theory lagrange totals: 11 formulas, 11 attempted, 11 succeeded
(3.57 s)
```

```
Proof summary for theory factor_groups
    IMP_normal_subgroups_TCC1.............proved - complete
[shostak](0.22 s)
    p0....................................proved - complete
[shostak](0.27 s)
    prep..................................proved - complete
[shostak](0.26 s)
    mult_prep.............................proved - complete
[shostak](0.27 s)
    mult_TCC1.............................proved - complete
[shostak](0.29 s)
    mult_lem_TCC1.........................proved - complete
[shostak](0.21 s)
    mult_lem_TCC2.........................proved - complete
[shostak](0.23 s)
    mult_lem..............................proved - complete
[shostak](0.52 s)
    mult_in...............................proved - complete
[shostak](0.28 s)
    mult_is_coset.........................proved - complete
[shostak](0.25 s)
    N_is_identity_TCC1....................proved - complete
[shostak](0.25 s)
    N_is_identity.........................proved - complete
[shostak](0.36 s)
    left_cosets_group_TCC1................proved - complete
[shostak](0.26 s)
    left_cosets_group_TCC2................proved - complete
[shostak](0.25 s)
    left_cosets_group.....................proved - complete
[shostak](0.47 s)
    over_TCC1.............................proved - complete
[shostak](0.23 s)
    Theory factor_groups totals: 16 formulas, 16 attempted, 16
succeeded (4.63 s)

Proof summary for theory normal_subgroups
    IMP_cosets_TCC1.......................proved - complete
[shostak](0.21 s)
    normal_prep...........................proved - complete
[shostak](0.46 s)
    normal_left_is_right..................proved - complete
[shostak](0.33 s)
    normal_subgroup_is_subgroup...........proved - complete
[shostak](0.34 s)
    nsg_prop..............................proved - complete
[shostak](0.28 s)
```

```
    nsg_prop2.............................proved - complete
[shostak](0.27 s)
    lc_gen_normal_TCC1....................proved - complete
[shostak](0.27 s)
    lc_gen_normal_TCC2....................proved - complete
[shostak](0.22 s)
    lc_gen_normal.........................proved - complete
[shostak](0.32 s)
    abelian_normal........................proved - complete
[shostak](0.28 s)
    Theory normal_subgroups totals: 10 formulas, 10 attempted, 10
succeeded (2.98 s)

Proof summary for theory cosets
    IMP_group_TCC1........................proved - complete
[shostak](0.20 s)
    congruence_is_equivalence.............proved - complete
[shostak](0.35 s)
    left_coset_subset.....................proved - complete
[shostak](0.24 s)
    right_coset_subset....................proved - complete
[shostak](0.24 s)
    left_coset_one........................proved - complete
[shostak](0.26 s)
    right_coset_one.......................proved - complete
[shostak](0.23 s)
    left_coset_assoc......................proved - complete
[shostak](0.26 s)
    right_coset_assoc.....................proved - complete
[shostak](0.27 s)
    lr_coset_assoc........................proved - complete
[shostak](0.38 s)
    subset_left_coset.....................proved - complete
[shostak](0.24 s)
    subset_right_coset....................proved - complete
[shostak](0.23 s)
    right_coset_TCC1......................proved - complete
[shostak](0.22 s)
    right_coset_image_TCC1................proved - complete
[shostak](0.25 s)
    right_coset_image.....................proved - complete
[shostak](0.24 s)
    right_coset_is........................proved - complete
[shostak](0.42 s)
    right_coset_def.......................proved - complete
[shostak](0.23 s)
    nonempty_right_coset..................proved - complete
[shostak](0.23 s)
```

```
    right_coset_correspondence_TCC1.......proved - complete
[shostak](0.24 s)
    right_coset_correspondence............proved - complete
[shostak](0.56 s)
    left_coset_TCC1.......................proved - complete
[shostak](0.23 s)
    left_coset_image......................proved - complete
[shostak](0.24 s)
    left_coset_def........................proved - complete
[shostak](0.21 s)
    lc_gen_TCC1...........................proved - complete
[shostak](0.22 s)
    lc_gen_def_TCC1.......................proved - complete
[shostak](0.98 s)
    lc_gen_def............................proved - complete
[shostak](0.23 s)
    rc_gen_TCC1...........................proved - complete
[shostak](0.23 s)
    rc_gen_def_TCC1.......................proved - complete
[shostak](0.97 s)
    rc_gen_def............................proved - complete
[shostak](0.22 s)
    lc_eq.................................proved - complete
[shostak](0.25 s)
    lc_is_eq..............................proved - complete
[shostak](0.30 s)
    rc_eq.................................proved - complete
[shostak](0.24 s)
    rc_is_eq..............................proved - complete
[shostak](0.30 s)
    Theory cosets totals: 32 formulas, 32 attempted, 32 succeeded (9.91
s)

Proof summary for theory cyclic_group
    IMP_group_TCC1........................proved - complete
[shostak](0.22 s)
    generated_by_lem......................proved - complete
[shostak](0.24 s)
    generated_is_subgroup.................proved - complete
[shostak](0.24 s)
    generated_by_is_finite................proved - complete
[shostak](0.38 s)
    cyclic_abelian........................proved - complete
[shostak](0.30 s)
    cyclic_subgroup.......................proved - complete
[shostak](1.02 s)
    is_cyclic.............................proved - complete
[shostak](0.27 s)
```

```
    Theory cyclic_group totals: 7 formulas, 7 attempted, 7 succeeded
(2.67 s)

Proof summary for theory zp_group
    Zn_group_TCC1.....................proved - complete
[shostak](0.24 s)
    Zn_group_TCC2.....................proved - complete
[shostak](0.24 s)
    Zn_group..........................proved - complete
[shostak](0.74 s)
    Zn_finite.........................proved - complete
[shostak](0.27 s)
    Zn_card_TCC1......................proved - complete
[shostak](0.24 s)
    Zn_card...........................proved - complete
[shostak](0.28 s)
    Theory zp_group totals: 6 formulas, 6 attempted, 6 succeeded (2.01
s)

Proof summary for theory cauchy_scaf
    set_seq_TCC1......................proved - complete
[shostak](0.26 s)
    emptyset_gives_emptyset...........proved - incomplete
[shostak](0.26 s)
    emptyset_gives_emptyset1..........proved - incomplete
[shostak](0.26 s)
    set_seq_singleton.................proved - incomplete
[shostak](0.29 s)
    set_seq_empty.....................proved - complete
[shostak](0.27 s)
    add_element_add_set...............proved - incomplete
[shostak](0.66 s)
    card_add_element_aux..............proved - incomplete
[shostak](0.46 s)
    card_add_element_TCC1.............proved - incomplete
[shostak](0.29 s)
    card_add_element..................proved - incomplete
[shostak](0.46 s)
    disjoint_add_set..................proved - incomplete
[shostak](0.54 s)
    add_set_is_add_ele................proved - incomplete
[shostak](0.56 s)
    add_set_is_finite_aux.............proved - incomplete
[shostak](0.44 s)
    add_set_is_finite.................proved - incomplete
[shostak](0.55 s)
    card_add_set_TCC1.................proved - incomplete
[shostak](0.27 s)
```

```
    card_add_set.......................proved - incomplete
[shostak](0.87 s)
    set_seq_is_finite..................proved - incomplete
[shostak](1.17 s)
    set_seq_is_add_set_TCC1............proved - complete
[shostak](0.25 s)
    set_seq_is_add_set_TCC2............proved - incomplete
[shostak](0.26 s)
    set_seq_is_add_set.................proved - incomplete
[shostak](1.35 s)
    card_set_seq_TCC1..................proved - incomplete
[shostak](0.25 s)
    card_set_seq_TCC2..................proved - complete
[shostak](0.27 s)
    card_set_seq.......................proved - incomplete
[shostak](0.82 s)
    Theory cauchy_scaf totals: 22 formulas, 22 attempted, 22 succeeded
(10.79 s)

Proof summary for theory top_rings
    Theory top_rings totals: 0 formulas, 0 attempted, 0 succeeded (0.00
s)

Proof summary for theory boolean_ring_homomorphisms
    S_TCC1.............................proved - complete
[shostak](0.33 s)
    img_hom_bool_ring..................proved - incomplete
[shostak]( n/a s)
    Theory boolean_ring_homomorphisms totals: 2 formulas, 2 attempted,
2 succeeded (0.33 s)

Proof summary for theory boolean_ring_def
    Theory boolean_ring_def totals: 0 formulas, 0 attempted, 0
succeeded (0.00 s)

Proof summary for theory chinese_remainder_theorem_Z
    nZ_mZ_comaximal_TCC1...............proved - complete
[shostak](0.34 s)
    nZ_mZ_comaximal....................proved - incomplete
[shostak]( n/a s)
    Intersection_add_first.............proved - incomplete
[shostak]( n/a s)
    nZ_fs_intersection.................proved - incomplete
[shostak]( n/a s)
    Chinese_Remainder_Theorem_for_int_TCC1...proved - incomplete
[shostak](0.35 s)
    Chinese_Remainder_Theorem_for_int_TCC2...proved - incomplete
[shostak](0.33 s)
```

```
    Chinese_Remainder_Theorem_for_int_TCC3...proved - incomplete
[shostak](0.35 s)
    Chinese_Remainder_Theorem_for_int_TCC4...proved - incomplete
[shostak](0.37 s)
    Chinese_Remainder_Theorem_for_int_TCC5...proved - incomplete
[shostak](0.36 s)
    Chinese_Remainder_Theorem_for_int_TCC6...proved - incomplete
[shostak](0.35 s)
    Chinese_Remainder_Theorem_for_int_TCC7...proved - incomplete
[shostak](0.45 s)
    Chinese_Remainder_Theorem_for_int_TCC8...proved - incomplete
[shostak](0.56 s)
    Chinese_Remainder_Theorem_for_int_TCC9...proved - incomplete
[shostak](0.57 s)
    Chinese_Remainder_Theorem_for_int_TCC10...proved - incomplete
[shostak](0.45 s)
    Chinese_Remainder_Theorem_for_int.....proved - incomplete
[shostak]( n/a s)
    gcd_lcm_property.....................proved - incomplete
[shostak]( n/a s)
    Theory chinese_remainder_theorem_Z totals: 16 formulas, 16
attempted, 16 succeeded (4.47 s)

Proof summary for theory chinese_remainder_theorem_rings
    oneSet_nonempty......................proved - complete
[shostak]( n/a s)
    surjective_aux_1_TCC1................proved - complete
[shostak](0.40 s)
    surjective_aux_1.....................proved - incomplete
[shostak]( n/a s)
    surjective_aux_2_TCC1................proved - complete
[shostak](0.34 s)
    surjective_aux_2_TCC2................proved - complete
[shostak](0.51 s)
    surjective_aux_2_TCC3................proved - complete
[shostak](0.51 s)
    surjective_aux_2_TCC4................proved - complete
[shostak](0.35 s)
    surjective_aux_2_TCC5................proved - complete
[shostak](0.53 s)
    surjective_aux_2_TCC6................proved - complete
[shostak](0.55 s)
    surjective_aux_2_TCC7................proved - complete
[shostak](0.52 s)
    surjective_aux_2_TCC8................proved - complete
[shostak](0.33 s)
    surjective_aux_2.....................proved - incomplete
[shostak]( n/a s)
```

```
    CRT_aux_1_TCC1.....................proved - complete
[shostak](0.44 s)
    CRT_aux_1_TCC2.....................proved - complete
[shostak](0.32 s)
    CRT_aux_1_TCC3.....................proved - complete
[shostak](0.70 s)
    CRT_aux_1_TCC4.....................proved - complete
[shostak](0.87 s)
    CRT_aux_1_TCC5.....................proved - complete
[shostak](0.64 s)
    CRT_aux_1_TCC6.....................proved - complete
[shostak](0.42 s)
    CRT_aux_1..........................proved - incomplete
[shostak]( n/a s)
    CRT_aux_2_TCC1.....................proved - complete
[shostak](0.37 s)
    CRT_aux_2..........................proved - incomplete
[shostak]( n/a s)
    Chinese_Remainder_Theorem_TCC1.....proved - complete
[shostak](0.31 s)
    Chinese_Remainder_Theorem_TCC2.....proved - incomplete
[shostak](0.39 s)
    Chinese_Remainder_Theorem_TCC3.....proved - incomplete
[shostak](0.40 s)
    Chinese_Remainder_Theorem_TCC4.....proved - incomplete
[shostak](0.39 s)
    Chinese_Remainder_Theorem_TCC5.....proved - incomplete
[shostak](0.35 s)
    Chinese_Remainder_Theorem_TCC6.....proved - complete
[shostak](0.74 s)
    Chinese_Remainder_Theorem_TCC7.....proved - complete
[shostak](0.91 s)
    Chinese_Remainder_Theorem_TCC8.....proved - complete
[shostak](0.94 s)
    Chinese_Remainder_Theorem_TCC9.....proved - complete
[shostak](0.71 s)
    Chinese_Remainder_Theorem..........proved - incomplete
[shostak]( n/a s)
    Theory chinese_remainder_theorem_rings totals: 31 formulas, 31
attempted, 31 succeeded (12.93 s)

Proof summary for theory product_finseq_sets_ring
    product_fs_rec_TCC1................proved - complete
[shostak](0.32 s)
    product_fs_rec_TCC2................proved - complete
[shostak](0.34 s)
    product_fs_rec_TCC3................proved - complete
[shostak](0.31 s)
```

```
    product_fs_TCC1.....................proved - complete
[shostak](0.34 s)
    product_of_sets_TCC1................proved - complete
[shostak](0.34 s)
    product_of_sets_TCC2................proved - complete
[shostak](0.34 s)
    product_fs_emptyseq.................proved - complete
[shostak]( n/a s)
    product_fs_1........................proved - complete
[shostak]( n/a s)
    product_fs_rec_caret_TCC1...........proved - complete
[shostak](0.47 s)
    product_fs_rec_caret................proved - complete
[shostak]( n/a s)
    product_fs_rec_mult_TCC1............proved - complete
[shostak](0.38 s)
    product_fs_rec_mult_TCC2............proved - complete
[shostak](0.43 s)
    product_fs_rec_mult.................proved - complete
[shostak]( n/a s)
    product_fs_split_TCC1...............proved - complete
[shostak](0.30 s)
    product_fs_split_TCC2...............proved - complete
[shostak](0.31 s)
    product_fs_split....................proved - complete
[shostak]( n/a s)
    Product_fs_o........................proved - incomplete
[shostak]( n/a s)
    Product_fs_o_split_TCC1.............proved - complete
[shostak](0.32 s)
    Product_fs_o_split_TCC2.............proved - complete
[shostak](0.31 s)
    Product_fs_o_split..................proved - incomplete
[shostak]( n/a s)
    product_fs_rec_in_ring..............proved - complete
[shostak]( n/a s)
    product_fs_rec_in_each_TCC1.........proved - complete
[shostak](0.59 s)
    product_fs_rec_in_each..............proved - complete
[shostak]( n/a s)
    Intersection_of_ideals_is_ideal.....proved - incomplete
[shostak]( n/a s)
    product_of_ideals_subset_of_each....proved - complete
[shostak]( n/a s)
    product_of_ideals_subset_intersection...proved - incomplete
[shostak]( n/a s)
    cartesian_product_fs_representative_TCC1...proved - complete
[shostak](0.37 s)
```

```
    cartesian_product_fs_representative_TCC2...proved - complete
[shostak](0.37 s)
    cartesian_product_fs_representative...proved - incomplete
[shostak]( n/a s)
    Theory product_finseq_sets_ring totals: 29 formulas, 29 attempted,
29 succeeded (5.84 s)

Proof summary for theory cartesian_product_quot_ring
    IMP_quotient_rings_TCC1..............proved - complete
[shostak](0.37 s)
    Sfs_TCC1.............................proved - complete
[shostak](0.47 s)
    Sfs_TCC2.............................proved - complete
[shostak](0.44 s)
    Sfs_TCC3.............................proved - complete
[shostak](0.41 s)
    Sfs_TCC4.............................proved - complete
[shostak](0.43 s)
    cartesian_product_quot_ring_is_ring_TCC1...proved - complete
[shostak](0.51 s)
    cartesian_product_quot_ring_is_ring_TCC2...proved - complete
[shostak](0.48 s)
    cartesian_product_quot_ring_is_ring_TCC3...proved - complete
[shostak](0.66 s)
    cartesian_product_quot_ring_is_ring_TCC4...proved - complete
[shostak](0.57 s)
    cartesian_product_quot_ring_is_ring_TCC5...proved - complete
[shostak](0.47 s)
    cartesian_product_quot_ring_is_ring...proved - complete
[shostak](2.51 s)
    Theory cartesian_product_quot_ring totals: 11 formulas, 11
attempted, 11 succeeded (7.34 s)

Proof summary for theory cartesian_product_finite
    cartesian_product_n_TCC1.............proved - complete
[shostak](0.38 s)
    cartesian_product_one_disjoint........proved - incomplete
[shostak](0.65 s)
    cartesian_product_one_emptyset........proved - incomplete
[shostak](0.50 s)
    cartesian_product_set_emptyset........proved - incomplete
[shostak](0.42 s)
    cartesian_product_n_emptyset..........proved - complete
[shostak](0.42 s)
    cartesian_product_n_add_is_union......proved - incomplete
[shostak](1.19 s)
    rest_card_fs..........................proved - incomplete
[shostak](0.93 s)
```

```
    add_card_fs_TCC1.....................proved - complete
[shostak](0.37 s)
    add_card_fs..........................proved - incomplete
[shostak](0.58 s)
    cartesian_product_one_finite.........proved - incomplete
[shostak](0.46 s)
    cartesian_product_one_card_TCC1.......proved - incomplete
[shostak](0.37 s)
    cartesian_product_one_card...........proved - incomplete
[shostak](0.58 s)
    cartesian_product_set_finite_aux......proved - incomplete
[shostak](0.46 s)
    cartesian_product_set_finite.........proved - incomplete
[shostak](0.47 s)
    cartesian_product_set_partition.......proved - incomplete
[shostak](0.44 s)
    cartesian_product_set_card_aux_TCC1...proved - incomplete
[shostak](0.38 s)
    cartesian_product_set_card_aux........proved - incomplete
[shostak](0.92 s)
    cartesian_product_set_card_TCC1.......proved - incomplete
[shostak](0.42 s)
    cartesian_product_set_card...........proved - incomplete
[shostak](0.55 s)
    cartesian_product_n_finite...........proved - incomplete
[shostak](1.24 s)
    cartesian_product_n_degenerated_TCC1...proved - complete
[shostak](0.37 s)
    cartesian_product_n_degenerated.......proved - complete
[shostak](0.52 s)
    cartesian_product_n_card_degenerated_TCC1...proved - complete
[shostak](0.36 s)
    cartesian_product_n_card_degenerated...proved - incomplete
[shostak](0.55 s)
    cartesian_product_n_card_TCC1.........proved - complete
[shostak](0.37 s)
    cartesian_product_n_card_TCC2.........proved - complete
[shostak](0.42 s)
    cartesian_product_n_card.............proved - incomplete
[shostak](1.96 s)
    Theory cartesian_product_finite totals: 27 formulas, 27 attempted,
27 succeeded (16.27 s)

Proof summary for theory lagrange_scaf
    partition_TCC1.......................proved - complete
[shostak](0.41 s)
    finite_partition_TCC1................proved - complete
[shostak](0.40 s)
```

```
    finite_partition_is_partition.........proved - complete
[shostak](0.44 s)
    card_Union_rest.......................proved - complete
[shostak](0.43 s)
    card_equal_partition_TCC1.............proved - complete
[shostak](0.37 s)
    card_equal_partition_TCC2.............proved - complete
[shostak](0.39 s)
    card_equal_partition..................proved - complete
[shostak](1.38 s)
    card_eq_part_TCC1.....................proved - complete
[shostak](0.48 s)
    card_eq_part_TCC2.....................proved - complete
[shostak](0.48 s)
    card_eq_part_TCC3.....................proved - complete
[shostak](0.37 s)
    card_eq_part_TCC4.....................proved - complete
[shostak](0.51 s)
    card_eq_part..........................proved - complete
[shostak](0.43 s)
    Theory lagrange_scaf totals: 12 formulas, 12 attempted, 12
succeeded (6.10 s)

Proof summary for theory sigma_R_below
    IMP_ring_TCC1.........................proved - complete
[shostak](0.36 s)
    R_sigma_below_TCC1....................proved - complete
[shostak](0.43 s)
    R_sigma_below_TCC2....................proved - complete
[shostak](0.45 s)
    R_sigma_below_TCC3....................proved - complete
[shostak](0.39 s)
    R_sigma_b_eq_arg......................proved - complete
[shostak](0.38 s)
    R_sigma_b_spl_TCC1....................proved - complete
[shostak](0.47 s)
    R_sigma_b_spl.........................proved - complete
[shostak](0.73 s)
    R_sigma_b_split_TCC1..................proved - complete
[shostak](0.39 s)
    R_sigma_b_split.......................proved - complete
[shostak](0.52 s)
    R_sigma_b_first_TCC1..................proved - complete
[shostak](0.39 s)
    R_sigma_b_first.......................proved - complete
[shostak](0.41 s)
    R_sigma_b_last_TCC1...................proved - complete
[shostak](0.37 s)
```

```
    R_sigma_b_last_TCC2.................proved - complete
[shostak](0.37 s)
    R_sigma_b_last.....................proved - complete
[shostak](0.38 s)
    R_sigma_b_middle_TCC1..............proved - complete
[shostak](0.37 s)
    R_sigma_b_middle...................proved - complete
[shostak](0.42 s)
    R_sigma_b_left_aux.................proved - complete
[shostak](0.51 s)
    R_sigma_b_left.....................proved - complete
[shostak](0.39 s)
    R_sigma_b_right_aux................proved - complete
[shostak](0.50 s)
    R_sigma_b_right....................proved - complete
[shostak](0.39 s)
    R_sigma_b_inv_aux..................proved - complete
[shostak](0.53 s)
    R_sigma_b_inv......................proved - complete
[shostak](0.39 s)
    R_sigma_b_eq_k_aux_TCC1............proved - complete
[shostak](0.37 s)
    R_sigma_b_eq_k_aux_TCC2............proved - complete
[shostak](0.49 s)
    R_sigma_b_eq_k_aux.................proved - complete
[shostak](0.89 s)
    R_sigma_b_eq_k_TCC1................proved - complete
[shostak](0.36 s)
    R_sigma_b_eq_k_TCC2................proved - complete
[shostak](0.48 s)
    R_sigma_b_eq_k.....................proved - complete
[shostak](0.48 s)
    R_sigma_b_in_ideal_aux_TCC1........proved - complete
[shostak](0.38 s)
    R_sigma_b_in_ideal_aux.............proved - complete
[shostak](0.65 s)
    R_sigma_b_in_ideal_TCC1............proved - complete
[shostak](0.37 s)
    R_sigma_b_in_ideal.................proved - complete
[shostak](0.45 s)
    R_sigma_b_add_zero_aux.............proved - complete
[shostak](0.42 s)
    R_sigma_b_add_zero.................proved - complete
[shostak](0.39 s)
    Theory sigma_R_below totals: 34 formulas, 34 attempted, 34
succeeded (15.27 s)

Proof summary for theory comaximal_finseqs_ideals
```

```
    IMP_ring_cosets_lemmas_TCC1..........proved - complete
[shostak](0.35 s)
    IMP_ring_with_one_TCC1..............proved - complete
[shostak](0.43 s)
    comaximal_ideals_equiv..............proved - complete
[shostak](0.54 s)
    Theory comaximal_finseqs_ideals totals: 3 formulas, 3 attempted, 3
succeeded (1.32 s)

Proof summary for theory finite_integral_domain
    surj_equiv_inj_fin_sets..............proved - complete
[shostak]( n/a s)
    zero_ring_is_fin_int_dom.............proved - incomplete
[shostak]( n/a s)
    nzx_member_S.........................proved - incomplete
[shostak]( n/a s)
    auxiliar_map_TCC1....................proved - incomplete
[shostak]( n/a s)
    auxiliar_map.........................proved - incomplete
[shostak]( n/a s)
    building_one.........................proved - incomplete
[shostak]( n/a s)
    one_is_member_S......................proved - incomplete
[shostak]( n/a s)
    fin_int_domain_is_ring_with_one.......proved - incomplete
[shostak]( n/a s)
    fin_int_domain_is_mult_group..........proved - incomplete
[shostak]( n/a s)
    fin_int_domain_is_field..............proved - incomplete
[shostak]( n/a s)
    Theory finite_integral_domain totals: 10 formulas, 10 attempted, 10
succeeded (0.00 s)

Proof summary for theory integral_domain
    IMP_commutative_ring_TCC1............proved - complete
[shostak](0.36 s)
    integral_domain_TCC1.................proved - complete
[shostak](0.42 s)
    integral_domain_is...................proved - complete
[shostak](0.38 s)
    integral_domain_is_ring..............proved - complete
[shostak](0.54 s)
    Theory integral_domain totals: 4 formulas, 4 attempted, 4 succeeded
(1.70 s)

Proof summary for theory commutative_ring
    IMP_ring_TCC1........................proved - complete
[shostak](0.38 s)
```

```
    commutative_ring_TCC1.................proved - complete
[shostak](0.43 s)
    times_commutative.....................proved - complete
[shostak](0.40 s)
    commutative_ring_is_ring..............proved - complete
[shostak](0.55 s)
    commutative_subrings..................proved - complete
[shostak](0.43 s)
    sq_times..............................proved - complete
[shostak](0.41 s)
    Theory commutative_ring totals: 6 formulas, 6 attempted, 6
succeeded (2.59 s)

Proof summary for theory ring_nz_closed
    IMP_ring_TCC1.........................proved - complete
[shostak](0.38 s)
    ring_nz_closed_TCC1...................proved - complete
[shostak](0.43 s)
    ring_nz_closed_is.....................proved - complete
[shostak](0.38 s)
    ring_nz_closed_is_ring................proved - complete
[shostak](0.55 s)
    nz_T_times_nz_T_is_nz_T...............proved - complete
[shostak](0.61 s)
    negate_nz_T_is_nz_T...................proved - complete
[shostak](0.40 s)
    times_is_zero.........................proved - complete
[shostak](0.39 s)
    nz_T_times............................proved - complete
[shostak](0.39 s)
    times_nz_T............................proved - complete
[shostak](0.39 s)
    nz_T_times_nz_T_is_not_zero...........proved - complete
[shostak](0.39 s)
    sq_nz_is_nz...........................proved - complete
[shostak](0.39 s)
    sq_eq_zero............................proved - complete
[shostak](0.40 s)
    Theory ring_nz_closed totals: 12 formulas, 12 attempted, 12
succeeded (5.11 s)

Proof summary for theory ring_binomial_theorem
    IMP_ring_with_one_basic_properties_TCC1...proved - complete
[shostak](0.38 s)
    F_bino_TCC1...........................proved - complete
[shostak](0.60 s)
    F_bino_TCC2...........................proved - complete
[shostak](0.42 s)
```

```
    R_bino_theo.........................proved - incomplete
[shostak](3.08 s)
    Theory ring_binomial_theorem totals: 4 formulas, 4 attempted, 4
succeeded (4.48 s)

Proof summary for theory zero_ring
    groupoid_plus_equiv.................proved - complete
[shostak](0.43 s)
    groupoid_times_equiv................proved - complete
[shostak](0.43 s)
    zero_ring...........................proved - complete
[shostak](0.76 s)
    zero_ring_with_one..................proved - complete
[shostak](0.51 s)
    Theory zero_ring totals: 4 formulas, 4 attempted, 4 succeeded (2.13
s)

Proof summary for theory ring_general_results_extras
    IMP_ring_homomorphism_lemmas_TCC1.....proved - complete
[shostak](0.39 s)
    IMP_ring_homomorphism_lemmas_TCC2.....proved - complete
[shostak](0.38 s)
    IMP_ring_general_results_TCC1.........proved - complete
[shostak](0.49 s)
    no_prop_id_mono_TCC1.................proved - complete
[shostak](0.44 s)
    no_prop_id_mono_TCC2.................proved - complete
[shostak](0.43 s)
    no_prop_id_mono_TCC3.................proved - complete
[shostak](0.44 s)
    no_prop_id_mono_TCC4.................proved - complete
[shostak](0.41 s)
    no_prop_id_mono_TCC5.................proved - complete
[shostak](0.43 s)
    no_prop_id_mono......................proved - complete
[shostak](1.79 s)
    mono_no_prop_id......................proved - complete
[shostak](0.50 s)
    Theory ring_general_results_extras totals: 10 formulas, 10
attempted, 10 succeeded (5.70 s)

Proof summary for theory ring_general_results
    homomorphism_Z_to_R_TCC1.............proved - complete
[shostak](0.42 s)
    homomorphism_Z_to_R..................proved - complete
[shostak]( n/a s)
    gen_times_char_one...................proved - complete
[shostak]( n/a s)
```

```
    nz_closed_char_prime.................proved - incomplete
[shostak]( n/a s)
    field_zero_maximal_ideal.............proved - complete
[shostak]( n/a s)
    maximal_ideal_iff_proper_id..........proved - complete
[shostak]( n/a s)
    proper_id_zero_maximal_ideal.........proved - complete
[shostak]( n/a s)
    Theory ring_general_results totals: 7 formulas, 7 attempted, 7
succeeded (0.42 s)

Proof summary for theory ring_characteristic_def
    IMP_ring_basic_properties_TCC1........proved - complete
[shostak](0.39 s)
    charac_TCC1..........................proved - complete
[shostak](0.50 s)
    times_char...........................proved - complete
[shostak](0.49 s)
    member_N_or_zero_TCC1................proved - complete
[shostak](0.49 s)
    member_N_or_zero.....................proved - complete
[shostak](0.39 s)
    multiple_char........................proved - complete
[shostak](0.88 s)
    char_1_zero_ring.....................proved - complete
[shostak](0.51 s)
    Theory ring_characteristic_def totals: 7 formulas, 7 attempted, 7
succeeded (3.64 s)

Proof summary for theory ring_with_one_homomorphism_extras
    R_TCC1...............................proved - complete
[shostak](0.40 s)
    S_TCC1...............................proved - complete
[shostak](0.38 s)
    isomorphic_fields_charac.............proved - complete
[shostak](0.85 s)
    ring_w_one_isomorphic_groupoid........proved - complete
[shostak](1.18 s)
    Theory ring_with_one_homomorphism_extras totals: 4 formulas, 4
attempted, 4 succeeded (2.81 s)

Proof summary for theory ring_with_one_homomorphism
    IMP_ring_homomorphism_lemmas_TCC1.....proved - complete
[shostak](0.44 s)
    IMP_ring_homomorphism_lemmas_TCC2.....proved - complete
[shostak](0.43 s)
    R_TCC1...............................proved - complete
[shostak](0.50 s)
```

```
    S_TCC1.............................proved - complete
[shostak](0.41 s)
    epi_maps_ones_TCC1.................proved - complete
[shostak](0.71 s)
    epi_maps_ones......................proved - complete
[shostak](0.68 s)
    isomorphic_fields..................proved - complete
[shostak](1.50 s)
    Theory ring_with_one_homomorphism totals: 7 formulas, 7 attempted,
7 succeeded (4.68 s)

Proof summary for theory ring_homomorphism_lemmas_extras
    IMP_quotient_rings_TCC1............proved - complete
[shostak](0.41 s)
    zero_natural_isomorphism_TCC1......proved - complete
[shostak](0.52 s)
    zero_natural_isomorphism_TCC2......proved - complete
[shostak](0.48 s)
    zero_natural_isomorphism_TCC3......proved - complete
[shostak](0.47 s)
    zero_natural_isomorphism_TCC4......proved - complete
[shostak](0.47 s)
    zero_natural_isomorphism...........proved - complete
[shostak](1.08 s)
    Theory ring_homomorphism_lemmas_extras totals: 6 formulas, 6
attempted, 6 succeeded (3.42 s)

Proof summary for theory ring_2nd_3rd_isomorphism_theorems
    IMP_quotient_rings_TCC1............proved - complete
[shostak](0.41 s)
    S_TCC1.............................proved - complete
[shostak](0.46 s)
    second_isomorphism_th_ax_TCC1......proved - complete
[shostak](0.46 s)
    second_isomorphism_th_ax_TCC2......proved - complete
[shostak](0.46 s)
    second_isomorphism_th_ax_TCC3......proved - complete
[shostak](0.47 s)
    second_isomorphism_th_ax_TCC4......proved - complete
[shostak](0.45 s)
    second_isomorphism_th_ax_TCC5......proved - complete
[shostak](0.72 s)
    second_isomorphism_th_ax...........proved - complete
[shostak](0.00 s)
    second_isomorphism_th_TCC1.........proved - complete
[shostak](0.44 s)
    second_isomorphism_th_TCC2.........proved - complete
[shostak](0.44 s)
```

```
    second_isomorphism_th_TCC3............proved - complete
[shostak](0.44 s)
    second_isomorphism_th_TCC4............proved - complete
[shostak](0.44 s)
    second_isomorphism_th_TCC5............proved - complete
[shostak](0.45 s)
    second_isomorphism_th_TCC6............proved - complete
[shostak](0.45 s)
    second_isomorphism_th_TCC7............proved - complete
[shostak](0.44 s)
    second_isomorphism_th.................proved - complete
[shostak](5.36 s)
    third_isomorphism_th_ax_TCC1..........proved - complete
[shostak](0.44 s)
    third_isomorphism_th_ax_TCC2..........proved - complete
[shostak](0.44 s)
    third_isomorphism_th_ax_TCC3..........proved - complete
[shostak](0.43 s)
    third_isomorphism_th_ax_TCC4..........proved - complete
[shostak](0.42 s)
    third_isomorphism_th_ax_TCC5..........proved - complete
[shostak](0.43 s)
    third_isomorphism_th_ax_TCC6..........proved - complete
[shostak](0.42 s)
    third_isomorphism_th_ax_TCC7..........proved - complete
[shostak](0.43 s)
    third_isomorphism_th_ax_TCC8..........proved - complete
[shostak](0.42 s)
    third_isomorphism_th_ax_TCC9..........proved - complete
[shostak](0.58 s)
    third_isomorphism_th_ax...............proved - complete
[shostak](2.94 s)
    third_isomorphism_th_TCC1.............proved - complete
[shostak](0.54 s)
    third_isomorphism_th_TCC2.............proved - complete
[shostak](0.64 s)
    third_isomorphism_th_TCC3.............proved - complete
[shostak](0.64 s)
    third_isomorphism_th_TCC4.............proved - complete
[shostak](0.43 s)
    third_isomorphism_th_TCC5.............proved - complete
[shostak](0.64 s)
    third_isomorphism_th_TCC6.............proved - complete
[shostak](0.60 s)
    third_isomorphism_th_TCC7.............proved - complete
[shostak](0.44 s)
    third_isomorphism_th_TCC8.............proved - complete
[shostak](0.42 s)
```

```
    third_isomorphism_th_TCC9.............proved - complete
[shostak](0.42 s)
    third_isomorphism_th..................proved - complete
[shostak](5.61 s)
    Theory ring_2nd_3rd_isomorphism_theorems totals: 36 formulas, 36
attempted, 36 succeeded (29.23 s)

Proof summary for theory ring_1st_isomorphism_theorem
    IMP_ring_homomorphism_lemmas_TCC1.....proved - complete
[shostak](0.37 s)
    IMP_ring_homomorphism_lemmas_TCC2.....proved - complete
[shostak](0.37 s)
    first_isomorphism_th_aux_1_TCC1.......proved - complete
[shostak](0.50 s)
    first_isomorphism_th_aux_1_TCC2.......proved - complete
[shostak](0.49 s)
    first_isomorphism_th_aux_1_TCC3.......proved - complete
[shostak](0.43 s)
    first_isomorphism_th_aux_1_TCC4.......proved - complete
[shostak](0.40 s)
    first_isomorphism_th_aux_1_TCC5.......proved - complete
[shostak](0.40 s)
    first_isomorphism_th_aux_1............proved - complete
[shostak](2.65 s)
    first_isomorphism_th_aux_2............proved - complete
[shostak](0.47 s)
    first_isomorphism_th_aux_3_TCC1.......proved - complete
[shostak](0.70 s)
    first_isomorphism_th_aux_3............proved - complete
[shostak](1.01 s)
    first_isomorphism_th_aux_4_TCC1.......proved - complete
[shostak](0.58 s)
    first_isomorphism_th_aux_4............proved - complete
[shostak](0.49 s)
    first_isomorphism_th_aux_5............proved - complete
[shostak](1.79 s)
    first_isomorphism_th_aux_6............proved - complete
[shostak](0.47 s)
    first_isomorphism_th_TCC1.............proved - complete
[shostak](0.50 s)
    first_isomorphism_th_TCC2.............proved - complete
[shostak](0.44 s)
    first_isomorphism_th_TCC3.............proved - complete
[shostak](0.43 s)
    first_isomorphism_th_TCC4.............proved - complete
[shostak](0.41 s)
    first_isomorphism_th..................proved - complete
[shostak](3.52 s)
```

```
    Theory ring_1st_isomorphism_theorem totals: 20 formulas, 20
attempted, 20 succeeded (16.41 s)

Proof summary for theory ring_homomorphism_lemmas
    R_homo_plus_TCC1.....................proved - complete
[shostak](0.64 s)
    R_homo_plus..........................proved - complete
[shostak]( n/a s)
    R_homo_mult_TCC1.....................proved - complete
[shostak](0.94 s)
    R_homo_mult..........................proved - complete
[shostak]( n/a s)
    R_homo_equiv_TCC1....................proved - complete
[shostak](0.49 s)
    R_homo_equiv_TCC2....................proved - complete
[shostak](0.73 s)
    R_homo_equiv.........................proved - complete
[shostak]( n/a s)
    zero_to_zero_TCC1....................proved - complete
[shostak](0.36 s)
    zero_to_zero.........................proved - complete
[shostak]( n/a s)
    inv_to_inv_TCC1......................proved - complete
[shostak](0.37 s)
    inv_to_inv...........................proved - complete
[shostak]( n/a s)
    epi_commutative......................proved - complete
[shostak]( n/a s)
    image_homo_is_subring................proved - complete
[shostak]( n/a s)
    R_homo_image_subring.................proved - complete
[shostak]( n/a s)
    R_homo_inv_image_subring.............proved - complete
[shostak]( n/a s)
    R_kernel_is_subring..................proved - complete
[shostak]( n/a s)
    R_kernel_is_subgroup.................proved - complete
[shostak]( n/a s)
    monomorphism_charac..................proved - complete
[shostak]( n/a s)
    inv_iso_is_iso_TCC1..................proved - complete
[shostak](2.00 s)
    inv_iso_is_iso.......................proved - complete
[shostak]( n/a s)
    R_isomorphic_groupoid_is_ring........proved - complete
[shostak]( n/a s)
    R_kernel_is_ideal....................proved - complete
[shostak]( n/a s)
```

```
    R_epimorphism_image_ideal.............proved - complete
[shostak]( n/a s)
    R_homo_inv_image_ideal................proved - complete
[shostak]( n/a s)
    R_kernel_in_inverse_image.............proved - complete
[shostak]( n/a s)
    inv_image_image_sum...................proved - complete
[shostak]( n/a s)
    inv_image_image_subring_TCC1..........proved - complete
[shostak](0.55 s)
    inv_image_image_subring...............proved - complete
[shostak]( n/a s)
    ring_natural_homo_TCC1................proved - complete
[shostak](0.33 s)
    ring_natural_homo_TCC2................proved - complete
[shostak](0.37 s)
    ring_natural_homo_TCC3................proved - complete
[shostak](0.51 s)
    ring_natural_homo_TCC4................proved - complete
[shostak](0.35 s)
    ring_natural_homo_TCC5................proved - complete
[shostak](0.37 s)
    ring_natural_homo_TCC6................proved - complete
[shostak](0.48 s)
    ring_natural_homo_TCC7................proved - complete
[shostak](0.35 s)
    ring_natural_homo.....................proved - complete
[shostak]( n/a s)
    Theory ring_homomorphism_lemmas totals: 36 formulas, 36 attempted,
36 succeeded (8.86 s)

Proof summary for theory ring_homomorphisms_def
    Theory ring_homomorphisms_def totals: 0 formulas, 0 attempted, 0
succeeded (0.00 s)

Proof summary for theory homomorphisms_def
    homomorphism?_TCC1....................proved - complete
[shostak](0.34 s)
    Theory homomorphisms_def totals: 1 formulas, 1 attempted, 1
succeeded (0.34 s)

Proof summary for theory ring_euclidean_gcd_algorithm_Z
    Z_TCC1................................proved - complete
[shostak](0.28 s)
    f_phi_Z_TCC1..........................proved - complete
[shostak](0.30 s)
    f_phi_Z_TCC2..........................proved - complete
[shostak](0.31 s)
```

```
    phi_Z_and_f_phi_Z_ok_TCC1.............proved - complete
[shostak](0.50 s)
    phi_Z_and_f_phi_Z_ok_TCC2.............proved - incomplete
[shostak](2.21 s)
    phi_Z_and_f_phi_Z_ok..................proved - incomplete
[shostak]( n/a s)
    euclidean_gcd_alg_correctness_in_Z_TCC1...proved - complete
[shostak](0.38 s)
    euclidean_gcd_alg_correctness_in_Z_TCC2...proved - complete
[shostak](0.33 s)
    euclidean_gcd_alg_correctness_in_Z_TCC3...proved - complete
[shostak](0.35 s)
    euclidean_gcd_alg_correctness_in_Z_TCC4...proved - incomplete
[shostak](0.59 s)
    euclidean_gcd_alg_correctness_in_Z_TCC5...proved - complete
[shostak](0.47 s)
    euclidean_gcd_alg_correctness_in_Z....proved - incomplete
[shostak]( n/a s)
    Theory ring_euclidean_gcd_algorithm_Z totals: 12 formulas, 12
attempted, 12 succeeded (5.72 s)

Proof summary for theory ring_euclidean_gcd_algorithm_Zi
    Zi_is_ring............................proved - incomplete [shostak]
(  n/a s)
    Zi_is_integral_domain_w_one..........proved - incomplete [shostak]
(  n/a s)
    sq_abs_Re_Im_integer_rational_pred_TCC1...proved - incomplete
[shostak]( 0.76 s)
    sq_abs_Re_Im_integer_rational_pred...proved - incomplete [shostak]
(  n/a s)
    times_conjugate_is_Zi................proved - incomplete [shostak]
(  n/a s)
    phi_Zi_TCC1..........................proved - incomplete [shostak]
( 0.62 s)
    phi_Zi_is_multiplicative_TCC1........proved - incomplete [shostak]
( 0.53 s)
    phi_Zi_is_multiplicative.............proved - incomplete [shostak]
(  n/a s)
    div_rem_appx_TCC1....................proved - complete   [shostak]
( 0.55 s)
    div_rem_appx_TCC2....................proved - complete   [shostak]
( 0.61 s)
    div_rev_appx_correctness.............proved - incomplete [shostak]
(  n/a s)
    f_phi_Zi_TCC1........................proved - incomplete [shostak]
( 0.55 s)
    f_phi_Zi_TCC2........................proved - incomplete [shostak]
( 0.58 s)
```

```
    f_phi_Zi_TCC3.....................proved - incomplete [shostak]
( 0.55 s)
    f_phi_Zi_TCC4.....................proved - incomplete [shostak]
( 1.09 s)
    f_phi_Zi_TCC5.....................proved - incomplete [shostak]
( 2.09 s)
    phi_Zi_and_f_phi_Zi_ok_TCC1.......proved - incomplete [shostak]
( 0.52 s)
    phi_Zi_and_f_phi_Zi_ok_TCC2.......proved - incomplete [shostak]
(11.08 s)
    phi_Zi_and_f_phi_Zi_ok_TCC3.......proved - incomplete [shostak]
( 2.43 s)
    phi_Zi_and_f_phi_Zi_ok............proved - incomplete [shostak]
(  n/a s)
    euclidean_gcd_alg_in_Zi_TCC1......proved - incomplete [shostak]
( 0.42 s)
    euclidean_gcd_alg_in_Zi_TCC2......proved - incomplete [shostak]
( 0.40 s)
    euclidean_gcd_alg_in_Zi_TCC3......proved - incomplete [shostak]
( 0.40 s)
    euclidean_gcd_alg_in_Zi_TCC4......proved - incomplete [shostak]
( 0.42 s)
    euclidean_gcd_alg_in_Zi_TCC5......proved - incomplete [shostak]
( 0.88 s)
    euclidean_gcd_alg_in_Zi_TCC6......proved - incomplete [shostak]
( 0.43 s)
    euclidean_gcd_alg_in_Zi...........proved - incomplete [shostak]
(  n/a s)
    Theory ring_euclidean_gcd_algorithm_Zi totals: 27 formulas, 27
attempted, 27 succeeded (24.89 s)

Proof summary for theory ring_euclidean_algorithm
    euclidean_gcd_algorithm_TCC1.....proved - complete
[shostak](0.66 s)
    euclidean_gcd_algorithm_TCC2.....proved - complete
[shostak](0.48 s)
    euclidean_gcd_algorithm_TCC3.....proved - complete
[shostak](0.51 s)
    euclidean_gcd_algorithm_TCC4.....proved - complete
[shostak](0.48 s)
    euclidean_gcd_algorithm_TCC5.....proved - complete
[shostak](0.58 s)
    euclidean_gcd_algorithm_TCC6.....proved - complete
[shostak](0.46 s)
    euclidean_gcd_algorithm_TCC7.....proved - complete
[shostak](0.59 s)
    euclidean_gcd_algorithm_TCC8.....proved - complete
[shostak](0.52 s)
```

```
    euclidean_gcd_algorithm_TCC9.....proved - complete
[shostak](6.54 s)
    euclidean_gcd_algorithm_TCC10.....proved - complete
[shostak](0.60 s)
    euclidean_gcd_algorithm_TCC11.....proved - complete
[shostak](0.50 s)
    euclidean_gcd_algorithm_TCC12.....proved - complete
[shostak](0.62 s)
    Euclid_theorem_TCC1.................proved - complete
[shostak](0.36 s)
    Euclid_theorem_TCC2.................proved - complete
[shostak](0.37 s)
    Euclid_theorem......................proved - complete
[shostak]( n/a s)
    euclidean_gcd_alg_correctness_TCC1....proved - complete
[shostak](0.38 s)
    euclidean_gcd_alg_correctness.....proved - complete
[shostak]( n/a s)
    Theory ring_euclidean_algorithm totals: 17 formulas, 17 attempted,
17 succeeded (13.64 s)

Proof summary for theory euclidean_domain
    integers_is_euclidean_domain.....proved - complete
[shostak]( n/a s)
    field_is_euclidean_domain.....proved - complete
[shostak]( n/a s)
    euclidean_is_unique_factorization_domain_TCC1...proved -
complete   [shostak](0.63 s)
    euclidean_is_unique_factorization_domain...proved - incomplete
[shostak]( n/a s)
    Theory euclidean_domain totals: 4 formulas, 4 attempted, 4
succeeded (0.63 s)

Proof summary for theory euclidean_domain_def
    Theory euclidean_domain_def totals: 0 formulas, 0 attempted, 0
succeeded (0.00 s)

Proof summary for theory ring_zn
    nZ_add_TCC1.........................proved - complete
[shostak](0.44 s)
    Z_ring..............................proved - complete
[shostak]( n/a s)
    Z_TCC1..............................proved - complete
[shostak](0.35 s)
    Z1_is_Z.............................proved - complete
[shostak]( n/a s)
    Z_commutative_ring_w_one.....proved - complete
[shostak]( n/a s)
```

```
    nZ_ideal.............................proved - complete
[shostak]( n/a s)
    Z_nz_closed..........................proved - complete
[shostak]( n/a s)
    Z_integral_domain_w_one..............proved - complete
[shostak]( n/a s)
    Zn_finite_set........................proved - complete
[shostak]( n/a s)
    Zn_card_n_TCC1.......................proved - complete
[shostak](0.32 s)
    Zn_card_n............................proved - complete
[shostak]( n/a s)
    Zn_commutative_ring_w_one_TCC1........proved - complete
[shostak](0.35 s)
    Zn_commutative_ring_w_one_TCC2........proved - complete
[shostak](0.35 s)
    Zn_commutative_ring_w_one_TCC3........proved - complete
[shostak](0.35 s)
    Zn_commutative_ring_w_one_TCC4........proved - complete
[shostak](0.34 s)
    Zn_commutative_ring_w_one_TCC5........proved - complete
[shostak](0.37 s)
    Zn_commutative_ring_w_one.............proved - complete
[shostak]( n/a s)
    equal_cosets_div.....................proved - complete
[shostak]( n/a s)
    nZ_mZ_sum_TCC1.......................proved - complete
[shostak](0.32 s)
    nZ_mZ_sum............................proved - incomplete
[shostak]( n/a s)
    nZ_mZ_intersection_TCC1..............proved - incomplete
[shostak](0.38 s)
    nZ_mZ_intersection...................proved - incomplete
[shostak]( n/a s)
    nZ_mZ_rel_prime_intersection.........proved - incomplete
[shostak]( n/a s)
    Zn_charac............................proved - incomplete
[shostak]( n/a s)
    Z2_charac............................proved - incomplete
[shostak]( n/a s)
    Zp_prime_is_nz_closed................proved - incomplete
[shostak]( n/a s)
    Zp_nz_closed_is_prime_or_one.........proved - incomplete
[shostak]( n/a s)
    Zp_prime_is_division_ring............proved - incomplete
[shostak]( n/a s)
    Zp_prime_is_field....................proved - incomplete
[shostak]( n/a s)
```

```
    nZ_mZ_subset.........................proved - complete
[shostak]( n/a s)
    power_sum_nat........................proved - complete
[shostak]( n/a s)
    power_sum_int........................proved - complete
[shostak]( n/a s)
    nZ_is_cyclic.........................proved - complete
[shostak]( n/a s)
    mZ_nZ_is_cyclic_TCC1.................proved - complete
[shostak](0.44 s)
    mZ_nZ_is_cyclic_TCC2.................proved - complete
[shostak](0.39 s)
    mZ_nZ_is_cyclic_TCC3.................proved - complete
[shostak](0.43 s)
    mZ_nZ_is_cyclic_TCC4.................proved - complete
[shostak](0.37 s)
    mZ_nZ_is_cyclic_TCC5.................proved - complete
[shostak](0.40 s)
    mZ_nZ_is_cyclic......................proved - complete
[shostak]( n/a s)
    Theory ring_zn totals: 39 formulas, 39 attempted, 39 succeeded
(5.59 s)

Proof summary for theory prop_primes_extra
    fs_rel_prime?_TCC1...................proved - complete
[shostak](0.39 s)
    lcm_div_TCC1.........................proved - complete
[shostak](0.35 s)
    lcm_div_TCC2.........................proved - incomplete
[shostak](0.65 s)
    lcm_div..............................proved - incomplete
[shostak](1.11 s)
    primes_lcm_div_TCC1..................proved - complete
[shostak](0.35 s)
    primes_lcm_div.......................proved - incomplete
[shostak](0.42 s)
    fs_rel_prime_fixed_TCC1..............proved - complete
[shostak](0.35 s)
    fs_rel_prime_fixed_TCC2..............proved - incomplete
[shostak](0.37 s)
    fs_rel_prime_fixed...................proved - incomplete
[shostak](1.04 s)
    fs_rel_prime_i_TCC1..................proved - incomplete
[shostak](0.43 s)
    fs_rel_prime_i.......................proved - incomplete
[shostak](0.46 s)
    Theory prop_primes_extra totals: 11 formulas, 11 attempted, 11
succeeded (5.92 s)
```

```
Proof summary for theory division_ring_extras
    xyx_division_ring.....................proved - complete
[shostak]( n/a s)
    div_ring_nz_unit_TCC1.................proved - complete
[shostak](0.39 s)
    div_ring_nz_unit......................proved - complete
[shostak]( n/a s)
    no_prop_l_ideal_div_ring..............proved - complete
[shostak]( n/a s)
    no_prop_r_ideal_div_ring..............proved - complete
[shostak]( n/a s)
    div_ring_no_prop_ideal................proved - complete
[shostak]( n/a s)
    Theory division_ring_extras totals: 6 formulas, 6 attempted, 6
succeeded (0.39 s)

Proof summary for theory ring_w_one_xyx_is_x
    IMP_ring_xyx_is_x_TCC1................proved - complete
[shostak](0.35 s)
    IMP_ring_with_one_nz_closed_TCC1......proved - complete
[shostak](0.41 s)
    xyx_one_is_member.....................proved - complete
[shostak](0.42 s)
    xyx_ring_with_one.....................proved - complete
[shostak](0.39 s)
    xyx_R_unit............................proved - complete
[shostak](0.81 s)
    unit_xyx_R_TCC1.......................proved - complete
[shostak](0.45 s)
    unit_xyx_R............................proved - complete
[shostak](0.52 s)
    unit_nz_closed........................proved - complete
[shostak](0.37 s)
    Theory ring_w_one_xyx_is_x totals: 8 formulas, 8 attempted, 8
succeeded (3.72 s)

Proof summary for theory ring_xyx_is_x
    IMP_ring_nz_closed_aux_TCC1...........proved - complete
[shostak](0.34 s)
    xyx_is_x_nz_divisor...................proved - complete
[shostak](0.53 s)
    xyx_is_x_nz_closed....................proved - complete
[shostak](0.43 s)
    yxy_is_y..............................proved - complete
[shostak](0.58 s)
    xyx_has_identity......................proved - complete
[shostak](0.84 s)
```

```
    Theory ring_xyx_is_x totals: 5 formulas, 5 attempted, 5 succeeded
(2.72 s)

Proof summary for theory ring_xyx_is_x_def
    Theory ring_xyx_is_x_def totals: 0 formulas, 0 attempted, 0
succeeded (0.00 s)

Proof summary for theory ring_with_one_nz_closed
    IMP_ring_with_one_TCC1.............proved - complete
[shostak](0.35 s)
    IMP_ring_nz_closed_aux_TCC1.........proved - complete
[shostak](0.43 s)
    subring_nz_closed_one_TCC1..........proved - complete
[shostak](0.37 s)
    subring_nz_closed_one...............proved - complete
[shostak](0.00 s)
    Theory ring_with_one_nz_closed totals: 4 formulas, 4 attempted, 4
succeeded (1.15 s)

Proof summary for theory lcm
    lcm_1...............................proved - incomplete
[shostak](0.49 s)
    lcm_same............................proved - incomplete
[shostak](0.47 s)
    lcm_sym.............................proved - incomplete
[shostak](0.65 s)
    lcm_divides.........................proved - incomplete
[shostak](0.54 s)
    lcm_is_min..........................proved - incomplete
[shostak](0.46 s)
    lcm_times...........................proved - incomplete
[shostak](1.16 s)
    lcm_rel_prime_TCC1..................proved - complete
[shostak](0.35 s)
    lcm_rel_prime.......................proved - incomplete
[shostak](0.74 s)
    lcm_gdm_rel.........................proved - incomplete
[shostak](1.20 s)
    lcm_absorption......................proved - incomplete
[shostak](0.73 s)
    divides_lcm.........................proved - incomplete
[shostak](0.94 s)
    Theory lcm totals: 11 formulas, 11 attempted, 11 succeeded (7.72 s)

Proof summary for theory euclidean_ring
    IMP_ring_principal_ideal_TCC1.........proved - complete
[shostak](0.20 s)
    euclidean_ring_ideal_is_gen...........proved - incomplete
```

```
[shostak](0.70 s)
    euclidean_ring_is_principal_ideal.....proved - complete
[shostak](0.25 s)
    euclidean_ring_has_one................proved - complete
[shostak](0.16 s)
    Theory euclidean_ring totals: 4 formulas, 4 attempted, 4 succeeded
(1.32 s)

Proof summary for theory euclidean_ring_def
    euclidean_ring?_TCC1..................proved - complete
[shostak](0.28 s)
    euclidean_ring?_TCC2..................proved - complete
[shostak](0.34 s)
    euclidean_ring?_TCC3..................proved - complete
[shostak](0.15 s)
    euclidean_ring?_TCC4..................proved - complete
[shostak](0.14 s)
    euclidean_pair?_TCC1..................proved - complete
[shostak](0.62 s)
    euclidean_pair?_TCC2..................proved - complete
[shostak](0.74 s)
    euclidean_pair?_TCC3..................proved - complete
[shostak](0.43 s)
    euclidean_pair?_TCC4..................proved - complete
[shostak](0.45 s)
    euclidean_f_phi?_TCC1.................proved - complete
[shostak](0.44 s)
    euclidean_f_phi?_TCC2.................proved - complete
[shostak](0.44 s)
    Theory euclidean_ring_def totals: 10 formulas, 10 attempted, 10
succeeded (4.04 s)

Proof summary for theory ring_unique_factorization_domain
    IMP_ring_principal_ideal_domain_TCC1...proved - complete
[shostak](0.26 s)
    UFD_prime_iff_irreducible.............proved - incomplete
[shostak]( n/a s)
    PID_is_UFD_TCC1.......................proved - complete
[shostak](0.65 s)
    PID_is_UFD............................proved - incomplete
[shostak]( n/a s)
    Theory ring_unique_factorization_domain totals: 4 formulas, 4
attempted, 4 succeeded (0.92 s)

Proof summary for theory ring_unique_factorization_domain_def
    unique_factorization_domain?_TCC1.....proved - complete
[shostak](0.57 s)
    unique_factorization_domain?_TCC2.....proved - complete
```

```
[shostak](1.76 s)
    unique_factorization_domain?_TCC3.....proved - complete
[shostak](0.75 s)
    Theory ring_unique_factorization_domain_def totals: 3 formulas, 3
attempted, 3 succeeded (3.08 s)

Proof summary for theory ring_principal_ideal_domain
    PID_maximal_prime_ideal.............proved - complete   [shostak]
(  n/a s)
    el_max_iff_one_gen_maximal..........proved - complete   [shostak]
(  n/a s)
    PID_prime_el_iff_irreducible_TCC1....proved - complete   [shostak]
( 0.61 s)
    PID_prime_el_iff_irreducible.........proved - complete   [shostak]
(  n/a s)
    nonzero_nonunit_irreducible_divides_TCC1...proved - complete
[shostak]( 0.46 s)
    nonzero_nonunit_irreducible_divides...proved - incomplete
[shostak](  n/a s)
    non_fact_el_set_TCC1................proved - complete   [shostak]
( 0.41 s)
    non_fact_el_set_TCC2................proved - complete   [shostak]
( 0.60 s)
    empty_non_fact_el_set_aux1_TCC1.....proved - complete   [shostak]
( 0.66 s)
    empty_non_fact_el_set_aux1_TCC2.....proved - complete   [shostak]
( 0.66 s)
    empty_non_fact_el_set_aux1..........proved - incomplete [shostak]
(  n/a s)
    phi_TCC1............................proved - complete   [shostak]
( 0.68 s)
    phi_TCC2............................proved - complete   [shostak]
( 0.51 s)
    phi_TCC3............................proved - complete   [shostak]
( 0.34 s)
    phi_TCC4............................proved - incomplete [shostak]
( 0.35 s)
    empty_non_fact_el_set_aux2_TCC1.....proved - complete   [shostak]
( 0.33 s)
    empty_non_fact_el_set_aux2_TCC2.....proved - complete   [shostak]
( 0.38 s)
    empty_non_fact_el_set_aux2..........proved - incomplete [shostak]
(  n/a s)
    empty_non_fact_el_set_aux3_TCC1.....proved - complete   [shostak]
( 0.34 s)
    empty_non_fact_el_set_aux3..........proved - incomplete [shostak]
(  n/a s)
    empty_non_fact_el_set...............proved - incomplete [shostak]
```

```
(  n/a s)
    PID_factorization_existence.........proved - incomplete [shostak]
(  n/a s)
    PID_factorization_uniqueness_TCC1....proved - complete   [shostak]
( 0.58 s)
    PID_factorization_uniqueness_TCC2....proved - complete   [shostak]
(19.44 s)
    PID_factorization_uniqueness_TCC3....proved - complete   [shostak]
( 6.22 s)
    PID_factorization_uniqueness.........proved - incomplete [shostak]
(  n/a s)
    Theory ring_principal_ideal_domain totals: 26 formulas, 26
attempted, 26 succeeded (32.58 s)

Proof summary for theory ring_with_one_maximal_ideal
    ring_one_maximal_prime_ideal..........proved - complete
[shostak]( n/a s)
    maximal_ideal_quot_field_TCC1.........proved - complete
[shostak](0.44 s)
    maximal_ideal_quot_field_TCC2.........proved - complete
[shostak](0.38 s)
    maximal_ideal_quot_field_TCC3.........proved - complete
[shostak](0.39 s)
    maximal_ideal_quot_field_TCC4.........proved - complete
[shostak](0.38 s)
    maximal_ideal_quot_field_TCC5.........proved - complete
[shostak](0.36 s)
    maximal_ideal_quot_field..............proved - complete
[shostak]( n/a s)
    quot_div_ring_maximal_ideal_TCC1......proved - complete
[shostak](0.37 s)
    quot_div_ring_maximal_ideal_TCC2......proved - complete
[shostak](0.37 s)
    quot_div_ring_maximal_ideal_TCC3......proved - complete
[shostak](0.37 s)
    quot_div_ring_maximal_ideal_TCC4......proved - complete
[shostak](0.35 s)
    quot_div_ring_maximal_ideal_TCC5......proved - complete
[shostak](0.34 s)
    quot_div_ring_maximal_ideal...........proved - complete
[shostak]( n/a s)
    maximal_ideal_charac_TCC1.............proved - complete
[shostak](0.35 s)
    maximal_ideal_charac_TCC2.............proved - complete
[shostak](0.37 s)
    maximal_ideal_charac_TCC3.............proved - complete
[shostak](0.36 s)
    maximal_ideal_charac_TCC4.............proved - complete
```

```
[shostak](0.35 s)
    maximal_ideal_charac_TCC5.............proved - complete
[shostak](0.36 s)
    maximal_ideal_charac..................proved - complete
[shostak]( n/a s)
    nonzero_ring_exists_maximal_ideal_aux...proved - incomplete
[shostak]( n/a s)
    nonzero_ring_exists_maximal_ideal.....proved - incomplete
[shostak]( n/a s)
    Theory ring_with_one_maximal_ideal totals: 21 formulas, 21
attempted, 21 succeeded (5.55 s)

Proof summary for theory ring_with_one_basic_properties
    IMP_ring_with_one_TCC1................proved - complete
[shostak](0.36 s)
    IMP_ring_basic_properties_TCC1........proved - complete
[shostak](0.44 s)
    power_commute_aux.....................proved - complete
[shostak](0.38 s)
    power_commute.........................proved - complete
[shostak](0.38 s)
    gen_times_int_one.....................proved - complete
[shostak](0.36 s)
    ring_w_one_is_idempotent..............proved - complete
[shostak](0.44 s)
    one_diff_zero_monad...................proved - complete
[shostak](0.41 s)
    Theory ring_with_one_basic_properties totals: 7 formulas, 7
attempted, 7 succeeded (2.77 s)

Proof summary for theory ring_with_one_prime_ideal
    prime_ideal_charac_TCC1...............proved - complete
[shostak](0.42 s)
    prime_ideal_charac_TCC2...............proved - complete
[shostak](0.37 s)
    prime_ideal_charac_TCC3...............proved - complete
[shostak](0.36 s)
    prime_ideal_charac_TCC4...............proved - complete
[shostak](0.36 s)
    prime_ideal_charac_TCC5...............proved - complete
[shostak](0.34 s)
    prime_ideal_charac....................proved - complete
[shostak]( n/a s)
    Theory ring_with_one_prime_ideal totals: 6 formulas, 6 attempted, 6
succeeded (1.85 s)

Proof summary for theory quotient_rings_with_one
    IMP_quotient_rings_TCC1...............proved - complete
```

```
[shostak](0.34 s)
    IMP_ring_with_one_ideal_TCC1.....proved - complete
[shostak](0.39 s)
    quotient_ring_with_one_TCC1.....proved - complete
[shostak](0.40 s)
    quotient_ring_with_one_TCC2.....proved - complete
[shostak](0.38 s)
    quotient_ring_with_one_TCC3.....proved - complete
[shostak](0.39 s)
    quotient_ring_with_one_TCC4.....proved - complete
[shostak](0.38 s)
    quotient_ring_with_one_TCC5.....proved - complete
[shostak](0.37 s)
    quotient_ring_with_one.............proved - complete
[shostak](0.51 s)
    fullset_quot_ring_with_one.....proved - complete
[shostak](0.79 s)
    one_diff_zero_coset.................proved - complete
[shostak](0.38 s)
    Theory quotient_rings_with_one totals: 10 formulas, 10 attempted,
10 succeeded (4.32 s)

Proof summary for theory ring_maximal_ideal
    maximal_prime_ideal.................proved - complete
[shostak]( n/a s)
    Theory ring_maximal_ideal totals: 1 formulas, 1 attempted, 1
succeeded (0.00 s)

Proof summary for theory ring_maximal_ideal_def
    Theory ring_maximal_ideal_def totals: 0 formulas, 0 attempted, 0
succeeded (0.00 s)

Proof summary for theory field_def
    field?_TCC1.........................proved - complete
[shostak](0.37 s)
    Theory field_def totals: 1 formulas, 1 attempted, 1 succeeded (0.37
s)

Proof summary for theory division_ring_def
    Theory division_ring_def totals: 0 formulas, 0 attempted, 0
succeeded (0.00 s)

Proof summary for theory ring_principal_ideal_domain_def
    Theory ring_principal_ideal_domain_def totals: 0 formulas, 0
attempted, 0 succeeded (0.00 s)

Proof summary for theory ring_unit
    proper_id_iff_no_unit_TCC1.....proved - complete
```

```
[shostak](0.40 s)
    proper_id_iff_no_unit.................proved - complete
[shostak]( n/a s)
    no_prop_l_ideal_nz_unit_TCC1..........proved - complete
[shostak](0.45 s)
    no_prop_l_ideal_nz_unit...............proved - complete
[shostak]( n/a s)
    no_prop_r_ideal_nz_unit_TCC1..........proved - complete
[shostak](0.45 s)
    no_prop_r_ideal_nz_unit...............proved - complete
[shostak]( n/a s)
    Theory ring_unit totals: 6 formulas, 6 attempted, 6 succeeded (1.30
s)

Proof summary for theory ring_prime_element
    IMP_ring_with_id_one_generator_TCC1...proved - complete
[shostak](0.25 s)
    IMP_ring_prime_ideal_TCC1.............proved - complete
[shostak](0.33 s)
    prime_el_iff_prime_ideal..............proved - complete
[shostak]( n/a s)
    el_irred_iff_one_gen_maximal_TCC1.....proved - complete
[shostak](0.54 s)
    el_irred_iff_one_gen_maximal..........proved - complete
[shostak]( n/a s)
    prime_el_is_irreducible_TCC1..........proved - complete
[shostak](0.47 s)
    prime_el_is_irreducible...............proved - complete
[shostak]( n/a s)
    assoc_irreducible_is_irreducible_TCC1...proved - complete
[shostak](0.44 s)
    assoc_irreducible_is_irreducible_TCC2...proved - complete
[shostak](0.52 s)
    assoc_irreducible_is_irreducible_TCC3...proved - complete
[shostak](0.49 s)
    assoc_irreducible_is_irreducible......proved - complete
[shostak]( n/a s)
    assoc_prime_is_prime_TCC1.............proved - complete
[shostak](0.47 s)
    assoc_prime_is_prime..................proved - complete
[shostak]( n/a s)
    irreducible_el_divisors_charac_TCC1...proved - complete
[shostak](0.38 s)
    irreducible_el_divisors_charac_TCC2...proved - complete
[shostak](0.47 s)
    irreducible_el_divisors_charac........proved - complete
[shostak]( n/a s)
    prime_el_divides_TCC1.................proved - complete
```

```
[shostak](0.41 s)
    prime_el_divides_TCC2.................proved - complete
[shostak](0.36 s)
    prime_el_divides......................proved - incomplete
[shostak]( n/a s)
    prime_el_divides_last_pos_TCC1........proved - complete
[shostak](0.58 s)
    prime_el_divides_last_pos.............proved - incomplete
[shostak]( n/a s)
    irreducible_prod_not_unit_TCC1........proved - complete
[shostak](0.46 s)
    irreducible_prod_not_unit_TCC2........proved - complete
[shostak](0.36 s)
    irreducible_prod_not_unit.............proved - incomplete
[shostak]( n/a s)
    irreducible_prod_unit_length_0_TCC1...proved - complete
[shostak](0.33 s)
    irreducible_prod_unit_length_0........proved - incomplete
[shostak]( n/a s)
    irreducible_prod_not_zero.............proved - complete
[shostak]( n/a s)
    prod_unit_irreducible_is_irreducible_TCC1...proved - complete
[shostak](0.40 s)
    prod_unit_irreducible_is_irreducible_TCC2...proved - complete
[shostak](0.44 s)
    prod_unit_irreducible_is_irreducible_TCC3...proved - complete
[shostak](1.09 s)
    prod_unit_irreducible_is_irreducible...proved - complete
[shostak]( n/a s)
    prod_unit_irreducible_is_associates_TCC1...proved - complete
[shostak](2.42 s)
    prod_unit_irreducible_is_associates...proved - complete
[shostak]( n/a s)
    Theory ring_prime_element totals: 33 formulas, 33 attempted, 33
succeeded (11.21 s)

Proof summary for theory ring_prime_element_def
    R_prime_element?_TCC1.................proved - complete
[shostak](0.22 s)
    R_prime_element?_TCC2.................proved - complete
[shostak](0.28 s)
    Theory ring_prime_element_def totals: 2 formulas, 2 attempted, 2
succeeded (0.50 s)

Proof summary for theory ring_irreducible_element_def
    Theory ring_irreducible_element_def totals: 0 formulas, 0
attempted, 0 succeeded (0.00 s)
```

```
Proof summary for theory ring_prime_ideal
    prime_ideal_prop1.....................proved - complete
[shostak]( n/a s)
    prime_ideal_prop2.....................proved - complete
[shostak]( n/a s)
    prime_ideal_prod_closed..............proved - complete
[shostak]( n/a s)
    prime_ideal_nz_closed_TCC1............proved - complete
[shostak](0.38 s)
    prime_ideal_nz_closed_TCC2............proved - complete
[shostak](0.36 s)
    prime_ideal_nz_closed_TCC3............proved - complete
[shostak](0.35 s)
    prime_ideal_nz_closed_TCC4............proved - complete
[shostak](0.32 s)
    prime_ideal_nz_closed.................proved - complete
[shostak]( n/a s)
    Theory ring_prime_ideal totals: 8 formulas, 8 attempted, 8
succeeded (1.41 s)

Proof summary for theory ring_prime_ideal_def
    Theory ring_prime_ideal_def totals: 0 formulas, 0 attempted, 0
succeeded (0.00 s)

Proof summary for theory quotient_rings
    add_charac_TCC1.......................proved - complete
[shostak](0.36 s)
    add_charac_TCC2.......................proved - complete
[shostak](0.30 s)
    add_charac............................proved - complete
[shostak]( n/a s)
    add_is_coset..........................proved - complete
[shostak]( n/a s)
    coset_add.............................proved - complete
[shostak]( n/a s)
    product_charac........................proved - complete
[shostak]( n/a s)
    lprod_equal_rprod_TCC1................proved - complete
[shostak](0.31 s)
    lprod_equal_rprod_TCC2................proved - complete
[shostak](0.31 s)
    lprod_equal_rprod_TCC3................proved - complete
[shostak](0.31 s)
    lprod_equal_rprod_TCC4................proved - complete
[shostak](0.30 s)
    lprod_equal_rprod.....................proved - complete
[shostak]( n/a s)
    product_is_coset......................proved - complete
```

```
[shostak]( n/a s)
    coset_product.....................proved - complete
[shostak]( n/a s)
    quotient_group_is_abelian_group_TCC1...proved - complete
[shostak](0.34 s)
    quotient_group_is_abelian_group_TCC2...proved - complete
[shostak](0.32 s)
    quotient_group_is_abelian_group_TCC3...proved - complete
[shostak](0.32 s)
    quotient_group_is_abelian_group.......proved - complete
[shostak]( n/a s)
    quotient_group_is_ring_TCC1..........proved - complete
[shostak](0.33 s)
    quotient_group_is_ring...............proved - complete
[shostak]( n/a s)
    fullset_quot_group_is_ring...........proved - complete
[shostak]( n/a s)
    inv_charac_TCC1......................proved - complete
[shostak](0.00 s)
    inv_charac_TCC2......................proved - complete
[shostak](0.34 s)
    inv_charac...........................proved - complete
[shostak]( n/a s)
    coset_subring_TCC1...................proved - complete
[shostak](0.31 s)
    coset_subring_TCC2...................proved - complete
[shostak](0.32 s)
    coset_subring_TCC3...................proved - complete
[shostak](0.31 s)
    coset_subring_TCC4...................proved - complete
[shostak](0.31 s)
    coset_subring_TCC5...................proved - complete
[shostak](0.30 s)
    coset_subring........................proved - complete
[shostak]( n/a s)
    coset_ideal_TCC1.....................proved - complete
[shostak](0.30 s)
    coset_ideal_TCC2.....................proved - complete
[shostak](0.30 s)
    coset_ideal_TCC3.....................proved - complete
[shostak](0.31 s)
    coset_ideal_TCC4.....................proved - complete
[shostak](0.33 s)
    coset_ideal..........................proved - complete
[shostak]( n/a s)
    commutative_quotient_ring_TCC1........proved - complete
[shostak](0.34 s)
    commutative_quotient_ring_TCC2........proved - complete
```

```
[shostak](0.33 s)
    commutative_quotient_ring_TCC3........proved - complete
[shostak](0.32 s)
    commutative_quotient_ring_TCC4........proved - complete
[shostak](0.40 s)
    commutative_quotient_ring.............proved - complete
[shostak]( n/a s)
    lcoset_power_nat......................proved - complete
[shostak]( n/a s)
    lcoset_power_int......................proved - complete
[shostak]( n/a s)
    Theory quotient_rings totals: 41 formulas, 41 attempted, 41
succeeded (7.72 s)

Proof summary for theory ring_cosets_lemmas
    IMP_ring_ideal_TCC1...................proved - complete
[shostak](0.33 s)
    lcoset_iff_rcoset.....................proved - complete
[shostak](0.43 s)
    lcoset_iff_coset......................proved - complete
[shostak](0.35 s)
    lcos_eq_rcos..........................proved - complete
[shostak](0.37 s)
    self_coset............................proved - complete
[shostak](0.52 s)
    gen_is_any............................proved - complete
[shostak](0.52 s)
    lcos_eq...............................proved - complete
[shostak](0.35 s)
    lcos_eq2..............................proved - complete
[shostak](0.45 s)
    lc_gen_eq_TCC1........................proved - complete
[shostak](0.35 s)
    lc_gen_eq.............................proved - complete
[shostak](0.45 s)
    ring_lcos_subset......................proved - complete
[shostak](0.38 s)
    ring_rcos_subset......................proved - complete
[shostak](0.38 s)
    left_zero.............................proved - complete
[shostak](0.36 s)
    right_zero............................proved - complete
[shostak](0.36 s)
    ideal_is_coset........................proved - complete
[shostak](0.38 s)
    sum_subring_ideal.....................proved - complete
[shostak](0.52 s)
    sum_ideal_ideal.......................proved - complete
```

```
[shostak](0.33 s)
    sum_is_ideal_TCC1.....................proved - complete
[shostak](0.35 s)
    sum_is_ideal..........................proved - complete
[shostak](0.43 s)
    Theory ring_cosets_lemmas totals: 19 formulas, 19 attempted, 19
succeeded (7.59 s)

Proof summary for theory product_coset_def
    product_TCC1..........................proved - complete
[shostak](0.38 s)
    product_TCC2..........................proved - complete
[shostak](0.36 s)
    Theory product_coset_def totals: 2 formulas, 2 attempted, 2
succeeded (0.74 s)

Proof summary for theory ring_with_one_ideal
    IMP_ring_with_one_TCC1................proved - complete
[shostak](0.39 s)
    IMP_ring_ideal_TCC1...................proved - complete
[shostak](0.37 s)
    l_ideal_w_one_is_R....................proved - complete
[shostak](0.47 s)
    r_ideal_w_one_is_R....................proved - complete
[shostak](0.24 s)
    ideal_w_one_is_R......................proved - complete
[shostak](0.27 s)
    no_prop_l_ideal_nz_closed.............proved - complete
[shostak](0.65 s)
    no_prop_r_ideal_nz_closed.............proved - complete
[shostak](0.52 s)
    set_of_ideals_bounded_above...........proved - incomplete
[shostak](0.77 s)
    set_of_ideals_has_maximal.............proved - incomplete
[shostak](0.97 s)
    Theory ring_with_one_ideal totals: 9 formulas, 9 attempted, 9
succeeded (4.66 s)

Proof summary for theory ring_principal_ideal
    IMP_ring_one_generator_TCC1...........proved - complete
[shostak](0.41 s)
    gen_is_member.........................proved - complete
[shostak](0.21 s)
    principal_ideal_is_ideal..............proved - complete
[shostak](0.33 s)
    principal_ideal_charac................proved - complete
[shostak](0.50 s)
    comm_principal_ideal_charac...........proved - complete
```

```
[shostak](0.12 s)
    principal_ideal_subset.............proved - complete
[shostak](0.22 s)
    stable_chain.......................proved - complete
[shostak](0.00 s)
    Theory ring_principal_ideal totals: 7 formulas, 7 attempted, 7
succeeded (1.80 s)

Proof summary for theory ring_principal_ideal_def
    Theory ring_principal_ideal_def totals: 0 formulas, 0 attempted, 0
succeeded (0.00 s)

Proof summary for theory ring_divides
    divides_subset_TCC1................proved - complete
[shostak](0.32 s)
    divides_subset.....................proved - complete
[shostak]( n/a s)
    divides_equal_TCC1.................proved - complete
[shostak](0.32 s)
    divides_equal_TCC2.................proved - complete
[shostak](0.32 s)
    divides_equal......................proved - complete
[shostak]( n/a s)
    associates_equiv_relation..........proved - complete
[shostak]( n/a s)
    unit_divides_TCC1..................proved - complete
[shostak](0.33 s)
    unit_divides.......................proved - complete
[shostak]( n/a s)
    one_gen_unit_R.....................proved - complete
[shostak]( n/a s)
    quot_unit_associates...............proved - complete
[shostak]( n/a s)
    int_domain_assoc_quot_unit_TCC1....proved - complete
[shostak](0.54 s)
    int_domain_assoc_quot_unit.........proved - complete
[shostak]( n/a s)
    x_divides_x_TCC1...................proved - complete
[shostak](0.41 s)
    x_divides_x........................proved - complete
[shostak]( n/a s)
    int_domain_assoc_unit_TCC1.........proved - complete
[shostak](0.45 s)
    int_domain_assoc_unit_TCC2.........proved - complete
[shostak](0.54 s)
    int_domain_assoc_unit_TCC3.........proved - complete
[shostak](0.49 s)
    int_domain_assoc_unit..............proved - complete
```

```
[shostak]( n/a s)
    div_member_fseq_div_op_finseq_TCC1....proved - complete
[shostak](1.69 s)
    div_member_fseq_div_op_finseq.........proved - complete
[shostak]( n/a s)
    Theory ring_divides totals: 20 formulas, 20 attempted, 20 succeeded
(5.41 s)

Proof summary for theory ring_with_id_one_generator
    IMP_ring_one_generator_TCC1...........proved - complete
[shostak](0.39 s)
    IMP_ring_with_one_TCC1................proved - complete
[shostak](0.39 s)
    member_center_r_prod_is_one_gen.......proved - complete
[shostak](0.95 s)
    member_center_l_prod_is_one_gen.......proved - complete
[shostak](0.82 s)
    member_center_l_prod_is_r_prod........proved - complete
[shostak](0.41 s)
    commutative_id_one_gen_charac.........proved - complete
[shostak](0.37 s)
    Theory ring_with_id_one_generator totals: 6 formulas, 6 attempted,
6 succeeded (3.33 s)

Proof summary for theory ring_one_generator
    IMP_ring_basic_properties_TCC1........proved - complete
[shostak](0.38 s)
    F_one_gen_TCC1........................proved - complete
[shostak](0.13 s)
    gen_is_member_one_gen.................proved - complete
[shostak](0.24 s)
    one_gen_is_sum_closed_TCC1............proved - complete
[shostak](0.17 s)
    one_gen_is_sum_closed.................proved - complete
[shostak](1.14 s)
    inv_one_gen_TCC1......................proved - complete
[shostak](0.07 s)
    inv_one_gen...........................proved - complete
[shostak](0.58 s)
    one_gen_is_ideal......................proved - complete
[shostak](0.00 s)
    R_sigma_of_comm_factor_of_gen.........proved - complete
[shostak](0.24 s)
    commutative_one_gen_charac............proved - complete
[shostak](0.23 s)
    commutative_one_gen_is_ideal..........proved - complete
[shostak](0.19 s)
    F_one_gen_r_comm......................proved - complete
```

```
[shostak](0.47 s)
    R_sigma_over_center_l.................proved - complete
[shostak](0.49 s)
    F_one_gen_l_comm......................proved - complete
[shostak](0.24 s)
    R_sigma_over_center_r.................proved - complete
[shostak](0.26 s)
    subset_product_one_gen_TCC1...........proved - complete
[shostak](0.46 s)
    subset_product_one_gen................proved - complete
[shostak](0.61 s)
    subset_prod_one_gen_ideal_prop1.......proved - complete
[shostak](0.49 s)
    subset_prod_one_gen_ideal_prop2.......proved - complete
[shostak](0.90 s)
    sum_strict_subset_one_gen.............proved - complete
[shostak](0.50 s)
    Theory ring_one_generator totals: 20 formulas, 20 attempted, 20
succeeded (7.80 s)

Proof summary for theory ring_ideal
    left_ideal_equiv......................proved - complete
[shostak]( n/a s)
    right_ideal_equiv.....................proved - complete
[shostak]( n/a s)
    ideal_equiv...........................proved - complete
[shostak]( n/a s)
    self_ideal............................proved - complete
[shostak]( n/a s)
    zero_ideal............................proved - complete
[shostak]( n/a s)
    ideal_transitive_TCC1.................proved - complete
[shostak](0.58 s)
    ideal_transitive......................proved - complete
[shostak]( n/a s)
    intersection_subring_ideal_TCC1.......proved - complete
[shostak](0.55 s)
    intersection_subring_ideal............proved - complete
[shostak]( n/a s)
    r_prod_is_sum_closed..................proved - complete
[shostak]( n/a s)
    l_prod_is_sum_closed..................proved - complete
[shostak]( n/a s)
    inv_is_member_l_prod..................proved - complete
[shostak]( n/a s)
    inv_is_member_r_prod..................proved - complete
[shostak]( n/a s)
    l_prod_is_r_ideal.....................proved - complete
```

```
[shostak]( n/a s)
    r_prod_is_l_ideal.....................proved - complete
[shostak]( n/a s)
    ideal_iunion_ideal....................proved - complete
[shostak]( n/a s)
    chain_ideal_union_ideal..............proved - incomplete
[shostak]( n/a s)
    Theory ring_ideal totals: 17 formulas, 17 attempted, 17 succeeded
(1.13 s)

Proof summary for theory ring_ideal_def
    Theory ring_ideal_def totals: 0 formulas, 0 attempted, 0 succeeded
(0.00 s)

Proof summary for theory cosets_def
    lc_gen_TCC1...........................proved - complete
[shostak](0.34 s)
    rc_gen_TCC1...........................proved - complete
[shostak](0.35 s)
    gen_TCC1..............................proved - complete
[shostak](0.34 s)
    lcos_is_left_coset....................proved - complete
[shostak](0.33 s)
    rcos_is_right_coset...................proved - complete
[shostak](0.32 s)
    add_TCC1..............................proved - complete
[shostak](0.34 s)
    Theory cosets_def totals: 6 formulas, 6 attempted, 6 succeeded
(2.02 s)

Proof summary for theory ring_with_one
    IMP_ring_TCC1.........................proved - complete
[shostak](0.34 s)
    IMP_monoid_TCC1.......................proved - complete
[shostak](0.34 s)
    ring_with_one_TCC1....................proved - complete
[shostak](0.18 s)
    one_times.............................proved - complete
[shostak](0.23 s)
    times_one.............................proved - complete
[shostak](0.20 s)
    unique_left_identity..................proved - complete
[shostak](0.42 s)
    unique_right_identity.................proved - complete
[shostak](0.43 s)
    minus_one_times.......................proved - complete
[shostak](0.20 s)
    times_minus_one.......................proved - complete
```

```
[shostak](0.39 s)
    minus_one_sq_is_one.................proved - complete
[shostak](0.45 s)
    ring_with_one_is_ring...............proved - complete
[shostak](0.56 s)
    ring_with_one_is_monoid.............proved - complete
[shostak](0.53 s)
    Theory ring_with_one totals: 12 formulas, 12 attempted, 12
succeeded (4.27 s)

Proof summary for theory ring_center
    center_subring......................proved - complete
[shostak]( n/a s)
    commutative_ring_equal_center.........proved - complete
[shostak]( n/a s)
    Theory ring_center totals: 2 formulas, 2 attempted, 2 succeeded
(0.00 s)

Proof summary for theory center_def
    center_def..........................proved - complete
[shostak](0.19 s)
    center_subset.......................proved - complete
[shostak](0.17 s)
    Theory center_def totals: 2 formulas, 2 attempted, 2 succeeded
(0.37 s)

Proof summary for theory ring_unit_def
    l_r_inv_equal.......................proved - complete
[shostak](0.27 s)
    Theory ring_unit_def totals: 1 formulas, 1 attempted, 1 succeeded
(0.27 s)

Proof summary for theory ring_with_one_def
    commutative_ring_with_one?_TCC1.......proved - complete
[shostak](0.19 s)
    finite_commutative_ring_with_one?_TCC1...proved - complete
[shostak](0.37 s)
    Theory ring_with_one_def totals: 2 formulas, 2 attempted, 2
succeeded (0.56 s)

Proof summary for theory ring_nz_closed_aux
    IMP_ring_basic_properties_TCC1........proved - complete
[shostak](0.34 s)
    nz_times_is_zero....................proved - complete
[shostak](0.29 s)
    nzd_R_cancel_left...................proved - complete
[shostak](0.45 s)
    nzd_R_cancel_right..................proved - complete
```

```
[shostak](0.27 s)
    subring_nz_closed.....................proved - complete
[shostak](0.44 s)
    Theory ring_nz_closed_aux totals: 5 formulas, 5 attempted, 5
succeeded (1.79 s)

Proof summary for theory ring_basic_properties
    IMP_ring_TCC1.........................proved - complete
[shostak](0.33 s)
    zero_is_member_R......................proved - complete
[shostak](0.07 s)
    inv_is_member_R.......................proved - complete
[shostak](0.09 s)
    R_sum_star_closed.....................proved - complete
[shostak](0.12 s)
    R_prod_star_closed....................proved - complete
[shostak](0.06 s)
    l_plus_zero...........................proved - complete
[shostak](0.36 s)
    r_plus_zero...........................proved - complete
[shostak](0.35 s)
    no_singleton_nzx......................proved - complete
[shostak](0.37 s)
    card_gt_one_nzx.......................proved - complete
[shostak](0.68 s)
    no_singleton_card.....................proved - complete
[shostak](0.39 s)
    subring_transitive_TCC1...............proved - complete
[shostak](0.33 s)
    subring_transitive....................proved - complete
[shostak](0.33 s)
    subring_equiv.........................proved - complete
[shostak](0.30 s)
    times_member..........................proved - complete
[shostak](0.05 s)
    left_times............................proved - complete
[shostak](0.33 s)
    right_times...........................proved - complete
[shostak](0.32 s)
    inv_times_neg.........................proved - complete
[shostak](0.15 s)
    inv_times_inv.........................proved - complete
[shostak](0.34 s)
    times_inv_neg.........................proved - complete
[shostak](0.34 s)
    times_int_zero........................proved - complete
[shostak](0.18 s)
    times_int_one.........................proved - complete
```

```
[shostak](0.05 s)
    times_sum.............................proved - complete
[shostak](0.09 s)
    times_o...............................proved - complete
[shostak](0.24 s)
    times_product.........................proved - complete
[shostak](0.19 s)
    R_sigma_TCC1..........................proved - complete
[shostak](0.07 s)
    R_sigma_TCC2..........................proved - complete
[shostak](0.14 s)
    R_sigma_first.........................proved - complete
[shostak](0.47 s)
    R_sigma_eq_k..........................proved - complete
[shostak](0.40 s)
    R_sigma_eq............................proved - complete
[shostak](0.06 s)
    R_sigma_eq2...........................proved - complete
[shostak](0.37 s)
    R_sigma_sum...........................proved - complete
[shostak](0.45 s)
    ast_R_sigma...........................proved - complete
[shostak](0.14 s)
    R_sigma_ast...........................proved - complete
[shostak](0.14 s)
    R_sigma_inv...........................proved - complete
[shostak](0.19 s)
    R_sigma_o.............................proved - complete
[shostak](0.44 s)
    R_sigma_R_sigma_TCC1..................proved - complete
[shostak](0.09 s)
    R_sigma_R_sigma_TCC2..................proved - complete
[shostak](0.25 s)
    R_sigma_R_sigma.......................proved - complete
[shostak](2.11 s)
    R_sigma_is_member_R...................proved - complete
[shostak](0.37 s)
    nlzd_TCC1.............................proved - complete
[shostak](0.34 s)
    nzd_cancel_left.......................proved - complete
[shostak](0.39 s)
    nzd_cancel_right......................proved - complete
[shostak](0.39 s)
    Theory ring_basic_properties totals: 42 formulas, 42 attempted, 42
succeeded (12.88 s)

Proof summary for theory ring
    IMP_abelian_group_TCC1................proved - complete
```

```
[shostak](0.33 s)
    ring_TCC1.............................proved - complete
[shostak](0.05 s)
    plus_associative......................proved - complete
[shostak](0.07 s)
    plus_commutative......................proved - complete
[shostak](0.10 s)
    times_associative.....................proved - complete
[shostak](0.07 s)
    right_distributive....................proved - complete
[shostak](0.08 s)
    left_distributive.....................proved - complete
[shostak](0.07 s)
    zero_plus.............................proved - complete
[shostak](0.04 s)
    plus_zero.............................proved - complete
[shostak](0.04 s)
    negate_is_left_inv....................proved - complete
[shostak](0.04 s)
    negate_is_right_inv...................proved - complete
[shostak](0.04 s)
    cancel_right_plus.....................proved - complete
[shostak](0.06 s)
    cancel_left_plus......................proved - complete
[shostak](0.08 s)
    negate_negate.........................proved - complete
[shostak](0.05 s)
    cancel_right_minus....................proved - complete
[shostak](0.07 s)
    cancel_left_minus.....................proved - complete
[shostak](0.36 s)
    negate_zero...........................proved - complete
[shostak](0.33 s)
    negate_plus...........................proved - complete
[shostak](0.06 s)
    times_plus............................proved - complete
[shostak](0.24 s)
    idempotent_add_is_zero................proved - complete
[shostak](0.06 s)
    zero_times............................proved - complete
[shostak](0.06 s)
    times_zero............................proved - complete
[shostak](0.06 s)
    negative_times........................proved - complete
[shostak](0.08 s)
    times_negative........................proved - complete
[shostak](0.08 s)
    negative_times_negative...............proved - complete
```

```
[shostak](0.34 s)
    ring_is_abelian_group.................proved - complete
[shostak](0.43 s)
    subring_is_ring.......................proved - complete
[shostak](0.35 s)
    sq_rew................................proved - complete
[shostak](0.33 s)
    sq_neg................................proved - complete
[shostak](0.33 s)
    sq_plus...............................proved - complete
[shostak](0.37 s)
    sq_minus..............................proved - complete
[shostak](0.37 s)
    sq_neg_minus..........................proved - complete
[shostak](0.34 s)
    sq_zero...............................proved - complete
[shostak](0.33 s)
    Theory ring totals: 33 formulas, 33 attempted, 33 succeeded (5.68
s)

Proof summary for theory abelian_group
    IMP_group_TCC1........................proved - complete
[shostak](0.33 s)
    abelian_group_TCC1....................proved - complete
[shostak](0.37 s)
    abelian_group_is_group................proved - complete
[shostak](0.41 s)
    abelian_group_is_commutative_monoid...proved - complete
[shostak](0.40 s)
    abelian_subgroups.....................proved - complete
[shostak](0.41 s)
    finite_abelian_group_TCC1.............proved - complete
[shostak](0.38 s)
    finite_abelian_group_is_abelian_group...proved - complete
[shostak](0.42 s)
    finite_abelian_group_is_finite_group...proved - complete
[shostak](0.43 s)
    finite_abelian_subgroups..............proved - complete
[shostak](0.35 s)
    Theory abelian_group totals: 9 formulas, 9 attempted, 9 succeeded
(3.50 s)

Proof summary for theory group
    IMP_monoid_TCC1.......................proved - complete
[shostak](0.33 s)
    group_TCC1............................proved - complete
[shostak](0.05 s)
    group_is_monoid.......................proved - complete
```

```
[shostak](0.39 s)
    finite_group_TCC1.....................proved - complete
[shostak](0.42 s)
    finite_group_is_group.................proved - complete
[shostak](0.39 s)
    finite_group_is_finite_monoid.........proved - complete
[shostak](0.40 s)
    finite_subgroups......................proved - complete
[shostak](0.34 s)
    one_is_group..........................proved - complete
[shostak](0.35 s)
    one_finite_group......................proved - complete
[shostak](0.33 s)
    one_group_TCC1........................proved - complete
[shostak](0.44 s)
    group_card_gt_0.......................proved - complete
[shostak](0.36 s)
    inv_exists............................proved - complete
[shostak](0.05 s)
    inv_TCC1..............................proved - complete
[shostak](0.08 s)
    inv_left..............................proved - complete
[shostak](0.04 s)
    inv_right.............................proved - complete
[shostak](0.05 s)
    cancel_right..........................proved - complete
[shostak](0.10 s)
    cancel_left...........................proved - complete
[shostak](0.08 s)
    inv_inv...............................proved - complete
[shostak](0.07 s)
    cancel_right_inv......................proved - complete
[shostak](0.07 s)
    cancel_left_inv.......................proved - complete
[shostak](0.41 s)
    inv_one...............................proved - complete
[shostak](0.06 s)
    inv_star..............................proved - complete
[shostak](0.08 s)
    unique_inv............................proved - complete
[shostak](0.07 s)
    inv_member............................proved - complete
[shostak](0.37 s)
    inv_in................................proved - complete
[shostak](0.33 s)
    divby.................................proved - complete
[shostak](0.34 s)
    product_in............................proved - complete
```

```
[shostak](0.36 s)
    one_is_subgroup.....................proved - complete
[shostak](0.43 s)
    group_is_subgroup...................proved - complete
[shostak](0.37 s)
    subgroup_is_group...................proved - complete
[shostak](0.33 s)
    subgroup_def........................proved - complete
[shostak](0.50 s)
    inv_power...........................proved - complete
[shostak](0.17 s)
    power_inv_right.....................proved - complete
[shostak](0.32 s)
    power_inv_left......................proved - complete
[shostak](0.32 s)
    caret_TCC1..........................proved - complete
[shostak](0.05 s)
    caret_TCC2..........................proved - complete
[shostak](0.05 s)
    expt_0..............................proved - complete
[shostak](0.18 s)
    expt_1..............................proved - complete
[shostak](0.05 s)
    expt_m1.............................proved - complete
[shostak](0.35 s)
    one_expt............................proved - complete
[shostak](0.21 s)
    expt_neg............................proved - complete
[shostak](0.08 s)
    inv_expt............................proved - complete
[shostak](0.09 s)
    expt_def1...........................proved - complete
[shostak](0.28 s)
    expt_def2...........................proved - complete
[shostak](0.17 s)
    expt_mult...........................proved - complete
[shostak](0.41 s)
    expt_div............................proved - complete
[shostak](0.36 s)
    expt_expt...........................proved - complete
[shostak](0.38 s)
    expt_commutes.......................proved - complete
[shostak](0.38 s)
    expt_inv_right......................proved - complete
[shostak](0.34 s)
    expt_inv_left.......................proved - complete
[shostak](0.35 s)
    expt_member.........................proved - complete
```

```
[shostak](0.11 s)
    generated_by_TCC1.....................proved - complete
[shostak](0.49 s)
    generated_by_lem......................proved - complete
[shostak](0.34 s)
    generated_is_subgroup.................proved - complete
[shostak](0.34 s)
    generated_by_is_finite................proved - complete
[shostak](0.48 s)
    center_TCC1...........................proved - complete
[shostak](0.41 s)
    center_def............................proved - complete
[shostak](0.38 s)
    center_subgroup.......................proved - complete
[shostak](0.54 s)
    one_left..............................proved - complete
[shostak](0.04 s)
    one_right.............................proved - complete
[shostak](0.04 s)
    assoc.................................proved - complete
[shostak](0.34 s)
    Theory group totals: 61 formulas, 61 attempted, 61 succeeded (16.03
s)

Proof summary for theory integral_domain_with_one_def
    Theory integral_domain_with_one_def totals: 0 formulas, 0
attempted, 0 succeeded (0.00 s)

Proof summary for theory integral_domain_def
    Theory integral_domain_def totals: 0 formulas, 0 attempted, 0
succeeded (0.00 s)

Proof summary for theory ring_nz_closed_def
    Theory ring_nz_closed_def totals: 0 formulas, 0 attempted, 0
succeeded (0.00 s)

Proof summary for theory op_finseq
    IMP_monoid_TCC1.......................proved - complete
[shostak](0.26 s)
    op_fseq_singleton.....................proved - complete
[shostak]( n/a s)
    op_fseq_composition...................proved - incomplete
[shostak]( n/a s)
    op_subfseq_closed.....................proved - complete
[shostak]( n/a s)
    op_fseq_closed........................proved - complete
[shostak]( n/a s)
    op_fseq_split_TCC1....................proved - complete
```

```
[shostak](0.28 s)
    op_fseq_split_TCC2.....................proved - complete
[shostak](0.27 s)
    op_fseq_split.........................proved - incomplete
[shostak]( n/a s)
    op_fseq_split_commute_TCC1............proved - complete
[shostak](0.27 s)
    op_fseq_split_commute_TCC2............proved - complete
[shostak](0.28 s)
    op_fseq_split_commute.................proved - incomplete
[shostak]( n/a s)
    op_fseq_split_delete..................proved - incomplete
[shostak]( n/a s)
    op_fseq_same_replace_first_TCC1.......proved - complete
[shostak](0.29 s)
    op_fseq_same_replace_first............proved - incomplete
[shostak]( n/a s)
    op_fseq_same_replace_last_TCC1........proved - complete
[shostak](0.31 s)
    op_fseq_same_replace_last.............proved - incomplete
[shostak]( n/a s)
    Theory op_finseq totals: 16 formulas, 16 attempted, 16 succeeded
(1.96 s)

Proof summary for theory op_finseq_def
    op_fseq_TCC1..........................proved - complete
[shostak](0.18 s)
    op_fseq_TCC2..........................proved - complete
[shostak](0.19 s)
    op_fseq_TCC3..........................proved - complete
[shostak](0.28 s)
    Theory op_finseq_def totals: 3 formulas, 3 attempted, 3 succeeded
(0.66 s)

Proof summary for theory monoid
    IMP_monad_TCC1........................proved - complete
[shostak](0.33 s)
    IMP_semigroup_TCC1....................proved - complete
[shostak](0.34 s)
    monoid_TCC1...........................proved - complete
[shostak](0.04 s)
    monoid_is_monad.......................proved - complete
[shostak](0.37 s)
    monoid_is_semigroup...................proved - complete
[shostak](0.38 s)
    power_0...............................proved - complete
[shostak](0.04 s)
    power_1...............................proved - complete
```

```
[shostak](0.05 s)
    one_power.............................proved - complete
[shostak](0.19 s)
    power_def.............................proved - complete
[shostak](0.14 s)
    power_mult............................proved - complete
[shostak](0.51 s)
    power_power...........................proved - complete
[shostak](0.47 s)
    power_commutes........................proved - complete
[shostak](0.39 s)
    power_member..........................proved - complete
[shostak](0.13 s)
    one_is_monoid.........................proved - complete
[shostak](0.36 s)
    generated_is_submonoid................proved - complete
[shostak](0.45 s)
    generated_set_card_1..................proved - complete
[shostak](0.38 s)
    finite_monoid_TCC1....................proved - complete
[shostak](0.50 s)
    finite_monoid_is_monoid...............proved - complete
[shostak](0.41 s)
    finite_monoid_is_finite_monad.........proved - complete
[shostak](0.40 s)
    finite_submonoids.....................proved - complete
[shostak](0.33 s)
    commutative_monoid_TCC1...............proved - complete
[shostak](0.44 s)
    commutative_monoid_is_monoid..........proved - complete
[shostak](0.38 s)
    commutative_monoid_is_commutative_monad...proved - complete
[shostak](0.38 s)
    commutative_submonoids................proved - complete
[shostak](0.37 s)
    Theory monoid totals: 24 formulas, 24 attempted, 24 succeeded (7.77
s)

Proof summary for theory monad
    monad_TCC1............................proved - complete
[shostak](0.04 s)
    one_member............................proved - complete
[shostak](0.07 s)
    one_in................................proved - complete
[shostak](0.33 s)
    left_identity.........................proved - complete
[shostak](0.05 s)
    right_identity........................proved - complete
```

```
[shostak](0.06 s)
    unique_left_identity.................proved - complete
[shostak](0.33 s)
    unique_right_identity................proved - complete
[shostak](0.34 s)
    one_is_monad.........................proved - complete
[shostak](0.39 s)
    trivial_monad_TCC1...................proved - complete
[shostak](0.39 s)
    monad_is_groupoid....................proved - complete
[shostak](0.35 s)
    sing_one_finite_monad................proved - complete
[shostak](0.36 s)
    finite_monad_TCC1....................proved - complete
[shostak](0.37 s)
    commutative_monad_TCC1...............proved - complete
[shostak](0.39 s)
    finite_commutative_monad_TCC1.........proved - complete
[shostak](0.39 s)
    order_TCC1...........................proved - complete
[shostak](0.38 s)
    order_is_1...........................proved - complete
[shostak](0.53 s)
    finite_monad_is_monad................proved - complete
[shostak](0.38 s)
    commutative_monad_is_monad...........proved - complete
[shostak](0.36 s)
    finite_commutative_monad_is_commutative_monad...proved -
complete   [shostak](0.37 s)
    finite_commutative_monad_is_finite_monad...proved - complete
[shostak](0.39 s)
    Theory monad totals: 20 formulas, 20 attempted, 20 succeeded (6.27
s)

Proof summary for theory semigroup
    fullset_is_semigroup_TCC1............proved - complete
[shostak](0.05 s)
    semigroup_TCC1.......................proved - complete
[shostak](0.34 s)
    semigroup_TCC2.......................proved - complete
[shostak](0.32 s)
    associative..........................proved - complete
[shostak](0.08 s)
    semigroup_is_groupoid................proved - complete
[shostak](0.33 s)
    Theory semigroup totals: 5 formulas, 5 attempted, 5 succeeded (1.12
s)
```

```
Proof summary for theory groupoid
    fullset_is_groupoid.................proved - complete
[shostak](0.32 s)
    groupoid_TCC1.......................proved - complete
[shostak](0.34 s)
    closed..............................proved - complete
[shostak](0.34 s)
    star_closed.........................proved - complete
[shostak](0.34 s)
    Theory groupoid totals: 4 formulas, 4 attempted, 4 succeeded (1.34
s)

Proof summary for theory ring_gcd_def
    gcd?_TCC1...........................proved - complete
[shostak](0.42 s)
    gcd?_TCC2...........................proved - complete
[shostak](0.52 s)
    Theory ring_gcd_def totals: 2 formulas, 2 attempted, 2 succeeded
(0.94 s)

Proof summary for theory ring_divides_def
    associates?_TCC1....................proved - complete
[shostak](0.24 s)
    associates?_TCC2....................proved - complete
[shostak](0.27 s)
    Theory ring_divides_def totals: 2 formulas, 2 attempted, 2
succeeded (0.51 s)

Proof summary for theory ring_def
    ring?_TCC1..........................proved - complete
[shostak](0.06 s)
    ring?_TCC2..........................proved - complete
[shostak](0.06 s)
    commutative_ring?_TCC1..............proved - complete
[shostak](0.05 s)
    finite_commutative_ring?_TCC1.........proved - complete
[shostak](0.35 s)
    Theory ring_def totals: 4 formulas, 4 attempted, 4 succeeded (0.52
s)

Proof summary for theory semigroup_def
    semigroup?_TCC1.....................proved - complete
[shostak](0.33 s)
    finite_commutative_semigroup?_TCC1....proved - complete
[shostak](0.32 s)
    Theory semigroup_def totals: 2 formulas, 2 attempted, 2 succeeded
(0.65 s)
```

```
Proof summary for theory group_def
    abelian_group?_TCC1...................proved - complete
[shostak](0.06 s)
    finite_abelian_group?_TCC1............proved - complete
[shostak](0.35 s)
    finite_group_surj.....................proved - complete
[shostak](0.34 s)
    Theory group_def totals: 3 formulas, 3 attempted, 3 succeeded (0.75
s)

Proof summary for theory monoid_def
    power_TCC1............................proved - complete
[shostak](0.07 s)
    power_TCC2............................proved - complete
[shostak](0.04 s)
    generated_set_lem.....................proved - complete
[shostak](0.33 s)
    monoid?_TCC1..........................proved - complete
[shostak](0.05 s)
    commutative_monoid?_TCC1..............proved - complete
[shostak](0.20 s)
    finite_commutative_monoid?_TCC1.......proved - complete
[shostak](0.34 s)
    Theory monoid_def totals: 6 formulas, 6 attempted, 6 succeeded
(1.04 s)

Proof summary for theory monad_def
    monad?_TCC1...........................proved - complete
[shostak](0.04 s)
    monad?_TCC2...........................proved - complete
[shostak](0.05 s)
    commutative_monad?_TCC1...............proved - complete
[shostak](0.33 s)
    finite_commutative_monad?_TCC1........proved - complete
[shostak](0.33 s)
    Theory monad_def totals: 4 formulas, 4 attempted, 4 succeeded (0.74
s)

Proof summary for theory groupoid_def
    commutative_groupoid?_TCC1............proved - complete
[shostak](0.31 s)
    finite_commutative_groupoid?_TCC1.....proved - complete
[shostak](0.32 s)
    Theory groupoid_def totals: 2 formulas, 2 attempted, 2 succeeded
(0.63 s)

Proof summary for theory operator_defs_more
    Theory operator_defs_more totals: 0 formulas, 0 attempted, 0
```

```
succeeded (0.00 s)

Proof summary for theory top
    Theory top totals: 0 formulas, 0 attempted, 0 succeeded (0.00 s)

Grand Totals: 1566 proofs, 1566 attempted, 1566 succeeded (664.85 s)
```