DERIVATION FROM TNE (2022-4-16)

For SAPH, the likelihood for a single observation is:

$$\underbrace{\ell_i(\hat{\theta}_{iF} \mid \Psi)}_{\text{evaluated at } t} = -\frac{1}{2}\log(2\pi V_i) - \frac{1}{2V_i}(t-\mu)^2 - \log \Phi(\tilde{c}_i)$$

exactly as in SAPH paper

where $V_i = T^2 + \sigma_i^2$ and $\tilde{c}_i = \dfrac{c\sigma_i - \mu}{\sqrt{V_i}}$.

($\tilde{c}_i$ is "z_b" in TNE notation and "mustarU" in init_stan_SAPH)

Take derivatives for Fisher info:

$$\frac{\partial^2 \ell_i}{\partial \mu^2} = \frac{1}{V_i}\left[r^2 + r\tilde{c}_i - 1\right]$$

(Exactly same as TNE b/c V_i is just a constant when taking deriv wrt μ)

$$\frac{\partial \ell_i}{\partial \mu \partial T} = \underbrace{\frac{\partial \ell_i}{\partial \mu \partial \sqrt{V_i}} \cdot \frac{\partial \sqrt{V_i}}{\partial T}}_{\text{Chain rule}} = \underbrace{\frac{1}{V_i}\left[r\tilde{c}_i^2 + r^2\tilde{c}_i + r\right]}_{(*)} \cdot \frac{T}{\sqrt{T^2+\sigma_i^2}}$$

$$\frac{\partial \sqrt{V_i}}{\partial T} = \frac{\partial}{\partial T}\sqrt{T^2+\sigma_i^2} = \frac{1}{2}(T^2+\sigma_i^2)^{-1/2} \cdot 2T = \frac{T}{\sqrt{T^2+\sigma_i^2}}$$

(*) from TNE, where we took deriv wrt the <u>total</u> SD, there called "σ"

$$\frac{\partial \ell_i}{\partial T^2} = \frac{\partial \ell_i}{\partial (\sqrt{V_i})^2} \cdot \left(\frac{\partial \sqrt{V_i}}{\partial T}\right)^2 = \frac{1}{V_i}\left[r\tilde{c}_i^3 + r\tilde{c}_i + (r\tilde{c}_i)^2 - 2\right] \cdot \left(\frac{T}{\sqrt{T^2+\sigma_i^2}}\right)^2$$

In the RHS of each of these, the black terms are a direct simplification of TNE under single-truncation. I confirmed this in helper_SAPH.R :: E_fisher_TNE_check. :)

Now, since the Fisher info for multiple <u>independent</u> (but not necessarily iid) RVs is just the sum of each obs' Fisher info, we get the terms in SAPH main text.