

```
import networkx as nx
import matplotlib.pyplot as plt
import sys
from collections import defaultdict

# Gabow's algorithm to find strongly
connected components
def gabow_scc(graph):
    index = [0]
    S = []
    P = []
    stack = []
    lowlink = {}
    scc_found = []
    scc_result = []

    def gabow(v):
        lowlink[v] = index[0]
        index[0] += 1
        stack.append(v)
        S.append(v)
```

```

P.append(v)

for w in graph[v]:
    if w not in lowlink:
        gabow(w)
    elif w not in scc_found:
while lowlink[P[-1]] > lowlink[w]:
    P.pop()

    if P[-1] == v:
        scc = []
        while True:
            w = S.pop()
            scc_found.append(w)
            scc.append(w)
            if w == v:
                break
        scc_result.append(scc)
        P.pop()

for v in graph:

```

```
if v not in lowlink:  
    gabow(v)
```

```
return scc_result
```

```
# Jens Schmidt's algorithm to check for  
articulation points in undirected graph
```

```
def jens_schmidt_ap(graph):
```

```
def dfs(u, parent, discovery_time, low, ap,  
        visited, time, child_count):
```

```
    visited[u] = True
```

```
    discovery_time[u] = low[u] = time[0]
```

```
    time[0] += 1
```

```
    children = 0
```

```
    for v in graph[u]:
```

```
        if not visited[v]:
```

```
            parent[v] = u
```

```
            children += 1
```

```
            dfs(v, parent, discovery_time, low,  
                ap, visited, time, child_count)
```

```
        low[u] = min(low[u], low[v])
    if parent[u] is None and children > 1:
        ap[u] = True
    if parent[u] is not None and low[v]
        >= discovery_time[u]:
        ap[u] = True
        elif v != parent[u]:
        low[u] = min(low[u],
            discovery_time[v])
```

```
    nodes = list(graph.nodes())
    visited = {node: False for node in nodes}
    discovery_time = {node: float('inf') for
        node in nodes}
    low = {node: float('inf') for node in nodes}
    parent = {node: None for node in nodes}
    ap = {node: False for node in nodes}
    time = [0]
```

```
        for node in nodes:
            if not visited[node]:
```

```
dfs(node, parent, discovery_time,  
     low, ap, visited, time, 0)
```

```
articulation_points = [node for node,  
                        is_ap in ap.items() if is_ap]  
return articulation_points
```

```
# Main function to check if graph is 2-  
vertex strongly biconnected  
def
```

```
is_2_vertex_strongly_biconnected(graph):  
    # Check if the graph is strongly  
    connected  
    sccs = gabow_scc(graph)  
    if len(sccs) > 1:  
        return False
```

```
# Convert directed graph to undirected  
graph  
undirected_graph = graph.to_undirected()
```

```
# Check for articulation points in the
    undirected graph
    articulation_points =
jens_schmidt_ap(undirected_graph)
    if articulation_points:
        return False

    return True
```

```
# Example usage
if __name__ == "__main__":
    # Example graph
    G = nx.DiGraph()
    G.add_edges_from([(1, 2), (2, 3), (3, 1),
                      (2, 4), (4, 5), (5, 2)])
```

```
# Check if the graph is 2-vertex strongly
                                biconnected
                                result =
is_2_vertex_strongly_biconnected(G)
print("The graph is 2-vertex strongly
```

```
biconnected:", result)
```

```
# Plot the graph for visual verification
```

```
    nx.draw(G, with_labels=True,  
node_color='lightblue', edge_color='gray')  
    plt.show()
```