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# Perspective, Anamorphosis and Vision

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Fig. 1 Piero della Francesca, *Flagellation*. Urbino, Galleria Nazionale delle Marche

**1. Introduction**

In order to view properly a painting in linear perspective we need to stand with an eye at the height of the picture's horizon level, directly in front of its central vanishing point and at a distance equal to that indicated by its distance point (figs. 1-5). By contrast, if a painting is in anamorphosis[1] we need to view it from a position off to the side (to the right in the case of fig. 19). Most of us simply take these rules for granted.

This essay opens with a survey of literature con-firming that the realities of pictorial composition are more complex. Classic examples of linear perspective including Piero della Francesca's *Flagellation* are then re-examined. It is demonstrated that such paintings can also profitably be viewed from positions off to the side. It is claimed that

Renaissance paintings with a central vanishing point were intended to be seen from multiple points of view, and not just from directly in front.

A contrast follows between undesired distortions of linear perspective and planned distortions in anamorphosis. This leads to an outline history of anamorphic and non-linear projection methods, including the use of mirrors and other devices. Connections between optics, surveying and cylindrical projection methods are examined as are discrepancies between theories of vision and representation overlooked by Panofsky and other cultural historians. The significance of Renaissance linear perspective (and anamorphosis) is then reassessed.

**2. Survey of Literature**

In 1824, William Wollaston writing "On the Apparent Direction of Eyes in a Portrait", drew attention to the way in which eyes in a portrait follow one even if one views it from the side.[2] In 1828, the Frenchman, Raymond, also addressed this phenomenon, attempting to identify its causes.[3] Giuseppe Ovio, Director of the Institute of Clinical Ophthalmology at Modena had different motives for examining this problem of viewpoints in paintings. Convinced that the laws of linear perspective must correspond generally with those of optics, he made a series of experiments on perceptual effects produced by vertically and laterally displaced picture planes.[4] His work has not received the attention it deserves. In 1931, a psychologist, Robert H. Thouless, studied how objects continue to look right even when they are seen obliquely from off side. He coined this „Phenomenal Regression to the Real Object".[5]

The relationship between laws of optics and linear perspective was also a starting point of Maurice Pirenne's studies. Scholars such as Panofsky (1927)[6] and White (1949, 1951)[7] had argued that linear perspective was merely a convention, at variance with optical laws. Pirenne set out to demonstrate experimentally that linear perspective had scientific laws in accordance with those of optics (1952).[8] These studies led him to compare effects of *Optics, Painting and Photography* (1970).[9] With a pinhole camera he painstakingly explored and analysed distortion effects with lateral displacement of three-dimensional and painted objects, including the famous trompe l'oeil ceiling in Il Gésu. In 1975 he examined what happens to a pinhole image of Piero della Francesca's *Flagellation* as the viewpoint is moved from in front to off side[10]; a commentary as it were on his friend B.A.R. Carter's earlier perspectival analysis of the same picture (1953).[11] Pirenne's excellent work deserves much more study.

In the meantime, however, the opinions of a philosopher with no understanding of the technicalities of perspective, attracted much more attention. Nelson Goodman (1968) felt linear perspective was a convention and insisted that an „artist who wants to produce a spatial representation that the present day Western eye will accept as faithful must defy the laws of geometry".[12] This was a direct challenge to Gibson (1960)[13], whose response came in the form of a new definition of pictures (1971).[14]

In London, Ernst Gombrich had also become interested in these problems. In his essay in honour of Nelson Goodman (1972)[15] he referred to the experiment by Thouless and drew attention to constancies involved when perspectival drawings or photographs are viewed from positions off to the side. The following year, Gombrich (1973) also explored these constancies in the case of photographs of Hobbema's *Avenue*.[16] This confirmed that like the eyes studied by Wheatstone, painted streets also follow a viewer as he moves off to the side. These writings of Gibson, Goodman, Gombrich and Pirenne were sources for Kenneth R. Adams „Perspective and the Viewpoint" (1972) in which he analysed mathematically how geometrical picture space changes with the observer's viewpoint.[17]

Considered together, it is striking that these authors have all concentrated on a negative point: namely, that paintings in linear perspective when seen from viewpoints off to the side are not as distorted as one would expect. By contrast, the essay that follows will focus on a positive claim[18]: that paintings in linear perspective acquire new meaning when seen from viewpoints off to the side.

Fig. 2 Giuliano da Sangallo and Domenico Ghirlandaio (?), Ideal City. Urbino, Galleria Nazionale della Marche

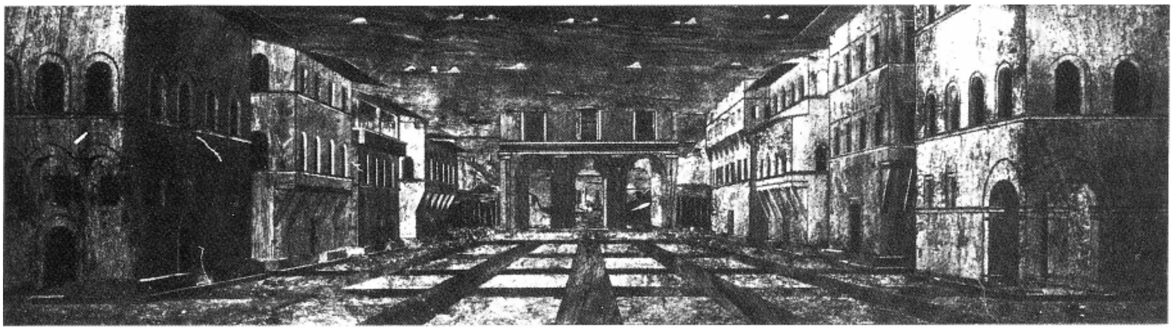


Fig. 3 Anonymous, *Ideal City*, Florence, Palazzo Pitti

Fig. 4 Pietro del Donzello (?), *Ideal City*, Berlin, Bode Museum

Fig. 5 Architect linked with Sangallo and the workshop of Cosimo Rosselli, *Ideal City*, Baltimore, Walters Art Gallery

### 3. Looking from the Side

What happens if we look at a perspectival painting such as Piero della Francesca's *Flagellation* (fig. 1) from a point off to the left side of the painting? From this viewpoint the three foreground figures standing on the right fade into the back-ground. The geometrical pattern of the pavement suddenly comes into focus. We can see clearly an eight pronged star-like figure, in front of Christ who stands in the centre of a black circle. The red tiles of the pavement in the foreground which had appeared rectangular are now recognized as squares. The pavement as a whole combines to give an unexpected spatial effect which focusses our attention on the flagellation scene.

This could of course be a freak case. We shall show, however, that it is the rule. The view of an *Ideal City* now in Urbino (fig. 2) offers a fine testcase. A classic example of a painting in linear perspective, it has often been attributed to Piero della Francesca.[19] Seen from directly in front everything appears to be correct, although the central building is somewhat squat. Viewed from a position off to the left side, or the right, the scene changes drastically. The central building becomes considerably taller, sleeker and rounder. The hitherto rectangular patterns of the pavement are now recognized as squares which evoke a sense of depth, enhanced in turn by the rows of buildings on either side.

A similar city view in intarsia is in the Pitti Palace in Florence (fig. 3). Viewed from directly in front, the pavement is covered with broad rectangles. From a viewpoint off to the side - either right or left - these are transformed into a series of foreshortened squares. What had formerly appeared a wider piazza now narrows into a regular street the pavement of which combines with the flanking houses to produce an unexpected sense of depth.

The related Berlin panel (fig. 4) is no less interesting in this respect. Viewed from off to the left, the dozen black foreground rectangles appear as twelve squares and the flattened lozenges they enclose are recognized as diamond shapes, or squares tilted at 45° relation to the horizon. What had been a rectangled pavement leading to the port now appears as a broad avenue of squares, which again combines with adjacent buildings to create a sense of depth. The four foreground columns function spatially, while the ceiling they support, which had seemed a strange play of light and shade, comes into focus as a surface with inverted pyramids reminiscent of the facade of the Palazzo dei Diamanti in Ferrara.

In the Baltimore panel (fig. 5) these changes are no less surprising. Viewed from the left or from the right side the dark rectangle surrounding the central fountain shrinks into a square. The piazza in the middle-ground shrinks into a boulevard culminating in perspectival stairs. The colisseum-like building to the left of the triumphal arch becomes round and the squat octangular building to the right of the arch appears considerably more elegant. The complete scene now evokes an impressive sense of depth.

Piero della Francesca's *Brera Altarpiece* (fig. 6) is a rather different painting. When viewed from off to the left side the vault and inverted scallop shell evoke a sense of depth. Something unexpected also happens: the suspended (ostrich)[20] egg trans-forms itself into a perfect sphere. This could seem far-fetched were it not that Piero himself gives careful instructions how to achieve this effect in his theoretical treatise *De prospectiva pingendi*.[21]

When Velazquez' *Rokeby Venus* (fig. 7) is viewed from directly in front it appears as if the artist were unaware of optical laws. The painted mirror is so positioned that it could not possibly reflect the face of Venus as it does. If, however, the painting is viewed from a point off to the right the orientation of the mirror aligns itself with the face of Venus such that the painting now appears correct.

Classic examples of linear perspective thus possess distortions when seen frontally which decrease when seen from viewpoints off to the side. The laws of linear perspective provide an explanation for this paradox. The inverse size-distance law of perspective applies only to objects positioned parallel to the picture plane. These objects diminish without distortion (fig. 8). When objects are not parallel to the picture plane each point on their surface is at a different distance from this plane with the result that they diminish with distortion. Regular shapes such as pavements, ceilings and walls positioned at right angles to the picture plane have two sides which are parallel to this plane while the other two are not.

A simple example helps make this clear. Take the case of a square ABCD (fig. 9) positioned on the ground directly behind a picture plane. The sides AB and CD of this square are parallel to the picture plane and therefore diminish without distortion (i.e. the original size CD diminishes to C, D, on the picture plane). The other two sides AD and BC are not parallel to the picture plane and therefore diminish with distortion (i.e. AD to AD, and BC to BC).

This distortion is also effected by the height of the vanishing point (fig. 9 ii-v). When the vanishing point is higher, the perspectival images of squares have a greater resemblance to the original shapes. However, such a higher vanishing point, if it be near the picture plane, leads to 1. a great discrepancy between the perspectival images of the first and second squares (cf. or and re in fig. 34) and 2 to a scene dominated by geometrical patterns of pavements and/or ceilings. To avoid these disadvantages artists usually choose a lower viewpoint where perspectival distortion is more coherent, although greater (fig. 9 i-iv, cf. figs. 1-5).


Fig. 6 Piero della Francesca, Brera Altarpiece. Milan, Brera

Fig. 7 Velazquez, Rokeby Venus. London, National Gallery

When a painting with such distortion is viewed from the side, lines parallel to the picture plane (AB and C, D, in fig. 9.1) in turn undergo a greater diminution than the orthogonals (AD, and BC). This serves as a corrective for the distortion such that the original shapes can now be recognized more clearly. This is why standard perspectival paintings are profitably viewed from the side.

Piero della Francesca must have known this. The dramatic effect of his egg in the *Brera Altarpiece* depends on its being seen from directly in front as well as from the side. In other words he must have intended that his perspectival paintings be seen from multiple points of view. (So too must Holbein when he painted *The Ambassadors* fifty years later). Piero's contemporaries responsible for the Baltimore, Berlin and Urbino panels were there-fore almost certainly aware that these works gained in apparent depth when not seen from a position directly in front of the central vanishing point.

Leonardo da Vinci is often criticized for having constructed his *Last Supper* (1495-1497) in such a way that its central vanishing point could only be seen directly if one stood on a ladder or related platform. The evidence of Piero and his contemporaries suggests, however, that Leonardo chose his inaccessible vanishing point consciously, thus inviting viewers to explore the depth enhancing effects of viewpoints off centre. In which case the *Last Supper* can be seen as a demonstration how a work in linear perspective can be viewed by a number of persons at once from multiple viewpoints. Or as the modern psychologists would say: it is a demonstration of the constancies.

On manuscript A $40^V$ (1492) Leonardo distinguishes clearly between regular perspectival works which can be viewed by a number of persons at once and other (we would say anamorphic) pictures which have their viewpoint controlled by a peep-hole[22] (*buso*). In this context the *Last Supper* can be seen as a challenge to the assumption that works in perspective need to be seen from a single position opposite their central vanishing point.

## 4. Perspective and Anamorphosis

What then is the difference between a regular perspectival painting and an anamorphic one if both contain distortions and both are profitably viewed from the side? In the case of linear perspective an artist is usually intent on rendering an entire scene spatially. Here distortion of certain parts is an undesired byproduct.

The term „anamorphosis" is often used loosely to refer to distortion in general. It is more appropriately used to refer to those cases where distortion is a desired effect. In anamorphosis an artist usually limits himself to a single object and deliberately projects it (or changes its scale) so that it is deformed or even unrecognizable when viewed from directly in front. Sometimes such objects are integrated into an otherwise normal scene, as for example, with Piero's egg (fig. 6) or Holbein's skull. Frequently the deformed object dominates and is surrounded by regular perspectival landscapes, as in the case of Schrot's portrait of Edward VI (fig. 19).

## 5. Origins of Anamorphosis

What prompted artists from the latter 15th century onwards to distort images deliberately? At the outset necessity must have played a role. By the early 15`h century it had become fashionable to cover the walls of side chapels with frescoes. The closest a member of the congregation (E in fig. 10) could come to such a chapel was by kneeling in front of the altar (F in fig. 10) from which position he saw two walls in extremely foreshortened form (AB and CD in fig. 10).

It is likely that some of the first deliberate distortions were designed to compensate for such extreme viewpoints. Indeed it may be no coincidence that some frescoes in inverted perspective, such as those in Santa Croce in Florence, look suprisingly convincing when seen from such view-points off to the side. Van Thiel[23] has drawn attention to a sixteenth century version of this problem. Tintoretto's *Last Supper* in San Giorgio Maggiore in Venice looks distorted when seen from directly in front. When seen off side from a position directly in front of the altar, the painting appears correct.

Shadows are another factor that led artists to study distorted projections.[24] For instance Leonardo da Vinci explores what happens when shadows from a circular object are projected onto an oblique plane (fig. 11) or onto a combination of oblique and vertical planes (fig. 12).

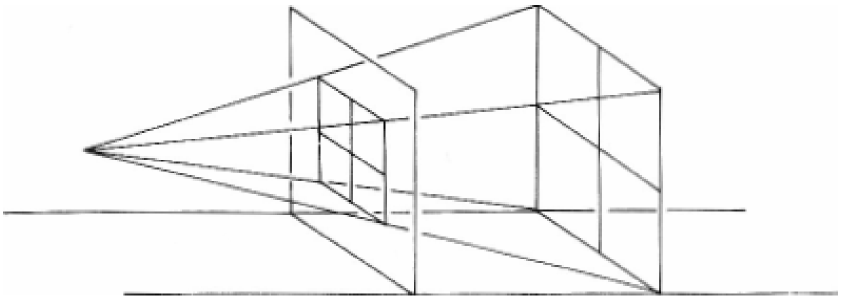


Fig. 8 Perspectival projection onto a parallel picture plane

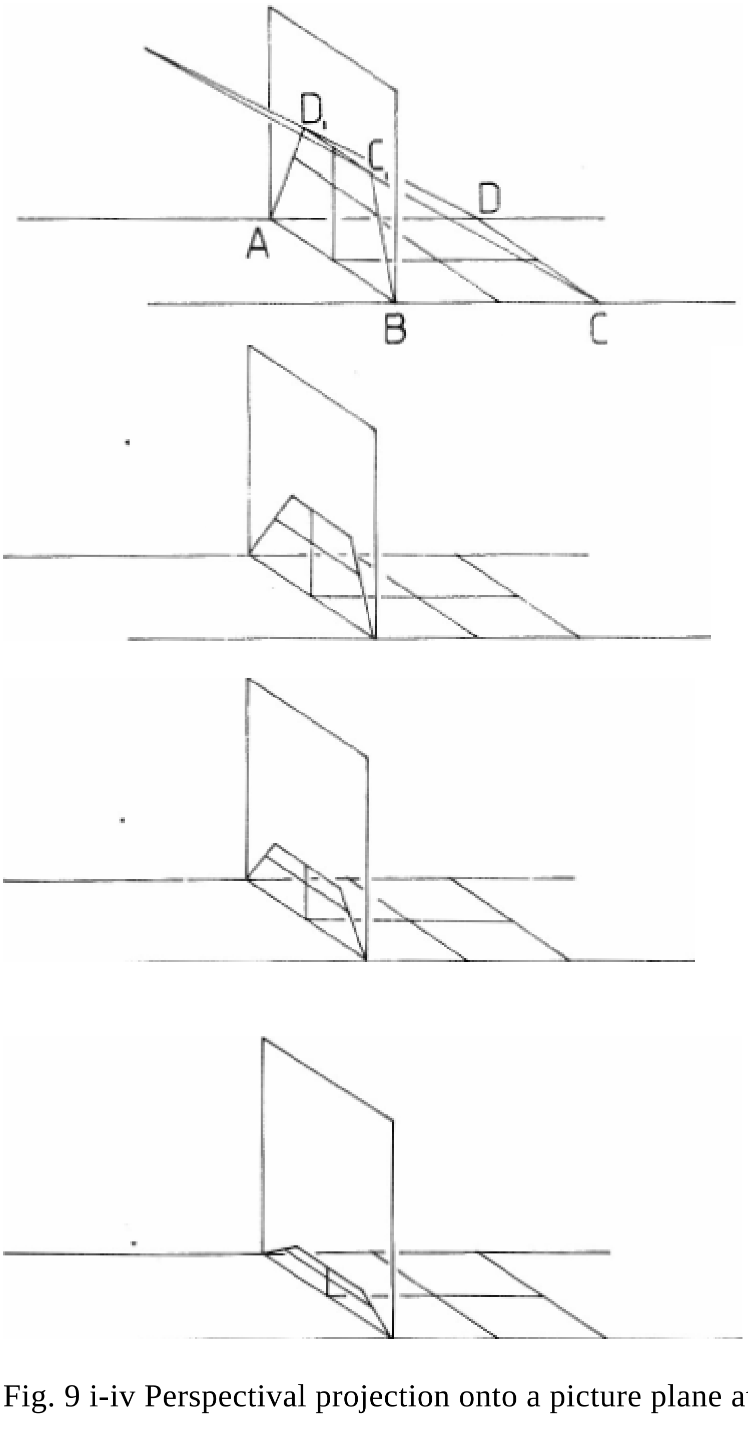



Fig. 9 i-iv Perspectival projection onto a picture plane at right angles.

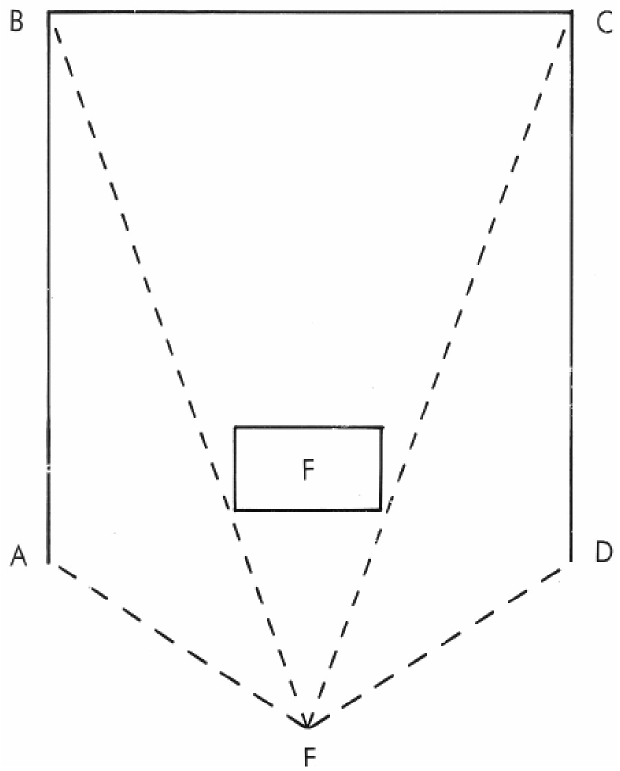


Fig. 10 Geometrical sketch of a viewer at E in front of an altar F, viewing from the side the walls AB and CD of a chapel.

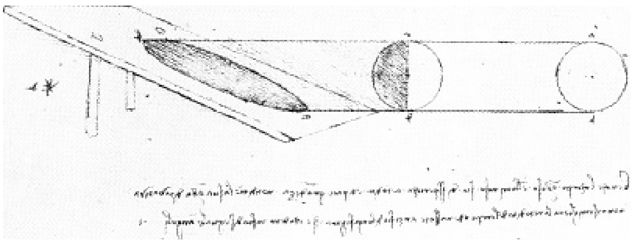


Fig. 11 Leonardo da Vinci, *Manuscript C*, fol. $18^{r}$, Paris. Institut de France

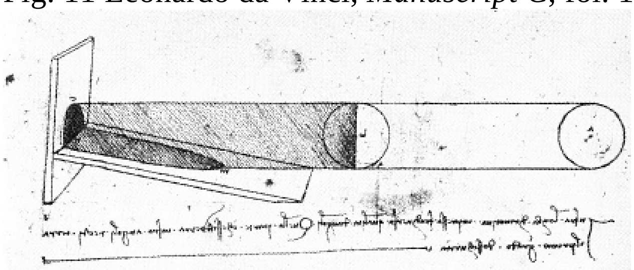


Fig. 12 Leonardo da Vinci, *Manuscript C*, fol. $11^{v}$, Paris. Institut de France

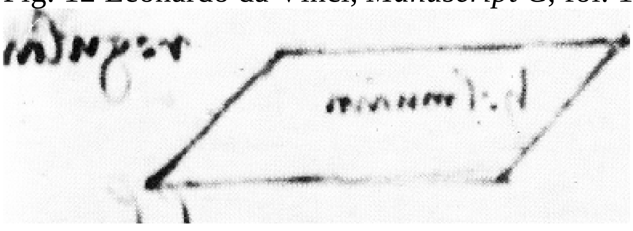


Fig. 13 Leonardo, *Madrid 11* $139^{r}$.

Geometry provides another context for interest in anamorphosis. For example, Campanus of Novara's Arabic based terms for a parallelogram (*hel muain* oder *el main*) become a starting point for Leonardo da Vinci's *el main* parallelogram (fig. 13) and his figure relating to anamorphic distortion (fig. 14).

Euclid in his *Elements* had described (I: 36) how any two parallelograms between parallel lines and with equal bases are equal in area (i.e. ABCD = BOE in fig. 15). Leonardo studies this proposition (fig. 16) and explores its implications for circular forms (figs. 17-18). In his approach geometry and anamorphic transformations of a circle go hand in hand.

Euclid had concentrated on geometrical laws of two dimensional shapes. His interest in three-dimensional shapes was limited to the five Platonic solids. Piero della Francesca, and Leonardo more so, are concerned with controlling and predicting transformations of both two and three-dimensional shapes. Thus Piero's famous egg (fig. 6) is both a mathematical demonstration of control and a visual demonstration of geometrical play in the profound sense (*ludo geometrico*).

The repertoire of shapes, and the complexity of the projection planes which were used, evolved slowly. Piero della Francesca describes three shapes in his *De prospectiva pingendi* (c. 1480): a sphere projected as an egg, a goblet and a ring.[25] Leonardo adds a child's face (CA 35va). Schrot draws a boy's face: Edward VI (fig. 19). Egnazio Danti, in his edition of Vignola's *Due regole della prospettiva pratica* (Rome, 1583) publishes a drawing of a man's face which he claims is the invention of Tommaso Laureti (fig. 20).[26] Marolois, in his treatise (1614) replaces the face with a dog[27] (fig. 21).

**6. Anamorphosis and Alternatives to Linear Perspective**

In the exploration of anamorphosis in cases involving complex projection planes, practical incentives again played a role. For instance, Leonardo da Vinci describes the problem of drawing pictures in the corner of a room.[28] Marolois (1614) is the first to publish this problem (figs. 23-24), which is later adapted for a peep-show in Copenhagen (figs. 25-26).[29] Ceilings posed related problems with respect to semi-cylindrical surfaces. Leonardo discusses two solutions in his *Treatise of Painting*.[30] Abraham Bosse is one of the first to publish at length on this theme.[31] A fashion also developed of projecting images onto complex surfaces such as the interiors and exteriors of square and round versions of the visual pyramid (figs. 27-28).

7**. Anamorphosis, Mirrors and Special Devices**

This fashion extended to the use of mirrors. An early 16`*h* century invention, attributed to Leonardo, involved an image projected onto triangular bars, requiring a plane mirror to be seen properly.[32] Egnazio Danti is the first to publish this device.[33] *33* In the 1630's the calculation of anamorphosis in connection with conical and cylindrical mirrors becomes a mathematical game for thinkers such as Vaulezard.[34] Playing with the practical results soon becomes the fashion in Paris society.[35] Leupold (1739) claims to be the first to invent mechanical devices which plot these projections automatically[36] (figs. 22, 29). While pragmatism and fashion are important for the development of these complex projection methods, surveying practice and optical theory also play a significant role.

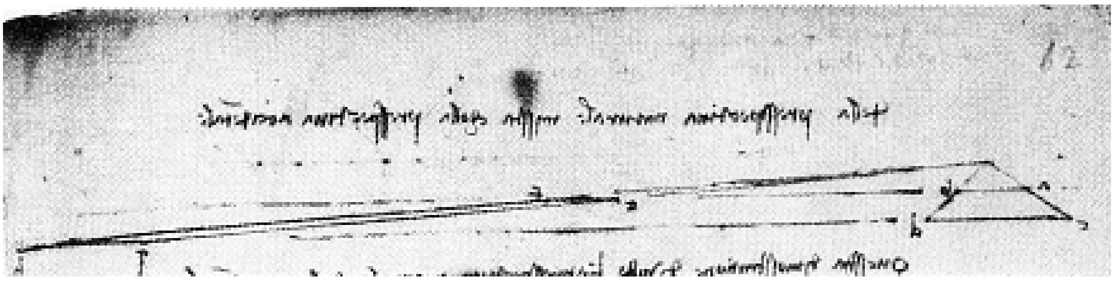


Fig. 14 Leonardo, *Codex Arundel*, 62v, London, British Library

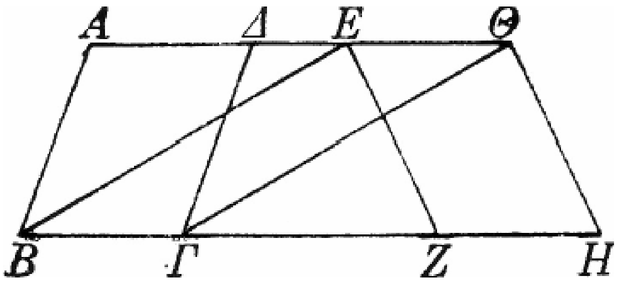


Fig. 15 Euclid, *Elementa* I:36


Fig. 16 Leonardo, *Madrid 11* 115v

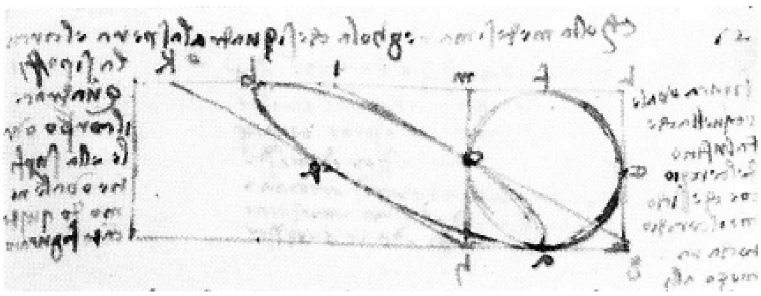


Fig. 17 Leonardo, *Madrid II* 72r


Fig. 18 Leonardo, *K*52 [3]v, Paris. Institut de France

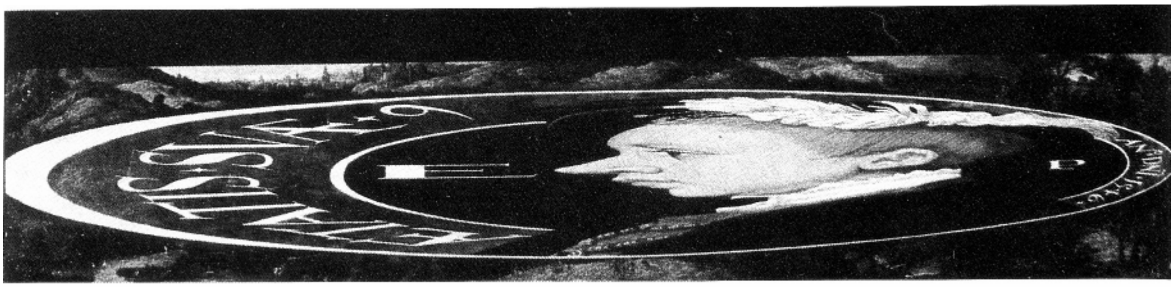


Fig. 19 W. Schrot, *Portrait of Edward VI*, London, National Portrait Gallery, 1546

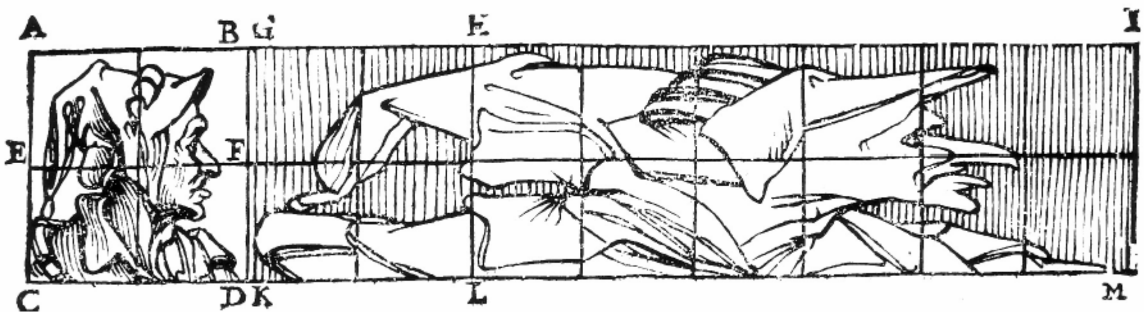



Fig. 20 Egnazio Danti, ed. Anamorphic Drawing of a Man in *Le due regole della prospettiva pratica*, Rome: Zanetti 1583

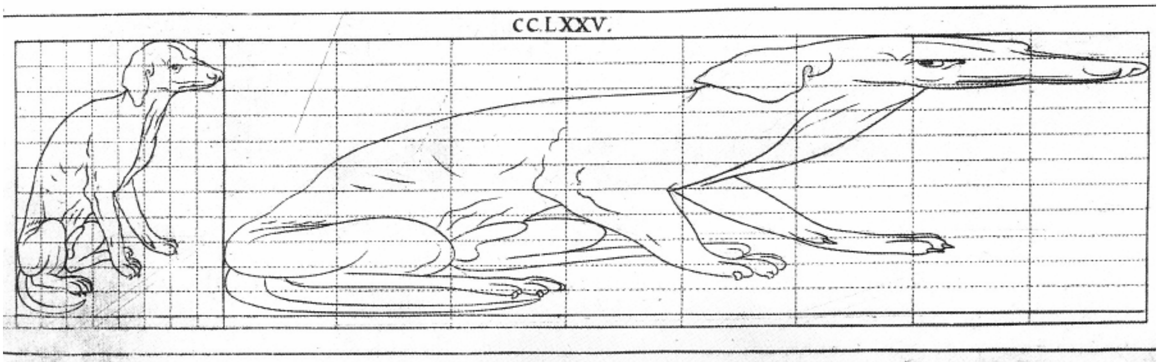



Fig. 21 Samuel Marolois, „Perspective"; Ibid, *Opera mathematica*, Hagae: Hondius, 1614, Fig. CCLXXV

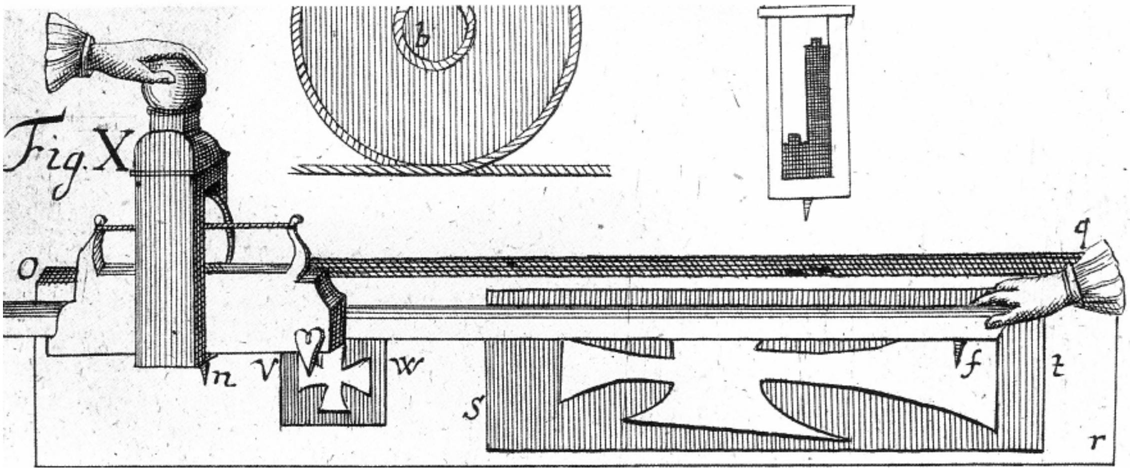



Fig. 22 Jacob Leupold, Th*eatrum mechanicarum. Supplementum* Leipzig: Authore, 1739, Tab. XVIII, Fig. X

## 8. Surveying and Perspective

From the outset there were close links between surveying and linear perspective. In Germany, it was assumed that the term „perspective" was synonymous with the „art of measurement".[37] Whence it is no coincidence that Dürer discusses perspective in his *Underweysung der Messung* (1525). At least since the time of Hero of Alexandria round tablets had been used for surveying purposes.[38] Alberti in the 15*th* century described the use of such a round tablet in his *Descriptio urbis Romae*. Gerard de Jode (1594)[39] describes the use of visual angles in surveying to record a scene or landscape.

In 1508 Leonardo explores the systematic diminution of visual angles in surveying under the heading of perspective.[40] By 1583, Danti, in his edition of Vignola's treatise on perspective, notes how these visual angles diminish proportionally with distance.[41] An interplay between surveying and perspective thus helps to account for the origins of the proportional compass (cf. Galileo). Whence it is not surprising that several key figures at the turn of the seventeenth century, e.g. Guidobaldo del Monte, Jobst Bürgi and Levinus Hulsius, are writing both on perspective and proportional compasses. Nor is it surprising that such compasses should be used by the mathematician Vaulezard[42] for both perspectival and anamorphic drawings. Indeed a tradition emerges in which proportional compasses are discussed in connection with perspective (e.g. Lambert[43]). In other words, the interplay of surveying and perspective led, though the proportional compass, to both linear perspectival and anamorphic projection methods.

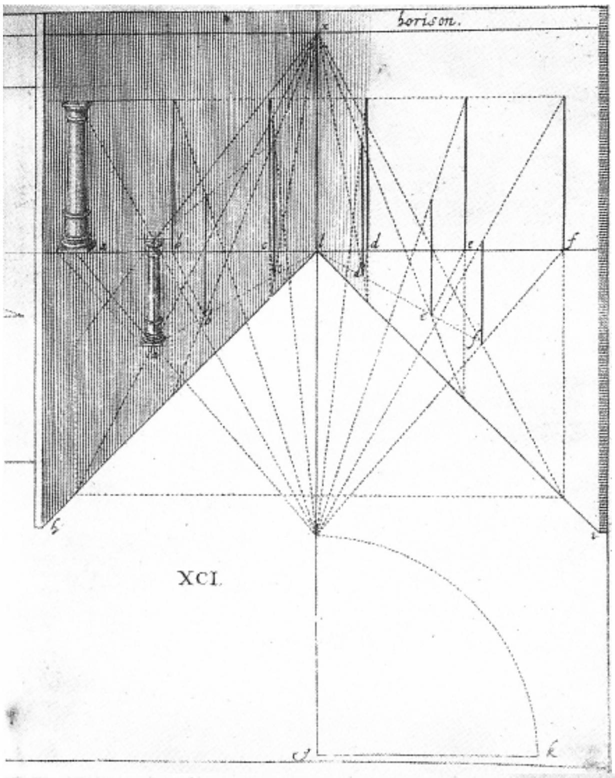
## 9. Surveying, Perspective and Cylindrical Projections

The interplay between surveying and perspective also led to such circular tablets becoming linked with cylindrical projection methods. An example is Baldassare Lanci's instrument, dated 1567, now in the Museum for the History of Science in Florence (fig. 30). It is partly intended to function as a traditional surveying instrument, as confirmed by its compass, two rods for measuring angles and the text inscribed around its circumference: "Agents and mathematicians can find use in this instrument both for geography and topography since it is especially easy with this kind of instrument to take any site whatever and have a perfect notation and make a picture of every distance."[44]

In addition the instrument has a series of small holes around its circumference designed for mounting a hemi-cylindrical projection plane which has since been lost (cf. fig. 31). In the centre of the circular tablet is a swivel device with a sighting tube and needle-like marker. This combination permits one to look through the sighting tube and trace its position directly onto the hemi-cylindrical projection plane.

Daniele Barbaro saw this instrument - or one very much like it - at Baldassare Lanci's home and gives a published account of its use for drawing purposes in his treatise on perspective[45] (1569). Egnazio Danti, in his edition of Vignola's treatise, provides an illustration of the instrument as it once was (fig. 31), a detailed account of its use, plus a critique of its shortcomings.[46]

Danti mentions that Lanci's device "was used by many others". An unpublished drawing (fig. 32) by Johann Kretzmayer (Krezmaier), in a manuscript with extracts of treatises on



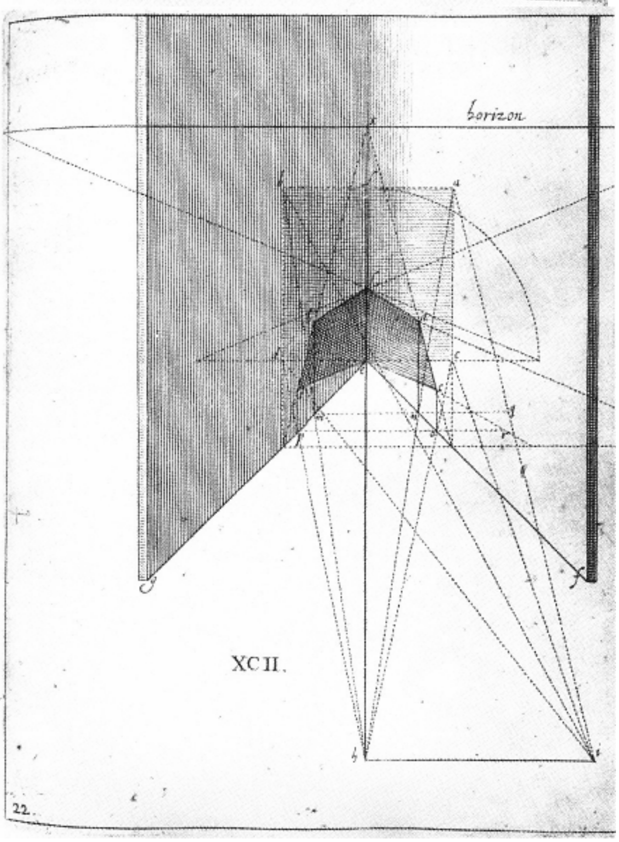




Fig. 23-24 Samuel Marolois, Perspective: Ibid, Opera mathematica. Hagae: Hondius, 1614, Figs. XCI-XCII

perspective, offers evidence that this instrument was used North of the Alps.[47] In Marolois' treatise the hemi-cylindrical projection plane recurs on its own (fig. 35). Hence the surveying tradition provided incentives for the development of both linear and cylindrical projection methods.

## 10. Optics versus Perspective

Another incentive for cylindrical projection methods came from the optical tradition. Euclid in theorem 10 of his *Optics* had stated: “It is obvious also that planes situated higher up will appear concave"[48]. Witelo, in his twelfth century optical thesaurus, developed this statement (IV:37): “And it follows from this that with the eye raised, many a surface of a plane far from the eye will appear concave for the forms of such points tend to the eye by means of a circumference around the centre of sight because of the equality of the visual power."[49] Hence a series of points equidistant from the eye defines a concave surface. Witelo discusses it but does not draw it.

Leonardo da Vinci takes up this problem in the mid 1480's (fig. 33) and discusses it in some detail in 1492. He draws (fig. 34) an eye at m looking at the objects ov and vx. The first object ov appears on the vertical picture plane as or; the second, radically smaller as re. In order to compensate for this radical change in projected size, Leonardo suggests a peep-hole (*buso*).[50]

Alternatively he recommends a curved (cylindrical) plane op which is equidistant from the eye m.[51] On this plane the objects ov and vx are projected as ot and ts. He explores this concept of a curved plane, equidistant from the eye, in several later sketches. His diagrams suggest that he intended the curved plane to be cylindrical rather than spherical. There is no firm evidence that he ever constructed either a cylindrical or a spherical plane for purposes of representation.

In any case these writings confirm how Leonardo's interest in optical theory led him to explore curved projection methods as alternatives to linear perspective. He makes it clear that these alternatives only come into question under extreme conditions and recommends that these extreme conditions be avoided in order that linear perspective can be used.[52] However, he makes no attempt to reconcile his theory of vision with his theory of representation, as we might be tempted to assume he should.

From a modern standpoint it is clear that the perspectival law of planes applies whether objects are higher up or off to the side. Hence in figure 36, the objects O, P, and Q, are projected equally as O, P and Q respectively because they are all in one object plane equidistant from the picture plane. Prior to Desargues (1636), however, theoreticians treated the perspectival representation of scenes as one problem, the use of optical adjustments with objects higher up as a second and objects off to the side as a third and quite distinct problem.

With respect to objects higher up, Euclidean optical theory offered a pragmatic solution: an object higher up subtends a smaller angle at the eye. A higher object must therefore be made correspondingly larger if it is to subtend the same visual angle at the eye as a lower object. This reasoning appears to have evolved by the fourth century BC in Greece.[53] In the Renaissance it is implicitly introduced by Leonardo in the 1490's in a number of sketches without text.[54] Dürer publishes the problem.[55]

Thereafter this method of optical adjustments becomes a dominant solution in perspectival treatises.[56] It appears also in surveying treatises where the required height of objects and the angles they subtend become problems for calculation (figs. 41-43). With authors such as Marolois, who write on both topics, these surveying aspects are integrated within his treatise on perspective (figs. 38-39).

In painting practice a curious compromise results. Artists tend to paint landscapes, scenes and buildings in accordance with linear perspective and then paint individual objects within this context in accordance with optical adjustments methods. Hence the figures on a building in a landscape by Claude are, for instance, radically smaller than those on the ground although they are both in a same plane (fig. 45 cf. Giotto's pre-perspectival solution, fig. 44).

In the period 1640-1660 Girard Desargues and Abraham Bosse repeatedly attack this method of optical adjustments on the grounds that it contradicts the laws of linear perspective." If, for instance (as in our fig. 37), a man stands at EF and looks at the objects AB, AH, HI, IK, KL, LM and MN, the angle that they subtend at the eye decreases, but their perspectival size on the interposed plane CG remains constant (i.e. AB : A, B = NM : N, M).

This geometrical paradox led Bosse to insist that artists should draw what is there and not what they see.[57] This claim met with disfavour in the Academy and played a role in his being dismissed as professor of perspective.[58] The optical adjustments method, sanctioned by the Academy, thus held its ground and remained a standard solution throughout the eighteenth century as is illustrated by an example from Werner's treatise on perspective (fig. 40).

Meanwhile theoreticians continued to treat the problems of objects off to the side as a fully distinct problem. Piero della Francesca's *De prospectiva pingendi* (c. 1480) is the first treatise in which the problem of columns off to the side is discussed[59] in some detail (fig. 46). Leonardo devotes several pages to the problem and describes concrete experiments.[60] Luca Pacioli devises a mathematical formula for calculating the diminution of objects off to the side.[61] In the sixteenth century the problem is discussed by authors such as Cousin.[62] Marolois (1614) outlines a complex method for calculating diminution off to the side based on his surveying experience (fig. 47). Pirenne (1970) describes a nineteenth century solution and analyses why the perspectival rendering of columns and spherical objects remains problematic to this day.[63]

Hence although linear perspective theoretically introduces the possibility of a uniform treatment of space, painting practice continues to be much more complex. Compromises are made in the case of objects below, above and off to the side. Sometimes these compromises are made in accordance with a theory of vision and sometimes not. There is no necessary connection between theory of representation, practice of representation, theory of vision and actual visual experience.

## 11. Conclusions

Erwin Panofsky[64], influenced by the Neo-Kantians, assumed that there must be necessary connections between the world view of a given culture, its theory of vision and its theory of projection and representation. He believed that a finite world view had con-strained the Greeks to develop both a curvilinear theory of vision and practice of representation. He was not clear whether this curvilinear method should be spherical or cylindrical. He assumed, in any case, that it would result in the fishbone-like alignment of vanishing points along a central axis (fig. 48). Neither a spherical nor a cylindrical projection plane placed in front of a model room with squares will produce such an effect (figs. 49-51). Panofsky also believed that the development of an infinite world view had led Brunelleschi to discover linear perspective which introduced a concept of uniform space in painting.

This new method of representation theoretically precluded alternatives involving cylindrical, curvilinear or angular projection methods. Moreover, linear perspective theoretically required a revision in the Euclidean theory of vision in order that apparent size be based on projected size (on planes) rather than on visual angles.

The historical evidence points to a considerably more complex situation. With respect to Antiquity there is evidence of at least three methods of representation: 1. optical adjustments methods, 2. what Panofsky termed fish-bone perspective and 3. those Pompeian examples that approximate the effects of linear perspective and which John White[65] believes are examples of linear perspective.

If one accepts John White's claim it makes nonsense of Panofsky's framework. For linear perspective would then be a manifestation of both a finite an infinite world view. If one rejects White's claim then the Pompeian examples are a variant of the fish-bone or axial perspective. This method does not link with a cylindrical or spherical projection method. The method appears instead to have been empirical and to have been used both by the Romans and artists in the fourteenth and fifteenth centuries.

With respect of optical adjustments methods we have evidence of their being used in sculpture, stage scenery and wall painting in Antiquity. Although these ancient methods were based on optical theory we have no concrete evidence of their having been linked with any systematic cylindrical or spherical projection method. We have evidence, however, that optical adjustments methods remained in use throughout the Renaissance, indeed throughout the seventeenth and eighteenth centuries (e.g. figs. 38-40) and that they were sometimes linked with cylindrical projection methods during this period.
In short, Panofsky assumed that a given culture should have one world view, one theory of vision, one method of projection and one method of representation and that all these should be linked. In Antiquity this was certainly not the case, and such methods as there were recurred in the Renaissance.

With respect to the Renaissance, Brunelleschi's experiments introduced neither a sudden revolution in spatial theory nor in painting practice. Change was gradual. As late as the 1470's Piero was denying the inverse size distance law of linear perspective. Leonardo in the 1490's was the first to state it and demonstrate it experimentally. Even so the full implications of perspective's law of planes (cf. fig. 36) were not understood by Leonardo or even his sixteenth century followers.

It was not until the seventeenth century that individuals such as Desargues and Bosse became aware of discrepancies between the law of planes of linear perspective and their Euclidean theory of vision based on visual angles (contrast figs. 37 and 40). They did not, however, adjust their theory of vision. Instead they insisted that one must draw „what is there" and not „what one sees". Or to put it in modern terms, they were saying that linear perspective is a question of geometry (objective projections) independent of visual theory or psychology (subjective impressions).

Thus contrary to Panofsky's assumptions, the development of linear perspective introduces no new harmony, but rather a separation between theory of vision and theory of representation, a separation which in turn helps to explain one of the paradoxes of modern culture. On the one hand we are told that the scientific revolution demanded a denial of common sense sight.[66] On the other hand we are told that this scientific revolution depended on a new committment to observation.[67]

Linear perspective achieved both of these requirements through the distinction that it provoked: a distinction between the geometry and psychology of a situation. Thereby Euclidean theories of vision could be denied or rather separated. Gombrich calls this the „how" of vision.[68] At the same time the geometry or visual information of a situation could be asserted. Gombrich terms this the „what" of vision[69], and it is this that scientists mean when they stress the importance of observation.

According to Panofsky's explanation the development of linear perspective should have replaced alternatives such as cylindrical or spherical projection methods. Anamorphosis ought actually never to have taken place. The evidence of the texts shows, however, that anamorphosis cannot be dismissed as a marginal problem. Those who explore its principles do so in their treatises on linear perspective: e.g. Piero della Francesca, Leonardo da Vinci, Dürer, Danti, Vaulezard, Desargues and Bosse.

Moreover, most of these thinkers have interests in cylindrical, spherical and conic projections that go far beyond their artistic concerns. Leonardo da Vinci and Dürer are concerned with cartographic projections. Egnazio Danti has interest also in planisphere, astrolabe and sundial projections. Vaulezard and Desargues are concerned with the projection principles of conic sections. They approach these apparently disparate fields as examples of projection principles in general (they would have simply have said geometry). This is why they see less opposition than do we between linear perspective and anamorphosis. Both are three dimensional expressions of geometrical play („ludo geometrico"): a profound game of systematically mastering all changes of form.

We need a new approach to linear perspective that relates it to the more general development of projection methods, and yet something more than that provided by nineteenth century historians of mathematics and science who were searching for the origins of descriptive geometry. It is not just a question of how the laws were discovered. Needed is a history of how these laws became recognized as being independent from Euclidean theories of vision; a history of how laws of projection and theories of vision were and were not applied to the „visual" arts.
Such a history will need to emphasize that the Renaissance fascination with projection, which included linear perspective and anamorphosis, did not straight-jacket artists into

becoming mechanical copiers of nature. Indeed it led gradually to a distinction between technical drawing and what we now term „fine art".

Paradoxically, the projection principles of perspective had an effect that was the reverse of taking away the artists' freedom. Perspective introduced new possibilities of playing with forms, not only in the natural world but equally in the mind.

The most famous examples of Renaissance perspective all involve idealized places, new fictions of the mind, rather than photographic-like copies of street scenes of actual places: e.g. the Baltimore, Berlin and Urbino panels (figs. 2-5), Piero's „*Flagellation*" (fig. 1), Leonardo's *Last Supper*, or the *Annunciation* scenes of Masolino, Domenico Veneziano, Fra Angelico, Fra Filippo Lippi, Vincenzo Foppa, Carlo Crivelli or Raphael's *School of Athens*.

The Renaissance masters knew that they were not bound to the monotony of geometrical lines imposed by regular man-made objects. Leonardo explored such lines in his Uffizi study of the *Adoration*, but left them aside in his later version. After 1497 he abandoned man made objects altogether. Hence the *Mona Lisa*, the *Virgin and St. Anne*, the *Bacchus* and *St. John* have no constraining vanishing points. His successors followed suit.

During the Renaissance there evolved links between theories of vision, projection and representation. These links led some thinkers to recognize that here were clear distinctions to be made between subjective theories of vision and objective methods of projection. In this matter, lesser thinkers were less clear, whence the historical realities are more complex than Panofsky's assumptions allowed him to see.

Neo-Kantians may be disappointed, but they ought not to be. Such complexities of history confirm that every discovery of new laws and rules, inspires new exceptions thereto. That is why the discovery of laws of perspective and anamorphosis is a chapter in the discovery of human freedom.

**Addendum**

The original article provided exclusively Black/White images. The web version has added colour images from Google images. A list of credits follows:

1. WebMuseum Paris, Piero della Francesca.
See: http://mexplaza.udg.mx/wm/paint/auth/piero/

2. Ettore Aldrovando.
See: http://web.math.fsu.edu/~ealdrov/

4. World Architectural History
See: http://www.brynmawr.edu/Acads/Cities/wld/04200/04200x3a.jpg

6.Web Gallery of Art
See: keptar.demasz.hu/arthp/html/ p/piero/francesc/altar/

7. ARC Art Renewal Centre
See:
http://www.artrenewal.org/images/artists/v/Velasquez/large/Velazquez_Venus_at_her_Mirror.jpg

44. Index of gallery/Giotto
See: http://www.mystudios.com/gallery/giotto/20-massacre-innocents.jpg

45 ARC Art Renewal Centre
See:
http://www.artrenewal.org/images/artists/l/Lorrain_Claude/large/Landscape_with_Aeneas_at_Delos_WGA.jpg

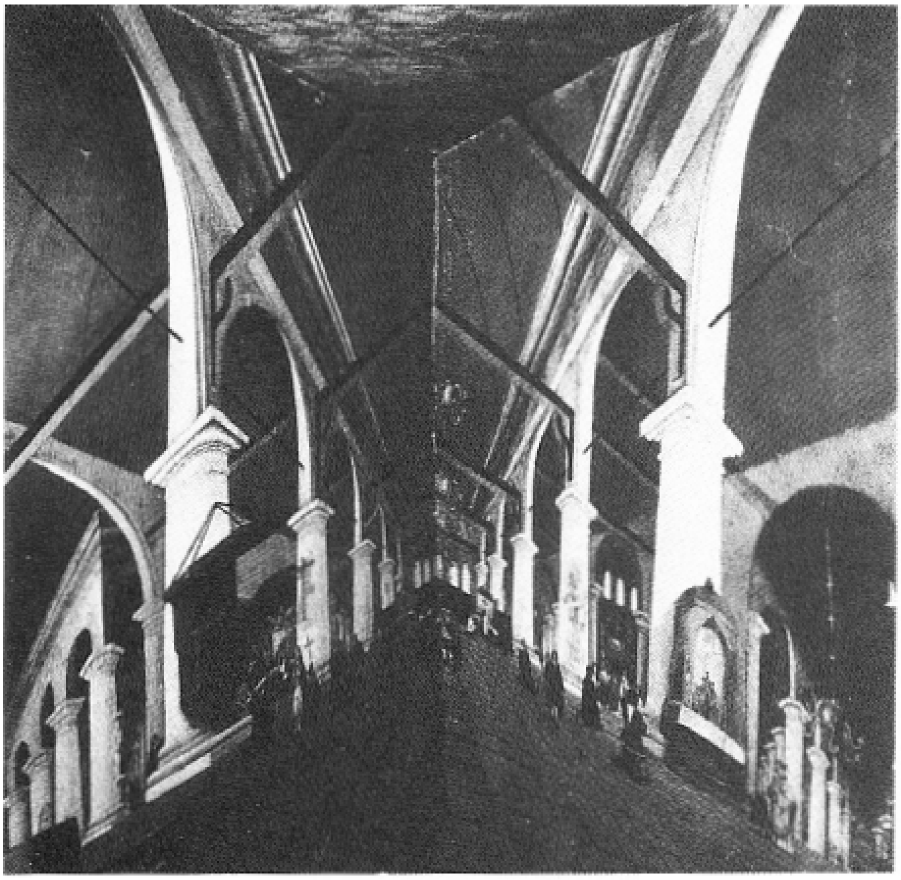


Fig. 25 Interior view of a perspectival peep-show. Copenhagen, Nationalmuseum.

Fig. 26 A view of the same through peep-hole

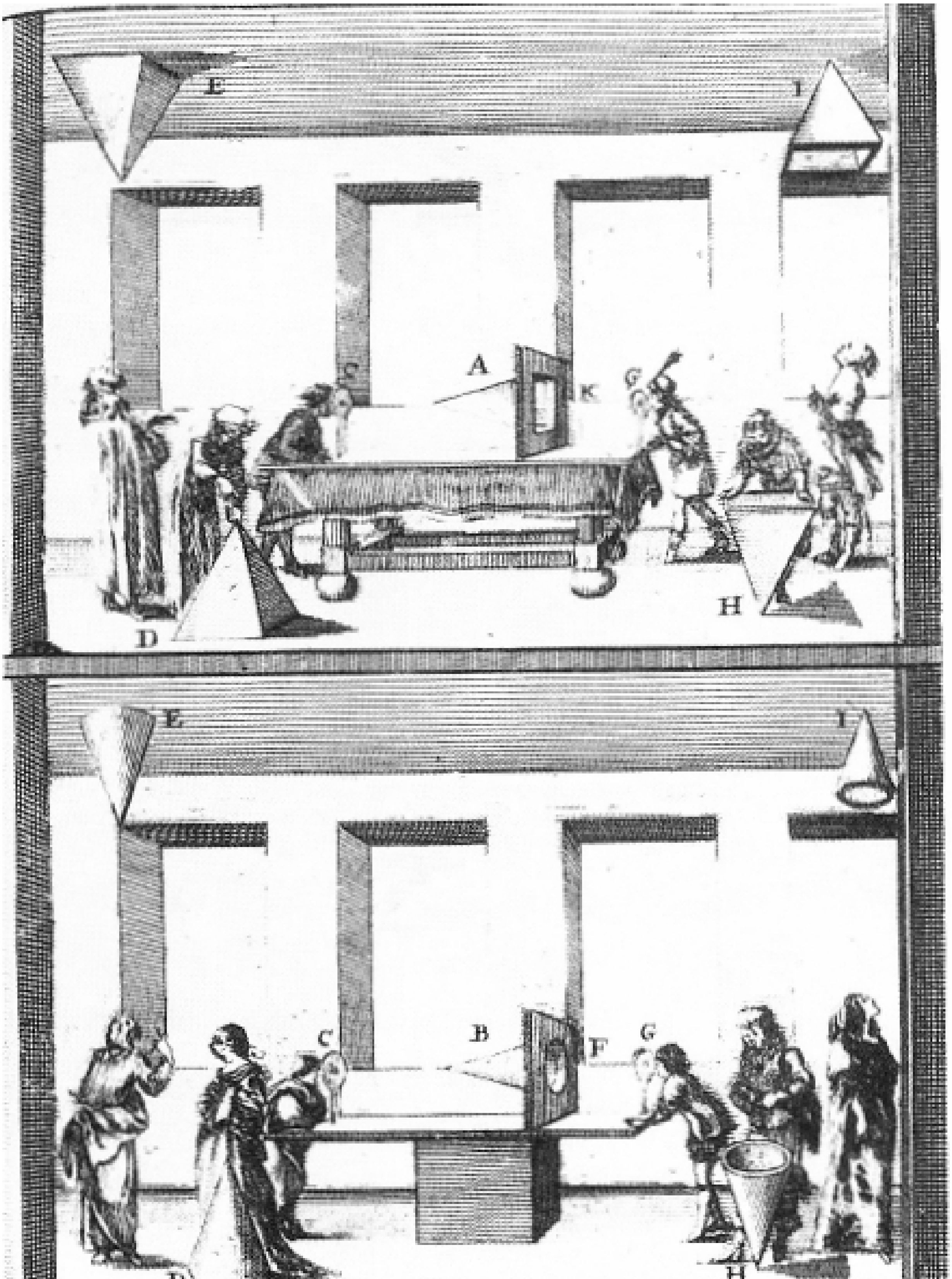



Fig. 27-28 Jean Dubreuil, *La perspective pratique troisiéme partie*. Paris: chez la veufve de F. L'Anglois, 1649

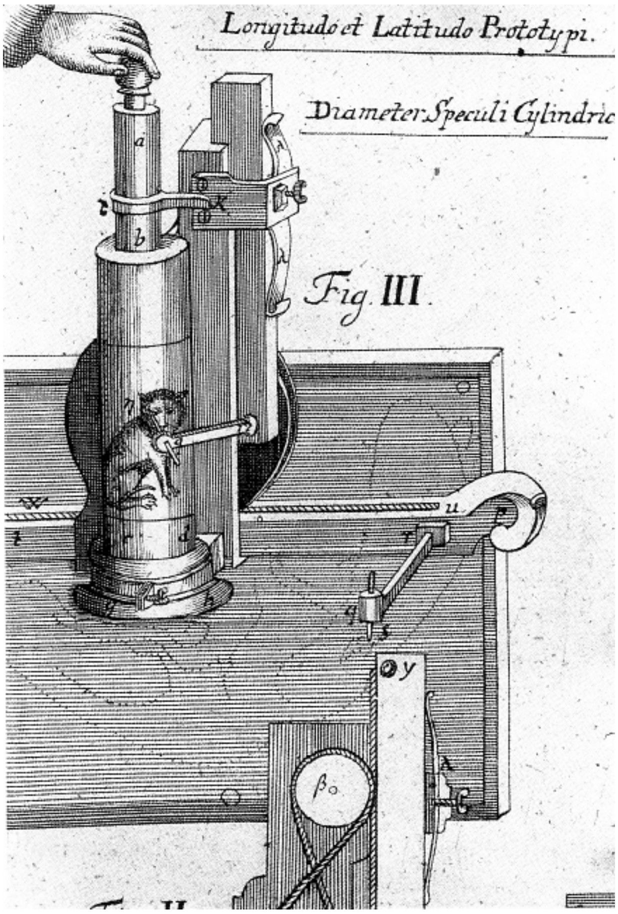



Fig. 29 Jacob Leupold, *Theatrum mechanicarum. Supplementum*. Leipzig: Authore, 1739, Tab. 16, Fig. 3

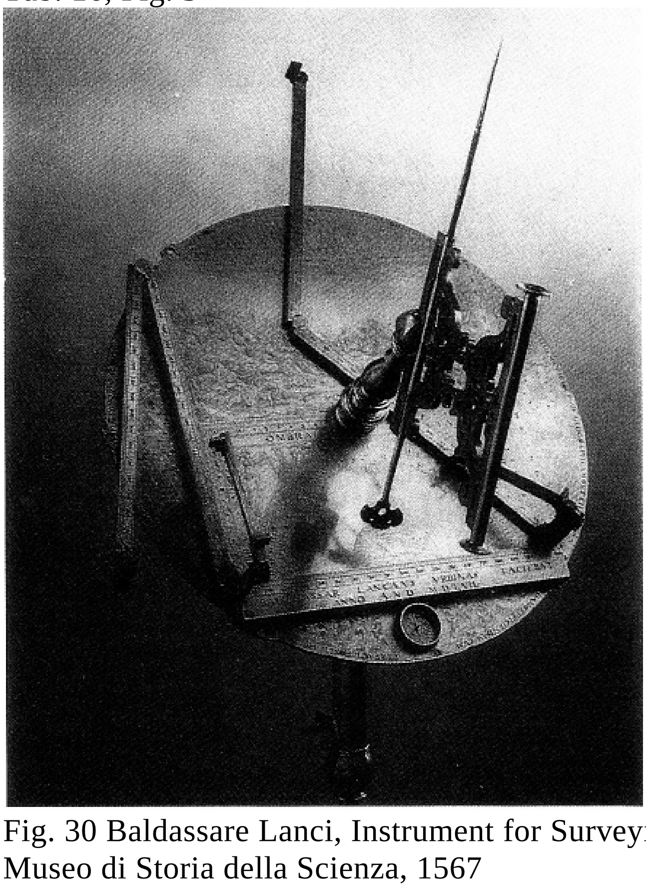
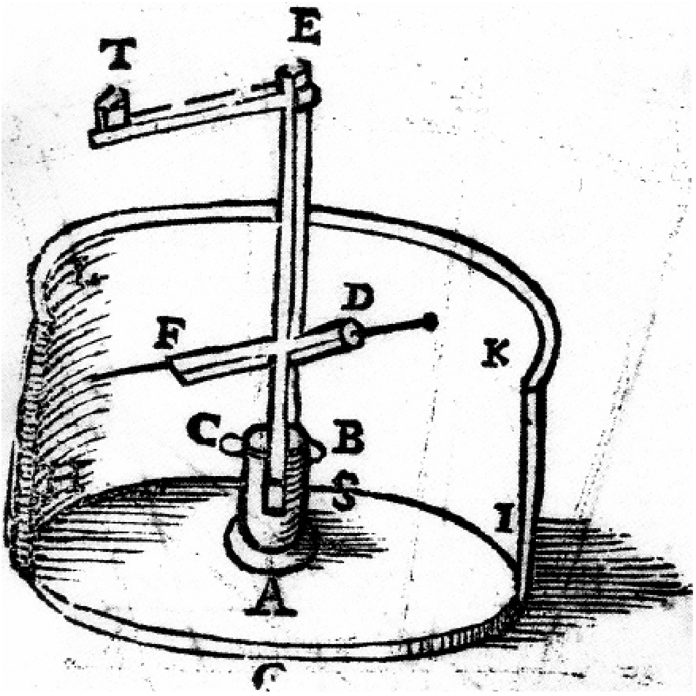


Fig. 30 Baldassare Lanci, Instrument for Surveying and Curvilinear Perspective, Florence, Museo di Storia della Scienza, 1567



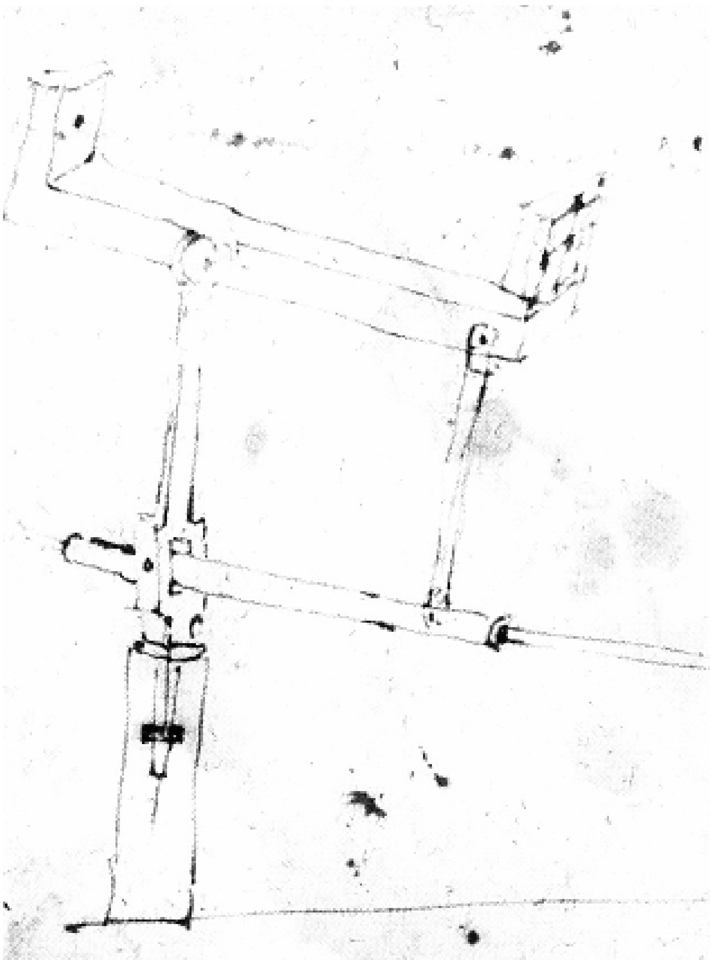



Fig. 31 Egnazio Danti, ed., *Le due regole della prospettiva pratica*, Rome: Zanetti, 1583, p. 61;  Fig. 32. Johann Kretzmayer, Instrument for Perspective, Wolfenbüttel, Herzog August Bibliothek, *Cod. Guelf. Extrav. 84 f.*, fol. 2o

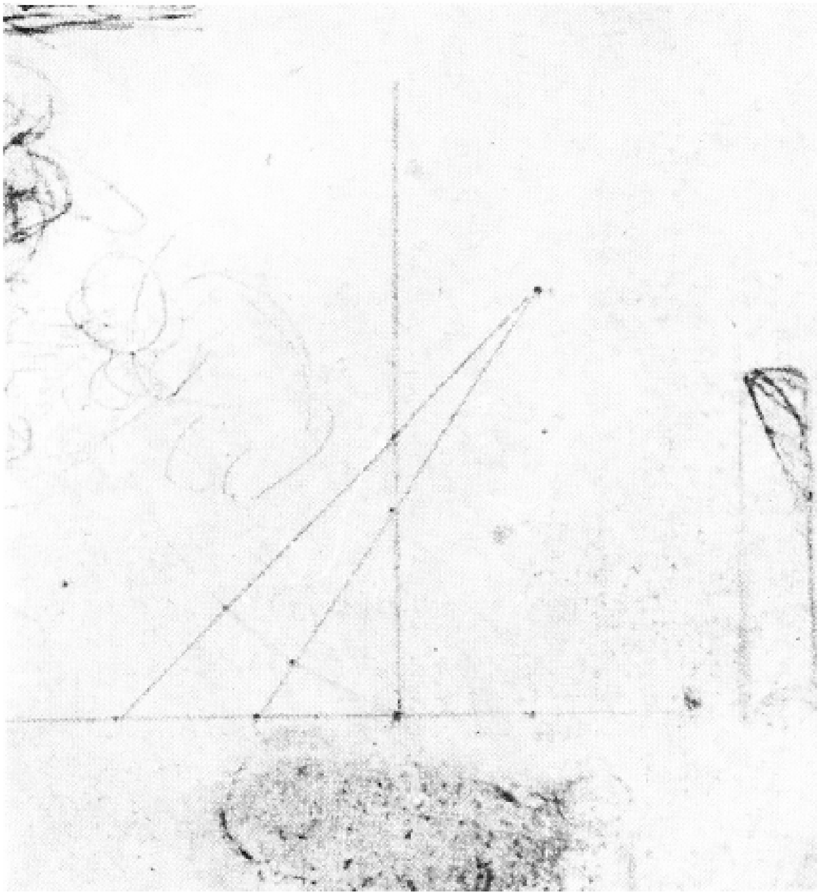


Fig. 33 Leonardo da Vinci, Detail from a *Study for the Annunciation*. New York, Metropolitan Museum, c. 1480

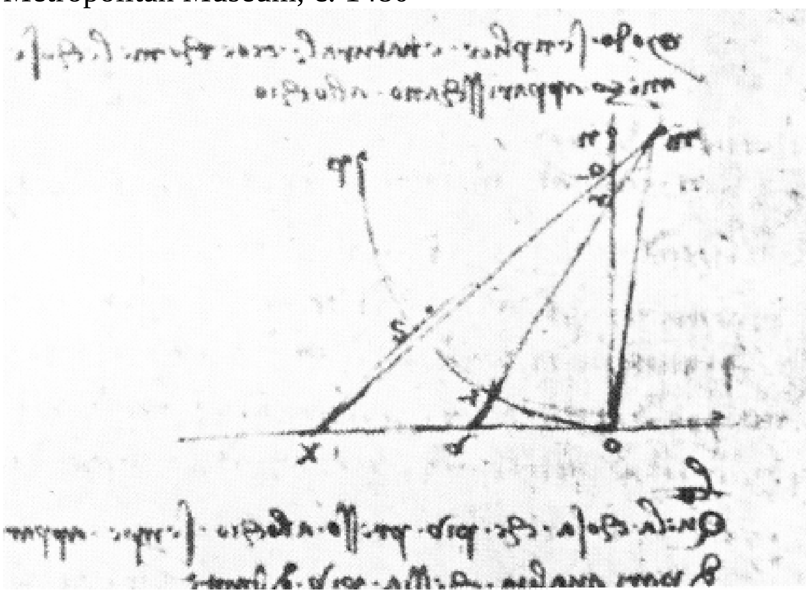


Fig. 34 Leonardo da Vinci, Manuscript A, fol 38'. Paris, Institut de France, 1492

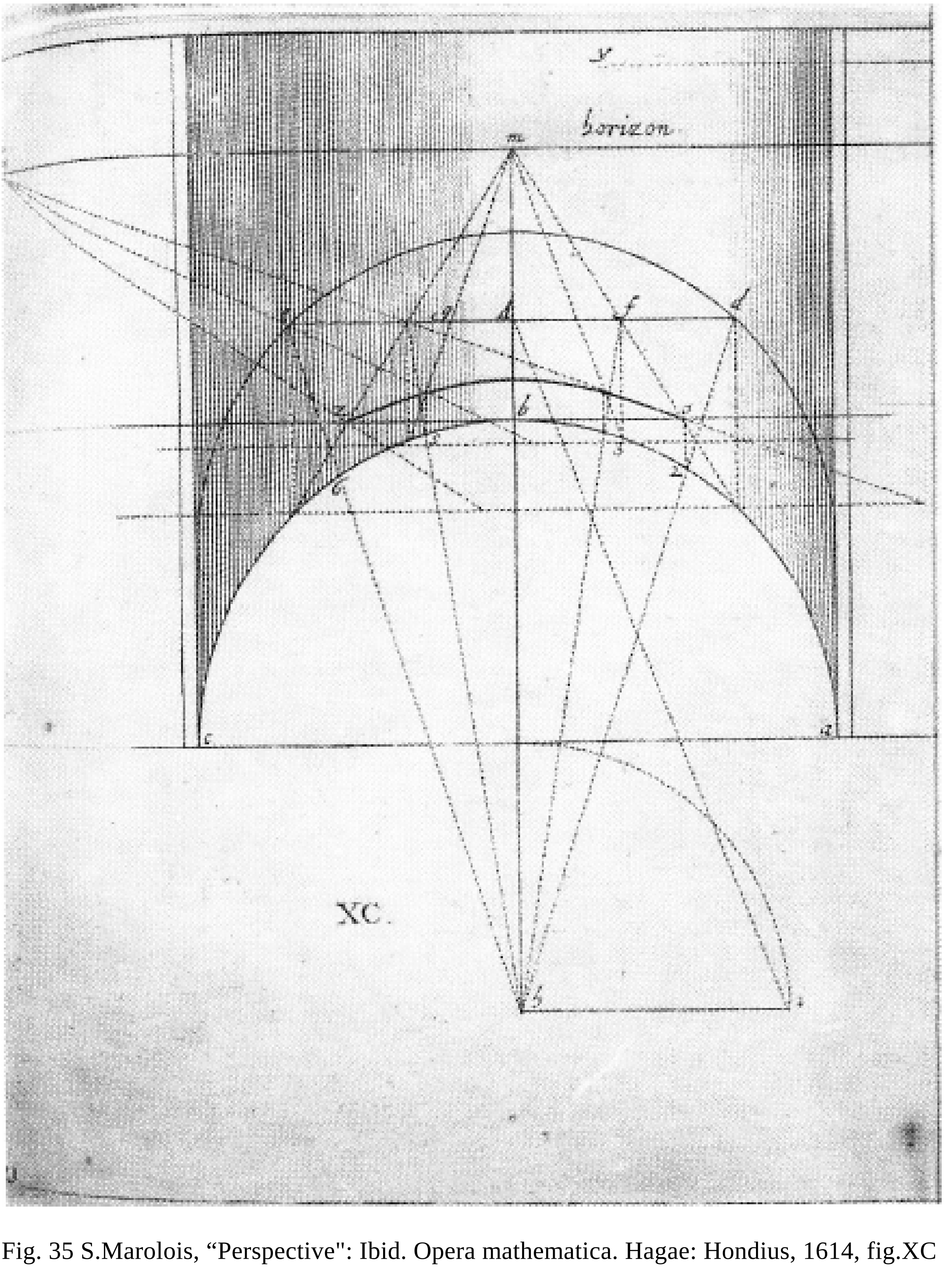



Fig. 35 S.Marolois, “Perspective'': Ibid. Opera mathematica. Hagae: Hondius, 1614, fig.XC

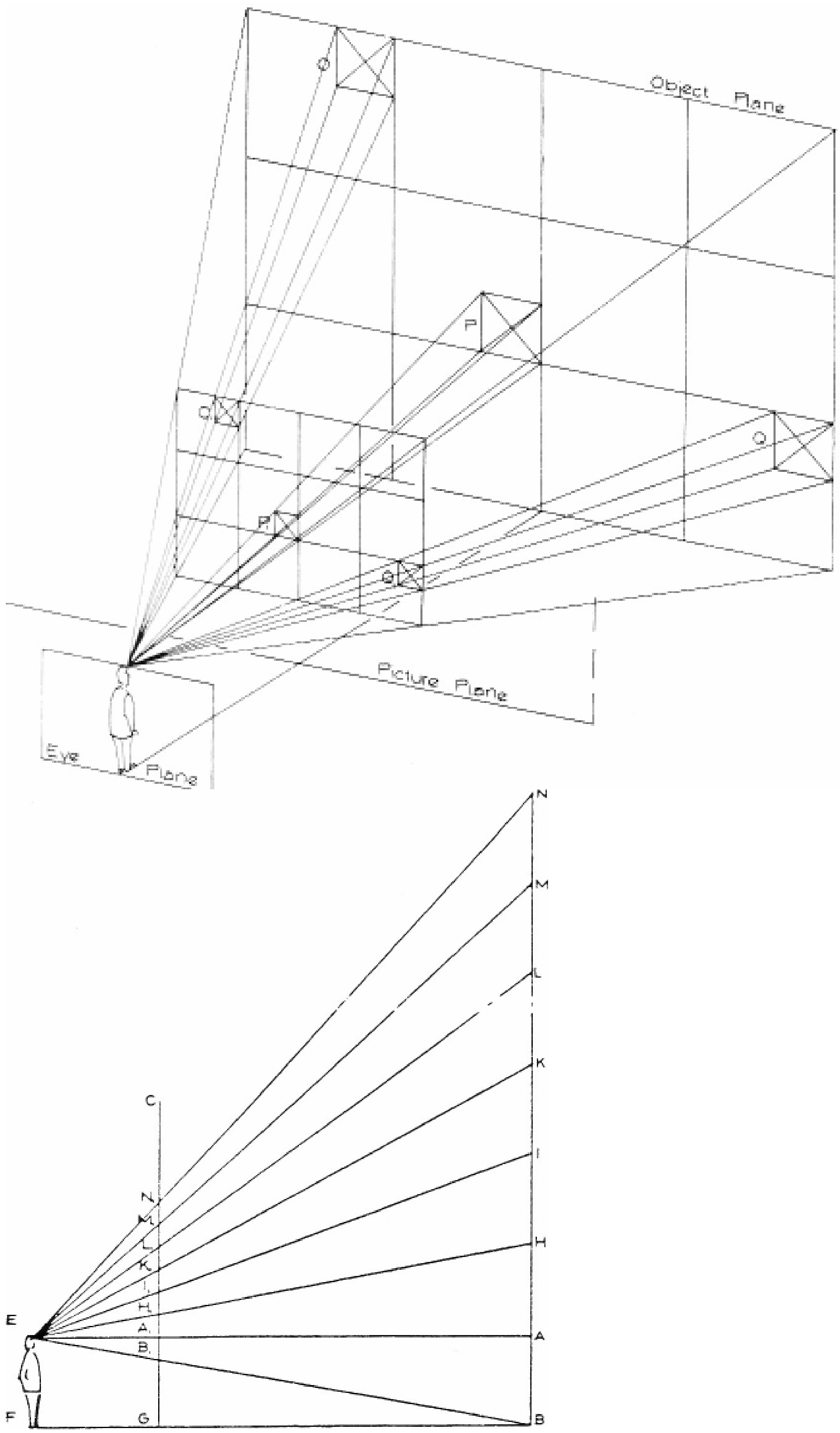
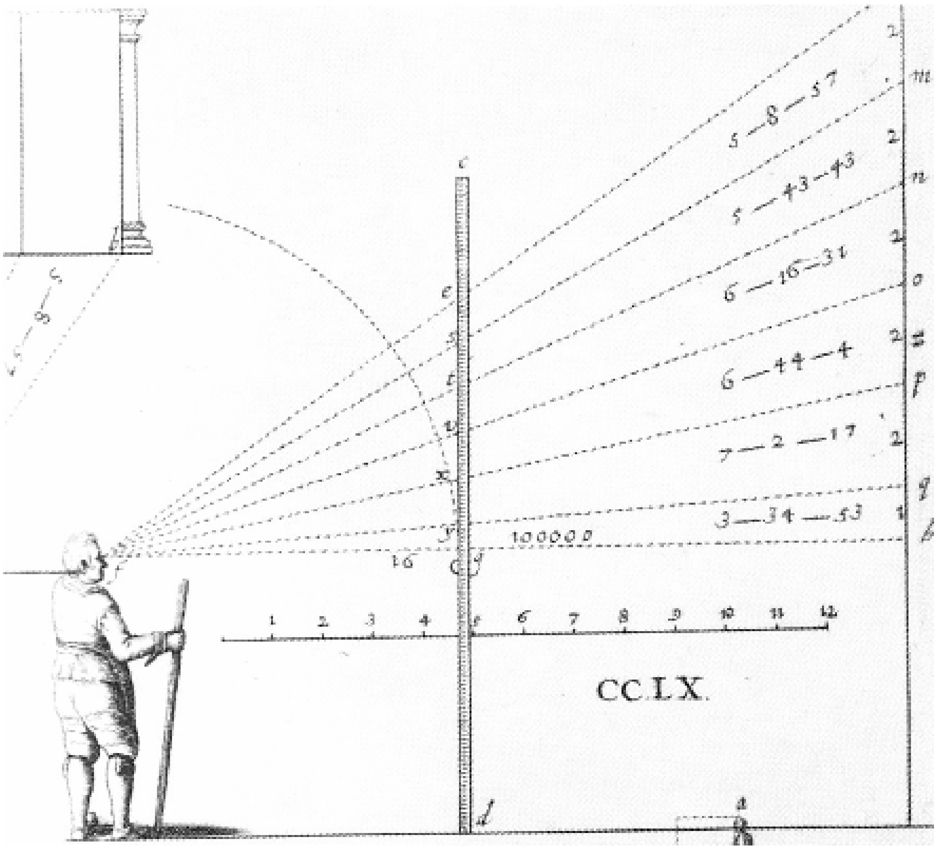



Fig. 36-37. Two demonstrations how size in perspective is governed by the distance of planes, and not by visual angles



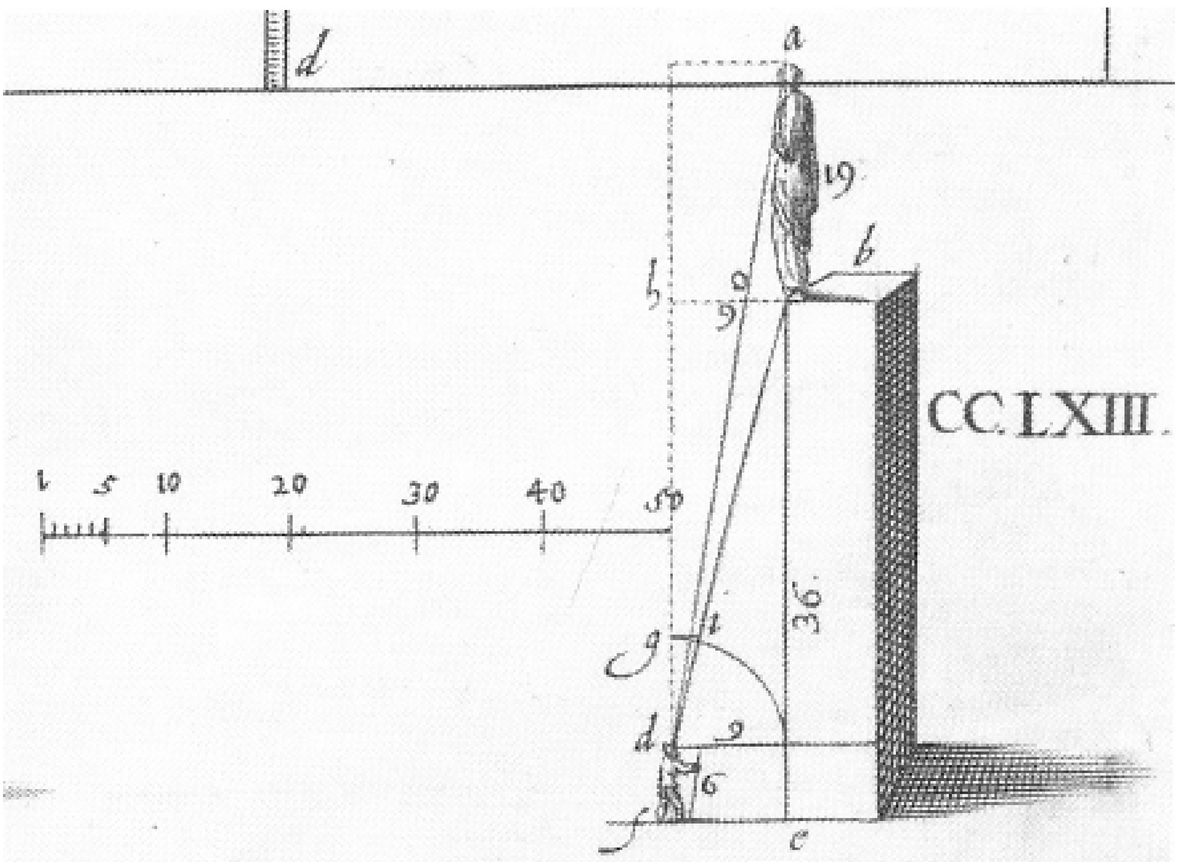
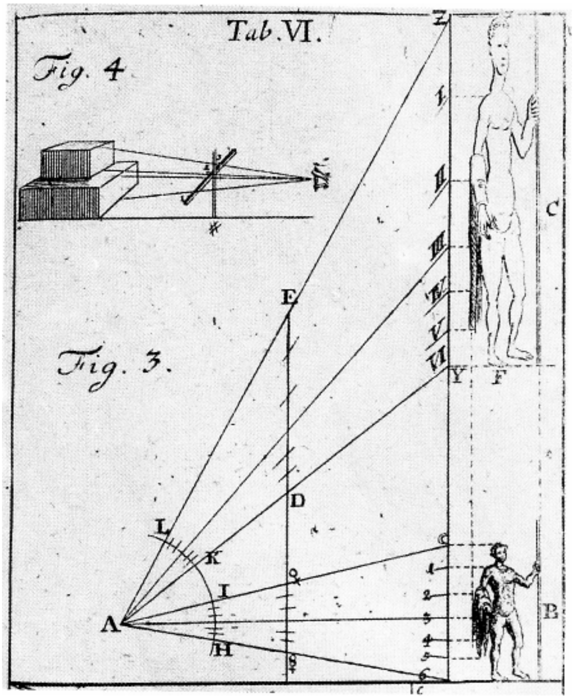




Fig. 38-39 Samuel Marolois, „Perspective"; Ibid, *Opera mathematica*. Hagae: Hondius, 1614, figs. CCLX, CCLXIII



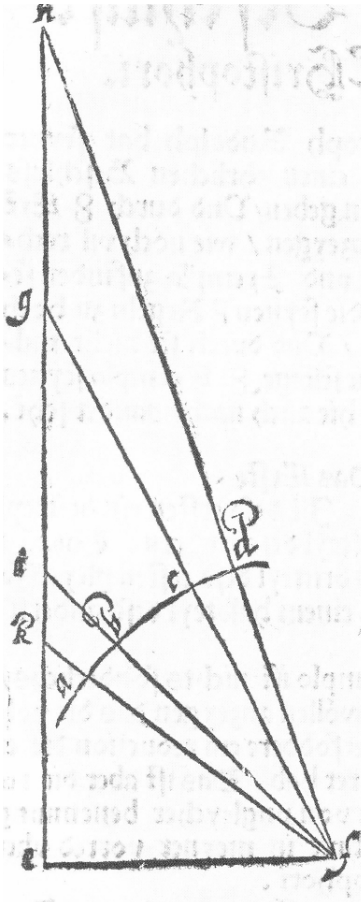
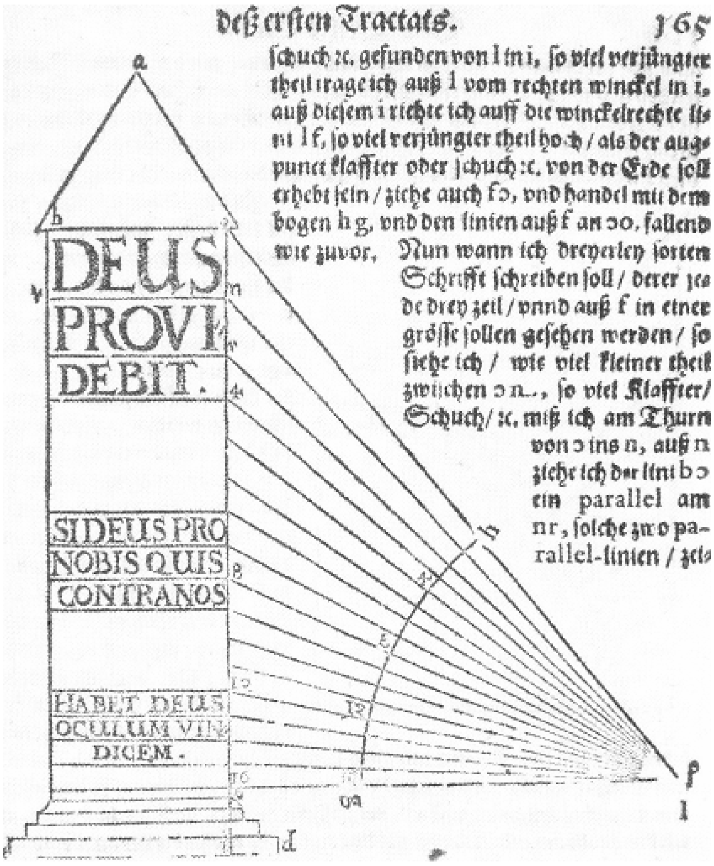



Fig. 40 Georg H. Werner, Gründliche Anweisung zur Zeichenkunst, Göttingen: Schneider 1796, Tab. VI, fig. 3.

Fig. 41 Christoff Rudolf, *Die Coß*, Königsberg: A. Lutomylensem 1553, fol. 475"



deß ersten Tractats. 165

schuch ꝛc. gefunden von l in i, so viel verjüngter theil trage ich auß l vom rechten winckel in i, auß diesem i richte ich auff die winckelrechte lini l f, so viel verjüngter theil hoch / als der augpunct klaffter oder schuch ꝛc. von der Erde soll erhebt sein / ziehe auch f o, vnd handel mit dem bogen h g, vnd den linien auß f an o o, fallend wie zuvor. Nun wann ich dreyerley sorten Schrifft schreiben soll / derer jede drey zeil / vnnd auß f in einer grösse sollen gesehen werden / so siehe ich / wie viel kleiner theil zwischen o n, so viel Klaffter / Schuch / ꝛc. miß ich am Thurn von o ins n, auß n ziehe ich der lini b o ein parallel am nr, solche zwo parallel-linien / zei-



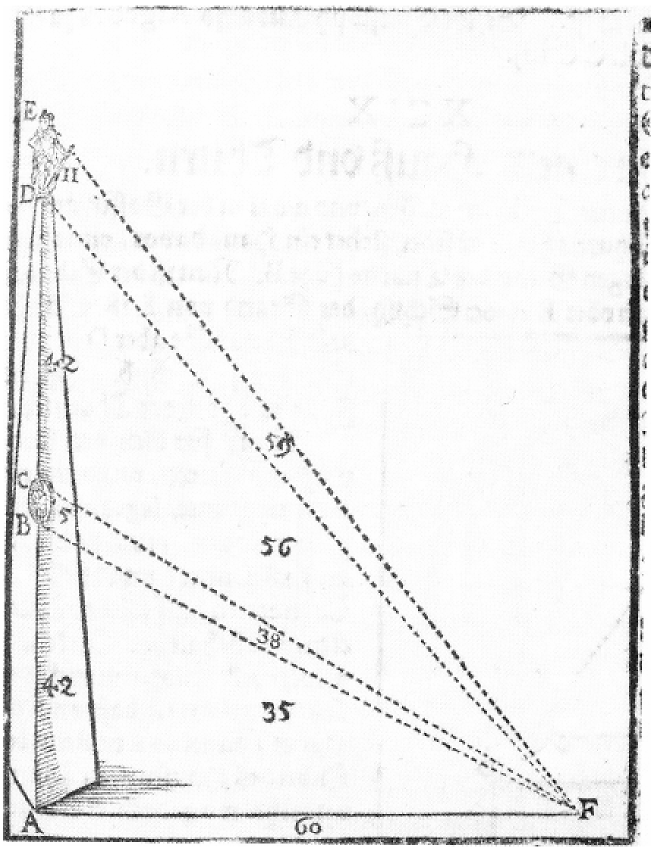



Fig. 42 D.Schwenter, *Geometriae practicae novae*,Nürnberg: S.Halbmayern,1618,165
Fig. 43 G. Galgemayr, ed., *Ein newer Proportional Circ*kel, Nürnberg: S. Halbmayern 1614, p. 85


Fig. 44: Giotto, Massacre of the Innocents, Padua, Scrovegni Chapel.


Fig. 45: Claude, Landscape Aeneas at Delos. London, National Gallery

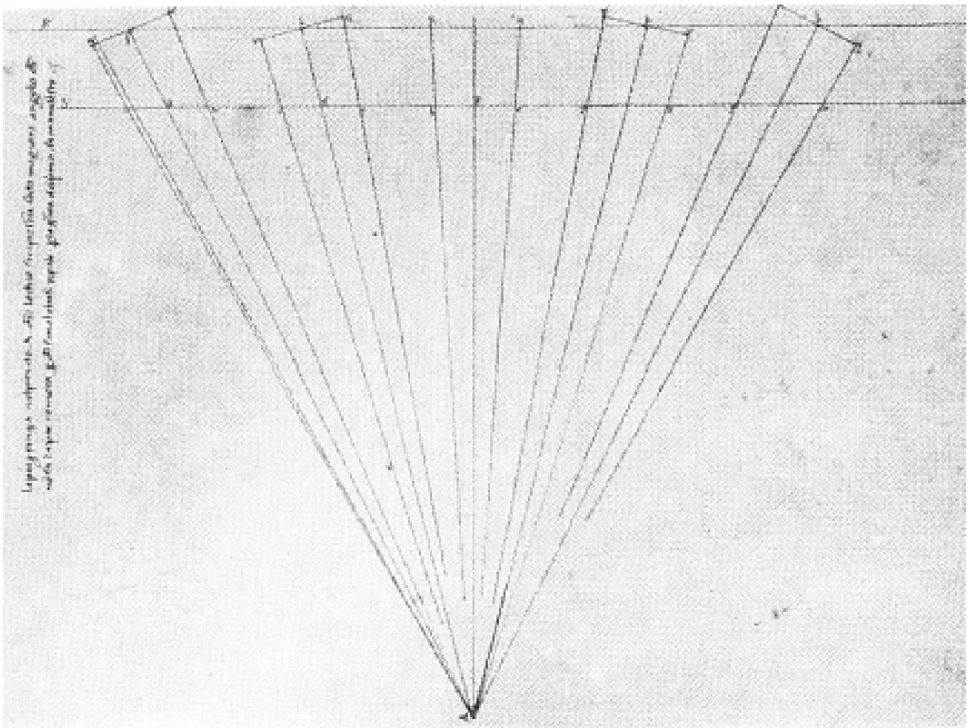


Fig. 46: Piero della Francesca. De prospectiva pingendi. Fig. XLIV. c. 1480. Paris: chez la vefue de P. Langlois, 1649, fol.

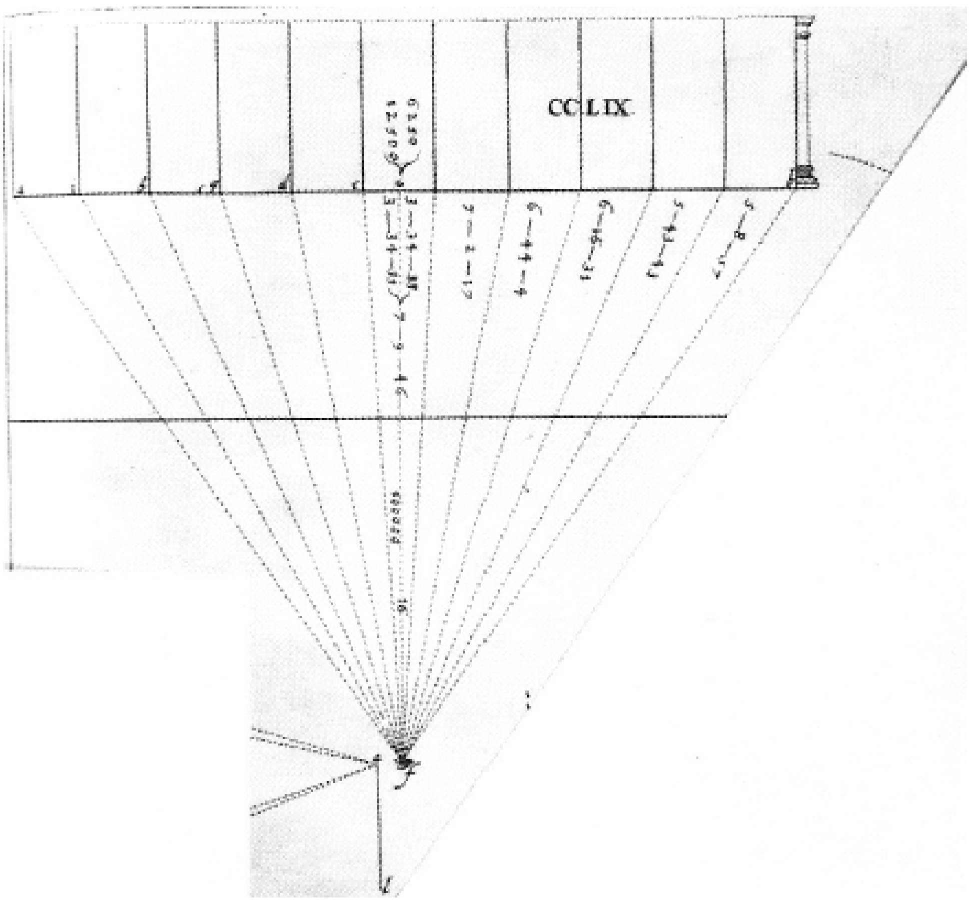



Fig. 47: Samuel Marolois, „Perspective": Ibid, Opera mathematica. Hagae: Hondius, 1614, Fig. CCLIX

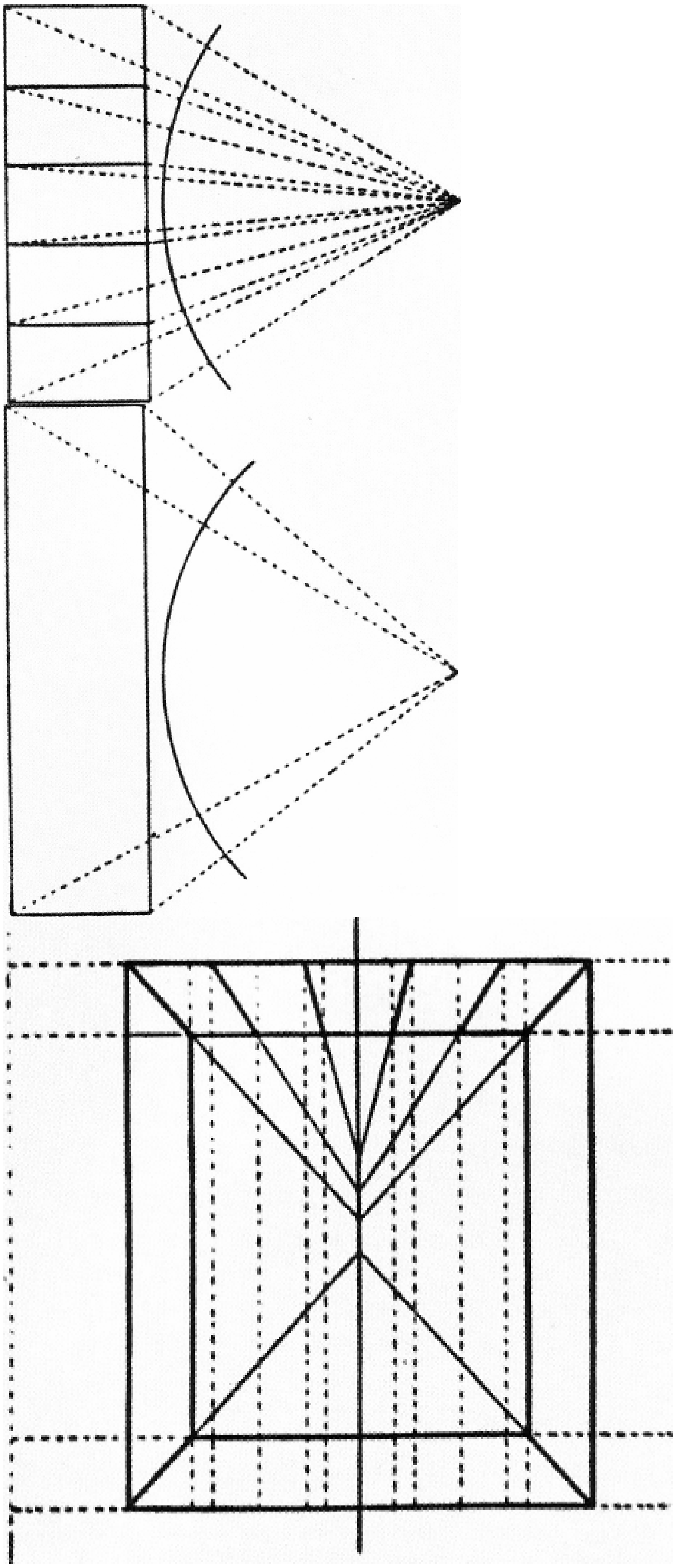


Fig. 48: Panofsky's angle-perspectival method which he attributed to the Greeks. From his „Die Perspektive als symbolische Form", *Vorträge der Bibliothek Warburg*, Leipzig, Berlin 1927, Textfig. 1.

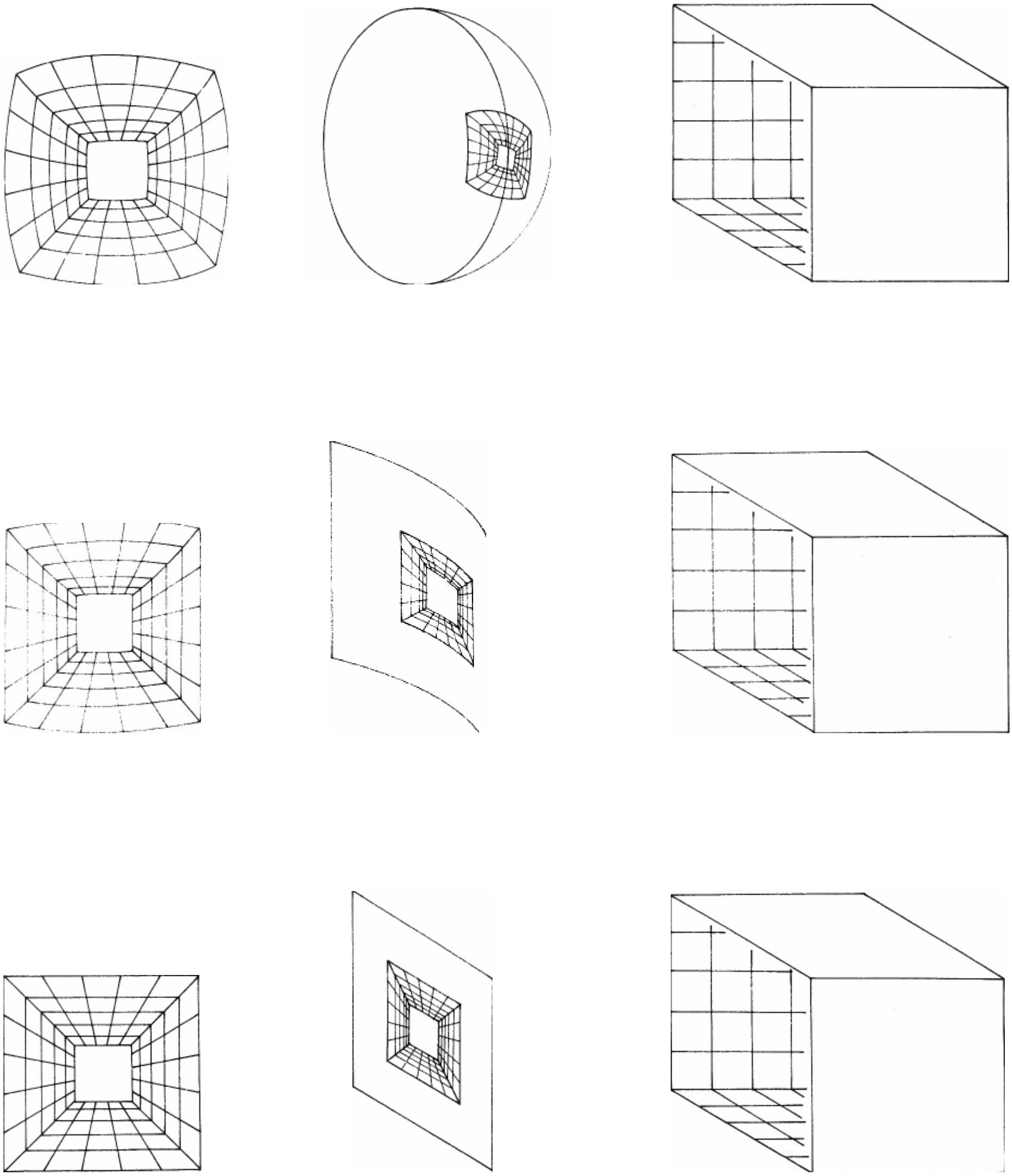


Fig. 49-51 Comparison of spherical, cylindrical and linear pespectival projections of a room. Drawings by Ute Barner.

**Notes**

[1] For a standard work on the history of anamorphosis see: Jurgis Baltrusaitis, *Anamorphoses ou perspectives curieuses*, Paris: Vrin, 1957.

[2] William Hyde Wollaston, “On the Apparent Direction of Eyes in a Portrait", *Philosophical Transactions*, London, 1824, pp. 247-256, plates IX-XII.

[3] M.G.-M. Raymond, „Sur les causes de la mobilité apparente du regard dans les yeux d'un portrait", *Mémoires de la Société Académique du Savoie*, Savoie, tome III, 1828, pp. 109-126.

[4] Guiseppe Ovio, „Effetto della prospettiva sull'acutezza visiva", *Memorie della reale accademia di scienze, lettere ed arti in Modena*, Modena, serie III, vol. X, parte prima, 1912, pp. 103-201.

[5] Robert H. Thouless, „Phenomenal Regression to the Real Object. I", *British Journal of Psychology*, 21, 1931, pp. 339-359 and „Phenomenal Regression to the Real Object. II", *British Journal of Psychology*, 22, 1931, pp. 1-30 with 1 fig. and 6 tables.

[6] Erwin Panofsky, „Die Perspektive als symbolische Form", *Vorträge der Bibliothek Warburg 1924-1925*, Leipzig, Berlin 1927, S. 258-330.

[7] John White, “Developments in Renaissance Perspective", *Journal of the Warburg and Courtauld Institutes*, London, vol. 12, 1949, pp. 58-79, and vol. 14, 1951, pp. 42-69.

[8] M.H. Pirenne, "The Scientific Basis of Leonardo da Vinci's Theory of Perspective", *British Journal for the Philosophy of Science*, London, vol. III, no. 10, 1952, pp. 169-185.

[9] M.H. Pirenne, *Optics, Painting and Photography*, Cambridge: University Press, 1970.

[10] M.H. Pirenne, “Vision and Art", *Seeing*, ed. E.C. Carterette and M.P. Friedman, New York: Academic Press, 1975, pp. 433-490. See especially pp. 454-459.

[11] B.A.R. Carter, “The Perspective of Piero della Francesca's Flagellation", *Journal of the Warburg and Courtauld Institutes*, London, vol. 16, 1953, pp. 292-302. This is part two of a paper by R. Wittkower and Carter.

[12] Nelson Goodman, *Languages of Art: An Approach to a Theory of Symbols*, New York: Bobbs Merrill, 1968, p. 12.

[13] James J. Gibson, „Pictures, Perspective and Perception", *Daedalus*, Cambridge Mass. vol. 89, 1960, p. 227.

[14] James J. Gibson, ”The Information Available in Pictures", *Leonardo*, Oxford, vol. 4, 1971, pp. 27-35.

[15] E.H. Gombrich, “The 'What' and the 'How': Perspective and the Phenomenal World", *Logic and Art: Essays in Honor of Nelson Goodman*, ed. R. Rudner and I. Scheffler, New York: Bobbs-Merrill, 1972, pp. 129-149.

On p.142 Gombrich notes: “I have discussed this intriguing phenomenon twice, once in Art and Illusion, and again in a later essay on Perception and Visual Deadlock", *Meditations on a Hobby Horse* (London: Phaidon Press, 1963), but I have never been satisfied that my explanation was exhaustive."

[16] E.H. Gombrich, „Illusion and Art", *Illusion in Nature and Art*, ed. R.L. Gregory and E.H. Gombrich, London: Duckworth, 1973, pp. 192-243, especially pp. 230-231.

[17] Kenneth R. Adams, „Perspective and the Viewpoint, *Leonardo*, Oxford, vol. 5, 1972, pp. 209-217.

[18] Professor Michael Kubovy, Department of Psychology, Rutgers, who is also working on this problem, refers to it as ,,the robustness of perspective".

[19] See, for instance, *Piero della Francesca. L'opera completa*, ed. O. Del Buono, P.-L. de Vecchi, Milano: Rizzoli,1967, p. 104. For the more recent attribution to Giuliano da Sangallo and Domenico Ghirlandaio, see: *Spettacolo e musica nella Firenze Medicea.*

*Documenti e restituzioni 1: Il luogo teatrale a Firenze*, ed. A.M. Petriolo Tofani, et al., Milano: Electa Editrice, pp. 76-80.

[20] Millard Meiss (1954) suggested that this was an ostrich egg. Since then a number of articles have appeared on the problem as, for example; Creighton Gilbert, « The Egg Reopened Again", *The Art Bulletin*, New York, vol. 56, no. 2, 1974, pp. 252-258.

[21] Piero della Francesca, *De prospectiva pingendi*, a cura di G. Nicco Fasola, Firenze: G.C. Sansoni, 1942, pp. 210-211 (fol. 82°-82`). Cf. vol. II, Tav. XLVIII (fig. LXXVIII).

[22] Leonardo da Vinci, *Manuscript A*, Paris: Institut de France, fol. 40`:

> *...onde bisognierebe fare una finestra de la grandeza del tuo volto, o veramente uno buso ... altrementi non te ne inpacciare, se giá tu non faciessi la tua veduta il meno 20 volte lontana ... e equesta sadisfara a ogni riguardatore situato in ogni contraposta parte a detta opera.*

A full analysis of Leonardo's perspectival and anamorphic studies is found in the author's book, written in consultation with Dr. K. D. Keele, *Linear Perspective and the Visual Dimensions of Science and Art*, Munich: Deutscher Kunstverlag, 1986.

[23] Pieter Jacobus Johannes van Thiel, *Vijf Studies over Haarlemse Schilder - en Tekenkunst*, Proefschrift ... te Utrecht, Rotterdam: Drukkerij Bronder-Offset N.V. Rotterdam, 1969, Stelling 2.

[24] Cf. Leon Battista Alberti, *On Painting*, ed. John R. Spencer, New Haven: Yale University Press, 1966, p. 71:

> I continue the circle in the painting with my mind guiding the lines from point to point. Would it perhaps be briefer to derive it from a shadow? Certainly, if the body which made the shadow were in the middle, located by rule in its place.

[25] Piero della Francesca, as in note 21 above, pp. 210-215.

[26] Daniele Barbaro, *La pratica della perspettiva*, Venetia: Appresso Camillo & Rutilio Borgominieri Fratelli, 1569, p. 161 discusses a related problem. Egnazio Danti is aware of this and mentions it in his commentary, pp. 61-62:

> E sebbene Daniel Barbaro nella quinta parte della sua prospettiva insegna un modo di far simili pitture con 1e carte ... si vedra non dimeno tal modo non aver quel fondamento, che ha il presente, mostratomi dal sopranominato Tommaso Laureti...

[27] This could be a visual pun on the name of Marolois' friend and collaborator Hondt or Hondius.

[28] Leonardo da Vinci, *Manuscript A*, Paris: Institut de France, fol. 42`. Cf. Leonardo, *Codex Atlanticus*, Milan: Ambrosiana, fol. 204[vb].

[29] For a discussion see F. Leemann, ed., *Anamorphosen*, Köln: Dumont, 1975, p. 88.

[30] Leonardo da Vinci, *Treatise on Painting (Codex Urbinas 1270*), Tr. and annotated A. Philip McMahon, Princeton: Princeton University Press, 1956, 139r-140r (497-498). In the Ludwig edition these passages are nos. 436-437.

[31] See, for instance, A. Bosse, *Moyen universel de pratiquer la perspective sur les tableaux ou surface irréguliéres*, Paris: chez ledit Bosse, en l'Is1e du Palais sur le Quay qui regarde celuy de 1a Megisserie, 1653.

[32] See, Carlo Pedretti, "Un ritratto anamorfosi di Francesco I, de probabile invenzione vinciana", *Documenti e memorie riguardante Leonardo da Vinci a Bologna e in Emilia,* Bologna: Editoriale Fiammenghi, 1923, pp. 121-123.

[33] Giacomo Barozzi il Vignola, *Le due regole della prospettiva pratica*, ed. Egnazio Danti, Roma: F. Zanetti, 1583, pp. 61-62.

[34] Jean Louis, Sieur de Vaulezard, *Perspective cilindrique et conique; ou traité des apparences veues par le moyen des miroirs cilindriques*, Paris: J. Jacquin, 1630; ibid., *Abrégé ou racourcy de la perspective par l'imitation,* Paris: chez l'auteur, 1631.
[35] Cf. Baltrusaitis, as in note 1 above, pp. 48-51.
[36] Jacob Leupold, *Theatrum mechanicarum, Supplementum,* Leipzig: Authore, pp. 55-57, Tab. 16,18
[37] See, for instance, the title to Hieronymus Rodler or Johann II v. Pfalz-Simmern: *Ein schön nötzlich Büchlin und under-weisung der Kunst des Messens ... Zu nutz allen ... Malern, Bildhawern ... auch allen ander, so sich der Kunst des Mes-sens (Perspectiva zu latin gnant) zu brauchen lust haben*, Siemeren: inn Verlegung Hieronimi Rodler, 1531.
[38] See: H. Schöne, „Die Dioptra des Heron", *Jahrbuch des Archäologischen Instituts*, Berlin, Bd. 14, 1899, S. 91-103.
[39] C. de Judaeis (i.e. Gerard de Jode*), De quadrante geometrico,* Norimbergae: Typis Christophori Lochneri, 1594, p. 47.
[40] Leonardo da Vinci, *Codex Arundel*, London: British Museum, fol. 279-281. These passages have been analysed in detail elsewhere. Cf. note 22 above.
[41] Egnazio Danti, as in note 33 above, p. 45.
[42] 42 Vaulezard, as in note 34 above.
[43] See: Johann Heinrich Lambert, *Schriften zur Perspektive*, Hrsg. ... Max Steck, Berlin: Dr. Georg Lüttke Verlag, 1943, pp. 180-185.
[44] „*Hoc uti possunt instrumento turn agentes turn matematici et ad geometriam et ad chorographiam efficiendam cum per-facile sit huius modo instrumento situm quevis sapere perfectamque habere notitiam omnis distantiae in picturam redigere*."
[45] Daniele Barbaro, as in note 26 above, Part Nine, Chapter IIII:
My translation is followed by the original Italian:

> Construction of Another Instrument by Baldessara Lanci. Baldessara Lanci, an ingenious engineer, when I was in Siena showed me an instrument invented by him to put things in perspective. This (if I remember correctly), is in this manner. On a round table of brass a peg was fitted in the centre which could turn around and it was two fingers above the table and parted in the middle to a certain extent, such that it produced (something) like two ears, in the middle of which was an-other peg, a foot high and this peg was held by a little peg which passed from one ear to the other and it was possible to bring this peg into play and twist it and move it as was needed. On the top of the peg was fixed a copper pipe with a small aperture which might have been half a foot long and stood like the letter T, fixed on the peg in the middle of which was fixed another pipe of brass parallel to the first in such a way that, lowering the peg on the little peg, both pipes were lowered and they always stood at the same distance from one another. The purpose of the upper pipe was for looking through; the purpose of the lower pipe was to receive a thin iron rod behind its canal in order to thrust onto a plane elevated at right angles to the circumference of the instrument. Which plane was not higher than the second pipe, that is, it did not come to the height of the first because it was necessary that the first was free and not impeded in order to be able to see. In this plane things were to be put into perspective which is done in this way. Having placed the instrument on some flat surface, one regards through the upper pipe the contours of something point by point and where one fixes the eye in the upper pipe one makes the iron rod pass through the lower pipe which will touch the plane opposite and there it makes a point. And in this same way, moving the peg and directing it to the other parts of the things, looking through, one marks every point

with the iron rod on the plane. This instrument is fine as far as invention goes, but as for usefulness it needs to be better formed and made larger with more instructions which I leave to the inventor who tells me he wishes to reform it."

„FABRICA D'UN ALTRO INSTRUMENTO DI BAL-DESSARA LANCI CAP. IIII. BALDESSARA Lanci ingenioso ingegneri essendo io in Siena, mi mostro uno instrumento ritrovato da lui da porre in Perspettiva. Il quale (se bene mi ricordo) é di questa maniera. Sopra una tavola ritonda di ottone nel mezza era fitto uno pirone, ilquale si poteva volgere a torno, era alto dalla tavola due ditta, & partito nel mezzo fino ad uno certo termine, di modo che egli faceva come due orrechie, tra'1 mezzo dellequali v'era un'altro pirone, alto uno piede, & questo pirone era tenuto da uno pironzino, che passava da un' orecchia all'altra, & poteva in detto pirone giocare pie-garsi, & drizzarsi secondo il bisogno, sopra la cima del pirone v'era saldata una canna di rame con uno piccolo bucco, laquale poteva essere longa mezzo piede, & stava come la lettera T. saldata sopra l'pirone, nel mezzo delquale v'era saldata un'altra canna di ottone equalmente distante alla prima, di modo che abbassandosi il pirone sopra il pironzino, amendue le canne s'abbassavano & sempre stavano in pari distanza l'una dall'altra. L'ufficio della canna di sopra era per traguardare, l'ufficio della canna di sotto era per rice-vere uno sottile ferruccio, dentro al suo canale per poterlo spignere fin ad uno piano elevato ad anguli giusti nella cir-conferenza dello instrumento, ilqual piano non era piu alto della seconda canna, cioé non veniva all'altezza della prima, perche bisognaua, che la prima fusse libera & non impedita per potere traguardare. In questa piana si hanno a ponere le cose in Perspettiva, ilche si fa in questo modo posto lo instrumento sopra qualche piano, si guarda per la canna di sopra i contorni d'alcuna cosa a punto per punto, & dove si ferma il vedere nella canna di sopra, si fa trapassare il ferruccio per 1a canna di sotto, il quale va a ferire nella piana opposta, & ivi si fa punto, & con la medesima via movendo il pirone & volgendolo alle altre parti delle cose, traguard-ando si segna ogni punto con il ferruccio nella piana.

Questo instrumento quanto alla inventione é bello, ma quanto all'uso ha bisogno di essere meglio formato, & fatto maggiore, & con piu avvertimenti, i quali lascio all'inventore, che mi disse di volerlo rifformare."

[46] Egnazio Danti, as in note 33 above, pp. 61-62:

My translation is again followed by the original Italian: „This instrument, which Daniel Barbaro claims to have seen

in Siena at the home of Baldassare Lanci of Urbino and which was used by many others is made as follows. To a circle similar to a platter a round table is attached, as would be a piece of the box of a drum of a circle of a large tin, as is seen (our fig. 31) in HLKI, which is attached to the round table GHSI. And then in the centre of this table is fitted a base which, at the centre A turns around and at the points C (and) B the ruler SE is positioned in such a manner that it turns in this circle. And at the summit of the ruler one places a little pipe or little ruler with two viewing holes at right ang-les in order to be able to see from nearby or from afar the things which need to be put into perspective. And lower down, that is, almost at the meeting point of the centre of the wooden circle one attaches to the said ruler SE another little pipe of copper, DF, which also stands at right angles to this ruler such that it is parallel to the one positioned above at the point E and such that when the one above turns or is raised or lowered, while the ruler SE turns around the points DB, this one below, DF, also turns rises or lowers. Then one attaches along the section of the circle HLKI a piece of paper and looking through the viewing holes E (and) T at what one wants to see, one puts a thread of iron in front of the little pipe DF and one makes a point on the paper attached to the circle; then following step by step until one has finished marking every point. And when one looks at the paper with the perspective that has been made, I say that, if it is taken from the circumference of the circle and reduced in a plane, that everything is false. And I show this as follows.

Let there be the sizes (our fig. 4) AF, FE, ED and DB and let the instrument with which we wish to draw in perspective be GL and let the eye be at the summit of the ruler at the point C, looking through which the above mentioned points are marked by the style at the points LKIHG of the paper. Now if the paper with the perspective were always to stand atta-ched to the circle, seen from the point C, everything would be well and the sizes, as in the case of AF and LK, being seen under a same angle ACE would appear equal and would show themselves to be the same. But when the paper is taken from the circumference LIG and is reduced to a plane along the line QOM, then everything is altered and confounded because point F [he means E] is seen as before at point O, but point A, which should be seen at point S, is seen at point Q, outside its position and similarly point F, at point P and the other two points D (and) B are equally seen beyond their position at the points N (and) M and must be seen at the points Z (and) R which parts from point C, being seen under equal angles along the circumference LIG, will be equal. But on the line SR they will be seen unequal, for if they were equal, as they are on the paper QOM, by the eye standing at the point C they would be seen under unequal angles. Since we have shown at prop. 36 that of equal sizes those appear larger which are closer to the eye and thus of equal sizes which are on the paper QOM, the two PO and ON will appear greater than the two QP and NM.

Hence the two angles PCO and OCN will be greater than (the two) QCP and NCM (and) hence the sizes AF, FE, ED and DB will not be seen under the four equal angles that they make at the point C as one supposes, which is false. And thus the sizes which are drawn on the paper LIG of the circle and correspond to those of the line AB as the paper is reduced to a straight line, they will be outside their true position and will not be shown as true in the section of the visual pyramid.

And thus this instrument is refuted as false and useless. But he who would reduce the instrument properly, which can be useful, leaving the ruler with the viewer in the same way that they are, will make the tables of the base square and in exchange for the piece of the circle HLKI, one draws a straight panel and one attaches a piece of paper to it and for the rest one proceeds as was said and all will succeed well."

„Questo strumento, che Daniel Barbaro dice haver visto in Siena á Baldassare Lanci da Urbino, & che da molti altri e usato, é fatto cosi. A un tondo simile a un tagliere é attaccata una tavoletta torta, come sarebbe un pezzo della cassa d'un tamburo, o d'un cerchio di scatola grande, come qui si vede la HLKI, che é attaccata alla tavola tonda GHSI. & poi nel centro d'essa tavola é fitto un piede, che nel punto A, si gira intorno, & nelli punti C, B, sta inchiodato il regolo SE, di maniera che in esso chiodo vi giri, & nella sommitá del regolo si mette una cannelletta, ó un'altro regoletto, con due mire ad angoli retti, per poter con esso traguardare da presso, ó di lontano, le cose che si hanno a mettere in Prospettiva: & piu á basso, cioé quasi all'incontro del mezo del cerchio di legno si attacca a1 prefato regolo SE, un'altra cannelletta di rame DF, che stia anche essa col regolo ad angoli retti, acció sia parallela á quella, che di sopra s'é posta nel punto E, & secondo che quella di sopra gira, ó salza, ó abbassa, mentre che il regolo SE, gira netti punti CB, questa di sotto DF, giri, & s'alzi, ó abbassi ancor ella. Dipoi si attacca nel pezzo di cerchio HLKI, una carta, & traguardando per le mire ET, quello che si vuol vedere, si spinge un filo di fero, che é den-tro alla cannella DF, & si fa un punto nella carta che é attac-cata a1 cerchio, seguitando poi di mano in mano finche sia finito di segnare ogni cosa, & si spicca 1a carta con la Prospet-tiva che vi e é fatta, la qual dico che come si lieve dalla circon-ferenza del cerchio & si riduce in piano, che ogni cosa vien falsa, & lo mostro cosi. Siano le grandezze AF, FE, ED, DB, & lo strumento con il quale le vogliamo levare in Prospettiva, sia GIL, & l'occhio stia alla sommitá del regolo nel punto C, per il quale mirando li sopradetti punti, siano segnati dallo stiletto netti punti della carta LKIHG. Hora se la cara có la Prospettiva dovesse star sempre nel cerchio attaccata, mirandola dal punto C, riuscirebbe ogni cosa bene, & le grandezze, poniam caso AF, & LK, essendo viste sotto il medisemo angolo ACF, ci apparirebbono uguali, & mostrerebbono d'essere le medesime. Ma come la carta si spicca della circonferenza LIG, & si riduce in piano nella linea QOM, all'hora si altera & confondo ogni cosa: perche il punto F, si vede come prima nel punto O, ma il punto A, che si doverebbe vedere nel punto S, si vede nel punto Q, fuor del suo luogo, e similmente il punto F, nel punto P, & g'altri due punti D, B, si vedranno parimente fuor del sito loro nelli punti N, M, & doverebbono essere netti punti ZR, lequali parti essendo dal punto C, viste sotto angoli uguali nella circonferenza LIG, saranno uguali: ma nella linea SR, saranno viste disuguali, perche se fussero uguali, si come stanno nella carta QOM, dall'occhio che sta nel punto C, sarebban viste sotto angoli disuguali, havendo noi dimo-strato alla prop. 31. che delle grandezze digradate uguali, quelle appariscano maggiori, che sono piu á dirimpetto all'occhio, & peró delle grandezze uguali, che sono nella carta QOM, le due PO, & ON, appariranno maggiori che non fanno le due QP & NM, adunque le grandezze AF, FE, ED, & DB, non saranno viste sotto li quattro angoli, che si fanno nel punto C, uguali, si come si suppone, il che é falso: & cosi le grandezze che nella carta LIG, del cerchio sono digradate, & rispondono á quella della linea AB, come la carta si riduce á dirittura in piano saranno fuori del sito loro, & non ci mostreranno il vero nella settione della piramide visuale: & peró questo strumento come falsa & inutile si rifuta. Ma chi volesse ridurre questo instrumento giusto, che potesse servire, lasciando li regoli con la mira nel medesimo modo che stanno, facciasi la tavola della basa della strumento quadra, in cambio del pezzo di cerchio HLKI, si pigli una tavoletta piana, & vi si attacchi la carta, & nel resto si operi come si é detto, & riuscirá ogni cosa bene."

Some scholars have sought to relate this method described by Danti to Leonardo da Vinci's perspectival methods: e.g. Carlo Pedretti, „Leonardo on Curvilinear Perspective", *Bibliothéque d'humanisme et renaissance*, Genève, vol. 25, 1963, pp. 77, 81, 87; Corrado Maltese, „La prospettiva curva di Leonardo da Vinci e uno strumento di Baldassare Lanci", *La prospettiva rinascimentale. Codificazioni e trasgressioni*, ed. Marisa Dalai Emiliani, Firenze: Centro Di, 1980, pp. 417-425.

[47] *Extract der Geometriae and Perspectivae gemalet geschrie-ben und abgezeichnet von Johann Krezmaier, anno 1611 Wolfenbüttel*, Herzog August Bibliothek, Cod. Guelf. Extrav. 84f, fol. 2'.
[48] Euclide, *L'Optique* ... trad. Paul ver Eecke, Paris: Librairie ... Albert Blanchard, 1959, p. 9: "Il est manifeste aussi que les plans situés en élévation paraîtront concaves."
[49] *Vitellonis ... Opticae libri decem ... instaurati* ... Federico Risnero, Basileae: ex officina episcopiana, 1572, p. 135: “Et patet ex hoc, quod multum exaltato visu superficies planae iacentes longe á visu concavae videbuntur: tendunt enim formae talium punctorum ad visum per modum circumferentias circa centrum visus propter aeqvalitatem virtutis visivae. Patet ergo propositum."
[50] Leonardo da Vinci, *Manuscript A*, Paris, Institut de France, fol. 38`:
“Onde se pure volessi metterlo in opera, ti bisognierebbe che essa prospettiva si vedessi da uno solo buso, il quale fussi ne loco m ..."
[51] Ibid:„La pariete op per l'essere sempre equidistante a l'ochio a uno modo, renderá le cose bene e atte a essere vedute da lloco a lloco."
[52] Leonardo, Manuscript E, Paris, Institut de France, fol. 16[vr]
[53] It is probably this practice which Plato criticizes in *The Sophist*, ed. E. Hamilton and H. Cairnes in *The Collected Dialogues of Plato*, New York: Pantheon Books, 1961, pp. 978-979 (235 D-236 C).
On the question of Greek perspective see: G.M.A. Richter, *Perspective in Greek and Roman Art*, London and New York: Phaidon, 1970; A.M.G. Little, *Roman Perspective Painting and the Ancient Stage*, Shiremanstown, Pa.: Moretus, 1971; John White, *The Birth and Rebirth of Pictorial Space*, London: Faber and Faber, 1972.
[54] Leonardo da Vinci, *Codex Adanticus*, Milan, Ambrosiana, fol. 13[ra, va], 191[ra, rb], 212[ra].
[55] Albrecht Dürer, *Underweysung der Messung*, Nürnberg, 1525, fol. K". Cf. Baltrusaitis, as in note 1, pp. 14.
[56] Cf. Baltrusaitis, as in note 1, pp. 11-15. 57 Ibid, pp. 71-75.
[57] Cf., for example, Abraham Bosse, *Traité des maniéres de dessiner*, Paris: Bosse, 1664, II:„
„Par ainsi il faut conclure que c'est une Erreur tres grossiére, de vouloir faire au perspectif grand ou petit, cette diminu-tion puis que ce seroit la mesme Erreur, que de desseigner et peindre comme l'Oeil voit: Et pour le Geometral qua moins de regarder de le hauteur d'Oeil et des lieux déterminez les desseins et modelles qui sont faits par ce raisonnement ils ne plairont point a l'Oeil, mais bien l'ouvrage Effectif, et que le contraire arrive a ceux qui sont faits autrement."
Cf. Baltrusaitis, as in note 1, p. 72, fig. 49:„Pour prouver qu'il ne faut pas dessiner n'y peindre comme l'oeil voit."
[58] On this problem see: Bernard Téyssedre, *Roger de Piles et les débats sur le coloris au siècle de Louis XIV*, Paris; Bibliothéque des arts, 1965; H.W. van Helsdingen, *Historier en peindre. Poussins Opvatingen over Kunst in het Licht van de dicussies in de Franse Kunstlitteratur in de tweede helft van de zeventiende eeuw*, PhD Utrecht, Rotterdam: Bronder Offset, 1971.
[59] Piero della Francesca, as in note 21 above, vol. 1, pp. 125-126, vol. II, Tav. XX, Fig. XLIV
[60] e.g. Leonardo da Vinci, *Manuscript A*, fol. 40"-41`
[61] Luca Pacioli, *Summa di arithmetica, geometria, proportioni e proportionalitá*, Venetüs: Paganinus de Paganinis, 1494, Part II, fol. 6.
[62] Jean Cousin, *Livre de perspective*, Paris: De l'Imprimerie de Iehan le Royer 1560, Civr-D i`.
[63] M.H. Pirenne, as in note 9 above, pp. 124-130.

[64] E. Panofsky as in note 6 above. Panofsky subsequently repeated his ideas in other publications. For a review of these see the author's „Panofsky's Perspective: A half Century Later", *La prospettiva rinascimentale: codificazione e trasgressioni*, ed. Marisa Dalai Emiliani, Firenze: Centro Di, 1980, pp. 565-584.

[65] John White, as in note 53 above, pp. 236-273.

[66] See, for example, Morris Kline, *Mathematics in Western Culture*, New York: Oxford University Press, 1964, p. 121.

[67] See, for instance, Jakob C. Burckhardt, „The Discovery of the World and of Man", *The Rise of Modern Science*, ed. George Basalla, Lexington, Mass: D.C. Heath and Co., 1968, p. 8.

[68] E.H. Gombrich, as in note 15 above.

[69] Ibid.