Anamorphic Projections

A Senior Project submitted to
The Division of Science, Mathematics, and Computing
of
Bard College

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Abstract

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1 Introduction

1.1 Background

An anamorphosis (or anamorphic projection/image) is an image that is intentionally distorted so that the original image can be recovered only when looked at from a certain point of view or using a special device, for example a mirror. Origins of anamorphosis can be traced back to the 16th century art where artists, mathematicians and philosophers, fascinated by the idea of perspective, experimented with the notions of illusion, truth and reality. [1] Some of the notable examples of anamorphosis included Jean-Francois Niceron's methods for geometrical construction that generated multiple types of anamorphic transforms which involved both exact and approximate methods. [1, 2] One of the most famous anamorphic paintings is Hans Holbein's *The Ambassadors* (Figure 1.1.1), in which the artist embedded an anamorphic image of a skull (Figure 1.1.2) at the bottom of the painting as a *memento mori*, a recurring motif in many artworks in the Renaissance period.

Anamorphosis is still prevalent to this day, and is incorporated in many new forms of art, most noticeably street art. The undying fascination for anamorphosis is manifested

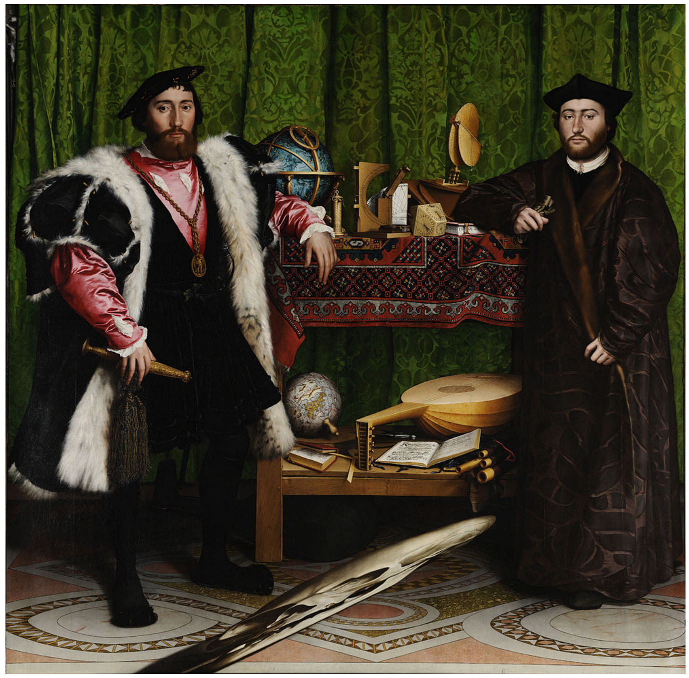

Figure 1.1.1: Hans Holbein's *The Ambassadors*, 1533 ¡Add reference¿

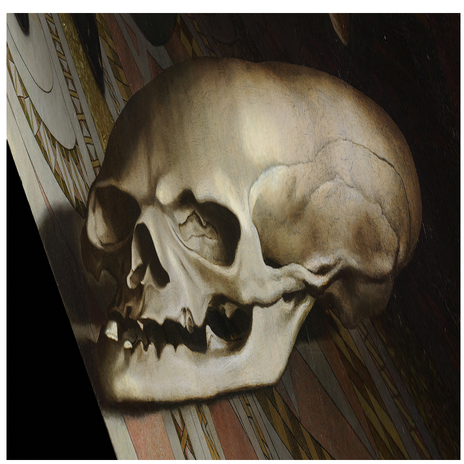

Figure 1.1.2: Undistorted skull from H. Holbein's *The Ambassadors* ¡Add reference¿

in numerous artworks centered around the anamorphic principle and using it to create a unique piece of art.

Beyond the esthetic values, anamorphosis has found its uses in many practical settings, such as road signs that are elongated so that drivers' looking at them from a small angle above the ground can see the signs correctly (Figure 1.1.3), or inverted "ambulance" signs that are meant to be reflected correctly in the rear-view mirrors.

However, despite the widespread uses, there is a surprising lack of detailed explanation of the mathematical principles behind anamorphosis, and even with the advanced technology there is hardly any computer software generating the anamorphic projections. The goal of this project is two-fold. First of all, we need to build a comprehensive documentation describing the most common types of anamorphosis. Next, we will apply the acquired knowledge to create a software that will automate the process of generating anamorphic projections.

Figure 1.1.3: Road sign in the shape of an elongated bike. Left: view from the top; right: view from a driver's perspective. [1]

1.2 Definitions

In this section we define some basic concepts and terminology needed to describe anamorphic projections. In particular, we introduce homogeneous representations of geometrical objects used in computer vision to facilitate computing geometrical transformations.

1.2.1 Basic geometry

In mathematics, geometric elements in a coordinate system are most commonly represented in Cartesian coordinates.

Definition 1.2.1 (Cartesian point)**.** Let P be a point in n dimensions positioned in a coordinate system. The n-tuple $(x_1, x_2, ..., x_n)$ is its representation in Cartesian coordinates if x_i is the distance from point P to the origin along the i^{th} axis for all $i \leq n$.

Definition 1.2.2 (Cartesian line)**.** ¡def line¿

1.2.2 Homogeneous representation

Homogeneous coordinates were first defined by August Ferdinand Möbius as ¡... why?¿. [3] They are widely used in computer vision, as they offer a convenient way to perform projective transformations.

Definition 1.2.3 (Homogeneous point)**.** Let P be a point in n dimensions, and $(x_1, x_2, ..., x_n)$ be its representation in Cartesian coordinates. The homogeneous representation of P is any $(n+1)$-tuple $(x_1\omega, x_2\omega, ..., x_n\omega, \omega)$, where ω is called the homogeneous coordinate.

A Cartesian point can be easily written as a homogeneous point by appending $\omega = 1$ as the $(n+1)^{th}$ coordinate. A homogeneous point can be converted to Cartesian coordinates by dividing each of its coordinates by ω, and removing the homogenous coordinate.

Lemma 1.2.4. *When a homogeneous point is multiplied by a scalar, it is the same point.*

¡prove this lemma?¿ Should this be a lemma or just a "narrative"?

Example 1.2.5. Let P_1 be a point in three dimensions with Cartesian coordinates (x, y, z). A possible homogeneous representation of P_1 is $(x, y, z, 1)$, or more generally $(x\omega, y\omega, z\omega, \omega)$ for any ω (¡$\omega \geq 0$???¿). Let P_2 be another point in three dimensions with homogeneous representation (x, y, z, ω). P_2 can be represented in Cartesian coordinates as $\left(\frac{x}{\omega}, \frac{y}{\omega}, \frac{z}{\omega}\right)$.

Definition 1.2.6 (Homogeneous line in 2D)**.** Let l be a straight line in two dimensions defined by the equation $ax + by + c = 0$. Then (a, b, c) is the homogeneous representation of l.

Lemma 1.2.7 (Intersection of lines in 2D)**.** *A point P is the intersection of two-dimensional lines l_1 and l_2 if and only if P is the dot product of l_1 and l_2 in homogeneous coordinates.*

Proof. Let l_1 and l_2 be two-dimensional lines represented as (a_1, b_1, c_1) and (a_2, b_2, c_2) respectively. □

Definition 1.2.8 (Degrees of freedom)**.** Let P be a point in n dimensions, and $(x_1, x_2, ..., x_n)$ be its representation in Cartesian coordinates. The homogeneous representation of P is any $(n+1)$-tuple $(x_1\omega, x_2\omega, ..., x_n\omega, \omega)$, where ω is called the homogeneous coordinate.

1.2.3 2D Transformations

Definition 1.2.9 (Translation)**.**

Definition 1.2.10 (Scaling)**.**

Definition 1.2.11 (Rotation)**.**

Definition 1.2.12 (Warping)**.**

Definition 1.2.13 (Forward Warping).

Definition 1.2.14 (Backward Warping).

Definition 1.2.15 (Homography). A homography or projective transformation is a transformation of a two-dimensional image I to another two-dimensional image I' (called the anamorphic image), such that all lines in I are preserved in I', or more rigorously. The homography can be represented as a 3×3 matrix H. The anamorphic image is obtained by applying H to every point in the original image to obtain the corresponding point in the anamorphic image:

$$p' = Hp, \text{ for all points } x \in I.$$

Line preservation means that the points $p_1, p_2, p_3 \in I$ lie on the same line if and only if Hp_1, Hp_2, Hp_3 are on the same line.

1.2.4 Types of anamorphosis

Definition 1.2.16 (Perspective anamorphosis). Let S be a surface and V be a viewer at some distance d looking directly at the surface S at some angle θ. Let I be some two-dimensional image. I is an anamorphic projection of I on the surface S for the viewer V if the resulting image that V sees when looking at S' is S.

Definition 1.2.17 (Planar anamorphosis). This is my definition of planar anamorphosis.

2
Anamorphic Projections

2.1 Planes

The relation between the anamorphic (distorted) image on a plane and the correct image that the viewer observes can be described using a 3D homography matrix.

2.2 Multiple Planes

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2.3 Cylinders

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2.4 Cones

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2.5 Mirrors

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3
Methods

Def. Least squares

Ransac

Corner detection

3.1 ¡Title of First Section¿

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3.2 ¡Title of Second Section¿

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