Angular spectrum method with correction of anamorphism for numerical reconstruction of digital holograms on tilted planes

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Abstract: We present a new method for numerically reconstructing digital holograms on tilted planes. The method is based on the angular spectrum of plane waves. Fast Fourier transform algorithm is used twice and coordinate rotation in the Fourier domain enables to reconstruct the object field on the tilted planes. Correction of the anamorphism resulting from the coordinate transformation is performed by suitable interpolation of the spectral data. Experimental results are presented to demonstrate the method for a single-axis rotation. The algorithm is especially useful for tomographic image reconstruction.

OCIS code: (090.0090) Holography; (090.2880) Holographic Interferometry; (100.3010) Image reconstruction techniques; (070.4790) Optical spectrum analysis.

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1. Introduction

The basic advantage of digital holography is that it makes possible to extract quantitatively the three-dimensional (3D) information of the object from the numerical reconstruction of a single digitally recorded hologram [1-3]. This open the field to a variety applications, such as 3D microscopic investigations of biological specimens where wavelength-scanning digital interference holography [4] is used to reconstruct the 3D volume from a set of scanning tomographic images, hybrid holographic microscopy [5], three-dimensional microscopy by optical scanning holography [6], phase shifting digital holography, particle holography, just to quote few of them. Accurate reconstruction of the 3D image volume requires reconstructing the object field at several axial distances from the recording plane and this is a demanding task from the numerical point of view since, for example, if we use the Fresnel transform method (FTM) [2,3] for hologram reconstructions, reconstructed images at different distances need to be resized to account of the dependence of the reconstruction pixel on the distance [7].

Although various methods have been devised for controlling the lateral resolution in the reconstruction plane [7,8], the task is still demanding when many reconstructions along the axial distance have to be stuck together and the axial resolution differs from the lateral one. However, if one is interested in inspecting characteristics of the object on a plane or a set of planes tilted with respect to the recording hologram one, such as in tomographic applications, it is more efficient to develop a method capable of reconstructing the hologram at arbitrarily tilted planes. Basically, this means simulating light propagation through calculation of the diffraction between arbitrarily oriented planes. Leseberg and Frère were the first [9] that addressed the problem of using the Fresnel approximation to find the diffraction pattern of a tilted plane. Later on, a general-purpose numerical method for analyzing optical systems by use of full scalar diffraction theory was proposed [10]. FTM for numerical reconstruction of digital holograms with changing viewing angles was described [11]. In this letter we present a different method for digital holography on tilted plane (DHT) that makes use of the angular spectrum of plane waves [12] for fast calculation of optical diffraction. Fast Fourier transformation is used twice and coordinate rotation of the spectrum enables to reconstruct the hologram on the tilted plane. Interpolation of the spectral data is shown to be effective for correcting the anhamorphism of the reconstructed image.

2. Experimental approach

The experimental set-up for DHT shown in Fig. 1 is a Mach–Zender interferometer designed for reflection imaging of microscopic objects. A linearly polarised collimated beam from a diode-pumped, frequency-doubled Nd:YVO_4 laser with $\lambda=532\ nm$ is divided by the combination of half-wave plate $\lambda/2$ and polarizing beam splitter into two beams.

In each arm, spatial filters and beam expanders are introduced to produce plane waves. The object beam is reflected by the test object consisting of the lithographic patterned logo ''MEMS'' covering an area of $(644 \times 1568)\ \mu m^2$ on a reflective silicon substrate. As shown in Fig. 1, the object is tilted with respect to the hologram plane $(\xi - \eta)$. The interfering object beam and the reference beam are tilted at small angle with respect to each other to produce off-axis holograms. The digital holograms are recorded by a standard black and white CCD camera and digitized into a square array $NxN = (1024\ x\ 1024)\ pixel^2$, $\Delta\xi \times \Delta\eta = (6.7 \times 6.7)\mu m^2$ pixel width.

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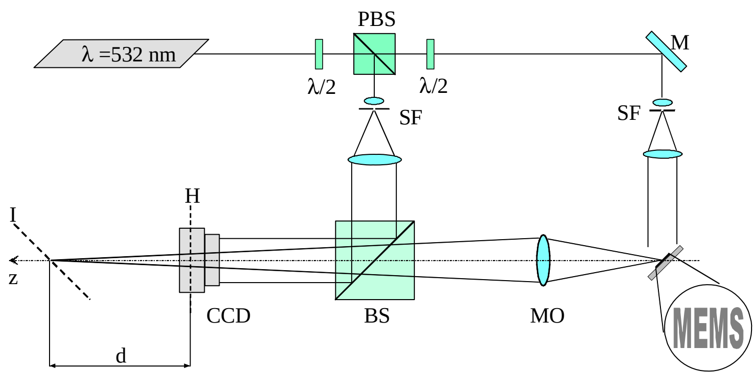

Fig. 1. Experimental set-up fro recording off-axis digital holograms. PBS polarizing beam splitter; BS beam splitter; SF's spatial filters; MO microscope objective. The hologram are recorded by the CCD camera.

The microscope produces a magnified image of the object and the hologram plane is located between the MO and the image plane *I* at a distance d from the recording hologram plane *H*. In digital holographic microscopy we can consider the object wave emerging from the magnified image and not from the object itself [13].

The geometry of the reconstruction algorithm is illustrated in Fig. 2. The image plane *I* has coordinates $(\hat{x}-\hat{y})$ and its origin located along the optical *z* axis at a distance d from the hologram plane $(\xi-\eta)$. The plane *I* is tilted at angle θ with respect to the plane $(x-y)$, that is $(x-y)$ is parallel to the hologram plane and share the same origin of the tilted plane *I*. Reconstructing by digital holography the complex wave distribution $o(\hat{x},\hat{y})$ on the tilted image plane means simulating light propagation from the hologram plane to the image plane through calculation of the diffraction of the object wave distribution $o(x,y)$ on the plane *H* (at *z=0*) to the tilted plane.

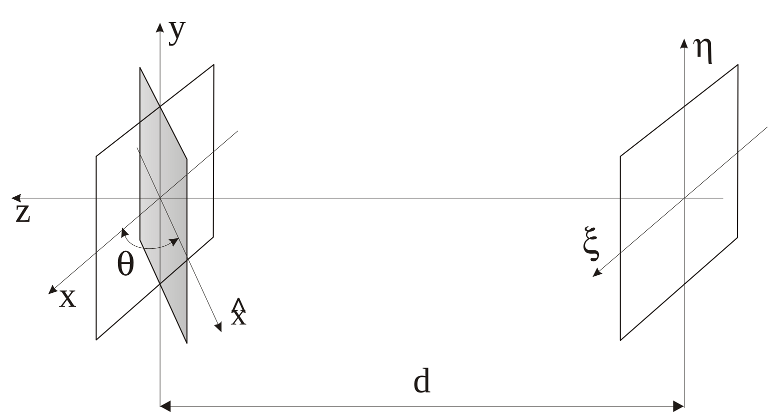

Fig. 2. Schematic illustration of the ASA algorithm for reconstructing digital holograms on tilted planes.

The angular spectrum-based algorithm (ASA), for reconstructing the wave field $o(\hat{x},\hat{y})$, consists basically of two steps. In the first stage we reconstruct the wave distribution on the intermediate plane $(\hat{x}-\hat{y})$ at distance d and calculate the spectrum $\hat{O}(d;u,v)$ where $u=\xi/\lambda d$ and $v=\eta/\lambda d$ denotes the Fourier frequencies in the intermediate plane. A convenient way of performing the spectrum calculation is through the application of the so called convolution method (CM) for reconstructing the wave field at the translational distance d [1]. The formulation makes use of the convolution theorem to calculate the spectrum as follows :

$$\hat{O}(d;u,v)=O(u,v)\exp\left[i2\pi d\left(\lambda^{-2}-u^2-v^2\right)\right] \tag{1}$$

where $O(u,v)=\Im\{o(x,y)\}$ is the spectrum of the object wave distribution $o(x,y)$ at $z=0$ is defined by the Fourier transform

$$O(u,v)=\iint o(x,y)\exp[-i2\pi(ux+vy)]dxdy \tag{2}$$

In CM the field distribution at distance d is derived by taking the inverse Fourier transform of the propagated angular spectrum (1), but in our case we have to take into account of the tilting of the reconstruction plane $(\hat{x}-\hat{y})$ with respect to the intermediate plane. Standard transformation matrix is used to rotate the wave vector coordinates $\mathbf{k}=2\pi[u,v,w]$ where $w=\left(\lambda^{-2}-u^2-v^2\right)^{1/2}$, on the y axis with the angle of θ to determine the spatial frequencies $\hat{u}$ and $\hat{v}$ associated to the components of the rotated wave vector components $\mathbf{k}=2\pi[\hat{u},\hat{v},\hat{w}]$, namely $\hat{u}=u\cos\theta-w\sin\theta$ and $\hat{v}=v$. According to the spatial frequencies transformation, the rotated angular spectrum (RAS) can be written as $\hat{O}(d;\hat{u},\hat{v})=\hat{O}(d;\hat{u}\cos\theta+\hat{w}\sin\theta,\hat{v})$.

In the second stage of ASA, the rotated spectrum is inverse Fourier transformed to calculate the reconstructed wave field $o(\hat{x},\hat{y})$ on the tilted plane, namely

$$o(\hat{x},\hat{y})=\Im^{-1}\left[\hat{O}(d;\hat{u}\cos\theta+\hat{w}\sin\theta,\hat{v})\right] \tag{3}$$

It should be remarked that reconstructing the field according to Eq. (3) is valid if we stay within the paraxial approximation, as it is applies to our experimental conditions. However, Eq. (3) can be generalized to include frequency dependent terms of the Jacobian associated to the RAS. However, the inclusion of these terms does not affect the reconstruction, since they are small in the paraxial case. Equation (3) is a good approximation for the numerical calculation of the wave field on tilted plane from a digitally recorded hologram as it will be shown in the discussion of the experimental results.

3. Experimental results

Figure 3(a) shows the hologram of the reflective target recorded in off-axis configuration. The plane reference wave interferes with the object waves at small angle (≤ 0.5), as required by the sampling theorem. The recording distance d is set to be 265 mm Figs. 3(b), 3(c) and 3(d) show a sequence of amplitude images reconstructed at different z planes located at distances 240 mm, 265 mm and 290 mm, respectively, from the hologram plane. The real and unsharp virtual images of the object are separated because of the off-axis geometry. The real amplitude reconstructions, calculated by the FTM with the zero-order term filtered out, can be seen in the bottom-left part of the reconstructed area.

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Note the three reconstructions by FTM cannot be compared each other, their size being different in each case, since the reconstruction pixel given by $RP = \lambda d / N\Delta\xi$ depends on the reconstruction distance d .

Looking at the reconstructions shown in the Fig. 3, different features of the amplitude images are more or less highlighted or darkened as the reconstruction distance is changed. At 240 mm the left border of the letter "S" is in- focus; at 265 mm the "E" is in-focus in the middle of the logo script and at 290 mm the "M" letter on the left is in-focus. The result of application of ASA is shown in Fig. 3(e). The target is reconstructed on a plane tilted at angle $\theta = 45^{\circ}$. Now the features of the patterned letters appear to be highlighted uniformly but the reconstructed image appears clearly shrunk in the horizontal *x* direction, since the pixel size $\Delta x \times \Delta y$ is different along the *x* and *y* directions of the tilted plane.

In fact, the spatial frequency resolution $\Delta\hat{u}$ in the horizontal *x* direction is related to the pixel size Δx by the relation $\Delta\hat{u} = 1/N\Delta x$ and, neglecting the w dependent term in the transformation law of $\hat{u}$, $\Delta\hat{u}$ is given to a first approximation by $\Delta\hat{u} = \Delta u \cos\theta$. Since $\Delta u = \Delta\xi/\lambda d$ we readly obtain $\Delta x = \lambda d/(N\Delta\xi\cos\theta)$. However, the spatial frequency along the *y* direction is unchanged, i.e. $\Delta\hat{v} = \Delta v = \Delta\eta/\lambda d$, and the pixel size along the *y* direction is $\Delta y = \lambda d/N\Delta\eta = \Delta x\cos\theta$, where in the last step we have assumed $\Delta\xi = \Delta\eta$.

Clearly, the greater the tilt angle, the larger the size of the reconstruction pixel in the tilted plane and the anamorphism of the reconstructed image.

Let us discuss a simple method for compensating this anamorphism which works straightforwardly in our case. In ASA we are in general confronted with the numerical problem of applying FFT algorithm to the RAS (see. Eq. (3)). The spacing of the sampling points of the RAS is not equal, due to the transformation law of the spatial frequencies and a method of interpolation must be unavoidable introduced into the sampled values of the RAS, given that FFT algorithm works properly for an equidistant sampled grid.

In order to obtain an evenly spaced grid of $N \times N$ interpolated values of the RAS, we use $\Delta\hat{v}$ as a frequency sampling interval for both the $\hat{u}$ and $\hat{v}$ ranges of the RAS. Since the $\hat{u}$ range is less than $\hat{v}$ range, a zero is introduced into the spectral array when the corresponding sampled $\hat{u}$ frequency does not fit into the $\hat{u}$ range. Taking the inverse Fourier transform we obtain a reconstructed image with a square pixel of size $\Delta x \times \Delta y = \lambda d/N\Delta\eta \times \lambda d/N\Delta\eta$ with corrected anamorphism as shown in Fig. 3(f). This correction procedure works in the Fourier space essentially the same the zero-padding method [14] does in the hologram space variables, to recover image resolution by FTM.

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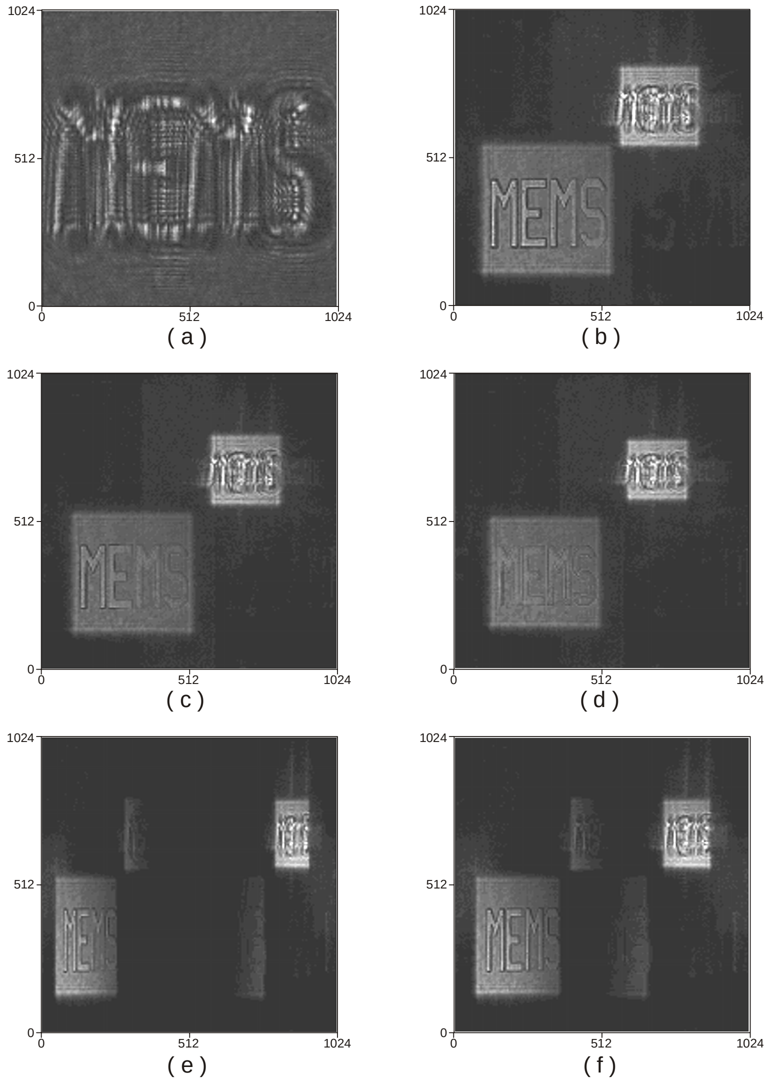

Fig. 3. (a) digitized hologram of the logo ”MEMS”; (b) amplitude image reconstruction at 240 mm; (c) amplitude image reconstruction at 265 mm; (d) amplitude image reconstruction at 290 mm; (e) reconstruction by the angular spectrum-based algorithm; (e)compensation of the anamorphism.

4. Conclusion

In conclusion, a two stage reconstruction algorithm for DHT has been proposed and experimentally demonstrated. The method is implemented by using FFT twice and coordinate rotation of the angular spectrum in the Fourier domain. The correction of the anamorphism resulting from the coordinate transformation is performed by suitable interpolation of the spectral data. It is worth to point out that although the described algorithm consider single-axis rotation of the reference coordinate it can be easily extended to more general case. The proposed method can be fruitfully exploited as a computationally fast method in tomographic applications by reconstructing the object field along a set of selected planes or for observing special features of an assembly of objects along selected directions, as in particle holography.