Computer vision and computer graphics analysis of paintings and drawings: An introduction to the literature

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ABSTRACT

In the past few years, a number of scholars trained in computer vision, pattern recognition, image processing, computer graphics, and art history have developed rigorous computer methods for addressing an increasing number of problems in the history of art. In some cases, these computer methods are more accurate than even highly trained connoisseurs, art historians and artists. Computer graphics models of artists' studios and subjects allow scholars to explore "what if" scenarios and determine artists' studio praxis. Rigorous computer ray-tracing software sheds light on claims that some artists employed optical tools. Computer methods will not replace tradition art historical methods of connoisseurship but enhance and extend them. As such, for these computer methods to be useful to the art community, they must continue to be refined through application to a variety of significant art historical problems.

Keywords: pattern recognition, computer image analysis, brush stroke analysis, painting analysis, image forensics, compositing, computer graphics reconstructions

1. INTRODUCTION

There is a long history of the use of sophisticated imaging and, in the past several decades *digital* imaging, in the study of art.[1] Shortly after the 19th-century discovery of x-rays such rays were used to reveal underdrawings and *pentimenti*. Later, infra-red photography and reflectography were exploited to similar ends; multispectra, fluoroesence and ultra-violet imaging have become a widespread, and used in revealing pigment composition and more.[2–5]

In such techniques, the resulting image is generally interpretted by an art scholar. In the past few years, however, we have entered a new era: one where some of the image interpretation relies in great part upon sophisticated algorithms developed from computer vision, the discipline seeking to make computers "see."[6,7] In some circumstances, computers can analyze certain aspects of perspective, lighting, color, the subtleties of the shapes of brush strokes better than even a trained art scholar, artist, or connoisseur. Rather than replacing connoisseurship, these methods—like other scientific methods such as imaging and material studies[8]—hold promise to enhance and extend it, just as microscopes extend the powers of biologists.

The source of the power of these computer methods arises from the following:

- The computer methods can rely on visual features that are hard to determine by eye, for instance subtle relationships among the structure of a brush stroke at different scales or colors, as in Perugino's *Holy family*, or lighting in de la Tour's *Christ in the carpenter's studio*, or perspective anomalies in van Eyck's *Arnolfini portrait*.
- The computer methods can abstract information from lots of visual evidence, for example, in principle *every* brush stroke executed by van Gogh and his contemporaries—a wealth of information that few scholars even experience, much less fully exploit.

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- Computer methods are objective—which need not mean they are "superior" to subjective methods, but rather promise to extend the language of to include terms that are not highly ambiguous. While today an art historian may describe a brush stroke as "bold" or "tentative" or "fluid" someday this scholar may also use technical terms and mathematical measures derived from computer image analysis, terms that other scholars will understand as well.
- Rigorous computer graphics techniques can reveal new three-dimensional views based on two-dimensional artwork, and provide new views into tableaus by dewarping the images reflected in mirrors depicted within a painting.

This brief paper lists some of these new computer techniques and how they have been used in the study of art. The set of topics and reference works here is by no means complete but is meant to show art scholars the power of these method and to encourage art scholars to propose new art historical problems amenable to attach through computer methods.[9–14] We shall not consider many other areas of computer use in arts, for instance computer art databases and retrieval, nor the task of *imaging* of art—the lighting, spectral filtering, exposure, and so on. Instead, we focus on the application of computer vision, image analysis and computer graphics algorithms to process and understand digital images of scanned art, particularly paintings and drawings.

We begin by describing traditional point- or pixel-based processes such as color adjustment, then consider algorithms based on a number of pixels in a digital image of a painting, and then successively more complex methods of high-level computer vision and graphics, such as dewarping, perspective analysis, lighting analysis, and three-dimensional computer modelling.

2. POINT-BASED PROCEDURES

Here and below we assume we have a digital image of a painting or drawing, the format required for computer analysis. The conceptually simplest class of computer image methods in the study of art are point- or pixel-based processing, that is, methods to alter the color and brightness of each pixel based solely on the color of that pixel. Such algorithms are better described as image *processing* than as image *analysis*.[15,16] Multispectral imaging and processing has been used for pigment analysis, color rejuvenation and predicting the effects of curatorial treatment.[17–22] Pixel-based image processing has been used to adjust the relative weights of different spectral bands to enhance readability,[23] as in the Archimedes palimpsest,[24–28] and to reveal details and structure in art that otherwise difficult to discern by the unaided eye.[29]

The range of light intensities in the natural world—from a darkened room to bright sunlight—spans as much as a factor of 10^{14} while the dynamic range in oil painting might be a factor of merely 10^2, even in the works of Caravaggio, de la Tour, Joseph Wright of Derby and others who exploited *chiaroscuro*. As such all artists must compress the luminance range in their works.[30] Graham and Field explored the nonlinear compression of the dynamic range in several classes of realist art work, a process that is based on the individual pixel values.[31,32]

3. AREA-BASED PROCEDURES

A very large class of image processing algorithms involve *filtering* a source image, where the color or grayscale value of a pixel is a function of the values of pixels in an area or region of the input image. In linear filtering the output value (color or gray level) is a linear combination of the values of the input pixels, while in non-linear filtering allows arbitrary functions of the input pixels. Typically the input image is a photograph of a painting and the output image a digital image processed to reveal some properties that are difficult to discern by unaided eye. Such filtering can remove gradual variations in the color across a painting and leave or even enhance the edges or contours as, for instance, the (nonlinear) Canny edge detector.[33–35] The Chamfer transform (or distance transform) is useful for quantifying similarity or difference between two shapes, for example when testing the fidelity that artists can achieve using different copying methods.[36,37]

Another class of non-linear filters are the morphological operators. Such operators are generally used on binary (black and white) images rather than color or grayscale, where the *shape* (rather than the color) is the matter of interest. For example, a *skeletonization* operator yields a single-pixel-wide curve down the center of a black brush stroke, regardless of the varying width of the stroke.[38] Other popular morphological operators implement *erosion*, *dilation*, *opening* and *closing*. For instance, Stork, Meador and Noble compared the shapes of different passages of brick work in a painting from the Dutch Golden Age despite variations and irregularities in the width of the painted lines. To this end they preprocessed a high-resolution photograph of the painting using a morphological filter to create an image where the mortar lines were of uniform thickness. They then computed the cross-correlation of this image to search for repeated patterns.[33, 39]

4. PERSPECTIVE ANALYSIS

The analysis of perspective, scale and geometry has a long and important history in the study of realist art, particularly in art history of the Renaissance.[40] Most of these analytic methods involve simple drawing of perspective lines, finding horizon lines, vanishing points, and so on and can be done without computers. Recently, however, a number of sophisticated computer-based methods for analysis of perspective and geometry have been developed. Criminisi and his colleagues have pioneered rigorous methods for recovering three-dimensional space from single "uncalibrated" images, such as in paintings. These methods have produced three-dimensional virtual spaces of artworks such as Masaccio's *Trinità*.[41–44] Smith, Stork and Zhang reconstructed the three-dimensional space of the tableau based on multiple views depicted in plane reflections within a single painting.[45] This method also reveals spatial inconsistencies between direct and reflected views and thereby sheds light on the artist's working methods.

While one can use simple commercial software, such as Adobe *Photoshop*, to perform perspective transformation between two images or passages—for instance two arms on the chandelier in van Eyck's *Arnolfini portrait*[46]—but such a technique suffers from a number of drawbacks, the most severe is that the experimenter can arbitrarily choose which portions of one image should match their partners in the other image. Criminisi derived rigorous, principled methods for finding optimal transformations that minimized the shape differences, thus eliminating this drawback.[36, 42, 47] Such analysis of perspective, perspective inconsistencies, and subtleties in shape have shed light on a number of topics, including the question of whether an artist used optical aids.[48–58]

5. ANAMORPHIC ART

Anamorphic art is distorted art that appears distorted when viewed directly but undistorted when viewed from a special position or in reflection from a curved surface. Such a mirrored cylinder or cone is called an "anamorphoscope." The root word comes from the Greek $\alpha\nu\alpha$, "again," and $\mu o\rho\phi\eta$, "form"—the image in anamorphic art is *formed again*. The earliest surviving deliberate anamorphic image appears to be a sketch of a slant anamorphic eye, drawn by Leonardo around 1485. Perhaps the most celebrated example of such slant anamorphoses is the skull along the bottom in Hans Holbein's *The ambassadors*.[59, 60] There was much experimentation, mathematical analysis and flourishing of anamorphic art in the seventeenth and eighteenth centuries, particularly in France. Today the transformations required by anamorphic art are easily performed by computer. The optics and perspective underlying such art appears in a number of basic texts,[30, 61] but perhaps the most complete and rigorous explanation is given by Hunt, Nickel and Gigault.[62, 63]

6. DEWARPING OF CURVED ART

Many frescos and mosaics on architectural spandrels, barrel vaults and markings on pottery are curved and warped, and art scholars use computer methods to dewarp them to better study the images.[64, 65] Often such digital dewarping requires an estimate of the camera position and the curvature properties of the surface itself. Another class of dewarping centers on dewarping the virtual image appearing in depictions of curved mirrors, such as in Parmigianino's *Self portrait in a convex mirror*, van Eyck's *Arnolfini portrait*, and Campin's *Heinrich von Werl and St. John the Baptist*. Here one models the optics of the spherical or parabolic mirror and adjusts parameters to yield an undistorted image.[41, 66, 67] The mirror properties (curvature and focal length) inferred by such a technique have been used to address claims that artists used such mirrors for optical projectors.[49, 50, 54, 55] On can also use computer graphics methods (cf., Sect. **??**, below) to dewarp reflected images, for instance in the analysis of Hans Memling's *van Nieuwenhove Diptych*.[67]

7. ANALYSIS OF LIGHTING AND ILLUMINATION

Some problems in art history require knowing or estimating the position and direction of lighting in a tableau. Such knowledge can be used for determining the studio conditions when the painting was executed; significant differences in the lighting on different subjects within a tableau may indicate the different studio conditions or presence of different hands, for example. Moreover, this information may indicate whether the artist used optical aids: if the light source was local rather than distant solar illumination, for instance, then it is highly unlikely projections were used.[68]

If the illuminant can be assumed to be small and relatively distant from the tableau, the simple method of cast-shadow analysis is particularly effective in locating the illuminant: one merely draws a line from a point on a cast shadow, through its associated occluder. This line should pass through the position of the illuminant.[69] Several such lines, from a set of occluder-shadow pairs, will intersect at the position of the illuminant.

A more sophisticated method, occluding-contour analysis, derives from forensic analysis of digital photographs, and is based on the pattern of light along an object's outer boundary or occluding contour.[70] Stork and Johnson recently extended the technique to apply to the case of *diffuse* illumination, where the pattern of illumination along an occluding boundary is described as a weighted set of spherical harmonics. If two figures in a painting differ significantly in their sets of coefficients, then they were likely painted under different studio lighting conditions. They studied the lighting in different figures by realist portraitist Garth Herrick and showed that different subjects were executed under different illumination conditions.[71] Stork and Kale modeled the physics flat surfaces, and thereby inferred the position of the illuminant from the floor in Georges de la Tour's *Christ in the carpenter's studio* and Caravaggio's *The calling of St. Matthew*.[72,73] This analysis showed that the light source in these works was likely *local*, rather than distant solar illumination, a result that rebuts claims that these works were executed by means of optical projections of very bright tableaus. Bayesian statistical methods can be used to integrate estimates derived from different sources, for example cast shadows and occluding contours, thereby refining and improving overall estimates.[70,74]

Computer shape-from-shading methods infer the properties of illumination given a known or assumed three-dimensional model of objects and a shaded image of those objects, such as in a highly realistic painting. One can assume a generic three-dimensional model (for instance the face in *Girl with a pearl earring*) and refine both the model and the direction to illumination.[75] Another method for estimating the lighting in a realist tableau is to create a full computer graphics model of the scene and adjust the positions of the virtual illuminants so that the rendered image matches the painting as closely as possible. This has been used to estimate the direction of illumination in Vermeer's *Girl with a pearl earring*, and Georges de la Tour's *Christ in the carpenter's studio*,[75,76] as described in Sect. 12.

8. ANALYSIS OF BRUSH STROKES AND MARKS

One of the most extensively explored areas of computer analysis of art is the analysis of marks and drawing tools. The basic approach is to use techniques of statistical pattern recognition to learn visual properties of brush strokes that correspond to a particular painter or marking tool.[33,38,77–95] In related techniques, Hedges analyzed the changes in marks in Renaissance copperplate prints as the plates were cleaned; his method yielded an image-based "clock" for estimating the age of such prints.[96,97] Shahram, Stork and Donoho developed the `De-pict` algorithm, which removed successive layers of brush strokes in a digital image of a painting, such as van Gogh's *Self portrait in a grey felt hat*. When such a sequence of images is displayed in "reverse" (i.e., showing the sequence of brush strokes as likely added to the painting), scholars can see and better understand the aesthetic choices made by the artist.[98]

A somewhat separate class of mark analyses are those for the analysis of dripped painting, particularly in the works of American Abstract Expressionist Jackson Pollock. Here the analyses are generally based on fractals, a mathematical structure that exhibits regularities at different scales or sizes.[99] Taylor and his colleagues first proposed that Pollock's drip paintings exhibited fractal structure,[100] though a number of scholars have questioned the recent claim that simple fractal dimension suffices to distinguish genuine Pollocks from forgeries or other apparently random images.[101–106] Recent work has returned to the application of traditional image measures for the analysis of Pollock's works, for instance curvature, connected components, and statistical pattern classification methods.[?,107]

9. OPTICAL ANALYSIS OF ART

It is well known that some artists used optical aids during the execution of passages in some of their works, e.g., Canaletto, Thomas Eakins, photo-realists such as Richard Estes and Robert Bechtle, and many others. Computer image analysis has addressed claims that some artists used optical aids when executing some passages in some of their works, for instance that Lorenzo Lotto secretly used a concave mirror projector to execute *Husband and wife*,[108] that a wide range of artists as early as 1430 secretly traced optical images,[109] that Jan Vermeer traced images projected in a camera obscura,[110] and so on. While there are a number of perspective and lighting analyses brought to bear on such claims,[42,57,111–114] as well as textual and material analyses,[115,116] the first and only analysis of paintings done by sophisticated computer ray tracing programs was by Stork and Robinson.[117,118] This research analyzed the aberrations and other properties of the setup in purported use of optics for Lorenzo Lotto and ultimately questioned the claim he used optics.

10. ANALYSIS OF CRAQUELURE

Craquelure is the pattern of fine cracks on the surface of paintings and several scholars have used computer image analysis to characterize the patterns for art imagery retrieval.[119–121] There remain opportunities for additional algorithms, for instance to detect and classify changes to craquelure due to injury to paintings.

11. ANALYSIS OF COMPOSITION

Many artists have characteristic compositional styles and it is natural that computer methods be applied to learning and classifying these styles. A particularly attractive oeuvre is that of the neo-plastic abstractionist Piet Mondrian, where the formal elements are simple (horizontal and vertical lines, rectangles of a small number of colors, etc.) and the two-dimensional composition of central importance. A few scholars have approached this problem for Mondrian[122,123] but there are additional painters whose works might yield information through these methods, such as the large Abstract Expressionist works of Franz Kline.

12. COMPUTER GRAPHICS

Computer graphics allows scholars to understand realist artists' working methods by exploring "what if" scenarios. Note that creating a three-dimensional model based on a two-dimensional painting is formally "ill-posed," that is, an infinite number of three-dimensional tableaus are consistent with a given two-dimensional projection.[124] As such, the creation of a *tableau virtuel* is part art, part science. Nevertheless, the weal the (2D) image information such as occlusion and physical constraints such as objects toughing or being supported on a given floor, and lighting consistency, strongly constrain the three-dimensional models. It is important that the assumption—for instance that bodies have normal proportions, that faces are approximately left-right symmetric, and so on—not bias or favor one conclusion over another.

Stork and Furuichi built a full three-dimensional model of Georges de la Tour's *Christ in the carpenter's studio* and adjusted the location of the virtual illuminant in the *tableau virtuel* until the digitally rendered image matched the painting as closely as possible. In this way, these authors found the illuminant was likely at the position of the candle, rather than in place of the figures, and thereby rebutted the claim that painting was executed by means of optical projections.[76] Savarese and colleagues built a very simple model of the convex mirror and a planar model of the tableau in the left panel of Hans Memling's *van Nieuwenhove Diptych* to test the consistency between the tableau and the image depicted in the convex mirror. The discrepancies between the mirror and the simple tableau suggested that Memling added the mirror later, as an afterthought.[67] Johnson and colleagues built a computer model of Vermeer's *Girl with a pearl earring* to estimate the direction of illumination.[75] Most recently, Stork and Furuichi created a model of both the tableau in Diego Velàsquez' *Las meninas* as well as the viewer's space to explore the relationship between these two spaces, for instance whether the position of the viewer corresponded to that of the king and queen visible in the plane mirror.[125]

13. WEBSITES

There are a few of websites addressing computer image analysis of art.

- Computer image analysis in the study of art: `www.diatrope.com/stork/FAQs.html`
- Digital painting analysis: `digitalpaintinganalysis.org`
- IAPR computer vision in cultural heritage applications: `iapr-tc19.prip.tuwien.ac.at/`
- Antonio Criminisi's publications: `research.microsoft.com/∼antcrim/papers.htm`
- Christopher W. Tyler's research on perception of art by humans and machines: `www.diatrope.com/projects`
- Computers in the history of art: `www.chart.ac.uk`

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