Linear Regression with Multiple Features

Boston houses dataset

In [1]:

```
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
```

In [2]:

```
from sklearn.datasets import load_boston
```

In [3]:

```
boston = load_boston()

X = boston.data
y = boston.target
```

In [4]:

```
X
```

Out[4]:

```
array([[6.3200e-03, 1.8000e+01, 2.3100e+00, ..., 1.5300e+01, 3.9690e+02,
        4.9800e+00],
       [2.7310e-02, 0.0000e+00, 7.0700e+00, ..., 1.7800e+01, 3.9690e+02,
        9.1400e+00],
       [2.7290e-02, 0.0000e+00, 7.0700e+00, ..., 1.7800e+01, 3.9283e+02,
        4.0300e+00],
       ...,
       [6.0760e-02, 0.0000e+00, 1.1930e+01, ..., 2.1000e+01, 3.9690e+02,
        5.6400e+00],
       [1.0959e-01, 0.0000e+00, 1.1930e+01, ..., 2.1000e+01, 3.9345e+02,
        6.4800e+00],
       [4.7410e-02, 0.0000e+00, 1.1930e+01, ..., 2.1000e+01, 3.9690e+02,
        7.8800e+00]])
```

In [5]:

```
y
```

Out[5]:

```
array([24. , 21.6, 34.7, 33.4, 36.2, 28.7, 22.9, 27.1, 16.5, 18.9, 15. ,
       18.9, 21.7, 20.4, 18.2, 19.9, 23.1, 17.5, 20.2, 18.2, 13.6, 19.6,
       15.2, 14.5, 15.6, 13.9, 16.6, 14.8, 18.4, 21. , 12.7, 14.5, 13.2,
       13.1, 13.5, 18.9, 20. , 21. , 24.7, 30.8, 34.9, 26.6, 25.3, 24.7,
       21.2, 19.3, 20. , 16.6, 14.4, 19.4, 19.7, 20.5, 25. , 23.4, 18.9,
       35.4, 24.7, 31.6, 23.3, 19.6, 18.7, 16. , 22.2, 25. , 33. , 23.5,
       19.4, 22. , 17.4, 20.9, 24.2, 21.7, 22.8, 23.4, 24.1, 21.4, 20. ,
       20.8, 21.2, 20.3, 28. , 23.9, 24.8, 22.9, 23.9, 26.6, 22.5, 22.2,
       23.6, 28.7, 22.6, 22. , 22.9, 25. , 20.6, 28.4, 21.4, 38.7, 43.8,
       33.2, 27.5, 26.5, 18.6, 19.3, 20.1, 19.5, 19.5, 20.4, 19.8, 19.4,
       21.7, 22.8, 18.8, 18.7, 18.5, 18.3, 21.2, 19.2, 20.4, 19.3, 22. ,
       20.3, 20.5, 17.3, 18.8, 21.4, 15.7, 16.2, 18. , 14.3, 19.2, 19.6,
       23. , 18.4, 15.6, 18.1, 17.4, 17.1, 13.3, 17.8, 14. , 14.4, 13.4,
       15.6, 11.8, 13.8, 15.6, 14.6, 17.8, 15.4, 21.5, 19.6, 15.3, 19.4,
       17. , 15.6, 13.1, 41.3, 24.3, 23.3, 27. , 50. , 50. , 50. , 22.7,
```

```
       25. , 50. , 23.8, 23.8, 22.3, 17.4, 19.1, 23.1, 23.6, 22.6, 29.4,
       23.2, 24.6, 29.9, 37.2, 39.8, 36.2, 37.9, 32.5, 26.4, 29.6, 50. ,
       32. , 29.8, 34.9, 37. , 30.5, 36.4, 31.1, 29.1, 50. , 33.3, 30.3,
       34.6, 34.9, 32.9, 24.1, 42.3, 48.5, 50. , 22.6, 24.4, 22.5, 24.4,
       20. , 21.7, 19.3, 22.4, 28.1, 23.7, 25. , 23.3, 28.7, 21.5, 23. ,
       26.7, 21.7, 27.5, 30.1, 44.8, 50. , 37.6, 31.6, 46.7, 31.5, 24.3,
       31.7, 41.7, 48.3, 29. , 24. , 25.1, 31.5, 23.7, 23.3, 22. , 20.1,
       22.2, 23.7, 17.6, 18.5, 24.3, 20.5, 24.5, 26.2, 24.4, 24.8, 29.6,
       42.8, 21.9, 20.9, 44. , 50. , 36. , 30.1, 33.8, 43.1, 48.8, 31. ,
       36.5, 22.8, 30.7, 50. , 43.5, 20.7, 21.1, 25.2, 24.4, 35.2, 32.4,
       32. , 33.2, 33.1, 29.1, 35.1, 45.4, 35.4, 46. , 50. , 32.2, 22. ,
       20.1, 23.2, 22.3, 24.8, 28.5, 37.3, 27.9, 23.9, 21.7, 28.6, 27.1,
       20.3, 22.5, 29. , 24.8, 22. , 26.4, 33.1, 36.1, 28.4, 33.4, 28.2,
       22.8, 20.3, 16.1, 22.1, 19.4, 21.6, 23.8, 16.2, 17.8, 19.8, 23.1,
       21. , 23.8, 23.1, 20.4, 18.5, 25. , 24.6, 23. , 22.2, 19.3, 22.6,
       19.8, 17.1, 19.4, 22.2, 20.7, 21.1, 19.5, 18.5, 20.6, 19. , 18.7,
       32.7, 16.5, 23.9, 31.2, 17.5, 17.2, 23.1, 24.5, 26.6, 22.9, 24.1,
       18.6, 30.1, 18.2, 20.6, 17.8, 21.7, 22.7, 22.6, 25. , 19.9, 20.8,
       16.8, 21.9, 27.5, 21.9, 23.1, 50. , 50. , 50. , 50. , 50. , 13.8,
       13.8, 15. , 13.9, 13.3, 13.1, 10.2, 10.4, 10.9, 11.3, 12.3,  8.8,
        7.2, 10.5,  7.4, 10.2, 11.5, 15.1, 23.2,  9.7, 13.8, 12.7, 13.1,
       12.5,  8.5,  5. ,  6.3,  5.6,  7.2, 12.1,  8.3,  8.5,  5. , 11.9,
       27.9, 17.2, 27.5, 15. , 17.2, 17.9, 16.3,  7. ,  7.2,  7.5, 10.4,
        8.8,  8.4, 16.7, 14.2, 20.8, 13.4, 11.7,  8.3, 10.2, 10.9, 11. ,
        9.5, 14.5, 14.1, 16.1, 14.3, 11.7, 13.4,  9.6,  8.7,  8.4, 12.8,
       10.5, 17.1, 18.4, 15.4, 10.8, 11.8, 14.9, 12.6, 14.1, 13. , 13.4,
       15.2, 16.1, 17.8, 14.9, 14.1, 12.7, 13.5, 14.9, 20. , 16.4, 17.7,
       19.5, 20.2, 21.4, 19.9, 19. , 19.1, 19.1, 20.1, 19.9, 19.6, 23.2,
       29.8, 13.8, 13.3, 16.7, 12. , 14.6, 21.4, 23. , 23.7, 25. , 21.8,
       20.6, 21.2, 19.1, 20.6, 15.2,  7. ,  8.1, 13.6, 20.1, 21.8, 24.5,
       23.1, 19.7, 18.3, 21.2, 17.5, 16.8, 22.4, 20.6, 23.9, 22. , 11.9])
```

In [6]:

```
print(X.shape, y.shape)
```

```
(506, 13) (506,)
```

In [7]:

```
df = pd.DataFrame(X)
df.head()
```

Out[7]:

	0	1	2	3	4	5	6	7	8	9	10	11	12
0	0.00632	18.0	2.31	0.0	0.538	6.575	65.2	4.0900	1.0	296.0	15.3	396.90	4.98
1	0.02731	0.0	7.07	0.0	0.469	6.421	78.9	4.9671	2.0	242.0	17.8	396.90	9.14
2	0.02729	0.0	7.07	0.0	0.469	7.185	61.1	4.9671	2.0	242.0	17.8	392.83	4.03
3	0.03237	0.0	2.18	0.0	0.458	6.998	45.8	6.0622	3.0	222.0	18.7	394.63	2.94
4	0.06905	0.0	2.18	0.0	0.458	7.147	54.2	6.0622	3.0	222.0	18.7	396.90	5.33

In [8]:

```
dfY = pd.DataFrame(y)
dfY.head()
```

Out[8]:

	0
0	24.0
1	21.6
2	34.7
3	33.4
4	36.2

0

In [9]:

```
boston.feature_names
```

Out[9]:

```
array(['CRIM', 'ZN', 'INDUS', 'CHAS', 'NOX', 'RM', 'AGE', 'DIS', 'RAD',
       'TAX', 'PTRATIO', 'B', 'LSTAT'], dtype='<U7')
```

In [10]:

```
print(boston.DESCR)
```

```
.. _boston_dataset:

Boston house prices dataset
---------------------------

**Data Set Characteristics:**

    :Number of Instances: 506

    :Number of Attributes: 13 numeric/categorical predictive. Median Value (attribute 14)
is usually the target.

    :Attribute Information (in order):
        - CRIM     per capita crime rate by town
        - ZN       proportion of residential land zoned for lots over 25,000 sq.ft.
        - INDUS    proportion of non-retail business acres per town
        - CHAS     Charles River dummy variable (= 1 if tract bounds river; 0 otherwise)
        - NOX      nitric oxides concentration (parts per 10 million)
        - RM       average number of rooms per dwelling
        - AGE      proportion of owner-occupied units built prior to 1940
        - DIS      weighted distances to five Boston employment centres
        - RAD      index of accessibility to radial highways
        - TAX      full-value property-tax rate per $10,000
        - PTRATIO  pupil-teacher ratio by town
        - B        1000(Bk - 0.63)^2 where Bk is the proportion of blacks by town
        - LSTAT    % lower status of the population
        - MEDV     Median value of owner-occupied homes in $1000's

    :Missing Attribute Values: None

    :Creator: Harrison, D. and Rubinfeld, D.L.

This is a copy of UCI ML housing dataset.
https://archive.ics.uci.edu/ml/machine-learning-databases/housing/


This dataset was taken from the StatLib library which is maintained at Carnegie Mellon Un
iversity.

The Boston house-price data of Harrison, D. and Rubinfeld, D.L. 'Hedonic
prices and the demand for clean air', J. Environ. Economics & Management,
vol.5, 81-102, 1978.   Used in Belsley, Kuh & Welsch, 'Regression diagnostics
...', Wiley, 1980.   N.B. Various transformations are used in the table on
pages 244-261 of the latter.

The Boston house-price data has been used in many machine learning papers that address re
gression
problems.

.. topic:: References

   - Belsley, Kuh & Welsch, 'Regression diagnostics: Identifying Influential Data and Sou
rces of Collinearity', Wiley, 1980. 244-261.
   - Quinlan,R. (1993). Combining Instance-Based and Model-Based Learning. In Proceedings
on the Tenth International Conference of Machine Learning, 236-243, University of Massach
usetts, Amherst. Morgan Kaufmann.
```

In [11]:

```
boston.filename
```

Out[11]:

```
'c:\\users\\mayank\\desktop\\python\\lib\\site-packages\\sklearn\\datasets\\data\\boston_
house_prices.csv'
```

In [12]:

```
df = pd.DataFrame(X)
df.columns = boston.feature_names
df.head()
```

Out[12]:

	CRIM	ZN	INDUS	CHAS	NOX	RM	AGE	DIS	RAD	TAX	PTRATIO	B	LSTAT
0	0.00632	18.0	2.31	0.0	0.538	6.575	65.2	4.0900	1.0	296.0	15.3	396.90	4.98
1	0.02731	0.0	7.07	0.0	0.469	6.421	78.9	4.9671	2.0	242.0	17.8	396.90	9.14
2	0.02729	0.0	7.07	0.0	0.469	7.185	61.1	4.9671	2.0	242.0	17.8	392.83	4.03
3	0.03237	0.0	2.18	0.0	0.458	6.998	45.8	6.0622	3.0	222.0	18.7	394.63	2.94
4	0.06905	0.0	2.18	0.0	0.458	7.147	54.2	6.0622	3.0	222.0	18.7	396.90	5.33

In [13]:

```
df.describe()
```

Out[13]:

	CRIM	ZN	INDUS	CHAS	NOX	RM	AGE	DIS	RAD	TA
count	506.000000	506.000000	506.000000	506.000000	506.000000	506.000000	506.000000	506.000000	506.000000	506.0000
mean	3.613524	11.363636	11.136779	0.069170	0.554695	6.284634	68.574901	3.795043	9.549407	408.2371
std	8.601545	23.322453	6.860353	0.253994	0.115878	0.702617	28.148861	2.105710	8.707259	168.5371
min	0.006320	0.000000	0.460000	0.000000	0.385000	3.561000	2.900000	1.129600	1.000000	187.0000
25%	0.082045	0.000000	5.190000	0.000000	0.449000	5.885500	45.025000	2.100175	4.000000	279.0000
50%	0.256510	0.000000	9.690000	0.000000	0.538000	6.208500	77.500000	3.207450	5.000000	330.0000
75%	3.677083	12.500000	18.100000	0.000000	0.624000	6.623500	94.075000	5.188425	24.000000	666.0000
max	88.976200	100.000000	27.740000	1.000000	0.871000	8.780000	100.000000	12.126500	24.000000	711.0000

In [14]:

```
u = np.mean(X, axis = 0)
std = np.std(X, axis = 0)
print("Mean:", u)
print("Std:", std)
print(u.shape, std.shape)
```

```
Mean: [3.61352356e+00 1.13636364e+01 1.11367787e+01 6.91699605e-02
 5.54695059e-01 6.28463439e+00 6.85749012e+01 3.79504269e+00
 9.54940711e+00 4.08237154e+02 1.84555336e+01 3.56674032e+02
 1.26530632e+01]
Std: [8.59304135e+00 2.32993957e+01 6.85357058e+00 2.53742935e-01
 1.15763115e-01 7.01922514e-01 2.81210326e+01 2.10362836e+00
 8.69865112e+00 1.68370495e+02 2.16280519e+00 9.12046075e+01
 7.13400164e+00]
(13,) (13,)
```

In [15]:

```
X = (X - u) / std
```

In [16]:

```
X
```

Out[16]:

```
array([[-0.41978194,  0.28482986, -1.2879095 , ..., -1.45900038,
         0.44105193, -1.0755623 ],
       [-0.41733926, -0.48772236, -0.59338101, ..., -0.30309415,
         0.44105193, -0.49243937],
       [-0.41734159, -0.48772236, -0.59338101, ..., -0.30309415,
         0.39642699, -1.2087274 ],
       ...,
       [-0.41344658, -0.48772236,  0.11573841, ...,  1.17646583,
         0.44105193, -0.98304761],
       [-0.40776407, -0.48772236,  0.11573841, ...,  1.17646583,
         0.4032249 , -0.86530163],
       [-0.41500016, -0.48772236,  0.11573841, ...,  1.17646583,
         0.44105193, -0.66905833]])
```

In [17]:

```
df = pd.DataFrame(X)
df.columns = boston.feature_names
df.head()
```

Out[17]:

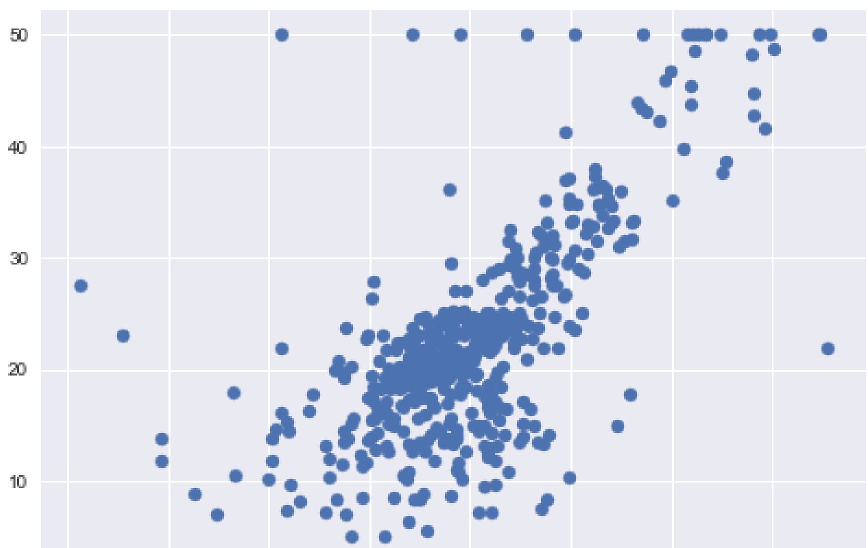
	CRIM	ZN	INDUS	CHAS	NOX	RM	AGE	DIS	RAD	TAX	PTRATIO	B	
0	-0.419782	0.284830	-1.287909	-0.272599	-0.144217	0.413672	-0.120013	0.140214	-0.982843	-0.666608	-1.459000	0.441052	1.
1	-0.417339	-0.487722	-0.593381	-0.272599	-0.740262	0.194274	0.367166	0.557160	-0.867883	-0.987329	-0.303094	0.441052	0.
2	-0.417342	-0.487722	-0.593381	-0.272599	-0.740262	1.282714	-0.265812	0.557160	-0.867883	-0.987329	-0.303094	0.396427	1.
3	-0.416750	-0.487722	-1.306878	-0.272599	-0.835284	1.016303	-0.809889	1.077737	-0.752922	-1.106115	0.113032	0.416163	1.
4	-0.412482	-0.487722	-1.306878	-0.272599	-0.835284	1.228577	-0.511180	1.077737	-0.752922	-1.106115	0.113032	0.441052	1.

In [18]:

```
plt.style.use('seaborn')
```

In [19]:

```
plt.scatter(X[:, 5], y)
plt.show()
```



Implementation

In [12]:

```
# Make the Data X of 14 columns by appending a ones column
print(X.shape)
ones = np.ones((X.shape[0], 1))
X = np.hstack((ones, X))
print(X.shape)
```

```
(506, 13)
(506, 14)
```

In [13]:

```
X[:5, :5]
```

Out[13]:

```
array([[ 1.        , -0.41978194,  0.28482986, -1.2879095 , -0.27259857],
       [ 1.        , -0.41733926, -0.48772236, -0.59338101, -0.27259857],
       [ 1.        , -0.41734159, -0.48772236, -0.59338101, -0.27259857],
       [ 1.        , -0.41675042, -0.48772236, -1.30687771, -0.27259857],
       [ 1.        , -0.41248185, -0.48772236, -1.30687771, -0.27259857]])
```

In [68]:

```
def hypothesis(x, theta):
    y_ = 0.0
    for i in range(x.shape[0]):
        y_ += x[i] * theta[i]
    return y_
```

In [69]:

```
def error(X, y, theta):
    e = 0.0
    m = X.shape[0]
    for i in range(m):
        y_ = hypothesis(X[i], theta)
        e += (y_ - y[i]) ** 2
    return e / m
```

In [70]:

```
def gradient(X, y, theta):
    m, n = X.shape
    grad = np.zeros((n,))

    for i in range(m):
        y_ = hypothesis(X[i], theta)
        for j in range(n):
            grad[j] += (y_ - y[i]) * X[i][j]

    return grad / m
```

In [71]:

```
def gradient_descent(X, y, lr = 0.1, max_epochs = 300):
```

```python
    m, n = X.shape
    error_list = []
    theta_list = []
    theta = np.zeros((n,))
    for i in range(max_epochs):
        e = error(X, y, theta)
        error_list.append(e)
        theta_list.append(theta)

        grad = gradient(X, y, theta)
        for i in range(n):
            theta[i] = theta[i] - lr * grad[i]

    return theta, error_list, theta_list
```

In [72]:

```python
theta, error_list, theta_list = gradient_descent(X, y)
```

In [73]:

```python
theta
```

Out[73]:

```
array([ 2.25328063e+01, -9.03091692e-01,  1.03815625e+00,  1.53477685e-02,
        6.99554920e-01, -2.02101672e+00,  2.70014278e+00, -1.93085233e-03,
       -3.10234837e+00,  2.34354753e+00, -1.72031485e+00, -2.04614394e+00,
        8.47845679e-01, -3.73089521e+00])
```

In [74]:

```python
error_list
```

Out[74]:

```
[592.1469169960473,
 462.46044906845856,
 375.385275095727,
 307.6933342962929,
 253.39168634679373,
 209.55988275906455,
 174.1302049153626,
 145.47824715741677,
 122.30009891984608,
 103.54456174692926,
 88.36328034470087,
 76.07131291555939,
 66.11549356461218,
 58.04896715358394,
 51.51067344934862,
 46.208810615526716,
 41.90750109402469,
 38.41603601946844,
 35.58019677482987,
 33.275250473323695,
 31.40029493330841,
 29.873691975646707,
 28.629378705367383,
 27.61388730734378,
 26.78393675990612,
 26.104486327537224,
 25.547161996504887,
 25.08898417710864,
 24.711338824412547,
 24.399145276935656,
 24.14018310253681,
 23.924547492728298,
 23.744208598563592,
 23.592654924717937,
 23.464604712017078,
 23.355772318479673,
 23.2626790967033
```

```
23.262679096703 58,
23.182500275377254,
23.1129409769508,
23.052135816286278,
22.998567586378837,
22.951001395282688,
22.908431312249895,
22.870037142264884,
22.835149402084745,
22.803220938113167,
22.773803923546097,
22.746531212641212,
22.721101224522528,
22.697265686396182,
22.674819693511417,
22.65359364637796,
22.633446709280467,
22.614261501761465,
22.595939789505692,
22.578398985402426,
22.561569307476553,
22.545391469463727,
22.52981480336543,
22.514795732401403,
22.50029652823866,
22.486284298899058,
22.47273016389668,
22.45960858137694,
22.446896798695192,
22.43457440326862,
22.42262295491569,
22.411025684441583,
22.399767246104748,
22.388833513931242,
22.37821141373254,
22.36788878421599,
22.357854261818904,
22.348097184906404,
22.338607513789555,
22.329375763684727,
22.320392948272374,
22.31165053195109,
22.303140389236233,
22.294854770041447,
22.28678626981419,
22.27892780368662,
22.271272583957163,
22.263814100343527,
22.256546102550608,
22.249462584777735,
22.242557771859154,
22.235826106785343,
22.229262239397954,
22.22286101608726,
22.216617470351263,
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```
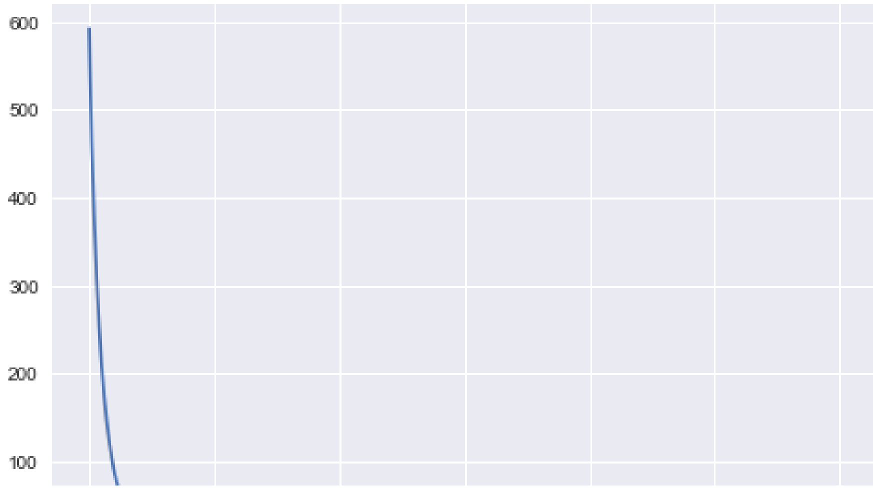
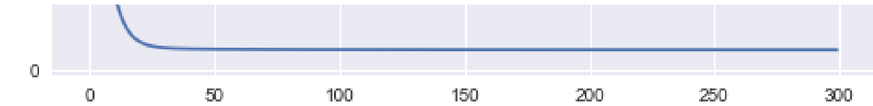

In [75]:

```
plt.plot(error_list)
```

Out[75]:

```
[<matplotlib.lines.Line2D at 0x1ca4deb0>]
```





In [76]:

```
import time
start = time.time()
theta, error_list, theta_list = gradient_descent(X, y)
end = time.time()

print("Time taken: ", end - start)
```

```
Time taken:  8.01843547821045
```

In [77]:

```
theta
```

Out[77]:

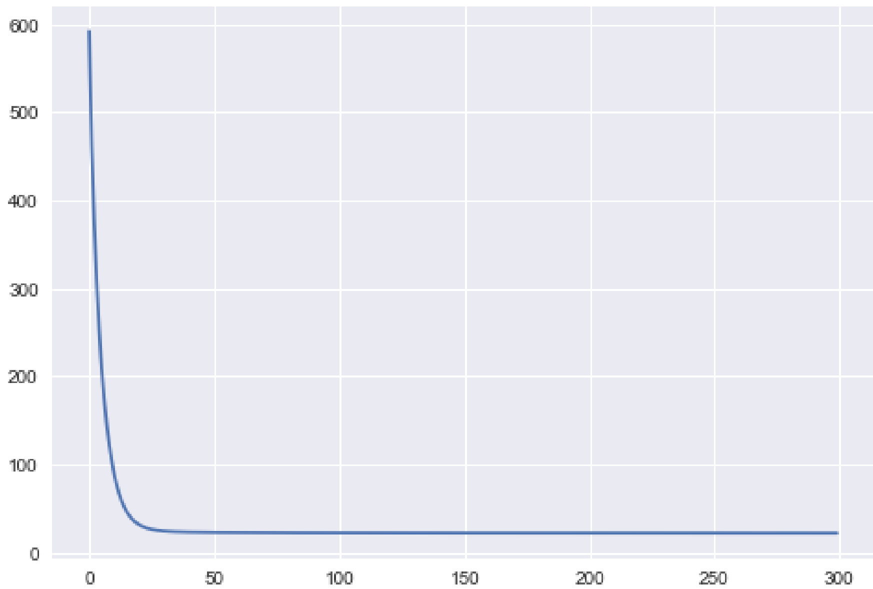
```
array([ 2.25328063e+01, -9.03091692e-01,  1.03815625e+00,  1.53477685e-02,
        6.99554920e-01, -2.02101672e+00,  2.70014278e+00, -1.93085233e-03,
       -3.10234837e+00,  2.34354753e+00, -1.72031485e+00, -2.04614394e+00,
        8.47845679e-01, -3.73089521e+00])
```

In [79]:

```
plt.plot(error_list)
```

Out[79]:

```
[<matplotlib.lines.Line2D at 0x1c8aac28>]
```



In [80]:

```
y_ = []
for i in range(X.shape[0]):
    pred = hypothesis(X[i], theta)
    y_.append(pred)
print(y_)
```

```
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```

In [81]:

```python
y_ = np.array(y_)
print(y_)
```

```
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 20.89285327 19.60128149 19.32367393 19.3458686  20.59832729 16.95783793
 16.23493881 18.44548854 12.55479461 17.70925422 15.88547043 13.84190443
 15.71570947 13.4205508  15.49775358 14.75818241 19.59955913 20.93977682
 11.49745095 18.09361635  8.88301038 14.31610222 13.75983633 23.77730951
 22.30988941 23.08772244 22.90940545 31.27562517 34.14934567 27.99150715
 25.15161107 24.56195221 22.86665722 22.01299699 20.35079995 17.94239765
  9.0032359  17.10959314 21.16463059 23.8404942  27.56319999 23.94281148
 15.70766609 30.96324445 24.93746243 32.91466583 21.63293708 20.92400291
 17.6977369  18.32617385 23.82616369 22.4283064  23.34895856 30.37522177
 25.5255783  21.2306401  17.52571154 20.89494847 25.20175729 21.72374982
 24.54156678 24.03905863 25.62679124 24.06381642 23.02484813 23.44984561
 21.35657377 22.51629435 28.40848967 26.96899155 26.0263214  25.03478105
 24.79279036 27.79575063 22.17067399 25.88154887 30.78085348 30.95800581
 27.22453062 27.51066854 28.71288943 28.859444   26.79054644 28.75132406
 24.83596211 35.94281395 35.29511986 32.40179358 24.75516584 25.77444454
 19.95640618 20.46644109 21.58632495 18.68176932 17.33656725 20.90711862
 22.80555026 19.93114262 20.83065748 26.72618732 20.94596258 20.89490035
 25.35381775 20.60731806 23.5681151  23.86484071 20.52156256 20.97469607
 21.46430039 22.01300726 20.09753035 15.91329192 20.09521555 22.02074575
 14.15984655 15.25545131 19.03740999 14.1352851  20.13025782 19.50313176
 20.15288628 15.83693395 13.33844552 17.36216855 15.96622551 19.46024179
 13.90315188 16.54388879 13.68013927  4.07198338 14.72017174 12.20784414
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 20.8952471  18.32765139 20.21117474 17.33271057 22.49615131 20.25459101
 13.65952896 33.2468105  28.98648563 25.62406378 32.74562707 36.7823308
 40.64269162 41.95843663 24.75521343 25.35466422 37.22852041 23.0666828
 26.38364858 26.63892348 22.52389982 24.25575334 22.97600489 29.09572209
 26.53800723 30.78324212 25.65654447 29.16112396 31.47969566 32.86492694
 34.68152029 27.68209514 33.81087102 30.89257238 22.60237111 24.6856437
 35.86195874 33.60287616 32.58804086 34.70805199 30.9599932  30.47235491
 33.11862186 32.15190019 31.58307316 40.7789065  36.23403224 32.77576042
 34.81194629 30.32367884 30.88109842 29.39755706 37.24480147 41.83064886
 42.99421839 22.62968833 23.61607964 17.76509228 23.48946808 16.95873754
 22.36303304 17.02544724 22.73071165 25.16655046 11.08457985 24.44301282
 26.49588    28.17670655 24.78916776 29.56428433 33.1752172  23.75561209
```

```
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 35.44820177 31.18054207 24.38893517 33.22734434 38.02023676 37.136191
 31.71171598 25.17935464 30.08970257 32.66118869 28.37608803 28.36666955
 27.23977663 23.65256421 24.04779992 27.35469295 16.25582102 13.34172873
 19.98853241 19.80620596 21.26183873 24.08247968 24.20210886 25.03563452
 24.93445899 30.00798592 24.03262586 21.76211251 37.36399601 43.31706217
 36.44279771 34.92929383 34.78322719 37.13926091 40.99074091 34.41612486
 35.79504137 28.1727436  31.19888966 40.83676536 39.288083   25.63769699
 22.17456441 27.09685757 28.38539604 35.46717966 36.06036707 33.67574658
 35.57670377 34.80227754 30.23276262 35.15620445 38.6547765  34.17960958
 40.27116049 44.62943331 31.62777158 27.46303705 20.06657863 26.90212328
 27.06757675 26.79818362 33.2601829  34.23933167 31.65529255 25.69684394
 24.2895399  28.34371519 27.23067951 19.38951302 29.16824677 31.99858441
 30.8269393  28.87794391 28.82920298 32.75779147 33.03623723 30.56272488
 35.37125142 32.50005623 28.63810791 23.57437039 18.53177462 26.88111528
 23.26016057 25.53297274 25.47905049 20.51632102 17.60898892 18.36840885
 24.2989913  21.33484616 24.86836994 24.84700456 22.84025836 19.39821704
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 21.97057077 20.07479807 23.43733111 22.04503869 21.46088043 20.51923232
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 22.54704194 27.81843045 28.68415534 16.7673417  15.00298937 25.32056825
 27.48492114 22.43006026 20.71716445 20.80542326 17.13133769 25.08441022
 14.38628919 16.66407668 19.68860641 22.75865294 22.23952396 19.17293283
 22.63144217 18.89898144 18.15059782 20.25540711 37.59798667 14.12492034
 15.44545915 10.69037333 23.6532781  32.63561679 34.61091787 24.86570161
 25.99542188  6.06180234  0.71215143 25.32082269 17.73873273 20.2120143
 15.84398124 16.8168905  14.58452918 18.46416478 13.37107784 13.00277178
  3.23209322  8.03047544  6.09178153  5.60426784  6.39907926 14.13868221
 17.14671178 17.25536638  9.82922179 20.17590698 17.89942848 20.26838217
 19.25481427 16.27530756  6.58723159 10.87430865 11.8912482  17.79163586
 18.23023728 12.95565459  7.43244283  8.32802386  7.97864605 19.90471826
 13.62476144 19.82469081 15.24712007 16.9292683   1.64962962 11.75673818
 -4.2422687   9.59547725 13.37128374  6.89080314  6.2886687  14.63272108
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 21.46341362 17.00279126 15.94079571 13.37168725 14.52722865  8.82709859
  4.8841789  13.02914465 12.71318484 17.28040573 18.69901555 18.04405036
 11.498378   11.97862086 17.65443998 18.10166746 17.4819     17.19808049
 16.5091233  19.38287045 18.54146673 22.49198974 15.27495005 15.79847469
 12.6455792  12.84583944 17.16031146 18.46994646 19.02408657 20.12821931
 19.73964122 22.38802915 20.27846759 17.83103569 14.31089914 16.86064979
 16.93728254 18.53081349 20.08306855 22.86851111 22.38272261 25.53964704
 16.28690641 16.03889051 20.47078681 11.47360361 19.14215843 21.81114569
 23.38202469 27.0173952  28.49714159 20.98934056 19.383023   22.15757864
 19.57601477 21.24893652 12.38837802  8.76205574  4.20127822 14.30651575
 16.48193898 20.7541228  20.75988486 17.04111566 14.13159609 19.23104385
 21.43031199 18.56811902 20.59285253 23.57836567 22.39616551 27.66240175
 26.15939095 22.35625671]
```

In [88]:

```
y = np.array(y)
print(y)
```

```
[24.  21.6 34.7 33.4 36.2 28.7 22.9 27.1 16.5 18.9 15.  18.9 21.7 20.4
 18.2 19.9 23.1 17.5 20.2 18.2 13.6 19.6 15.2 14.5 15.6 13.9 16.6 14.8
 18.4 21.  12.7 14.5 13.2 13.1 13.5 18.9 20.  21.  24.7 30.8 34.9 26.6
 25.3 24.7 21.2 19.3 20.  16.6 14.4 19.4 19.7 20.5 25.  23.4 18.9 35.4
 24.7 31.6 23.3 19.6 18.7 16.  22.2 25.  33.  23.5 19.4 22.  17.4 20.9
 24.2 21.7 22.8 23.4 24.1 21.4 20.  20.8 21.2 20.3 28.  23.9 24.8 22.9
 23.9 26.6 22.5 22.2 23.6 28.7 22.6 22.  22.9 25.  20.6 28.4 21.4 38.7
 43.8 33.2 27.5 26.5 18.6 19.3 20.1 19.5 19.5 20.4 19.8 19.4 21.7 22.8
 18.8 18.7 18.5 18.3 21.2 19.2 20.4 19.3 22.  20.3 20.5 17.3 18.8 21.4
 15.7 16.2 18.  14.3 19.2 19.6 23.  18.4 15.6 18.1 17.4 17.1 13.3 17.8
 14.  14.4 13.4 15.6 11.8 13.8 15.6 14.6 17.8 15.4 21.5 19.6 15.3 19.4
 17.  15.6 13.1 41.3 24.3 23.3 27.  50.  50.  50.  22.7 25.  50.  23.8
 23.8 22.3 17.4 19.1 23.1 23.6 22.6 29.4 23.2 24.6 29.9 37.2 39.8 36.2
 37.9 32.5 26.4 29.6 50.  32.  29.8 34.9 37.  30.5 36.4 31.1 29.1 50.
 33.3 30.3 34.6 34.9 32.9 24.1 42.3 48.5 50.  22.6 24.4 22.5 24.4 20.
 21.7 19.3 22.4 28.1 23.7 25.  23.3 28.7 21.5 23.  26.7 21.7 27.5 30.1
 44.8 50.  37.6 31.6 46.7 31.5 24.3 31.7 41.7 48.3 29.  24.  25.1 31.5
 23.7 23.3 22.  20.1 22.2 23.7 17.6 18.5 24.3 20.5 24.5 26.2 24.4 24.8
```

```
 29.6 42.8 21.9 20.9 44.  50.  36.  30.1 33.8 43.1 48.8 31.  36.5 22.8
 30.7 50.  43.5 20.7 21.1 25.2 24.4 35.2 32.4 32.  33.2 33.1 29.1 35.1
 45.4 35.4 46.  50.  32.2 22.  20.1 23.2 22.3 24.8 28.5 37.3 27.9 23.9
 21.7 28.6 27.1 20.3 22.5 29.  24.8 22.  26.4 33.1 36.1 28.4 33.4 28.2
 22.8 20.3 16.1 22.1 19.4 21.6 23.8 16.2 17.8 19.8 23.1 21.  23.8 23.1
 20.4 18.5 25.  24.6 23.  22.2 19.3 22.6 19.8 17.1 19.4 22.2 20.7 21.1
 19.5 18.5 20.6 19.  18.7 32.7 16.5 23.9 31.2 17.5 17.2 23.1 24.5 26.6
 22.9 24.1 18.6 30.1 18.2 20.6 17.8 21.7 22.7 22.6 25.  19.9 20.8 16.8
 21.9 27.5 21.9 23.1 50.  50.  50.  50.  50.  13.8 13.8 15.  13.9 13.3
 13.1 10.2 10.4 10.9 11.3 12.3  8.8  7.2 10.5  7.4 10.2 11.5 15.1 23.2
  9.7 13.8 12.7 13.1 12.5  8.5  5.   6.3  5.6  7.2 12.1  8.3  8.5  5.
 11.9 27.9 17.2 27.5 15.  17.2 17.9 16.3  7.   7.2  7.5 10.4  8.8  8.4
 16.7 14.2 20.8 13.4 11.7  8.3 10.2 10.9 11.   9.5 14.5 14.1 16.1 14.3
 11.7 13.4  9.6  8.7  8.4 12.8 10.5 17.1 18.4 15.4 10.8 11.8 14.9 12.6
 14.1 13.  13.4 15.2 16.1 17.8 14.9 14.1 12.7 13.5 14.9 20.  16.4 17.7
 19.5 20.2 21.4 19.9 19.  19.1 19.1 20.1 19.9 19.6 23.2 29.8 13.8 13.3
 16.7 12.  14.6 21.4 23.  23.7 25.  21.8 20.6 21.2 19.1 20.6 15.2  7.
  8.1 13.6 20.1 21.8 24.5 23.1 19.7 18.3 21.2 17.5 16.8 22.4 20.6 23.9
 22.  11.9]
```

In [92]:

```python
a = np.vstack((y, y_))
print(a)
```

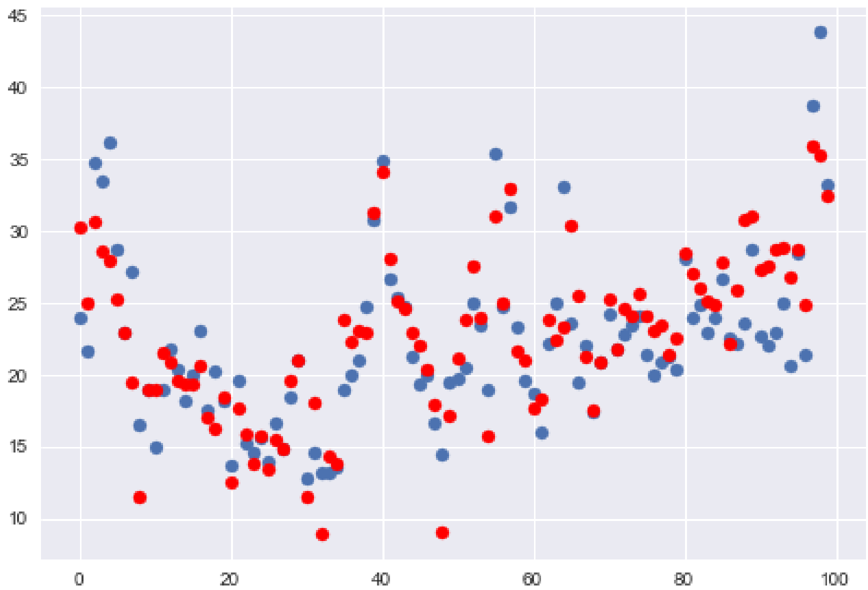
```
[[24.         21.6        34.7        ... 23.9        22.
  11.9       ]
 [30.18633694 24.99095484 30.56568098 ... 27.66240175 26.15939095
  22.35625671]]
```

In [97]:

```python
x = np.arange(100)
plt.scatter(x, y[:100])
plt.scatter(x, y_[:100], color='red')
```

Out[97]:

```
<matplotlib.collections.PathCollection at 0x1cdbb190>
```



In [83]:

```python
def r2_score(y, y_):
    num = np.sum((y - y_) ** 2)
    denom = np.sum((y - y.mean()) ** 2)
    score = (1 - num / denom)
    return score * 100
```

In [85]:

```python
#Score
```

```
r2_score(y, y_)
```

Out[85]:

```
74.04541323942743
```

Slow implementation, only for 506 samples, improve this

Efficient code using Vectorization

In [17]:

```
print(X.shape)
print(X[:5, :5])
```

```
(506, 14)
[[ 1.         -0.41978194  0.28482986 -1.2879095  -0.27259857]
 [ 1.         -0.41733926 -0.48772236 -0.59338101 -0.27259857]
 [ 1.         -0.41734159 -0.48772236 -0.59338101 -0.27259857]
 [ 1.         -0.41675042 -0.48772236 -1.30687771 -0.27259857]
 [ 1.         -0.41248185 -0.48772236 -1.30687771 -0.27259857]]
```

In [28]:

```
def hypothesis(X, theta):
    return np.dot(X, theta)
```

In [29]:

```
def error(X, y, theta):
    m = X.shape[0]
    e = 0.0
    y_ = hypothesis(X, theta)
    e = np.sum((y_ - y) ** 2)
    return e / m
```

In [30]:

```
def gradient(X, y, theta):
    m, n = X.shape
    grad = np.zeros((n,))
    y_ = hypothesis(X, theta)
    grad = np.dot(X.T, y_ - y)
    return grad / m
```

In [31]:

```
def gradient_descent(X, y, lr = 0.1, max_epochs = 300):
    m, n = X.shape
    theta = np.zeros((n,))
    error_list = []
    theta_list = []
    for i in range(max_epochs):
        e = error(X, y, theta)
        error_list.append(e)
        theta_list.append(theta)
        grad = gradient(X, y, theta)
        theta = theta - lr * grad

    return theta, error_list, theta_list
```

In [32]:

```python
import time
start = time.time()
theta, error_list, theta_list = gradient_descent(X, y)
end = time.time()
print("Time taken:", end - start)
```

```
Time taken: 0.025261402130126953
```

In [33]:

```python
theta
```

Out[33]:

```
array([ 2.25328063e+01, -9.03091692e-01,  1.03815625e+00,  1.53477685e-02,
        6.99554920e-01, -2.02101672e+00,  2.70014278e+00, -1.93085233e-03,
       -3.10234837e+00,  2.34354753e+00, -1.72031485e+00, -2.04614394e+00,
        8.47845679e-01, -3.73089521e+00])
```

In [34]:

```python
error_list
```

Out[34]:

```
[592.1469169960474,
 462.46044906845816,
 375.3852750957273,
 307.6933342962929,
 253.3916863467937,
 209.5598827590644,
 174.13020491536253,
 145.47824715741686,
 122.300098919846,
 103.54456174692925,
 88.3632803447008,
 76.0713129155594,
 66.11549356461222,
 58.04896715358396,
 51.51067344934865,
 46.20881061552668,
 41.90750109402472,
 38.41603601946843,
 35.58019677482985,
 33.275250473323695,
 31.400294933308405,
 29.873691975646697,
 28.629378705367408,
 27.613887307343795,
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 26.10448632753721,
 25.547161996504908,
 25.088984177108614,
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 24.39914527693565,
 24.140183102536813,
 23.92454749272831,
 23.744208598563592,
 23.592654924717976,
 23.464604712017074,
 23.35577231847968,
 23.26267909670338 3,
 23.182500275377258,
 23.1129409769508,
 23.05213581628629,
 22.998567586378826,
 22.95100139528269,
 22.9084313122499,
 22.87003714226488,
 22.835149402084745,
 22.80322093811317
```

22.80322093811317,
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22.093481235376775,

22.093481235376775,
22.090195047864643,
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22.08073969295409,
22.077716477051652,
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22.071852962412873,
22.069009529249705,
22.06622296926763,
22.063491832849987,
22.060814714567442,
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22.038939608673694,
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21.990013443996812,
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21.986035319233707,
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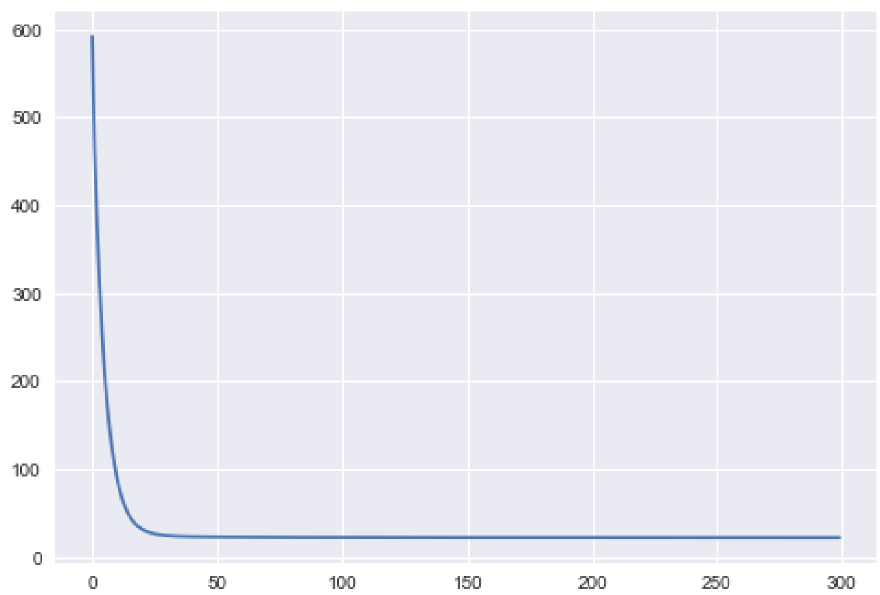
In [52]:

```python
plt.style.use('seaborn')
plt.plot(error_list)
```

Out[52]:

```
[<matplotlib.lines.Line2D at 0x5b74a18>]
```



In [43]:

```python
theta_list
```

Out[43]:

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```

In [48]:

```
theta_df = pd.DataFrame(theta_list)
theta_df.tail(20)
```

Out[48]:

	0	1	2	3	4	5	6	7	8	9	10	11
280	22.532806	-0.899100	1.031325	-0.001601	0.701938	-2.012471	2.704598	-0.005382	-3.098974	2.299647	-1.672755	-2.043503
281	22.532806	-0.899317	1.031696	-0.000703	0.701812	-2.012955	2.704353	-0.005195	-3.099180	2.301982	-1.675272	-2.043648
282	22.532806	-0.899532	1.032064	0.000189	0.701687	-2.013431	2.704111	-0.005009	-3.099381	2.304302	-1.677775	-2.043792
283	22.532806	-0.899745	1.032429	0.001076	0.701562	-2.013902	2.703871	-0.004825	-3.099579	2.306607	-1.680262	-2.043934
284	22.532806	-0.899956	1.032790	0.001957	0.701438	-2.014366	2.703634	-0.004642	-3.099772	2.308896	-1.682735	-2.044075

	0	1	2	3	4	5	6	7	8	9	10	11
285	22.532806	-0.900165	1.033148	0.002834	0.701315	-2.014824	2.703399	-0.004461	-3.099961	2.311170	-1.685192	-2.044214
286	22.532806	-0.900372	1.033502	0.003704	0.701193	-2.015276	2.703166	-0.004282	-3.100146	2.313430	-1.687635	-2.044352
287	22.532806	-0.900578	1.033854	0.004570	0.701071	-2.015722	2.702936	-0.004104	-3.100327	2.315674	-1.690062	-2.044489
288	22.532806	-0.900781	1.034203	0.005430	0.700950	-2.016162	2.702708	-0.003928	-3.100504	2.317904	-1.692475	-2.044624
289	22.532806	-0.900983	1.034548	0.006285	0.700830	-2.016597	2.702483	-0.003753	-3.100677	2.320119	-1.694874	-2.044758
290	22.532806	-0.901183	1.034890	0.007134	0.700710	-2.017026	2.702259	-0.003580	-3.100846	2.322320	-1.697258	-2.044890
291	22.532806	-0.901382	1.035230	0.007979	0.700591	-2.017449	2.702038	-0.003409	-3.101012	2.324506	-1.699627	-2.045021
292	22.532806	-0.901578	1.035566	0.008818	0.700473	-2.017866	2.701819	-0.003239	-3.101174	2.326678	-1.701982	-2.045151
293	22.532806	-0.901773	1.035900	0.009652	0.700356	-2.018278	2.701602	-0.003070	-3.101333	2.328836	-1.704323	-2.045280
294	22.532806	-0.901967	1.036231	0.010481	0.700240	-2.018685	2.701388	-0.002903	-3.101488	2.330979	-1.706650	-2.045407
295	22.532806	-0.902158	1.036559	0.011305	0.700124	-2.019086	2.701175	-0.002738	-3.101639	2.333108	-1.708962	-2.045533
296	22.532806	-0.902348	1.036884	0.012123	0.700009	-2.019483	2.700965	-0.002573	-3.101788	2.335224	-1.711260	-2.045657
297	22.532806	-0.902536	1.037206	0.012937	0.699894	-2.019874	2.700756	-0.002411	-3.101933	2.337325	-1.713545	-2.045781
298	22.532806	-0.902723	1.037525	0.013746	0.699780	-2.020260	2.700550	-0.002249	-3.102074	2.339413	-1.715815	-2.045903
299	22.532806	-0.902908	1.037842	0.014549	0.699667	-2.020641	2.700345	-0.002089	-3.102213	2.341487	-1.718072	-2.046024

In [37]:

```
y_ = hypothesis(X, theta)
print(y_)
print(type(y_))
```

```
[30.18633694 24.99095484 30.56568098 28.6220126  27.9619464  25.24452832
 22.96581931 19.4959112  11.48031987 18.87883585 18.97129884 21.54039453
 20.89285327 19.60128149 19.32367393 19.3458686  20.59832729 16.95783793
 16.23493881 18.44548854 12.55479461 17.70925422 15.88547043 13.84190443
 15.71570947 13.4205508  15.49775358 14.75818241 19.59955913 20.93977682
 11.49745095 18.09361635  8.88301038 14.31610222 13.75983633 23.77730951
 22.30988941 23.08772244 22.90940545 31.27562517 34.14934567 27.99150715
 25.15161107 24.56195221 22.86665722 22.01299699 20.35079995 17.94239765
  9.0032359  17.10959314 21.16463059 23.8404942  27.56319999 23.94281148
 15.70766609 30.96324445 24.93746243 32.91466583 21.63293708 20.92400291
 17.6977369  18.32617385 23.82616369 22.4283064  23.34895856 30.37522177
 25.5255783  21.2306401  17.52571154 20.89494847 25.20175729 21.72374982
 24.54156678 24.03905863 25.62679124 24.06381642 23.02484813 23.44984561
 21.35657377 22.51629435 28.40848967 26.96899155 26.0263214  25.03478105
 24.79279036 27.79575063 22.17067399 25.88154887 30.78085348 30.95800581
 27.22453062 27.51066854 28.71288943 28.859444   26.79054644 28.75132406
 24.83596211 35.94281395 35.29511986 32.40179358 24.75516584 25.77444454
 19.95640618 20.46644109 21.58632495 18.68176932 17.33656725 20.90711862
 22.80555026 19.93114262 20.83065748 26.72618732 20.94596258 20.89490035
 25.35381775 20.60731806 23.5681151  23.86484071 20.52156256 20.97469607
 21.46430039 22.01300726 20.09753035 15.91329192 20.09521555 22.02074575
 14.15984655 15.25545131 19.03740999 14.1352851  20.13025782 19.50313176
 20.15288628 15.83693395 13.33844552 17.36216855 15.96622551 19.46024179
 13.90315188 16.54388879 13.68013927  4.07198338 14.72017174 12.20784414
  8.76929608 12.12027516 15.86629239  8.55791283  9.77073593 14.86013526
 20.8952471  18.32765139 20.21117474 17.33271057 22.49615131 20.25459101
```

```
 13.65952896 33.2468105  28.98648563 25.62406378 32.74562707 36.7823308
 40.64269162 41.95843663 24.75521343 25.35466422 37.22852041 23.0666828
 26.38364858 26.63892348 22.52389982 24.25575334 22.97600489 29.09572209
 26.53800723 30.78324212 25.65654447 29.16112396 31.47969566 32.86492694
 34.68152029 27.68209514 33.81087102 30.89257238 22.60237111 24.6856437
 35.86195874 33.60287616 32.58804086 34.70805199 30.9599932  30.47235491
 33.11862186 32.15190019 31.58307316 40.7789065  36.23403224 32.77576042
 34.81194629 30.32367884 30.88109842 29.39755706 37.24480147 41.83064886
 42.99421839 22.62968833 23.61607964 17.76509228 23.48946808 16.95873754
 22.36303304 17.02544724 22.73071165 25.16655046 11.08457985 24.44301282
 26.49588    28.17670655 24.78916776 29.56428433 33.1752172  23.75561209
 32.13861685 29.65742786 38.33728061 39.79578234 37.53634615 32.32855843
 35.44820177 31.18054207 24.38893517 33.22734434 38.02023676 37.136191
 31.71171598 25.17935464 30.08970257 32.66118869 28.37608803 28.36666955
 27.23977663 23.65256421 24.04779992 27.35469295 16.25582102 13.34172873
 19.98853241 19.80620596 21.26183873 24.08247968 24.20210886 25.03563452
 24.93445899 30.00798592 24.03262586 21.76211251 37.36399601 43.31706217
 36.44279771 34.92929383 34.78322719 37.13926091 40.99074091 34.41612486
 35.79504137 28.1727436  31.19888966 40.83676536 39.288083   25.63769699
 22.17456441 27.09685757 28.38539604 35.46717966 36.06036707 33.67574658
 35.57670377 34.80227754 30.23276262 35.15620445 38.6547765  34.17960958
 40.27116049 44.62943331 31.62777158 27.46303705 20.06657863 26.90212328
 27.06757675 26.79818362 33.2601829  34.23933167 31.65529255 25.69684394
 24.2895399  28.34371519 27.23067951 19.38951302 29.16824677 31.99858441
 30.8269393  28.87794391 28.82920298 32.75779147 33.03623723 30.56272488
 35.37125142 32.50005623 28.63810791 23.57437039 18.53177462 26.88111528
 23.26016057 25.53297274 25.47905049 20.51632102 17.60898892 18.36840885
 24.2989913  21.33484616 24.86836994 24.84700456 22.84025836 19.39821704
 25.10698231 24.67525381 23.67513357 19.32722116 21.5477069  24.64514863
 21.97057077 20.07479807 23.43733111 22.04503869 21.46088043 20.51923232
 20.04886555 19.1653249  22.06584752 21.14213423 21.3104903  30.47952808
 22.54704194 27.81843045 28.68415534 16.7673417  15.00298937 25.32056825
 27.48492114 22.43006026 20.71716445 20.80542326 17.13133769 25.08441022
 14.38628919 16.66407668 19.68860641 22.75865294 22.23952396 19.17293283
 22.63144217 18.89898144 18.15059782 20.25540711 37.59798667 14.12492034
 15.44545915 10.69037333 23.6532781  32.63561679 34.61091787 24.86570161
 25.99542188  6.06180234  0.71215143 25.32082269 17.73873273 20.2120143
 15.84398124 16.8168905  14.58452918 18.46416478 13.37107784 13.00277178
  3.23209322  8.03047544  6.09178153  5.60426784  6.39907926 14.13868221
 17.14671178 17.25536638  9.82922179 20.17590698 17.89942848 20.26838217
 19.25481427 16.27530756  6.58723159 10.87430865 11.8912482  17.79163586
 18.23023728 12.95565459  7.43244283  8.32802386  7.97864605 19.90471826
 13.62476144 19.82469081 15.24712007 16.9292683   1.64962962 11.75673818
 -4.2422687   9.59547725 13.37128374  6.89080314  6.2886687  14.63272108
 19.57294693 18.07517537 18.44554355 13.12415263 14.55111653  9.90466233
 16.29625138 14.12584991 14.23134913 13.02062831 18.10033763 18.6562879
 21.46341362 17.00279126 15.94079571 13.37168725 14.52722865  8.82709859
  4.8841789  13.02914465 12.71318484 17.28040573 18.69901555 18.04405036
 11.498378   11.97862086 17.65443998 18.10166746 17.4819     17.19808049
 16.5091233  19.38287045 18.54146673 22.49198974 15.27495005 15.79847469
 12.6455792  12.84583944 17.16031146 18.46994646 19.02408657 20.12821931
 19.73964122 22.38802915 20.27846759 17.83103569 14.31089914 16.86064979
 16.93728254 18.53081349 20.08306855 22.86851111 22.38272261 25.53964704
 16.28690641 16.03889051 20.47078681 11.47360361 19.14215843 21.81114569
 23.38202469 27.0173952  28.49714159 20.98934056 19.383023   22.15757864
 19.57601477 21.24893652 12.38837802  8.76205574  4.20127822 14.30651575
 16.48193898 20.7541228  20.75988486 17.04111566 14.13159609 19.23104385
 21.43031199 18.56811902 20.59285253 23.57836567 22.39616551 27.66240175
 26.15939095 22.35625671]
<class 'numpy.ndarray'>
```

In [38]:

```
yy = np.array(y)
```

In [50]:

```
yy
```

Out[50]:

```
array([24. , 21.6, 34.7, 33.4, 36.2, 28.7, 22.9, 27.1, 16.5, 18.9, 15. ,
```

```
       18.9, 21.7, 20.4, 18.2, 19.9, 23.1, 17.5, 20.2, 18.2, 13.6, 19.6,
       15.2, 14.5, 15.6, 13.9, 16.6, 14.8, 18.4, 21. , 12.7, 14.5, 13.2,
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       19.4, 22. , 17.4, 20.9, 24.2, 21.7, 22.8, 23.4, 24.1, 21.4, 20. ,
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       23.6, 28.7, 22.6, 22. , 22.9, 25. , 20.6, 28.4, 21.4, 38.7, 43.8,
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       25. , 50. , 23.8, 23.8, 22.3, 17.4, 19.1, 23.1, 23.6, 22.6, 29.4,
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       32. , 29.8, 34.9, 37. , 30.5, 36.4, 31.1, 29.1, 50. , 33.3, 30.3,
       34.6, 34.9, 32.9, 24.1, 42.3, 48.5, 50. , 22.6, 24.4, 22.5, 24.4,
       20. , 21.7, 19.3, 22.4, 28.1, 23.7, 25. , 23.3, 28.7, 21.5, 23. ,
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       31.7, 41.7, 48.3, 29. , 24. , 25.1, 31.5, 23.7, 23.3, 22. , 20.1,
       22.2, 23.7, 17.6, 18.5, 24.3, 20.5, 24.5, 26.2, 24.4, 24.8, 29.6,
       42.8, 21.9, 20.9, 44. , 50. , 36. , 30.1, 33.8, 43.1, 48.8, 31. ,
       36.5, 22.8, 30.7, 50. , 43.5, 20.7, 21.1, 25.2, 24.4, 35.2, 32.4,
       32. , 33.2, 33.1, 29.1, 35.1, 45.4, 35.4, 46. , 50. , 32.2, 22. ,
       20.1, 23.2, 22.3, 24.8, 28.5, 37.3, 27.9, 23.9, 21.7, 28.6, 27.1,
       20.3, 22.5, 29. , 24.8, 22. , 26.4, 33.1, 36.1, 28.4, 33.4, 28.2,
       22.8, 20.3, 16.1, 22.1, 19.4, 21.6, 23.8, 16.2, 17.8, 19.8, 23.1,
       21. , 23.8, 23.1, 20.4, 18.5, 25. , 24.6, 23. , 22.2, 19.3, 22.6,
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       18.6, 30.1, 18.2, 20.6, 17.8, 21.7, 22.7, 22.6, 25. , 19.9, 20.8,
       16.8, 21.9, 27.5, 21.9, 23.1, 50. , 50. , 50. , 50. , 50. , 13.8,
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        7.2, 10.5,  7.4, 10.2, 11.5, 15.1, 23.2,  9.7, 13.8, 12.7, 13.1,
       12.5,  8.5,  5. ,  6.3,  5.6,  7.2, 12.1,  8.3,  8.5,  5. , 11.9,
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       20.6, 21.2, 19.1, 20.6, 15.2,  7. ,  8.1, 13.6, 20.1, 21.8, 24.5,
       23.1, 19.7, 18.3, 21.2, 17.5, 16.8, 22.4, 20.6, 23.9, 22. , 11.9])
```

In [34]:

```
def r2_score(y, y_):
    num = np.sum((y - y_) ** 2)
    denom = np.sum((y - y.mean()) ** 2)
    score = (1 - num / denom)
    return score * 100
```

In [41]:

```
r2_score(yy, y_)
```

Out[41]:

```
74.04541323942743
```

In [42]:

```
r2_score(y, y_)
```

Out[42]:

```
74.04541323942743
```

Sci-kit learn Linear Regression on Boston Housing dataset

In [22]:

```
boston = load_boston()
```

In [23]:

```
boston
```

Out[23]:

```
{'data': array([[6.3200e-03, 1.8000e+01, 2.3100e+00, ..., 1.5300e+01, 3.9690e+02,
         4.9800e+00],
        [2.7310e-02, 0.0000e+00, 7.0700e+00, ..., 1.7800e+01, 3.9690e+02,
         9.1400e+00],
        [2.7290e-02, 0.0000e+00, 7.0700e+00, ..., 1.7800e+01, 3.9283e+02,
         4.0300e+00],
        ...,
        [6.0760e-02, 0.0000e+00, 1.1930e+01, ..., 2.1000e+01, 3.9690e+02,
         5.6400e+00],
        [1.0959e-01, 0.0000e+00, 1.1930e+01, ..., 2.1000e+01, 3.9345e+02,
         6.4800e+00],
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         7.8800e+00]]),
 'target': array([24. , 21.6, 34.7, 33.4, 36.2, 28.7, 22.9, 27.1, 16.5, 18.9, 15. ,
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        23.6, 28.7, 22.6, 22. , 22.9, 25. , 20.6, 28.4, 21.4, 38.7, 43.8,
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        21.7, 22.8, 18.8, 18.7, 18.5, 18.3, 21.2, 19.2, 20.4, 19.3, 22. ,
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        23. , 18.4, 15.6, 18.1, 17.4, 17.1, 13.3, 17.8, 14. , 14.4, 13.4,
        15.6, 11.8, 13.8, 15.6, 14.6, 17.8, 15.4, 21.5, 19.6, 15.3, 19.4,
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        25. , 50. , 23.8, 23.8, 22.3, 17.4, 19.1, 23.1, 23.6, 22.6, 29.4,
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```

```
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 'feature_names': array(['CRIM', 'ZN', 'INDUS', 'CHAS', 'NOX', 'RM', 'AGE', 'DIS', 'RAD',
        'TAX', 'PTRATIO', 'B', 'LSTAT'], dtype='<U7'),
 'DESCR': ".. _boston_dataset:\n\nBoston house prices dataset\n-------------------------
--\n\n**Data Set Characteristics:**  \n\n    :Number of Instances: 506 \n\n    :Number of
Attributes: 13 numeric/categorical predictive. Median Value (attribute 14) is usually the
target.\n\n    :Attribute Information (in order):\n        - CRIM     per capita crime ra
te by town\n        - ZN       proportion of residential land zoned for lots over 25,000
sq.ft.\n        - INDUS    proportion of non-retail business acres per town\n        - CH
AS     Charles River dummy variable (= 1 if tract bounds river; 0 otherwise)\n        - N
OX      nitric oxides concentration (parts per 10 million)\n        - RM       average nu
mber of rooms per dwelling\n        - AGE      proportion of owner-occupied units built p
rior to 1940\n        - DIS      weighted distances to five Boston employment centres\n
- RAD      index of accessibility to radial highways\n        - TAX      full-value prope
rty-tax rate per $10,000\n        - PTRATIO  pupil-teacher ratio by town\n        - B
1000(Bk - 0.63)^2 where Bk is the proportion of blacks by town\n        - LSTAT    % lowe
r status of the population\n        - MEDV     Median value of owner-occupied homes in $1
000's\n\n    :Missing Attribute Values: None\n\n    :Creator: Harrison, D. and Rubinfeld,
D.L.\n\nThis is a copy of UCI ML housing dataset.\nhttps://archive.ics.uci.edu/ml/machine
-learning-databases/housing/\n\n\nThis dataset was taken from the StatLib library which i
s maintained at Carnegie Mellon University.\n\nThe Boston house-price data of Harrison, D
. and Rubinfeld, D.L. 'Hedonic\nprices and the demand for clean air', J. Environ. Economi
cs & Management,\nvol.5, 81-102, 1978.   Used in Belsley, Kuh & Welsch, 'Regression diagn
ostics\n...', Wiley, 1980.   N.B. Various transformations are used in the table on\npages
244-261 of the latter.\n\nThe Boston house-price data has been used in many machine learn
ing papers that address regression\nproblems.   \n     \n.. topic:: References\n\n   - Be
lsley, Kuh & Welsch, 'Regression diagnostics: Identifying Influential Data and Sources of
Collinearity', Wiley, 1980. 244-261.\n   - Quinlan,R. (1993). Combining Instance-Based an
d Model-Based Learning. In Proceedings on the Tenth International Conference of Machine L
earning, 236-243, University of Massachusetts, Amherst. Morgan Kaufmann.\n",
 'filename': 'c:\\users\\mayank\\desktop\\python\\lib\\site-packages\\sklearn\\datasets\\
data\\boston_house_prices.csv'}
```

In [24]:

```
X = boston.data
y = boston.target
```

In [25]:

```
print(X.shape)
print(y.shape)
```

```
(506, 13)
(506,)
```

In [26]:

```
dfX = pd.DataFrame(X, columns=boston.feature_names)
dfX.head()
```

Out[26]:

	CRIM	ZN	INDUS	CHAS	NOX	RM	AGE	DIS	RAD	TAX	PTRATIO	B	LSTAT
0	0.00632	18.0	2.31	0.0	0.538	6.575	65.2	4.0900	1.0	296.0	15.3	396.90	4.98
1	0.02731	0.0	7.07	0.0	0.469	6.421	78.9	4.9671	2.0	242.0	17.8	396.90	9.14
2	0.02729	0.0	7.07	0.0	0.469	7.185	61.1	4.9671	2.0	242.0	17.8	392.83	4.03
3	0.03237	0.0	2.18	0.0	0.458	6.998	45.8	6.0622	3.0	222.0	18.7	394.63	2.94
4	0.06905	0.0	2.18	0.0	0.458	7.147	54.2	6.0622	3.0	222.0	18.7	396.90	5.33

In [27]:

```
dfY = pd.DataFrame(y)
```

```
dfY.head()
```

Out[27]:

	0
0	24.0
1	21.6
2	34.7
3	33.4
4	36.2

In [28]:

```
from sklearn.linear_model import LinearRegression
```

In [29]:

```
model = LinearRegression()
```

In [30]:

```
model.fit(X, y)
```

Out[30]:

```
LinearRegression()
```

In [31]:

```
print(model.coef_)
print(model.intercept_)
```

```
[-1.08011358e-01  4.64204584e-02  2.05586264e-02  2.68673382e+00
 -1.77666112e+01  3.80986521e+00  6.92224640e-04 -1.47556685e+00
  3.06049479e-01 -1.23345939e-02 -9.52747232e-01  9.31168327e-03
 -5.24758378e-01]
36.459488385089855
```

In [32]:

```
y_ = model.predict(X)
```

In [33]:

```
print(type(y_))
print(y_.shape)
```

```
<class 'numpy.ndarray'>
(506,)
```

In [35]:

```
score = r2_score(y, y_)
print(score)
```

```
74.06426641094095
```

After normalisation

In [36]:

```
u = np.mean(X, axis = 0)
std = np.std(X, axis = 0)
print("Mean:", u)
print("Std:", std)
print(u.shape, std.shape)
```

```
Mean: [3.61352356e+00 1.13636364e+01 1.11367787e+01 6.91699605e-02
 5.54695059e-01 6.28463439e+00 6.85749012e+01 3.79504269e+00
 9.54940711e+00 4.08237154e+02 1.84555336e+01 3.56674032e+02
 1.26530632e+01]
Std: [8.59304135e+00 2.32993957e+01 6.85357058e+00 2.53742935e-01
 1.15763115e-01 7.01922514e-01 2.81210326e+01 2.10362836e+00
 8.69865112e+00 1.68370495e+02 2.16280519e+00 9.12046075e+01
 7.13400164e+00]
(13,) (13,)
```

In [37]:

```
X = (X - u) / std
```

In [38]:

```
X
```

Out[38]:

```
array([[-0.41978194,  0.28482986, -1.2879095 , ..., -1.45900038,
         0.44105193, -1.0755623 ],
       [-0.41733926, -0.48772236, -0.59338101, ..., -0.30309415,
         0.44105193, -0.49243937],
       [-0.41734159, -0.48772236, -0.59338101, ..., -0.30309415,
         0.39642699, -1.2087274 ],
       ...,
       [-0.41344658, -0.48772236,  0.11573841, ...,  1.17646583,
         0.44105193, -0.98304761],
       [-0.40776407, -0.48772236,  0.11573841, ...,  1.17646583,
         0.4032249 , -0.86530163],
       [-0.41500016, -0.48772236,  0.11573841, ...,  1.17646583,
         0.44105193, -0.66905833]])
```

In [44]:

```
model_ = LinearRegression()
model_.fit(X, y)
print(model_.coef_)
print(model_.intercept_)
y__ = model_.predict(X)
score_ = r2_score(y, y__)
print(score_)
```

```
[-0.92814606  1.08156863  0.1409      0.68173972 -2.05671827  2.67423017
  0.01946607 -3.10404426  2.66221764 -2.07678168 -2.06060666  0.84926842
 -3.74362713]
22.532806324110684
74.06426641094095
```

In [45]:

```
y
```

Out[45]:

```
array([24. , 21.6, 34.7, 33.4, 36.2, 28.7, 22.9, 27.1, 16.5, 18.9, 15. ,
       18.9, 21.7, 20.4, 18.2, 19.9, 23.1, 17.5, 20.2, 18.2, 13.6, 19.6,
       15.2, 14.5, 15.6, 13.9, 16.6, 14.8, 18.4, 21. , 12.7, 14.5, 13.2,
       13.1, 13.5, 18.9, 20. , 21. , 24.7, 30.8, 34.9, 26.6, 25.3, 24.7,
       21.2, 19.3, 20. , 16.6, 14.4, 19.4, 19.7, 20.5, 25. , 23.4, 18.9,
       35.4, 24.7, 31.6, 23.3, 19.6, 18.7, 16. , 22.2, 25. , 33. , 23.5,
       19.4, 22. , 17.4, 20.9, 24.2, 21.7, 22.8, 23.4, 24.1, 21.4, 20. ,
       20.8, 21.2, 20.3, 28. , 23.9, 24.8, 22.9, 23.9, 26.6, 22.5, 22.2,
       23.6, 28.7, 22.6, 22. , 22.9, 25. , 20.6, 28.4, 21.4, 38.7, 43.8,
       33.2, 27.5, 26.5, 18.6, 19.3, 20.1, 19.5, 19.5, 20.4, 19.8, 19.4,
       21.7, 22.8, 18.8, 18.7, 18.5, 18.3, 21.2, 19.2, 20.4, 19.3, 22. ,
       20.3, 20.5, 17.3, 18.8, 21.4, 15.7, 16.2, 18. , 14.3, 19.2, 19.6,
       23. , 18.4, 15.6, 18.1, 17.4, 17.1, 13.3, 17.8, 14. , 14.4, 13.4,
       15.6, 11.8, 13.8, 15.6, 14.6, 17.8, 15.4, 21.5, 19.6, 15.3, 19.4,
       17. , 15.6, 13.1, 41.3, 24.3, 23.3, 27. , 50. , 50. , 50. , 22.7,
       25. , 50. , 23.8, 23.8, 22.3, 17.4, 19.1, 23.1, 23.6, 22.6, 29.4,
```

```
       23.2, 24.6, 29.9, 37.2, 39.8, 36.2, 37.9, 32.5, 26.4, 29.6, 50. ,
       32. , 29.8, 34.9, 37. , 30.5, 36.4, 31.1, 29.1, 50. , 33.3, 30.3,
       34.6, 34.9, 32.9, 24.1, 42.3, 48.5, 50. , 22.6, 24.4, 22.5, 24.4,
       20. , 21.7, 19.3, 22.4, 28.1, 23.7, 25. , 23.3, 28.7, 21.5, 23. ,
       26.7, 21.7, 27.5, 30.1, 44.8, 50. , 37.6, 31.6, 46.7, 31.5, 24.3,
       31.7, 41.7, 48.3, 29. , 24. , 25.1, 31.5, 23.7, 23.3, 22. , 20.1,
       22.2, 23.7, 17.6, 18.5, 24.3, 20.5, 24.5, 26.2, 24.4, 24.8, 29.6,
       42.8, 21.9, 20.9, 44. , 50. , 36. , 30.1, 33.8, 43.1, 48.8, 31. ,
       36.5, 22.8, 30.7, 50. , 43.5, 20.7, 21.1, 25.2, 24.4, 35.2, 32.4,
       32. , 33.2, 33.1, 29.1, 35.1, 45.4, 35.4, 46. , 50. , 32.2, 22. ,
       20.1, 23.2, 22.3, 24.8, 28.5, 37.3, 27.9, 23.9, 21.7, 28.6, 27.1,
       20.3, 22.5, 29. , 24.8, 22. , 26.4, 33.1, 36.1, 28.4, 33.4, 28.2,
       22.8, 20.3, 16.1, 22.1, 19.4, 21.6, 23.8, 16.2, 17.8, 19.8, 23.1,
       21. , 23.8, 23.1, 20.4, 18.5, 25. , 24.6, 23. , 22.2, 19.3, 22.6,
       19.8, 17.1, 19.4, 22.2, 20.7, 21.1, 19.5, 18.5, 20.6, 19. , 18.7,
       32.7, 16.5, 23.9, 31.2, 17.5, 17.2, 23.1, 24.5, 26.6, 22.9, 24.1,
       18.6, 30.1, 18.2, 20.6, 17.8, 21.7, 22.7, 22.6, 25. , 19.9, 20.8,
       16.8, 21.9, 27.5, 21.9, 23.1, 50. , 50. , 50. , 50. , 50. , 13.8,
       13.8, 15. , 13.9, 13.3, 13.1, 10.2, 10.4, 10.9, 11.3, 12.3,  8.8,
        7.2, 10.5,  7.4, 10.2, 11.5, 15.1, 23.2,  9.7, 13.8, 12.7, 13.1,
       12.5,  8.5,  5. ,  6.3,  5.6,  7.2, 12.1,  8.3,  8.5,  5. , 11.9,
       27.9, 17.2, 27.5, 15. , 17.2, 17.9, 16.3,  7. ,  7.2,  7.5, 10.4,
        8.8,  8.4, 16.7, 14.2, 20.8, 13.4, 11.7,  8.3, 10.2, 10.9, 11. ,
        9.5, 14.5, 14.1, 16.1, 14.3, 11.7, 13.4,  9.6,  8.7,  8.4, 12.8,
       10.5, 17.1, 18.4, 15.4, 10.8, 11.8, 14.9, 12.6, 14.1, 13. , 13.4,
       15.2, 16.1, 17.8, 14.9, 14.1, 12.7, 13.5, 14.9, 20. , 16.4, 17.7,
       19.5, 20.2, 21.4, 19.9, 19. , 19.1, 19.1, 20.1, 19.9, 19.6, 23.2,
       29.8, 13.8, 13.3, 16.7, 12. , 14.6, 21.4, 23. , 23.7, 25. , 21.8,
       20.6, 21.2, 19.1, 20.6, 15.2,  7. ,  8.1, 13.6, 20.1, 21.8, 24.5,
       23.1, 19.7, 18.3, 21.2, 17.5, 16.8, 22.4, 20.6, 23.9, 22. , 11.9])
```

In [46]:

```
y_
```

Out[46]:

```
array([30.00384338, 25.02556238, 30.56759672, 28.60703649, 27.94352423,
       25.25628446, 23.00180827, 19.53598843, 11.52363685, 18.92026211,
       18.99949651, 21.58679568, 20.90652153, 19.55290281, 19.28348205,
       19.29748321, 20.52750979, 16.91140135, 16.17801106, 18.40613603,
       12.52385753, 17.67103669, 15.83288129, 13.80628535, 15.67833832,
       13.38668561, 15.46397655, 14.70847428, 19.54737285, 20.8764282 ,
       11.45511759, 18.05923295,  8.81105736, 14.28275814, 13.70675891,
       23.81463526, 22.34193708, 23.10891142, 22.91502612, 31.35762569,
       34.21510225, 28.02056414, 25.20386628, 24.60979273, 22.94149176,
       22.09669817, 20.42320032, 18.03655088,  9.10655377, 17.20607751,
       21.28152535, 23.97222285, 27.6558508 , 24.04901809, 15.3618477 ,
       31.15264947, 24.85686978, 33.10919806, 21.77537987, 21.08493555,
       17.8725804 , 18.51110208, 23.98742856, 22.55408869, 23.37308644,
       30.36148358, 25.53056512, 21.11338564, 17.42153786, 20.78483633,
       25.20148859, 21.7426577 , 24.55744957, 24.04295712, 25.50499716,
       23.9669302 , 22.94545403, 23.35699818, 21.26198266, 22.42817373,
       28.40576968, 26.99486086, 26.03576297, 25.05873482, 24.78456674,
       27.79049195, 22.16853423, 25.89276415, 30.67461827, 30.83110623,
       27.1190194 , 27.41266734, 28.94122762, 29.08105546, 27.03977365,
       28.62459949, 24.72744978, 35.78159518, 35.11454587, 32.25102801,
       24.58022019, 25.59413475, 19.79013684, 20.31167129, 21.43482591,
       18.53994008, 17.18755992, 20.75049026, 22.64829115, 19.7720367 ,
       20.64965864, 26.52586744, 20.77323638, 20.71548315, 25.17208881,
       20.43025591, 23.37724626, 23.69043261, 20.33578364, 20.79180873,
       21.91632071, 22.47107777, 20.55738556, 16.36661977, 20.56099819,
       22.48178446, 14.61706633, 15.17876684, 18.93868592, 14.05573285,
       20.03527399, 19.41013402, 20.06191566, 15.75807673, 13.25645238,
       17.26277735, 15.87841883, 19.36163954, 13.81483897, 16.44881475,
       13.57141932,  3.98885508, 14.59495478, 12.1488148 ,  8.72822362,
       12.03585343, 15.82082058,  8.5149902 ,  9.71844139, 14.80451374,
       20.83858153, 18.30101169, 20.12282558, 17.28601894, 22.36600228,
       20.10375923, 13.62125891, 33.25982697, 29.03017268, 25.56752769,
       32.70827666, 36.77467015, 40.55765844, 41.84728168, 24.78867379,
       25.37889238, 37.20347455, 23.08748747, 26.40273955, 26.65382114,
       22.5551466 , 24.29082812, 22.97657219, 29.07194308, 26.5219434 ,
       30.72209056, 25.61669307, 29.13740979, 31.43571968, 32.92231568,
```

```
       34.72440464, 27.76552111, 33.88787321, 30.99238036, 22.71820008,
       24.7664781 , 35.88497226, 33.42476722, 32.41199147, 34.51509949,
       30.76109485, 30.28934141, 32.91918714, 32.11260771, 31.55871004,
       40.84555721, 36.12770079, 32.6692081 , 34.70469116, 30.09345162,
       30.64393906, 29.28719501, 37.07148392, 42.03193124, 43.18949844,
       22.69034796, 23.68284712, 17.85447214, 23.49428992, 17.00587718,
       22.39251096, 17.06042754, 22.73892921, 25.21942554, 11.11916737,
       24.51049148, 26.60334775, 28.35518713, 24.91525464, 29.68652768,
       33.18419746, 23.77456656, 32.14051958, 29.7458199 , 38.37102453,
       39.81461867, 37.58605755, 32.3995325 , 35.45665242, 31.23411512,
       24.48449227, 33.28837292, 38.0481048 , 37.16328631, 31.71383523,
       25.26705571, 30.10010745, 32.71987156, 28.42717057, 28.42940678,
       27.29375938, 23.74262478, 24.12007891, 27.40208414, 16.3285756 ,
       13.39891261, 20.01638775, 19.86184428, 21.2883131 , 24.0798915 ,
       24.20633547, 25.04215821, 24.91964007, 29.94563374, 23.97228316,
       21.69580887, 37.51109239, 43.30239043, 36.48361421, 34.98988594,
       34.81211508, 37.16631331, 40.98928501, 34.44634089, 35.83397547,
       28.245743  , 31.22673593, 40.8395575 , 39.31792393, 25.70817905,
       22.30295533, 27.20340972, 28.51169472, 35.47676598, 36.10639164,
       33.79668274, 35.61085858, 34.83993382, 30.35192656, 35.30980701,
       38.79756966, 34.33123186, 40.33963075, 44.67308339, 31.59689086,
       27.3565923 , 20.10174154, 27.04206674, 27.2136458 , 26.91395839,
       33.43563311, 34.40349633, 31.8333982 , 25.81783237, 24.42982348,
       28.45764337, 27.36266999, 19.53928758, 29.11309844, 31.91054611,
       30.77159449, 28.94275871, 28.88191022, 32.79887232, 33.20905456,
       30.76831792, 35.56226857, 32.70905124, 28.64244237, 23.58965827,
       18.54266897, 26.87889843, 23.28133979, 25.54580246, 25.48120057,
       20.53909901, 17.61572573, 18.37581686, 24.29070277, 21.32529039,
       24.88682244, 24.86937282, 22.86952447, 19.45123791, 25.11783401,
       24.66786913, 23.68076177, 19.34089616, 21.17418105, 24.25249073,
       21.59260894, 19.98446605, 23.33888   , 22.14060692, 21.55509929,
       20.61872907, 20.16097176, 19.28490387, 22.1667232 , 21.24965774,
       21.42939305, 30.32788796, 22.04734975, 27.70647912, 28.54794117,
       16.54501121, 14.78359641, 25.27380082, 27.54205117, 22.14837562,
       20.45944095, 20.54605423, 16.88063827, 25.40253506, 14.32486632,
       16.59488462, 19.63704691, 22.71806607, 22.20218887, 19.20548057,
       22.66616105, 18.93192618, 18.22846804, 20.23150811, 37.4944739 ,
       14.28190734, 15.54286248, 10.83162324, 23.80072902, 32.6440736 ,
       34.60684042, 24.94331333, 25.9998091 ,  6.126325  ,  0.77779806,
       25.30713064, 17.74061065, 20.23274414, 15.83331301, 16.83512587,
       14.36994825, 18.47682833, 13.4276828 , 13.06177512,  3.27918116,
        8.06022171,  6.12842196,  5.6186481 ,  6.4519857 , 14.20764735,
       17.21225183, 17.29887265,  9.89116643, 20.22124193, 17.94181175,
       20.30445783, 19.29559075, 16.33632779,  6.55162319, 10.89016778,
       11.88145871, 17.81174507, 18.26126587, 12.97948781,  7.37816361,
        8.21115861,  8.06626193, 19.98294786, 13.70756369, 19.85268454,
       15.22308298, 16.96071981,  1.71851807, 11.80578387, -4.28131071,
        9.58376737, 13.36660811,  6.89562363,  6.14779852, 14.60661794,
       19.6000267 , 18.12427476, 18.52177132, 13.1752861 , 14.62617624,
        9.92374976, 16.34590647, 14.07519426, 14.25756243, 13.04234787,
       18.15955693, 18.69554354, 21.527283  , 17.03141861, 15.96090435,
       13.36141611, 14.52079384,  8.81976005,  4.86751102, 13.06591313,
       12.70609699, 17.29558059, 18.740485  , 18.05901029, 11.51474683,
       11.97400359, 17.68344618, 18.12695239, 17.5183465 , 17.22742507,
       16.52271631, 19.41291095, 18.58215236, 22.48944791, 15.28000133,
       15.82089335, 12.68725581, 12.8763379 , 17.18668531, 18.51247609,
       19.04860533, 20.17208927, 19.7740732 , 22.42940768, 20.31911854,
       17.88616253, 14.37478523, 16.94776851, 16.98405762, 18.58838397,
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