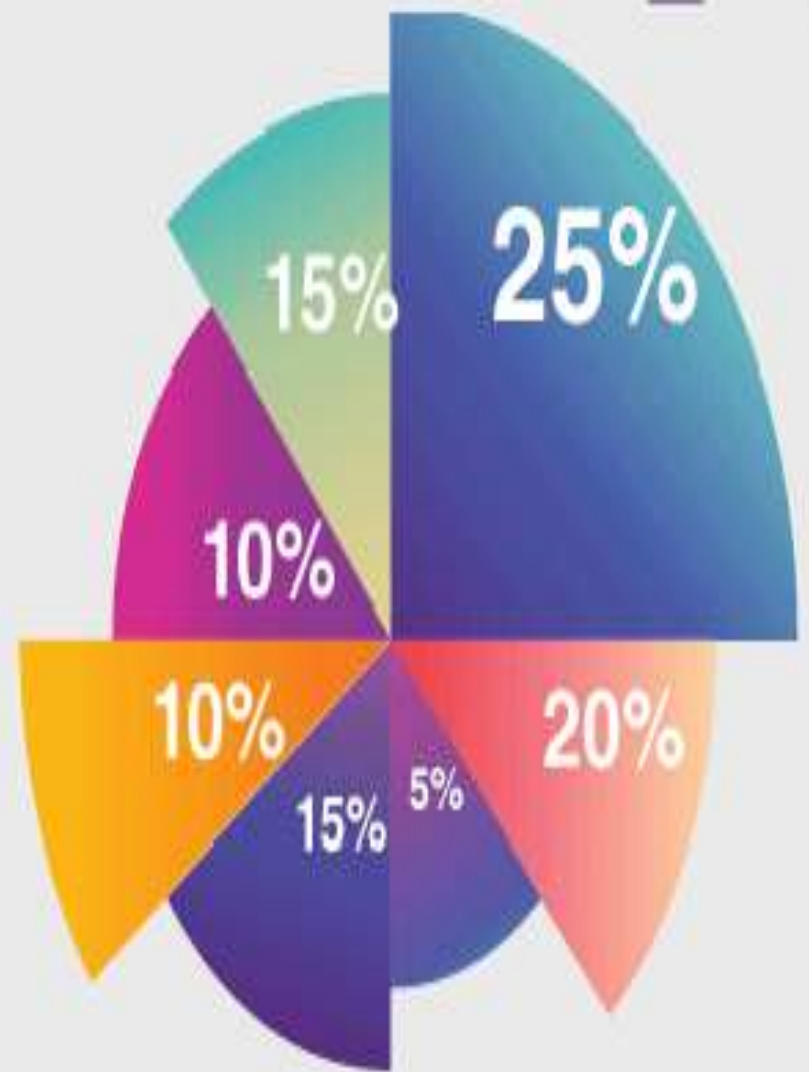


RATIOS — AND — PROPORTION



By Rahul Agrahari

Definition of Ratio

- In certain situations, the comparison of two quantities by the method of division is very efficient. We can say that the comparison or simplified form of two quantities of the same kind is referred to as ratio.
- **Key Points to Remember:-**
- The ratio should exist between the quantities of the same kind
- While comparing two things, the units should be similar
- There should be significant order of terms
- The comparison of two ratios can be performed, if the ratios are equivalent like the fractions

Definition of Proportion

- Proportion is an equation which defines that the two given ratios are equivalent to each other. In other words, the proportion states the equality of the two fractions or the ratios. In proportion, if two sets of given numbers are increasing or decreasing in the same ratio, then the ratios are said to be directly proportional to each other.
- For example, the time taken by train to cover 100km per hour is equal to the time taken by it to cover the distance of 500km for 5 hours. Such as $100\text{km/hr} = 500\text{km}/5\text{hrs}$.

Ratio and Proportion Formula

- Ratio Formula
- Assume that, we have two quantities (or two numbers or two entities) and we have to find the ratio of these two, then the formula for ratio is defined as;
- $a : b \Rightarrow a/b$
- where a and b could be any two quantities.
- Here, “a” is called the first term or **antecedent**, and “b” is called the second term or **consequent**.
- Example: In ratio 4:9, is represented by $4/9$, where 4 is antecedent and 9 is consequent.
- If we multiply and divide each term of ratio by the same number (non-zero), it doesn't affect the ratio.
- Example: $4:9 = 8:18 = 12:27$

Ratio and Proportion Formula

Proportion Formula

Now, let us assume that, in proportion, the two ratios are $a:b$ & $c:d$. The two terms 'b' and 'c' are called 'means or mean term,' whereas the terms 'a' and 'd' are known as 'extremes or extreme terms.'

$$a/b = c/d \text{ or } a : b :: c : d$$

Example: Let us consider one more example of a number of students in a classroom. Our first ratio of the number of girls to boys is 3:5 and that of the other is 4:8, then the proportion can be written as:

$$3 : 5 :: 4 : 8 \text{ or } 3/5 = 4/8$$

Here, 3 & 8 are the extremes, while 5 & 4 are the means.

Note: The ratio value does not affect when the same non-zero number is multiplied or divided on each term.

Important Properties of Proportion

- The following are the important properties of proportion:
- **Addendo** – If $a : b = c : d$, then $a + c : b + d$
- **Subtrahendo** – If $a : b = c : d$, then $a - c : b - d$
- **Dividendo** – If $a : b = c : d$, then $a - b : b = c - d : d$
- **Componendo** – If $a : b = c : d$, then $a + b : b = c + d : d$
- **Alternendo** – If $a : b = c : d$, then $a : c = b : d$
- **Invertendo** – If $a : b = c : d$, then $b : a = d : c$
- **Componendo and dividendo** – If $a : b = c : d$, then $a + b : a - b = c + d : c - d$

Difference Between Ratio and Proportion

S.No	Ratio	Proportion
1	The ratio is used to compare the size of two things with the same unit	The proportion is used to express the relation of two ratios
2	It is expressed using a colon (:), slash (/)	It is expressed using the double colon (::) or equal to the symbol (=)
3	It is an expression	It is an equation
4	Keyword to identify ratio in a problem is “to every”	Keyword to identify proportion in a problem is “out of”

Fourth, Third and Mean Proportional

- If $a : b = c : d$, then:
- d is called the fourth proportional to a, b, c .
- c is called the third proportion to a and b .
- Mean proportional between a and b is \sqrt{ab} .

Duplicate Ratios

- If $a:b$ is a ratio, then:
- $a^2:b^2$ is a duplicate ratio
- $\sqrt{a}:\sqrt{b}$ is the sub-duplicate ratio
- $a^3:b^3$ is a triplicate ratio

QUESTION

- Are the ratios 4:5 and 8:10 said to be in Proportion?
- Are the two ratios 8:10 and 7:10 in proportion?

EXPLANATION

- ✓ $4:5 = 4/5 = 0.8$ and $8:10 = 8/10 = 0.8$
 - Since both the ratios are equal, they are said to be in proportion.
-
- ✓ $8:10 = 8/10 = 0.8$ and $7:10 = 7/10 = 0.7$
 - Since both the ratios are not equal, they are not in proportion.

SSC CHSL

The first, second and third terms of the proportion are 42, 36, 35. Find the fourth term.

EXPLANATION

Let the fourth term be x .

Thus 42, 36, 35, x are in proportion.

Product of extreme terms = $42 \times x$

Product of mean terms = 36×35

Since, the numbers make up a proportion

Therefore, $42 \times x = 36 \times 35$

or, $x = (36 \times 35)/42$

or, $x = 30$

Therefore, the fourth term of the proportion is 30.

QUESTION

□ Given ratio are-

- $a:b = 2:3$
- $b:c = 5:2$
- $c:d = 1:4$

Find $a: b: c$?

EXPLANATION

- Multiplying the first ratio by 5, second by 3 and third by 6, we have
- $a:b = 10:15$
- $b:c = 15:6$
- $c:d = 6:24$
- In the ratio's above, all the mean terms are equal, thus
- $a:b:c:d = 10:15:6:24$

RRB JE

- If $2A = 3B = 4C$, find $A : B : C$

EXPLANATIONA

- Let $2A = 3B = 4C = x$

So, $A = x/2$ $B = x/3$ $C = x/4$

The L.C.M of 2, 3 and 4 is 12

Therefore, $A : B : C = x/2 \times 12 : x/3 \times 12 : x/4 \times 12 = 12$

$= 6x : 4x : 3x$

$= 6 : 4 : 3$

Therefore, $A : B : C = 6 : 4 : 3$

QUESTION

Rs. 5130 is distributed among A, B and C such that A gets $\frac{6}{13}$ rd of what B and C together get and B gets $\frac{4}{14}$ of what A and C together get. Find C's share.

- a. 2160
- b. 1620
- c. 1080
- d. 1800

EXPLANATION

$$A+B+C = 5130$$

$$A = 6/13 (B+C)$$

$$B = 5/14 (A+C)$$

$$A = 6, B = 5, C = 8, \text{ Total} = 19$$

$$A/(B+C) = 6/13$$

$$A_s = 6/19 \times 5130 = 1620$$

$$B/(A+C) = 5/14$$

$$B_s = 5/19 \times 5130 = 1350$$

$$C_s = 8/19 \times 5130 = 2160$$

QUESTION

Rs. 3000 is distributed among A, B and C such that A gets $\frac{2}{3}$ rd of what B and C together get and C gets $\frac{1}{2}$ of what A and B together get. Find C's share.

- a. 1500**
- b. 1000**
- c. 1200**
- d. 1800**

EXPLANATION

$$A+B+C = 3000$$

$$A = \frac{2}{3} (B+C)$$

$$C = \frac{1}{2} (A+B)$$

$$A/(B+C) = \frac{2}{3}$$

$$A_s = \frac{2}{5} \times 3000 = 1200$$

$$C/(A+B) = \frac{1}{2}$$

$$C_s = \frac{1}{3} \times 3000 = 1000$$

$$B_s = 3000 - (1200+1000) = 800$$

QUESTION

5 pens cost as much as 7 pencils. 21 pencils cost as much as 10 books, 16 books cost as much as 9 boxes the cost of two boxes is equal to 64 Rs. What is the cost of 2 pens?

EXPLANATION

$$X = 2 \text{ Pens}$$

$$5 \text{ Pens} = 7 \text{ Pencils}$$

$$21 \text{ Pencils} = 10 \text{ Books}$$

$$16 \text{ Books} = 9 \text{ Boxes}$$

$$2 \text{ Boxes} = 64 \text{ Rs.}$$

$$X = (2 \times 7 \times 10 \times 9 \times 64) / (5 \times 21 \times 16 \times 2)$$

$$X = 24 \text{ Rs. (Ans.)}$$

Direct Proportion Statements

$$y \propto x$$

$$y = kx$$

Statement	formula for y in terms of x
y is proportional to x	$y = kx$
y is proportional to the square of x	$y = kx^2$
y is directly proportional to x cubed	$y = kx^3$
y varies directly with the square root of x	$y = k\sqrt{x}$

Inverse Proportion Statements

$$y \propto \frac{1}{x}$$

$$y = \frac{k}{x}$$

Statement	formula for y in terms of x
y is inversely proportional to x	$y = \frac{k}{x}$
y is inversely proportional to the square of x	$y = \frac{k}{x^2}$
y is inversely proportional to x cubed	$y = \frac{k}{x^3}$
y varies inversely with the square root of x	$y = \frac{k}{\sqrt{x}}$

QUESTION

Cost of diamond is directly proportion to square of its weight. The cost of 10 desi gram of the diamond is 1600 Rs. Diamond break in to two pieces having weight in 3:2. find the value of loss?

EXPLANATION

$$D_c \propto W^2$$

$$1600 \propto (10)^2$$

$$1600 = K \cdot 100$$

$$K = 16$$

$$2:3$$

$$D_1 = k_1 \cdot 4^2 = k_1 \cdot 16$$

$$D_2 = k_2 \cdot 6^2 = k_2 \cdot 36$$

$$\text{After Break the cost of diamond} = D_1 + D_2 = 52 K$$

$$\text{Loss} = 100K - 52K = 48K = 48 \times 16 = 768$$

CONCEPT

Cost \propto weight²

$$(a+b)^2K - (a^2 + b^2) = 2abK$$

Cost \propto weight³

$$(a+b)^3K - (a^3 + b^3)K = 3ab(a+b) K$$

QUESTION

Reduction in speed of a railway engine is directly Proportional to square root of no of compartment attached. If the maximum speed of the engine is 42 Km/hr when no compartment is attached and the speed is 24km/hr when 9 compartment are attached then maximum number of compartment that can be carried throw by the engine?

EXPLANATION

Speed Reduction $\propto (N)^{1/2}$

Speed Reduction = $K (N)^{1/2}$

$SR = K \times (N)^{1/2}$

$S_{\max} - S = K \times (N)^{1/2}$

$V_2 - V_1 = K (9)^{1/2}$

$42 - 24 = K \times 3$

$K = 6$

$42 - V = 6 (N)^{1/2}$

$N = 49$

Questions

1. $A:B = 1:2$, $B:C = 1:3$, $C:D = 2:1$, $D:E = 3:1$, $E:F = 1:2$. Then what is $A:F$, $E:B$, $A:D$, $F:B$, $A:E$?
2. In a college, the ratio of intake across 4 branches (ME, IT, CS, EC) is $3:2:4:5$. The total intake in an year is 560. it is proposed to increase the intake as $33\frac{1}{3}\%$, 50%, 25%, 20% respectively. Find the ratio of new students after the increase ?
3. A bag contains 1rs, 2rs, 5rs, 10rs coins in the ratio of $2 : 3: 4: 5$. Find the number of 5rs coins in the bag if total value of currency in bag is 1560/-?

1. $A:B = 1:2$, $B:C = 1:3$, $C:D = 2:1$, $D:E = 3:1$, $E:F = 1:2$. Then what is $A:F$, $E:B$, $A:D$, $F:B$, $A:E$?

Ans :

$$\begin{array}{r}
 A : B : C : D : E : F \\
 \hline
 1 : 2 \\
 \quad 1 : 3 \\
 \hline
 1 : 2 : 6 \\
 \quad \quad 2 : 1 \\
 \hline
 2 : 4 : 12 : 6 \\
 \quad \quad \quad 3 : 1 \\
 \hline
 6 : 12 : 36 : 18 : 6 \\
 \quad \quad \quad \quad 1 : 2 \\
 \hline
 6 : 12 : 36 : 18 : 6 : 12
 \end{array}$$

$$\begin{array}{r}
 A : B : C \\
 1 : 2 \quad \times 3 \\
 3 : 4 \quad \times 2 \\
 \hline
 3 : 6 : 8
 \end{array}$$

$$\begin{aligned}
 A:F &= 6 : 12 = 1:2 \\
 A:E &= 6 : 6 = 1:1 \\
 E:B &= 6 : 12 = 1:2 \\
 A:D &= 6 : 18 = 1:3 \\
 F:B &= 12 : 12 = 1:1
 \end{aligned}$$

2. In a college, the ratio of intake across 4 branches (ME, IT, CS, EC) is 3:2:4:5. The total intake in an year is 560. It is proposed to increase the intake as $33\frac{1}{3}\%$, 50%, 25%, 20% respectively. Find the percentage increase in students ?

Ans : 3 : 2 : 4 : 5 \Rightarrow sum = 14 i.e out of 14

After increase with respective percentages
The ratio becomes,

4 : 3 : 5 : 6 \Rightarrow sum =18 i.e. out of 18

$$\begin{aligned}\text{Therefore percentage increase in students is } G &= \frac{18 - 14}{14} \times 100 \\ &= \frac{200}{7} \%\end{aligned}$$

3. A bag contains 1rs, 2rs, 5rs, 10rs coins in the ratio of 2 : 3 : 4 : 5. Find the number of 5rs coins in the bag if total value of currency in bag is 1560/-?

Ans :

2 : 3 : 4 : 5

1 2 5 10

2 6 20 50 \Rightarrow sum = 78

Number of coins

78	-	20
1560	--	400

For sum = 1560, 5rs coins should be ??

Number of 5rs coins = $\frac{400}{5} = 80$
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TCS

- The sum of Rs 4,50,000 is to be distributed among A, B, C and D such that

$$A:B = 1:2$$

$$B:C = 1:2$$

$$C:D = 1:2$$

if so, then what is share of C?

A sum of 4,50,000 has to be distributed among A, B, C, D such that

$$A:B = 1:2$$

$$B:C = 1:2$$

$$C:D = 1:2 \quad \text{if so, then what is the share of C?}$$

Ans :

$$A : B : C : D$$

$$1 : 2$$

$$1 : 2$$

$$1 : 2 : 4$$

$$1 : 2$$

$$1 : 2 : 4 : 8$$

$$\Rightarrow \frac{4}{15} \times 4,50,000 = 1,20,000$$

$\text{Share of C} = 1,20,000$

UPSC

- Divide \$370 into three parts such that second part is $\frac{1}{4}$ of the third part and the ratio between the first and the third part is 3 : 5. Find each part.

Solution

- Let the first and the third parts be $3x$ and $5x$.

Second part = $\frac{1}{4}$ of third part.

$$= \left(\frac{1}{4}\right) \times 5x$$

$$= \frac{5x}{4}$$

Therefore, $3x + \left(\frac{5x}{4}\right) + 5x = 370$

$$\frac{(12x + 5x + 20x)}{4} = 370$$

$$\frac{37x}{4} = 370$$

$$x = \frac{(370 \times 4)}{37}$$

$$x = 10 \times 4$$

$$x = 40$$

Therefore, first part = $3x$

$$= 3 \times 40$$

$$= \$120$$

Second part = $\frac{5x}{4}$

$$= \frac{5 \times 40}{4}$$

$$= \$50$$

Third part = $5x$

$$= 5 \times 40$$

$$= \$200$$

GATE 2002

- If the ratio of Income of A and B is in the ratio of 5:4 and expenditure in the ratio of 3:2. If each saved Rs 300. Then find the Income of A and B.

Solution

Income

A:B

5:4

Expense A:B

3:2

$$\frac{A_g - A_g}{B_g - B_g} = \frac{A_E}{B_E} \Rightarrow \frac{5x - 300}{4x - 300} = \frac{3}{2} \Rightarrow \underline{\underline{x = 150}}$$

A's Income = $5x = \text{Rs } 750$

B's Income = $4x = \text{Rs } 600$

UPSC

- If the ratio of the income of A, B and C is 11:9:7 and the ratio of their expenditure is 8:6:5. If A saved Rs 2600 out of Rs 6600. Find the saving of B and C.

Solution

Income of A = Rs 6600

Saved by A = Rs 2600

Exp. of A = Rs 4000

8 = 4000

1 = 500

6 = 3000

5 = 2500

11 = 6600

1 = 600

9 = 5400

7 = 4200

B's ~~income~~ ^{Saving} = 5400 - 3000 = Rs 2400

C's ^{Saving} = 4200 - 2500 = Rs 1700

Thank
you