PERMUTATION AND COMBINATION

Collect the resources and use them.

Permutation

per·mu·ta·tion
A way, esp. one of several possible variations, in which a set or number of things can be ordered or arranged.

Definition:
A permutation is an arrangement in a definite order of a number of objects taken some or all at a time.

Note:
Whenever we deal with permutations order is important.

Combinations

com·bi·na·tion
The act or an instance of combining; the process of being combined.

Definition:
A Combination is a selection of some or all of a number of different objects. It is an un-ordered collection of unique sizes.

Note:
Whenever we deal with combinations order is not important.

FUNDAMENTAL PRINCIPLE OF COUNTING

1- Additive Rule:- Only one thing at a time. (or)

Exp:- 10 Boys and 12 Girls in a class room. How many way to select one monitor.

2-Product Rule:- More than one thing at a time.(and)

Exp:- 10 Boys and 12 Girls in a class room. How many way to select one boys monitor and one girls monitor.

CONCEPT

Number of way from 1 to 3 via 2?

1 2 3

EXPLANATION

Number of ways 1 to 3 via 2

AP BP CP

AQ BQ CQ

AR BR CR

AS BS CS

1 to 2 and 2 to 3

3 X 4 = 12 Ways.

Number of ways 1 to 2 or 2 to 3

3 + 4 = 7 Ways

ABC

At a time pick two alphabet.

SELECTION	ARRANGEMENT
AB = BA	AB ≠ BA
BC = CB	BC ≠ CB
CA = AC	CA ≠ AC
3 WAYS	6 WAYS
Order is not important	Order is important.
Combination	Permutation

CONCEPT

$$C_{(n,r)} = \frac{n!}{r!\,(n-r)!}$$

$$P_{(n,r)} = \frac{n!}{(n-r)!}$$

n = set size: the total number of items in the sample

r = subset size: the number of items to be selected from the sample

$${}^{n}P_{r} = {}^{n}C_{r} \times r!$$

$${}^{n}C_{0} = 1 \qquad {}^{n}P_{0} = 1$$

$${}^{n}C_{1} = n \qquad {}^{n}P_{1} = n$$

$${}^{n}C_{n} = 1 \qquad {}^{n}P_{n} = n!$$

$${}^{n}C_{n-r} = {}^{n}C_{r} \qquad {}^{n}P_{n-1} = n!$$

REDUCTION RULE

$^{7}P_{2} = 7!/(7 - 2)! = 7!/5! = 7 \text{ X } 6 = 42$

$^{7}P_{2} = 7 \text{ X } 6$

$^{n}P_{r} = n(n-1)(n-2)(n-3)\ldots\ldots\ldots\ldots r \text{ terms}$

$^{7}C_{2} = {}^{7}P_{2}/2! = 7X6/2 = 21$

$^{6}C_{3} = {}^{6}P_{3}/3! = (6X5X4)/(3X2X1) = 20$

QUESTION

How many words with or without meaning can be formed by using the letter of the word CAT. Using each letter exactly ones.(Repetition is not allowed)

EXPLANATION

First Method:-

ACT

CAT

CTA

ATC

TAC

TCA (6 Ways)

Second Method:-

3	2	1

$^{3}C_{1} \times {}^{2}C_{1} \times {}^{1}C_{1} = 3X2X1=6$

Third Method:-

$^{n}C_{r} \times r!$

$^{3}C_{3} \times (3X2X1) = 1 \times 6 = 6$

Selection Arrangement

Fourth Method:-

$^{3}P_{3} = 3! = 6$

Arrangement

QUESTION

1- GATE

2- GENCO

REPETATION ALLOWED

CAT

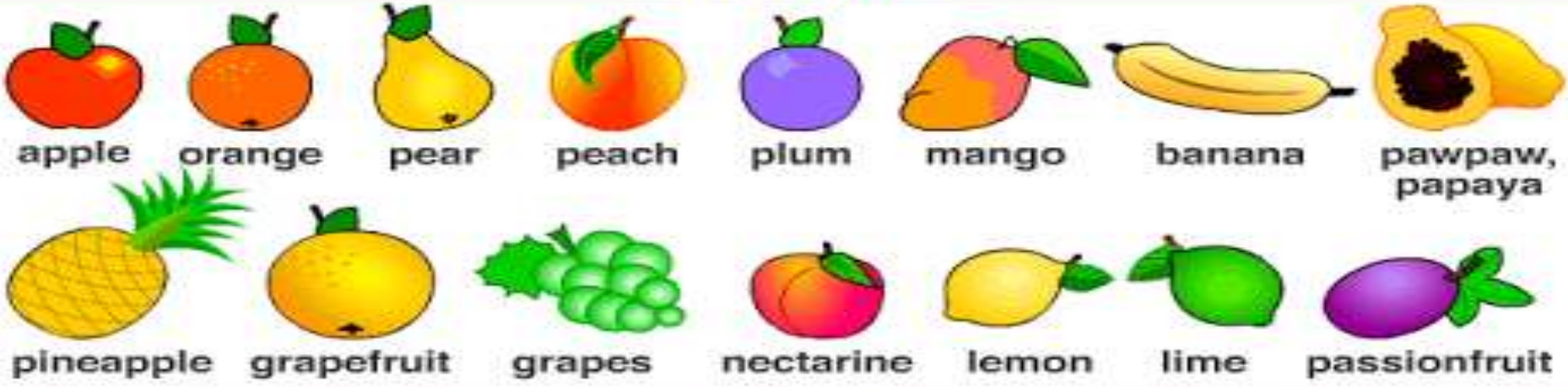
3	3	3

$^{3}C_{1} \times {}^{3}C_{1} \times {}^{3}C_{1} = 3 \times 3 \times 3 = 3^{3} = 27$

GATE :-

GENCO :-

example



2. Using a selected number of the items.

How many ways can a combination of 4 different pieces of fruit can be chosen from a set of 15?

$$C = \frac{n!}{r!(n - r)!}$$

$$C = \frac{15!}{4!11!}$$

$$C = \frac{1307674368000}{958003200}$$

$$C = 1365$$

There are 1,365 different ways 4 fruits can be chosen.

Combinations are arrangements where order is not important. Permutations are about arrangements in ordered sets.

QUESTION

How many different 4 alphabet word can be form by using the letters of the word LOGARITHMS using each letters exactly once.

EXPLANATION

10	9	8	7

$^{10}C_1 \times {}^{9}C_1 \times {}^{8}C_1 \times {}^{7}C_1 = 5040$

$^{10}C_4 \times 4! = 5040$

Selection Arrangement

$^{10}P_4 = 10 \times 9 \times 8 \times 7 = 5040$

If repetition are allowed :-

10	10	10	10

$^{10}C_1 \times {}^{10}C_1 \times {}^{10}C_1 \times {}^{10}C_1 = 10^4$

UPSC

A painter has to paint a 4 digit number by using the digit 1,2,3,4……..up to 9. how many such number can form if the repetition of digit is

(i) Not allowed

(ii) Allowed

EXPLANATION

S = {1,2,3,4,5,6,7,8,9}

9	8	7	6

(i) ${}^{9}C_{1} \times {}^{8}C_{1} \times {}^{7}C_{1} \times {}^{6}C_{1} = 3024$

9	9	9	9

(ii) ${}^{9}C_{1} \times {}^{9}C_{1} \times {}^{9}C_{1} \times {}^{9}C_{1} = 94 = 6561$

QUESTION

In how many different ways the world APPLE can be arranged?

EXPLANATION

AP_1P_2LE

AP_2P_1LE (Duplicate value)

${}^5P_5 / {}^2P_2 = 5!/2!$ (To filter the duplicate value)

$= 60$

QUESTION

(i) TATA

(ii) GOOGLE

(iii) ENGINEERING

EXPLANATION

(i) TATA

4!/(2! X 2!) = 6

(i) GOOGLE

6!/(2! X 2!) = 180

(i) ENGINEERING

11!/(3! X 3! X 2! X 2!) = 277200

QUESTION

In how many different ways the word SIGNATURE arrange so that

(i) All the vowels should come together

(ii) All the vowels should not come together

EXPLANATION

(i) All the vowels should come together

SIGNATURE

SGNTR IAUE

12345 6

6! X 4! (Internal arrangement with in the vowels)

= 17280

(ii) All the vowels should not come together

= All – All the vowels should come together

= 9! – 17280 = 345600

UPSC

Q:- In how many ways 4 boys and 4 girls can be seated in a row so that all the girls should not sit together?

EXPLANATION

4B & 4G

All girls seated together = 5! X 4! = 2880

All girls not seated together = All – All girls seated together

= 8! – (5! X 4!) = 37440

UPSC

Q:- In how many ways the word CORPORATION be arranged so that all the vowels

(i) Should come together?

(ii) Should not come together?

EXPLANATION

CORPORATION

3O,2R,CPATIN

3O,A,I 2R,C,P,T,N

(i) (7! X 5!)/(2! X 3!) = 50400

(ii) All – Unwanted

= 11!/(2! X 3!) – (7! X 5!)/(2! X 3!)

= 3326400 – 50400 = 3276000

ONGC

Q:- In how many ways the word DELHI can be arranged by taking three alphabet at a time so that two alphabet D and E always exist. when the repetition of alphabet is not allowed?

EXPLANATION

DE LHI

$1 \times {}^{3}C_{1} \times 3! = 18$

TCS

Q:- In how many ways the word SIGNATURE by taking 4 alphabet at a time so that 2 alphabet S and G always exist. when the repetition of alphabet is not allowed?

EXPLANATION

SIGNATURE

SG INATURE

$1 \times {}^{7}C_{2} \times 4! = 504$

WIPRO

Q:- In how many different ways BUFFER can be arranged by taking 4 alphabet at a time so that 2 F always exist and any of the remaining alphabet should not be repeated in the remaining place.

EXPLANATION

BUFFER

1 method to select 2F.

$1 \times {}^{4}C_{2} \times 4!/2! = 72$

CHALLANGING

Q:- In how many ways the word MADAM can be arranged taking 3 alphabet at a time?

EXPLANATION

MADAM 2M,2A,D

Case1:- Three different alphabet (MAD)

$^{3}P_{3} = 3! = 6$

Case2:- 2 same and 1 other

(i) 2M and 1 (A/D)

$(^{2}C_{1} \times 3!/2!) = 6$

(ii) 2A and 1 (M/D)

$(^{2}C_{1} \times 3!/2!) = 6$

Total = 6 + 6 + 6 = 18

CAT

Q:- In how many ways the word OFFICIAL can be arranged taking 4 alphabet at a time?

EXPLANATION

OFFICIAL 2F,2I,OCAL

Case1:- All different alphabet

6P4 = 6X5X4X3 = 360

Case2:- 2 same and 2 other

(i) 2F and 2 different

$({}^5C_2$ X 4!/2!) = 120

(ii) 2I and 2 different

$({}^5C_2$ X 4!/2!) = 120

Case 3:- 2I and 2F

4!/(2! X 2!) = 6

Total =360 + 240 + 6 = 606

Thank you