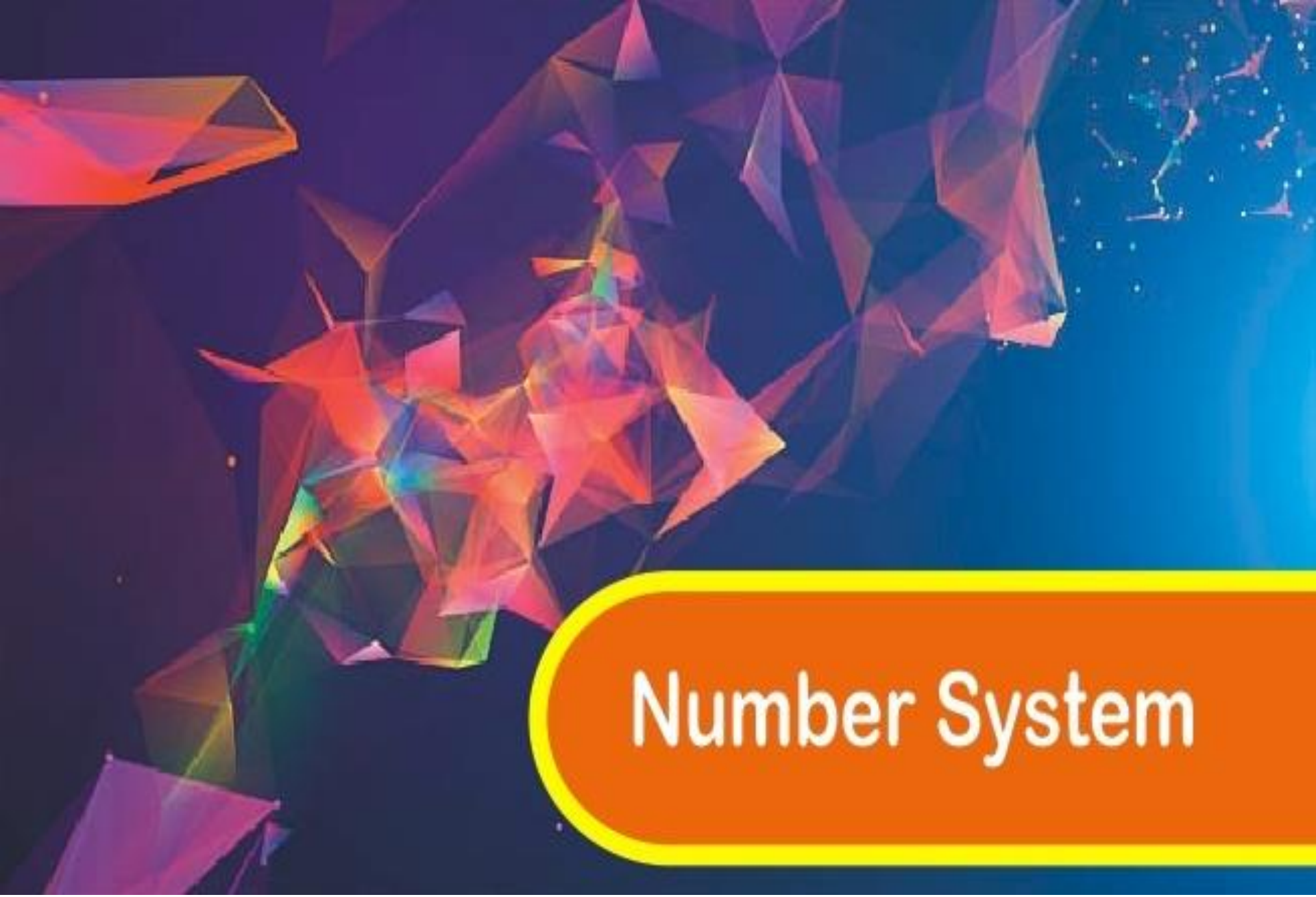

By-Rahul Agrahari

NUMBER SYSTEM

- **Factors**
- **Factorial**
- **Base System**
- **Remainder**
- **Cyclicity (Unit place digit)**
- **L.C.M. and H.C.F. (SELF STUDY)**

FACTOR

Factor is the set of number which will divide given number completely.

Example:-

10=1,2,5,10=4 factors

72= 1,2,3,4,6,8,9,12,18,24,36,72=12 factors

120=1,2,3,4,5,6,8,10,12,15,20,24,30,40,60,120=16

TRICK

72= 1,2,3,4,6,8,9,12,18,24,36,72=12 factors

$72=2^3 \times 3^2 = (3+1) \times (2+1) = 4 \times 3 = 12$

120=1,2,3,4,5,6,8,10,12,15,20,24,30,40,60,120=16

$120=2^3 \times 3^1 \times 5^1 = (3+1)(1+1)(1+1) = 4 \times 2 \times 2 = 16$

PRIME NUMBER

Prime Number:- A number that is divisible only by itself and 1.

Example:- 2,3,5,7,11……ect.

FORMULAS

$N = a^p \times b^q \times c^r$

Total number of Factor = $(p+1)(q+1)(r+1)$

Total prime factor= $p+q+r$

Different prime fcator = 3

Where a,b and c are distinct prime number and p,q and r are the natural number.

$N=2^3 \times 3^2 \times 5^3$

1-Total factor

2-Total prime factor

3-Different prime factor

4-Odd factor

5-Even factor

6-Perfect square factor

7-Perfect cube factor

8-Sum of all factor

9-Product of all factor

FACTORIAL

Factorial is the product of natural numbers.

FACTORIAL

Factorial is the product of natural number.

N!=1X2X3X4X5……(N-1)XN

0!=1

1!=1

2!=1X2=2

3!=1X2X3=6

4!=1X2X3X4=24

5!=1X2X3X4X5=120

6!=1X2X3X4X5X6=720

QUESTION

Q. 1! +2! +3! +4! +5! +6!.....+100!

What is the unit place digit?

EXPLANATION

1! = 1

2! = 1 X 2 = 2

3! = 1 X 2 X 3 = 6

4! = 1 X 2 X 3 X 4 = 24

5! = 1 X 2 X 3 X 4 X 5 =120

1 + 2 + 6 + 24 + 120 + 0 + 0 + 0 + 0 + 0 + 0 = 3

(Unit Place Digit)

CONCEPT

NOTE:-

1-5! and beyond it every factorial will end with at least one zero.

2-10! and beyond it every factorial will end with at least two zeros.

3-15! and beyond it every factorial will end with at least three zeros, and so on……

QUESTION

Q:- 100! trails with how many zeros?

EXPLANATION

100!=1X2X3X4X5X6X7X8……99X100

100/5 = 20 (5,10,15,20,25,30,35,40,45,50,55,60,65,70

75,80,85,90,100)

20/5 = 4 (25,50,75,100)

QUESTION

Q:- In 100! how many times does 2 appear?

EXPLANATION

100/2 = 50 (2,4,6,8,10,12,14,16,18,20,22,24,....)

50/2 = 25 (4,8,12,16,20,24,28,32,36.....)

25/2 = 12 (8,16,24,32,40.....)

12/2 = 06 (16,32,48,64,80,96)

6/2 = 03 (32,64,96)

3/2 = 01 (64)

TOTAL = 97

QUESTION

Q:- In 100! how many times does 3 appear?

EXPLANATION

100/3 = 33 (3,6,9,12,15,18,21,24,….)

33/3 = 11 (9,18,27,36,45,54,63…….)

11/3 = 3 (27,54,81)

3/3 = 1 (81)

TOTAL = 48

QUESTION

Q:- In 100! What is the maximum power of 12?

EXPLANATION

$100! = 1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7 \times 8 \times 9 \times 10 \times 11$

$\times 12 \ldots \times \ldots 24 \ldots \times \ldots\ldots 36 \ldots\ldots \times 48 \ldots\ldots\ldots \times 96$

$12 = 2^2 \times 3^1$

$100! = 2^{97} \times 3^{48} = (2^2)^{48} \times 3^{48} = (2^2 \times 3^1)^{48} = 12^{48}$

IAS

Q. $40!^{40!}$ trails with how many zeros?

EXPLANATION

40/5 = 8 (5,10,15,20,25,30,35,40)

8/5 = 1 (25)

TOTAL = (8+1) = 9

10^2 = 100

10^3 = 1000

Ans:- 9 X 40!

BASE SYSTEM

BASE SYSTEM

$(25)_{10} = (11001)_2 = (121)_4 = (41)_6$

BASE	DIGIT	MAXIMUM DIGIT
2	0,1	1
4	0,1,2,3	3
N	0,1,2,3……N	N-1

QUESTION

32+24 = 100 find out the base?

EXPLANATION

b^1b^0 b^1b^0 $b^2b^1b^0$

3 2 + 2 4 = 1 0 0

$(3b + 2) + (2b + 4) = b^2$

$b^2 – 5b – 6 = 0$

$(b – 6)(b + 1) = 0$

$b = 6, - 1$

SHORT CUT :-

32 + 24 = 100

2 + 4 = 0 + b

b = 6

NOTE

Base can never be negative and fraction.

GATE-2010

Q. If 137+276=435 how much is 731+672=?

(a) 534

(b) 1403

(c) 1623

(d) 1531

EXPLANATION

137 + 276 = 435

7+6 = 5 + b

b = 8 (Base)

731 + 672 = 1623

QUESTION

If 10001-2222=4446, then value of 2342-1656=?

(a) 453

(b) 353

(c) 893

(d) 686

EXPLANATION

10001 – 2222 = 4446

1 – 2 = 6 – b

b = 7

2342 – 1656 = 0353

QUESTION

44 X 11=1034 find out base?

EXPLANATION

44

X 11

44

44 X

1034

4 + 4 = 3 + b

b = 5 (Ans)

REMAINDER

Any number in the world can be written as:

N = (Re) Mod (Divisor)

80 = 8 Mod (9)

24 = 4 Mod (5)

26 = 5 Mod (7)

CONCEPT

If $X = Y \bmod C$

Then $X-Y = 0 \bmod C$

$80 = -1 \bmod (9)$

$24 = -1 \bmod (5)$

$26 = -2 \bmod (7)$

RULE-1

$A = B \bmod C$

$D = E \bmod C$

$F = G \bmod C$

$A \times D \times F = B \times E \times G \bmod C$

$B \times E \times G < C$

Note:- This rule is applicable only on 3 operations (addition, subtraction and multiplication).

EXAMPLE

(1421 X 1423 X1425) ÷ 12

what is the remainder?

EXPLANATION

1421 = 5 Mod 12

1423 = 7 Mod 12

1425 = 9 Mod 12

1421 X 1423 X 1425 = 5 X 7 X 9 Mod 12

= 315 Mod 12

Re = 3

RULE-2

If $A = B \ Mod \ C$

Then $A^n = B^n \ Mod \ C$

$B^n < C$

EXAMPLE

Q. $2^{600} \div 15$

What is the remainder?

EXPLANATION

$2^1 = 2 \text{ Mod } 15$

$2^2 = 4 \text{ Mod } 15$

$2^3 = 8 \text{ Mod } 15$

$2^4 = 1 \text{ Mod } 15$---------- [i]

According to rule 2 Both side power 150

$(2^4)^{150} = 1^{150} \text{ Mod } 15$

$2^{600} = 1 \text{ Mod } 15$

Re = 1 (Ans.)

QUESTION

$(10^{10} + 10^{1000} + 10^{10000} - 10^{100}) \div 3$

What is the remainder?

EXPLANATION

$10 = 1 \text{ Mod } 3$

Using rule number 1 and 2

$10^{10} = 1 \text{ Mod } 3$

$10^{1000} = 1 \text{ Mod } 3$

$10^{10000} = 1 \text{ Mod } 3$

$10^{100} = 1 \text{ Mod } 3$

$(10^{10} + 10^{1000} + 10^{10000} - 10^{100}) = 2 \text{ Mod } 3$

Re = 2 (Ans.)

QUESTION

Q. $5^{625} \div 7$

What is the remainder?

EXPLANATION

$5^3 = -1 \text{ Mod } 7$

$(5^3)^{208} = (-1)^{208} \text{ Mod } 7$

$5^{624} = 1 \text{ Mod } 7$

$5 = 5 \text{ Mod } 7$

$5^{625} = 5 \text{ Mod } 7$

Re = 5 (Ans.)

CONCEPT

$f(x) = X^2 - 5X + 6$

$f(x) = (X - 2)(X - 3)$

$f(2) = 0$

$f(3) = 0$

CASE - 1

$(a^n + b^n) = (a + b)\ K$

$a + b = 0$

$a = -b$

$(-b)^n + b^n = 0 \qquad \{\because n = \text{odd}\}$

QUESTION

$(15^{23} + 23^{23}) \div 19$

What is the remainder?

EXPLANATION

$(15^{23} + 23^{23}) = (15 + 23\ )\ K$

$= 38\ K\ /\ 19 = Re = 0$

QUESTION

$(16^3 + 17^3 + 18^3 + 19^3) \div 70$

What is the remainder?

EXPLANATION

$(16^3 + 17^3 + 18^3 + 19^3) \div 70$

$(16^3 + 19^3) + (17^3 + 18^3) \div 70$

$35\ K_1 + 35\ K_2 = 35\ (K_1 + K_2) \div 70$

$35\ X\ 2\ (K_3) \div 70$

$70\ K_3 \div 70$

$Re = 0$

CASE - 2

$(a^n - b^n) = (a + b)\ K$

$(a + b) = 0$

$a = -\ b$

$(-b)^n - b^n = 0$ ---------- {∴ n = Even}

CASE - 3

$(a^n – b^n) = (a - b)\ K$

$(a - b) = 0$

$a = b$

$(b)^n - b^n = 0$ ---------- {❖ n = Any natural number}

CASE - 4

$(a^n + b^n) = (a - b)\ K$

$a - b = 0$

$a = b$

$(\ b)^n + b^n = 0$ {∴ n = Never}

UNIT PLACE DIGIT

$2^1 = 2$	$3^1 = 3$	$4^1 = 4$	$7^1 = 7$	$8^1 = 8$	$9^1 = 9$
$2^2 = 4$	$3^2 = 9$	$4^2 = 16$	$7^2 = 49$	$8^2 = 64$	$9^2 = 81$
$2^3 = 8$	$3^3 = 27$	$4^3 = 64$	$7^3 = 343$	$8^3 = 512$	$9^3 = 729$
$2^4 = 16$	$3^4 = 81$	$4^4 = 256$	$7^4 = 2401$	$8^4 = 4096$	$9^4 = 6561$
$2^5 = 32$	$3^5 = 243$	$4^5 = 1024$	$7^5 = 16807$	$8^5 = 32768$	$9^5 = 59049$
$2^6 = 64$	$3^6 = 729$	$4^6 = 4096$	$7^6 = 117649$	$8^6 = 262144$	$9^6 = 531441$
$2^7 = 128$	$3^7 = 2187$	$4^7 = 16384$	$7^7 = 823543$	$8^7 = 2097152$	$9^7 = 4782969$
$2^8 = 256$	$3^8 = 6561$	$4^8 = 65536$	$7^8 = 5764801$	$8^8 = 16777216$	$9^8 = 43046721$

CYCLICITY

BASE DIGIT	*2*	*3*	*4*	*7*	*8*	*9*
4N+1	2	3	4	7	8	9
4N+2	4	9	6	9	4	1
4N+3	8	7		3	2	
4N+4 OR 4N	6	1		1	6	

QUESTION

Unit place digit of 23252^{765438}

EXPLANATION

765438 ÷ 4 = Re = 2 That means (4N+2) form.

$2^{(4N + 2)}$ = 4 (Unit place digit)

QUESTION

Q:- Find unit place digit $2^{72} \times 3^{83} \times 7^{42} \times 8^{56}$

EXPLANATION

$2^{72} \times 3^{83} \times 7^{42} \times 8^{56}$

$72 \div 4 = Re = 0\ (4N + 4)$ form

$83 \div 4 = Re = 3\ (4N + 3)$ form

$42 \div 4 = Re = 2\ (4N + 2)$ form

$56 \div 4 = Re = 0\ (4N + 4)$ form

$2^{(4N + 4)} \times 3^{(4N + 3)} \times 7^{(4N + 2)} \times 8^{(4N + 4)}$

$6 \times 7 \times 9 \times 6 = 8$

QUESTION

Q:- Find unit place digit $23^{32!}$

EXPLANATION

Q:- Find unit place digit $23^{32!}$

$32! = 1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7 \times 8 \ldots\ldots\ldots \times 32$

$3^{(4N+4)} = 1$ (Unit place digit)

QUESTION

Find unit place digit $(32!)^{23}$

EXPLANATION

Find unit place digit $(32!)^{23}$

32! = 1X2X3X4X5X6X7X8X9……..X32 =xyz……0

Ans:- 0

QUESTION

Find unit place digit

1!+2!+3!+4!+5!+6!.....+100!

QUESTION

Find unit place digit

$1^2+2^2+3^2+4^2+5^2\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots+99^2$

EXPLANATION

$1^2+2^2+3^2+4^2+5^2$……$+99^2$

= N(N+1)(2N+1)/6

= 99(99+1)(2X99+1)/6

= (99 X 100 X 199)/6

= 0 Unit place digit

QUESTION

Find unit place digit

$$1^1+2^2+3^3+4^4+5^5\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots+10^{10}$$

EXPLANATION

$1^1+2^2+3^3+4^4+5^5\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots+10^{10}$

1+4+7+6+5+6+3+6+9+0 =37

Thank you