Permutations & Combinations

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QUESTION

A three digit number is formed by using the digit 1,2,3 without repeating any of them. What is the sum of all such possible three digit number?

EXPLANATION

123+132+213+231+312+321=1332

Short Cut :-

Basic sum X Visibility in the column X Sum of place value

(1+2+3) X (2) X (100+10+1) = 1332

Visibility in the column = N!/ Number of digit

= 3! / 3 = 2

GATE-2014

Q:- A five digit number is formed using the digits 1,3,5,7 and 9 without repeating any of them what is the sum of all such possible five digit numbers?

(a) 6666660

(b) 6666600

(c) 6666666

(d) 6666606

EXPLANATION

Basic sum X Visibility in the column X Sum of place value

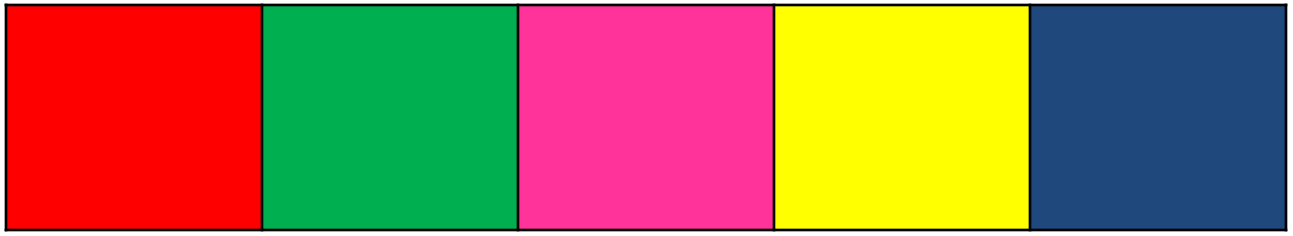
(1+3+5+7+9) X 5!/5 X (10000+1000+100+10+1) = 9999900

QUESTION

A new flag is to be design having 5 vertical strips selected colour (R,G,B,Y,P) how many such flag are there in which no two adjacent strip will have the same colour.

EXPLANATION

5	4	4	4	4



5 X 4 X 4 X 4 X 4 = 1280

PSU

10 identical balls distributed among 3 friends. How many ways can be distribution done? (whole number solution)

EXPLANATION

A	B	C	=10
	0	10	
	1	9	
	2	8	
	3	7	
	4	6	
0	5	5	
	6	4	
	7	3	
	8	2	
	9	1	
	10	0	

CONT……

A	B	C	10
	0	9	
	1	8	
	2	7	
	3	6	
1	4	5	
	5	4	
	6	3	
	7	2	
	8	1	
	9	0	

CONT.....

A	B	C	10
9	0	1	
	1	0	
10	0	0	

11+10+9+.....+1=

n(n+1)/2=11 X 12/2 = 66

n identical object are to distributed among r people can be done in

$^{n+r-1}C_{r-1}$ $^{10+3-1}C_{3-1}$= 66

NOTE:-

1- Identical object.

2- Whole number solution.

NATURAL NUMBER SOLUTION

$A+B+C=10$

$A+B+C=7$

$^{7+3-1}C_{3-1} = {}^{9}C_{2} = 9x8/2 = 36$

CAT

Q:- 15 identical ball are to be distributed among 4 friends (A,B,C,D) in how many way distribution is made if A to get at least 3 balls, B at least 2 balls and C at least 1 ball?

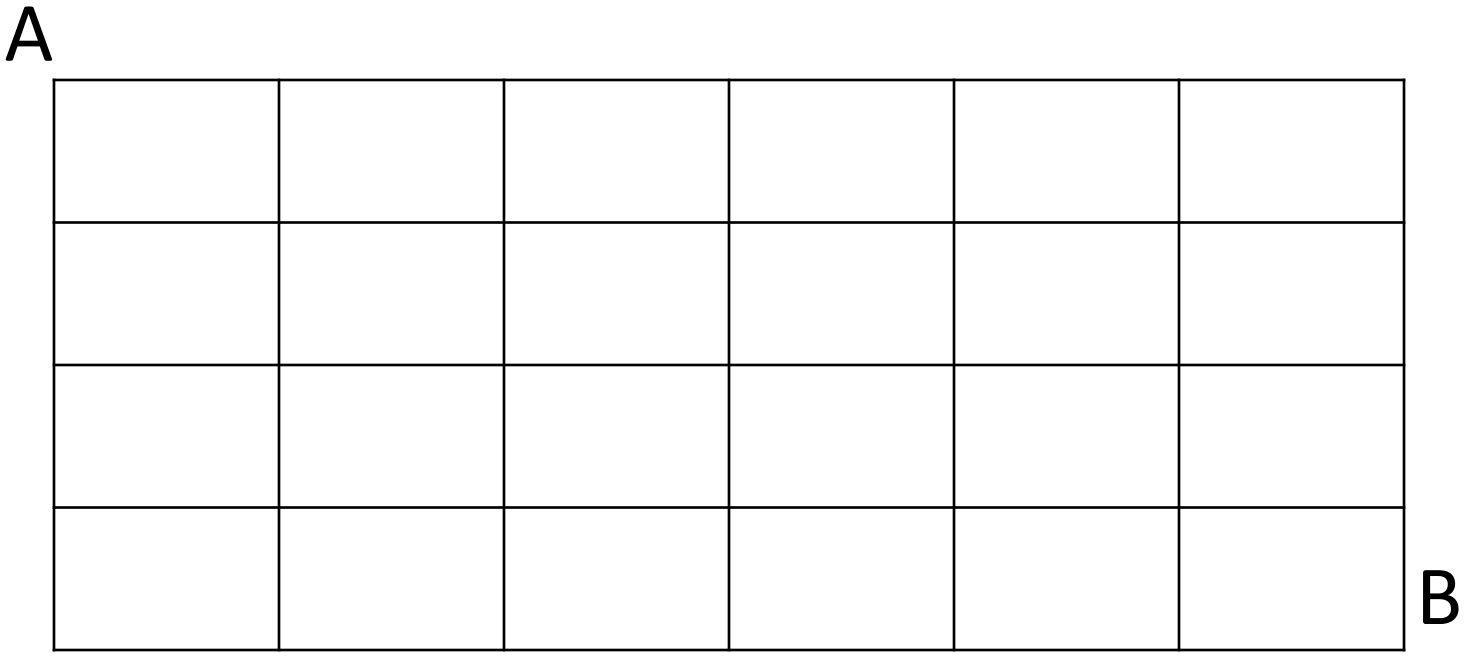
EXPLANATION

$A + B + C + D = 15$

$A + B + C + D = 9$

${}^{9+4-1}C_{4-1} = {}^{12}C_{3} = 220$

PSU,ONGC,NTPC,GMAT



How many different shortest route from A to B?

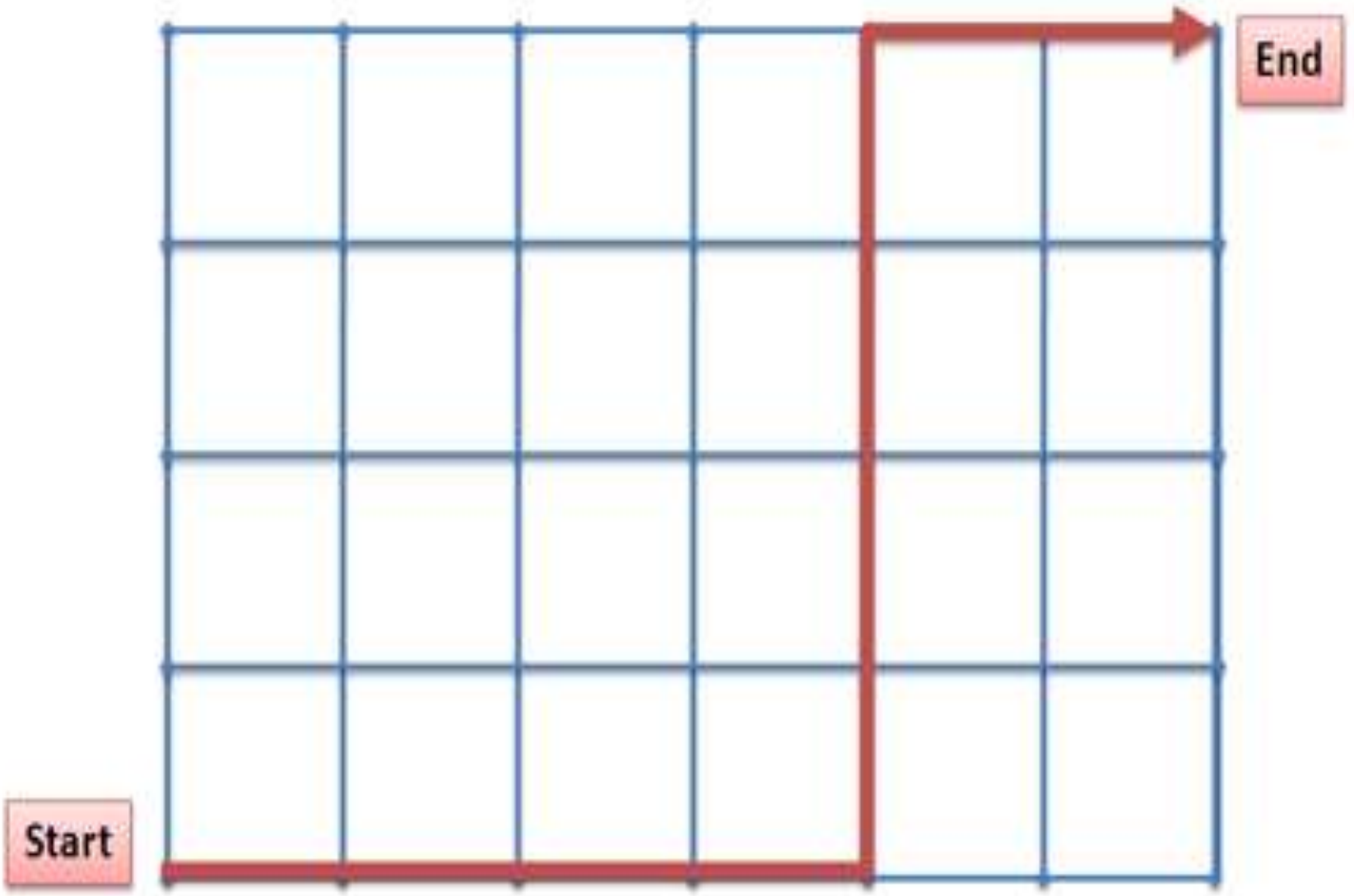
EXPLANATION

Formula = (H + D)!/H! X D!

= (6 + 4)!/6! X 4!

= 10!/6! X 4!

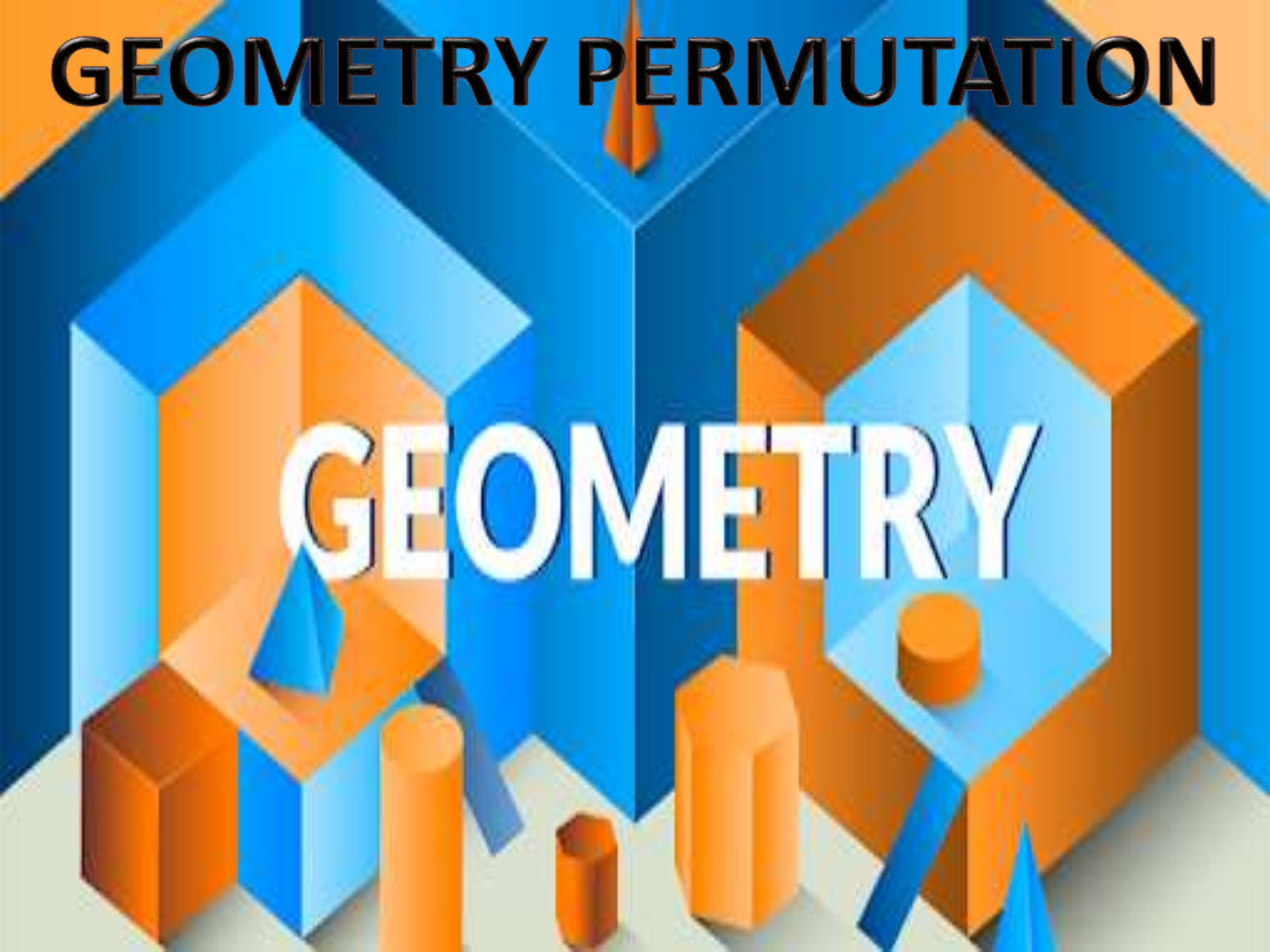
How Many Paths?



EXPLANATION

10!/6! X 4! = 210

GEOMETRY PERMUTATION



QUESTION

Q:- 12 points how many straight line?

EXPLANATION

$$^{12}C_2 = 12 \times 11/2 \times 1 = 66$$

QUESTION

Q:- 12 points how many straight line, in this question 5 points collinear?

(a) 56

(b) 66

(c) 21

(d) 57

EXPLANATION

$$^{12}C_2 - (^5C_2) + 1 = 57$$

QUESTION

Q:- 12 points how many triangle ?

EXPLANATION

$^{12}C_3 = 12 \times 11 \times 10 / 3 \times 2 \times 1 = 220$

QUESTION

Q:- 12 points how many triangle, in this question 5 points are collinear?

(a) 220

(b) 210

(c) 200

(d) NOT

EXPLANATION

$^{12}C_3 - (^5C_3) = 220 - 10 = 210$

UPSC

Q:- Line L parallel to line M, L has 10 points M has 11 points. How many triangle form?

EXPLANATION

$^{21}C_3 - {}^{10}C_3 - {}^{11}C_3 = 1045$

10 Points

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11 Points

$^{10}C_2 \times {}^{11}C_1 + {}^{11}C_2 \times {}^{10}C_1 = 1045$

COGNIZANT

10 points lie in a plane out of which 4 points are collinear. Barring this 4 points no 3 points are collinear. How many distinct quadrilateral can be drawn from these points.

EXPLANATION

Out of 10 points 4 are collinear so remaining

non-collinear points are 6

No. of quadrilateral:-

4 points from 6 non- collinear points

3 points from non-collinear and 1 from collinear points

2 from collinear and 2 from non-collinear points

$= {}^{6}C_{4} + {}^{6}C_{3} * {}^{4}C_{1} + {}^{6}C_{2} * {}^{4}C_{2}$

$= 15 + 80 + 90$

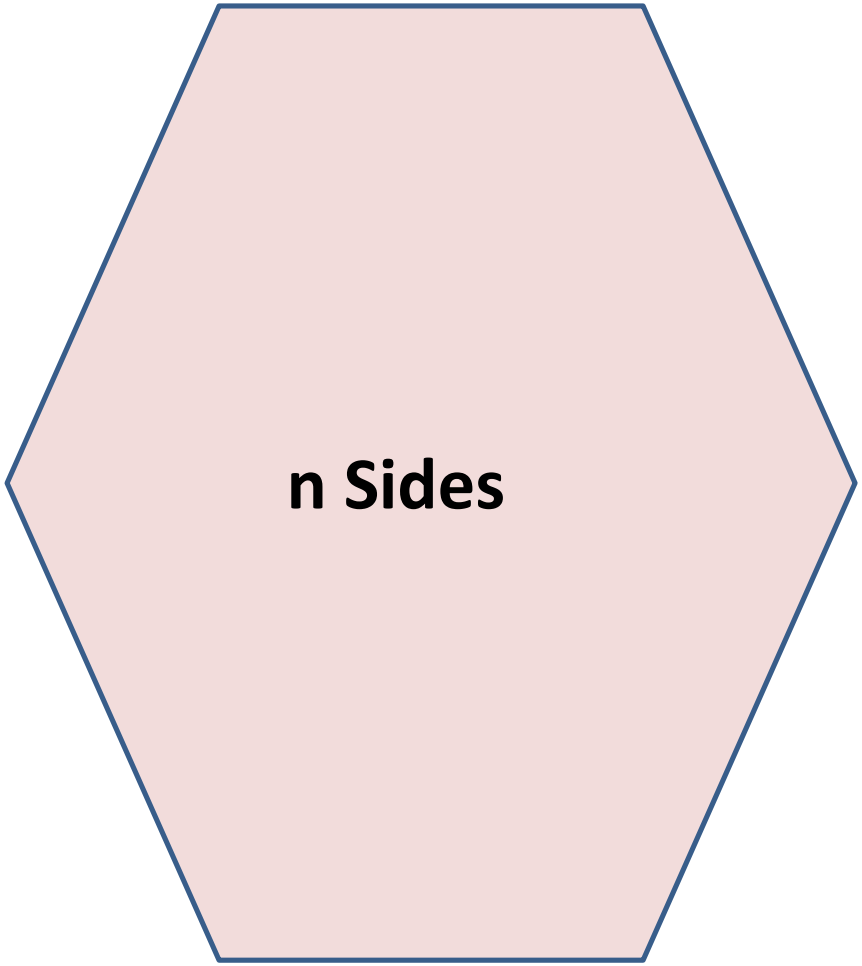
$= 185$

CONCEPT

Number of diagonal of any n – sides polygon

$$= n(n-3)/2$$

PROOF



$^{n}C_{2}$ = All sides + All diagonal

All diagonal = $^{n}C_{2} - n$

$= n(n-1)/2 - n$

$= n(n-3)/2$

CAT

Q:- If number of diagonal of n sides polygon is 50% more than its number of side. Then the polygon is ?

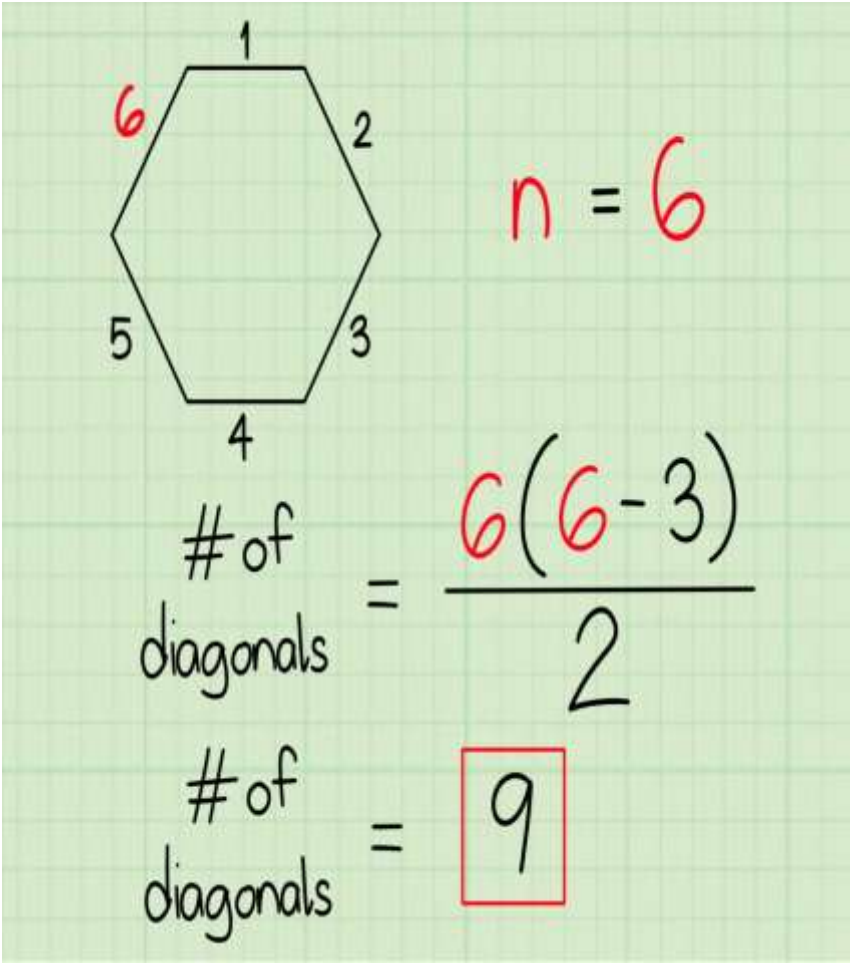
(a)Pentagon

(b)Hexagon

(c)Heptagon

(d)Triangle

EXPLANATION



n(n-3)/2 = 1.5 n

n(n-3) = 3n

n-3 = 3

n = 6 (Hexagon)

CIRCULAR PERMUTATION

LINEAR PERMUTATION:-

A,B,C = N!

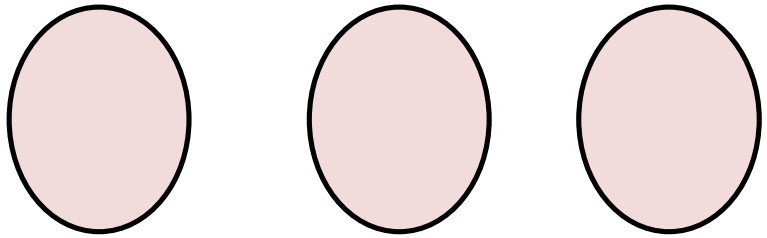
3 X 2 X 1 = 6

3	2	1

CIRCULAR PERMUTATION:-

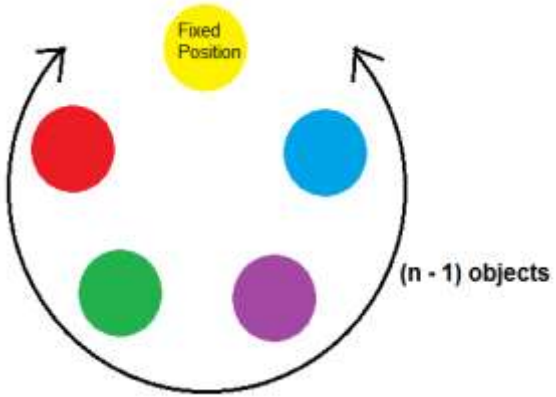
A,B,C = (N-1)!

(3-1)! = 2! = 2



Circular Gardening:-

To plant N different Flowers
(N-1)!/2
In Gardening Clockwise and Anticlockwise is considered same



IB 2006

In how many ways 5 cricketer and 4 basket ball players can be seated around a table if.

I. **There is no restriction.**

II. **All the 4 basket ball player sit together.**

III. **None of the basket ball player sit together.**

EXPLANATION

I. (N-1)! = (9-1)! = 8!

II.5! X 4!

III.8! – (5! X 4!)

WIPRO

A gardener want to plant 6 different types of Flowers in his garden in a circle. In how many different ways can he plant these Flowers.

EXPLANATION

The order of planting flowers is important as the flowers are of different types.

As the order is important we use **Permutation**

The number of permutation of P object arranged in circle is (P-1)!/2

Hence 6 plant in(6-1)!/2

i.e. **60 ways.**

QUESTION

Q:- A couple invite there 10 friends to a dinner party to be held a cross a circular dinning table having 12 chair, in how many way can be get sheeted if there has to be exactly 1 friend in between the couple?

(a) 10 X 9!

(b) 10!

(c) 2 X 10!

(d) NOT

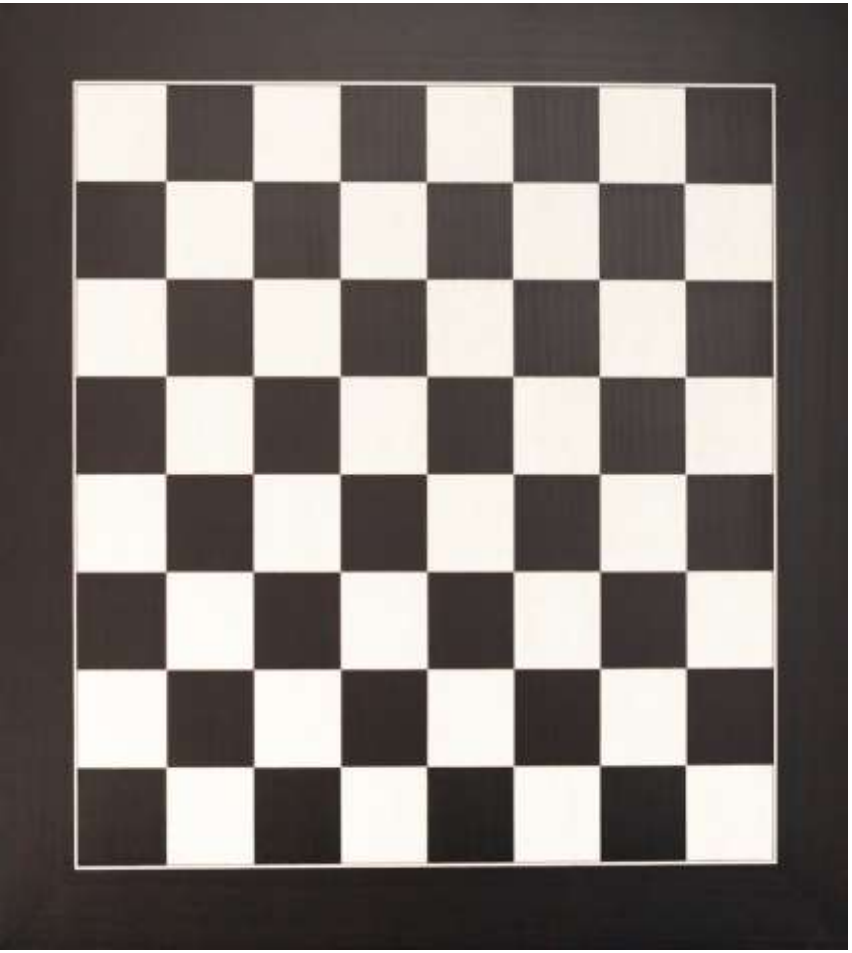
EXPLANATION

H F_1W + 9 F

(10-1)! X 10 X 2 = 10! X 2 (Ans.)

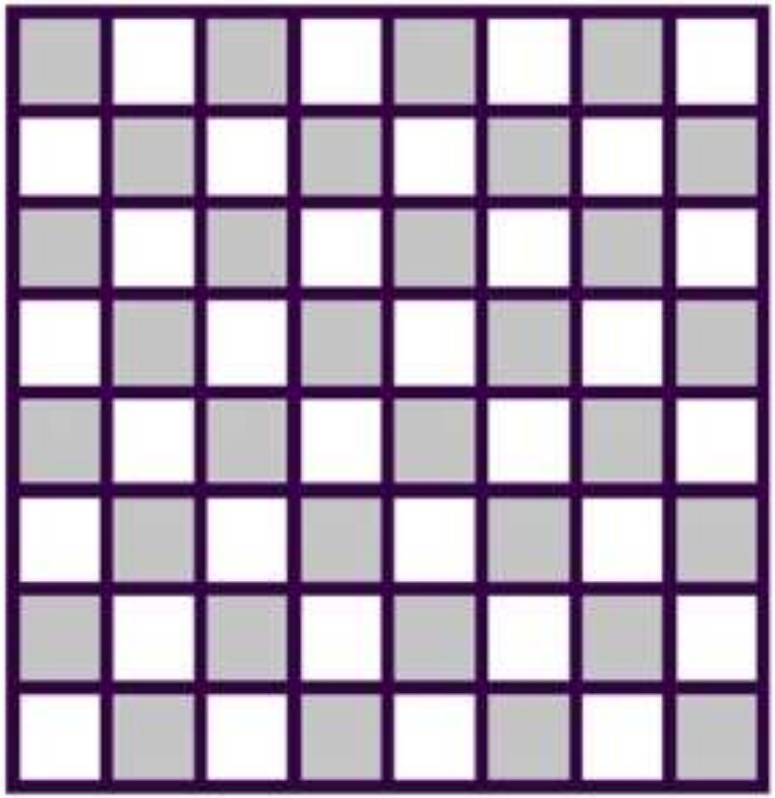
CHESS BOARD

QUESTION



1Q:- How many Rectangle?

EXPLANATION



9 H , 9 V line

$^{9}C_{2}$ X $^{9}C_{2}$ = 36 X 36 =

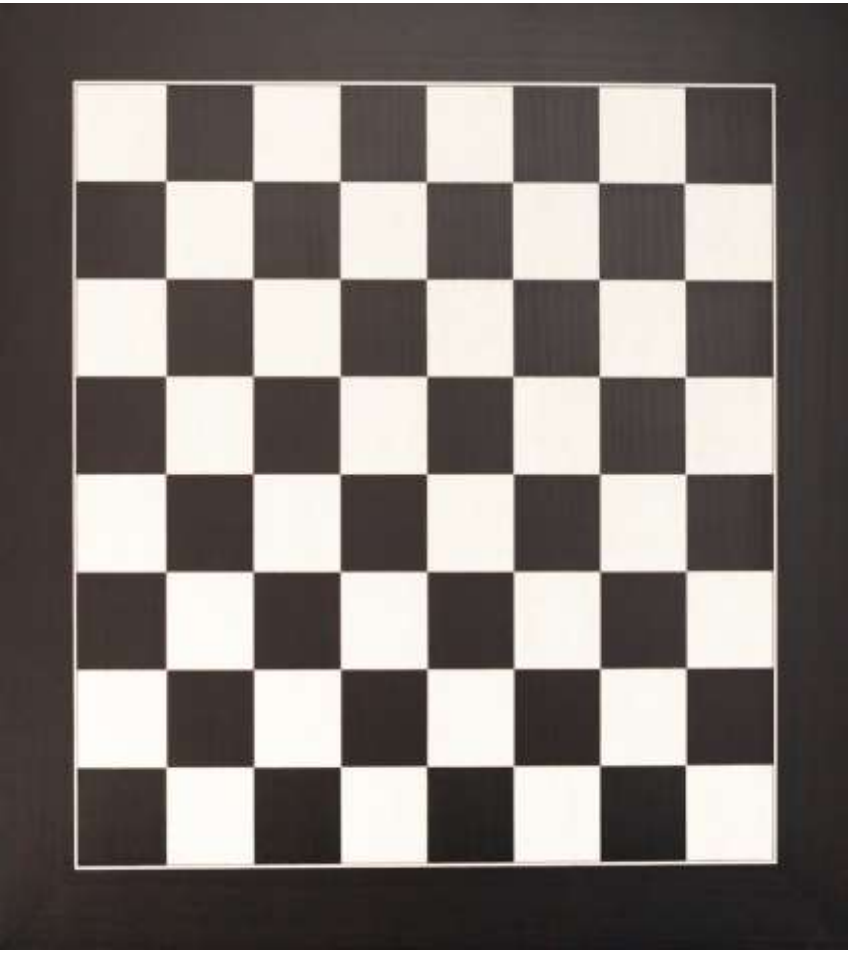
1296

Alternate method

$$= \frac{N^2(N+1)^2}{4}$$

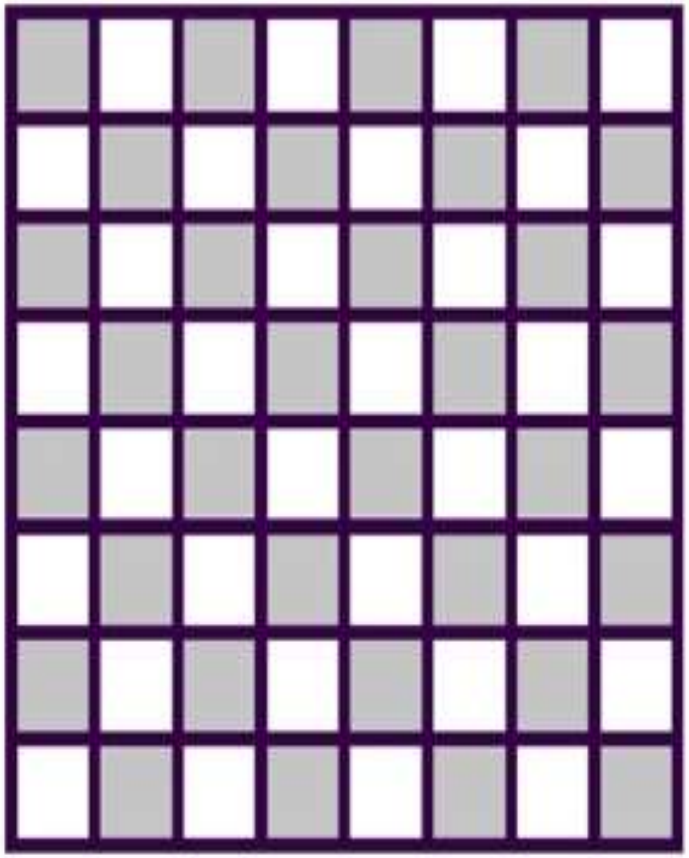
Were N is boxes

QUESTION



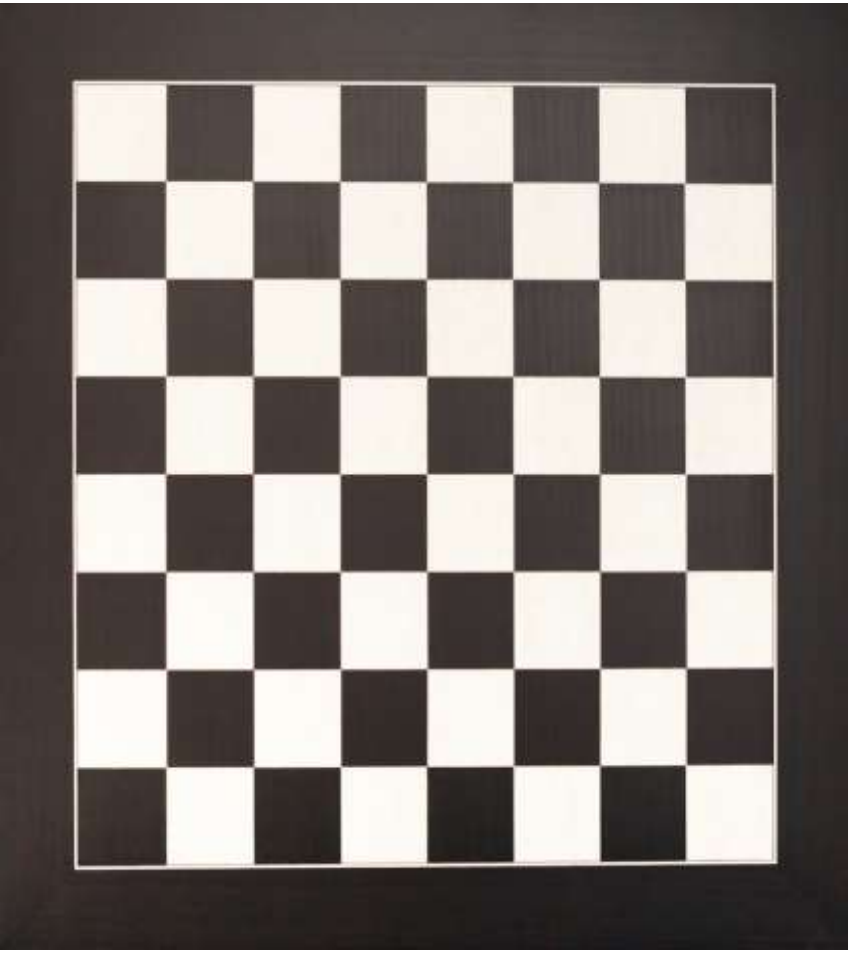
Q:- How many square?

EXPLANATION



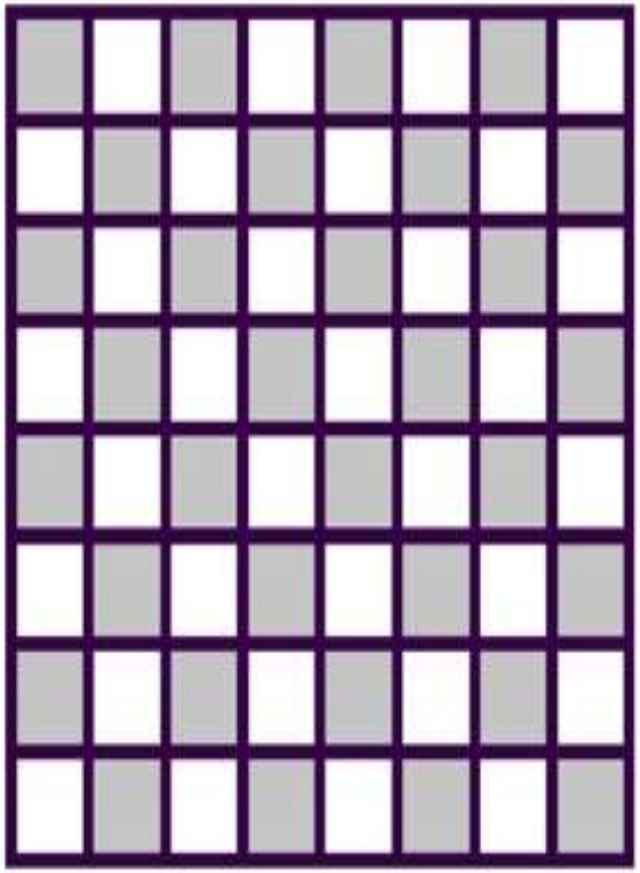
Size	Vertical positions	Horizontal positions	Total Positions
1*1	8	8	64
2*2	7	7	49
3*3	6	6	36
4*4	5	5	25
5*5	4	4	16
6*6	3	3	9
7*7	2	2	4
8*8	1	1	1
		Total	204

QUESTION



Q:- How many different type of rectangle ?

EXPLANATION



Different type of rectangle=

1X1,1X2,1X3,1X4,1X5,1X6,1X7,1X8= 8

2X2,2X3,2X4,2X5,2X6,2X7,2X8 = 7

3X3,3X4,3X5,3X6,3X7,3X8 = 6

,, ,, ,, ,, ,, ,, ,,,

8X8 = 1

8+7+6+5+4+……..+1 = n(n+1)/2

= (8 X 9)/2 = 36

THANK YOU!