

The background is a complex, abstract pattern of blue lines forming a grid that curves and spirals inward, creating a tunnel-like effect. A solid white rectangle is centered horizontally and vertically, serving as a backdrop for the title text.

Probability

By – Rahul Agrahari

What Is Probability?

- **Probability** is a measure of how likely it is for an event to happen.
- We name a **probability** with a number from 0 to 1.
- If an event is certain to happen, then the **probability** of the event is 1.
- If an event is certain not to happen, then the **probability** of the event is 0.
- We can even express **probability** in percentage.

<i>TERM</i>	<i>DEFINITION</i>	<i>EXAMPLE</i>
Sample Space	The set of all the possible outcomes to occur in any trial	1.Tossing a coin, Sample Space (S) = {H,T}
Experiment or Trial	A series of actions where the outcomes are always uncertain.	The tossing of a coin, Selecting a card from a deck of cards, throwing a dice.
Event	It is a single outcome of an experiment.	Getting a Heads while tossing a coin is an event.
Outcome	Possible result of a trial/experiment	T (tail) is a possible outcome when a coin is tossed.

CONCEPT



$$\text{probability} = \frac{\text{event/s}}{\text{number of outcomes}}$$

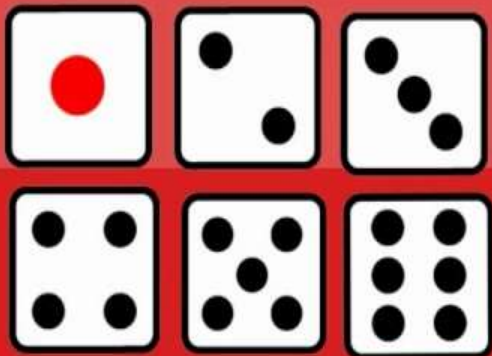
CONCEPT

Which Sides Are Greater Than 3?



Number of Favorable Outcomes = 3

How Many Possibilities Are There?



Total Number of Possibilities = 6

Number of Favorable Outcomes / Total Number of Outcomes = $3/6 = 50\%$



TYPE OF EVENT

1 – INDEPENDENT EVENT

2 – DEPENDENT EVENT

What is an independent event?

- Independent events are events where the outcome of one event **does not affect** the outcome of the other events
 - Example:
 - Tossing a coin and rolling a number cube are independent events.





Mutually Exclusive and Dependence Relationship

- **Mutually Exclusive Events** (events that have NO common outcomes) are ALWAYS **dependent** on Each other!
- **Ex 1: Say you roll a di once.**
- A = even #, then $A = \{2, 4, 6\}$ and B = odd #, then $B = \{1, 3, 5\}$ are Both **Mutually Exclusive events**. They are also Dependent because:
 - $P(A)$ does not equal $P(A|B)$.
 - $3/6$ does not equal 0.

4.6 Independent Versus Dependent Events

- Two events are said to be **Independent** if the occurrence of one does **NOT** affect the Probability of the other.
- **Example:** Rolling a die and tossing a coin.
- **Definition:** 2 events, A and B, are Independent if either:
 $P(A | B) = P(A)$ or $P(B | A) = P(B)$
If one above is true, then the other one is true. They mean that the
Conditional Prob.= Marginal Prob.

SAMPLE SPACE FOR PAIR OF DICE

	1	2	3	4	5	6
1	(1, 1)	(1, 2)	(1, 3)	(1, 4)	(1, 5)	(1, 6)
2	(2, 1)	(2, 2)	(2, 3)	(2, 4)	(2, 5)	(2, 6)
3	(3, 1)	(3, 2)	(3, 3)	(3, 4)	(3, 5)	(3, 6)
4	(4, 1)	(4, 2)	(4, 3)	(4, 4)	(4, 5)	(4, 6)
5	(5, 1)	(5, 2)	(5, 3)	(5, 4)	(5, 5)	(5, 6)
6	(6, 1)	(6, 2)	(6, 3)	(6, 4)	(6, 5)	(6, 6)

TCS

Three fair coins are tossed simultaneously. What is the probability of getting at least 2 tails?

EXPLANATION

When 3 coins are tossed, the possible outcomes can be

{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT}.

Thus, total number of possible outcomes = 8

Getting at least 2 tails includes

{HTT, THT, TTH, TTT} outcomes.

So number of desired outcomes = 4

Therefore, probability of getting at least 2 tails =
 $4/8 = 1/2$

QUESTION

Two dice are rolled, find the probability that the sum is:

I. Equal to 1

II. Equal to 4

III. Greater than 8

EXPLANATION

$$1) P(E) = n(E) / n(S) = 0/36 = 0$$

$$2) E = \{(1,3),(2,2),(3,1)\}$$

$$\text{Hence, } P(E) = n(E) / n(S) = 3/36 \\ = 1/12$$

$$3) P(E) = n(E) / n(S) = 10/36 = \\ 5/18$$

CAT

If two dice are thrown simultaneously, the probability that the sum of the numbers on them is at least 10 is

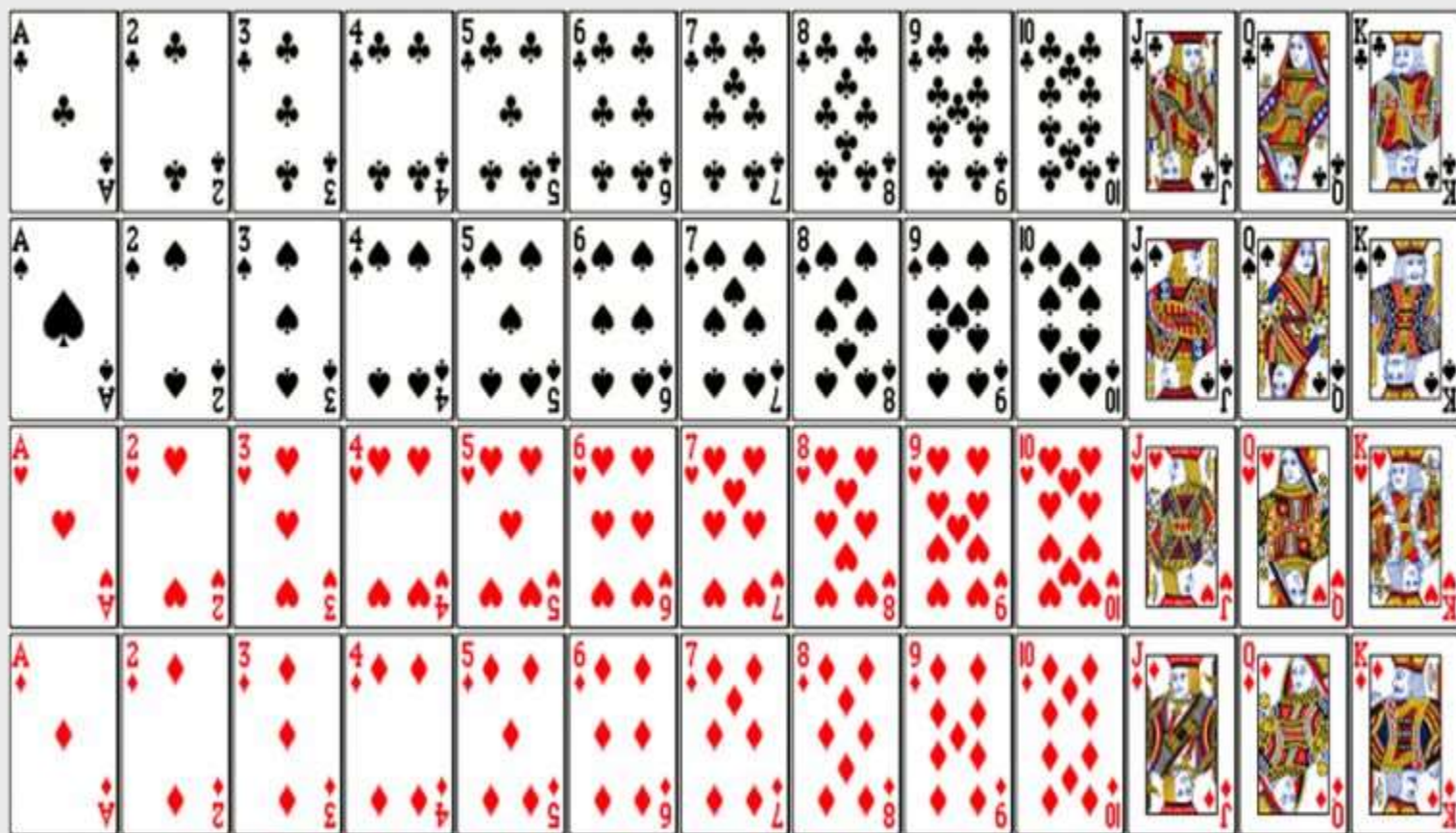
EXPLANATION

(6,4) (5,5) (4,6) (6,5) (5,6)

(6,6)

$$6/36 = 1/6$$

Standard Deck of Playing Cards



52 cards, 4 suits, 13 cards per suit
Ace-10, Jack, Queen, King (face cards)

QUESTION

Two cards are drawn at random from a pack of 52 cards. What is the chance that first is King and second is Queen if the card is not replaced.

- (a) $1/13$
- (b) $4/663$
- (c) $1/169$
- (d) $1/221$

EXPLANATION

$${}^4C_1 \times {}^4C_1 / {}^{52}C_1 \times {}^{51}C_1 \\ = 4 \times 4 / 52 \times 51 = 4/663$$

UPSC

Two cards are drawn from a well shuffled deck of 52 cards one after the other without replacement. The probability of first card being a spade and the second a black king is

(a) $1/104$

(b) $25/2652$

(c) $3/338$

(d) $26/2652$

EXPLANATION

$${}^{13}C_1 \times {}^2C_1 - {}^1C_1 / {}^{52}C_1 \times {}^{51}C_1 \\ = 25/2652$$

QUESTION

A speak truth in 60% cases and B in 75% cases. While stating a same fact in either Yes or No only. They are likely to contradict in what percentage case?

EXPLANATION

$$P(A) = 3/5 \quad P(\bar{A}) = 2/5$$

$$P(B) = 3/4 \quad P(\bar{B}) = 1/4$$

$$P(A) \times P(B) + P(\bar{A}) \times P(B)$$

$$3/5 \times 1/4 + 2/5 \times 3/4 = 9/20$$

$$(9/20) \times 100 = 45\% \text{ (Ans)}$$

GATE-2017

Q:- A couple has 2 children. The Probability that both children are boys if the older one is a boy is

- (a) $1/4$**
- (b) $1/3$**
- (c) $1/2$**
- (d) 1**

EXPLANATION

$$SS = \{B, G\}$$

$$P(B) = 1/2$$

QUESTION

A man and his wife appear in an interview for two vacancies in the same post. The probability of husband selection is $(1/7)$ and the probability of wife selection is $(1/5)$.

I- What is the assurance that only one of them is selected?

II- Both selected?

III- None?

EXPLANATION

$$P(H) = 1/7$$

$$P(W) = 1/5$$

Only one get the job :-

$$1/7 \times 4/5 + 6/7 \times 1/5 = 10/35$$

$$\text{Both :- } 1/7 \times 1/5 = 1/35$$

$$\text{None :- } 6/7 \times 4/5 = 24/35$$

TCS

**A bag contains 8 white and 6 red balls.
What is the probability of drawing
two balls of the same colour?**

(a) $28/29$

(b) $15/91$

(c) $43/91$

(d) None

EXPLANATION

8 W and 6 R

$${}^8C_2 + {}^6C_2 / {}^{14}C_2 = 43/91$$

QUESTION

A bag contains 5 red and 10 black balls. What is the probability that 2 balls drawn one black and one red is?

EXPLANATION

$${}^{10}C_1 \times {}^5C_1 / {}^{15}C_2 = \\ 10/21$$

QUESTION

The probability of India winning a test match against Pakistan is 0.5. Assuming independence of the result of various matches the chance that in a 5 match series. India second win occurs at 3rd test is

- (a) $2/3$
- (b) $1/4$
- (c) $1/8$
- (d) $1/2$

EXPLANATION

W **L** W or **L** W W

$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$

$$\frac{1}{8} + \frac{1}{8} = \frac{1}{4}$$

QUESTION

The probability that when 12 balls are put into three boxes the first will contain 3 ball is?

(a) $2^9/3^{12}$

(b) ${}^{12}C_3 \times (2/3)^{12}$

(c) $2^9 \times {}^{12}C_3 / 3^{12}$

(d) NOT

EXPLANATION

$^{12}C_3$		
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$$^{12}C_3 \times 2^9 / 3^{12}$$

CHALLENGING

Q:- A bag P contains 3 white and 4 black balls and another bag Q contains 4 white and three black balls. A ball is transferred (at random) from bag P to the bag Q and then a ball is transferred from bag Q to the bag P. A ball is then taken out from the bag P. The chance that it is a white ball is ?

EXPLANATION

P

3 White
4 Black

Q

4 White
3 Black

W W W or W B W or B W W or B B W

$$\frac{3}{7} \times \frac{5}{8} \times \frac{3}{7} + \frac{3}{7} \times \frac{3}{8} \times \frac{2}{7} + \frac{4}{7} \times \frac{4}{8} \times \frac{4}{7} + \frac{4}{7} \times \frac{4}{8} \times \frac{3}{7}$$

= 25/56

UP-PCS

A purse contains 4 copper coins and 3 silver coins another purse contains 6 copper coins and 2 silver coins. A coin is taken out from any purse, the probability that it is a copper coin is

- (a) $11/4$**
- (b) $37/56$**
- (c) $3/7$**
- (d) $3/4$**

EXPLANATION

$$= \frac{1}{2} [{}^4C_1 / {}^7C_1 + {}^6C_1 / {}^8C_1] =$$

$$37/56$$

UPSC

Q:- The probability that a students know the correct answer to a multiple choice question is $\frac{2}{3}$ if the student does not know the answer. The probability that the guessed answer being correct is $\frac{1}{4}$. Given that the student has answered correctly. The probability that the student know the correct answer is

(a) $\frac{2}{3}$

(b) $\frac{3}{4}$

(b) $\frac{5}{6}$

(d) $\frac{8}{9}$

EXPLANATION

Known 2/3	Unknown 1/3

$$\begin{aligned} & (2/3 \times 1) \\ & / (2/3 \times 1 + \\ & 1/4 \times 1/3) \\ & = 8/9 \end{aligned}$$

Binomial Distribution for Successive Events

Binomial distribution is one of the most important distributions in the probability theory. The probability distribution that summarizes the likelihood that a value will take one of two independent values under a given set of parameters or assumptions is called the binomial distribution.

Let probability of success in any trial be p and that of failure be q , then

$$p + q = 1$$

Then $(p + q)^n = {}^nC_0 P^n + C_1 P^{n-1}q + \dots + C_r p^{n-r}q^r + \dots + C_n q^n$

Then the probability of exactly k successes in n trials is given by

$$P_k = {}^nC_k q^{n-k} p^k$$

**The
number of
observati
ons n is
fixed**



**Mean is
 np &
variance
is npq**



**Binomial
Distributio
 n**



**Each trial
has the
same
probability
of success.**



**Each trial
is
mutually
exclusive.**

NTPC

If a fair coin is tossed four times. What is the probability that two heads and two tails will result

(a) $3/8$

(b) $1/2$

(c) $5/8$

(d) $3/4$

EXPLANATION

$$P(H) = \frac{1}{2}$$

$$P(T) = \frac{1}{2}$$

$$= {}^nC_r \times P(H)^r \times P(T)^{n-r} \quad (\text{Using Binomial Theorem})$$

$$n = 4, \quad r = 2$$

$${}^4C_2 \times (1/2)^2 \times (1/2)^{4-2} = 6 \times 1/16 = 3/8$$

IB 2014

80% of people those who purchase pet insurance are women. If the owners of 9 pet insurance are randomly selected, then find the probability that exactly 6 out of them are women.

EXPLANATION

$$n = 9, p = 0.8, q = 0.2, k = 6$$

$${}^nC_k q^{n-k} p^k$$

$${}^9C_6 0.2^3 0.8^6$$

$$84 \times 0.262144 \times 0.008 =$$

$$0.176$$

QUESTION

$X = [1, 100]$ What get probability to satisfied the inequality.

$$(X-40)(X-70)/(X-30) < 0$$

(a) $28/50$

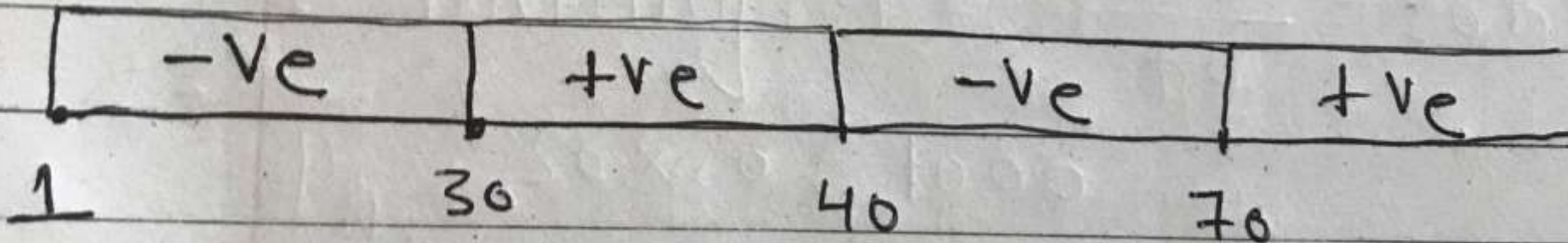
(b) $29/50$

(c) $30/50$

(d) $59/100$

EXPLANATION

$$\frac{(x-40)(x-70)}{(x-30)} < 0$$



$$\frac{29 + 29}{100} = \frac{29}{50}$$

QUESTION

$X, Y = [0, 10]$ (whole number) find probability to satisfied this equation

$$|X - Y| \leq 7$$

(a) $108/121$

(b) $109/121$

(c) $111/121$

(d) $107/121$

EXPLANATION

$$|X - Y| \leq 7$$

$$X, Y = [0, 10]$$

Unfavorable Chance:-

$$|X - Y| > 7$$

$$12/11 \times 11 = 12/121$$

Favorable Chance :-

$$1 - 12/121$$

$$= 109/121$$

X	Y	X	Y
10	0	0	10
10	1	1	10
10	2	2	10
9	0	0	9
9	1	1	9
8	0	0	8

GATE - 2013

Q:- A and B decided to meet between 6 PM and 7 PM on 14th February 2022. What is the probability that they will meet. Provided one can not wait for the other more than 20 Minutes?

EXPLANATION

Conditional Probability

A **conditional probability** is the probability of an event occurring, given that another event has already occurred. The conditional probability of event B occurring, given that event A has occurred, is denoted by $P(B/A)$ and is read as “probability of B, given A.”

Conditional Probability (cont.)

- To find the probability of the event **B** *given* the event **A**, we restrict our attention to the outcomes in **A**. We then find in what fraction of *those* outcomes **B** also occurred.

$$P(\mathbf{B}|\mathbf{A}) = \frac{P(\mathbf{A} \text{ and } \mathbf{B})}{P(\mathbf{A})}$$

- Note: $P(\mathbf{A})$ cannot equal 0, since we know that **A** has occurred.

QUESTION

**A dice is through at a random
what is the probability of get
prime number on the dice.
Provided dice as all ready
shown an even number?**

EXPLANATION

$$SS = \{1,2,3,4,5,6\}$$

$$EVEN = \{2,4,6\}$$

$$PRIME = \{2,3,5\}$$

$$SS = \{2,4,6\}$$

$$P(\text{Prime})/P(\text{Even}) = 1/3$$

BANK

An Aptitude teacher gave his class two tests. 25% of the class passed both tests and 42% of the class passed the first test. What percent of those who passed the first test also passed the second test?

EXPLANATION

$$P(\text{Second} | \text{First}) = \frac{P(\text{first/second})}{P(\text{First})} = 0.25 / 0.42 = 0.6 = 60\%$$

QUESTION

The two-way table below displays demographic data for the passengers aboard the Titanic.

	Man	Woman	Child	Total
First Class	175	144	6	325
Second Class	168	93	24	285
Third Class	462	165	79	706
Total	805	402	109	1316

Given that a passenger selected at random was a child, find the probability that the passenger traveled in the third class.

EXPLANATION

$P(\text{third class} \mid \text{child})$

$$= 79 / 109$$

GATE - 2013

An automobile plant contracted to buy shock absorbers from two suppliers X and Y. X supplies 60% and Y supplies 40% of the shock absorbers. All shock absorbers are subjected to a quality test. The ones that pass the quality test are considered reliable. Of X's shock absorbers, 96% are reliable. Of Y's shock absorbers, 72% are reliable.

The probability that a randomly chosen shock absorber, which is found to be reliable, is made by Y is

- (a) 0.288 (b) 0.334 (c) 0.667 (d) 0.720**

EXPLANATION

$$TC = 100$$

$$X = 60$$

$$Y = 40$$

$$\begin{aligned} X_R &= 0.96 \times 60 \\ &= 57.6 \end{aligned}$$

$$\begin{aligned} Y_R &= 0.72 \times 40 \\ &= 28.8 \end{aligned}$$

$$T_R = X_R + Y_R = 86.4$$

$$P = Y_R / T_R = 0.334$$

GATE-2013

Q:- Out of all 2-digit integers between 1 and 100, a 2-digit number has to be selected at random. What is the probability that the selected number is not divisible by 7?

(a) $13/90$

(b) $12/90$

(c) $78/90$

(d) $77/90$

EXPLANATION

2 Digit SS = {10.....99} = 90

**$100/7 = 14 - 1$ (Because 7 not consider)
= 13**

$P = 77/90$

GATE-2014

In any given year, the probability of an earthquake greater than magnitude 6 occurring in the Garhwali Himalayas is 0.04. The average time between successive occurrences of such earthquakes isYears.

EXPLANATION

The average time between successive occurrences of such earthquakes

$$= 1/0.04 = 25 \text{ Year}$$

GATE-2014

Q:- You are given three coins: one has heads on both faces, the second has tails on both faces, and the third has a head on one face and a tail on the other. You choose a coin at random and toss it, and it comes up heads. The probability that the other face is tails is

- (a) $1/4$**
- (b) $1/3$**
- (c) $1/2$**
- (d) $2/3$**

EXPLANATION

(H,H) (T,T) (H,T)

$$P = 1/3$$

GATE-2014

Q:- A batch of one hundred bulbs is inspected by testing four randomly chosen bulbs. The batch is rejected if even one of the bulbs is defective. A batch typically has five defective bulbs. The probability that the current batch is accepted is

EXPLANATION

Probability of bulb being defective = $5/100 = 0.05$

Probability of bulb being non defective = $1 - 0.05 = 0.95$

Probability that the batch is accepted =

None of the four bulb being defective = $(0.95)^4 = 0.8145$

GATE-2015

Q:- Ram and Ramesh appeared in an interview for two vacancies in the same department. The probability of ram's selection is $\frac{1}{6}$ and that of Ramesh is $\frac{1}{8}$ what is the probability that only one of them will be selected?

(a) $\frac{47}{48}$

(b) $\frac{1}{4}$

(c) $\frac{13}{48}$

(d) $\frac{35}{48}$

EXPLANATION

$$P(\text{Ram}) = 1/6 \quad P(\text{Ramesh}) = 1/8$$

Probability that only one of them will be selected =

$$1/6 \times 7/8 + 1/8 \times 5/6 = 12/48 = 1/4$$

GATE-2013

**Q:- What is the probability of getting
53 Sundays in a leap year**

(a) $1/7$

(b) $2/7$

(c) $3/7$

(d) $1/2$

EXPLANATION

Leap Year = 366 Days

$366 = 52 \times 7 + 2$ (odd days)

(**Sun**, Mon) (Mon, Tue) (Tue, Wed)
(Wed, Th) (Th, Fri) (Fri, Sat) (Sat, **Sun**)

$P = 2/7$

QUESTION

Four different pens (A, B, C, D) are to be distributed at random in four pen stands marked as 1, 2, 3, 4. what is the probability that none of the pen occupies the place corresponding to its number

- (a) $17/24$
- (b) $3/8$
- (c) $1/2$
- (d) $5/8$

EXPLANATION

$$P = 9/4! = 3/8$$

<i>A</i>	<i>B</i>	<i>C</i>	<i>D</i>	
2	1	4	3	
2	4	1	3	
2	3	4	1	
3	1	4	2	9
3	4	2	1	
3	4	1	2	
4	1	2	3	
4	3	2	1	
4	3	1	2	



THANK YOU!