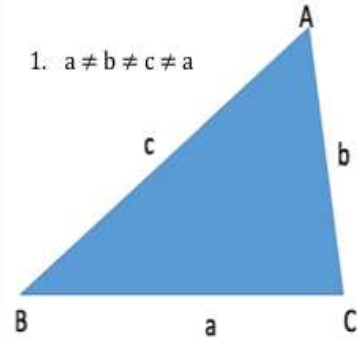
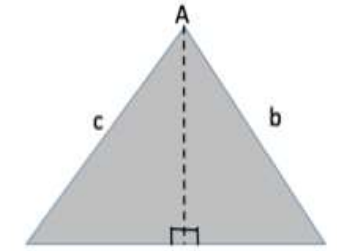
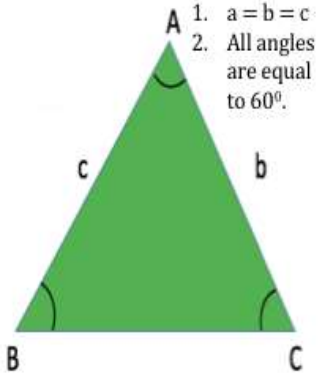
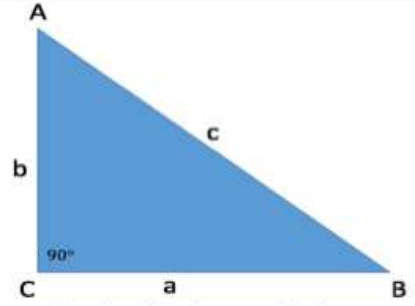
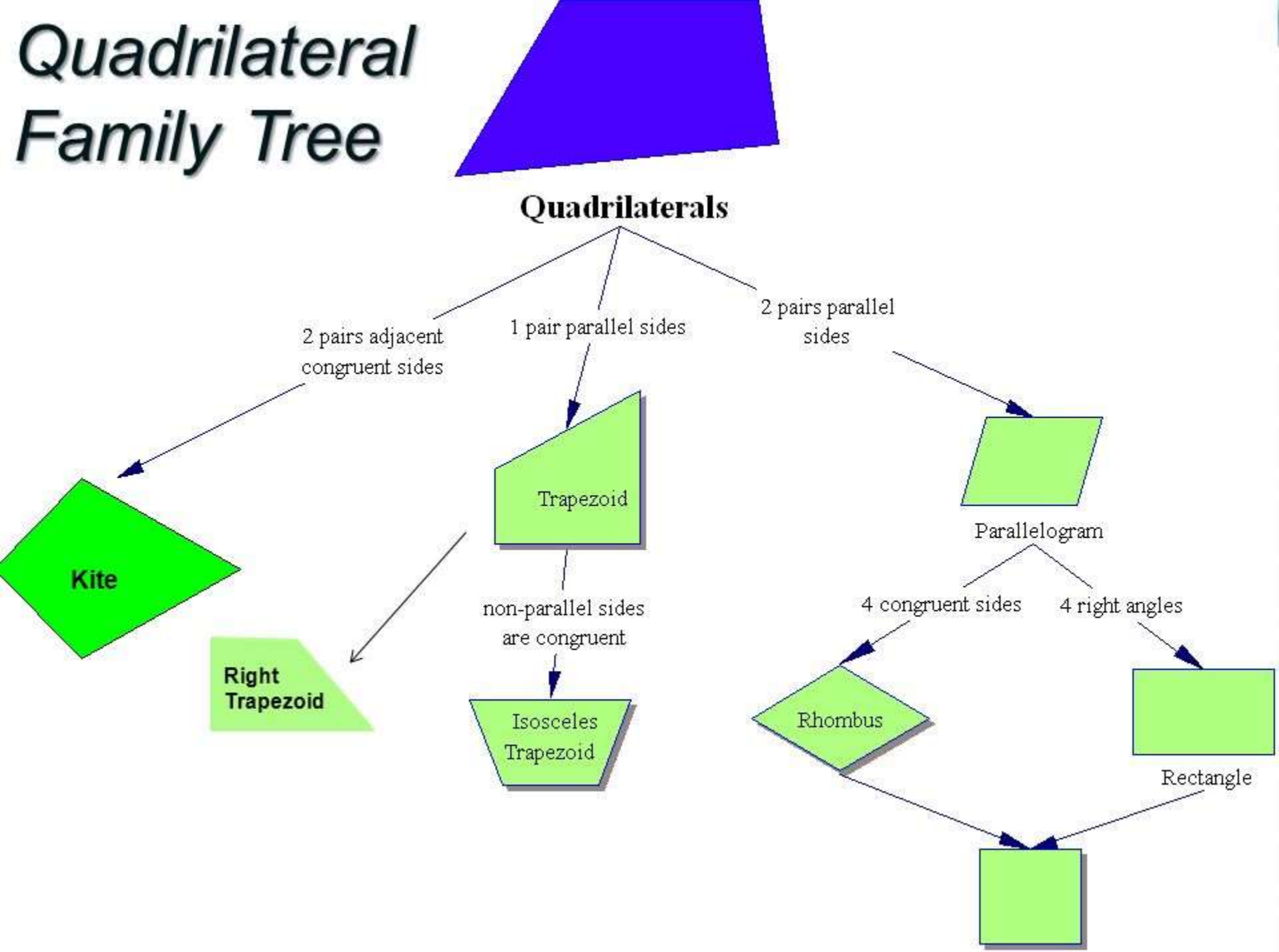

MENSURATION
By – Rahul Agrahari

Properties	Type of Triangles			
	Scalene	Isosceles	Equilateral	Right-Angled
Side Property	• None of the sides are equal. • $a \neq b \neq c \neq$,a where a, b and c are three sides of the triangle.	• Two side are equal. • $a \neq b = c$, where a, b and c are the three sides of the triangle.	• All the sides are equal. • $a = b = c$, where a, b and c are the three sides of the triangle.	• Two sides forming the right angle may or may not be equal. • If they are equal, the triangle is known as isosceles right –angled triangle.
Angle Property	• All the angles are distinct.	• Angles opposite to equal sides are equal.	• All angles are equal to 60^0.	• One of the angles is 90^0.
Area Formula	• Area = ½ x base x height • The above formula is applicable to triangles.		• Area = $\frac{\sqrt{3}}{4}$ x a^2, where 'a' is the length of a side of the triangle.	• Area = (½) x base x perpendicular
Special Property		• The perpendicular drawn to the unequal side, divides it into two equal parts.	• The perpendicular from any vertex, divides the opposite side into 2 equal halves.	• According to Pythagoras Theorem, $c^2 = a^2 + b^2$, where c is the largest side/hypotenuse
Diagrammatic Representation		 1. $a \neq b = c$ 2. The perpendicular from any vertex, divides the opposite side into 2 equal halves.		 1. $c^2 = a^2 + b^2$, where c is the largest side/hypotenuse 2. Angle ACB = 90^o

Quadrilateral Family Tree



Type	Example	Properties
Parallelogram A parallelogram is a quadrilateral with two pairs of parallel sides.		*opposite sides are parallel *opposite sides are congruent
Rectangle A rectangle is a parallelogram with four right angles.		*a rectangle has all the properties of a parallelogram *all angles are right angles *adjacent sides form right angles
Rhombus A rhombus is a parallelogram with four congruent sides		*a rhombus has all the properties of a parallelogram *all sides are congruent

The Properties of A Square

1. All sides of a square are the same length.

   $AB = CD = AD = BC$

2. Square has two pairs of parallel sides and have the same length.

   $AB \parallel CD$ and $AB = CD$

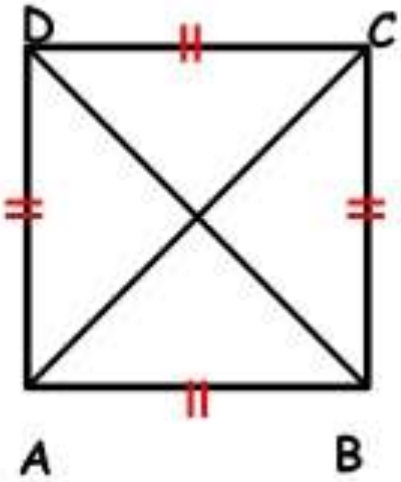
   $AD \parallel BC$ and $AD = BC$

3. All four angles of a square are right angles (90°).

   $\angle A = \angle B = \angle C = \angle D = 90°$.

4. Two diagonals of a square are the same length.

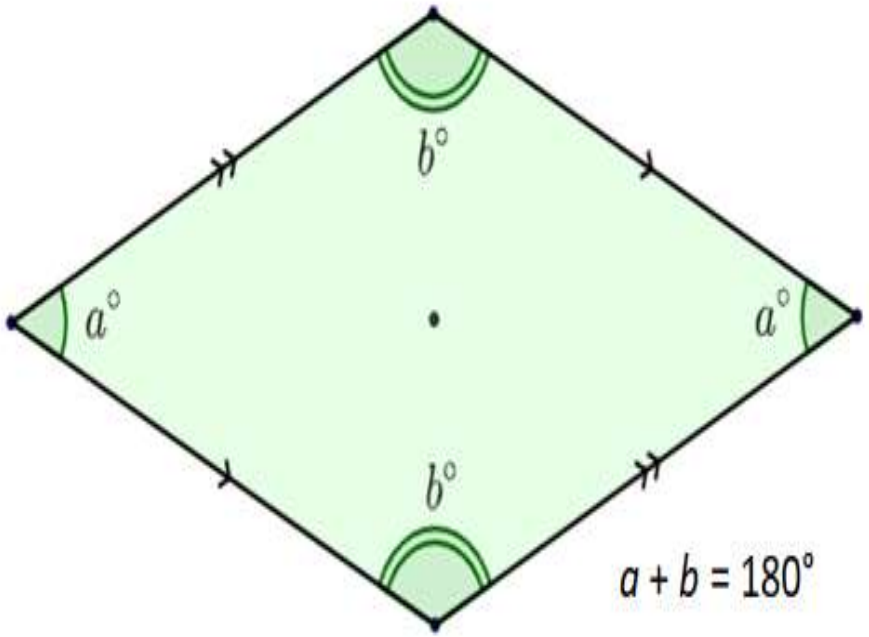
   $AC = BD$



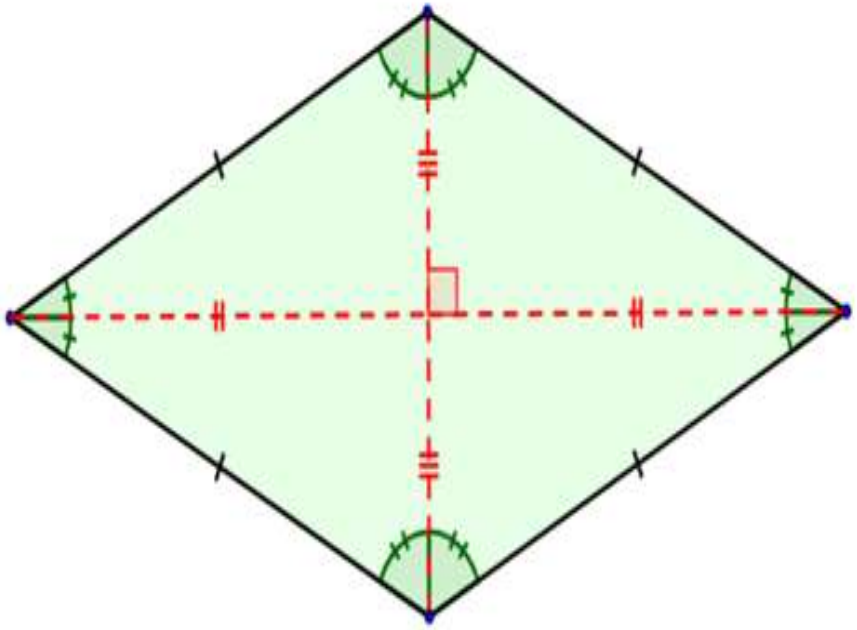
Based on the properties of the square on above, then:

A square is a rectangle that all four sides the same length.

Rhombus



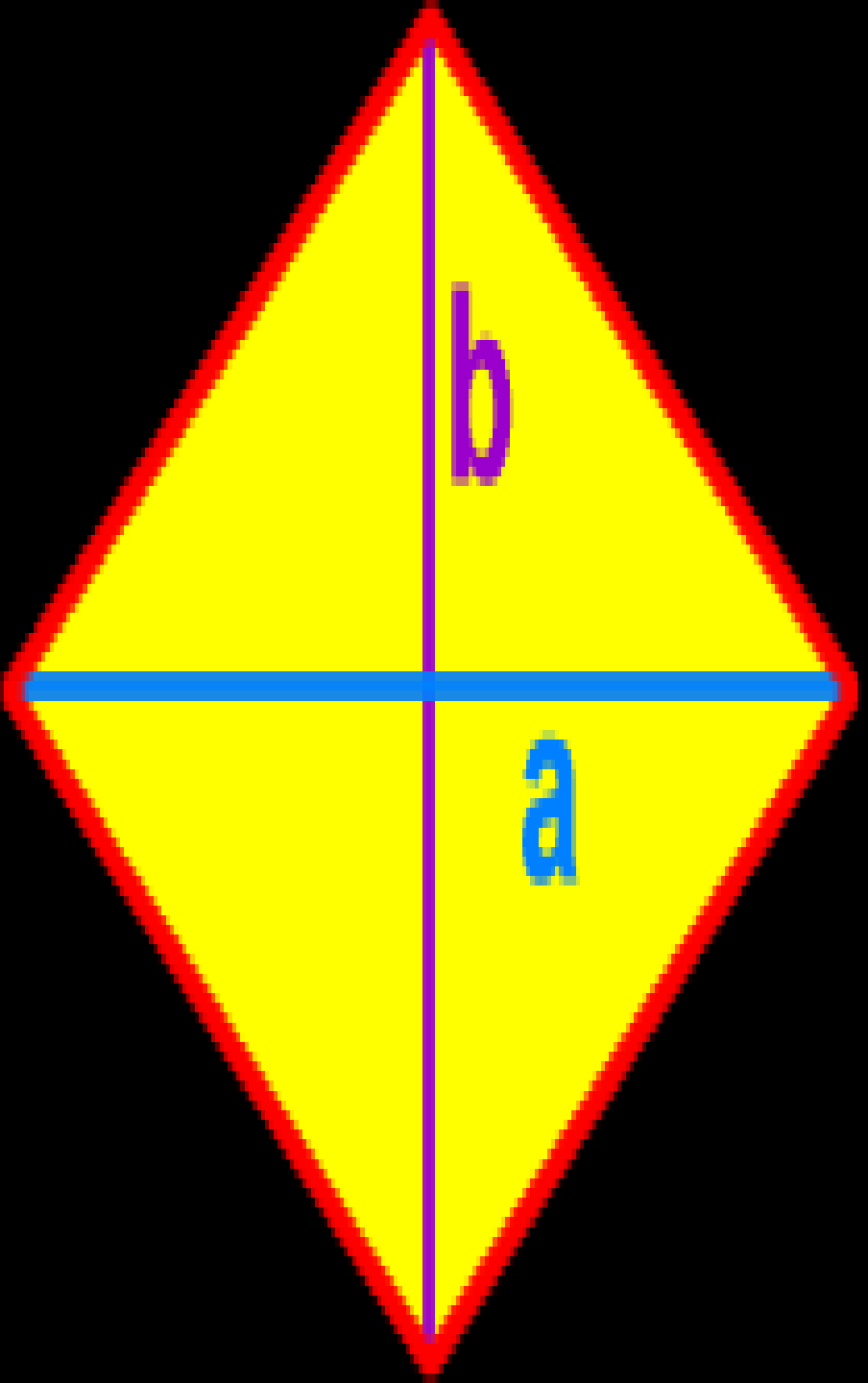
$a + b = 180^\circ$



A rhombus has all the properties of a quadrilateral
* 4 sides
* 4 vertices
* 4 angles that sum to 360°

A rhombus has all the properties of a parallelogram
* 2 sets of opposite parallel sides
* opposite angles are congruent
* consecutive angles sum to 180°

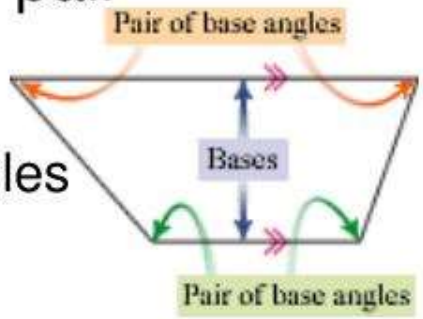
In addition, a rhombus has these special properties
* 4 congruent sides
* diagonals are perpendicular bisectors
* diagonals are angle bisectors



$$\text{Area} = \frac{1}{2}ab$$

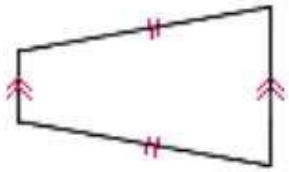
Trapezoid Properties

- A **trapezoid** is a quadrilateral with exactly one pair of parallel sides
  - The **bases** of a trapezoid are the parallel sides
  - The **base angles** of a trapezoid are the pairs of angles that share a common base



- A **trapezium** is a quadrilateral with no parallel sides
  - There are two bones in your wrist called the trapezoid and trapezium because of their shapes

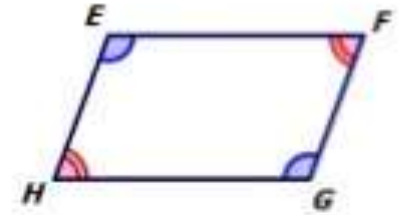
- An **isosceles trapezoid** is a trapezoid in which the two non-parallel sides are congruent



PARALLELOGRAM

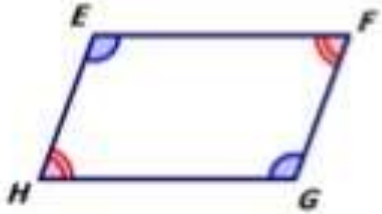
Properties of parallelograms:

- Opposite sides are parallel
- Opposite sides are congruent
- Opposite angles are congruent
- Consecutive angles are supplementary
- Diagonals are bisect each other



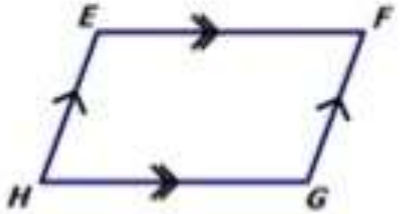
$\angle E \cong \angle G$ and $\angle H \cong \angle F$

Opposite angles are congruent



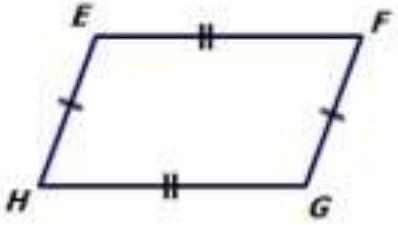
$\angle E + \angle F = 180°$
$\angle F + \angle G = 180°$
$\angle G + \angle H = 180°$
$\angle H + \angle E = 180°$

Consecutive angles are supplementary



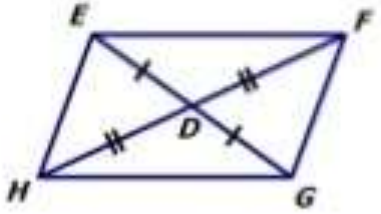
$EH \parallel FG$ and $EF \parallel HG$

Opposite sides are parallel



$EH \cong FG$ and $EF \cong HG$

Opposite sides are congruent



$HD \cong DF$ and $ED \cong DG$

Diagonals bisect each other

Mensuration Formulas

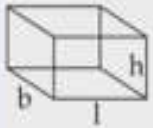
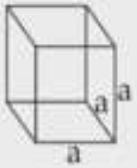
Perimeter	
Square	$P = 4s$
Rectangle	$P = 2(l + w)$

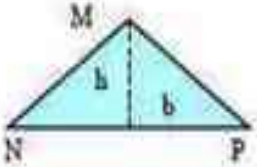
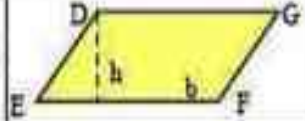
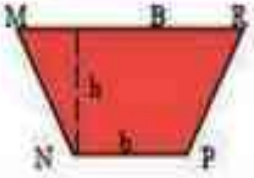
Circumference	
Circle	$C = 2\pi r$

Area	
Square	$A = s^2$
Rectangle	$A = lw$
Triangle	$A = \frac{1}{2}bh$
Trapezoid	$A = \frac{1}{2}(b_1 + b_2)h$
Circle	$A = \pi r^2$

Surface Area	
Cube	$SA = 6s^2$
Cylinder	$SA = 2\pi rh + 2\pi r^2$
Cone	$SA = \pi rl$
Sphere	$SA = 4\pi r^2$

Volume	
Cube	$V = s^3$
Cylinder	$V = \pi r^2 h$
Cone	$V = \frac{1}{3}\pi r^2 h$
Sphere	$V = \frac{4}{3}\pi r^3$

Name of the solid	Figure	Volume	Laterial/Curved Surface Area	Total Surface Area
Cuboid		lbh	2lh + 2bh or 2h(l+b)	2lh+2bh+2lb or 2(lh+bh+lb)
Cube		a^3	$4a^2$	$4a^2+2a^2$ or $6a^2$
Right circular cylinder		$\pi r^2 h$	$2\pi rh$	$2\pi rh + 2\pi r^2$ or $2\pi r(h+r)$
Right circular cone		$\frac{1}{3}\pi r^2 h$	πrl	$\pi rl + \pi r^2$ or $\pi r(l+r)$
Sphere		$\frac{4}{3}\pi r^3$	$4\pi r^2$	$4\pi r^2$
Hemisphere		$\frac{2}{3}\pi r^3$	$2\pi r^2$	$2\pi r^2+\pi r^2$ or $3\pi r^2$

NAME	FIGURE	AREA	PERIMETER CIRCUMFERENCE
TRIANGLE		$A=\frac{b\times h}{2}$	$P=MN+NP+PM$
PARALLELOGRAM		$A=b\times h$	$P=DE+EF+FG+GD$
RHOMBUS		$A=b\times h$	$P=b+b+b+b$ $P=4b$
RECTANGLE		$A=L\times w$	$P=L+w+L+w$ $P=2L+2w$
SQUARE		$A=l^2$	$P=l+l+l+l$ $P=4l$
TRAPEZOID		$A=\frac{(B+b)\times h}{2}$	$P=MN+NP+PR+RM$
CIRCLE		$A=\pi r^2$	$C=2\pi r=\pi d$

QUESTION

If the area of a rhombus is 15 cm^2 and the length of one diagonals is 5 cm, then find the length of the other diagonal?

EXPLANATION

Area of rhombus = ½ d_1 X d_2

15 = ½ X 5 X b

b = 6 cm

QUESTION

If the area of a trapezium, whose parallel sides are 6 cm and 10 cm is 32 cm^2 then find the distance between the parallel sides.

EXPLANATION

Area of trapezium = ½ (a + b) h

32 = ½ X (6 + 10) h

h = 4 cm

TCS NINJA-2018

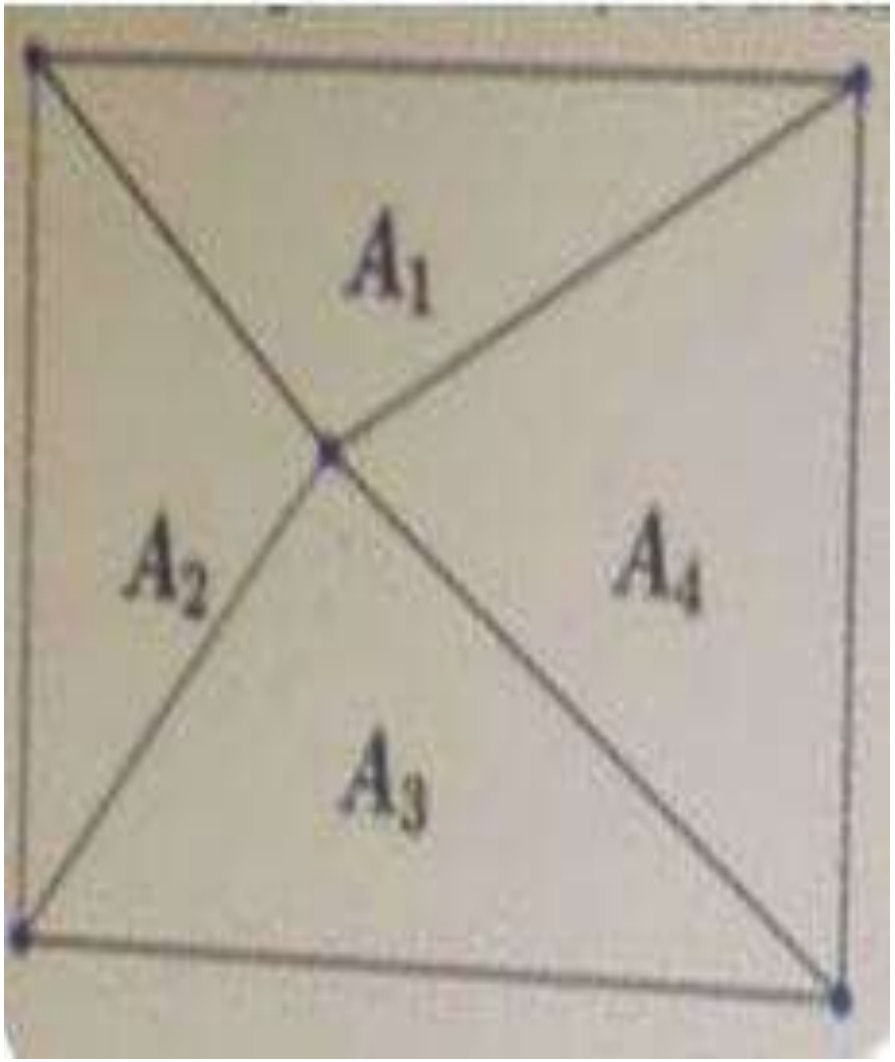
In the diagram below, the areas of the triangles are as follows: A_1=1024, A_2=1016, A_3=1057. What is the area of A_4?

a. 1032

b. 1036

c. 1020

d. 1065



EXPLANATION

$\frac{1}{2} ax = A_1 = 1024$ -----[i]

$\frac{1}{2} a(b-x) = A_3 = 1057$ ----[ii]

$A_1 + A_3 = 1024 + 1057$

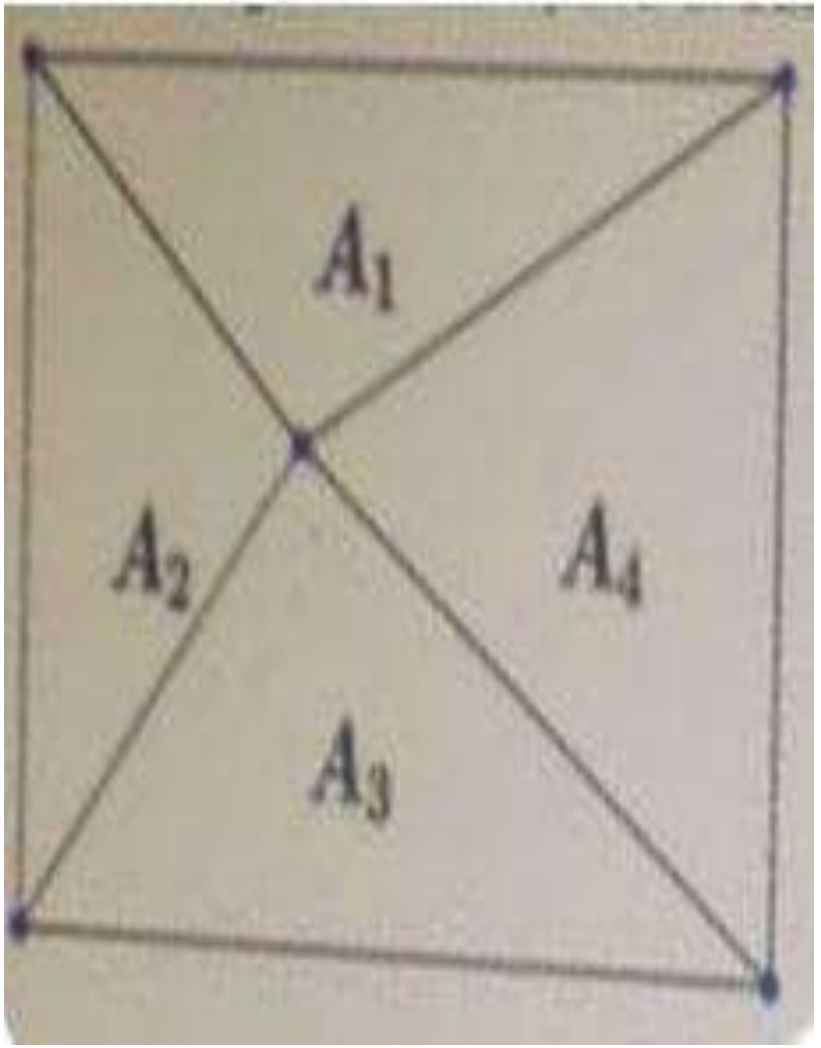
$\frac{1}{2} ax + \frac{1}{2} a(b-x) = 2081$

$\frac{1}{2} ab = 2081$

$ab = 4162$

$A_1 + A_2 + A_3 + A_4 = 4162$

$A_4 = 1065$



INFYTQ - 2020

The length width and height of a room are in 3:2:1.If the width and height are halved while length is double. Then the total surface are of the 4 walls in the room will be?

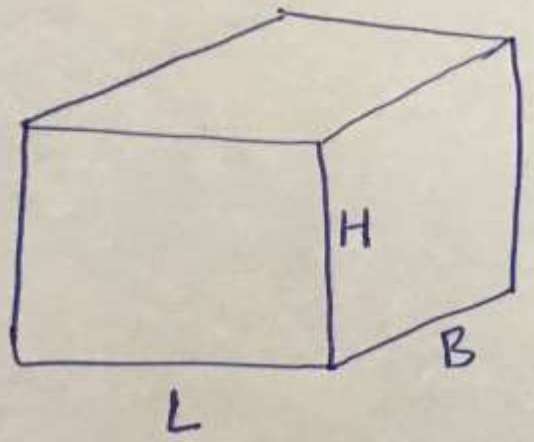
A. Remains the same

B. Decreased by 13%

C. Decreased by 30%

D. Decreased by 15

EXPLANATION



$L : B : H = 3 : 2 : 1$

	Initial	final
L	$3x$	$6x$
B	$2x$	x
H	x	$x/2$

4 Wall surface Area $= 2[LH + BH]$

$= 2[L+B]H$

$(S.A)_I = 2[3x+2x]x = 10x^2$

$(S.A)_F = 2[6x + x]\frac{x}{2} = 7x^2$

$\%\ \text{change} = \frac{10x^2 - 7x^2}{10x^2} \times 100 = 30\%.$

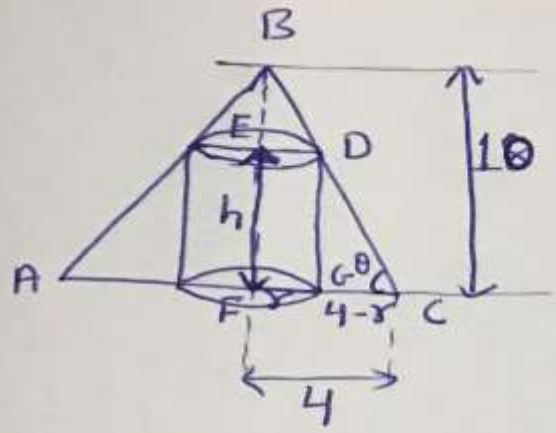
INFYTQ - 2020

Consider a right circular cone of radius 4 cm and height 10 cm. A cylinder is to be placed inside the cone such that one of the flat surface rests on the base of the cone. Find the largest possible total surface are of the cylinder?

(a)100π/3 **(b) 80π/3**

(c) 120π/7 **(d) 110π/7**

EXPLANATION



Total surface Area of cylinder

$A = 2\pi rh + 2\pi r^2$

$A = 2\pi r[r + h]$

ΔFBC and ΔDGC

$\tan\theta = \frac{FB}{FC} = \frac{DG}{GC}$

$= \frac{10}{4} = \frac{h}{4-r}$

$A = 2\pi r\left[r + \frac{20-5r}{2}\right]$

$h = \frac{20-5r}{2}$

$A = 20\pi r - 3\pi r^2$

$\frac{dA}{dr} = 20\pi - 6\pi r = 0$

$r = 10/3$

$A = 2\pi r\left[r + \frac{20-5r}{2}\right]$

$A = \frac{100\pi}{3}$

COGNIZANT

A hall is 15 m long and 12 m broad. If the sum of the areas of the floor and the ceiling is equal to the sum of the areas of four walls, the volume of the hall is:

a) 720

b) 1200

c) 900

d) 1800

EXPLANATION

According to the question,

2(15 + 12)*h = 2(15 * 12)

=> h = (180/27) = 20/3 meter

Therefore, the volume = l * b * h = (15 * 12 * 20/3) cubic-meter = 1200 (answer)

CAT

Q.3:- The length, breadth and height of a cube increases by 10%, 10% and 20% respectively. Find the percentage increase in the total surface area of cube.

(a) 10%

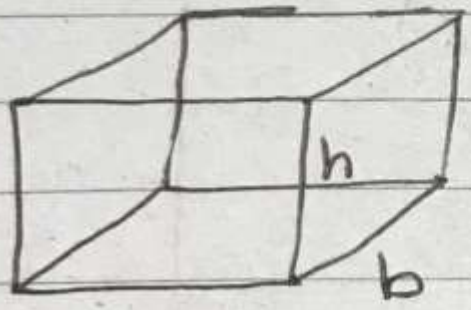
(b) 20%

(c) 22.22%

(d) 28.33%

EXPLANATION

3Ans:- Surface Area of cube $= 2(lb + bh + hl)$



Let initial

$l = 10,\ b = 10,\ h = 10$

then final

$1.1l = 11,\ 1.1b = 11,\ 1.2h = 12$

$(SA)_i = 2[10^2 + 10^2 + 10^2] = 2 \times 3 \times 10^2 = 600$

$(SA)_f = 2[11 \times 11 + 11 \times 12 + 12 \times 11] = 770$

$\% = \dfrac{770 - 600}{600} \times 100 = 28.33\%$

GATE-2019

Q.12:-The radius as well as the height of a circular cone increases by 10%. The percentage increase in its volume is_____.

(a) 17.1

(b) 21.0

(c) 33.1

(d) 72.8

EXPLANATION

12 Ans:- Volume of a circular cone

$$= \frac{1}{3}\pi r^2 h$$

$$V_i = \frac{1}{3}\pi r_1^2 h_1$$

$$V_f = \frac{1}{3}\pi (1.1 r_1)^2 \times 1.1 h_1$$

$$= 1.331 \left(\frac{1}{3}\pi r_1^2 h_1\right)$$

$$\% \text{ Change in Volume} = \frac{V_f - V_i}{V_i} \times 100$$

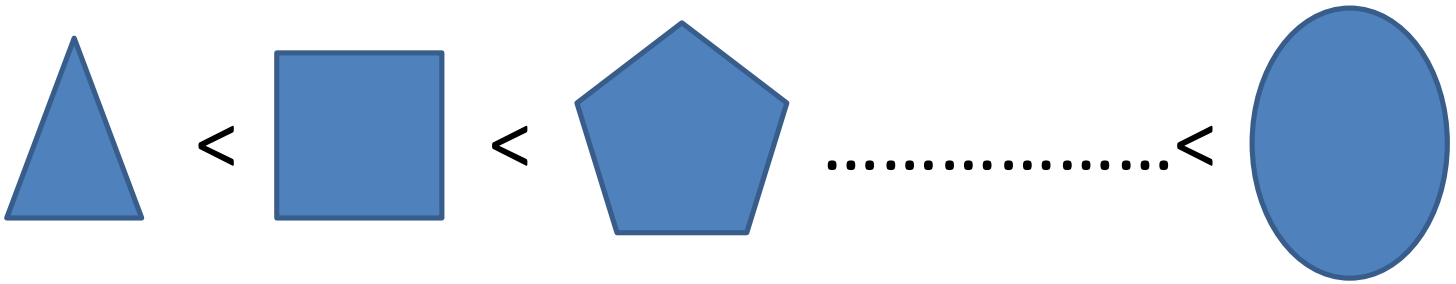
$$= 33.1\%$$

NTPC-2010

Q.:-From a rope of length 27 m as perimeter an equilateral triangle (A), a square (B), a circle (C) is being drawn ascending order of there area……

Explanation

Ascending order of there area……

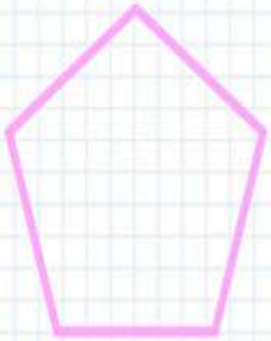


CONCEPT

NOTE:-If perimeter is fixed for all regular and convex polygon more the number of its sides higher will be the value of its area.

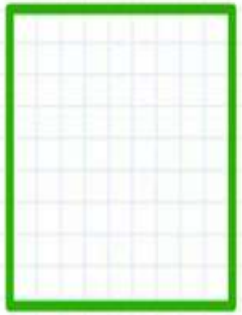
***Regular Polygon*:-Its side equal and angle equal.**

***Convex Polygon*:-There angle is not more than 180°.**



Convex

Has no interior angle greater than 180°



Simple

Does not have self-intersecting sides.

Irregular

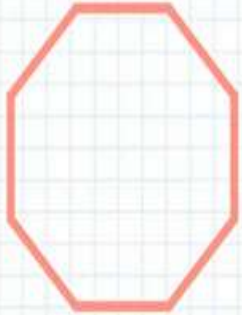
Does not have congruent sides and interior angles

Concave

Has 1 interior angle greater than 180°

Complex

Have self-intersecting sides.



Regular

Has congruent sides and interior angles

PERCENTAGE

Q.4:- During a heating process, the surface area of a sphere increases by 44%. Find the percentage change in the sphere's radius and volume

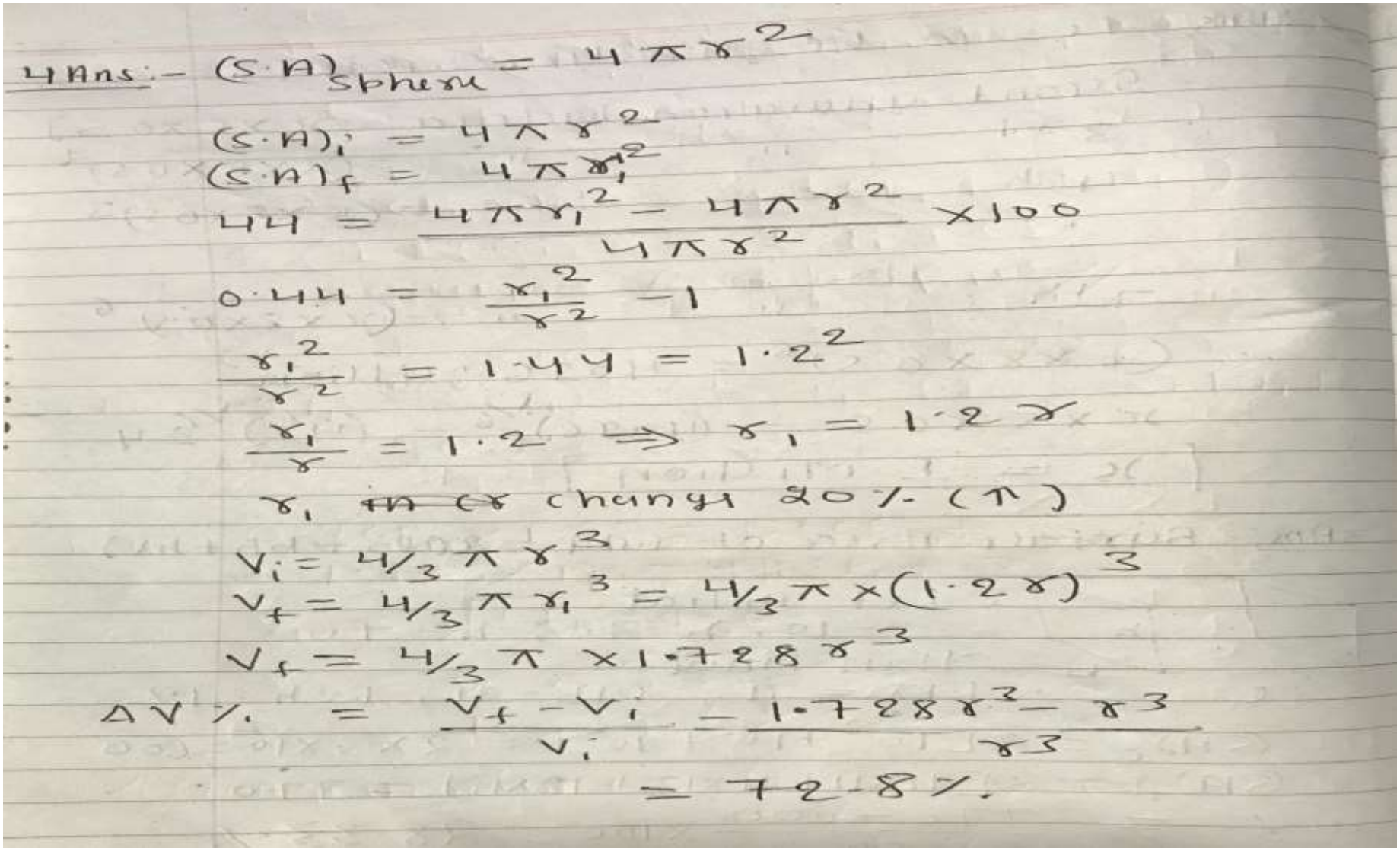
(a)11%, 48%

(b) 20%, 72.8%

(c) 22%, 66%

(d) 25%, 62.8%

EXPLANATION



THANK YOU!