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Steady Infiltration in Heterogeneous Soil

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Abstract. In this study, problems involving steady infiltration from periodic channels in heterogeneous soils are considered. The problems are governed by a Richards's equation. To study the problems, the equation is transformed into a modified Helmholtz equation. The modified Helmholtz equation is then solved numerically by employing a Dual Reciprocity Method (DRM). The method is applied to solve infiltration in different types of heterogeneous soils. Numerical solutions in terms of suction potential are compared. The solutions indicate that soil layers produce jumps in values of the suction potential. The results also indicate that the soil type influence to the water content in the soil.

1. Introduction

Problems involving infiltration in homogeneous soils has been considered by numerous researchers, such as Waechter and Mandal [8], Pullan and Collins [4], Clements and Lobo [3], and Solekhuudin [6, 7]. Waechter and Mandal [8] have studied infiltration from a semicircular cylindrical trench and hemispherical pond. Pullan and Collins considered steady infiltration from buried and surface cavities [4]. In other hand, Solekhuudin [6, 7] has considered the water flow from periodic channels in homogeneous soils. Clements and Lobo examined water ow from single channel in a homogeneous soil [3]. The works considered by such researchers were ow in homogeneous soil. Hence, a possible direction to continue their study is to study the infiltration in heterogeneous soils.

In this paper, a steady infiltration problem from periodic trapezoidal channels in three-layered soils is considered. The problem is governed by a Richards's equation. Since the governing equation may not be solved analytically, a numerical method called Dual Reciprocity Method (DRM) is employed. In order to employ the method, the governing equation is transformed into a modified Helmholtz equation using a set of transformation. The modified Helmholtz equation is then solved using the method. The results obtained are then used to compute values of suction potential.

2. Formulation of the problem

In this study, the researchers consider three-layered soils. The three-layered soils consist of Silt Loam (SL), Pima Clay Loam (PCL) and Touchet Silt Loam (TSL). The order of the layers, from the surface of soil, is SL-PCL-TSL. The depth of the first layer is 100 cm. The second layer has the same depth as the first layer. On the surface of the first layer, periodic trapezoidal channels are created. For every centimeter length, the channel has a sunken surface area of 100 cm². The distance between the centers of two adjacent channels is 200 cm. It is assumed that the channels are identical and sufficiently long. The channels are filled with water. It is also assumed that water fluxes are only from the surface of the channels, and such fluxes are constant. Given this situation, it is expected to determine the values of suction potential in the soils. To solve the problem, it was used a Cartesian coordinate system *OXYZ* with *OZ* vertically positive downwards. The problem described is illustrated in Figure 1.



Since the length of the channels are sufficiently long compared to the width of the channels, water flow may be assumed two-dimensional flow. From the assumptions, the problem is symmetrical about the plane $X = \pm 100k$, $k = 0, 1, 2, \dots$. Hence, it is sufficient to consider a semi-infinite region bounded by $0 \leq X \leq 100$ cm and $Z \geq 0$. R denotes such region where it is bounded by a curve C , as shown in Figure 2.

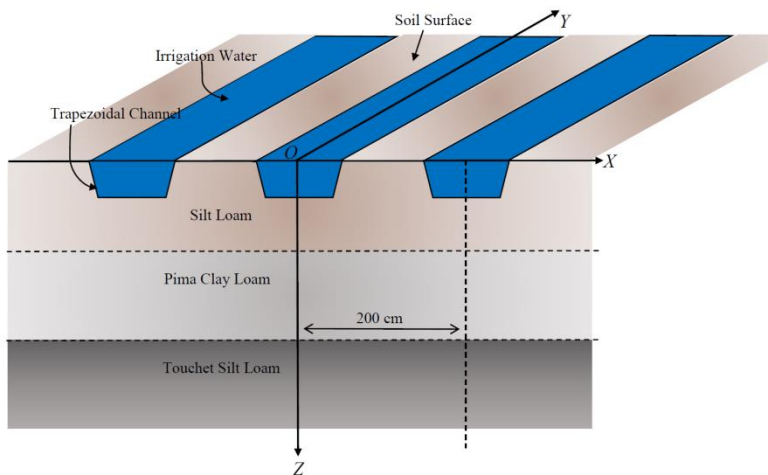


Figure 1. Geometry of periodic trapezoidal channels.

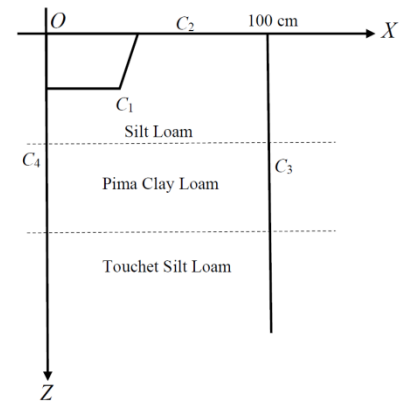


Figure 2. Region of the problem

From the assumption above, flux across the surface of channel is constant, v_0 , and flux across the surface of soil outside the channel is 0. Fluxes across $X = 0$ and $X = 100$ cm are 0, as the problem is symmetrical about these two lines. The derivatives $\frac{\partial \theta}{\partial X} \rightarrow 0$ and $\frac{\partial \theta}{\partial Z} \rightarrow 0$ as $X^2 + Z^2 \rightarrow \infty$ [2].

3. Basic equations

The governing equation of steady infiltration problems from periodic channels in homogeneous soil has been presented in [5]. Set of transformations needed to transform the governing equation into a modified Helmholtz equation are also presented and discussed in [5]. In this study, the governing equation and the method used are similar to those in [5]. However, for the completeness and the convenience of readers, some details of the equation and the method are presented. The governing equation of steady infiltration in three-layered soils is in the form of following Richard's equation.

$$\frac{\partial}{\partial X} \left(K \frac{\partial \psi}{\partial X} \right) + \frac{\partial}{\partial Z} \left(K \frac{\partial \psi}{\partial Z} \right) - \frac{\partial K}{\partial Z} = 0, \quad (1)$$

Where K is the hydraulic conductivity and ψ is the suction potential.

The Kirchhoff transformation

$$\Theta = \int_{-\infty}^{\psi} K(s) ds, \quad (2)$$

Where Θ is the Matric Flux Potential (MFP), and an exponential relationship between K and ψ

$$K(\psi) = K_0 e^{\alpha \psi}, \quad (3)$$

Where K_s is the saturated hydraulic conductivity and α is a constant soil parameter corresponding to the roughness of the soil, transform Equation (1) to

$$\frac{\partial^2 \Theta}{\partial X^2} + \frac{\partial^2 \Theta}{\partial Z^2} = \alpha \frac{\partial \Theta}{\partial Z}, \quad (4)$$

In the present study, there are distinct values of α in three different layers. Assume α_1, α_2 and α_3 are the values of α in the first, second, and third layer, respectively. Consequently, Equation (4) cannot be used to solve the problem described. Hence, it needs to overcome this issue. A method that may be employed to solve the problem is as that has been used by [9], which is presented as follows:

$$\alpha^* = \frac{\alpha_1 + \alpha_2 + \alpha_3}{3}.$$

Using α^* , Equation (4) now is written as

$$\frac{\partial^2 \Theta}{\partial X^2} + \frac{\partial^2 \Theta}{\partial Z^2} = \alpha^* \frac{\partial \Theta}{\partial Z}, \quad (5)$$

The flux normal to the surface with outward pointing normal $\mathbf{n} = (n_1, n_2)$ is

$$F = U n_1 + V n_2, \quad (6)$$

Where

$$U = -\frac{\partial \Theta}{\partial X} \text{ and } V = \alpha \Theta - \frac{\partial \Theta}{\partial Z} \quad (7)$$

Now, substituting the dimensionless variables

$$\begin{aligned} \Phi &= \frac{\pi}{50\alpha^*} \Theta, & x &= \frac{\alpha^*}{2} X, & z &= \frac{\alpha^*}{2\pi} Z, \\ u &= \frac{\pi}{25v_0\alpha^*} U, & v &= \frac{\pi}{25v_0\alpha^*} V, & f &= \frac{\pi}{25v_0\alpha^*} F, \end{aligned} \quad (8)$$

Into Equation (5), it is obtained

$$\frac{\partial^2 \Phi}{\partial x^2} + \frac{\partial^2 \Phi}{\partial z^2} - 2 \frac{\partial \Phi}{\partial z} = 0, \quad (9)$$

The dimensionless flux is

$$f = -\frac{\partial \Phi}{\partial x} n_1 + \left(2\Phi - \frac{\partial \Phi}{\partial z} \right) n_2. \quad (10)$$

Transformation

$$\Phi = \phi e^z, \quad (11)$$

Transforms Equation (9) into

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial z^2} - \phi = 0, \quad (12)$$

and

$$f = -e^z \left(\frac{\partial \phi}{\partial n} - \phi n_2 \right), \quad (13)$$

where $\frac{\partial \phi}{\partial n} = \frac{\partial \phi}{\partial x} n_1 + \frac{\partial \phi}{\partial y} n_2$.

Boundary conditions, which are described in the preceding section, are written in terms of $\partial \phi / \partial n$ is presented as follows.

$$\frac{\partial \phi}{\partial n} = \phi n_2 + \frac{\pi}{25\alpha} e^{-z}, \text{ on the surface of the channels,} \quad (14)$$

$$\frac{\partial \phi}{\partial n} = -\phi, \text{ on } z = 0 \text{ outside the channels,} \quad (15)$$

$$\frac{\partial \phi}{\partial n} = 0, \text{ on } x = 0 \text{ and } z \geq 0, \quad (16)$$

$$\frac{\partial \phi}{\partial n} = 0, \text{ on } x = 50\alpha \text{ and } z \geq 0, \quad (17)$$

$$\frac{\partial \phi}{\partial n} = -\phi, \text{ for } 0 \leq x \leq 50\alpha \text{ and } z = \infty. \quad (18)$$

Equation (12), which is a modified Helmholtz equation, subject to Boundary conditions (14) - (18) that may be solved numerically by using a DRM to obtain numerical values of ϕ . Using the values of ϕ obtained, from Equations (2), (3), (11) and Dimension less variables (8), values of ψ may be computed by the formula of:

$$\psi = \frac{1}{\alpha} \ln \left(\frac{50v_0\alpha\phi e^z}{\pi K_s} \right). \quad (19)$$

Readers may refer to [5] for the details of the DRM.

4. Results and discussion

In this section, the DRM is employed to solve a problem involving steady infiltration in three-layered soil. The first layer, which is Silt Loam, has a depth of 100 cm. Pima Clay Loam, the second layer, also has a depth of 100 cm. Touchet Silt Loam, which is the third layer, has an infinitely depth. It was set the width and the depth of the channels, which are $200/\pi$ cm and $150/2\pi$ cm, respectively. As discussed in the preceding section, the value of α^* must be computed by taking average of the values of α in each layer. Moreover, to compute ψ , it needs the values of K_s . The values of α and K_s of the soils are shown in Table 1. These values are as reported by Amoozegar-Fard et al [1].

Table 1. Values of α and K_s .

	α (cm ⁻¹)	K_s (cm/day)
Silt Loam	1.39×10^{-2}	4.9594
Pima Clay Loam	1.40×10^{-2}	9.9360
Touchet Silt Loam	1.56×10^{-2}	41.9904

To implement the method, the region R must be bounded by a simple closed curve. Hence, an imposed boundary is needed. After several computational experiments, it is obtained that the suitable imposed boundary is $z = 4$. The DRM needs discretization of the boundary and a number of interior collocation points in its implementation. To do so, the boundary is discretized into 197 elements, and 400 interior collocation points are chosen. The results obtained are presented in Table 2, Figure 3, and Figure 4.

Table 2. Values of ψ at selected points.

Soil Type	Suction potential (cm)
-----------	------------------------

(10 cm, 25 cm)	Silt Loam	-51.84549382
(10 cm, 75 cm)	Silt Loam	-66.91679566
(10 cm, 125 cm)	Pima Clay Loam	-119.99130522
(10 cm, 175 cm)	Pima Clay Loam	-121.26207966
(10 cm, 225 cm)	Touchet Silt Loam	-194.53090712
(10 cm, 275 cm)	Touchet Silt Loam	-194.48987849
(90 cm, 25 cm)	Silt Loam	-97.66829776
(90 cm, 75 cm)	Silt Loam	-80.29745086
(90 cm, 125 cm)	Pima Clay Loam	-123.80530011
(90 cm, 175 cm)	Pima Clay Loam	-122.36097650
(90 cm, 225 cm)	Touchet Silt Loam	-194.82079719
(90 cm, 275 cm)	Touchet Silt Loam	-194.58348375

Table 2 shows values of ψ at selected points or locations. In this table, values of ψ at locations under the channels, $X = 10$ cm, and further from the channels, $X = 90$ cm are presented. It can be seen that under the channels, at the first and the second layer, values of ψ decreases as soil goes deeper. In the first layer, which is Silt Loam, from $Z = 25$ cm to $Z = 75$ cm values of ψ declines about 29.07 %. In Pima Clay Loam, ψ drops about 1.06 %, from $Z = 125$ cm to $Z = 175$ cm. In the third layer, values of ψ are about the same. However, at locations further from the channels, $X = 90$ cm, ψ increases as Z increases. For the same values of Z , increasing in Silt Loam is about 17.79 %. The rise in ψ in Pima Clay Loam is about 1.17 %. In Touchet Silt Loam, the values of ψ are also about the same.

The results indicate that at location under the channels, in the first and second layer, suction potential drops as soil goes deeper. The drops in at locations near the surface of the soil are higher than those deeper. The drops get lower as Z increases, and eventually after reaching some value of Z , there are no drops in ψ . Conversely, at locations further from the channels, suction potential in the first and second layer increases as Z increases. The increase falls as soil goes deeper, and after reaching a level of soil depth, there are no more increase in ψ .

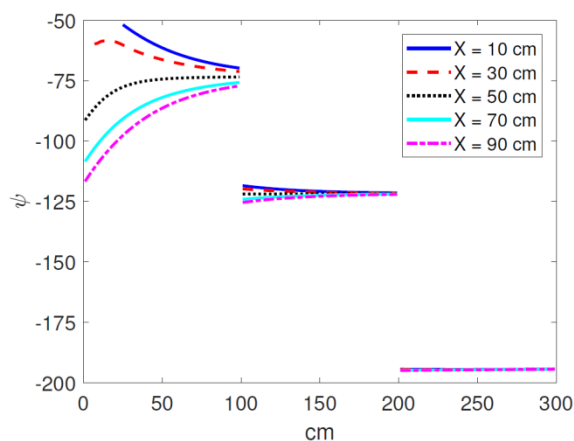


Figure 3. Graphs of ψ along Z-axis at selected values of X.

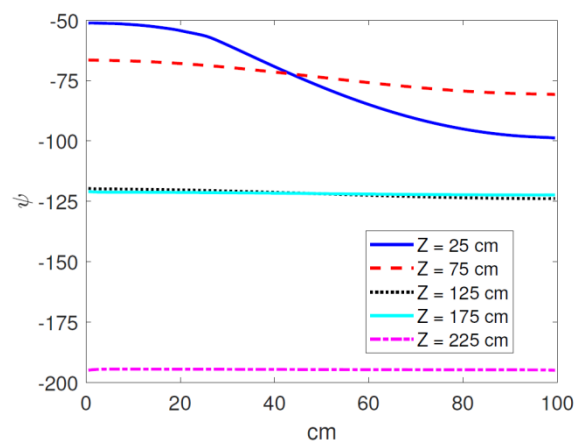


Figure 4. Graphs of ψ along X-axis at selected values of Z.

Graphs of ψ at selected values of X along Z-axis are presented in Figure 3. In the first layer, values of ψ at $X = 10$ cm, especially at the surface of channels, the highest compared to those at any locations. In other hand, values of ψ at $X = 90$ cm, especially at the surface of the soil, the lowest. These results indicate that the highest value of suction potential occurs at the location nearest the

center of the channels, and the lowest value of ψ is at the furthest away from the channels at surface of the soil.

As those in the first layer, in the second layer ψ reaches its highest value at $X = 10$ cm, and the lowest value of ψ is attained at $X = 90$ cm. In the third layer, it can be observed that values of ψ have reached their convergent value. These results are expected as $\partial\theta/\partial X \rightarrow 0$ and $\partial\theta/\partial Z \rightarrow 0$ as $Z \rightarrow \infty$.

It can also be observed that there are jumps in values of ψ at $Z = 100$ cm and $Z = 200$ cm. These due to the situation that soil type at $Z < 100$ cm is different from that $Z > 100$ cm. Similarly, the situation occurs at $Z = 200$ cm. Among the three layers, the first layer results in higher values of ψ . In other hand, the third layer cause in lower values of ψ . These imply that soil with higher values of α and K_s has lower values of ψ .

Figure 4 shows graphs of ψ at selected values of Z along X -axis. It can be seen that there are variation in values of ψ at $Z = 25$ cm. At $Z = 75$ cm, there are less variation in the values of ψ . As Z goes deeper, the variation of ψ along X -axis is lesser, and eventually after reaching a deep level there are no variation in values of ψ .

5. Concluding remarks

Problems involving steady infiltration from periodic trapezoidal channels have been solved numerically by transforming the governing equation and using a Dual Reciprocity Method (DRM) to solve the resulting partial differential equation. The method is applied to obtain numerical values of suction potential.

The results obtained indicate the effect of soil type to the suction potential. Soil with higher hydraulic conductivity and more porous, produce such higher suction potential than less porous medium and lower hydraulic conductivity. More decrease or increase in the values of suction potential occurs at shallower level of soil. There are no decrease or increase in values of suction potential after a level of soil depth. In shallower level of soil, there are more variation in values of the suction potential than those in deeper level of soil. Moreover, there are jumps in the values of suction potential. The jumps occur at locations where the soil type changes.

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