Basic Maths

1) Sequence And Series (A.P)

~~(A.P)~~

Q. If $a_1, a_2, a_3, \dots a_n$ are in AP where $a_i > 0$ for all i, Show that

$$\frac{1}{\sqrt{a_1}+\sqrt{a_2}} + \frac{1}{\sqrt{a_2}+\sqrt{a_3}} + \dots + \frac{1}{\sqrt{a_{n-1}}+\sqrt{a_n}} = \frac{(n-1)}{\sqrt{a_1}+\sqrt{a_n}}$$

Sol:- LHS $= \frac{1}{\sqrt{a_1}+\sqrt{a_2}} + \frac{1}{\sqrt{a_2}+\sqrt{a_3}} + \dots + \frac{1}{\sqrt{a_{n-1}}+\sqrt{a_n}}$

$$\frac{1}{\sqrt{a_2}+\sqrt{a_1}} + \frac{1}{\sqrt{a_3}+\sqrt{a_2}} + \dots + \frac{1}{\sqrt{a_n}+\sqrt{a_{n-1}}}$$

Multiplying & Dividing

$$\frac{\sqrt{a_2}-\sqrt{a_1}}{a_2-a_1} + \frac{\sqrt{a_3}-\sqrt{a_2}}{a_3-a_2} + \dots + \frac{\sqrt{a_n}-\sqrt{a_{n-1}}}{a_n-\cancel{\sqrt{}}a_{n-1})}$$

Now $a_2 - a_1 = a_3 - a_2 = a_4 - a_3 = \dots a_n - a_{n-1} = d$

$$\therefore \frac{1}{d}\left(\cancel{\sqrt{a_2}} - \sqrt{a_1} + \cancel{\sqrt{a_3}} - \cancel{\sqrt{a_2}} + \dots + \sqrt{a_n} - \cancel{\sqrt{a_{n-1}}}\right)$$

$$= \frac{1}{d}\left(\sqrt{a_n} - \sqrt{a_1}\right)$$

$$= \frac{1}{d}\,\frac{a_n - a_1}{\sqrt{a_n}+\sqrt{a_1}} \Rightarrow \frac{1}{d}\left(\frac{\cancel{a_1} + (n-1)d - \cancel{a_1}}{\cancel{d}\,(\sqrt{a_n}+\sqrt{a_1})}\right)$$

$$= \frac{1}{d}\left(\frac{(n-1)d}{\sqrt{a_n}+\sqrt{a_1}}\right)$$

$$= \frac{(n-1)}{\sqrt{a_1}+\sqrt{a_n}}$$

Q The interior angles of a polygon are arithmetic progression. The smallest angle is 120° and common difference is 5°. Find the number of sides of the polygon.

NOTE: Sum of interior angles of a polygon of n sides $= (n-2)\times 180^\circ$

Sol÷ Let no of sides of a polygon be n

$\therefore$ Sum of interior angles of a polygon $= (n-2)\pi$

given $a_1 = 120^\circ$, $d = 5^\circ$

S_n = Sum of Interior Angles

$$(n-2)\pi = \frac{n}{2}(2a+(n-1)d)$$

$$(n-2)180^\circ = \frac{n}{2}(2(120^\circ)+(n-1)5^\circ)$$

$$(n-2)180^\circ = \frac{n}{2}(240+5(n-1))$$

$$360^\circ(n-2) = 240n + 5(n-1)n$$

$$\cancel{\begin{bmatrix} 345n = 240 - 725 \\ 345n = -480 \end{bmatrix}}\ \times$$

$$360^\circ n - 720^\circ = 240n + 5n^2 - 5n$$

$12 \,|\, 144$, 12

$$\Rightarrow 5n^2 - 125n + 720 = 0$$

$2\times2\times3$

$$n^2 - 25n + 144 = 0$$

$2\times2\times3$

$$\cancel{\begin{bmatrix} n^2 - 12n - n + 144 = 0 \\ n(n-1) - 12(n-12) = 0 \end{bmatrix}}\ \times$$

$$n^2 - 9n - 16n + 144 = 0$$

$$n(n-9) - 16(n-9) = 0$$

$$(n-9)(n-16) = 0$$

$$n = 9, 16$$

put $n = 16$ ie largest angle

$$a_n = a+(n-1)d$$

$$= 120^\circ + (16-1)5^\circ$$

$$= 120^\circ + 75^\circ$$

$$= 195^\circ > 180^\circ \quad \text{(not possible)}$$

$\therefore n = 9$ valid.

Q₂ 4 different integers form an increasing AP. One of these numbers is equal to the sum of the squares of the other 3 numbers. Find the numbers.

Sol÷ Let 4 numbers be,

$$a-d,\ a,\ a+d,\ a+2d.$$

$$a, d \in I,\ d > 0$$

According to the hypothesis

$a+2d = (a-d)^2 + a^2 + (a+d)^2$

$a+2d = 3a^2 + 2d^2 - 2ad + 2ad$

$d^2 + 2a - 2d = 0$

$d^2 - 2d + 2a = 0$ (A quadratic eqn)

$d = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$d = \frac{+2 \pm \sqrt{4-8a}}{2}$

$= \frac{2 \pm \sqrt{4-8a}}{2}$

$= \frac{1}{2}[$ ×

$2d^2 - 2d + 3a^2 - a = 0$

$d = \frac{+2 \pm \sqrt{4 - 8(3a^2-a)}}{2\times2}$

$d = \frac{1}{2}[1 \pm \sqrt{1 - 2(3a^2-a)})$ (taking out 2 common)

$d = \frac{1}{2}(1 \pm \sqrt{1-6a^2+2a})$

$d = \frac{1}{2}(1 \pm \sqrt{1+2a-6a^2})$

$\because$ d is positive

$d > 0$

$1 + 2a - 6a^2 > 0$

$6a^2 - 2a - 1 < 0$

$a^2 - \frac{a}{3} - \frac{1}{6} < 0$

$[(a)^2 - 2 \cdot \frac{1}{6} \cdot a + (\frac{1}{6})^2 - (\frac{1}{6})^2 - \frac{1}{6} < 0]$ ×

$a = \frac{+\frac{1}{3} \pm \sqrt{\frac{1}{9} + \frac{4}{6}}}{2}$

$a = \frac{\frac{1}{3} \pm \sqrt{\frac{1}{9} + \frac{2}{3}}}{2}$

$a = \frac{\frac{1}{3} \pm \sqrt{\frac{1+6}{9}}}{2}$

$a = \frac{\frac{1}{3} \pm \frac{1}{3}\sqrt{7}}{2}$

$a = \frac{1 \pm \sqrt{7}}{6}$

$\left(a - \left(\frac{1+\sqrt{7}}{6}\right)\right)\left(a - \left(\frac{1-\sqrt{7}}{6}\right)\right) < 0$

$\therefore \left(\frac{1-\sqrt{7}}{6}\right) < a < \left(\frac{1+\sqrt{7}}{6}\right)$

$\because a$ is an integer

$\therefore a = 0$

then $d = \frac{1}{2}\left[1 \pm \sqrt{1+0-0}\right]$

$= \frac{1}{2}(1 \pm 1)$

$= 1, 0$

$\because d > 0$

$\therefore d = 1$

Hence no's are $-1, 0, 1, 2$

G.P

Q. The sum of the squares of 3 distinct real numbers which are in G.P is S^2. If their sum is αS. Show that $\alpha^2 \in \left(\frac{1}{3}, 1\right) \cup (1, 3)$

Sol:- Let 3 no's be $\frac{a}{r}, a, ar$

$\frac{a^2}{r^2} + a^2 + a^2r^2 = S^2$

$\frac{a^2(1+r^2+r^4)}{r^2} = S^2$ — (i)

$\frac{a}{r} + a + ar = \alpha S$

$\frac{a(1+r+r^2)}{r} = \alpha S$

$\frac{a(1+r+r^2)}{\alpha r} = S$ — (ii)

From eq^n (i) & (ii)

$\frac{\cancel{a^2}(1+r^2+r^4)}{\cancel{r^2}} = \frac{\cancel{a^2}(1+r+r^2)^2}{\alpha^2 \cancel{r^2}}$

$\alpha^2(1+r^2+r^4) = (1+r+r^2)^2$

$\alpha^2(1+r+r^2)(1-r+r^2) = (1+r+r^2)^2$

$\boxed{\because (1+r+r^2) \neq 0}$

$\alpha^2(1-r+r^2) = (1+r+r^2)$

$\alpha^2(1-r) + \alpha^2 r^2 = 1+r+r^2$

$(\alpha^2-1)r^2 - (\alpha^2+1)r + \alpha^2 - 1 = 0$

$r^2 - \frac{(\alpha^2+1)}{(\alpha^2-1)}r + 1 = 0$

$\boxed{\because r \text{ is real}}$

$\left(\frac{\alpha^2+1}{\alpha^2-1}\right)^2 - 4 > 0$ (no's in G.P are distinct)

$$\left(\frac{\alpha^2+1}{\alpha^2-1}\right)^2 - 4 > 0$$

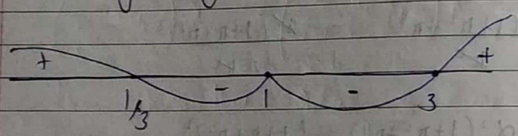
$$\left(\frac{\alpha^2+1}{\alpha^2-1} - 2\right)\left(\frac{\alpha^2+1}{\alpha^2-1} + 2\right) > 0$$

$$\cancel{(-\alpha^2+3)(3\alpha^2-1) > 0}$$

$$\frac{(3-\alpha^2)(3\alpha^2-1)}{(\alpha^2-1)^2} > 0$$

$$\frac{(\alpha^2-3)(\alpha^2-1/3)}{(\alpha^2-1)^2} < 0$$

Using Wavy Curve Method



$$\alpha^2 \in \left(\frac{1}{3}, 1\right) \cup (1, 3)$$

Q. Find natural number 'a' for which $\sum_{k=1}^{n} f(a+k) = 16(2^n-1)$ where function f satisfies $f(x+y) = f(x)f(y)$ for all natural no's x, y and further $f(1) = 2$

Sol. given, $f(x+y) = f(x)f(y)$ and $f(1) = 2$

$$f(1+1) = f(1)f(1)$$
$$f(2) = 2^2$$

$$f(1+2) = f(1)f(2) = 2 \cdot 2^2$$
$$f(3) = 2^3$$

$$\vdots$$

$f(k) = 2^k$ and $f(a) = 2^a$

Hence $f(a+k) = f(a)f(k)$

$$\sum_{k=1}^{n} f(a+k) = \sum_{k=1}^{n} f(a)f(k)$$

$$= f(a)\sum_{k=1}^{n} f(k)$$

$$= f(a)\left(f(1) + f(2) + f(3) + \dots + f(n)\right)$$

$$= f(a)\left(2 + 2^2 + 2^3 + \dots + 2^n\right)$$

$$= f(a)\left(\frac{2(2^n-1)}{2-1}\right)$$

$$= f(a)(2^n-1) \cdot 2$$

$$= 2^{a+1}(2^n-1)$$

but $\sum_{k=1}^{n} f(a+k) = 16(2^n - 1)$

$16(2^n-1) = 2^{a+1}(2^n-1)$

$2^4 = 2^{a+1}$

$a = 4+1$

$a = 3$

Q. If $a_i \in R$, $i = 1,2,3,\ldots\ldots n$ and all a_i's are distinct such that

$$\left(\sum_{i=1}^{n-1} a_i^2\right)x^2 + 2\left(\sum_{i=1}^{n-1} a_i a_{i+1}\right)x + \sum_{i=2}^{n} a_i^2 \leq 0$$

then show that $a_1, a_2, \ldots a_n \ldots$ are in G.P

Sol:- $$\left(\sum_{i=1}^{n-1} a_i^2\right)x^2 + 2\left(\sum_{i=1}^{n-1} a_i a_{i+1}\right)x + \sum_{i=2}^{n} a_i^2 \leq 0$$

$$\sum_{i=1}^{n-1}\left(a_i^2 x^2 + 2a_i a_{i+1} x + a_{i+1}^2\right) \leq 0$$

$$\sum_{i=1}^{n-1}\left(a_i x + a_{i+1}\right)^2 \leq 0$$

$\therefore a_i x + a_{i+1} = 0 \quad \forall\ i = 1,2,3,\ldots\ldots n-1$

$\frac{a_{i+1}}{a_i} = -x \quad \forall\ i = 1,2,3,\ldots\ldots n-1$

$\therefore \frac{a_2}{a_1} = \frac{a_3}{a_2} = \frac{a_4}{a_3} = \ldots\ldots \frac{a_n}{a_{n-1}}$

$\therefore a_1, a_2, a_3, \ldots$ are in G.P

H.P

No term of HP can be zero

Q. If $\frac{1}{a} + \frac{1}{c} + \frac{1}{a-b} + \frac{1}{c-b} = 0$. Prove that a,b,c are in HP, unless $b = a+c$

Sol. $$\frac{1}{a} + \frac{1}{c} + \frac{1}{a-b} + \frac{1}{c-b} = 0$$

$$\frac{a+c}{ac} + \frac{a+c-2b}{(a-b)(c-b)} = 0$$

Let $a+c = \lambda$

$$\frac{\lambda}{ac} + \frac{\lambda - 2b}{(a-b)(c-b)} = 0$$

$$\frac{\lambda(a-b)(c-b) + ac(\lambda - 2b)}{ac(a-b)(c-b)} = 0$$

$$\frac{\lambda(a-b)(c-b) + (\lambda-2b)(ac)}{ac(a-b)(c-b)} = 0$$

$$\lambda ac \overset{-\lambda ab}{\cancel{-\lambda ab}} - \lambda c^2 + \lambda bc + \lambda ac - 2abc = 0$$

$$2\lambda ac - \lambda ab - \lambda c^2 + \lambda bc - 2abc = 0$$

$$2ac(\lambda - b) - \lambda b(a-$$

$$\lambda ac - \lambda ab - \lambda bc + b^2 + \lambda ac - 2abc = 0$$

$$2\lambda ac - 2abc - \lambda ab - \lambda bc + b^2 = 0$$

$$\frac{a+c}{ac} + \frac{a+c-2b}{(a-b)(c-b)} = 0$$

$$\frac{a+c}{ac} + \frac{a+c-2b}{(ac-ab-bc+b^2)} = 0$$

$$\frac{a+c}{ac} + \frac{a+c-2b}{(ac-b(a+c)+b^2)} = 0$$

put $a+c = \lambda$

$$\frac{\lambda}{ac} + \frac{\lambda-2b}{(ac-\lambda b+b^2)} = 0$$

$$\lambda ac - \lambda^2 b + \lambda b^2 + \lambda ac - 2bac = 0$$

$$2\lambda ac - 2abc - \lambda^2 b + \lambda b^2 = 0$$

$$2ac(\lambda - b) - \lambda b(\lambda - b) = 0$$

$$(\lambda - b)(2ac - \lambda b) = 0$$

$\lambda = b$ or $\lambda = \frac{2ac}{b}$

$\therefore a+c = b$ or $a+c = \frac{2ac}{b}$

$a+c = b$ or $b = \frac{2ac}{a+c} = \frac{1}{a} + \frac{1}{c}$

Q. If $x_1, x_2, x_3, \ldots x_n$ are in HP, Prove that

$$x_1x_2 + x_2x_3 + x_3x_4 + \ldots x_{n-1}x_n = (n-1)x_1x_n$$

Sol: $\because x_1, x_2, x_3, \ldots x_n$ are in HP

$\therefore \frac{1}{x_1}, \frac{1}{x_2}, \frac{1}{x_3}, \ldots \frac{1}{x_n}$ are in AP

$\therefore$ Common difference $D = \frac{1}{x_2} - \frac{1}{x_1}$

$$D = \frac{x_1 - x_2}{x_2x_1}$$

$$x_1x_2 = \frac{x_1 - x_2}{D}$$

Similarly $x_2x_3 = \frac{x_2 - x_3}{D}, \ldots, x_{n-1}x_n = \frac{x_{n-1}\, x_n}{D}$

Adding all, we get

$x_1x_2 + x_2x_3 + x_3x_4 + \dots + x_{n-1}x_n = \frac{x_1 - x_n}{D}$

$= \frac{1}{D}\left(\frac{x_1 - x_n}{1}\right)$

Multiply and Divide by $x_1 x_n$

$= \frac{x_1 x_n}{D}\left(\frac{1}{x_n} - \frac{1}{x_1}\right)$

$\frac{x_1 x_n}{D}\left(\cancel{\frac{1}{x_1}} + (n-1)D - \cancel{\frac{1}{x_1}}\right)$

$\frac{x_1 x_n (n-1)\cancel{D}}{\cancel{D}}$

Result

$\boxed{x_1x_2 + x_2x_3 + \dots + x_{n-1}x_n = (n-1)x_1x_n}$

For $n = 4$

$x_1x_2 + x_2x_3 + x_3x_4 = 3x_1x_4$

$n = 6$

$x_1x_2 + x_2x_3 + x_3x_4 + x_4x_5 + x_5x_6 = 5x_1x_6$

If $\frac{a-b}{b-c} = \frac{a}{a}$ then a, b, c are in AP

$\frac{a-b}{b-c} = \frac{a}{b}$ then a, b, c are in GP

$\frac{a-b}{b-c} = \frac{a}{c}$ then a, b, c are in HP

Q. If $\sqrt[x]{a} = \sqrt[y]{b} = \sqrt[z]{c}$ and if a, b, c are in GP, then prove that x, y, z are in AP.

Sol. $\because$ a, b, c are in GP

$\therefore$ $\log a$, $\log b$, $\log c$ are in AP

$\therefore \log b = \frac{\log a + \log c}{2}$

$2\log b = \log a + \log c$

given, $\sqrt[x]{a} = \sqrt[y]{b} = \sqrt[z]{c}$

$a^{1/x} = b^{1/y} = c^{1/z}$

$\frac{1}{x}\log a = \frac{1}{y}\log b = \frac{1}{z}\log c = k$ (say)

$\therefore \log a = kx \quad \log b = ky \quad \log c = kz$

$\therefore \quad 2ky = kx + kz$

$2y = x + z$

$\therefore$ x, y, z are in A.P

Aptitude (GATE)

Permutations
& Combinations

Basics

$n!$ or $\lfloor n$ (product of first n natural numbers)

$n! = 1\cdot2\cdot3\cdot4 \ldots\ldots n$

Ex:- $3! = 3\cdot2\cdot1 = 6$

$4! = 4\cdot3\cdot2\cdot1 = 24$

$5! = 5\cdot4\cdot3\cdot2\cdot1 = 120$

* $\boxed{1! = 1 \quad 0! = 1}$

Note:- factorial of a -ve number doesn't exists.

$n! = n(n-1)(n-2) \ldots\ldots 2\cdot1$
$= n(n-1)!$

Ex: $(n+1)! - n!$
$(n+1)n! - n!$
$n!(n+1-1)$
$n!(n)$

$(n-1)! + n!$
$(n-1)! + n(n-1)!$
$(n-1)!(1+n)$
$(n+1)(n-1)!$

$${}^nP_r = \frac{n!}{(n-r)!}$$

$$= \frac{1\cdot2\cdot3\cdot4 \ldots\ldots (n-r)(n-r+1)(n-r+2) \ldots\ldots n}{1\cdot2\cdot3\cdot4 \ldots\ldots (n-r)}$$

$$= (n-r+1)(n-r+2) \ldots\ldots n$$

Ex: ${}^{10}P_4 = \frac{10!}{6!} = \frac{10\cdot9\cdot8\cdot7\cdot\cancel{6!}}{\cancel{6!}} = 10\cdot9\cdot8\cdot7 = 5040$

$${}^nC_r = \frac{n!}{(n-r)!\,r!} = \frac{{}^nP_r}{r!}$$

* $\boxed{{}^nC_r = \frac{{}^nP_r}{r!}}$

Important properties of ${}^nC_r$

1) ${}^nC_r = {}^nC_{n-r}$

Proof: ${}^nC_r = \frac{n!}{(n-r)!\,r!}$ ${}^nC_{n-r} = \frac{n!}{r!\,(n-r)!}$

LHS = RHS

Ex: ${}^nC_x = {}^nC_y$

either $x = y$

or

$y = n - x$ $(\because {}^nC_r = {}^nC_{n-r})$

* $\boxed{x + y = n}$

Maximum value of ${}^nC_r$

I> if n is even max value at $r = \frac{n}{2}$

II> if n is odd max value at either $r = \frac{n-1}{2}$ or $r = \frac{n+1}{2}$

Ex:- ${}^{10}C_6 = {}^{10}C_4 = \frac{10!}{6!\,4!} = \frac{10 \times \cancel{9} \times \cancel{8} \times 7}{4 \times 3 \times 2 \times 1} = 210$

${}^{10}C_9 = {}^{10}C_1 = 10$

Max value of ${}^{10}C_r$ will be at $r = 5$ ie ${}^{10}C_5$

Max value of ${}^{11}C_r$ will be either at $r = 6$ or $r = 5$

*** 2> ${}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$

Prove:- LHS:- ${}^nC_r + {}^nC_{r-1}$

$$\frac{n!}{r!\,(n-r)!} + \frac{n!}{(n-r+1)!\,(r-1)!}$$

$$= \frac{n!}{(r-1)!\,(n-r)!}\left(\frac{1}{r} + \frac{1}{(n-r+1)}\right)$$

$$= \frac{n!}{(r-1)!\,(n-r)!}\left(\frac{n-r+1+r}{(n-r+1)(r)}\right)$$

$$= \frac{n!\,(n+1)}{(r-1)!\,r\,(n-r)!\,(n-r+1)}$$

$$= \frac{(n+1)!}{r!\,(n-r+1)!}$$

$$= \frac{(n+1)!}{(n+1-r)!\,r!}$$

$$= {}^{n+1}C_r$$

= RHS

Hence Proved.

Ex: $^{10}C_3 + {}^{10}C_4 = ?$

$^{10}C_3 + {}^{10}C_4 = {}^{11}C_4$

$^{20}C_3 + {}^{20}C_4 = {}^{21}C_4$

3) $^{n}C_r = \frac{n}{r}\, {}^{n-1}C_{r-1}$

$^{10}C_3 = \frac{10!}{7!\,3!} = \frac{10\times 9\times 8\times \cancel{7!}}{\cancel{7!}\times 3\times 2\times 1} = \frac{10\cdot 9\cdot 8}{3\cdot 2\cdot 1}$

Prove:- LHS:- $^{n}C_r = \frac{n!}{(n-r)!\,r!}$

$= \frac{n(n-1)!}{(n-r)!\,r(r-1)!}$

$= \frac{n}{r}\,\frac{(n-1)!}{(n-r)!(r-1)!}$

$= \frac{n}{r}\,\frac{(n-1)!}{((n-1)-(r-1))!\,(r-1)!}$

$= \frac{n}{r}\,{}^{n-1}C_{r-1}$

$=$ RHS

Hence Proved.

Ex: $^{10}C_3 = \frac{10}{3}\times {}^{9}C_2$

$= \frac{10}{3}\times\frac{9}{2}\times {}^{8}C_1$

$= \frac{10}{3}\cdot\frac{9}{2}\cdot\frac{8}{1}\cdot {}^{7}C_0$

$= \frac{10}{3}\cdot\frac{9}{2}\cdot\frac{8}{1}\cdot 1 \quad (\because {}^{7}C_0 = 1)$

Note:- $\boxed{\begin{array}{l} ^{n}C_0 = {}^{n}C_n = 1 \\ ^{n}C_1 = {}^{n}C_{n-1} = n \end{array}}$

4) $\frac{^{n}C_r}{^{n}C_{r-1}} = \frac{n-r+1}{r}$

prove:- LHS: $\frac{^{n}C_r}{^{n}C_{r-1}} = \frac{n!}{r!\,(n-r)!} \times \frac{(n-r+1)!\,(r-1)!}{n!}$

$= \frac{(n-r+1)(n-r)!\,(r-1)!}{r(r-1)!\,(n-r)!}$

$= \frac{n-r+1}{r}$

$=$ RHS

Hence Proved

Important Result

$$\boxed{\begin{gathered} {}^nC_0 + {}^nC_1 + {}^nC_2 + \ldots\ldots + {}^nC_n = 2^n \\ {}^nC_0 + {}^nC_2 + {}^nC_4 + \ldots\ldots = {}^nC_1 + {}^nC_3 + {}^nC_5 + \ldots\ldots = 2^{n-1} \end{gathered}}$$

Ex:- ${}^{13}C_0 + {}^{13}C_1 + {}^{13}C_2 + \ldots\ldots {}^{13}C_{13} = 2^{13}$

${}^{13}C_1 + {}^{13}C_3 + {}^{13}C_5 + \ldots\ldots + {}^{13}C_{11} = 2^{13-1} = 2^{12}$

Q) ${}^nC_{r-1} = 36$ ${}^nC_r = 84$ ${}^nC_{r+1} = 126$

then what is n and r.

Sol:- ${}^nC_{r-1} = 36$ ${}^nC_r = 84$ ${}^nC_{r+1} = 126$

$\frac{{}^nC_r}{{}^nC_{r-1}} = \frac{\cancel{84}\,7}{\cancel{36}\,3} \Rightarrow \frac{7}{3} = \frac{n-r+1}{r}$

$7r = 3n - 3r + 3$

$10r = 3n + 3$ —(1)

$\frac{{}^nC_{r+1}}{{}^nC_r} = \frac{\cancel{126}\,7+3}{\cancel{84}\,14\,2} = \frac{n-r-1+1}{r+1} = \frac{n-r}{r+1}$

$\times\left[\begin{array}{l} 3r = 2n - 2r + 2 \\ 5r = 2n + 2 \quad —(2) \end{array}\right]$ $\begin{array}{l} 3r + 3 = 2n - 2r \\ 5r = 2n - 3 \quad —(2) \end{array}$

from eqn (1) and (2)

$10r = 3n + 3$ $\qquad 5r = 2n \cancel{+2} - 3$

$\times\left[\begin{array}{l} 5(5r) = 3n+3 \\ 5(2n+2) = 3n+3 \\ 10n + 10 = 3n+3 \end{array}\right]$ $\left[\begin{array}{l} 5(2n-3) = 3n+3 \\ 10n - 15 = 3n+3 \\ 7n = 18 \end{array}\right] \times$

$2(5r) = 3n+3$

$2(2n-3) = 3n+3$

$4n - 6 = 3n + 3$

$n = 9$

$\because 5r = 2n - 3$

$5r = 2(9) - 3$

$5r = 18 - 3 \Rightarrow r = 3$

Ans:- $n = 9,\ r = 3$

Q) ${}^{90}C_9 + \sum_{r=0}^{10} {}^{100-r}C_8 = ?$

Sol: ${}^{90}C_9 + \sum_{r=0}^{10} {}^{100-r}C_8$

${}^{100}C_8 + {}^{99}C_8 + {}^{97}C_8 + \ldots\ldots + \underbrace{{}^{90}C_8 + {}^{90}C_9}$

${}^{100}C_8 + {}^{99}C_8 + \ldots\ldots + \underbrace{{}^{91}C_8 + {}^{91}C_9}$ $\quad (\because {}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r)$

${}^{100}C_8 + {}^{99}C_8 + \ldots\ldots \underbrace{{}^{92}C_8 + {}^{92}C_9}$

Keep on combining terms

$= {}^{100}C_8 + \underbrace{{}^{99}C_8 + {}^{99}C_9}$

$= {}^{100}C_8 + {}^{100}C_9$

$= {}^{101}C_9$

Q. ${}^{50}C_{20} + 3\,{}^{50}C_{21} + 3\,{}^{50}C_{22} + {}^{50}C_{23}$

Sol:- ${}^{50}C_{20} + 3\,{}^{50}C_{21} + 3\,{}^{50}C_{22} + {}^{50}C_{23}$

$\underbrace{{}^{50}C_{20} + {}^{50}C_{21}} + 2\,{}^{50}C_{21} + 2\,{}^{50}C_{22} + \underbrace{{}^{50}C_{22} + {}^{50}C_{23}}$

${}^{51}C_{21} + 2\left({}^{50}C_{21} + {}^{50}C_{22}\right) + {}^{51}C_{23}$

$\underbrace{{}^{51}C_{21} + {}^{51}C_{22}} + \underbrace{{}^{51}C_{22} + {}^{51}C_{23}}$

$=$

$= {}^{52}C_{22} + {}^{52}C_{23}$

$= {}^{53}C_{23}$

Imp ★★★

Q. $1! + 4! + 7! + 10! + \dots + 301!$
What is the last digit of the sum and 2 last digits of sum?

Sol:- $1! + 4! + 7! + 10! + \dots + 301!$

5! will contain 0 as last term

$\forall n$, $n!$ which are greater than 5!

they all will contain 5! as a sub term in them. Hence any term after 5! will have 0 in its unit place.

$\therefore$ last digit of unit place will depend on terms less than 5!

ie $\forall n$ $n! < 5!$ (these $n!$ will affect the unit place)

$\therefore 1! + 4! = 1 + 24 = \underline{25}$ $\quad 5 + 0 = 5$

$\therefore$ last digit of sum is 5.

For last 2 digits, we need to observe that 10! will also contribute one more zero in addition to zero contributed by 5!. Hence any term greater than 10! will contain last two digits as 0.

These last 2 of digits will be affected by the terms less than 10!

$1! + 4! + 7! = 1 + 24 + 5040$
$= 50\underline{65}$ $\quad 65 + 00 = 65$

$\therefore$ last 2 digits will be 65.

Similar logic can be applied for last 3 digits we need to check for the terms less than 151 (151 will contribute one more addition 0 to the 2 zero's produced by 101) Hence all the terms greater than 151 will have 3 zero's at last digits.

Counting

If we have big problem, divide it into small problems, find the solution and appropriately either multiply or add.

Principle of addition (OR) & principle of multiplication (AND)

Suppose we have 3 red balls and 3 black balls in how many ways can we pick 1 ball.

From 3 Red balls there are 3 ways of picking 1 ball

Similarly from 3 black balls, there are 3 ways of picking 1 ball.

∴ no of ways we can pick 1 ball out of these 6 balls is (either from 3 Red or from 3 black balls)

$3+3=6$

Ex: Suppose we have 10 black suits and 20 blue suits. no of ways a person choose 1 suit / or pick 1 suit

10 black $\longrightarrow$ 10 ways

$+$ $= 30$ ways

20 blue $\longrightarrow$ 20 ways

Ex:- Suppose we have 2 shirts and 2 pants. no of ways we can dress up.

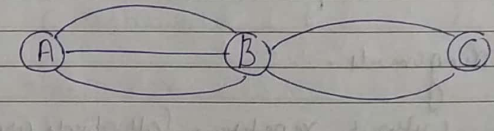
2 shirts (a, b) 2 pants (1, 2)

(a, 1) (b, 1)
(a, 2) (b, 2)

2 shirt $\rightarrow$ 2 ways of choosing shirt and from
2 pants $\rightarrow$ 2 ways of choosing pants.

∴ no of ways we can dress up $= 2\times2 = 4$.

Ex.



In how many ways we can go from A to C.

(A → B) AND (B → C)
3 × 2
= 6 ways

Concept choosing & arranging

O	O	O
O	O	O
O	O	O

If we have a box containing 9 objects then in how many ways we can pick choose 4 objects (ie the problem is combination)
$^9C_4$

Suppose we have chosen 4 objects, in how many ways we can arrange them

O O O O = 4!

Here we use permutation

Arrangements.

i) without repetition (all objects are distinct)
ii) with repetition (objects can be repeated)

Row arrangements (without repetition)

Ex. no of ways we can arrange 3 objects

a b c → 6 ways

a b c   b a c   c a b
a c b   b c a   c b a

arranging 3 objects means placing them at 3 available places

_ _ _
3 ways (ie 3 objects available), 2 ways (ie 2 objects available), 1 ways

∴ We can arrange n objects in n! ways.

_ _ _ _ . . . . _ _
n, (n-1), (n-2), (n-3) 2, 1

= n(n-1)(n-2) 2.1
= n! ways.

Example

Given word CINEMA

I> How many words can be formed

1> all the letters are distinct / 6-letter word

6! _ _ _ _ _ _
6 5 4 3 2 1

II> Restricted Permutation

1> always begin with 'C'

C _ _ _ _ _
↓ ↓ ↓ ↓ ↓
5 4 3 2 1

5!

2> Starts with 'C' and ends with 'A'

C _ _ _ _ A
↓ ↓ ↓ ↓
4 3 2 1

4!

3> 'C' and A should be at extremes

4! X2

2!

C/A _ _ _ _ C/A
↓ ↓ ↓ ↓
4 3 2 1

4> Vowels are always at even positions

3!

_ _ _ _ _ _
1 2 3 4 5 6   I EA

3! <- I E A

3! X 3!

5> 'C' and 'I' should always be together

$\boxed{CI}$ N E M A   5! X 2 ways

or

$\boxed{IC}$ N E M A

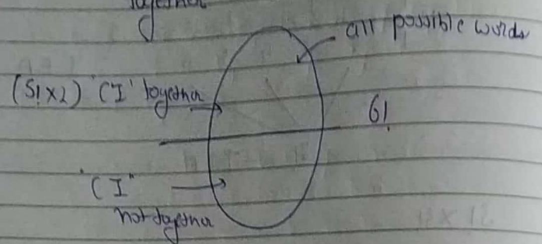
5 letters can be arranged in 5! ways
and $\boxed{CI}$ can be arranged in 2! ways

6> 'C', 'I', 'N' should always be together

$\boxed{CIN}$ E M A   4!
↓
3!

∴ 4! X 3!

7> 'C' and 'I' should never be together



$\therefore$ no of words such that "CI" are never together $= 6! - (5! \times 2)$

Note:- This complement method only works for 2 letters. ie Ex:- for "CIN" should never come together this method will not work.

8> C, I, N should always be together and 'I' always b/w 'C' and 'N'.

CIN or NIC

$\boxed{CIN}$ E M A $4!$

$2!$

$4! \times 2!$

Row Arrangements (with repetition)

ab n n n here 'n' is repeated 3 times

Now if we consider every n distinct and try to find out possible arrangements of these "n"

ab $n_1 n_2 n_3$		abnnn	
ab $n_1 n_3 n_2$		abnnn	
ab $n_2 n_1 n_3$	$\because$ all n's are same $\Rightarrow$	abnnn	abnnn
ab $n_2 n_3 n_1$		abnnn	
ab $n_3 n_2 n_1$		abnnn	
ab $n_3 n_1 n_2$		abnnn	

$\therefore$ We get ~~6!~~ 3! arrangements ~~wt~~ which are same

** If we have n objects out of which we have m repetitions then number of arrangements will be

$$\frac{n!}{m!}$$

Ex: MAYANK no of words $= \frac{6!}{2!}$

** If we have n different objects out of which we have m repetitions of object type 1, r repetitions of object type 2, p repetitions of object type 3.

such that $m+r+p = n$

no of arrangements $= \frac{n!}{m!\,r!\,p!}$

Ex: 10 balls, out of which 4 red balls, 3 green balls, 3 yellow balls.

$\therefore$ no of ways balls can be arranged

$= \frac{10!}{4!\,3!\,3!}$

Examples

Given word RAVINDRABABU

1) How many total words can be formed.

A – 3 times R – 2 times B → 2 times

$= \frac{12!}{3!\,2!\,2!}$

II) Restricted Permutations.

1) Starts with R

R _ _ _ _ _ _ _ _ _ _ _

A – 3 times
R → 2 times

$\frac{11!}{3!\,2!}$

R is not taken into consideration for repetitions, bcz one R is fixed.

2) Starts with 'R' ends with 'U'.

R _ _ _ _ _ _ _ _ _ _ U

A → 3
B → 2

$\frac{10!}{3!\,2!}$

3) Vowels are together, and consonants together

Vowels: $\boxed{\text{AIAAU}}$ → $\frac{5!}{3!}$

Consonants: $\boxed{\text{RVNDRBB}}$ → $\frac{7!}{2!\,2!}$

$2!$

$2! \times \frac{5!}{3!} \times \frac{7!}{2!\,2!}$

4) All 'A's should come together

$\boxed{AAA}$ RVINDRBBRU

$\frac{3!}{3!}$ $\frac{10!}{2!2!}$

5) All 'A's should come together
no 'B' should be together.

$\boxed{AAA}$ B _ B

all A's together consists of 2 subcases

Complementation Method

AAA — I case

II ← AAA BB AAA B _ B → III

III = (I − II)

II → all A's together all B's together

$\boxed{AAA}$ $\boxed{BB}$ RVINDRU

$\frac{9!}{2!}$

Ans:- $\left(\frac{10!}{2!2!} - \frac{9!}{2!}\right)$ ← III case (i.e. no B's together)

Imp ★★★

6) R's → not together
B's → not together

RAVINDRABABU

all possible words $\frac{12!}{3!2!2!}$

RR B_B — I
R_R BB — II
RR BB — III
R_R B B — IV

all R's together ← combine

V ← together B may or may not be together

RR — BB, B_B
R_R — BB, B_B

V case all R's together

$\boxed{RR}$ AVINDABABU

$\frac{11!}{3!2!}$

II case → (All B's together) − (R's not together & B's together)

$\boxed{BB}$ RAVINDRAAU

$\frac{11!}{3!2!}$

BB — RR, R_R

All R's together & All B's together — III

$\boxed{RR}$ $\boxed{BB}$ AV ITYDRAAU

$$\frac{10!}{3!}$$

II → R_R BB (R's not together, B's together)

$$\frac{11!}{3!2!} - \frac{10!}{3!}$$

IV — R_R and B_B.

total − (V + II)

Ans:- $$\frac{12!}{3!2!2!} - \left(\frac{11!}{3!2!} + \left(\frac{11!}{3!2!} - \frac{10!}{3!}\right)\right)$$

Digits

given 5 digits 2, 3, 4, 4, 5

I) total how many 5-digits numbers can be formed.

$$= \frac{5!}{2!}$$

II) ~~All~~ 5-digits even numbers

_ _ _ _ _
↑
2, 4, 4

Note:- Since even number should end with 2 or 4 we have 2 possiblity, but 4 is repeated. Hence if we have an object which need to be fixed, and that object is repeating, then separately cases should be analyzed.

_ _ _ _ _
↑
2

$$\frac{4!}{2!}$$

_ _ _ _ _
↑
4

4!

Since one 4 is fixed so no repetition.

$$\therefore \frac{4!}{2!} + 4!$$

III) ~~Odd~~ 5-digit odd numbers

_ _ _ _ _
↑
3 or 5

$$= \left(\frac{4!}{2!} \times 2\right)$$

Or we can also use complementation method

odd = Total − even

IV) even digits should be placed at odd places.

$$\underbrace{\_\ \_\ \_\ \_\ \_}_{2!}$$

① 2 ③ 4 ⑤

$\boxed{2,4,4}$

$\frac{3!}{2!}$

$= \frac{3!}{2!} \times 2!$

V) 5-digit numbers less than 40,000

_ _ _ _ _

can not be 5 or 4

either 2 or 3

$= \left(2 \times \frac{4!}{2!}\right)$

VI) 5-digit numbers less than 50,000

_ _ _ _ _

2,3,4,4

_ _ _ _ _

2 $\frac{4!}{2!}$

or

_ _ _ _ _

3 $\frac{4!}{2!}$

or

_ _ _ _ _

4 $4!$

Ans:- $\frac{4!}{2!} + \frac{4!}{2!} + 4!$

Or we can also use complementation

$\left(\text{no's less than } 50{,}000\right) = (\text{Total}) - (\text{no's} > 50{,}000)$

$= \left(\frac{5!}{2!} - \left(\frac{4!}{2!}\right)\right)$

Divisibility Model

given digits - 2, 3, 4, 4, 5

I > 5-digits numbers possible

$\frac{5!}{2!}$

II > 5-digit no's divisible by 2 (Even number)

_ _ _ _ _ ↑ 2 or 4

⇓

_ _ _ _ _ ↑ 2 $\frac{4!}{2!}$

or

_ _ _ _ _ ↑ 4 $4!$

$= \frac{4!}{2!} + 4!$

III > 5-digit no's divisible by 3

$2+3+4+4+5 = 18$ divisible by 3.

∴ all possible 5-digit numbers are divisible by 3. $= \frac{5!}{2!}$

III > 5-digit numbers divisible by 4.

_ _ _ _ _ last 2 digit must be divisible by 4.

(3, 4, 5) 2 4 → $3!$

(4, 4, 5) 3 2 → $3!/2!$

(2, 3, 5) 4 4 → $3!$

(3, 4, 4) 5 2 → $3!/2!$

Ans:- $3! + \frac{3!}{2!} + 3! + \frac{3!}{2!} = 2\times 3! + \frac{2\times 3!}{2!}$

IV > 5-digit no's divisible by 5.

_ _ _ _ _ ↑ 5 $\frac{4!}{2!}$

Special Case When Zero is also included.

no of digits given 0, ~~1~~, 2, 3, 4, 5

I > How many 5-digit numbers can be formed

_ _ _ _ _ options $4!$

↑ (2 or 3 or 4 or 5) 4 options

fixed

2 (0, 3, 4, 5)

3 (0, 2, 4, 5)

4 (0, 2, 3, 5)

5 (0, 2, 3, 4)

Ans:- $4 \times 4!$

Or We can also solve this using complementation method

5-digit = total no − (4-digit number)

$5! - 4!$; first place 0, remaining $4!$

$= (5! - 4!)$

$= 5.4! - 4! \Rightarrow 4!(5-1) \Rightarrow 4 \times 4!$

II > 5-digit no's divisible by 2.

We have 3 options and 3 blanks can be filled in 3! ways

0 → 4!

(4,3,5) → $\boxed{}$ _ _ _ 2 → $3 \times 3!$

(2,3,5) $\boxed{}$ _ _ _ 4 → $3 \times 3!$

2 (0,3,5)

3 (0,2,5)

5 (0,2,3)

Ans: $4! + 2 \times 3 \times 3!$

III > 5-digit no's divisible by 5.

0 → 4!

(2,3,4) → $\boxed{}$ _ _ _ 5 → ~~4x3x4~~ $3 \times 3!$

Ans: $(4! + 3 \times 3!)$

IV 5-digit no's divisible by 4.

(2,3,5) 0 4 → 3!

(3,4,5) 2 0 → 3!

(3,5) → $\boxed{}$ _ _ 2 4 → $2 \times 2!$

3 2 → $2 \times 2!$

4 0 → 3!

~~4 4 → 2x2!~~ $4 \times 4 \rightarrow 2 \times 2!$

5 2 → $2 \times 2!$

distinct 5-digit no. — no of are not distinct

Ans: $(3 \times 3! + \cancel{3}\,3 \times 2 \times 2!)$ — $(3 \times 3! + 4 \times 2 \times 2!)$

I RANK of a word (without Repetition)

given a word RAVI

no of words possible − 4!

What is the rank of RAVI among all the possible words?

I) Arrange the letters alphabetically

A I R V

To reach the word RAVI, we have to cross all the words starting with A, and I

Rank, to these words rank is given

A _ _ _ 3! = 6 (1-6)
I _ _ _ 3! = 6 (7-12)
[R]

Whenever we reach a letter we needed fix it

[R] [A] I V - 13
R A V I - 14

Q Find the rank of the word RANK

Sol: RANK

no of words possible → 4!

A, K, N, R ← alphabetically arrange

by reaching RANK we have to cross all the words starting with A, K, N

A _ _ _ 3! = 6 (1-6)
K _ _ _ 3! = 6 (7-12)
N _ _ _ 3! = 6 (13-18)
[R] [A] K N - 19
R A N K - 20

RANK

Find the rank of SOURAV

SOURAV

no of words → 6!

A, O, R, S, U, V ← alphabetically

A _ _ _ _ _ 5! = 120
O _ _ _ _ _ 5! = 120
R _ _ _ _ _ 5! = 120
[S] A _ _ _ _ 4! = 24 SOURAV
(by reaching SO we have to cross all the words starting with A)
[S] [O] A _ _ _ = 3! = 6 SOURAV
[S] [O] R _ _ _ 3! = 6
[S] [O] [U] A _ _ 2! = 2 AORSUV
[S] [O] [U] [R] A V → 1

Rank = 120 + 120 + 120 + 24 + 6 + 6 + 2 + 1 = 399

II *** RANK of word (With Repetition) Imp

given word IGNITE

E, G, I, I, N, T,

no of words possible = $\frac{6!}{2!}$

E, G, I, I, N, T

E _ _ _ _ _ 5! = 120
G _ _ _ _ _ 5! = 120
[I] E _ _ _ _ 4! = 24
[I] [G] E _ _ _ 3! = 6
[I] [G] [I] E _ _ 2! = 2
[I] [G] [I] [N] _ _

E, G, I, I, N, T
E G I I N T
E G I I N T

no repetition bcz one "I" is fixed

I G N I T E

E _ _ _ _ _ 5! = 120
G _ _ _ _ _ 5! = 120
[I] E _ _ _ _ 4! = 24
[I] [G] E

E G I I N T
E G I I N T

IGNIE
IGNII
IGNIET

E _ _ _ _ _ 5! = 120
G _ _ _ _ _ 5! = 120
[I] E _ _ _ _ 4! = 24
[I] [G] E _ _ _ 3! = 6
[I] [G] I _ _ _ 3! = 6
[I] [G] [N] E _ _ 2! = 2
[I] [G] [N] [I] E T ⇒ 1
[I] [G] [N] [I] T E ⇒ 1

E G I I N T

Rank = 120 + 120 + 24 + 6 + 6 + 2 + 1 + 1
= 160

Given Rank find the word.

Ex:- find the word at Rank 14.

A, I, R, V

A _ _ _ 3! = 6 (1-6)
I _ _ _ 3! = 6 (7-12)
R _ _ _ 3! = 6 (13-18)

Hence required word is present in R series.

[R] A _ _ 2! = 2 (13, 14)

R A I V - 13
R A V I - 14 ← req. word.

Ex: find the word at rank 20.

A, I, R, V

A _ _ _ 3! = 6 (1-6)
I _ _ _ 3! = 6 (7-12)
R _ _ _ 3! = 6 (13-18)
V _ _ _ 3! = 6 (19-24)

Hence req word is present in V series.

[V] A _ _ 2! = 2 (19, 20)

VAIR - 19
VARI - 20 ← req word

find the word at rank 23.

A, I, R, V

A _ _ _ 3! = 6
I _ _ _ 3! = 6
R _ _ _ 3! = 6
V _ _ _ 3! = 6 (19-24)

$\boxed{V}$ A _ _ 2! = 2 (19, 20)
V I _ _ 2! = 2 (21, 22)
~~VIAR~~
V R _ _ 2! = 2 (23, 24)
VRAI - 23 ← req word.

Here we have repetition of the given alphabets

find the ~~rank~~ of ~~the~~ word at 160th rank

E, G, I, I, N, T

E _ _ _ _ _ $\frac{5!}{2!}$ = 60
G _ _ _ _ _ $\frac{5!}{2!}$ = 60
I _ _ _ _ _ 5! = 120

Hence req word is present in I series.

$\boxed{I}$ E _ _ _ _ 4! = 24 (144)
$\boxed{I}$ G _ _ _ _ 4! = 24 168

Hence given word is present in IG series

$\boxed{I}\boxed{G}$ E _ _ _ 3! = 6 (150)
I G I _ _ _ 3! = 6 (156)
I G N _ _ _ 3! = 6 (162)

Hence given word is present in IGN series

$\boxed{I}\boxed{G}\boxed{N}$ E _ _ 2! = 2 (157, 158)
I G N I _ _ 2! = 2 (159, 160)
I G N I E T ← 159
I G N I T E ← ~~160~~ 160 ← req word.

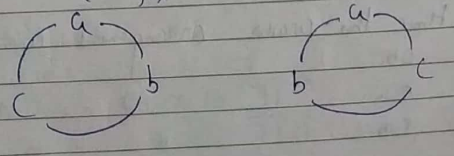
Circular ~~Permutation~~ Arrangement

given n objects, we can arrange them in (n-1)! ways

fix one object and w.r.t to the fixed object, arrange the remaining objects

Ex: abc

fix



Concept

Given n objects, each circular permutation will cover n row arrangements.

Ex: $n=3$ abc

abc
acb
bac
bca
cab
cba

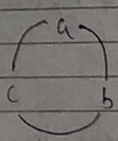
$1\ CP \to n$ arrangements

$n \to 1\ CP$

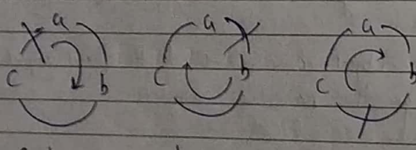
$1 \to \frac{1}{n}\ CP$

$n! \to \frac{n!}{n}\ CP$

$= (n-1)!$

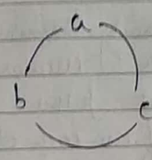


We can cut this ring at 3 different places and try to open it.

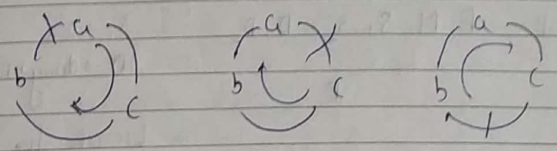


abc bca cab

Hence this circular arrangement covers
abc
bca
cab



This circular arrangement covers
abc
cba
bac



abc cba bac

In case of repetition, follow same concept which we followed in row arrangements

Repetition

Ex: RAVINDRA

Arrange given letters in circular fashion

$n = 8$

$(n-1)! = \frac{7!}{2!\,2!}$

R → 2 repeat
A → 2 repeat

Ex: - CIRCULAR

$n = 8$ $(n-1)! = \frac{7!}{2!\,2!}$

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Ex. Number of ways 6 family members can be arranged in a circular dinning table such that father and mother always face each other.

Sol: F, M, S_1, S_2, D_1, D_2

$S_1 \rightarrow$ Son

$D \rightarrow$ daughter

F

M

We can fix F & M and now to fill remaining 4 places we have 4 objects S_1, S_2, D_1, D_2

$\therefore$ 4! ways

Ex. Number of ways to arranged 4 boys and 4 girls in such a way that no 2 boys and no 2 girls sit together ?

Sol: B_1 B_2 B_3 B_4 G_1 G_2 G_3 G_4

B_1

B_4 B_2

B_3

$\rightarrow$ these places can be filled by the girls.

from 4 boys we will get 3! circular arrangement and each arrangement will give 4! arrangements of Girls

Ans. $3! \times 4!$

If we want to find the complement i.e at least 2 boys and 2 girls should sit together.

Total $-$ (no 2 boys and no 2 girls sit together)

$= 7! - 3!4!$

Ex. RAVINDRA

no of ways these letters can be arranged circularly ?

Sol: $\frac{7!}{2!2!}$

$R \rightarrow 2$ times repeated

$A \rightarrow 2$

ii) all A's should come together

R V I N D R $\boxed{AA}$

$= \frac{6!}{2!}$

iii) no A' should come together

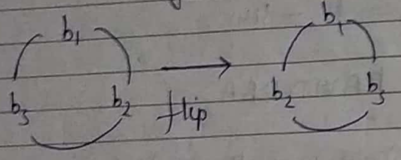
Total $-$ (A's come together)

$= \frac{7!}{2!2!} - \frac{6!}{2!}$

Garlands & Beads Circular Arrangements

If we have 3 beads,

Circular arrangements possible



Here both arrangements are same.

If we have n objects (beads / flowers) no of circular arrangements

$$\frac{(n-1)!}{2}$$

Ex: We have 3 flowers of same type and 2 ~~flowers~~ another flowers which are of same type. In total we have 9 flowers. In how many ways we can form a garland.

Sol: - $9\left(\frac{8!}{3!\,2!}\right)\left(\frac{1}{2}\right)$

circular arrangement

handling flip case

Combinations

^nC_r ie given n objects, number of ways we can select r objects.

Ex: $n=3$ $r=2$ $^nC_2$

$a\ b\ c \Rightarrow \{ab, bc, ac\}$

Ex: We have 10 boys, 8 girls in how many ways we can select 5 students for play / drama

Sol:- $^{18}C_5$ ways

B	G
0	5
1	4
2	3
3	2
4	1
5	0

<- all these cases are covered

ii) We need exactly 3 boys and 2 girls.

Sol:- $^{10}C_3 \times {}^8C_2$

iii) Majority boys

B	G
5	0
4	1
3	2

$= {}^{10}C_5 \times {}^8C_0 + {}^{10}C_4 \times {}^8C_1 + {}^{10}C_3 \times {}^8C_2$

iii) Majority girls

B	G
0	5
1	4
2	3

$= {}^{10}C_0 \times {}^{8}C_5 + {}^{10}C_1 \times {}^{8}C_4 + {}^{10}C_2 \times {}^{8}C_3$

Ex:- We have 22 players, 12 bats, 8 bowlers, 2 wicket keeps.

i) no of ways we can form a cricket team

Sol:- ${}^{22}C_{11}$ ways

ii) no of ways we can form a cricket team such that we need exactly 5 bats, and 5 bow, and 1 wk

Sol:- ${}^{12}C_5 \times {}^{8}C_5 \times {}^{2}C_1$

Ex:- We have to solve Maths question paper consisting of 12 questions. out of which we need to answer 8 questions to pass the exam.

i) no of ways we can attempt the question paper

Sol:- ${}^{12}C_8$ ways

Ex) Question paper is divided into 2 parts, Section A contains 6 questions and Section B contains 6 question.

i) We need to answer at least 3 questions from Section A

A	B
3	5
4	4
5	3
6	2

$= {}^{6}C_3 \times {}^{6}C_5 + {}^{6}C_4 \times {}^{6}C_4 + {}^{6}C_5 \times {}^{6}C_3 + {}^{6}C_6 \times {}^{6}C_2$

ii) We need to answer at most 3 questions from Section A

Sol:-

A	B
0	8 <- not possible
1	7 <- not possible
2	6
3	5

$= {}^{6}C_2 \times {}^{6}C_6 + {}^{6}C_3 \times {}^{6}C_5$

~~Imp~~

EX:- We have 10 couples.

10-Man 10-Woman

In how many ways we can form a team of 4 members

i> consisting of 2 M, 2 W

Sol:- ~~$^{20}C_4$~~ $^{10}C_2 \times {}^{10}C_2$ ways

ii> no couple should be present in team of 2M, 2W

Sol:- $^{10}C_2 \times {}^{8}C_2$

Concept - select 2 M, remove their corresponding ~~wife~~ wives. and select 2 W from the remaining

iii> Exactly one ~~coupl~~ couple should be present.

$H_1\ W_1$
$H_2\ W_2$
$H_3\ W_3$
⋮
$H_{10}\ W_{10}$

Sol:- $^{10}C_1 \times {}^{9}C_1 \times {}^{8}C_1$

↓ remove this selected man's wife.

iv> H_1 and W_5 should always be present

Sol:- $^{9}C_1 \times {}^{9}C_1$

***

v> If H_1 is selected then W_5 should be selected.

Sol: I case → H_1 is selected

$^{9}C_1 \times {}^{9}C_1$

II case:- H_2 is not selected. W_5 may or may not be present.

$^{9}C_2 \times {}^{10}C_2$

Sol:- ⇒ $^{9}C_1\, {}^{9}C_1 + {}^{9}C_2\, {}^{10}C_2$

vi> W_2, W_3 should not be selected simultaneously

Case I	Case II	Case - III	Case IV
W_2 W_3	W_2 W_3	W_2 W_3	W_2 W_3
✓ ✓	✓ ✗	✗ ✓	✗ ✗
$^{10}C_8$	$^{10}C_2 \times {}^{8}C_1$ ↓ W_3 should not be present	$^{10}C_2 \times {}^{8}C_1$ ↓ W_2 should not be present	$^{10}C_2 \times {}^{8}C_2$

Ans:- $(\underline{II} + \underline{III} + \underline{IV})$ $^{10}C_2\, {}^{8}C_1 + {}^{10}C_2\, {}^{8}C_1 + {}^{10}C_2\, {}^{8}C_2$

OR

Complementation method

~~$^{20}C_4 - {}^{10}C_2$~~ Total − I ⇒ $^{10}C_2 \times {}^{10}C_2 - {}^{10}C_8$

Imp Ex: We have 10 couples

10 − M   10 − W

i) How many ways Handshake can be performed.

Sol:- To perform handshake we need 2 people.

$^{20}C_2$ ways

ii) no handshake b/w the ~~oo~~ couples.
ie Husband & Wife should not perform handshake b/w them

Sol: $^{20}C_2 - 10$
→ no of handshake b/w couples (∵ we have 10 couples)

iii) Numbers are assigned to every couple ~~both husband and wife~~. How many handshake possible such that there is no handshake b/w prime numbered person.

$H_1\ H_2\ H_3 \ldots H_{10}$
$W_1\ W_2\ W_3 \quad W_{10}$

prime numbered $H_2\ H_3\ H_5\ H_7$
$W_2\ W_3\ W_5\ W_7$

$^{20}C_2 - {^8C_2}$
→ no of handshakes b/w prime numbered people

iv) number of handshakes such that, there ~~is~~ is no handshake b/w H_1 and W_5

Sol: $^{20}C_2 - 1$
→ handshake b/w H_1 & W_5

v) number of handshakes such that H_1 never handshakes with any one

Sol:- $^{19}C_2$ or $^{20}C_2 - 19$
→ H_1's handshake with 19 other people.

Q How Repetition concept is derived from combination?

Given n letters out of which r are repeating how many words we can form

Firstly we will select r places out ~~p~~ of n places available, and we will put ~~r~~ r repeating letters at those places.

∴ We can select r places in ^nC_r ways

now we take each possible arrangement and in each arrangement we will put the remaining $n-r$ objects.
$n-r$ objects can be placed in $(n-r)!$ ways

$${}^nC_r \times (n-r)! \Rightarrow \frac{n!}{(n-r)!\,r!} \times (n-r)! = \frac{n!}{r!}$$

Order

Given n objects out of which r object should occur in some specific order.

Concept → We need to pick r places out of available n places, and place the r objects in the specified order.

For every placement of r objects in specified objects remaining $n-r$ objects can be arranged in $(n-r)!$ ways.

Thus every combination of arrangement of r objects in specified order will generate $(n-r)!$ arrangements of remaining objects.

$\Rightarrow {}^nC_r\,(n-r)!$

Ex: Given 3 objects a, b, c how many arrangements are possible such that "a" always ~~come~~ comes b4 "b"

a b c ✓
a c b ✓
b c a
b a c
c a b ✓
c b a

Ex:- Given word PERMUTATION, how many words can be formed such all vowels should appear as (A, E, I, O, U) in this specific order.

Sol:- P.E to place 5 letter we need to choose 5 places out of available 11 places and to them we try to fix these 5 letters.

${}^{11}C_5$

remaining 6 letter can be arranged in $6!$ way for ever combination of 5 letters.

$$= \left({}^{11}C_5 \; \frac{6!}{2!}\right) \qquad T \to 2 \text{ repetition}$$

Imp

Ex: Given word RAVINDRA

i) Number of words possible such that "R" always appears b4 "A".

Sol: $^6C_2 \times (4!)$

Ex: RVINYAD
VIRNDA

ii) 3 letters ; R, A and V should appear in these order $R \to A \to V$
i.e "R" should always appear b4 "A" and "A" should always appear b4 "V"

Sol: $^6C_3 \times 3!$

iii) How many words are possible such that "R" should always appear b4 "A" and "V"

$R \to A$ and $R \to V$

Sol. Case I | Case II

$R \to A \to V$ | $R \to V \to A$

$^6C_3\,3!$ + $^6C_3\,3!$

Ans. $^6C_3\,3! + {}^6C_3\,3!$

Imp

iv) How many words are possible such that R should appear b4 "A" or b4 "V".

$R \to "A"$ or $R \to "V"$

Sol: We can use complementation to solve this

Total − ($R \to A$ and $R \to V$)

$= 6! - ({}^6C_3\,3! + {}^6C_3\,3!)$

Combination with repetition

Ex: given 4 alphabets a a b c . how many way can we select 2 letters

a a b c

Case I
both letters are same
↓
1 way
i.e here we have only 1 option to select a, a

both letters are different
↓
$^3C_2$
here we have 3 option a, b, c . although a is repeated, but we consider it only once while counting distinct alphabets.

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$^3C_2 + 1 = 3+1 = 4$ ways

Ex:- given RAVINDRA

i> How many ways we can select 2 letters

Case I
2 letters are same
↓
1
ie RR

Case II
2 letters are diff.
$^6C_2$
we have 6 distinct letters.

Ans: $1 + {}^6C_2$

ii> How many ways we can select 3 letters.

Case I
All 3 letters diff → $^6C_3$

Case II
2 letters same + 1 diff → $1 + {}^5C_1$

Case III
all 3 letters same → 0

Ans. $1 + {}^5C_1 + {}^6C_3$

Imp

Ex:- Given RAVINDRA BABU

R − 2 V − 1 U − 1
A − 3 I − 1 D − 1
B − 2 N − 1

i> In how many ways we can select 4 letters.

Case I

All are different − $^8C_4$

Case II

2 same + 2 diff → $^3C_1 \times {}^7C_2$

here we should not consider those letters which have been selected as same letters (RR, AA, BB)

If RR is selected then options we have are A, B, V, I, N, U, D

Case III

2 same + 2 same → $^3C_2$

Here we have to select 2 letters such that both are same, and we have 3 options RR, AA or BB.

Case IV

3 same + 1 diff → $1 \times {}^7C_1$

Case V

all 4 are same = 0

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Ans. $^8C_4 + {^3C_1} \times {^7C_1} + {^3C_2} + 1 \times {^7C_1}$

Imp Ex:- ASSASSINATION

S → 4
A - 3
N - 2
I - 2
T - 1
O - 1

How many way we can select 4 letters

Case I all letters are distinct - $^6C_4$

Case II 2 same + 2 diff. - $^4C_1 \times {^5C_2}$

no of ways to select a letter such that it is repeating

Case III 2 same + 2 same - $^4C_2$

no of ways to select 2 letters such that each is repeating.

Case IV → 3 same + 1 diff → $^2C_1 \times {^5C_1}$ (SSS, AAA)

Case V → 4 same → 1

Ans:- $^6C_1 + {^4C_1} \times {^5C_2} + {^4C_2} + {^2C_1} \times {^5C_1}$

Permutation and Combination Combined

Ex: COMPUTER ← given word.

How many 3 letter words can be formed

Sol: COMPUTER

Here we have to do both selection and arrangement.

First we select 3 letters from 8 letters, and after selecting ~~them~~ we need to arrange them

$= {^8C_3}\, 3!$

Ex:- given word RAVINDRABABU. How many 3 letter words can be formed

Sol: I case - all are different $^8C_3\, 3!$

R - 2 V - 1 D - 1
A - 3 I - 1 U - 1
B - 2 N - 1

II case - 2 same, 1 different. $^3C_1 \times {^7C_1}\, \frac{3!}{2!}$

2 letters are repeated

iii) all 3 same

$1 \times \frac{3!}{3!} = 1$

Ans:- ${}^{8}C_3\, 3! + {}^{3}C_1\, {}^{7}C_1 \frac{3!}{2!} + 1$

Ex: given a word ~~ASSAHA~~ ASSASSINATION how many 4 letter words can be formed.

ASSASSINATION

S - 4   T - 1
A - 3   O - 1
I - 2
N - 2

i) all diff. ${}^{6}C_4\, 4!$

ii) 2 same, 2 diff $\quad {}^{4}C_1 \times {}^{5}C_2 \; \frac{4!}{2!}$

iii) 2 same, 2 same $\quad {}^{4}C_2 \; \frac{4!}{2!\,2!}$

iv) 3 same, 1 diff $\quad {}^{2}C_1\, {}^{5}C_1 \; \frac{4!}{3!}$

v) 4 same $\rightarrow 1 \times \frac{4!}{4!} = 1$

Q Given 5 flags of different colours, how many different signals can be generated if each signal requires the use of 2 flags, one below the other?

Sol: F_1 $\boxed{}$ $\rightarrow$ 5 options $\quad 5 \times 4 = 20$
F_2 $\boxed{}$ $\rightarrow$ 4 options

Q In how many ways can 5 girls and 3 boys be seated in a row so that no two boys are together?

Sol: 5 - Girls   3 - boys

_ G _ G _ G _ G _ G _

boys can be seated at any of these places.

for girls $\rightarrow 5! = 120$

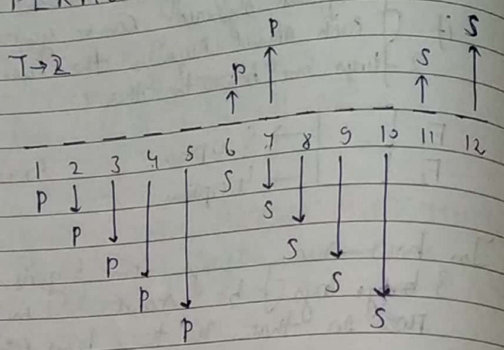
for boys $\rightarrow {}^{6}C_3\, 3! = 120$

$\therefore$ total no of ways $\rightarrow 120 \times 120 = 1440$

Q In how many ways can the letters of the word PERMUTATIONS be arranged if there are always 4 letters b/w P and S.

PERMUTATIONS

T→2



∵ positions of P and S are fixed.

∴ remaining letters can be arranged in

$\left(\frac{10!}{2!}\right)$

I case P is ahead of S

$7 \times \left(\frac{10!}{2!}\right)$

II case ~~P~~ is S is ahead of P.

$7 \times \left(\frac{10!}{2!}\right)$

Ans:- $\frac{7 \times 10!}{2!} + \frac{7 \times 10!}{2!}$

((Combinations)

Type I

Number of ways of n different things selecting at least one of them is

$\cancel{^nC_0}\ ^nC_1 + {^nC_2} + {^nC_3} + \dots + {^nC_n} = 2^n - 1$

Ex. Mohan has 8 friends. in how many ways he can invite one or more of them to dinner?

Sol:- $^8C_1 + {^8C_2} + {^8C_3} + {^8C_4} + {^8C_5} + {^8C_6} + {^8C_7} + {^8C_8} = 2^8 - 1 = 255$

Type II

$(p+q+r+t) \rightarrow$ things

$p \rightarrow$ are alike one kind
$q \rightarrow$ are alike ~~one kind~~ second kind.
$r \rightarrow$ are alike of third kind.
$t \rightarrow$ are different.

∴ total number of combinations

$(p+1)(q+1)(r+1)\,2^t - 1$

Ex: Find the number of combinations that can be formed with 5 oranges, 4 mangoes, and 3 bananas.

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ii) It is essential to take at least one fruit

Sol: Mangoes = 4 bananas = 3
Oranges = 5

$(5+1)(4+1)(3+1)2^0 - 1$
$= 6 \cdot 5 \cdot 4 \cdot 1 - 1$
$= 119$

iii) One fruit of each kind is essential to be take

$^5C_1 \times {}^4C_1 \times {}^3C_1$

Imp (Grate) Type - III

Let $N = p_1^{\alpha_1} p_2^{\alpha_2} p_3^{\alpha_3} \ldots p_k^{\alpha_k}$

$p_1, p_2, p_3 \ldots p_k$ are different primes
$\alpha_1, \alpha_2, \ldots \alpha_k$ are natural numbers

i) Total number of divisors of N

a) Including 1 and N

$(\alpha_1+1)(\alpha_2+1) \ldots (\alpha_k+1)$

b) Excluding 1 and N
$(\alpha_1+1)(\alpha_2+1) \ldots (\alpha_k+1) - 2$

c) excluding either 1 or N

$(\alpha_1+1)(\alpha_2+1) \ldots (\alpha_k+1) - 1$

ii) Sum of divisors.

$(p_1^0 + p_1^1 + p_1^2 + \ldots + p_1^{\alpha_1})(p_2^0 + p_2^1 + p_2^3 + \ldots + p_2^{\alpha_2}) \ldots (p_k^0 + p_k^1 + p_k^2 + \ldots + p_k^{\alpha_k})$

iii) Number of ways in which N can be resolved as a product of 2 factors

a) If N is perfect square

$\frac{1}{2}[(\alpha_1+1)(\alpha_2+1) \ldots (\alpha_k+1) + 1]$

b) If N is not perfect square

$\frac{1}{2}[(\alpha_1+1)(\alpha_2+1) \ldots (\alpha_k+1)]$

iv) Number of ways in which a composite number N can be resolved into two factors which are relatively prime to each other is equal to 2^{n-1} where n is number of different factors of N.

Ex. Find number of factor of 38808 (including 1 and N). Also find sum of divisors

Sol. $38808 = 2^3 \times 3^2 \times 7^1 \times 11^1$

$\therefore$ number of factors (including 1 and N)

$= (3+1)(2+1)(1+1)(1+1)$
$= (4)(3)(2)(2)$
$= 48$

Sum of divisors

$(2^0+2^1+2^2+2^3)(3^0+3^1+3^2)(7^0+7^1)(11^0+11^1)$
$= (15)(13)(8)(12)$

Q How many ways 10800 can be resolved as a product of 2 factors

Sol:- $N = 10800 = 2^4 \times 3^3 \times 5^2$

~~As~~ N is not perfect square

$\frac{1}{2}[(4+1)(3+1)(2+1)]$

$= \frac{1}{2}[5.4.3]$

$= 30$

Ex: How many ways 18900 can be split into two factors which are relative prime.

Sol: $18900 = 2^2 \times 3^3 \times 5^2 \times 7^1$

$2^{n-1} \Rightarrow 2^{4-1} \Rightarrow 2^3 \Rightarrow 8$

Imp

Division into groups

I> number of ways in which $(m+n)$ different things can be divided into two groups which contain m & n things respectively

$(m+n)$ diff things → m, n

$^{m+n}C_m \; ^nC_n = \frac{(m+n)!}{m!\,n!}$

If $m=n$

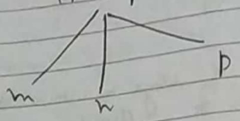
i> Order of group not Imp

$^{2n}C_n \; ^nC_n = \frac{(2n)!}{2!\,(n!)^2}$

ii> order of group is imp

$^{2n}C_n \; ^nC_n \times 2! = \frac{(2n)!}{2!\,(n!)^2} \times 2! = \frac{(2n)!}{(n!)^2}$

$(m+n+p)$ - diff things



$^{m+n+p}C_m \; ^{n+p}C_n \; ^pC_p = \frac{(m+n+p)!}{m!\,n!\,p!}$

If $m=n=p$

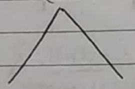
i) Order group not imp

$^{3p}C_p \; ^{2p}C_p \; ^pC_p = \frac{(3p)!}{3!\,(p!)^3}$

ii) Order group imp

$^{3p}C_p \; ^{2p}C_p \; ^pC_p \times 3! = \frac{(3p)!}{3!\,(p!)^3} \times 3! = \frac{(3p)!}{(p!)^3}$

(mn) - diff things



divide equally into m groups

$\frac{mn!}{(n!)^m m!}$

divide equally into m distinct groups

$\frac{(mn)!}{(n!)^m}$

Ex: In how many ways can a pack of 52 cards be equally among 4 players in order?

Sol:- $^{15}C.$ $^{52}C_{13}$ $^{39}C_{13}$ $^{26}C_{13}$ $^{13}C_{13}$

Ex. In how many ways can a pack of 52 cards be formed into 4 groups of 13 cards each?

Sol:- $\frac{^{52}C_{13} \times ^{39}C_{13} \times ^{26}C_{13} \times ^{13}C_{13}}{4!}$ (order not imp)

Ex: In how many ways can a pack of 52 cards be divided in 4 sets, three of them having 17 cards each and fourth just 1 card.

Sol:- $\frac{^{52}C_{17} \times ^{35}C_{17} \times ^{18}C_{17} \times ^{1}C_{1}}{3!}$ (order not imp)

II Arrangements in groups

n diff things can be arrange in r different places in

$^{n+r-1}P_n$ or $n!\;^{n-1}C_{r-1}$

blank group admissible

blank group not admissible

Ex. In how many ways 5 diff balls can be arranged into 3 diff boxes so that no box remains empty

Sol: $n=5$ $r=3$ $n! \, {}^{n-1}C_{r-1}$

$5! \, {}^{4}C_{2}$

blank group not admissible

III no of ways n diff things distributed into r diff groups *(Here blank group not allowed)

$r^n - {}^rC_1 (r-1)^n + {}^rC_2 (r-2)^n - {}^rC_3 (r-3)^n + \ldots (-1)^{r-1} \, {}^rC_{r-1}$

Ex. In how many ways 5 diff balls can be distributed into 3 diff boxes so that no box remains empty?

Sol: $n=5$ $r=3$

$(3)^5 - {}^3C_1 (3-1)^5 + {}^3C_2 (3-2)^5$

$= (3)^5 - 3(2)^5 + 3(1)^5$

$= 243 - 96 + 3$

$= 147 + 3$

$= 50$

(Great)

IV The no of ways n identical things can be distributed into r diff groups in

${}^{n+r-1}C_{r-1}$ or ${}^{n-1}C_{r-1}$

blank group admissible

blank group not admissible

Ex. In how many ways 5 identical balls can be distributed into 3 different boxes so that no box remains empty?

Sol:- ${}^{n-1}C_{r-1} \Rightarrow {}^{5-1}C_{3-1} \Rightarrow {}^4C_2 = 6$

Ex. $x + y + z + w = 20$

i) Find non-negative solutions

ii) Find positive solutions.

Sol: i) $x + y + z + w = 20$ $x \geq 0, y \geq 0, z \geq 0, w \geq 0$

$n = 20$ $r = 4$ $\Rightarrow {}^{n+r-1}C_{r-1}$ ${}^{20+4-1}C_{4-1}$

$= {}^{23}C_3$

Sol: ii) $x + y + z + w = 20$ $x > 0, y > 0, z > 0, w > 0$

${}^{20-1}C_{4-1} \Rightarrow {}^{19}C_3$

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Dearrangements.

n things arranged in a row, number of ways they can be dearranged so that no one of them occupy their original place

$$n!\left(1-\frac{1}{1!}+\frac{1}{2!}-\frac{1}{3!}+\dots(-1)^n\frac{1}{n!}\right)$$

Ex. A person writes a letters, to six friends and addresses the corresponding envelopes. In how many ways can letters be placed in the envelopes so that

i) all the letters are in wrong places.

ii) at least 2 of them are in wrong envelopes

Sol: i) $6!\left(1-\frac{1}{1!}+\frac{1}{2!}-\frac{1}{3!}+\frac{1}{4!}-\frac{1}{5!}+\frac{1}{6!}\right)$

~~Sol: ii) either 0 wrong place or 1 wrong place~~

Sol: ii) at least 2 in wrong place $= 6! - ($ all goes to right place + 5 goes to right place$)$

no. of ways to put all 6 letters $= 6!$

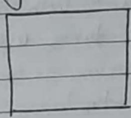
or

$6! - ($ 0 goes to wrong place + 1 letter goes to wrong place$)$

$= 6! - (1 + 0)$

$= 719$

Number of rectangles and Squares

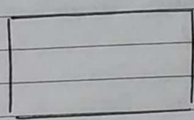
i) 

given square

$n \times n$

no of rectangles $= \sum_{r=1}^{n} r^3$

no of Square $= \sum_{r=1}^{n} r^2$

ii) 

given rectangle

$n \times p$

$n < p$

no of rectangles $= \frac{np}{4}(n+1)(p+1)$

no of squares $= \sum_{r=1}^{n}(n+1-r)(p+1-r)$

Important Results

i) n straight lines are drawn in the plane such that no two lines are parallel and no three are concurrent. Then the number of parts into which these lines ~~divided~~ divide the plane is equal to $1+\sum n$

ii) The sum of digits in the unit place of all numbers formed with the help of $a_1, a_2, \ldots a_n$ taken all at a time is

$(n-1)!\,(a_1+a_2+\ldots+a_n)$ (repetition digit not allowed)

Ex Find the sum of the digits in the unit place of all numbers formed with 3, 4, 5, 6 taken all at a time.

Sol:- $(4-1)!\,(3+4+5+6)$

$= 3!\,(18)$

$= 6 \times 18$

$= 108$

$\underline{\quad}\ \underline{\quad}\ \underline{\quad}\ \underline{3}$ — $3!$

$\underline{\quad}\ \underline{\quad}\ \underline{\quad}\ \underline{4}$ — $3!$

$\underline{\quad}\ \underline{\quad}\ \underline{\quad}\ \underline{5}$ — $3!$

$\underline{\quad}\ \underline{\quad}\ \underline{\quad}\ \underline{6}$ — $3!$

$3!3 + 3!4 + 3! \times 5 + 3!6$

$3!\,(3+4+5+6)$

iii) The sum of all digit numbers that can be formed using the digits $a_1, a_2, a_3 \ldots a_n$ (repetition of digits not allowed) is

$= (n-1)!\,(a_1+a_2+\ldots+a_n)\,\frac{(10^n-1)}{9}$

Ex: Find the sum of all 5-digit numbers that can be formed using the digits 1, 2, 3, 4 and 5 (repetition of digits not allowed)

Sol: $(5-1)!\,(1+2+3+4+5)\,\frac{(10^5-1)}{9}$

$= (4)!\,(15)\,\frac{(10^5-1)}{9}$

Exponent of Prime p in $n!$

$$E_p(n!) = \left[\frac{n}{p}\right] + \left[\frac{n}{p^2}\right] + \left[\frac{n}{p^3}\right] + \ldots \left[\frac{n}{p^s}\right]$$

Where s is the largest natural number such that $p^s \le n < p^{s+1}$

Ex:- Find number of zero's at the end of 100!

Sol:- number of 0's will depend on the exponent of 2 and 5. pairs of 2 and 5 will generate 0 $2 \times 5 = 10$

$E_2(100!) = \frac{100}{2} + \frac{100}{4} + \frac{100}{8} + \frac{100}{16} + \frac{100}{32} + \frac{100}{64}$

$= 50 + 25 + 12 + 6 + 3 + 1$

$= 97$

$E_5(100!) = \frac{100}{5} + \frac{100}{25}$

$= 20 + 4$

$= 24$

$\therefore (2)^{97}(5)^{24}$ pairs of $(2,5) = \left((2)^{24}(5)^{24}\right)(2)^{73}$

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∴ 24 pairs are there, hence we have 24 zero's.

Ex:- Find the prime factorization of 15!

Sol:- prime factors in 15! are 2, 3, 5, 7, 11, 13

we have to find exponent of each prime number.

$E_2(15!) = \frac{15}{2} + \frac{15}{4} + \frac{15}{8} \Rightarrow 7+3+1 \Rightarrow 11$

$E_3(15!) = \frac{15}{3} + \frac{15}{9} \Rightarrow 5+1 \Rightarrow 6$

$E_5(15!) = \frac{15}{5} \Rightarrow 3$

$E_7(15!) = \frac{15}{7} \Rightarrow 2$

$E_{11}(15!) = \frac{15}{11} \Rightarrow 1$

$E_{13}(15!) = \frac{15}{13} = 1$

$\therefore 15! = (2)^{11} \cdot (3)^6 \cdot (5)^3 \cdot (7)^2 \cdot (11)^1 \cdot (13)^1$.

* Combination with Repetition

From a set of n distinct elements, when repetition of r elements is allowed there are each

$^{n+r-1}C_r$ combinations possible

or

$^{n+r-1}C_{n-1}$

Ex:- How many ways are there to select 5 bills from a cash box containing \$1 bills, \$2 bills, \$5 bills, \$20 bills, \$50 bills, and \$100 bills? Assume that ~~the~~ the order in which bills are chosen does not matter, that the bills of each denomination are indistinguishable, and that there are at least five bills of each type.

Sol:- $n = 6$ $r = 5$ $^{n+r-1}C_r$

$\Rightarrow {}^{10}C_5$ $\frac{10 \times 9 \times 8 \times 7 \times 6}{5 \times 4 \times 3 \times 2 \times 1}$ = 252 ways

Percentages

$x\%$ of y = $y\%$ of x

$x\%$ of $y = \frac{x}{100} \times y \Rightarrow \frac{x \times y}{100} \Rightarrow y\%$ of x

i) 30% of 600 = ?

100% — 600

30% — $\frac{30}{100} \times 600$ 180

ii) 40% of ? = 160

40% ⟶ 160

100% ⟶ $\frac{100}{40} \times 160$ = 400

iii) ?% of 800 = 320

100% ⟶ 800

800 — 100%

320 — $\frac{320}{800} \times 100$ = 40

Q Santosh spends 20% of his monthly salary on house rent, 30% on food, 10% on transportation, 20% on household expenses, and 15% on medication. He saves the remaining amount of ₹ 6,750. What is his monthly salary ?

Sol. 20% + 30% + 20% + 15% = 95% spent

5% saves

5% of TA ⟶ 6750

5% — 6750

100% — $\frac{100}{5} \times 6750$ 1,35,000

Q Aman gave 40% of the amount he had to Rohan. Rohan in turn gave one-fourth of what he received from Aman to Sahil. After paying ₹200 to the taxi driver out of the amount he got from Rohan Sahil now has ₹ 600 left with him. How much amount did Aman have initially ?

Sol. Say Aman had ₹ 100

Rohan — 40 $\xrightarrow{1/4}$ 10

↑

Sahil

10 − 200 = 600

10 = 800

1 = 80

∴ 100 = 8000

∴ Aman initially had ₹8000

Percentage of percentage

$a\%$ of $b\%$ of $k = \left(\frac{ab}{100}\right)\%$ of k

Q Prashant decided to ~~donate~~ donate 10% of his salary to a charity. On the day of donation, he changed his mind ~~and~~ and donated ₹ 2100 which was 70% of what he decided earlier. How much is his salary?

Sol. Let salary be ₹100

10% ⟶ ₹10

70% of 10 — ₹7

7 ≡ 2100

1 ≡ 300

100 ≡ 100 × 300 = 30,000

∴ salary is ₹ 30,000

Q In an election, b/w two candidates, candidate A gets 76% of the votes, wins by a majority of 312 vote votes. What is the total number of votes polled?

Sol:- Let total votes polled be 100

A gets 76 votes. B gets 24 votes

A − B = 312

76 − 24 ≡ 312

52 ≡ 312

∴ 1 ≡ $\frac{312}{52}$ 6

∴ 100 ≡ 6 × 100 = 600 votes.

Q In a college elections b/w two candidates, one candidate got 55% of total valid votes. 10% of votes were found to be invalid. If the total number of votes were 8000. What is the number of valid votes the other candidate got?

Sol:- Let total no of votes be 100

∴ 100 ≡ 8000

1 ≡ 80

(90) Valid   invalid (10)

55% 45%

A B

$\frac{45}{100} \times 90 \times 80 \Rightarrow 72 \times 45 = 3240$

Q In a college election b/w two students, 10% of the votes cast were invalid. The winner gets 60% of the valid votes and defeats the loser by 1800 votes. How many votes were totally cast?

Sol:- Let total votes be 100

Valid (90)   invalid (10)

(54) W   L (36)

54 − 36 ≡ 1800 ⇒ 18 ≡ 1800 ⇒ 1 ≡ 100

∴ 100 ≡ 100 × 100 = 10,000 votes

x is what % of y = $\frac{x}{y} \times 100$

What % of y is x = $\frac{x}{y} \times 100$

What % ~~of~~ is x of y = $\frac{x}{y} \times 100$

If x is 90% of y, then what % of x is y?

$\frac{y}{x} \times 100 = 90$

$\frac{y}{x} = \frac{9}{10}$

$\frac{y}{x} \times 100 = \frac{1000}{9} = 111.11$

Q In an exam, Arjun scored 20% less than Balu and Chandra scored 30% less than Balu. Chandra's score is what % of Balu's score?

Sol: Let Balu score be 100
Arjun → 80
Chandra → 70

$\frac{70}{80} \times 100 = \frac{700}{8} = 87.5\%$

x is what % more/less than y (compared to y) $= \left(\frac{x-y}{y}\right) \times 100$

Q If A's salary is 25% more than B's salary, then by what percent is B's salary less than A's salary?

Sol: Let B' salary be 100
A → 125

$\frac{A-B}{A} \times 100 \Rightarrow \frac{125-100}{125} \times 100 = 20\%$

Q If A's salary is 25% less than B's salary, then by what % B's salary more than A's salary?

Sol: B's salary → 100
A's → 75

$\frac{100-75}{75} \times 100 = \frac{25}{75} \times 100 = 33.33\%$

Q Two students appeared for an examination, one of them scored 9 marks more than other and his marks was 56% of the sum of their marks. The marks obtained by them are:-

Let marks of one student be x
other student → $x+9$

$x+9 = 56\%$ of $(x+x+9)$

$x+9 = \frac{56}{100}(2x+9)$

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$100x + 900 = 112x + 504$

$396 = 12x$

$33 = x$

Q The difference of two numbers is 1800. If 5% of one number is 8% of the other number. find the number

Sol: Let one no. be x

other no = $x - 1800$

5% of x = 8% of $(x-1800)$

$\frac{5}{100}x = \frac{8}{100} \times (x-1800)$

$5x = 8x - 8 \times 1800$

$\frac{8 \times 1800}{3} = x$

$4800 = x$

Other no = 3000

☆ Effective percentage change = $\left(\frac{\text{final value} - \text{initial value}}{\text{initial value}}\right) \times 100$

Q The population of a city increased from 1200000 to 1500000 in a year. The percentage increase of population in that year is:

Sol:- % ↑ in population = $\frac{1500000 - 1200000}{1200000} \times 100$

$= \frac{300000}{1200000} \times 100$

$= 25\%$

Q The profit made by a company in the present year is ₹ 1800000. Two years ago, the profit made by the company was 2400000. What is the percentage change in the profit made by the company?

Sol:- % Change in profit = $\frac{2400000 - 1800000}{2400000} \times 100$

$= \frac{600000}{2400000} \times 100$

$= 25\%$ (decrease)

Q The revenue of a shop in the month of April was ₹ 40,000. The shopkeeper announced a discount of 30% in the month of May and hence the sales went up by 20%. what will be the revenue in the month of may?

Sol: revenue = no of sales × cost of each sale

Let initial sales be 100
Cost of each ~~sales of~~ sale be ₹100

$\therefore 40{,}000 = 100 \times 100$
$\cancel{4=1}$ $1 = 4$

Now in May, cost of each sale = 70
no of sales = 120

$\therefore$ revenue = $120 \times 70 = 8400 \times 1 = 8400 \times 4$
$= 33200$

$\therefore$ revenue is 33,200

Concept: $X = A \cdot B$
$\uparrow a\% \downarrow \quad \uparrow b\%$

$$X = \left(A + \frac{aA}{100}\right)\left(B + \frac{bB}{100}\right)$$

$$\boxed{X = AB + \left(a + b + \frac{ab}{100}\right) AB}$$

It is applicable for ↑ or ↓ in % as we appropriately place the value of "a" and "b".

Q If the length of a rectangle is decreased by 20% and the breadth of is increased by 40%, find the percentage change in the area of rectangle.

Sol:- $a = l \times b$

Let l be 100
"b" be 100

$\therefore$ initially $a = 10000$

$l = 80 \qquad b = 140$

$a_1 = 80 \times 140$
$a_1 = 11200$

$\therefore$ % change $= \left(\frac{11200 - 10000}{10000}\right) \times 100$

$= \frac{1200}{10000} \times 100$

$= 12\%$

Q The price of onions is increased by 30%, what should be the % decrease in the consumption of onions by a family such that their expenditure of onions remains the same?

Sol:- Expenditure = no of onions consumed × price of each onion

Let no of onions consumed "n" = 100

price of each onion "p" = 100

$\therefore E = 10000$

Now, $p_1 = 130$ $n_1 = ?$ $E = 10000$

$10000 = n_1 \times 130$

$n_1 = \frac{10000}{130}$

$n_1 = 76.92$

$\therefore$ % ↓ in the consumption $= \frac{100 - 76.92}{100} \times 100$

$= 23.08\ \%$

Averages

Q The avg. weight of a cricket team with 11 players is 60 kg. If the average weight increases by 0.5kg when a player leaves the team, what is the player's weight?

Sol. $\frac{a_1 + a_2 + a_3 + \dots a_{10} + a_{11}}{11} = 60$

$\frac{\frac{(a_1 + a_2 + a_3 + \dots + a_{10})}{10} \times 10 + a_{11}}{11} = 60$

$\frac{(60.5) \times 10 + a_{11}}{11} = 60$

$a_{11} = 55$

When player left, he increased the avg by 0.5, hence 0.5 must ~~be add~~ have been added to all the 10 player i.e a total of 5kg. Hence when player left he increased 5kg ~~for~~ of in the total weight. Hence player's weight must have been 60 − 5 = 55kg. (He must have borrowed 5kg from other's and ~~an~~ g. is maintained an avg weight of 60kg)

Q The avg weight of 8 persons increased by 3 kg when a person with 70 kg is replaced by a new person. What might be the weight of new person?

Sol:- The new person added an extra weight of 24 kg when replaced with 70 kg person. Hence this 24 kg came from new person. ∴ new person's weight is 70 + 24 = 94 kg.

$$\frac{(a_1+a_2+\dots+a_7+70\,kg)}{8} = x$$

$$\frac{a_1+a_2+\dots+a_7+a_8}{8} = x+3$$

$$\frac{(8x-70)+a_8}{8} = x+3$$

$$a_8 = 94\,kg$$

Q The average age of 12 persons in committee is increased by 1 year when two men aged 35 and 43 are replaced by two women. Find the average age of two women.

Sol:- 2 new women added 12 more ~~years~~ years extra increasing the avg, Hence they created a difference of ∴ 12 years

∴ avg ~~weight~~ age of 2 women $= \frac{(35+43)+12}{2} = \frac{78+12}{2} = 45$

Q Avg of 10 numbers is calculated as 30. It is discovered later that one number was wrongly entered as 36 instead of 63. What is the actual avg.?

Sol:- When wrong value is ~~↑ by actual~~ replaced by the actual value 63 (i.e. more than ~~cu~~ current avg.), it must have ↑ avg by ((63−36) = 27). This 27 must have been distributed over 10 number, hence each got 2.7. ∴ avg got increased by 2.7, hence actual avg is 32.7.

Q ☆ If the avg of 5 consecutive even numbers is 44. Find the highest number.

Sol:- if consecutive numbers are given, and they are in A.P. then ~~the~~ avg value is the middle number (in case of odd number of numbers), in case of even number of numbers, take ~~avg~~ middle value as avg of middle two numbers.

~~a + a₂ +~~ given 5 consecutive even numbers.

$a, a+2, a+4, a+6, a+8$

$avg = 44 = a+4$

$a = 40$

∴ highest number $= a+8 = 48$.

Q The avg of 5 consecutive odd numbers is 59. What will be the sum of next set of 5 consecutive odd numbers?

Sol. $a, a+2, a+4, a+6, a+8$ <- 5 consecutive odd numbers
10 | 10 | 10 | 10 | 10 |
$a+10, a+12, a+14, a+16, a+18$ <- every no differ by 10

$\therefore$ Sum of next consecutive 5 numbers is $5 \times 10 = 50$ more than sum of current consecutive odd numbers.

$\therefore 50 + (5 \times 59) \rightarrow$ sum of current consecutive 5 numbers
$= 50 + 295$
$= 345$

Q. Average temperature of for a week is noted as $27°C$. But later it was realised that one of the days temperature reading was taken as $30°C$ instead of $23°C$. What is the correct avg. temperature for the week

Sol:- $\frac{(a_1+a_2+\dots+a_6)+30}{7} = 27°C$

$\frac{(a_1+a_2+\dots+a_6)+30+(23-30)}{7} =$

$\frac{(a_1+a_2+\dots+a_6)+30}{7} + \frac{(23-30)}{7}$

$27° + \left(\frac{-7}{7}\right)$

$= 26°C$

If a_4 is replaced by new value a_5

$\frac{a_1+a_2+a_3+a_4}{4} = x$

$\frac{a_1+a_2+a_3+a_4+(a_5-a_4)}{4} =$

$\frac{a_1+a_2+a_3+a_4}{4} + \frac{a_5-a_4}{4}$

$= x + \frac{a_5-a_4}{4}$

$$\boxed{\text{New avg} = \text{old avg} + \frac{\text{new val} - \text{old val}}{n}}$$

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Ages

Q Santosh age after 10 years will be 4 times his age 5 years back. What is his present age?

Sol:-

$S+10 = 4(S-5)$

$S+10 = 4S-20$

$30 = 3S$

$S = 10$ years

Q The present age of father is 5 times the age of his daughter. Five years ago, the age of the father was 10 times the age of daughter at that time. What is the daughter's age after 5 years?

Sol:

$F = 5D$

$(5D-5) = 10(D-5)$

$5D-5 = 10D-50$

$45 = 5D$

$9 = D$

∴ D age after 5 years = 14 years

Q The sum of ages of 4 siblings born at the intervals of 3 years each is 50 years. What is the age of eldest sibling?

Sol:- $x,\ x+3,\ x+6,\ x+9$.

$x+x+3+x+6+x+9 = 50$

$4x+18 = 50$

$4x = 32$

$x = 8$

eldest = $8+9 = 17$ years

Q Father is 25 years elder to his son. Four years hence, the father's age will be 1 less than thrice the son's age. What is the father's present age?

Sol:-

$F = S+25$

$S+25+4 = 3(S+4)-1$

$S+29 = 3S+12-1$

$S+29 = 3S+11$

$18 = 2S$

$S = 9$ years.

$F = 34$ years

Q The ratio of present ages of Susmita and Prabha is 7:3 and the difference is 16 years. The sum of their ages is?

Sol:

$7-3 = 16$

$4 = 16$

$1 = 4$

Sum $= 28+12 = 40$.

Q The ratio of present ages of Sudeepa and Madhu is 3:4. The ratio of their ages will become 4:5 six years hence. Present age of Madhu is?

Sol:

$\frac{S}{M} = \frac{3}{4}$

$4S = 3M$

$\frac{S+6}{M+6} = \frac{4}{5}$

$5S+30 = 4M+24$

$5\times\frac{3}{4}M+30 = 4M+24$

$15M+120 = 16M+96 \Rightarrow M = 24$

$\frac{3+6}{4+6} = \frac{4}{5}$

$\frac{9}{10} = \frac{4}{5}$

$9 = 8$

Q. Ratio of Sriniva and Murali's age ~~an~~ an year ~~ago~~ ago was 6:7 respectively. It would become 7:8 four years hence. Murali's current age is

Sol:- S M $\frac{S-1}{M-1} = \frac{6}{7}$ $\frac{S+4}{M+4} = \frac{7}{8}$

$7S - 7 = 6M - 6$

$7S = 6M + 1$

$8S + 32 = 7M + 28$

$8S + 4 = 7M$

$8\left(\frac{6M+1}{7}\right) + 4 = 7M$

$48M + 8 + 28 = 49M$

$36 = M$

M = 36 years

Q. A father said to his son 'I was as old as you are at the present at the time of your birth', the father's age is 40 years now. Then the son's age 7 years back is.

Sol: Let son's present age is x years

at the time of son's birth

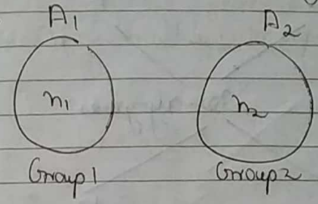
$40 - x = x$

$x = 20$

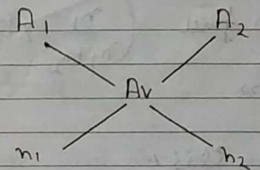
∴ Son's age 7 years back = 13 years

Mixture & Allegation

Averages →



n = no of elements



$A_1 > A_2$

$A_1 \leq Av \leq A_2$

$\frac{n_1}{n_2} = \frac{Av - A_2}{A_1 - Av}$

Q. Seed mixture 'X' is 40% rye grass and 60% bluegrass by weight. Seed mixture Y is 25% ryegrass and 75% fescue. If a mixture of 'X' and 'Y' contains 30% rye grass. What % of weight of this mixture is X?

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Sol:-

40% X   25% Y

30% Ryegrass

5   10

$\therefore \frac{n_1}{n_2} = \frac{5}{10} = \frac{1}{2}$

We have to find $\left(\frac{n_1}{n_1+n_2} \times 100\right)$

$\frac{1}{1+2} \times 100$

$= \frac{1}{3} \times 100 = 33.33\%$

Q Find the ratio in which juice costing Rs 192/liter is ~~mixture~~ mixed with juice costing Rs 150/liter so that mixture when sold ~~R~~ for Rs 194.40/liter gives a profit of 20%?

Sol:-

192   150

x

n_1 n_2

Let CP of mixture be x.

~~profit~~

~~actual~~ SP of mixture as per question

$SP = CP + 20\% CP$

$SP = x + \frac{20}{100}x$

$194.40 = x\left(\frac{120}{100}\right)$

$\frac{19440}{100} \times \frac{100}{120} = x$

$162 = x$

$\therefore$ Mixture price is 162

$\therefore n_1 = 12 \qquad n_2 = 30$

$\frac{n_1}{n_2} = \frac{2}{5}$

Q How much water must be added to 60 liters of milk at 1.5 l for Rs 20 to have a mixture of worth Rs $10\frac{2}{3}$ a liter?

Sol:- Cost of 1.5 l of milk $\rightarrow$ 20

1 l $\rightarrow \frac{20}{1.5} = \frac{40}{3}$

suppose x liters water is added

60 l milk

$\therefore$ cost of 60 l milk $= \frac{60 \times 40}{3} = 800$

Cost of mixture $= 10\frac{2}{3} = \frac{32}{3}$ per liter

cost of water is 0

$\frac{40}{3}$ $\qquad$ 0

$\frac{32}{3}$

$n_1 = \frac{32}{3}$ $\qquad$ $n_2 = \frac{8}{3}$

$\therefore \frac{n_1}{n_2} = \frac{4}{1}$

$\therefore$ milk : water = 4 : 1

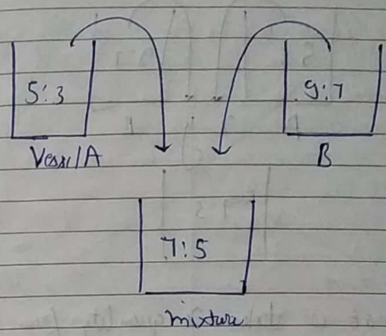
$\frac{60}{x} = \frac{4}{1}$

$x = 15\,l.$

$\therefore$ 15 l water is added.

Q Milk and water are mixed in a vessel "A" in the ratio 5:3, and in vessel B in ratio 9:7. In what ratio should quantities be taken from two vessels so as to form a mixture in which milk and water will be in the proportion of 7:5?

Sol:



Solving for milk

$\frac{5}{8}$ $\qquad$ $\frac{9}{16}$

$\frac{7}{12}$

$n_1 =$ $\qquad$ n_2

$n_1 = \frac{7}{12} - \frac{9}{16}$

$= \frac{28-27}{48}$

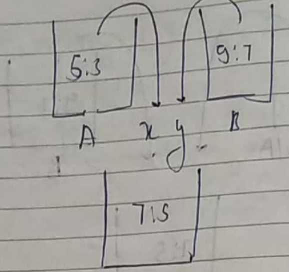
$= \frac{1}{48}$

$n_2 = \frac{5}{8} - \frac{7}{12}$

$= \frac{15-14}{24}$

$= \frac{1}{24}$

$\therefore n_1 : n_2 = 1/2$



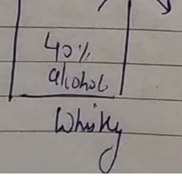
Suppose we take x quantity from A and y quantity from B and formed a mixture.

$$\frac{\frac{5}{8}x + \frac{9}{16}y}{\frac{3}{8}x + \frac{7}{16}y} = \frac{7}{5}$$

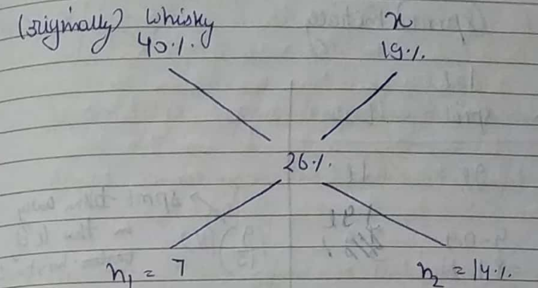
$$\frac{x}{y} = \frac{1}{2}$$

Q A jar full of whisky contains 40% alcohol. A part of this whisky is replaced by another containing 19% alcohol. Now the % of alcohol found to be 26%, the percentage of whisky replaced is?

Sol:-



Suppose x part is removed, and replaced



$\frac{n_1}{n_2} = \frac{1}{2}$

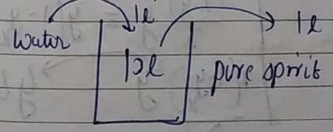
% of whisky replaced $\left(\frac{n_2}{n_1+n_2} \times 100\right) = ?$

$\left(\frac{2}{1+2} \times 100\right) = \frac{2}{3} \times 100 = 66.66\%$

Questions on replacement (Gate)

Q A container originally contains 10 l of pure sprit. From this container 1 liter of spirit is replaced with 1 litre of water. subsequently, 1 litre of mixture is replaced with 1 litre of water and this process is repeated one more time. How much sprit is now left in the container?

Sol.



(sprit) initially

10 l

	sprit	water	
1)	9l	1l	
2)	9-0.9 = 8.1 l	1.9l	$\left(\frac{9}{10}\right)(1l)$ sprit taken away in this 1l of ~~water~~ mixture.
3)	8.1-0.81 = 7.29	2.71l	$\left(\frac{8.1}{10}\right)(1l)$ sprit taken away in 1l of mixture.

We can generalize this case

if initially "x" l of pure sprit and "y" l is ~~replaced~~ removed and replaced with water every time.

find sprit left after n^{th} repetition.

x

	sprit	water	
1)	$(x-y)\,l$	$y\,l$	$y\,l$
2)	$(x-y)-\frac{(x-y)}{x}y$		$\frac{(x-y)}{x}\,y = y\left(1-\frac{y}{x}\right) \rightarrow y\,l$
3)	$x\left(1-\frac{y}{x}\right)^2$		$\frac{x\left(1-\frac{y}{x}\right)^2 y}{x} \rightarrow y\,l$
4)	$x\left(1-\frac{y}{x}\right)^3$		

$\therefore$ so on if we repeat this process till n times.

sprit remaining in the mixture $= x\left(1-\frac{y}{x}\right)^n$

Q 8 litres are drawn from a cask full of wine and is then filled with water. This operation is performed 3 more times. The ratio of the quantity of wine ~~now left~~ now left in cask to that of the cask hold originally is 16 : 81. How much wine did cask hold originally ?

Sol: $\because$ there are total 4 operation

$n = 4$ let initial wine be "x" litres

$y = 8l$

$$\therefore \frac{x\left(1-\frac{8}{x}\right)^4}{x} = \frac{16}{81}$$

$$1-\frac{8}{x} = \frac{2}{3}$$

$$\frac{1}{3} = \frac{8}{x}$$

$$x = 24\,l.$$

Q A can contains mixture of alcohol and water in the ratio of 7:5. When 9 litres of mixture are drawn off and the can is filled with water, the ratio of alcohol and water becomes 7:9. How many ~~litt~~ ~~litres~~ litres of alcohol was contained by the can initially?

Sol:-

7:5 → 9l drawn

alcohol	water	
$7x$	$5x$	$\left(\frac{7}{12}\right)9$
		$\left(\frac{5}{12}\right)9$

1) $7x-\left(\frac{7}{12}\right)9 \qquad 5x-\left(\frac{5}{12}\right)9 + 9$

$$\frac{7}{9} = \frac{7x-(7/12)9}{5x-(5/12)9+9}$$

$$\frac{7}{9} = \frac{7\left(x-\frac{9}{12}\right)}{5\left(x-\frac{9}{12}\right)+9}$$

$$5x - 5\left(\frac{9}{12}\right) + 9 = 9x - \frac{81}{12}$$

$$5x - \frac{45}{12} + 9 = 9x - \frac{81}{12}$$

$$\frac{81}{12} - \frac{45}{12} + 9 = 4x$$

$$\frac{36}{12} + 9 = 4x$$

$$12 = 4x$$

$$x = 3$$

Initially content of alcohol $= 7 \times 3 = 21\,l$

Profit & loss

Q Mr Deadpool purchased 80 chimichanges at a rate of Rs 13.50 each and he kept them in a box having 120 chimichangas each of cost is Rs 16. At what rate he should sell each of them to get a 16% profit?

Sol:- 80 — 13.50 Rs 120 — Rs 16

→ CP

$$CP = \frac{80(13.50) + 120(16)}{80+120}$$

$$= \frac{1080.00 + 1920}{200}$$

$$= \frac{300\cancel{0}\cancel{0}}{2\cancel{0}\cancel{0}} = 15$$

$$\begin{array}{r} 116 \\ \underline{15} \\ 1740 \end{array}$$

$$SP = \left(\frac{116}{100}\right) 15$$

= Rs 17.4

Q Captain Barbosa sold two jars of dirt to captain Jack-sparrow for \$990 each. On one he got 10% profit and on the other he got 10% loss. Barbosa's % profit or % loss in whole transaction was?

Sol. In such cases there is always a loss.

$$\text{loss \%} = \left(\frac{x}{10}\right)^2 = \cancel{\left(\frac{810}{120}\right)^2}$$

$$= \left(\frac{10}{10}\right)^2$$

$$= 1\%$$

Note:- Whenever we sell 2 items at same SP; and on one we have $x\%$ profit, while on other we have $x\%$ loss. Then overall there is always a loss.

$$\text{loss \%} = \left(\frac{x}{10}\right)^2$$

Q A man gains selling price of 11 toy cars by selling 33 toy cars. What is his % gain?

Sol: gain on 33 toys cars = SP of 33 toy cars − CP of 33 toy cars.

SP of 11 toy cars = gain on 33 toys cars = SP of 33 toy cars − CP of 33 toy cars

SP of 11 toy cars = SP of 33 toy cars − CP of 33 toy cars

CP of 33 toy cars = SP of 22 toy cars

$$\% \text{ gain} = \frac{\text{SP of 33 TC} - \text{CP of 33 TC}}{\text{CP of 33 TC}} \times 100$$

$$= \frac{\text{SP of 33 TC} - \text{SP of 22 TC}}{\text{SP of 22 TC}} \times 100$$

$$= \frac{\text{SP of 11 TC}}{\text{SP of 22 TC}} \times 100$$

$$= 50\%$$

Q A dishonest Shopkeeper uses 85 cm scale instead of metre scale and claims for sale at the cost price. What is his % profit?

Sol:-

$$\underbrace{|\!-\!-\!-\!-\!-\!-\!-\!-\!-}_{85\text{ cm}}|\!-\!-\!-\!-\!|^{100\text{ cm}}$$

CP = CP of 85 cm

SP = CP of 100 cm

$$\% \text{ profit} = \frac{SP - CP}{CP} \times 100$$

$$= \frac{\text{CP of 15 cm}}{\text{CP of 85 cm}} \times 100$$

$$= \frac{300}{17}$$

$$= 17.64\%$$

$$= 17.64\%$$

Note:- In x units, if there is error of E units

$$|\!-\!-\!-\!-\!-\!-\!-\!-\!-|\underbrace{-\!-\!-\!-}_{E}|$$

$$\% \text{ profit} = \frac{E}{x-E} \times 100$$

shopkeeper selling only $x-E$ units of items and false claiming.

Simple Interest

Principle [P]

$$x \xrightarrow{P} y \qquad y \xrightarrow{P+I} x$$

x lends P amount to "y" or

x gives loan of P amount to y or

y borrows P amount from x

Rate [R] — After some years "y" is suppose to return P along with some interest I.

The interest I, which we get is our profit.

Time [T] — Here we can consider P is our investment for T years, and after T years, we are going to get profit I on our investment.

Simple Interest [S.I]

Amount [A] = P + S.I

Relating to the concepts of profit and loss

Let Principle P be our CP, ie Cost we invested.

Let Amount A be our Selling Price, ie the price we get on P investment

$\therefore SP = CP\left(\frac{100 + P\%}{100}\right)$ this is profit

if interest is R% per annum ie every year we are going to get an interest, of R% on amount P. (or profit)

and we invested P amount for T years. then profit we make for T years is RT%.

$$A = P\left(\frac{100 + RT}{100}\right)$$

$$A = P\left(1 + \frac{RT}{100}\right)$$

$$A = P + \frac{PRT}{100} \quad \leftarrow SI.$$

Amount we invest

SI grows in A.P.

1st year $\quad A = P + \frac{PR}{100} \quad T = 1$

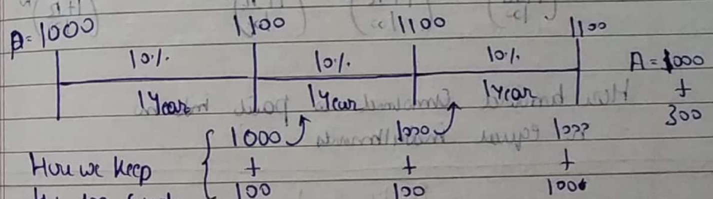
2nd year $\quad A = P + \frac{2PR}{100} \quad T = 2$

3rd year $\quad A = P + \frac{3PR}{100} \quad T = 3$

4th year $\quad A = P + \frac{4PR}{100} \quad T = 4$

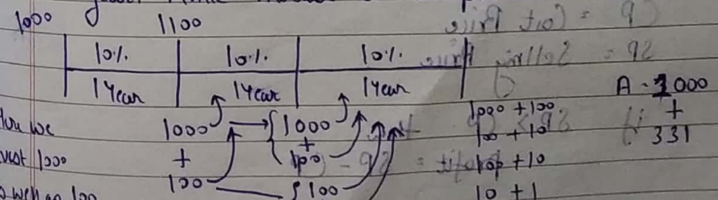
Compound Interest

In simple interest amount grows every year. but interest doesn't grow.



Here we keep this 100 and invest 1000 again

In Compound Interest, interest also grows every year. Hence interest is compounded here.



Here we invest 1000 as well as 100 also again.

In Compound Interest

$$A = P\left(1+\frac{R}{100}\right)^T$$

$$CI = A - P$$

Compound Interest grows in G.P.

$$CI = P\left(1+\frac{r \text{ per part}}{100}\right)^{\text{no of parts}}$$

Concept of Equal installments

$$P = \frac{x}{\left(1+\frac{r}{100}\right)} + \frac{x}{\left(1+\frac{r}{100}\right)^2} + \frac{x}{\left(1+\frac{r}{100}\right)^3} + \dots + \frac{x}{\left(1+\frac{r}{100}\right)^n}$$

Here borrowed amount to paid in n equal installments

LOGRATHMIC (Aptitude)

Properties

1) $\log_a(mn) = \log_a m + \log_a n$

2) $\log_a\left(\frac{m}{n}\right) = \log_a m - \log_a n$

3) $\log_a a = 1$

4) $\log_a(m^n) = n\log_a m$

5) $\log_{a^n} m = \frac{1}{n}\log m$

6) $\log_a b = \frac{\log_x b}{\log_x a}$

x may be any natural no

7) $\log_a 1 = 0$

8) $\log_b a = \frac{1}{\log_a b}$

9) $\log_b a \times \log_a b = 1$

10) $a^{(\log_a x)} = x$

Important points

→ $a^n + b^n$ is always divisible by $a+b$ if n is odd

→ $a^n + b^n$ is never divisible by $(a-b)$

→ $a^n - b^n$ is always divisible by $(a-b)$

→ $a^n - b^n$ is divisible by $(a+b)$ only when n is even

$$\sqrt{a\sqrt{a\sqrt{a\sqrt{a\sqrt{a\ldots}}}}}\ \text{n times} = a^{1-\frac{1}{2^n}}$$

$$\sqrt{a+\sqrt{a+\sqrt{a+\sqrt{a+\ldots\infty}}}} = P$$

then $P(P-1) = a$

$$\sqrt{a\sqrt{a\sqrt{a\sqrt{a\ldots\infty}}}} = a$$

$$\sqrt{(a+\sqrt{b})} = \pm(\sqrt{m}+\sqrt{n})$$

$m+n = a \quad 4mn = b$

Laws of Surds

1) $\sqrt[n]{a} = (a)^{1/n}$

2) $\sqrt[n]{ab} = \sqrt[n]{a}\,\sqrt[n]{b}$

3) $\sqrt[n]{a/b} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$

4) $(\sqrt[n]{a})^m = (\sqrt[n]{a^m})$

5) $\sqrt[n]{a^n} = a$

6) $\sqrt[m]{\sqrt[n]{a}} = \sqrt[mn]{a}$

7) $\sqrt[m]{\sqrt[n]{\sqrt[p]{a}}} = \sqrt[mnp]{a}$

Progression

1) AM > GM > HM

$$\frac{(a+b)}{2} > \sqrt{ab} > \frac{2ab}{a+b}$$

2) if AM, GM, HM form a GP

$$(GM)^2 = (AM)(HM)$$

Progression (AP)

$$T_n = S_n - S_{n-1}$$

$$S_n = AM \times n \quad \text{ie} \quad \frac{n}{2}(a+l)$$

GP

Product of all the n terms of a GP = $(GM)^n$

HP

$$T_n = \frac{1}{a+(n-1)d}$$

$$HM = \frac{2ab}{a+b}$$

Sum of natural numbers

$$\Sigma n = \frac{n(n+1)}{2}$$

$$\Sigma n^2 = \frac{n(n+1)(2n+1)}{6}$$

$$\Sigma n^3 = \left(\frac{n(n+1)}{2}\right)^2$$

☆ Questions on Polygon

~~Sum of in~~

Let there be n sides in a polygon

Sum of all interior angles of a polygon = $(n-2) * 180°$

Sum of all exterior angles of a polygon = $360°$

Highest power of 42 in 1456! = Highest power of 7 in 1456!

$42 = 2 \times 3 \times 7$

power of 7 $= \left[\frac{1456}{7}\right] + \left[\frac{1456}{49}\right] + \left[\frac{1456}{343}\right] + \cancel{\frac{1456}{}}$

$= 208 + 29 + 4$

$= 241$

When 7 fair coins are tossed simultaneously, in how many of the outcomes will at most four of the coins turn as head

$^7C_0 + {^7C_1} + {^7C_2} + {^7C_3} + {^7C_4} = 99$

Profit & loss

Types of Questions

CP = Cost Price

SP = Selling Price

if $SP > CP$ then

profit = $SP - CP$

if $CP > SP$ then loss = $CP - SP$

P% or L% are always on CP

$$P\% = \frac{P}{CP} \times 100 \qquad L\% = \frac{L}{CP} \times 100$$

$$SP = \frac{CP(100 + P\%)}{100} \qquad SP = \frac{CP(100 - L\%)}{100}$$

$$CP = \frac{SP \times 100}{(100 + P\%)} \qquad CP = \frac{SP \times 100}{(100 - L\%)}$$

Type - I Profit Equation

$SP = CP + \text{Profit}$

$= CP - \text{Loss}$

Q. SP of 5 articles is equal to CP of 4 articles find P% or L%.

Sol:- SP of 5 = CP of 4.

$$\frac{SP}{CP} = \frac{4}{5}$$

∴ if SP is 4 then CP is 5, hence loss of 1

$$\text{Loss}\% = \frac{1}{5} \times 100 = 20\%$$

Q While selling 5 pens a person gains profit equal to SP of 2 pens. Find P.% or L.%

Soln: 5 SP = 5 CP + Profit
↓
2SP

5SP = 5CP + 2SP

3SP = 5CP

$\frac{SP}{CP} = \frac{5}{3}$

∴ SP > CP

∴ profit % = $\frac{2}{3} \times 100$ = 66.66%

Q While selling 5 pens a person gets loss equal to CP of 2 pens. Find P.% or L.%

Soln: 5 SP = 5CP − loss

5 SP = 5CP − 2CP

5SP = 3CP

$\frac{SP}{CP} = \frac{3}{5}$

CP > SP → Loss

∴ Loss % = $\frac{2}{5} \times 100$ = 40%

Type-2

if CP of 2 items is same. And both items are sold.

I case > Item 1 — x% − P; Item 2 — y% − P

find avg = $\frac{x+y}{2}$ → +ve sign

Hence overall profit

II case > Item 1 — x% − P; Item 2 — y% − L

avg = $\frac{x-y}{2}$ → if sign is +ve then profit

else

if sign is −ve then loss.

III case > item 1 — x% − L; item 2 — y% − L

avg = $\frac{-x-y}{2}$ → −ve sign

∴ hence overall loss.

Q CP of 2 cows is equal

i) I cow — P% of 20%
2nd cow — L% of 20%

find over all P% or L%

Soln: I Method

Let CP of each cow be 100

			total
CP	100	100	= 200
	↓	↓	
SP	120	80	= 200

∴ no P% or L%

ii) I cow — P% of 20%

II cow — L% of 40%

Soln: Let CP of each cow be Rs 100

			total
CP	100	100	200
	↓	↓	
SP	120	60	180

∴ CP > SP

∴ loss % = $\frac{20}{200} \times 100 = 10\%$

Method-2

avg = $\frac{20-40}{2} = \frac{-20}{2} = -10$

∴ loss of 10%

Type-3

if SP of 2 items is equal and both the items are sold

I case) Item 1 — $x\%$ — P   Item 2 — $x\%$ — L

then there is always overall loss

loss % = $\frac{x^2}{100}$

II case) Item 1 — $x\%$ → P   item 2 — $y\%$ → L

Check the sign of expression

$$\frac{100(x-y) - 2xy}{(100+x)+(100-y)}$$

+ve → profit   -ve → loss.

III) Item 10% item 2% 10%

$x\%$ — P $y\%$ — P

Method

$$\frac{100(x+y)+2xy}{(100+x)+(100+y)} \to +ve$$

Overall profit

Q SP of 2 cows is equal

I - cow — P% 20%

II cow — L% 20%

find over all P% or L%.

Soln: I-Method

Let SP of each cow be 100

			total
SP	100	100	200
CP	$\frac{\cancel{100}^{50} \times \cancel{100}^{5}}{\cancel{120}_{6}}$	$\frac{\cancel{100}^{25} \times \cancel{100}^{5}}{\cancel{80}_{4}}$	$\frac{625}{3}$
	$\frac{250}{3}$	125	

$\therefore$ CP > SP

$$\text{Loss\%} = \frac{\frac{25}{3}}{\frac{625}{3}} \times 100 = 4\%$$

$$\frac{100(20-20)-2(400)}{120+80} = -ve$$

$$= \frac{-2 \times 4\cancel{00}}{2\cancel{00}} = -4\% \text{ loss}$$

Type-4 Chain Rule

Q A purchased an item at CP of Rs 2000 and sold it to B at ~~loss~~ profit of 20%. B spends Rs 100 in repair and sold the item to C at loss of 10%. Then what is the ~~at~~ amount paid by C.

Sol:

$$A \xrightarrow{P\%\ 20\%} B \xrightarrow{L \to 10\%} C$$

CP = 2000   +100

Selling price of A = CP of B.

CP of B = 2000 + 400 = 2400

SP of B = CP of C

$$\text{SP of B} = (2400+100) \times \frac{(100-L\%)}{100}$$

$$= 2500 \times \frac{(90)}{100}$$

= Rs 2250

Type-5 Part Selling

Q A person sells 20% of articles at profit of 20%. He sells 30% of remaining articles at L% of 25%. Remaining articles he sells at P% of 50%. Find overall P% or L%.

Soln Let there be 100 articles and CP of each be Rs 1

CP
100 × 1 = 100

20% articles i.e. 20 articles at P → 20%.

∴ on CP of 20 he gets P → 20%

∴ 100 → 20 — 4 +

P = 20% of 20 = $\frac{20}{100} \times 20$

remaining → 80 —30%— 24 — 6 +

L = 25% of 24 = $\frac{25}{100} \times 24$

↑ L – 25%

56 —— ~~28~~ 28 +

P – 50%

∴ On CP of Rs 100 he gets overall 26 + profit

∴ Profit → 26%

Type-6

Equal purchasing and selling in reverse order

Q A man buys 11 toys at a rate of Rs 10. He sells equal no of toys at a rate of 10 toys of Rs 11. Find over all P% or L%.

Soln: CP of 1 toy = $\frac{10}{11}$

SP of 1 toy = $\frac{11}{10}$

∴ SP > CP

∴ Profit % = $\frac{\frac{11}{10} - \frac{10}{11}}{\frac{10}{11}} \times 100 = 21\%$

Short trick

item price

buy 11 10

sells 10 11

Cross multiply

$= \frac{121 - 100}{100} \times 100$

$= 21\%$

Short trick

buying rate was x toys for Rs y

selling rate was m toys for Rs n

$$P/L\% = \frac{nx - my}{my} \times 100 \quad \begin{array}{l} + \text{ve} - P\% \\ -\text{ve} - L\% \end{array}$$

Type-7

Successive discount %.

Q. a shop keeper allows a successive discount of 60% + 20% + 20% on an article. find over all discount %.

Sol^n^: 60% + 20% + 20%

↓

means we have to pay 40% of CP of article.

Let CP be Rs 100

60% + 20% + 20%

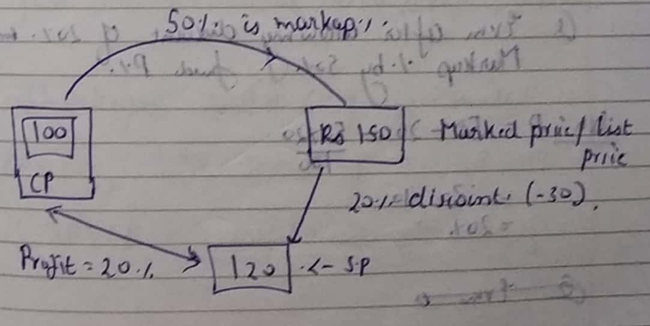
$$\left(\left(100 \times \frac{40}{100}\right) \times \frac{80}{100}\right) \times \frac{80}{100} =$$

ie we have to pay 80% of $\left(100 \times \frac{40}{100}\right)$

$$\left(\left(100 \times \frac{40}{100}\right) \times \frac{80}{100}\right) \times \frac{80}{100} = \frac{128}{5} = 25.6$$

∴ overall discount = 100 − 25.6 = 74.4%.

Concept of Marked price (list price), Markup % and discount %.



Suppose cost of manufacturing an article is Rs 100. The price on the label on article is Rs 150. Hence there is a markup of 50% on article. Shopkeeper allows a discount of Rs 20% on Marked price and sells it at Rs 120. Hence overall profit in the article is Rs 20%.

Market price (list price) - it is price specified on label.

NOTE:-
Mark up % is always on CP.
Discount % is always given on Marked price.

Relation b/w M% (Markup %), D% (discount) and P% (profit)/L% (loss).

$$P/L\% = M - D - \frac{M \cdot D}{100}$$

Q Even after allowing discount of 20% and Markup % by 50%. find P%.

Soln $P = 50 + 20 - \frac{50 \times 20}{100}$
$= 30 - 10$
$= 20\%$

~~Q Even~~

Q Even after allowing a discount of 20%. A person still gets a profit of 30%. find Markup %?

Soln $30 = P = M - D - \frac{MD}{100}$

$30 = M - 20 - \frac{M \times 20}{100}$

$50 = M - \frac{M}{5}$

$\frac{50 \times 5}{4} = M$

$M = 62.5\%$

Type-8 Faulty balances/ Dishonest seller

Q A balance shows 800gm as 1000gm. find P%/L% wrt to shopkeeper/customer

Soln Shopkeeper Let CP of 1gm be Rs 1.

$CP = 800$ $SP = 1000$

$\therefore SP > CP$ profit $= \frac{200}{800} \times 100 = 25\%$

Customer

$CP = Rs\ 1000$

↳ if in future he sells the goods. He will get paid for ~~Rs~~ 800gm

$SP = 800$

$\therefore SP < CP$

$loss = \frac{200}{1000} \times 100 = 20\%$

Q A balance reads 1000gm as 800gm

Sol^n Shopkeeper

$CP = 1000 \quad SP = 800$

$Loss = \frac{200}{1000} \times 100 = 20\%$

Customer

$CP = 800 \quad SP = 1000$

$profit = \frac{200}{800} \times 100 = 25\%$

Q A balance reads 800gm as 1000gm. Shopkeeper has already marked up price by 20%.
Find overall P/L % of shopkeeper

Sol^n:- Shopkeeper

~~SP~~ $= CP = 800 \quad SP = 1000$

$Profit_1 = \frac{200}{800} \times 100 = 25\%$

already markedup by 20% i.e he has already increased the price by 20%

$Profit\ 2 = 20\%$ Concept of successive %.

$\therefore$ overall $profit = a+b+\frac{ab}{100} = 25+20+\frac{25 \times 20}{100}$

$= 45+5$

$= 50\%$

Short tricks & Standard Problems

Type-I

Q purchase
11 apples for Rs 10
sells
10 apples for Rs 11

find P/L %.

Sol:- Q P

B	11	10
S	10	11

$$= \frac{121-100}{100} \times 100$$

= 21% profit

Q. Buys 8 apples for Rs 10

Sells 12 apples for Rs 9.

find P/L %.

Sol^n: Q P

B	8	10
S	12	9

$$= \frac{72-120}{120} \times 100$$

$$= \frac{-48}{120} \times 100$$

= 40% loss

II) When CP is same.

Q 2 cows CP is same

I → P.% 25%. find overall P/L %.
II → L.% 40%.

Soln: Here we take avg with sign.

$$avg = \frac{P-L}{2} = \frac{25-40}{2} = \frac{-15}{2} = -7.5\% \text{ loss}.$$

Q 3 cows CP equal

I → 25%. P
II → 40%. P find overall P/L %.
III → 35%. L

Soln:- $\frac{25+40-35}{3} = \frac{30}{3} = 10\%$ profit

Type-3 When SP of 2 articles is equal

Q 2 articles SP is equal

I → 20%. P find overall P/L %.
II → 20% L

Soln. Overall there is always loss

$$loss \% = \frac{(20)^2}{100} = 4\% \text{ loss}.$$

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Type-4 When SP is same

2 articles have same SP

I — 20% P
II — 10% L

find overall P/L %

Soln:- $\dfrac{100(P-L)-2PL}{(100+P)+(100-L)}$

$\dfrac{100(10)-2(20)(10)}{120+90}$

$\dfrac{1000-400}{210} = \dfrac{\cancel{600}^{20}}{\cancel{210}_{7}}$ 2.85% profit

Type-5 Dishonest Selling

Dishonest seller uses a weight of 960 gm for 1 kg.

find P% seller

Soln: $\boxed{P/L\% = \dfrac{\text{Error}}{\text{False weight}} \times 100}$

$\dfrac{\cancel{40}^{5}}{\cancel{960}_{24}} \times \cancel{100}^{5} = \dfrac{25}{6}$ 4.16% profit

Type-6

Q. A person sells rice at P% 10% and he uses a weight which is 20% less than the original value.

Sol: Let CP be 100

then 100 → SP 110

100 → 80

actual CP

∴ P% = $\dfrac{30}{80} \times 100$

$= \dfrac{300}{8} = 37.5\%$

Type-7

Q A person sells an article at P% of 20%. If he sells article for Rs 50 more he gets P% of 22%. What is CP?

Sol:- SP → P + 20%
+50 → P + 22%
There is a change of 2%.

P/L% is always taken w.r.t CP.
2% change is due to Rs 50 (ie we increased SP by Rs 50)

(2% of CP corresponds to Rs 50)

2% CP = 50
1% CP = 25
100% CP = 2500

$\therefore$ CP is Rs 2500

Q. A man sells an article at P% of 20%. If he decreases the price by Rs 50, he gets a profit of 15%. Find the CP of article?

Sol:- SP — 20%
-50 ↓) 5%
-50 — 15%

change of 5% is due to Rs 50

5% CP = 50
1% CP = 10
100% CP = 1000

CP is Rs 1000

Type - 8 (Imp)

Q. A man sells an article at P% of 10%. If he buys it for 10% less and ~~stated~~ sold it for Rs 3 more he gets a P% 25%. Find CP.

Soln:- Let CP be Rs 100

SP 110

90 — 100
CP

if CP is 90 P% — 25%

$SP = 90\left(\frac{125}{100}\right) = 112.5$

CP — 100 SP — 110) change of 2.5 — ie on selling for Rs 2.5 more

SP — 112.5

When change is of 2.5, CP is Rs 100 (ie if we sell at 2.5 more than CP is 100)

2.5 — 100 CP

1 — $\frac{100}{2.5}$ CP

3 — $\frac{3 \times 100}{2.5}$ CP = 120 CP

$\therefore$ CP is Rs 120

Type-9

Q. Cost price of 10 articles is equal to SP of 8 articles find P/L%

Soln: 10 CP = 8 SP

$\frac{CP}{SP} = \frac{8}{10}$

$P\% = \frac{2}{8} \times 100 = 25\%$

Q While selling 10 articles a person gets a profit equal to CP of 2 articles. find P/L%.

Sol:- $10\,SP = 10\,CP + P\%$
$10\,SP = 10\,CP + 2\,CP$
$10\,SP = 12\,CP$
$\frac{SP}{CP} = \frac{12}{10}$

$P\% = \frac{2}{10} \times 100 = 20\%$

Q While selling 10 articles a person gets a profit equal to SP of 2 articles. find P/L%.

Sol: Let SP of 1 article - 1
$2\,SP = 2$
$10\,SP = 10$
$\therefore CP = 8$
$\therefore P\% = \frac{2}{8} \times 100 = 25\%$

Q While selling 10 articles a person gets a loss equal to SP of 2 articles. find.

Sol:- $1\,SP = 1$ (Let SP of 1 article be Rs 1)
$2\,SP = 2$
$10\,SP = 10$
$CP = 12$

$L\% = \frac{2}{12} \times 100 = 16.66\%$

Type-10 (Imp)

Q. A person sells

table $- 12\frac{1}{2}\%$ P
Chair $- 8\frac{1}{3}\%$ L
} he gets overall profit 25 Rs

i) sells
table $- 8\frac{1}{3}\%$ L
chair $- 12\frac{1}{2}\%$ P
} Overall neither profit nor loss.

find CP?

Sol: Apply concept of Mixture & allegation

T	C
$8\frac{1}{3}\%$ L	$12\frac{1}{2}\%$ P

CP $\frac{25}{2}$ $\frac{25}{3}$

Ratio of CP of table chair $= 3:2$

Let Common ratio be x

$CP = 3x : 2x$

from 1st Condition

$3x \times \frac{25}{200} - 2x \times \frac{25}{300} = 25$

$\frac{25}{100}\left(\frac{3x}{2} - \frac{2x}{3}\right) = 25$

$\frac{5x}{6} = 100$

$x = 120$

$\therefore$ CP of table $= 360$
CP of chair $= 240$

Type-11 (Imp)

Q A man purchases 5 tables and 10 chairs for Rs 10000. He sells,

table − 15% P
chair − 10% L
} overall profit 375 Rs

find CP of 1 table and 1 chair.

Sol:

$$
\begin{array}{ccc}
T & & C \\
15\% & & -10\% \\
& 3.75\% & \\
13.75\% & & 11.25\%
\end{array}
$$

$P\% = \frac{375}{10,000} \times 100 = 3.75\%$

$\therefore$ ratio of CP's $= 13.75\% : 11.25\%$

$= \cancel{\frac{1375}{1125}} = \cancel{\frac{55}{45}} = \frac{11}{9}$

CP's $= 11 : 9$

$11 + 9 = 20 \equiv 10000$

$1 \equiv 500$

$\therefore$ 5 table − 11×500 10 chairs − 9×500

1 table − 1100 1 chair − 450

Type-12

Q A decrease of 20% in price of mango, it enables buyer to buy 25 mangoes more for Rs 500. Initial CP of mango?

Sol^n: 20% of Rs 500 = 100 we are spending Rs 500

When the price of mango is reduced,

In this Rs 100 we are able to buy 25 mangoes.

$\therefore$ reduced price of 1 mango $= \frac{100}{25} = 4$

if initial cost of 1 mango = Rs x

$\therefore$ after reducing, new price $= \frac{80}{100}x$

$\frac{80}{100}x = 4$

$x = 5$

$\therefore$ initial CP of mango is Rs 5.

Concept:- if the price of mango is reduced by 20% (And total we are spending Rs 500) total amount will also be reduced by 20%. (to purchase same no of mangoes)

20% of 500 = Rs 100

Here we can buy same no of mangoes that we are initially buying in Rs 400.

but we are spending Rs 500, and we are able to buy 25 mangoes more.

$\therefore$ ~~In Rs 100~~ Hence after reduction in price of mangoes, we ~~can buy~~ are able to buy 25 mangoes in Rs 100.

$\therefore$ reduced price of 1 mango is $\frac{100}{25}$ = Rs 4.

Type 13 (Imp)

Q. A man buy a certain no of apples at 3 apples for Rs 1 & another quality at 4 apples for Rs 1. He mix both the quality of apples and sell the mixture at Rs 1 for 3 apples. find P/L %.

Solⁿ:- take LCM of both the ~~quantities~~ qualities

LCM(3, 4) = 12

The man can purchase 12 apples of Quality Q1 at rate

1st kind - 12 apple — Rs 4 (cost per apple)

2nd kind 12 apple — Rs 3.

$\therefore$ after mixing = total 24 apple — Rs 7 ← CP

24 apple — Rs 8 ← SP

$\therefore$ P% $= \frac{1}{7} \times 100 = 14.28\%$

Type 14 Markup%, Discount%, Profit

$P\% = M - D - \frac{MD}{100}$

Q A person marked up the article by 20%. After allowing a discount of ~~20~~ 10%. find the overall P/L%.

Sol: $P\% = M - D - \frac{MD}{100}$

$= 20 - 10 - \frac{20 \times 10}{100}$

8%

Q after giving a discount of 25%, seller still makes a profit of 20%. Find marked up %.

Soln: $P\% = M - D - \frac{MD}{100}$

$20 = M - 25 - \frac{M \times 25}{100}$

$45 = \frac{3M}{4}$

$60 = M$

Type – 15

Q A man buys 2 horses for Rs 1350. he sells

I — 6% L
II — 7.5% P

overall neither gain nor loss

find CP of each horse?

Soln:

L		P
−6%		7.5%
	0	
7.5		6

ratio of CP's = ~~8~~ 5 : 4

Let common ratio be

$9 = 1350$
$1 = 150$

∴ CP of 1st horse = 750
2nd horse = ~~600~~ 600

Type – 16 (Imp)

Q A man sells 2 horses for Rs 3910 each

I — 15% P
II — 15% L

find total P/L ? Here absolute value is asked

Soln:- Overall L% = $\frac{15 \times 15}{100} = 2.25\%$

Let overall CP of horse be Rs 100

CP

$H_1 + h_2 = 100$

$SP = 100 - 2.25$
$= 97.75$

$97.75 \equiv 3910 + 3910$
$97.75 \equiv 7820$
$1 \equiv \frac{7820}{97.75} = 80$

$1 \equiv 80$

$\therefore CP = 8000$

$Loss = \frac{1\% \times CP}{100}$

$= \frac{2.25 \times 8000}{100} = 180$

$\therefore loss = 180$

Type - 17 (Imp)

What will be the P.% if article is sold at its actual price. Given that while selling an article at half of its actual price, there is a loss of 45%.

Sol: $P.\% = x(100 - L\%) - 100$

x = amount at which article is sold
ie $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}$ etc.

Here $x = 2$

$P.\% = 2(100 - 45) - 100$
$= 2 \times 55 - 100$
$= 110 - 100$
$= 10\%$

Type-18

Given buy 5 get 3 free. Find the discount %.

Sol:- total I am selling 8, customer is paying for 5 only.

$\therefore$ discount $= \frac{3}{8} \times 100 = 37.5\%$

Hence we are giving 3 articles for free on sell of 8 articles.

Type — 19 (Imp)

Q. A man sells an article for Rs 1000 and makes some profit. If he sells the article for Rs 1010 he gets 5% more profit. Find CP of article.

Soln:- 1000 — P.%
1010 — P.% + 5%

$5\%\ CP = 10$
$1\%\ CP = 2$
$100\%\ CP = 200$ Rs

Important Questions

Q D bought 100 kg of rice for Rs 1100 and sold it at a loss of as much money as he received for 20 kg rice. At what price did he sell the rice?

Sol:- Let SP be x Rs/kg

$$100x = 1100 - 20x$$

$$120x = 1100$$

$$x = \frac{1100}{120}$$

$$x = 9.16$$

Q A shopkeeper brought locks at the rate of 8 locks for Rs 34 and sold them at the rate of 12 locks for Rs 57. Calculate gain percent

Sol:-

	Q	P
B	8	34
S	12	57

$$P\% = \frac{8 \times 57 - 34 \times 12}{34 \times 12} \times 100 \Rightarrow \frac{456 - 408}{408} \times 100$$

$$= \frac{48}{408} \times 100 \Rightarrow \frac{200}{17}$$

Q A dishonest shopkeeper, at the time of selling and purchasing, weighs 10% less and 20% more per kg respectively. Find the % profit earned by treachery. (assuming he sells at CP)

Sol: Suppose, shopkeeper purchases quantity worth Rs 100.

According to Question, he weighs 20% more hence he purchases quantity worth Rs 120. and at time of selling he weighs 10% hence sells quantity worth Rs 90.

90 —— 100 —— 120

CP SP

$$P\% = \frac{120 - 90}{90} \times 100$$

$$= \frac{30}{90} \times 100$$

$$= 33.33\%$$

Q A merchant has 1000 kg of sugar, part of which he sells at 8% profit and the rest at 18% profit. He gains 14% on the whole. The quantity sold at 18% profit.

Sol:-

I		II
8%		18%
	14%	
4%		6%

ratio of 2 parts of sugar = 2:3

2+3 ≡ 1000 kg
5 ≡ 1000 kg
1 ≡ 200 kg

Quantity sold at 18% profit = 600 kg

Q. A firm dealing in furniture allows 4% discount on the marked price of each item. What price must be marked on a dining table that cost Rs 400 to assemble, so as to make a profit of 20%?

Sol:- $CP = 400$
to make a profit of 20%
$SP = 480$
∵ firm gives 4% discount

∴ Customer must be paying 96% of marked price

~~96%~~ ∴ MP = ?

96% of MP = 480

$MP = \frac{\cancel{480}^{5} \times 100}{\cancel{96}}$

= 500

Q. A watch dealer pays 10% custom duty on a watch that costs Rs 250 abroad. For how much should he mark it, if he desires to make a profit of 20% after giving a discount of 25% to the buyer?

Sol:- CP = 250 + custom duty
CP = 275

to make a profit of 20%

∴ $SP = \cancel{\frac{20}{100}} \times \cancel{275}$ $\frac{\cancel{275}^{55} \times \cancel{120}^{6}}{\cancel{100}_{2}} = 330$

∴ 75% of MP = 330

$MP = \frac{\cancel{330}^{110} \times \cancel{100}^{4}}{\cancel{75}_{3}}$

= 440

Q. Ramesh buys rice at Rs 10/kg and puts a price tag on it so as to earn a profit of 20%. However, her faulty balance shows 1000 gm when it is actually 800 gm. What is ~~the~~ his actual gain percentage?

Sol: ~~25%~~ Here profit is earned in 2 transactions

I → 20% — P on MP

II — Profit ~~on~~ due to faulty balance = $\frac{200}{800} \times 100 = 25\%$.

∴ Overall profit % = $20\% + 25\% + \frac{20 \times 25}{100}$

$= 50\%$

Q On selling tea at Rs 40 per kg a loss of 10% is incurred. Calculate the amount of tea (in kg) sold if total loss incurred is Rs 80.

Sol: ∵ In 1kg (ie on Rs 40) there is 10% loss
ie on Rs 40 there is loss of Rs 4.

∴ ~~the~~ if total loss incurred is Rs 80, then amount of tea sold is 20 kg

(Rs 4 loss — 1kg
Rs 80 loss — $\frac{80}{4}$ kg)

Q A trader purchases a watch and a wall clock for Rs 390. He sells them making a profit of 10% on the watch and 15% on the wall clock. He earn a profit of Rs 51.50. The difference b/w the original prices of the wall clock and the watch is equal to.

Sol: W WC
10% - P 15% - P

Let CP of W be Rs x

$\frac{10x}{100} + \frac{(390-x) \times 15}{100} = 51.50$

$10x + 5850 - 15x = 5150$
$700 = 5x$
$\frac{700}{5} = x$

$x = 140$ $390 - x = 250$

difference b/w prices $= 250 - 140$
$= 110$

Q A merchant earns a profit of 20% by selling a basket containing 80 apples whose cost is Rs 240. but he gives one-forth of it to his friend at CP and sells the remaining apples. In order to earn the same profit, at what price must he ~~sells~~ sell each apple?

Sol: CP = 240 Cost of each apple = 3

$\frac{1}{4} \times 80 = 20$ — $20 \times 3 = 60$

given earn 20% profit on 80 apple (ie on 240)

∴ SP = 288 ← this amount we have to earn

Rs 60 is earned by selling ~~from~~ to friends

∴ $288 - 60 = 228$ still need to be earned

∴ from remaining 60 apples we need to earn 228

∴ ~~cost of~~ each apple must be ~~sell~~ sold at Rs $\frac{228}{60}$ = Rs 3.80

Imp

Q. A owns a house worth Rs 10,000. He sells it to B at a profit of 15%. After some time, B sells it back to A at 15% loss. Find A's loss or gain percent.

Sol: A own house — 10,000 (CP)

$A \xrightarrow{15\%\ P} B$ CP = 11,500

SP = 11,500

1725

SP = 11500 − 1725
= 9775

∴ A makes a profit of 10,000 − 9775
= 225

i.e. P% = $\frac{225}{10,000} \times 100$ = 2.25%

Q A man buys two cycles for a total cost of Rs 900. By selling one for 4/5 of its cost and other for 5/4 of its cost, he makes a profit of Rs 90 on the whole transaction. Find the cost price of lower priced cycle.

Sol: $C_1 + C_2 = 900$

$C_1 \to x$, $C_2 \to (900-x)$

total SP

$\frac{4}{5}x + \frac{5}{4}(900-x) = 990$

$\frac{4}{5}x + 1125 - \frac{5}{4}x = 990$

$\frac{-9x}{20} = 990 - 1125$

$+\frac{9x}{20} = +135$

$x = 135 \times \frac{20}{9}$

$x = 300$

Simple Interest & Compound Interest

$S.I = \frac{P \times r \times t}{100}$

$P = \frac{SI \times 100}{r \times t}$

$r = \frac{SI \times 100}{P \times t}$

$t = \frac{SI \times 100}{P \times r}$

Type-I

Q: A sum amounts to Rs 2000 in 2 years 25% SI. Find P?

Sol: $P = \frac{A \times 100}{100 + rt}$

$P = \frac{2000 \times 100}{100 + 50} = \frac{2000 \times 100}{150}$

$P = \frac{4000}{3}$

Q find t = ?
given P = 2000, r = 15% pa SI = 400

$t = \frac{SI \times 100}{P \times r} = \frac{400 \times 100}{2000 \times 15} = \frac{4}{3}$ years

T-2 rate is diff. for different years

Q Sum for first 2 year r% — 15% pa
for next 2 years r% — 20% pa
for next 1 year r% — 10% pa
at end of 5 years total received Rs 16000 interest.
find P = ?

Sol:- $SI = P\left(\frac{r_1 t_1 + r_2 t_2 + r_3 t_3}{100}\right)$

$P = \frac{100 \times 16000}{30 + 40 + 10} = \frac{16000 \times 100}{80} = 20,000$

(Imp) T-3

A sum "A" is due after t years at r% SI. Find annual installment.

$\text{Installment} = \frac{100A}{100t + \frac{rt(t-1)}{2}}$

Q A sum Rs 848 is due after 4 years at r% 4% pa SI. Find installments.

Sol:- $\text{installment} = \frac{100 \times 848}{100 \times 4 + \frac{16(3)}{2}}$

$= \frac{100 \times 848}{424} = 200$ per year

Q A man pays annual installment of Rs 80 for 5 years at 5% SI. Find discharge loan.

Sol:- $\text{installment} = \frac{100A}{100t + \frac{rt(t-1)}{2}}$

$100A = \text{installment} \times \left(100t + \frac{rt(t-1)}{2}\right)$

$100A = 80 \times \left(100 \times 5 + \frac{25(4)}{2}\right)$

$A = \frac{80 \times 550}{100}$

440

T-4 A sum becomes 4 times in 12 years at SI. find r%.

(4 times → n, 12 years → t)

$r\% = \frac{(n-1) \times 100}{t}$

$r\% = \frac{(4-1) \times 100}{12} = 25\%$

Q. A sum becomes 5 times in 24 years at S.I. In how many years it becomes 25 times

Sol:- $R\% = \frac{(n-1)\times 100}{t}$

$= \frac{4\times 100}{24} = 20\%$

$t = \frac{24\times 100}{20}$

$t = 120$ years

T-5

Rs 2400

A — 20% p.a for 2 years

B — 15% p.a 3 years

C — 15% p.a for 4 years

at end of respective years S.I received is equal.

find sum borrowed by A, B and C respectively.

Sol:- ∵ S.I received is equal

Sum borrowed by A, B & C are in the ratio

$\frac{1}{r_1t_1} : \frac{1}{r_2t_2} : \frac{1}{r_3t_3}$

$\frac{1}{40} : \frac{1}{30} : \frac{1}{60}$ LCM (40, 30, 60) = 120

3 : 4 : 2

$A = 2400 \times \frac{3}{9}$ = 800

$B = 2400 \times \frac{4}{9} = \frac{3200}{3}$

$C = 2400 \times \frac{2}{9} = \frac{1600}{3}$

II-Case If at end of respective years. Amount received is equal

Sol Sum borrowed by A, B & C are in the ratio

$\frac{1}{100+r_1t_1} : \frac{1}{100+r_2t_2} : \frac{1}{100+r_3t_3}$

$\frac{1}{140} : \frac{1}{130} : \frac{1}{160}$ $\frac{1}{14} : \frac{1}{13} : \frac{1}{16}$

T-6 A sum amounts to Rs 2500 in 2 years SI and the same sum amounts to Rs 3000 in 4 years SI, find P & and R?

Sol:

500: 2500 —— 2 years; 3000 —— 4 years; 2 years

∴ interest for 2 years is 500

Given amount = 2500 after 2 years

∴ P = 2500 − 500 = 2000

$R\% = \frac{SI \times 100}{P \times t} = \frac{500 \times 100}{2000 \times 2} = 12.5\%$

(Imp) T-7

Rs 20,000

A: 10% p.a for 2 years

B: 15% p.a for 2 years

at the end of 2 years SI Rs 5200.

find ~~amount~~ sum loan to A & B.

Sol 2 years ~~total~~ interest Rs 5200

∴ interest for 1 years = 2600

Now we get a interest of 2600 on 20,000

∴ rate of interest $= \frac{2600}{20,000} \times 100 = 13\%$

this is avg rate of interest

10% 15%

13%

2 : 3

∴ amt borrowed by A & B are in the ratio 2:3

∴ $A = 2 \times \frac{20,000}{5} = 8000$ $B = 3 \times \frac{20000}{5} = 12000$

T-8 A sum was put on SI for 2 years. Had it been put on 3% more rate, it would fetch Rs 300 more SI, find P?

Sol: $P = \frac{\text{diff in Int} \times 100}{\text{diff in rate} \times \text{time}}$

$P = \frac{300 \times 100}{3 \times 2}$

$P = 5000$

(Imp) T-9

Q 2 equal sums are deposited in 2 banks each at 15% p.a for 3.5 year and 5 years. Difference in there interest is Rs 144, find P?

Sol:- $P = \frac{\text{Diff in Int} \times 100}{|r_1t_1 - r_2t_2|}$

$P = \frac{144 \times 100}{15 \times 1.5}$

$P = \frac{144 \times 100 \times 2}{15 \times 3}$

$P = 640$

T-10 Change in interest

Q $P = Rs\ 10,000$

$t = 2$ years

What is change in SI when rate of interest changes from 10% to 15%?

Sol:- Change in $SI = \frac{P \times t \times |r_1 - r_2|}{100}$

$= \frac{P \times r \times |t_1 - t_2|}{100}$

$= \frac{|P_1 - P_2| \times r \times t}{100}$

Change in $SI = \frac{10,000 \times 5 \times 2}{100}$

$= 1000$

Compound Interest

$A = P\left(1 + \frac{r}{100}\right)^t$

P = principle

r = rate of interest p.a

t = time in years

$CI = A - P$

Scanned by CamScanner

Suppose amount of Rs 1000 is loan at
r% 10%

P	SI in a year	total SI	Amount	P	CI in a year	Total CI	Amount
1000	100	100	1100	1000	100	100	1100
1000	100	200	1200	1100	110	210	1210
1000	100	300	1300	1210	121	331	1331

Imp results

i) For 1st year SI = CI

ii) In CI, interest earned in 2 consecutive years increases by r%.

iii) In CI, amount earned in 2 consecutive years increases by r%.

Q On a sum,
SI for 2 years is Rs 200
CI for 2 years is Rs 220

find r% ?

	SI	CI
I	100	100
II	100	120

$\Big\}$ 20

from 2nd result :- $r\% = \frac{20}{100} \times 100 = 20\%$

Q On a sum,
CI earned in 3rd year is Rs 2000
CI earned in 4th year is Rs 2200

find r% = ?

Soln :- 3rd year — 2000
4th year — 2200 $\Big\}$ 200

$r\% = \frac{\cancel{200}}{\cancel{2000}} \times 1\cancel{00} = 10\%$

Important results to Remember

$(1.1)^2 = 1.21$
$(1.1)^3 = 1.331$
$(1.2)^2 = 1.44$
$(1.2)^3 = 1.728$
$(1.3)^2 = 1.69$
$(1.3)^3 = 2.197$
$(21)^2 = 441$
$(21)^3 = 9261$

$(1.1)^4 = 1.4641$
$(1.4)^2 = 1.96$
$(1.5)^2 = 2.25$
$(1.6)^2 = 2.56$
$(1.7)^2 = 2.89$
$(1.8)^2 = 3.24$
$(1.9)^2 = 3.61$
$(2.0)^2 = 4.00$

$(2.2)^2 = 4.84$
$(2.5)^2 = \cancel{5.76}\ 5.29$
$(2.4)^2 = 5.76$
$(2.5)^2 = 6.25$
$(2.7)^2 = 7.29$
$(2.8)^2 = 7.84$
$(2.9)^2 = 8.41$
$(3.0)^2 = 9.00$

T-I time - years

$P = 10,000$
$r\% = 20\%$ p.a
$t = 2$ years 6 months

find Amnt ?

6 months $= \frac{6}{12}$ years

$A = P\left(1+\frac{r}{100}\right)^t$

$A = 10,000\left(1+\frac{20}{100}\right)^2\left(1+\frac{20}{100}\times\frac{1}{2}\right)^1$

II > $t = 2$ years 4 months

$\frac{4}{12} = \frac{1}{3}$ years

$A = 10,000\left(1+\frac{20}{100}\right)^2\left(1+\frac{20}{100}\times\frac{1}{3}\right)^1$

T-2 diff r% for diff. years

$P = 10000$

for first 2 years $r\% = 10\%$
for next 1 year $r\% = 20\%$
for next 1 year $r\% = 25\%$
at CI. What will be amt after 4 years?

Soln. $A = P\left(1+\frac{r_1}{100}\right)^{t_1}\left(1+\frac{r_2}{100}\right)^{t_2}\left(1+\frac{r_3}{100}\right)^{t_3}$

$A = 10,000\left(1+\frac{10}{100}\right)^2\left(1+\frac{20}{100}\right)^1\left(1+\frac{25}{100}\right)^1$

$= 10,000\left(\frac{11}{10}\right)^2\left(\frac{12}{10}\right)\left(\frac{5}{4}\right)$

$\cancel{10000}^{50} \times \frac{121}{100} \times \frac{3}{2}$

$= 150 \times 121$
$= 18150$

T-3

interest is compounded annually $= P\left(1+\frac{r}{100}\right)^t$

" " " half yearly $= P\left(1+\frac{r}{100}\times\frac{1}{2}\right)^{2t}$

" " " Quartly $= P\left(1+\frac{r}{100}\times\frac{1}{4}\right)^{4t}$

" " " after every month $= P\left(1+\frac{r}{100}\times\frac{1}{12}\right)^{12t}$

" " " after every moment $= Pe^{\frac{rt}{100}}$

Q. $P = 10,000$
$r\% = 20\%$ p.a
interest compounded half yearly for 2 year.
$A = ?$

$$A = P\left(1 + \frac{R}{100} \times \frac{1}{2}\right)^{2t}$$

$$= 10,000\left(1 + \frac{20}{100} \times \frac{1}{2}\right)^{2 \times 2}$$

$$= 10,000\left(1.1\right)^4$$

$$= 10,000 \times 1.4641$$

$$= 14641$$

Q $P = 10,000$
$R = 20\%$ p.a
Interest compounded quartly
find amnt after 9 months.

Sol: $P = 10,000$

$$A = 10,000\left(1 + \frac{20}{100} \times \frac{1}{4}\right)^3$$

$$10,000\left(\frac{21}{20}\right)^3$$

$$10,\cancel{000} \times \frac{9261}{8\cancel{000}}$$

$$= 92610$$

T-4 A sum amounts to twice in 3 years at CI In how many years it becomes 8 times at same rate?

Sol: given a sum becomes "m" times in "t" years ~~and~~ then after how many years it becomes "n" times

"m" times — t_1 years

"n" times — t_2 years

$$\boxed{m^{1/t_1} = n^{1/t_2}}$$

$$(2)^{1/3} = (8)^{1/t_2}$$

$$(2)^{1/3} = (2)^{3/t_2}$$

$$\frac{1}{3} = \frac{3}{t_2}$$

$$t_2 = 9$$

$\therefore$ in 9 years.

T-5 Diff b/w SI and CI in 2 years $= P\left(\frac{R}{100}\right)^2$

Diff b/w SI and CI in 3 years $= P\left(\frac{R}{100}\right)^2\left[3 + \frac{R}{100}\right]$

Q given $P = 1000$
$R\% = 10\%$
Find diff. b/w CI and SI after 2 years

i) after 2 years
ii) after 3 years

Sol: (i) After 2 years

$P\left(\frac{r}{100}\right)^2$

$1000\left(\frac{10}{100}\right)^2$

$1000 \times \frac{100}{10000}$

$= 10$

ii) After 3 years

$= P\left(\frac{r}{100}\right)^2\left(3+\frac{r}{100}\right)$

$= 1000\left(\frac{10}{100}\right)^2\left(3+\frac{10}{100}\right)$

$= 10\left(\frac{31}{10}\right)$

$= 31$

I-6 S.

SI on a sum for 2 years Rs 200
r% = 10%
find CI in 2 years

2 year SI = 200

	SI		CI
1^{st}	100	—	100
2nd	100		110
			210

After 2 years

I-7 A sum amts to Rs 5000 in 2 years at CI
The same sum amts to Rs 6000 in 4 years
at CI. find P and r% ?

Sol: $P\left(1+\frac{r}{100}\right)^2 = 5000$ — ①

$P\left(1+\frac{r}{100}\right)^4 = 6000$ — ②

After from ① and ②

$$\frac{P\left(1+\frac{r}{100}\right)^4}{P\left(1+\frac{r}{100}\right)^2} = \frac{6000}{5000}$$

$\left(1+\frac{r}{100}\right)^2 = \frac{6}{5}$ — ③

$\left(1+\frac{r}{100}\right) = \sqrt{1.2}$

from ① and ③

$P\left(1+\frac{r}{100}\right)^2 = 5000$

$P \times \frac{6}{5} = 5000$

$P = \frac{12500}{3}$

Terminology

present value = principle
future value = amount

future value = present value $\left(1+\frac{R}{100}\right)^t$

T-8 Effective rate of interest

$P = 1000$
$R\% = 20\%$ comp half yearly
$t = 1$ year.

find effective rate of interest

Sol:- Step 1 - Calculate interest after 1 year.

$CI = A - P$

$A = 1000\left(1+\frac{20}{100}\times\frac{1}{2}\right)^{2\times1}$

$1000\left(\frac{11}{10}\right)^2$

$= 1210$

$CI = 210$

∴ We are getting an interest of 210 on 1000.

Hence, effective rate of interest $= \frac{210}{1000}\times100$

$= 21\%$

T-9

CI on a certain sum in 2 years is Rs 2000
CI on a certain sum in 3 years is Rs 3200

find rate of interest R%.

$P\left(1+\frac{R}{100}\right)^2 - P = 2000$ — ①

$P\left(1+\frac{R}{100}\right)^3 - P = 3200$ — ②

dividing ① and ②

$$\frac{\left(1+\frac{R}{100}\right)^2 - 1}{\left(1+\frac{R}{100}\right)^3 - 1} = \frac{2000}{3200}$$

Let $1+\frac{R}{100} = x$

$$\frac{x^2-1}{x^3-1} = \frac{\cancel{2000}\,5}{\cancel{3200}\,8}$$

$$\frac{(x+1)(x-1)}{(x-1)(x^2+x+1)} = \frac{5}{8}$$

$$8(x+1) = 5(x^2+x+1)$$

$$8x+8 = 5x^2+5x+5$$

$$5x^2-3x-3=0$$

$$x = \frac{+3 \pm \sqrt{9+60}}{2\times5}$$

$$x = \frac{3 \pm \sqrt{69}}{10}$$

$$x = \frac{3 \pm 8.3}{10}$$

$$x = \frac{11.3}{10}$$

$$x = 1.13$$

$$1+\frac{r}{100} = 1.13$$

$$\boxed{r = 13\%}$$

Installments

Imp things to keep in mind

i) When money is due

ii) SI/CI

iii) installment paid at begining / end of year

Ex:- Suppose money due after 3 years is Rs10,000. And it is repaid in 3 installments. The amount was borrowed at r% = 10% p.a Comp annually

$$x\left(1+\frac{r}{100}\right)^2 + x\left(1+\frac{r}{100}\right) + x = 10,000$$

amount paid 2 years ago (i.e. just 1 year after borrow) | amount repaid 1 year's ago | amount paid on due date.

Q A man borrowed Rs 22000 at 10% p.a CI. And repaid in 2 annual installments at end of given year

~~Amount~~ Amount to be repaid after 2 years

$$= 22000\left(1+\frac{10}{100}\right)^2$$

$$= \cancel{22000}\left(\frac{121}{100}\right)$$

$$= 220 \times 121$$

$$= 26620$$

121
×22
242
242×
2662

$$x\left(1+\frac{r}{100}\right)^1 + x = 26620$$

$x\left(1+\frac{10}{100}\right)+x = 26620$

$x(1+0.1+1) = 26620$

$x = \frac{26620}{2.1} = 12676.19$

Q What annual installment will discharge a loan of Rs 66000 due in 3 years 10% CI (installment paid end of year)

Sol:- $x\left(1+\frac{10}{100}\right)^2 + x\left(1+\frac{10}{100}\right)^1 + x = 66000$

$x\left((1.1)^2 + (1.1) + 1\right) = 66000$

$x = \frac{66000}{(1.21+1.1+1)}$

$x = \frac{66000}{3.31}$

II case if loan of Rs 66000 due in 3 years 10% Simple Interest (S.I)

$x + \frac{x \times 10 \times 2}{100} + x + \frac{x \times 10 \times 1}{100} + x = 66000$

$x(1+0.2+1+0.1+1) = 66000$

$x = \frac{66000}{3.3}$

Percentages

T-1

A	B
↓	↓
120	150

i) A is what % of B
ii) B is what % of A
iii) B is how % more than A
iv) A is how % less than B
v) By what % A should be increased equal B?
vi) By what % B should be decreased equal A?

Sol:- i) Concept - $\left(\frac{\text{What we compare}}{\text{To whom we compare}}\right) \times 100$

Sol: i) $\frac{12\cancel{0}}{15\cancel{0}} \times 100 = 80\%$

ii) $\frac{15\cancel{0}}{12\cancel{0}} \times 100 = 125\%$

iii) $\frac{3\cancel{0}}{12\cancel{0}} \times 100 = 25\%$

iv) $\frac{30}{150} \times 100 = 20\%$

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v) $\frac{30}{120} \times 100 = 25\%$

vi) $\frac{30}{150} \times 100 = 20\%$

Ex: if A is 25% more than B, B is how % less than A

Let B be 100, then A = 125

$\frac{25}{125} \times 100 = 20\%$

T-2

Important Values (Remember)

$100\% = \frac{100}{100} = 1$

$\frac{1}{2} = 50\%$

$\frac{1}{3} = 33.33\%$

$\frac{1}{4} = 25\%$

$\frac{1}{5} = 20\%$

$\frac{1}{6} = 16.66\%$

$\frac{1}{7} = 14.28\%$

$\frac{1}{8} = 12.5\%$

$\frac{1}{9} = 11.11\%$

$\frac{1}{10} = 10\%$

$\frac{1}{11} = 9.09\%$

$\frac{1}{12} = 8.33\%$

$\frac{1}{13} = 7.69\%$

$\frac{1}{14} = 7.14\%$

$\frac{1}{15} = 6.66\%$

$\frac{1}{18} = 5.55\%$

$\frac{2}{9} = 22.22\%$ $\frac{2}{3} = 66.66\%$

$\frac{3}{9} = 33.33\%$ $\frac{4}{9} = 44.44\%$

$\frac{5}{6} = 83.33\%$

$\frac{3}{8} = 37.5\%$

$\frac{5}{8} = 62.5\%$

$\frac{7}{8} = 87.5\%$

Q 137.5% of 266.66% of 240

$(100+37.5)\%$ of $(200+66.66)\%$ of 240

$\left(1+\frac{3}{8}\right)\left(2+\frac{2}{3}\right)(240)$

$\frac{11}{8} \times \frac{8}{3} \times 240$

880

T-3 % increase

% decrease

$A \longrightarrow n\% \uparrow = A \times \left(\frac{100+n\%}{100}\right)$

$A \longrightarrow n\% \downarrow \quad A \times \left(\frac{100-n\%}{100}\right)$

$A \rightarrow x\% \uparrow \rightarrow y\% \uparrow \rightarrow z\% \downarrow$

$A \rightarrow A \times \frac{(100+x)}{100} \times \frac{(100+y)}{100} \times \frac{(100-z)}{100}$

Ex:- population of a city

in year 2000 — 5 lac
2001 — 20% ↑
2002 — ~~decre~~ 30% ↓

find population at the end of 2002

Sol: $5 \text{lac} \times \frac{120}{100} \times \frac{70}{100} = 4.2$ lac

T-4

Q A student gets 30% marks failed by 10 marks
A student gets 35% marks. he gets 5 marks more than passing.

find max. marks
Passing marks
Passing %

Sol:-
I — 30% −10
5% II — 35% +5
$(5-(-10)) = 15$

5% marks = 15
100% marks = 300

∴ max marks = 300

passing marks = 30% of 300 + 10
= 90 + 10
= 100

passing % = $\frac{100}{300} \times 100 = 33.33\%$

Q If price of mango decreases by 20%. it enables buyer to buy 10 mangoes more for Rs 100. find initial price of mangoes

Sol:- If price is decreased by 20%, then buyer can ~~so~~ buy same no of mangoes for Rs 80.

x — Rs 100
decrease of price ↓ 20% ↓ 20
x — Rs 80

∴ In Rs 20 ~~buyer can~~ Buyer can ~~be~~ buy 10 mangoes

∴ decreased price of mango is 2

Let initial price be x $\xrightarrow{20\% \downarrow} \frac{80}{100}x$

∴ $\frac{80x}{100} = 2$

$x = 2.5$

T-5 Expenditure/Income

Income − Expenditure = Saving

Q If ratio of I : E is 3 : 2
next year income increases by 20%
and expenditure decreases by 30%
find % change in savings.

Sol:-

I	E	S
3	2	1
3.6	1.4	2.2

20% ↑ ; 1.2

% change in savings = $\frac{1.2}{1} \times 100 = 120\%$

T-6

Q A is 20% more than B
B is how % less than A ?

Let B = 100
A = 120

= $\frac{20}{120} \times 100 = 16.66\%$

Q A is 20% less than B. B is how % more than A

Sol: $\frac{20}{80} \times 100 = 25\%$

T-7 Successive % change

Ex- 100 ⟶ 20% ↑ —— 30% ↑ —— total % change

total % change = $a + b + \frac{ab}{100}$

$= 20 + 30 + \frac{20 \times 30}{100}$

$= 56\%$ increase

Ex. A ⟶ 30% ↑ —— 20% ↓

total % change = $30 - 20 - \frac{30 \times 20}{100}$

= 4% increase

Ex. A → 30% ↓ → 20% ↑

total % change = $-30 + 20 - \frac{30 \times 20}{100}$

= −10 − 6

= −16

∴ 16% decrease

Solve these pair first

Ex. A $\rightarrow$ 20% ↑ $\rightarrow$ 30% ↓ $\rightarrow$ 10% ↑

total % change =

$20 - 30 - \frac{20 \times 30}{100}$

$= -10 - 6$

$= -16$

$= -16 + 10 - \frac{16 \times 10}{100}$

$= -6 - 1.6$

$= -7.6$

7.6% decrease

total % change $= a+b+c + \frac{ab+bc+ca}{100} + \frac{abc}{10000}$

Ex. A $\rightarrow$ 20% ↑ $\rightarrow$ 20% ↓

total % change $= 20 - 20 - \frac{20 \times 20}{100}$

$= -4\%$

$= 4\%$ decrease

Application of Successive % change

Volume of cylinder $= \pi r^2 h$ (i.e. $\pi \times r \times r \times h$)

Rectangle = L × B = area

Square area = side × side

Area circle = $\pi \times r \times r$

Volume cube = $a \times a \times a$

cuboid = $l \times b \times h$

Expenditure = price × quantity purchased

Total revenue = Price of ticket × no of tickets sold

Ex:- area rect = L × B   Overall % change

20% ↑ 20% ↑

% change in area $= 20 + 20 + \frac{20 \times 20}{100}$

$= 44\%$

area increases by 44%

Q if side of square increases by 30%

Sol:- ~~% area of square change~~

% change in area of square

$= 30 + 30 + \frac{30 \times 30}{100}$

$= 69\%$

Q if radius of circle increased by 20%

area $= \pi r \times r$

Sol :- % change in area of square $= 20 + 20 + \frac{20 \times 20}{100}$

$= 44\%$

Q. If length of rectangle increases by 20%. but area remains same. Find % change in breadth.

Sol:- ~~total % change in area~~ ~~= 20+~~

Area = $l \times b$

20% ↑

Let % change in B be x

$0 = 20 + x + \frac{20 \times x}{100}$

$-20 = \frac{6x}{5}$

$\frac{-50}{3} = x$

$x = 16.66\%$ decrease

∴ breadth decreases by 16.66%.

I-case

A = L × B

↓ Same ↓ r% increase ↓ decreases $\left(\frac{r}{100+r} \times 100\right)$

II-case

A = L × B

↓ Same ↓ r% decrease ↓ increase $\left(\frac{r}{100-r} \times 100\right)$

Q. If price of kerosene oil increases by 10%. By what % consumption should be decreased so that overall expenditure should remain same.

Sol. Expenditure = price × consumption

↓ 20% ↑ ↓ decrease

$\frac{20}{120} \times 100 = 16.66\%$

I-I

Percentage point

↓

it is an absolute value (quantity)

Suppose a number A is 20% of say B.
We want to increase A by 10%.

10% of 20% = 2%

∴ A is 22% of B.

if given we want to increase A by 10% point.

Then $A = 20+10 = 30\%$ of B.

T-8

appreciation – increase
depreciation – decrease

Q A machine purchased for Rs 5 lac. every year, value depreciate by 20%. find ~~value~~ value after 3 years

Sol:- $5\,\text{lac}\left(\frac{80}{100}\right)\left(\frac{80}{100}\right)\left(\frac{80}{100}\right)$

$5\,\text{lac} \times \frac{8 \times 8 \times 8}{1000} = \frac{64}{25}\,\text{lac} = 2.56$ lacks.

Problems on Ages

T-1

Q 10 years ago age of father was 5 times of age of son. If present age of father is 3 times of age of son. find present age of son.

Sol:- $\frac{T_1(x-1)}{|x-y|}$ ← we take absolute value

$\frac{10(5-1)}{|5-3|} = \frac{10(4)}{2} = 20$

∴ age of son is 20 years.

T-2

Present age of father is 4 times of age of son. 10 years hence age of father becomes twice age of son. find present age of son?

Sol:- $\frac{T_1(x-1)}{|x-y|} = \frac{10(2-1)}{|2-4|} = \frac{10(1)}{2} = 5$

∴ age of son = 5 years

T-3

10 years ago age of father was 5 times of age of son. 10 years hence age of father becomes twice age of son. find present age of son

Sol: $\frac{T_1(x-1) + T_2(y-1)}{|x-y|} = \frac{10(5-1) + 10(2-1)}{|5-2|}$

$= \frac{40+10}{3} = \frac{50}{3} =$

T-4 $\downarrow T_1$

One year ago ratio ages of A and B (4 : 3) $\leftarrow x$

One year hence ratio ages of A and B (5 : 4) $\leftarrow y$

find present age of B.

Soln:- $B = \dfrac{T_1(x-1) + T_2(y-1)}{|x-y|}$

$$= \frac{1\left(\frac{4}{3}-1\right) + 1\left(\frac{5}{4}-1\right)}{\left|\frac{4}{3}-\frac{5}{4}\right|}$$

$$= \frac{\frac{1}{3}+\frac{1}{4}}{\frac{1}{12}}$$

$B = 7$

Q ratio of ages of father and son is 6:1
after 5 years ratio becomes 7:2.
find present age of son?

Soln:- $\dfrac{T_1(x-y)}{|x-y|}$ $\dfrac{5\left(\frac{7}{2}-1\right)}{\left|\frac{7}{2}-\frac{6}{1}\right|}$

$$\frac{\frac{25}{2}}{\frac{5}{2}} =$$

Son = 5 years

Alligation & Mixtures:-

Avg age of boys → 20 years

Avg age of girls → 25 years

if avg age of class → 22 years.

find ratio of boys : girls in the class?

Soln:-

B		G
20		25
	22	
3	:	2

$\therefore$ B : G = 3 : 2

Q 2 varities of sugar

I → Rs 5/kg II → Rs 7/kg

mixed, and the mixture is sold at Rs 6.5/kg
in what ratio both the varities are ~~mixture~~
mixed?

Soln:-

I		II
5		7
	6.5	
.5		1.5

I : II = 1 : 3

T-2

Q A man mix 80kg of Ist variety sugar costing Rs 7 with another variety costing Rs 12. Cost of mixture is Rs 10. How much sugar of IInd variety he mixed?

Soln.

I		II
7		12
	10	
2		3

$\frac{I}{II} = \frac{2}{3} = \frac{80}{x} \Rightarrow x = 120\,kg$

T-3

Avg price (cost) of Ist variety Rs 20
" " " IInd variety Rs 35
he sold the mixture at Rs 40 at P%
of $33\frac{1}{3}\%$. In what ratio he mixed both the varieties

Soln

	I		II
CP :-	20		35
		CP → 30	
	5		10

Given SP of mixture = 40 P% = $33\frac{1}{3}\%$

$CP = \frac{SP \times 100}{(100+P\%)} = \frac{40 \times 100}{100+33.33\%} = \frac{30}{}\frac{4000}{133} = 30$

I : II = 1 : 2

Try

2 solutions of milk and water

I — 60% milk

II — 95% milk

mixed and mixture contains 70% milk. In what ratio he mixed both?

Soln

milk I		milk II	water I		water II
60%		95%	40%		5%
	70%			30%	
25%		10%	25%		10%

I : II = 5 : 2

I : II = 5 : 2

~~Q 2m~~

(Imp)

Q. 2 solutions
milk : water

I 4 : 1

II 1 : 2

in what ratio he mixed both so that now in mixture m : w = 2 : 1

	m	w	m	w
Sol: I	4	1	→ 4/5	4/5
II	1	2	→ 1/3	2/3

I II

4/5 1/3

2/3

1/3 2/15

Ans I : II = 5 : 2

Concept → here we take in fractions and then solve

T-5

2 solutions

m : w

I — 3 : 2

II — 5 : 3

If he mixes both in equal quantity then what is the ratio of m : w in the resulted mixture?

Sol: ∵ mixes in equal quantity, we take LCM to simplify calculations

	m	w			m	w
I —	3	2	5	40l	24	16
II —	5	3	8	40l	25	15
					49	31

LCM(5, 8) = 40

∴ In mixing m : w = 49 : 31

Q 3 solution

m : w

I 3 : 2

II 5 : 3

III 1 : 3

He mix them in ratio 5 : 3 : 2, find ratio of m : w in the mixture?

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	m	w	Lt	m	w
Sol: I	3 : 2		— 5L	3	2
II	5 : 3		— 3l	15/8	9/8
III	1 : 3		— 2l	1/2	3/2
				45/8	37/8

mixture — 5 : 3 : 2

∴ m : w in mixture = 45 : 37

T-6 Removal & Replacement (Imp)

Q 90 l 80% of milk
9l of sol is removed
and replaced with water. The process is repeated for 3 times. find quantity of milk at end?

Sol:- 9l of soln is removed from 90l in every step

∴ 10% is removed in every step.
hence after every step 90% of soln is remaining (ie left)

∴ quantity of milk at end = $90 \times \frac{90}{100} \times \frac{90}{100} \times \frac{90}{100}$

= 65.61 l.

Ratio - Proportion & Variation

T-1 In a 80l 80% of milk and water ratio of m : w is 2 : 3. How much milk should be added to make ratio of m : w = 4 : 1?

Sol:- 2+3 = 5 ≡ 80
1 ≡ 16

m : w — 2 : 3
m = 2×16 = 32
w = 3×16 = 48

Say x l of milk need to be added

$\frac{32+x}{48} = \frac{4}{1}$ 32+x = 192
x = 160 l

II> How much water should be added to make ratio m:w = 1:4.

$\frac{32}{48+x} = \frac{1}{4}$ => 128 = 48+x => x = 80 l

∴ 80 l water must be added.

Q 2 positive no's are in ratio of 5:6
If 16 is subtracted from 2 no's, new ratio is 5:4. find both the no's.

classmate

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Sol:- $\frac{n_1}{n_2} = \frac{5}{6}$

$\frac{5x-16}{6x-16} = \frac{3}{4}$

$~~20x-64~~$ $20x - 64 = 18x - 48$

$2x = 16$

$x = 8$

$n_1 = 40$

$n_2 = 48$

T-2

Q Ratio of Income : Expenditure is 5:3.
Next year income increases by 20%
and saving increases by 10%.
find ratio of I : E next year?

Sol:- I : E = 5 : 3

I	E	S
5	3	2
6	3.8	2.2

Ans:- I : E = ~~15:19~~ 30 : 19

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Q 2 persons A and B
A B
ratio of income 2 : 3
Expenditure 3 : 5
each person saves Rs 500
What is their incomes?

Sol:-

	A	B	g
I	2	3	500
E	3	5	500
S	500	500	

$A \rightarrow 2x - 3y = 500$ $\quad B \rightarrow 3x - 5y = 500$

① — $6x - 9y = 1500$ $\quad 6x - 10y = 1000$ — ②

① - ②

$y = 500$ $\quad x = 1000$

∴ income A = 2000
B = 3000

T-3 Equality of

If $2a = 3b = 5c = 1$

find $a : b : c$

Sol:- $a:b:c$

$2a = 3b = 5c = 1$

$a = \frac{1}{2}$ $b = \frac{1}{3}$ $c = \frac{1}{5}$

$a:b:c = \frac{1}{2} : \frac{1}{3} : \frac{1}{5} = 15:10:6$

T-4 4 Standard ratios

Given $a:b$

Duplicate ratio $a^2 : b^2$

Triplicate ratio $a^3 : b^3$

Subduplicate ratio $a^{1/2} : b^{1/2}$

Subtriplicate ratio $a^{1/3} : b^{1/3}$

Q given subduplicate 2:3

find triplicate ratio

Sol:- $\frac{\sqrt{a}}{\sqrt{b}} = \frac{2}{3}$ $\frac{a}{b} = \frac{4}{9}$ $\frac{a^3}{b^3} = \frac{64}{729}$

T-5 Joint ratios

A : B = 5 : 2 , B : C = 2 : 3 , C : D = 5 : 3

$\therefore$ A : B : C = 5 : 2 : 3 C : D = 5 : 3

mult by 5 mult by 3

A : B : C : D = 25 : 10 : 15 : 9

find A : D = ? A : D = 25 : 9

Q In a race 100 m, A beats B by 20 m and C by 30 m. Then in a race of 100 m, by how many meters will B beats C ?

Sol: A B C

100 80 70

A covers 100, then B covers 80, and C covers 70

if race is of 80 m, then B beats C by 10 m

B C

80 70

80 m — 10

1 m — $\frac{10}{80}$

100 m — $\frac{10}{80} \times 100$

= 12.5 m

Q In a race of 100m, A beats B by 20m
In a race of 100m, B beats C by 30m
Then in a race of 100m, by how many metres will A beat C.

Sol:- A B B C

100 80 100 70

A : B = B : C
10 8 10 : 7
5 : 4

A : B : C = 50 : 40 : 28

1) race is of 50m, A beats C by 22m

50m — 22m
$100m — \frac{100}{50} \times 22 = 44m$

T-6 Comparison

Proper fraction $\frac{a}{b}$ $a < b$

Improper fraction $\frac{a}{b}$ $a > b$

Proper fraction $\boxed{\frac{a-x}{b-x} < \frac{a}{b} < \frac{a+x}{a+x}}$

Improper fraction

$\boxed{\frac{a-x}{b-x} > \frac{a}{b} > \frac{a+x}{b+x}}$

T-7 Value Substitution

$x : y = 3 : 4$

find $\frac{2x+3y}{3x+2y}$

Sol:- if degree in N and D is same, then put $x = 3$ and $y = 4$

$\frac{2(3)+3(4)}{3(3)+2(4)} = \frac{6+12}{9+8} = \frac{18}{17}$

Q. $x : y = 3 : 4$

find $\frac{2x^2+3y}{3x+2y}$

Sol:- can not be solved $deg(N) > deg(D)$

Proportion

Equality of 2 ratios

2 : 3 = 4 : 6

given ~~ratio a : b~~ 2 no's a, b

Mean proportion = $\sqrt{ab}$

Third proportion = $\frac{b^2}{a}$

Fourth proportion = $\frac{bc}{a}$ given a, b, c

$\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = k$

if same changes are brought in both N and D, then ratio remains same

$\frac{a+2c-3e}{b+2d-3f} = k$

Q given $\frac{a}{b+c} = \frac{b}{c+a} = \frac{c}{a+b} = k$

find k?

Sol:- $a+b+c = k(2a+2b+2c)$

$k = \frac{1}{2}$

Componendo & Dividendo

$\frac{a}{b} = \frac{3}{2}$

Componendo $\frac{a}{b}+1 = \frac{3}{2}+1$

$\frac{a+b}{b} = \frac{5}{2}$

dividendo $\frac{a}{b}-1 = \frac{3}{2}-1$

$\frac{a-b}{b} = \frac{1}{2}$

Applying both together

$\frac{a+b}{a-b} = \frac{5}{1}$

Variation

Directly proportional

If change is same and it is in same direction.

Q Height of a person is directly prop. with square root of age.

If H = 5ft age is 36 years

What is height if age is 64 years?

Sol:- $H \propto \sqrt{A}$

$H = k\sqrt{A}$

$5 = k\sqrt{36}$

$k = \frac{5}{6}$

$H = k\sqrt{64}$

$H = \frac{5}{6} \times 8$

$H = \frac{20}{3} = 6.66$ ft

Inverse Proportional

$A \propto \frac{1}{B}$

becomes k times

becomes $\frac{1}{k}$ times

Q Value of diamond is inversely prop with weight

If V = Rs 10,000 W = 5gm

find value when W = 20gm

Sol:- $V \propto \frac{1}{W}$

$V = \frac{k}{W}$

$10,000 \times 5 = k$

$V = \frac{\cancel{10,000}^{2500} \times 5}{\cancel{20}_{4}}$ $V = 2500$

Joint Variation

$A \propto B$ $A \propto \frac{1}{C^2}$ $A \propto D$

$$\boxed{A = k\frac{BD}{C^2}}$$

Averages

$\text{Avg} = \frac{\text{Sum of observations}}{\text{total no of observations}}$

i) a, b, c, d, e — max — avg = f

min

a < f < e

(ii) if all no's are multiplied by x avg $= f x$
" " " " added by x avg $= f + x$
" " " " subtracted by x avg $= f - x$
" " " " divided by n avg $= f/n$

Important points

1) Sum of first n natural no's. $= \frac{n(n+1)}{2}$

$1+2+3+\dots+n = \frac{n(n+1)}{2}$

2) $1^2+2^2+3^2+\dots+n^2 = \frac{n(n+1)(2n+1)}{6}$

3) $1^3+2^3+3^3+\dots+n^3 = \left[\frac{n(n+1)}{2}\right]^2$

4) Sum of first n odd no's $= n^2$

5) Sum of first n even no's $= n(n+1)$

A.P

$a, a+d, a+2d, a+3d, \dots\dots\dots a+(n-1)d$

$T_n = a+(n-1)d$

$S_n = \frac{n}{2}(2a+(n-1)d) = \frac{n}{2}(a+l)$

T-1) A man goes from home to office at speed of 20 km/hr and he comes back at a speed of 30 km/hr. Find avg speed?

Soln:- avg speed $= \frac{2ab}{a+b}$

$= \frac{2\times 20\times 30}{20+30}$

$= \frac{2\times 20\times 30^{6}}{50}$

$= 24$ km/hr

T-2) In a class of 20 students avg age is 15 years. If age of teacher is included avg age increases by 2 years. What is age of teacher?

Sol:- due to teacher avg age increases by 2 years

∴ she must have given 2 years to every student from her age & since from remaining left, she must have maintained avg age of 17.

∴ age of teacher $= 2\times 20+17$
$= 57$ years

Q In a class of 20 students avg age is 15 years. If age of a student leaves avg age decreases by $\frac{1}{2}$ years. What is the age of student left?

Sol:- age of student $= 0.5 \times 19 + 15$
$= 9.5 + 15$
$= 24.5$ years

T-3

Q avg marks of 20 students is 50. By mistake marks of 2 students were wrongly taken as 65 and 55 instead of 35 and 45. find correct avg marks

Sol: $50 + \left[\frac{(35+45) - (65+55)}{20}\right]$

$50 + \left[\frac{80 - 120}{20}\right]$

$50 + (-2)$

$= 48$

T-4

Q Avg temp Mon to Wed $\rightarrow 25°C$
Avg temp Wed to Sunday $\rightarrow 30°C$
If avg temp of whole week $\rightarrow 28°C$
find temp on Wednesday?

Sol:- Mon + Tues + Wed $= 75°C$ — ①
Wed + Thur + Fri + Sat + Sun $= 150°C$ — ②
Mon + Tues + ... + Sun $= 196°C$ — ③

∴ ① + ②

(Mon + Tues + ... + Sun) + Wed $= 225°C$

from ③

$196 + \text{Wed} = 225$

$\text{Wed} = 29°C$

Partnerships

Suppose there are 2 people A and B and they invested capital in a business

T-1

A B

10,000 15,000

for 1 year for ~~8 month~~ 1 year

I - Case

Profit sharing ratio = ratio of investment

= 10,000 : 15,000

= 2 : 3

II - Case for 1 year for 8 months

P. sharing ratio = capital invested × time.

120 000 : 120 000

$\boxed{\text{P. sharing} = \text{C.I} \times \text{time}}$

T-2 3 partners A B C

C I 1 : 2 : 3

Time 8M 12M 6M

P. Sharing = 8 : 24 : 18

= 4 : 12 : 9

T-3 A B C

P. Sharing 3 : 2 : 1

Time 6M 8M 12M

C I = $\frac{1}{2}$: $\frac{1}{4}$: $\frac{1}{12}$ (∵ ~~P. Sharing = C.I × time~~) (P. sharing = C.I × time)

= 6 : 3 : 1

T-4 (Imp)

	A	B	C
1st April	10,000	20,000	15,000
	10,000×6 = 60,000	×6 = 120,000	×9 = 135000
1st Oct.	+ 10,000	- 10,000	
	20,000 × 3	10,000 × 3	
	= 60,000	30,000	

	A	B	C
1st Jan	45000		45000
	25000 × 3		10,000 × 3
	75000		30,000
March End ↓	195000	180,000	165000

Profit sharing

P. sharing ratio = 195000 : 180000 : 165000

= 39 : 36 : 33

= 13 : 12 : 11

T. S (Imp)

A	B
15000	20000
8 months	12 months

B was paid a monthly salary of 200 P.M. At the end of year profit earned was Rs 20,000. find P. sharing ratio.

Sol:-

A	B
15000	20000
8 months	12 months

P. sharing = 120,000 : 240,000

1 : 2

Profit = Rs 20,000

salary paid to B = 12 × 200 = 2400

20,000 − 2400 = 17600

This 17600 profit is ~~equ~~ divided in the ratio 1 : 2

$A = 1 \times \frac{17600}{3}$; $B = 2 \times \frac{17600}{3} + 2400$

Number Systems (LCM & HCF)

24 is a multiple of all these no's

24 = 1, 2, 3, 4, 6, 8, 12, 24 — factor of 24 i.e these

Multiple means all the no's which are no's divisible by the given no. divides 24 completely without leaving any remainder

~~factor~~ Ex: multiple of 5

5, 10, 15, 20, 25 ....

factors Ex: 120 — 1, 2, 3, 4, 5, 6, 8, 10, 12, 15, 20, 24, 30, 40, 60, 120.

(those no's which divide the given no)

Given a no x

multiples of x → $\geq x$

factors of x → $\leq x$

LCM(a,b) → smallest such no which is completely divisible by both a & b.

HCF(a,b) ⇒ biggest such no, which completely divides both "a" and "b".

LCM & HCF

find LCM 36, 42, 56

1) Method - 1 Traditional

2	36, 42, 56
2	18, 21, 28
2	9, 21, 14
3	9, 21, 7
3	3, 7, 7
7	1, 7, 7
	1, 1, 1

$LCM = 2 \times 2 \times 2 \times 3 \times 3 \times 7$
$= 8 \times 9 \times 7$
$= 504$

2) Method-2 Prime factorization Method

LCM - all prime no's with max power

$36 = 2^2 \times 3^2$
$42 = 2 \times 3 \times 7$
$56 = 2^3 \times 7$

$LCM = 2^3 \times 3^2 \times 7$
$= \underline{504}$

Q find LCM of 120, 150, 200

Sol. $120 = 12 \times 10 = 2^2 \times 3 \times 2 \times 5 = 2^3 \times 3 \times 5$
$150 = 2 \times 3 \times 5^2$
$200 = 2^3 \times 5^2$

$LCM = 2^3 \times 3 \times 5^2$
$= 600$

find HCF of 288, 432, 768

1) Method-1 Traditional Method by repeated Division

pick any 2 no's

$$
\begin{array}{l}
432\overline{)768}(1 \\
\quad\ \ 432 \\
\quad 336\overline{)432}(1 \\
\qquad\quad 336 \\
\qquad\ \ 96\overline{)336}(3 \\
\qquad\qquad\ 288 \\
\qquad\qquad\ \ 48\overline{)96}(2 \\
\qquad\qquad\qquad\ 96 \\
\qquad\qquad\qquad\ \ 0
\end{array}
$$

$\therefore HCF(432, 768) = 48$

$$
\begin{array}{l}
48\overline{)288}(6 \\
\quad\ \ 288 \\
\qquad 0
\end{array}
$$

$\therefore HCF(288, 432, 768) = \underline{48}$

2) Method-2 Prime factorization

HCF = take all common prime no's with min powers

288, 432, 768

$$\begin{array}{r|l} 2 & 288 \\ \hline 12 & 144 \\ \hline 12 & 12 \\ \hline & 1 \end{array}$$

$288 = 2^5 \times 3^2$

$$\begin{array}{r|l} 2 & 432 \\ \hline & 216 \end{array}$$

$216 = 2^4 \times 3^3$

$$\begin{array}{r|l} 2 & 768 \\ \hline 2 & 384 \\ \hline 2 & 192 \\ \hline 2 & 96 \\ \hline 2 & 48 \\ \hline 2 & 24 \\ \hline 2 & 12 \\ \hline 2 & 6 \\ \hline & 3 \end{array}$$

$768 = 2^8 \times 3$

$HCF = 2^4 \times 3^1$

$= 48$

LCM & HCF for fractions

$\frac{a}{b}, \frac{c}{d}, \frac{e}{f}$

$LCM = \frac{LCM \text{ of } N}{HCF \text{ of } D}$ $\quad HCF = \frac{HCF \text{ of } N}{LCM \text{ of } D}$

Q find LCM & HCF of 1.8, 2, 2.4

Sol:- LCM 1.8, 2, 2.4 $= \frac{18}{10}, \frac{2}{1}, \frac{24}{10}$

$\frac{9}{5}, \frac{2}{1}, \frac{12}{5}$

$LCM = \frac{LCM(9,2,12)}{HCF(5,1,5)} = \frac{12}{1}$

$HCF = \frac{HCF(9,2,12)}{LCM(5,1,5)} = \frac{1}{5}$

Important Properties

i) product of 2 no's = (LCM of no's) × (HCF of no's)

ii) HCF of always divides given no exactly

iii) HCF of 2 no = x

We can assume no's to be xa, xb such that a, b are coprime.

LCM = x ab   a, b are coprime

T-1

Q Given LCM of 2 no's is 120.
HCF of 2 no's is 15
one no is 48. find other no?

Sol: Let other no be x

$48 \times x = 120 \times 15$
$x = 40$

according to (ii) property HCF exactly divided no., but 15 does not divide 40.
∴ second no does not exists

T-2

Q LCM of 2 no = 196
HCF of 2 no's = 7
if difference b/w 2 no's is 21
find larger no?

Sol:- Let 2 no be 7x, 7y   x, y are coprime

LCM = 7xy

7xy = 196

xy = 28

xy = 28   given x, y are coprime

✓	1	28	28
	2	14	14
✓	4	7	7

x, y = 1, 28
4, 7

∴ possible pairs of no's are (7, 196) and (28, 49)

given diff b/w 2 no is 21
∴ Valid pair is (28, 49)

larger no is 49.

Q HCF of 2 no = 6
Product of 2 no's is 4320
How many such pairs no's exists?

Sol: Let no be 6x, 6y   x, y are coprime

$6x \times 6y = 4320$

$xy = \frac{4320}{6 \times 6}$

xy = 120

$xy = 120$ x, y are coprime

- 1×120 ✓
- 2×60
- 3×40 ✓
- 4×30
- 5×24 ✓
- 6×20
- 8×15 ✓
- 10×12

__Note:__ We will not consider reverse order pairs bcz, question is not asking about __ordered pairs__.

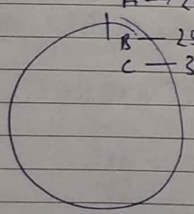
$\therefore$ 4 pairs are possible

$(1, 120), (3, 40), (5, 24), (8, 15)$

Application of LCM

i) __Circular Races__

$A \rightarrow 20$ sec
$B - 25$ sec
$C - 30$ sec



If they start together when will they meet first time at starting point.

Sol: $LCM(20, 25, 30) = 300$ sec

$\therefore$ they meet first time at starting point after 300 sec.
Second time meet = 600 sec
3rd time meet = 900 sec

ii) __Lights / Bells__

light after every
A — Legal $\longrightarrow$ 10 sec
B — Point $\longrightarrow$ 20 sec
C — Foundation $\longrightarrow$ 25 sec

Initially light together, find after how much time will they light again together.

$LCM(10, 20, 25) = 100$ sec

Q
		total time
$A \longrightarrow 10$ sec $\rightarrow$ for 5 sec	=	15 sec
$B \longrightarrow 20$ sec $\rightarrow$ for 5 sec		25 sec
$C \longrightarrow 25$ sec $\longrightarrow$ for 10 sec		35 sec

find after how much time will they light again together?

$LCM(15, 25, 35) = \cancel{1050\text{ sec}}$ 525 sec

Imp

Q Bells

A → 10 sec

B → 15 sec

C → 20 sec

i) after how much time they will toll together?

LCM (10, 15, 20) = 60 sec

ii) Between 6:00 PM to 7:00 PM how many time bells will toll together, including starting point

60 + 1 = 61 times
↑ starting point

Numbers

basic concept

Q What should be subtracted from largest 4 digit no so that it is divisible by 21.

Soln: largest 4 digit = 9999

$$
\begin{array}{r}
21 \overline{)9999} (476 \\
84 \\
\hline
159 \\
147 \\
\hline
129 \\
126 \\
\hline
3
\end{array}
$$

∴ 3 must be subtracted to make it divisible by 21.

Q What should be added to smallest 4 digit no so that it is divisible by 17

Sol:- Smallest 4 digit no = 1000

$$
\begin{array}{r}
17 \overline{)1000} (58 \\
85 \\
\hline
150 \\
136 \\
\hline
14
\end{array}
$$

∴ 17 − 14 = 3 should be added to 1000 to make it divisible by 17.

T-1 Basic Questions (Imp)

i) Smallest no exactly divisible by 3, 4, 5

i.e. we have find no, which is multiple of 3, 4, 5 and it should be smallest.

60) 1000 (16
60
400
360
40

	2nd smallest	Smallest 4-digit no
$LCM(3,4,5) = 60$		
i) Smallest no exactly divides 3, 4, 5 $LCM(3,4,5) = 60$ general no = $60k$	put $k = 2$ 120	1020
ii) Smallest no when divided by 3, 4, 5 it always leaves remainder 1 in each case $LCM(3,4,5) = 60 + 1$ general no = $60k + 1$	put $k = 2$ 121	1021
iii) Smallest no when divided by 3, 4, 5 leaves remainder 1, 2, 3 respectively. $3 - 1 = 2$ $4 - 2 = 2$ $5 - 3 = 2$ $LCM(3,4,5) = 60 - 2 = 58$ general no = $60k - 2$	put $k = 2$ 118	1018

iv) find smallest no which when divided by 11, 13 leaves remainder 7, 12.

Sol:- $\therefore$ no must be of the form

$\cancel{11x+7}$ $13x + 12$

When divided by 11 leaves remainder 7

$\cancel{\frac{11x+7}{13}}$ $\cancel{13x+12}$ $\frac{13x+12}{11} \longrightarrow 7$

express $13x+12$ in terms of multiple of 11

$$\frac{11x + 11 + 2x + 1}{11}$$

$$\frac{(11x+11)}{11} + \frac{(2x+1)}{11} \longrightarrow 7$$

always divisible by 11

We have find x such that rem. is 7

put $x = 3$ $\frac{7}{11} \longrightarrow 7$

$\therefore$ Smallest no is $13(3) + 12 = 39 + 12 = 51$

General no = $LCM(11,13)\,k + 51$
$= \boxed{143k + 51}$

2nd Smallest no put $k = 1$
$143 + 51 = 194$

(Imp)

T-2 Find smallest no which divided by 12, 16 or 18 it leaves rem 1 & when no is divided by 17, rem is 0.

Sol:- LCM(12, 16, 18) = 144 + 1 = 145

General no = 144k + 1

We have to satisfy 2nd condition also

$$\frac{144k+1}{17} = \frac{136k + 8k + 1}{17}$$

$$\frac{136k}{17} + \frac{8k+1}{17}$$

always divisible

find k such that it is divisible by 17

put k = 2

smallest no = 144(2) + 1
= 288 + 1
= 289

Application of HCF

Q find largest no which divides 1920, 640, 1320 exactly

Sol:-

2	1920
2	960
2	480
2	240
2	120
2	60
2	30
3	15
5	5

ans = $2^7 \times 3 \times 5$

2	1320
2	660
2	330
5	165
3	33
11	11
	1

$2^3 \times 3 \times 5 \times 11$

8	640
8	80
2	10
	5

$= 2^7 \times 5$

HCF(1920, 1320, 640) = $2^3 \times 5$ = 40

T-2 (Imp)

Boys	Girls
289	340

Divide boys and girls in different sections such that each section has equal no. of students. And each section has either boys or girls. What is min no of sections?

Sol:- Since, sections have to be min
Hence, each section must have as maximum no of students as possible. i.e. no of students must be multiple of 289 and 340. and must be as max as possible

Hence HCF(289, 340)
= 17

$\therefore$ no. of sections = $\frac{289}{17} + \frac{340}{17}$

$= 17 + 20$

$= 37$

Q Three different containers contain 496 l, 403 l, 713 ~~litres~~ l of mixtures of milk and water respectively. What is ~~the~~ biggest measure can measure all the different quantities exactly?

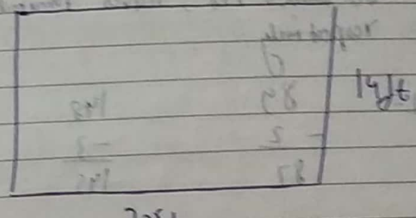
Sol:- 496 l, 403 l, 713 l

biggest measure HCF(496, 403, 713)

$$\begin{array}{r|l} 2 & 496 \\ \hline 2 & 248 \\ \hline 2 & 124 \\ \hline 2 & 62 \\ \hline 31 & 31 \\ \hline & 1 \end{array} \quad 2^4 \times 31 \qquad \begin{array}{r|l} 31 & 403 \\ \hline 13 & 13 \\ \hline & 1 \end{array} = 31 \times 13 \qquad \begin{array}{r|l} 31 & 713 \\ \hline 23 & 23 \\ \hline & 1 \end{array} \; 31 \times 23$$

$\therefore$ biggest measure = 31 l

Q3 Suppose we have a piece of cloth



piece of cloth have to be cut into small equal squares so that no piece of cloth gets wasted.

find i) max. no of squares

ii) min no of squares

Sol:- i) Infinite (side should be as min as possible)

ii) min no of squares, hence side can be as big as possible. hence side of square should be multiple of 14 & 20 and it should be as big as possible.

side of square =

HCF (20, 14) = 2

$\therefore$ no of squares = $\frac{20 \times 14}{2 \times 2} = 70$

T-4 (Imp)

Q find largest no when ~~divided~~ divides by 89 and 148, leaves remainder 2 and 3 respectively.

Sol:-

89	148
− 2	− 3
87	145

We have to find largest no which divides 87 and 145

$3 | 87$, $29 | 29$, 1 $5 | 145$, $29 | 29$, 1 HCF = 29

(Imp)

Q find the largest no when ~~divided~~ divides by 444, 804, 1344 leaves same remainder in each case

Sol:- 444, 804, 1344

I Step → take pair wise difference

$(804 - 444) = 360$
$(1344 - 804) = 540$
$(1344 - 444) = 900$

II Step = find HCF of 360, 540, 900

Sol:- $360 = 2^3 \times 3^2 \times 5$
$540 = 2^2 \times 3^3 \times 5$
$900 = 2^2 \times 3^2 \times 5$

HCF = $2^2 \times 3^2 \times 5^1 = 180$ ← Ans

Successive Division Concept

$5 | 72$
$3 | 14 - 2$
$2 | 4 - 2$
$2 - 0$

We can say that 72 is successively divided by 5, 3, 2 and leaves remainder 2, 2, 0 respectively

Q find general no when successively divided by 5, 3, 2 it leaves rem 3, 2, 1 respectively.

Sol:- $5 | 30x+28$
$3 | 6x+5 - 3$
$2 | 2x+1 - 2$
$x - 1$

always take last quotient as x

general no = $30x + 28$

Smallest possible no = put x = 0 i.e. 28

2nd smallest no = put x = 1 → 58

88

Q find rem. when 2479 is successively divided by 3, 5, 7, 2

Sol:

3	2479	
5	826	− 1
7	165	− 1
2	23	− 4
	11	− 1

Rem are 1, 1, 4, 1

Q A no. is divided successively by 3, 5, 6 leaves rem 1, 3, 2 resp. Find rem. when order of division is reversed?

Sol.

3	40	
5	13	− 1
6	2	− 3
	0	− 2

6	40	
5	6	− 4
3	1	− 1
	0	− 1

We can assume last quotient to be any no say we take 0. or we can take 1 also

∴ Rem are 4, 1, 1

Imp
Q A no when successively divided by 6, 4 it leaves rem. 5, 2 resp. find largest 3 digit no.?

Sol:

6	$24x+17$	
4	$4x+2$	− 5
	x	− 2

∴ no is of the form $24x+17$

largest 3 digit no 999

$24)\,999\,(41$
96
39
24
15

$999 - 15 = 984$

if we add 17 then 1001 which is 4 digit. Hence we have to take multiple of 24 less than 984

i.e $24 \times 40 = 960$

$960 + 17 = 977$

Imp
Q A no is successively divided by 3, 5, 6 leaves remainder 1, 3, 2 resp. find how many such no's are there which are less than 1000.

Sol:

3	$90x+40$	
5	$30x+13$	− 1
6	$6x+2$	− 3
	x	− 2

General no = $90x+40$

put $x = 11$ no = 1030 (> 1000)

$x = 0, 1, 2, 3, \ldots, 10$

∴ 11 such no are possible

Factorial

T-1 find Max power of a prime no in

i) max power of 2 in 100!

$\frac{100}{2} = \frac{50}{2} = \frac{25}{2} = \frac{12}{2} = \frac{6}{2} = \frac{3}{2} = 1$

$1+3+6+12+25+50 = 97$

ii) max power of 5

$\frac{100}{5} = \frac{20}{5} = 4$

$4+20 = 24$

iii) max power of 7 in 150!

$\frac{150}{7} = \frac{21}{7} = 3$

$21+3 = 24$

T-2 Trailing zeros in n!

0 is generated by a pair of (5,2). Here to find no of zero's we need to find power of 5 and 2. And then check for (5,2) pairs

In n! power of 5 < power of 2.

∴ to find trailing zeros find max power of 5.

Q find no of trailing zero's in 100!

pow (2) = 97 $\quad$ (2,5) = 24. ∴ 24 pairs of (5,2)
pow (5) = 24

∴ 24 zero's are there

T-3 Power of Composite Number

Ex: find power of 12 in 100!

$12 = (2^2 \times 3)$

we have to find how many such pairs exists

pow(2) = 97 $\quad 2^{97} = (2^2)^{48} \cdot 2^1$

$\frac{100}{3} = \frac{33}{3} = \frac{11}{3} = \frac{3}{3} = 1$

pow(3) = 48 $\quad 3^{48}$

∴ max power of $12 = (2^2)^{48} \, 2^1 \, 3^{48} = (2^2 \cdot 3)^{48} \cdot 2$
$= 48$

find power of 24 in 100!

$24 = 2^3 \times 3$

pow(2) = 97 $\quad (2)^{97} = (2^3)^{32} \cdot 2^1$

pow(3) = 48

pow of 24 = $(2^3)^{32} \cdot 2^1 \cdot 3^{48}$

$= (2^3 \cdot 3)^{32}$

Standard Question

Q find total no of trailing zeros in product of first 100 multiples of 10.

Sol:- 10, 20, 30, 40, ..., 1000

(10x1), (10x2), (10x3), ..., (10x100)

$(10)^{100}(1 \times 2 \times 3 \times 4 \times \ldots \times 100)$

$(10)^{100}(100)!$ $\quad \frac{100}{5} + \frac{20}{5} = 24$

= 100 + 24 $\quad$ pow(5) = 24

Ans = 124

Q find total no of trailing zero's in product of first 100 multiples of 5.

Sol:- 5, 10, 15, 20, ... 500

(5x1), (5x2), (5x3) ...... (5x100)

$(5)^{100}(100)!$

$= (5)^{100} \cdot (5)^{24}(2)^{97}$

$= (2)^{97}(5)^{124}$

$= (2 \cdot 5)^{97} (5)^{27}$

97 = trailing zeros.

Q find remainder when

1! + 2! + 3! + 4! + ... + 50! is divided by 20.

$\frac{1 + 2 + 6 + 24 + 120 + \ldots + 50!}{20}$

all the no's after 4! will have term 20 in them, hence they will be divisible by 20.

∴ Remainder depends on $\frac{1! + 2! + 3! + 4!}{20}$

$\frac{33}{20} = 13$

Ans Rem = 13.

Divisibility Rule & Remainder

Divisibility rule of 2, 4, 8, 16, 32, ... 2^n

2^1 2 we check last digit — divisible by 2 or 0

2^2 4 — — last 2 digit divisible by 4 or 00

2^3 8 — — — last 3 digit divisible by 8 or 000

2^4 16 — — — — last 4 digit divisible by 16 or 0000

Q Check divisibility

× 32758 → 4 2

× 328564 → 8 4

✓ 1257856 → 16 0

Q Let $x = 1111$ .... 491 times

When x is divided by 16, find remainder?

Sol: check last 4 digits $\frac{1111}{16}$ → 7 rem.

Divisibility by 3 and 9

sum of digits of the no is divisible by 3 or 9.

Q $x = 1111$.... 100 times. When divided by 9 find rem?

$\frac{100}{9}$ = 1 ← rem.

Divisibility rule of 5

last digit 5 or 0

Divisibility rule by 11

Add alternate digits starting from right to left.

3 5 2 7 8 9 1 = (1+8+2+3) − (5+7+9)
= 14 − 21

∴ no not divisible by 11.

$\left(\text{add digit at even places}\right) - \left(\text{add digits at odd places}\right) = 0$ or multiple of 11.

Concept of positive remainder & negative remainder

$\frac{7}{5} = 2$ $\qquad \frac{5+2}{5} = 2$ +ve rem

$\frac{10-3}{5}$ -ve rem

Convert +ve rem → -ve rem

$2 - 5 = -3$ (subtract divisor)

Convert -ve rem → +ve rem

$-3 + 5 = 2$ (add divisor)

Divisibility rule of 9, 99, 999, ...

1 2 3 2 3 1 5 2 1 3 2 1

divisible by 9 → add single digit $= \frac{26}{9} = 8$

∴ not divisible, rem → 8.

divisible by 99 → add pair of 2 digits. take pairs from right.

12 32 31 52 13 21 = 12+32+31+52+13+21 → 161

$\frac{161}{99} = 62$ ← rem

Q. $x = 1111\ldots$ 100 times. find remainder when divided by 999.

Sol:- $\boxed{\text{note:- pair of 6 1's 111111 is divisible by 999}}$ ✗

We have to take pair of 3 1's and add them pair wise

Sum = 33 × (111) + 1
= 3663 + 1
= 3664

$999 \overline{)3664}$ 3
2997
667

$\frac{3664}{999}$ = 667 ← rem.

Divisibility by 11, 111, 1001, 10001, 100001, ...

1 2 3 2 5 2 3 1 2

$(1+1+2+2+2) - (1+3+5+3+2)$
$8 - 14$
$= -6$

not divisible by 11, rem → $-6+11 = 5$

check divisiblity by 101

1 2 3 2 5 2 3 1 21

$(85) - (63) = 22$

$\therefore$ not divisible by 101, rem $\rightarrow$ 22

Q $n = 1111 \ldots\ldots$ 100 times

To find remainder when divided by 1001

Sol:- pair of 6 $111111 = 0$
is divisible by 1001

We have to form pairs of 6

$16 \times (111111) + 1111 = 110$

1111 → 1001, rem.

divisible by 1001

☆ Important

$1001 = 7 \times 11 \times 13$

For divisibility by 7, 11 and 13 we can apply same rule as for 1001.

ie take pairs of 3 digits → find difference.

Number Tree

Complex Numbers

Real no
(any no that can be represented on a number line)

imaginary
(can not be represented on a number line)

$\sqrt{-1} = i$, $i^2 = -1$, $i^3 = -i$

$i^4 = 1$

$i^{720} = (i^4)^{180} = (1)^{180} = 1$

→ Integers

positive, non-negative, negative, non-positive

→ ~~rat~~ fractions

terminating | non-terminating and repeating | non-terminating and non-repeating

Ex:- $\frac{1}{2}, \frac{1}{4}, \frac{1}{5}$

Ex:- π, e, $\log 2$, $\sqrt{2}$

rational number

Irrational number

Ex:- $\frac{1}{3}, \frac{1}{6}, \frac{1}{9}$

(-2, -3, 0, 1, 2, 3, ...) $\left(\frac{1}{2}, \frac{1}{4}, \frac{1}{5}, \frac{1}{8}\right)$ $\left(\frac{1}{3}, \frac{1}{6}, \frac{1}{9}, \ldots\right)$

Integers — terminating fractions — non-terminating repeating

(π, e, log 2, $\sqrt{2}$, $\sqrt{3}$ etc.)

non-terminating and non-repeating

irrational nos

Integers

(... -3, -2, -1, 0, 1, 2, 3, ...)

positive - 1, 2, 3, 4, ...
negative - -1, -2, -3, -4, ...
zero - does not have any sign
non-positive - 0, -1, -2, -3, ...
non-negative - 0, 1, 2, 3, ...

Q: $x + y = 5$

find no of integral solutions

I → x, y are integers

x	y
0	5
1	4

Sol: $x + y = 5$

x	y
0	5
1	4
2	3
3	2
4	1
5	0
6	-1

Ans → infinite solutions

II > x, y are positive integers $x, y \in Z^+$

$x + y = 5$

x	y
1	4
2	3
3	2
4	1

2 pairs exists (1, 4) (2, 3)
4 ordered pairs exists

III > x, y are non-negative integers

$x + y = 5$

x	y
0	5
1	4
2	3
3	2
4	1
5	0

3 pairs exists - (0, 5), (4, 1), (2, 3)
6 ordered pairs exists.

Prime Number (Very Important)

no divisible by 1 and no itself.

2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47
53, 59, 61, 67, 71, 73, 79, 83, 89, 97

☆> 1 is neither prime nor composite

☆> 2 is single even prime no.

☆> Any prime no greater than 5 when divided by 6 gives remainder 1 or 5.
but vice-versa not true.

generally prime no is assumed to be of the form
$6k \pm 1$ & (2, 3) are only consecutive prime no's.

Counting prime no

1 - 50 - 15 prime
1 - 100 - 25 prime
1 - 200 - 46 prime
1 - 1000 - 168 prime

☆> a, a+2, a+4 are prime. Then there is only one possible value of a and that is 3.

Twin Prime ÷ pair of prime no's whose difference is 2.
Ex:- (3, 5) (11, 13) etc.

Coprime no - pair of no's which are relatively prime to each other.

How to check Prime no

check 191 prime or not?

I step - find sq rt of 191 $\sqrt{191} \simeq 14$

II step ÷ Write all prime no's less than 14
2, 3, 5, 7, 11, 13

III step :- divide 191 by every prime no one by one.

if 191 is divisible by any prime no then
191 is not prime.
else if not divisible by any prime no then,
191 is a prime no.

∴ 191 is not divisible by any of the prime no.
∴ 191 is prime.

Composite numbers

A number which has more than 2 factors.

Smallest composite no is 4.

Properties of Even and Odd

I
e + e = e
o + o = e
e + o = o
o + e = o

II
e − e = e
o − o = e
e − o = o
o − e = o

III
e × e = e
e × o = e
o × o = o

IV
$e^n = e$ if $n \in N$
$o^n = o$, $n \in N$

Q given a, b are odd no and c is even no.
find nature of expression

$(a+b)^c + (b+c)^a + (c+a)^b$ → even

Sol:- = e + o + o
= o + o
= e

Fractions

Rational Numbers → non-terminating & repeating

Ex: $\frac{1}{3} = 0.\bar{3}$ $\frac{1}{6} = 0.1\bar{6}$

Convert into rational no i) $0.\overline{12}$ ii) $2.2\overline{34}$

Sol: i) $x = 0.\overline{12}$
$100x = 12.\overline{12}$
$99x = 12$
$x = \frac{12}{99}$

ii) $x = 2.2\overline{34}$
$10x = 22.\overline{34}$
$1000x = 2234.\overline{34}$
$990x = 2212$
$x = \frac{2212}{990}$

Short trick

for Numerator

i) Write the no. except decimal and bar.
ii) Subtract non-repeating part.

for Denominator

i) Write as many 9's as repeating digits.
ii) Write as many 0's as no of digits b/w decimal and repeating part.

Ex:- $1.2\overline{3}$

$$\frac{123-12}{90} = \frac{111}{90}$$

Ex:- $12.\overline{82}$ $= \frac{1282-12}{99} = \frac{1270}{99}$

Perfect no

A number whose sum of factors (except the number itself) is equal to the number

Ex:- 28 = $\underbrace{1, 2, 4, 7, 14}$, 28

$1+2+4+7+14 = 28$

Remainder Theorem

T-1 find remainder — We find rem in individual division

1) $\frac{1956 \times 1955 \times 1954}{19}$

$= \frac{18 \times 17 \times 16}{19} \Rightarrow \frac{(-1) \times (-2) \times (-3)}{19}$

rem → $= -6 + 19 = 13$

(ii) $\frac{2^{40}}{3}$

Sol- $\frac{2^{40}}{3} = \frac{(-1)^{40}}{3} = 1$

(iii) $\frac{2^{41}}{3} = \frac{(-1)^{41}}{3} = -1+3 = 2$

NOTE:- In most of the cases we try to express Numerator, such a way that we get rem. either 1 or -1.

(iv) $\frac{(16)^{70}}{7} = \frac{(2)^{70}}{7} = \frac{(2^3)^{23}(2)}{7} = \frac{(1)^{23}(2)}{7} = 2$

(v) $\frac{(35)^{20}}{17} = \frac{(1)^{20}}{17} = 1$

(vi) $\frac{(9)^{81}}{7} = \frac{(2)^{81}}{7} = \frac{(2^3)^{27}}{7} = \frac{(1)^{27}}{7} = 1$

II Cancellation Rules

$\frac{18 \times 41}{6 \quad 3}$ rem → $1 \times 2 = 2$

Whatever we cancel, we multiply it to rem. in end. Here we cancelled 2, so we multiply 2 to rem. at end.

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$\boxed{2^5}$

(i) $\frac{2^{70}}{96} = \frac{2^{70}}{32\times 3} = \frac{2^{70}}{2^5\times 3} = \frac{2^{65}}{3}$

$\frac{(-1)^{65}}{3} = \frac{(-1)}{3} \cancel{\times 15 \times 3}$ $(-1)+3 = 2 \times 2^5 = \underline{64}$

(ii) $\frac{3^{85}}{6} = \frac{3^{85}}{2\times 3} = \frac{3^{84}}{2} = \frac{(1)^{84}}{2} = 1\times 3 = 3$

$\boxed{3}$

<u>Fermat's Theorem</u> (if divisor is prime no apply this method)

$\frac{a^{p-1}}{p}$ p is prime, a, p coprime

1 rem $\rightarrow$ 1

(i) $\frac{2^{85}}{83} = \frac{2^{82}\times 2^3}{83} = \frac{(1)\times 2^3}{83} = 8$

(ii) $\frac{2^{106}}{53} = \frac{2^{52}\times 2^{52}}{53} \times 2^2 = \frac{(1)\times(1)\times 4}{53} = 4$

(iii) $\frac{26^{57}}{29} = \frac{26^{28}\times 26^{28}\times 26}{29} = \frac{(1)\times(1)\times 26}{29} = 26$

<u>CRT</u> (Chinese Remainder Theorem)

$\frac{N}{x*y}$ x, y are coprime

<u>Steps</u> find rem by individual division

When N divided by x rem is a
When N divided by y rem is b

Final Rem is given by

smallest no which when divided x, y gives remainder a and b respectively

(i) $\frac{(128)^{100}}{153} = \frac{(128)^{100}}{17\times 9}$

$\rightarrow \frac{(128)^{100}}{17} = \frac{((128)^{16})^6 (128)^4}{17} = \frac{(1)^6 (128)^4}{17} = \frac{(9)^4}{17}$

$= \frac{(81)(81)}{17} = \frac{(-4)(-4)}{17} = 16$

$\rightarrow \frac{(128)^{100}}{9} = \frac{(2)^{100}}{9} = \frac{(8)^{33}(2)}{9} = \frac{(-1)(2)}{9} = (-2)+9 = 7$

Rem $\rightarrow$ smallest no when divided by 17, 9 leaves rem 16, 7 respectively.

$17 \longrightarrow 16$
$9 \longrightarrow 7$

$\frac{17x+16}{9}$

$\boxed{\frac{9x+9}{9} + \frac{8x+7}{9}}$

$\therefore$ no is of the form

$17x+16$ & $9y+7$

$\therefore$ Smallest no is 16

final rem $\to 16$

Wilson Theorem

N $\to$ factorial
D $\to$ prime no.

When $\frac{(p-1)!}{p}$ is divided p!

rem $\to$ p-1

Ex: $\frac{(28)!}{29}$ = rem $\to$ 28

Important points (GATE)

$a^n + b^n$ is divisible by $(a+b)$ if n is odd

$a^n - b^n$ is divisible by $(a+b)$ if n is even

$a^n - b^n$ is always divisible by $(a-b)$, irrespective of "n"

Ex: $(17)^{21} + 10^{21} \longrightarrow$ divisible by 27.

Base System

Binary (base 2) - 0, 1

Decimal (base 10) - 0,1,2,3,4,5,6,7,8,9

(base 3) - 0, 1, 2

(base 8) "Octal" - 0,1,2,3,4,5,6,7

(base 16) Hexadecimal - 0,1,2,3,4,5,6,7,8,9, A, B, C, D, E, F

(base 12) Duodecimal - 0,1,2,3,4,5,6,7,8,9, A, B

A = 10, B = 11, C = 12, D = 13, E = 14, F = 15

Conversion

$(\quad)_{10} \longrightarrow (\quad)_{\text{other base}}$

$(120)_{10} \longrightarrow (\quad)_2$

2	120	
2	60	0
2	30	0
2	15	0
2	7	1
2	3	1
	1	1

$(1111000)_2$

$(120)_{10}$ — $(\quad)_3$

3	120
3	40 - 0
3	13 - 1
3	4 - 1
	1 - 1

$(11110)_3$

$(\quad) \longrightarrow (\quad)$
any other base

i) $(11011)_2$ — $(\quad)_{10}$

$(((1\times2+1)\times2+0)\times2+1)\times2+1 = (27)_{10}$

ii) $(440)_5$ — $(120)_{10}$

$4\times5+4 = 24\times5+0 = (120)_{10}$

iii) $(A2A)_{11} \longrightarrow (\quad)_{10}$

$10\times11+2 = 112\times11+10 = (1242)_{10}$

Decimal Numbers

$(24.375)_{10} \longrightarrow (11000.011)_2$

2	24
2	12 0
2	6 0
2	3 0
	1 1

$(11000.$

fraction $- 0.375$

$0.375\times2 = 0.750$

$0.750\times2 = 1.500$

$0.5\times2 = 1.0$

(ii) $(0.375)_{10}$ — $(.12)_4$

$0.375\times4 = 1.500$

$0.5\times4 = 2.0$

$(11000.011)_2 \longrightarrow (24.375)_{10}$

$1\times2+1 = 3\times2+0 = 6\times2+0 = 12\times2+0 = 24$

fraction $.011 = 0\times2^{-1} + 1\times2^{-2} + 1\times2^{-3}$

$= \frac{1}{4} + \frac{1}{8}$

$= \frac{3}{8}$

$.375$

Addition

$$(345)_6 + (234)_6 = (1023)_6$$

9-6=3
8-6=2
6-6=0

$(737)_{10}$
$(666)_{10}$
1403

13-10=3
10-10=0
14-10=4

(ii) $(376)_8 + (437)_8 = (1035)_8$

Subtraction

$(832)_{10} - (137)_{10} = 695$

Whenever we borrow carry we borrow base
10+2=12-7=5
10+2=12-3=9
7-1=6

i) $(832)_9 - (137)_9 = (684)_9$

9+2-5=8

ii) $(1066)_9 - (278)_9 = 677$

9+6-8=7
9+5-7=7

Some Standard Problems

Q Find the value of k such that

$$(524)_8 = (664)_k$$

Sol:- $k > 6$

$340 = 6k^2 + 6k + 4$
$6k^2 + 6k - 336 = 0$
$k^2 + k - 56 = 0$
$k = -8, 7$

$\therefore k = 7$

Q Express product of $(45)_6$ and $(20)_4$ in base 12

Sol:- $(45)_6 = (29)_{10}$ $\quad 29 \times 8 = (232)_{10}$

$(20)_4 = (8)_{10}$

$(232)_{10} \longrightarrow (174)_{12}$

12 | 232
12 | 19 - 4
   | 1 - 7

Q

Q LCM of $(41)_k$, $(36)_{k+2}$, is $(150)_{10}$
HCF of 2 no.'s is $(5)_{10}$
find k ?

Sol: $(4k+1)_{10}(3k+12)_{10} = (150)_{10} \times (5)_{10}$

$(4k+1)(k+4) = 150 \times 5$

$4k^2 + 17k + 4 = 250$

$4k^2 + 17k - 246 = 0$

$4k^2 + 41k - 24k - 246 = 0$

$k(4k+41) - 6(4k+41) = 0$

$(4k+41)(k-6) = 0$

$k = 6$ ← Ans

2 | 246
3 | 123
  | 41

$4 \times 2 \times 3 \times 4$
24

✱ How to find Unit Place

Cyclicity

2 − 2, 4, 8, 6
3 − 3, 9, 7, 1
7 − 7, 9, 3, 1
8 − 8, 4, 2, 6

n	odd	even
4^n	4	6
9^n	9	1

0, 1, 5, 6 ⟶ their cyclicity is 1
ie $0^n = 0$ $1^n = 1$ $5^n = 5$ $6^n = 6$

find unit digit of product
$373 \times 298 \times 324 \times 326$

4 6 Ans − 6

i) 2^{83} ii) 2^{82} v) 3^{983}

iii) 2^{84} iv) 2^{85} vi) 7^{542}

Concept :- divide pow by cyclicity
rem → 1, 2, 3
if rem → 0, make cyclicity as power

Sol: (i) $2^{83} = 2^3 = 8$

(ii) $2^{82} = 2^2 = 4$

(iii) $2^{84} = 2^4 = 6$ ← Here Rem. → 0

(iv) $2^{85} = 2^1 = 2$

(v) $3^{983} = 3^3 = 7$

(vi) $7^{542} = 7^2 = 9$

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Q $6^{223} \times (253)^{121} \times (747)^{782}$

Sol: $6 \times 3 \times 9$

8

Ans → 2

★ How to find last two digits (Not in Grate)

T-1

Numbers ending with 1,3,7,9.

We use following property

$$(\ldots\ldots x1)^{n} = \underset{\downarrow}{\_}\ \underline{1}$$

$(n \times x)$

Ex:- $(331)^{8} = 8\underline{4}\underline{1}$

$\downarrow$

$3 \times 8 = 2\underline{4}$

$(21)^{923} = \underline{6}\ \underline{1}$

$\downarrow$

$2 \times 923 = 184\underline{6}$

We pick unit digit of $(n \times x)$.

if no ends with 3,7,9 we try to express no in such a way that it ends with 1

$(3)^4 = 81$ $\quad (7)^4 = 2401$ $\quad (9)^2 = 81$

Ex:- $3^{723} = (3^4)^{180} \times 3^3$

$= (81)^{180} \times 27$

$= \underline{01} \times 27$

$= 27$

Ex:- $9^{820} = (9^2)^{410} = (81)^{410} = \underline{01}$

8×410

T-2

Numbers ending with 2, 4, 6, 8.

$(2)^{20} = 1048576$ $\quad$ better $\quad 2^{10} = 1024$

$(24)^{odd} \to 24$

$(24)^{even} \to 76$

We use unique property of the no 76

$(76)^n = 76$ ie any power of 76, will always yield last 2 digits as 76.

Ex:- $2^{105} = (2^{20})^5 \cdot 2^5$

$= (1048576)^5 \cdot 32$

$76 \cdot 32$

$= 32$

76
32
152
228
2432

Ex:- $4^{80} = (2^2)^{80} = (2^{160}) = (2^{20})^8 = (1048576)^8 = 76$

T-3 Number ending with 6

Q $6^{80} = 2^{80} \times 3^{80}$
$= (2^{20})^4 \times (3^4)^{20}$
$= (76)^4 \times (81)^{20}$
$= 76 \times 01$
$= 76$

T-4 Number ending with 5

No. ending with 5 will always have last 2 digits as 25.

$5^2 = 25$ $\quad$ $5^{80} = 25$
$5^3 = 125$
$5^4 = 625$

Some Standard Problems

Q find unit digit

$1^{421} + 2^{421} + 3^{421} + 4^{421} + 5^{421} + 6^{421} + 7^{421} + 8^{421} + 9^{421}$

Sol:- $1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9$
$= 45$ (use cyclicity property)
$= \underline{5}$

Q find the right most non-zero digit.

770^{3520}

Sol:- There are 3520 trailing zeros.

$(77)^{3520} \times (10)^{3520}$

↓

right-most non-zero is the unit digit of this no.

$(77)^{4\times 880} = (77^4)^{880} = (7)^4 = 2401$

right-most non-zero digit is 1.

Concept of factors (Gate)

Let $N = 120$

factor is a no which exactly divides the number

∴ factors of 120 are

1, 2, 3, 4, 5, 6, 8, 10, 12, 15, 20, 24, 30, 40, 60, 120

$N = a^p \times b^q \times c^r$ Where a, b, c are prime no.
$120 = 2^3 \times 3^1 \times 5^1$

Possible Questions on factors

1) Total no of factors ?

N = 120

Note:- always express N in terms of prime factors.

$N = 120 = 2^3 \times 3^1 \times 5^1$ ie $(3+1)(1+1)(1+1)$

$= 4 \times 2 \times 2$

$= 16$

$\therefore N = a^p \times b^q \times c^r$

$$\boxed{\text{total no of factors} = (p+1)(q+1)(r+1)}$$

2) What is sum of factors ?

$120 = 2^3 \times 3^1 \times 5^1$

$$\text{Sum of all factors} = \boxed{\frac{(a^{p+1}-1)}{a-1} \times \frac{(b^{q+1}-1)}{b-1} \times \frac{(c^{r+1}-1)}{c-1}}$$

3) What is product of factors ?

$$\boxed{(N)^{\frac{\text{total factors}}{2}}}$$

4) How many factors are odd ?

to find odd factors remove even prime no. ie 2.

$120 = 2^3 \times 3^1 \times 5^1$

odd factors $= (1+1) \times (1+1)$

$= 2 \times 2$

$= 4$

5) How many factors are even ?

= (total no of factors) − (odd factors)

6) How many factors are perfect square ?

A no is said to be a perfect square if its power is either 0 or multiple of 2

$120 = 2^3 \times 3^1 \times 5^1$

factors of $2^3 = 2^0 \times 2^1 \times 2^2 \times 2^3 \times$

$2^3 \times 3^1 \times 5^1$

factors of 2^3: $2^0, 2^1, 2^2, 2^3$

$\therefore$ factors of 2^3 are 4.

2^0 3^0 5^0

2^1 3^1 5^1

2^2

2^3

$\therefore$ factors which are perfect square $= 2 \times 1 \times 1$

$= 2$

7) Q How many factors are perfect cube?

A no. is said to be a perfect cube if its power is either 0 or multiple of 3.

$120 = 2^3 \times 3^1 \times 5^1$

(2^0) (3^0) (5^0)

2^1 3^1 5^1

2^2

(2^3)

$= 2 \times 1 \times 1$

$= 2$

8) In how many ways no. can be represented as a product of 2 factors?

$\frac{\text{total no of factors}}{2}$

9) number of coprimes which are less than the given no. ?

$X = (a^p - a^{p-1})(b^q - b^{q-1})(c^r - c^{r-1})$

10) Sum of all such coprimes ?

$= \frac{N}{2} * (X)$

Ex:- $N = 900$

$N = 2^2 \times 3^2 \times 5^2$

i) total no of factors $= (2+1) \times (2+1) \times (2+1)$

$= 27$

ii) sum of factors $= \left(\frac{2^3-1}{2-1}\right)\left(\frac{3^3-1}{3-1}\right)\left(\frac{5^3-1}{5-1}\right)$

$= 7 \times 13 \times 31$

$= 91 \times 31 = 221$

iii) product of factors $= (900)^{27/2} = (900)^{13.5}$

iv) odd factors $= (2+1)(2+1) = 9$

v) even factors $= 27 - 9 = 18$

vi) no of factors which are perfect $= 2^2 \times 3^2 \times 5^2$

(2^0) (3^0) (5^0)

2^1 3^1 5^1

(2^2) (3^2) (5^2)

$= 2 \times 2 \times 2$

$= 8$

vii) no of factors which are perfect cube $= 2^0 \; 3^0 \; 5^0$

$2^1 \; 3^1 \; 5^1$

$2^2 \; 3^2 \; 5^2$

$= 1 \times 1 \times 1$

$= 1$

viii) no of coprimes which are less than 900

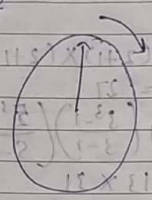
$= (2^2 - 2^1)(3^2 - 3^1)(5^2 - 5^1)$

$= 2 \times 6 \times 20$

$= 240$

Reasoning

Clock



Hour Hand (HH)
Minute Hand (MH)

MH 1hr → 360° HH 12hr → 360°

60min → 360° 1hr → 30°

1min → 6° 60min → 30°

speed of MH = 6°/min 1min → $\frac{1}{2}$°

speed of HH = $\frac{1}{2}$°/min

Their relative speed = $6° - \frac{1}{2} = \frac{11}{2}$°/min (Clock wise)

1) Angle b/w 2 hands of clock

hr min

Time a : b

$\theta = \left|30a - \frac{11}{2}b\right|$

i) 7:15

$\theta = 30 \times 7 - \frac{11}{2}(15)$

$= 210 - \frac{165}{2} \Rightarrow 210 - 82.5 = 127.5°$

ii) 8:30

$\theta = 30 \times 8 - \frac{11}{2} \times 30$

$= 240 - 165$

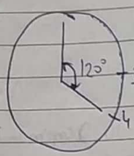
$= 75°$

Important Points to remember

1) In 1hr MH & HH coincide — once
2) In 12hr MH & HH meet (coincide) − 11 times
3) In 1hr MH & HH will be opposite to each other − once
4) In 12hr MH & HH will be opposite to each other − 11 times.
5) In 1hr MH & HH will be in straight line 2 times
6) In 12hr MH & HH will be in straight line 22 times
7) In 1hr MH & HH will make 90° angle 2 times.
8) In 12hr MH & HH will make 90° angle 22 times.

II Ex. b/w 4:00 PM – 5:00 PM

i) at what time both MH & HH meet?



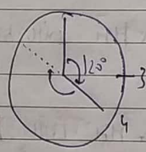
1hr — 30°
4hr → 120°

In order to meet MH has to cover 120°.

$\therefore t = \frac{120^\circ}{\frac{11^\circ}{2}} = \frac{120 \times 2}{11} = \frac{240}{11} = 21\frac{9}{11}$ min

at 4:21 min

ii) at what time both hands will be opposite to each other?



for opposite ∠ b/w them should be 180°
we know MH travels faster than HH.

$\therefore$ MH travels $= 120^\circ + 180^\circ = 300^\circ$

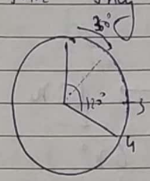
$t = \frac{300}{\frac{11^\circ}{2}} = \frac{600^\circ}{11} =$ 54 min (approx)

Ans 4:55 min

iii) At what time both hands will make 90°.

In 1hr they make 90° twice

I



MH has to travel 30°

$t = 30 \times \frac{2}{11}$

$= \frac{60^\circ}{11}$

$= 5$

at 4:05 min

MH has to travel $120 + 90 = 210^\circ$

$t = \frac{210}{\frac{11}{2}} = \frac{210 \times 2}{11}$

$= \frac{420}{11}$

$= 38$

at 4:38 min

III A man left the house at time ~~somewh~~ some where b/w 6:00 - 7:00 PM. When he returned, ~~he s~~ the clock showed time b/w 7:00 - 8:00 PM. For how long he was out?

Sol. Suppose man was out for x min

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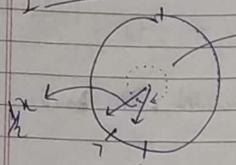
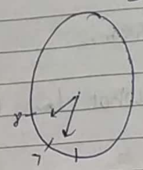
I - When he left

II When he came back
positions of the hands were exchanged





HH $\longrightarrow \frac{1}{2}°/\text{min}$

$\therefore$ in x min HH $\longrightarrow \frac{1}{2}°x$

MM in x min $\longrightarrow 6x°$

$\therefore \frac{1}{2}x + 6x = 360°$

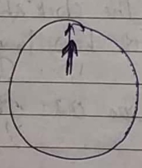
$x = 360 \times \frac{2}{13}$

$x = \frac{720}{13}$ min

$x = 55$ min (approx)

IV Suppose HH & MM meet at some time. After how much time will they meet again?

Sol.



To meet again MH has to cover 360°.

$\therefore$ they meet after $\frac{360}{\frac{11}{2}} = \frac{720}{11} = 65\frac{5}{11}$ min

$\therefore$ We can say that

Both the hands of the clock meets after every $65\frac{5}{11}$ min.

Trick 20%

Important Question

Q. Find sum of all 5 digit no's that can be formed using the digits 1, 3, 5, 7, 9 each digit exactly once.

Concept

Sol:- $\frac{n!}{n}(1+2+3+\ldots+n)[10^{n-1}+10^{n-2}+\ldots\ldots+10^0]$

how many times each no appears in every column

sum of all the digits in every column

Ex. 1, 2, 3

12	12	12
1	2	3
1	3	2
2	1	3
2	3	1
3	1	2
3	2	1
13	3	2

$2 \cdot (1+2+3)$

each digit appears exactly 2 times in every column

1200 + 120 + 12
= 1332

$2 \cdot (1+2+3)[100+10+1]$

for 5 digits 1, 2, 3, 4, 5

Sum of all 5 digits no

$= \frac{5!}{5}(1+2+3+4+5)[10^4+10^3+10^2+10^1+10^0]$

$= 24(15)(11111)$

$= 3999960$

CBT

Q What is the probability that divisor 10^{99} is multiple of 10^{96}?

Sol:- # of divisors of $10^{99} = 2^{99} \times 5^{99}$

$= (99+1)(99+1) = 10^4$

$10^{96} \times 1000 = 10^{99}$

$1000 = 2^3 \times 5^3 = (3+1)(3+1) = 16$

1000 will contain 16 divisors of 10^{99} which are multiple of ~~10~~ 10^{96}.

Required prob $= \frac{16}{10000} = 0.0016$

Blood Relation Concept

□ → male

○ → Female

□—○ Husband - wife

Important Questions

Q A person travels 285 km in 6 hr. In 2 stages.

I - stage — travel by bus at speed 40 km/hr

II - stage — travel by train at speed 55 km/hr

find distance he travelled by train?

Sol:- bus — 40 km/hr Train — 55 km/hr

avg speed $= \frac{285}{6} = 47.5$ km/hr

40 55

47.5

7.5 : 7.5 = 1 : 1

Here we have taken speeds i.e. km/hr

1 : 1 → ratio of time.

$2 \equiv 6$

$1 \equiv 3$

time ~~taken~~ travelled by train = 3 hr with 55 km/hr

distance = 3 × 55 = 165 km

Q. The present population of a town is 5000. If the number of males is ↑ by 10% & no. of females ↑ by 15%, the population of town increases to 5600. find # of males

i) by increase
ii) after increase

Sol:- 5000 ⟶ 5600
initially (finally)

Male ⟶ 10% ↑
Female ⟶ 15% ↑

% ↑ in population $\frac{600}{5000} \times 100 = 12\%$

M F
10% 15%

12%

3 2 = 3 : 2

ratio of M:F initially 3:2

according to ratio, population 3+2 = 5 parts

5 ≡ 5000
1 ≡ 1000

Male initially = 3000
finally male = (1.10) 3000 = 3300

Q. ~~Milk & Water are mixed~~. In a certain mixture of milk & water, 40% water. When 21 l of mixture is removed and then 21 l of water is added the water becomes 75% in the mixture. Find

i) Final Quantity of mixture

ii) Initial Quantity of mixture

Sol:- Mixture $\xrightarrow{-21l}$ new mixture
M : W ⟵ M : W
60% : 40% +21l water 25% : 75%

let quantity of mixture be x l

$\frac{4}{10}$ $\frac{3}{4}$

x − 21 (21 l)
$\frac{2}{5}$ 1

$\frac{3}{4}$

$\frac{1}{4}$: $\frac{7}{20}$

ratio of quantities = $\frac{1}{4} \times \frac{20}{7} = \frac{5}{7}$

$\frac{5}{7} = \frac{x-21}{21}$ 36 = x

initial quantity = 36 l
final quantity = 36 − 21 = 15 l

Q How much water be mixed with 30 l of pure milk costing Rs 16/l so that on selling the mixture at Rs 9/l a profit of 20% would be made?

Sol:- Pure milk 30 l ₹ 16/l ← CP

new mix SP = 9/l Profit = 20%

CP × (1.2) = SP

$CP = \frac{9}{1.2} = \frac{45}{6}$

(30) (x)

16 0

$\frac{45}{6}$

$\frac{96 - 45}{51}$

$-\frac{45}{6}$ $-\frac{51}{6}$

ratio of quantities = $-\frac{45}{6} \times \frac{6}{-51} = \frac{15}{17}$

$\frac{15}{17} = \frac{30}{x}$

$x = 17 \times 2 = 34 l$

Q A milk vendor has 2 cans of milk. The first contains 25% water and the rest milk. The second contains 50% water. How much milk should be mix from each of the containers so as to get 12 l of milk such that the ratio of water to milk is 3:5?

Sol:- Can 1 Can 2

M : W M : W Milk

75% 25% 50% : 50% $\frac{1}{2}$

3 : 1 1 : 1

Milk new mix 12 l milk

$\frac{3}{4}$ M : W

5 : 3

Can 2 Can 1 $\frac{3}{4} - \frac{5}{8}$

$\frac{1}{2}$ $\frac{5}{8}$ $\frac{3}{4}$ $\frac{5}{8} - \frac{1}{2}$

$\frac{1}{8}$ $\frac{1}{8}$

Ratio of quantities of milk = $\frac{1}{8} \times \frac{8}{1} = 1:1$

1 + 1 = 12

2 = 12

1 = 6 l

A 6 l from each

Q Gita buys a plot of land for ₹ 96000. She sells 2/5 of it at a loss of 6%. She wants to make a profit of 10% on the whole transaction by selling the remaining land. Gain % on the ~~whole~~ remaining land is?

Sol. SP $\frac{2}{5}$th barrel —— 6% loss

$1 - \frac{2}{5} = \frac{3}{5}$ remaining —— ? profit

Overall profit 10%

$\left(\frac{2}{5}\right)$ $\left(\frac{3}{5}\right)$

-6 $+x$

10

$x-10$ 16 $= x-10 : 16$

$\frac{x-10}{16} = \frac{2/5}{3/5}$

$\frac{x-10}{16} = \frac{2}{3}$

$3x - 30 = 32$

$x = \frac{62}{3}$ 20.67%

Q. A man purchases same mangoes at the rate of 3 for ₹40 and 5 for Rs 60. If he sells all the mangoes at the rate of 3 for ₹50, find his gain or loss %.

Sol:- T1 ⟶ 3 mangoes — ₹40
T2 ⟶ 5 mangoes — ₹60

We don't know how many mangoes of each type he purchased

for simplicity,

mangoes of each type = LCM(3,5)
= 15

T1 ⟶ $CP = \frac{15}{3} \times 40 = 200$

T2 — $CP = \frac{15}{5} \times 60 = 180$

total CP = 380 of 30 mangoes

SP — $\frac{30}{3} \times 50 = 500$

$P = 500 - 380 = 120$

$P\% = \frac{\cancel{120}^{6}}{\cancel{380}_{19}} \times 100 = \frac{600}{19}\%$

Method 2. With Alligation

CP (T1): 3 m — ₹40, 1m — $\frac{40}{3}$

CP (T2): 5m — ₹60, 1m — $\frac{60}{5}$

SP: 3 m — ₹50, 1 m — ₹$\frac{50}{3}$

(1) T2: 12 (1) T1: $\frac{40}{3}$

x ← this must also be CP

$\frac{40}{3} - x$ $x - 12$

$\frac{\frac{40}{3} - x}{x - 12} = \frac{1}{1}$

$$\frac{40/3 - x}{x - 12} = \frac{1}{1}$$

$40 - 3x = 3x - 36$

$76 = 6x$

$x = \frac{\cancel{76}^{38}}{\cancel{6}_3}$ ← CP of 1 mango

SP → $\frac{50}{3}$ CP → $\frac{38}{3}$

$P = \frac{50}{3} - \frac{38}{3} = \frac{12}{3}$

$P\% = \frac{12/3}{38/3} \times 100 = \frac{12}{38} \times 100 = \frac{600}{19}$

Q When 1l of water is added to a mixture of milk ~~and~~ water the new mixture contains 20% milk. When 1l of milk is added to the new mixture the resulting mixture contains 33% milk. The percentage (%) of milk in the original mixture was?

Sol. (Original) $x \xrightarrow{+1l\ water}$ (1) water

M : W (x+1)l

new mix $\xrightarrow{+1l\ milk}$ (1) milk

M : W

1 : 4

new mix

M : W

33% 67%

↓

$\frac{100}{33}$

(x+1) (1) milk

M : W M

1 : 4 1

(x+1) 1l

$\frac{1}{5}$ 1

$\frac{33}{100} - \frac{1}{5}$

$\frac{\cancel{100}}{\cancel{33}}$ $\frac{33}{100}$

$33 - 20$

$\frac{67}{100}$ $\frac{13}{100}$

$\frac{67}{13} = \frac{x+1}{1}$

$67 = 13x + 13$

$50 = 13x$

$x = \frac{50}{13}$ 4 (approx)

Original quantity = 4l

4 l 1 l water

x 1

$\frac{4}{5}$

$\frac{1}{5}$ $\frac{4}{5} - x$ ⇒ $\frac{1}{5} : (4-5x)/5$

$\frac{1}{5} \times \frac{5}{4-5x} = \frac{1}{4-5x}$ ⇒ $1 : (4-5x)$

$\frac{1}{4-5x} = \frac{\cancel{2}4}{1} = 1 = 16 - 20x$

$x = \cancel{\frac{15}{20}}$ $\frac{15}{20}$ $\frac{3}{4}$

~~% milk = $\frac{1}{5} \times$ ... = 20%.~~ $\frac{1}{5} \times 4 = \frac{4}{5} \times 100 = 80\%$.

% milk = $\frac{1}{4} \times 100 = 25\%$

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APTITUDE - 2

1. Surdes & Indices
2. Logrithmics
3. Time, Speed & Distance
4. Time & Work
5. PnC & Probability
6. Progression
7. Geometry & Mensuration

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Surds & Indices

Index → power

1) $a^m \times a^n = a^{m+n}$

2) $\frac{a^m}{a^n} = a^{m-n}$

3) $a^m \times b^m = (ab)^m$

4) $\frac{a^m}{b^m} = \left(\frac{a}{b}\right)^m$

5) $a^m + a^n = a^m(1+a^{n-m})$
$a^n(a^{m-n}+1)$

6) $a^{m^n} \neq a^{m\times n} = (a^m)^n$

$(2^3)^4 = 2^{12}$

7) $a^0 = 1$

8) $\sqrt{x} = x^{1/2}$ $\sqrt[3]{x} = x^{1/3}$ $\sqrt[n]{x} = (x)^{1/n}$

9) $\sqrt[n]{\sqrt[m]{x}} = (x)^{\frac{1}{mn}} = \sqrt[m]{\sqrt[n]{x}}$

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$a^n = b \Rightarrow a = b^{1/n}$

$2^5 = 32 \quad 2 = (32)^{1/5}$

Q Simplify

i) $\left(\frac{9^{-7}}{27^{-6}}\right)^{4/3}$

Sol:- $\left(\frac{9^{-7}}{27^{-6}}\right)^{4/3} = \left(\frac{3^{-14}}{3^{-18}}\right)^{4/3} \left(3^{-14+18}\right)^{4/3}$

$= (3)^{16/3}$

(ii) $12^5 \times 20^7 \times 18^4 = 2^x \times 3^y \times 5^z$ find $x+y+z$

$(2^{10} \times 3^5) \times (2^{14} \times 5^7) \times (2^4 \times 3^8) = 2^x \times 3^y \times 5^z$

$(2^{28} \times 3^{13} \times 5^7) = 2^x \times 3^y \times 5^z$

$x = 28, \; y = 13, \; z = 7$

$\therefore x+y+z = 48$

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Q $2^a = 3^b = 6^c$ find c in terms of "a" and "b"

Sol: $2^a = 3^b = 6^c = k$

$2 = k^{1/a} \quad 3 = k^{1/b} \quad 6 = k^{1/c}$

$2 \times 3 = 6 \Rightarrow k^{1/a} \times k^{1/b} = k^{1/c}$

$k^{\frac{1}{a}+\frac{1}{b}} = k^{1/c}$

$\frac{1}{a} + \frac{1}{b} = \frac{1}{c}$

$c = \frac{ab}{a+b}$

Q $2^{4m+2} = 4^{6m-4}$ find m?

Sol:- $4m+2 = 2(6m-4)$

$4m+2 = 12m-8$

$10 = 8m$

$m = 5/4$

Q find x

$x = \frac{2^n + 2^{n-2}}{2^{n+1} + 2^n}$

Sol:-

$$x = \frac{2^n + 2^{n-2}}{2^{n+1} + 2^n}$$

$$x = \frac{\cancel{2^n}(1 + 2^{-2})}{\cancel{2^n}(2^1 + 1)}$$

$$x = \frac{1 + 2^{-2}}{2^1 + 1}$$

$$x = \frac{5}{4} \times \frac{1}{3}$$

$$x = \frac{5}{12}$$

Q $\left(\frac{x^a}{x^b}\right)^{(a+b)} \left(\frac{x^b}{x^c}\right)^{(b+c)} \left(\frac{x^c}{x^a}\right)^{(c+a)}$

Sol:- $(x)^{(a-b)(a+b)} (x)^{(b-c)(b+c)} (x)^{(c-a)(c+a)}$

$= x^{a^2-b^2+b^2-c^2+c^2-a^2}$

$= x^0$

$= 1$

Surds - Irrational Number

1) Rationalization

$$= \frac{1}{2-\sqrt{2}+\sqrt{3}} \times \frac{2+(\sqrt{2}-\sqrt{3})}{2+(\sqrt{2}-\sqrt{3})}$$

$$= \frac{2+\sqrt{2}-\sqrt{3}}{4-(5+2\sqrt{6})} = \frac{2+\sqrt{2}-\sqrt{3}}{2\sqrt{6}-1} \times \frac{2\sqrt{6}+1}{2\sqrt{6}+1}$$

$$= \frac{4\sqrt{6}+2+2\sqrt{12}+\sqrt{2}-2\sqrt{18}-\sqrt{3}}{24-1}$$

$$= \frac{4\sqrt{6}+2+4\sqrt{3}+\sqrt{2}-6\sqrt{2}-\sqrt{3}}{23}$$

$$= \frac{4\sqrt{6}+3\sqrt{3}-5\sqrt{2}+2}{23}$$

2) Comparison

$\sqrt{2}$, $\sqrt[3]{3}$ $\sqrt[4]{6}$

Sol: $\sqrt{2}$, $\sqrt[3]{3}$, $\sqrt[4]{6}$

$(2)^{1/2}$, $(3)^{1/3}$ $(6)^{1/4}$

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take LCM (2,3,4) = 12

$(2)^6$, $(3)^4$, $(6)^3$

64, 81, 216

$\sqrt{2} < \sqrt[3]{3} < \sqrt[4]{6}$

(ii) $\sqrt{7}+\sqrt{8}$, $\sqrt{11}+\sqrt{4}$, $\sqrt{10}+\sqrt{5}$

Sol: $\sqrt{m}+\sqrt{n} = m+n \rightarrow$ small for all

$\sqrt{7}+\sqrt{8} = 7+8=15$
$\sqrt{11}+\sqrt{4} = 11+4=15$
$\sqrt{10}+\sqrt{5} = 10+5=15$

Squaring

$15+2\sqrt{56}$, $15+2\sqrt{44}$, $15+2\sqrt{50}$

$\downarrow$ $\downarrow$ $\downarrow$

$\sqrt{56}$, $\sqrt{44}$ $\sqrt{50}$

1 3 2

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$(\sqrt{11}+\sqrt{4}) < (\sqrt{10}+\sqrt{5}) < (\sqrt{7}+\sqrt{8})$

(ii) $\sqrt{9}-\sqrt{7}$, $\sqrt{5}-\sqrt{3}$, $\sqrt{13}-\sqrt{11}$

$\sqrt{m}-\sqrt{n} = m-n \rightarrow$ ~~same~~ Same for all

$\sqrt{9}-\sqrt{7} = 9-7=2$
$\sqrt{5}-\sqrt{3} = 5-3=2$
$\sqrt{13}-\sqrt{11} = 13-11=2$

Squaring all terms

$2-2\sqrt{63}$, $2-2\sqrt{15}$, $2-2\sqrt{143}$

2 3 1

$(\sqrt{5}-\sqrt{3}) < (\sqrt{9}-\sqrt{7}) < (\sqrt{13}-\sqrt{11})$

$\sqrt{1}=1$	$\sqrt{2}=1.414$	$\sqrt{2}-\sqrt{1}= .414$
$\sqrt{2}=1.414$	$\sqrt{3}=1.732$	$\sqrt{3}-\sqrt{2}= .318$
$\sqrt{3}=1.732$	$\sqrt{4}=2$	$\sqrt{4}-\sqrt{3}= .268$
$\sqrt{4}=2$	$\sqrt{5}=$ ~~2.23~~ 2.24	$\sqrt{5}-\sqrt{4}= .240$

We can clearly see that, as the number increases, difference b/w 2 consecutive numbers decreases.
So bigger the numbers, lesser the the value of difference b/w them

∴ $\sqrt{13}-\sqrt{11} < \sqrt{9}-\sqrt{7} < \sqrt{5}-\sqrt{3}$

(iii) $3^{3^{3^{3}}}$, 33^{33}, 333^{3}, $3^{3^{33}}$, 3^{333}
a, b, c, d, e

Concept ÷ chote no ki badi power bada hota h, bade no ki choti power se

∴ a, d, e > b, c

a		d
$3^{3^{27}}$	$<$	$3^{3^{33}}$

d > a > e

b	c
33^{33}	333^{3}

Try to expree them in nearest power of 3

$33 \rightarrow (3)^4$

$((3)^4)^{33} = (3)^{132}$

$333 \rightarrow (3)^6$

$(333)^3 = (3^6)^3 = (3)^{18}$

∴ b > c

∴ d > a > e > b > c

$3^{3^{33}} > 3^{3^{3^{3}}} > 3^{333} > 33^{33} > 333^{3}$

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Infinite Problems

(i) $\sqrt{20+\sqrt{20+\sqrt{20+\ldots\infty}}}$

$x = \sqrt{20+x}$

(ii) $\sqrt{20-\sqrt{20-\sqrt{20-\ldots\infty}}}$

$x = \sqrt{20-x}$

(iii) $\sqrt{5\sqrt{5\sqrt{5\sqrt{5\ldots\infty}}}}$ = Ans = 5

$x = \sqrt{5x}$

$x^2 = 5x$

(iv) $x = \cfrac{2}{2+\cfrac{2}{2+\cfrac{2}{2+\cfrac{2}{\ldots\infty}}}}$

$x = \dfrac{2}{2+x}$

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Imp

(V) $x = \sqrt{3\sqrt{3\sqrt{3\ldots}}}$ 10 times

Generalize solution

$\sqrt{a\sqrt{a\sqrt{a\ldots}}}$ n times $= (a)^{1-\left(\frac{1}{2}\right)^n}$

$\sqrt{3\sqrt{3\sqrt{3\ldots}}}$ 10 times $= (3)^{1-\left(\frac{1}{2}\right)^{10}} \Rightarrow (3)^{\frac{1023}{1024}}$

Square Root of Irrational Number

i) $\sqrt{5+\sqrt{24}}$ $\sqrt{2+3+2\sqrt{6}} = (\sqrt{2}+\sqrt{3})^2$

ii) $\sqrt{7+\sqrt{48}} = \sqrt{3+4+2\sqrt{12}} = (\sqrt{3}+\sqrt{4})^2$

logarithmic

Imp Properties

i) $a^x = y \qquad x \log a = \log y$

ii) $\frac{\log y}{\log a} = \log_a y$

iii) $a^x = y \qquad x = \log_a y$

iv) $\log_a y = \frac{1}{\log_y a}$

v) $\log_a y = \log_{a^m} y^m$

Ex:- $\log_{100} 16 = \log_{10} 4$

vi) $\log_{a^n} y^m = \frac{m}{n} \log_a y$

vii) $\log 1 = 0$

if no base is mentioned, then take it 10.

viii) $\log 10 = 1 \qquad \log 2 = .3010 \qquad \log 3 = .4771$

$\log 5 = .6990$

ix) $\log a + \log b = \log (ab)$

x) $\log a - \log b = \log \left(\frac{a}{b}\right)$

Ex: $\log a + \log b - \log c = \log \left(\frac{ab}{c}\right)$

V.V Imp

i) $a^{\log_a y} = y$

ii) $a^{\log_b y} = y^{\log_b a}$

$\boxed{\text{ii) } a^{\log_b y} = b^{\log_a y} \quad \times}$

Q find $\log 120$

$\log 120 = \log(10 \cdot 2^2 \cdot 3)$

$= \log 10 + \log 2^2 + \log 3$

$= 1 + 2 \times (0.3010) + 0.4771$

$= 1 + 0.6020 + 0.4771$

$= 1 + 1.0791 = 2.0791$

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Q $\log(x+3) + \log(x-3) = 72$

find x?

Sol:- $\log(x+3) + \log(x-3) = 72$

$\log((x+3)(x-3)) = 72$

$(x^2-9) = 10^{72}$

$x^2 = 10^{72} + 9$

$x = \pm\sqrt{10^{72}+9}$

Q $3\log x = 4\log y + 5$ find relation b/w x, y

Sol. $3\log x = 4\log y + 5$

$\log x^3 - \log y^4 = 5$

$\log\left(\frac{x^3}{y^4}\right) = 5$

$10^5 = \frac{x^3}{y^4}$

$x^3 = 10^5 y^4$

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Q $\log_2(\log_{\sqrt{2}}(\log_x 6561)) = 2$ find x?

Sol:- $\log_{\sqrt{2}} \log_x 6561 = 2^2$

$\log_x 6561 = 16$

$x^{16} = 6561$

$x^{16} = (81)^2$

$x^{16} = (3^4)^2$

$x^{16} = (3)^8$

$x = (3)^{1/2}$

Ans: $x = \sqrt{3}$

Q $\log_x(\log_5(\sqrt{x+5} + \sqrt{x})) = 0$ find x?

Sol:- $\log_5(\sqrt{x+5} + \sqrt{x}) = x^0$

$5 = \sqrt{x+5} + \sqrt{x}$

$5 \cancel{\sqrt{x+5}} - \sqrt{x} = \sqrt{x+5}$

$(5-\sqrt{x})^2 = (\sqrt{x+5})^2$

$25 + \cancel{x} - 10\sqrt{x} = \cancel{x} + 5$

$20 = 10\sqrt{x}$

$\sqrt{x} = 2$

$x = 4$

Q find x if $\log_2\left(1 - \frac{1}{2^x}\right) = x - 2$

Sol:- $2^{x-2} = 1 - \frac{1}{2^x}$

let $2^x = m$

$\frac{m}{2^2} = 1 - \frac{1}{m}$

$\frac{m}{4} = 1 - \frac{1}{m}$

$m^2 = 4m - 4$

$m^2 - 4m + 2 = 0$

$(m-2)^2 = 0$

$m = 2$

$\therefore 2^x = 2$

Ans $x = 1$

Q $\log_{(x+y)}(x-y) = 3$ ← given

What is value of $\log_{x^2-y^2}(x^2+2xy+y^2)$

Sol:- given $\log_{(x+y)}(x-y) = 3$

$\log_{(x^2-y^2)}(x^2+2xy+y^2) = \log_{(x^2-y^2)}(x+y)^2$

$\Rightarrow 2\log_{x^2-y^2}(x+y)$

$\Rightarrow 2\left(\frac{1}{\log_{(x+y)}(x+y)(x-y)}\right)$

$\Rightarrow \frac{2}{\log_{(x+y)}(x+y) + \log_{(x+y)}(x-y)}$

$\Rightarrow \frac{2}{1+3}$

$= \frac{1}{2}$

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Q $\log_{10}(2x+3) - 1 = \log_{10} x$

Sol:- $\log_{10}(2x+3) - \log_{10} x = 1$

$\log_{10}\left(\frac{2x+3}{x}\right) = 1$

$\frac{2x+3}{x} = 10$

$2x+3 = 10x$

$\frac{3}{8} = x$

Q given $x = y^2 = z^3 = w^4 = v^5$

find $\log_x xyzwv = ?$

Sol:- $\log_x (x \cdot x^{1/2} \cdot x^{1/3} \cdot x^{1/4} \cdot x^{1/5})$

$\log_x x^{1+1/2+1/3+1/4+1/5}$

$= \left(1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{5}\right) \log_x x$

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$= 1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{5}$

$= \frac{60+30+20+15+12}{60}$

$= \frac{137}{60}$

Q $\frac{1}{3}\log_7 x - 3\log_7 y = 1 + \log_{.125} 2$

Sol:- $\log_7\left(\frac{x^{1/3}}{y^3}\right) = 1 + \log_{1/8} 2$

$\log_7\left(\frac{x^{1/3}}{y^3}\right) = 1 + \log_{2^{-3}} 2$

$\log_7\left(\frac{x^{1/3}}{y^3}\right) = 1 - \frac{1}{3}\log_2 2$

$\log_7\left(\frac{x^{1/3}}{y^3}\right) = 1 - \frac{1}{3}$

$\frac{x^{1/3}}{y^3} = (7)^{2/3}$

$x = y^9 \times 49$

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Standard Problem

If $\log 2 = 0.3010$ $\log 3 = 0.4771$

find total no of digits in 12^{1000}

Sol:- total no of digits = characteristic value + 1

$x = 12^{1000}$

$\log x = 1000 \log 12$

$\log x = 1000 (\log 2^2 + \log 3)$

$1000 (2 \times 0.3010 + 0.4771)$

$= 1000 \; (0.6020 + 0.4771)$

$= 1000 (\cancel{0.0791} \; 1.0791)$

$= 1079.1$

$= 1079.1$ mantissa

Characteristic

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$\therefore$ total no of digits $= 1079 + 1 = 1080$

Q given $n = 40!$

find

$\frac{1}{\log_2 n} + \frac{1}{\log_3 n} + \frac{1}{\log_4 n} + \cdots\cdots + \frac{1}{\log_{64} n}$

Sol: $\frac{1}{\log_2 n} + \frac{1}{\log_3 n} + \cdots\cdots + \frac{1}{\log_{64} n}$

$= \log_n 2 + \log_n 3 + \log_n 4 + \cdots\cdots + \log_n 64$

$= \log_n (2 \cdot 3 \cdot 4 \cdot 5 \ldots 64)$

$= \log_n 40!$

$= 1$

Q If $\frac{\log a}{b-c} = \frac{\log b}{c-a} = \frac{\log c}{a-b}$

find $a^{(b+c)} \cdot b^{(c+a)} \cdot c^{(a+b)}$

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Sol: $\frac{\log a}{b-c} = \frac{\log b}{c-a} = \frac{\log c}{a-b}$

$a^{(b+c)} b^{(c+a)} c^{(a+b)} = x$

$(b+c)\log a + (c+a)\log b + (a+b)\log c = \log x$

$\log a = k(b-c)$

$\log b = (c-a)k$

$\log c = (a-b)k$

$(b+c)k(b-c) + (c+a)k(c-a) + (a+b)k(a-b) = \log x$

$= k(b^2-c^2+c^2-a^2+a^2-b^2) = \log x$

$k \cdot 0 = \log x \Rightarrow 10^0 = x$

$x = 1$

Q $\log_x y = \frac{5}{4}$ $\log_z y = \frac{5}{6}$ $\log_2 x = 3M$

find M?

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$\log_x y = \frac{5}{4} \Rightarrow \frac{\log y}{\log x} = \frac{5}{4} \Rightarrow \frac{\log x}{\log y} = \frac{4}{5}$ — ①

$\log_z y = \frac{5}{6} \Rightarrow \frac{\log y}{\log z} = \frac{5}{6}$ — ②

Multiplying ① × ②

$\frac{\log x}{\log z} = \frac{4}{5} \times \frac{5}{6}$

$\log_z x = \frac{2}{3} \Rightarrow 3M = \frac{2}{3} \Rightarrow M = \frac{2}{9}$

Q $\log_2 3 \times \log_3 4 \times \log_4 5 \times \dots \times \log_{63} 64$

Sol: $\frac{\cancel{\log 3}}{\log 2} \times \frac{\cancel{\log 4}}{\cancel{\log 3}} \times \frac{\cancel{\log 5}}{\cancel{\log 4}} \times \dots \times \frac{\cancel{\log 63}}{\cancel{\log 62}} \times \frac{\log 64}{\cancel{\log 63}}$

$= \frac{\log 64}{\log 2} =$

Ans = 6

SPEED TIME DISTANCE

Km/hr $\longrightarrow$ m/sec $\times \frac{5}{18}$

m/sec $\longrightarrow$ Km/hr $\times \frac{18}{5}$

distance = speed $\times$ time

if time = constant

d $\propto$ speed

ie as $s\uparrow$ $d\uparrow$

if distance = constant

speed $\propto \frac{1}{\text{time}}$ ie as $s\uparrow$ $t\downarrow$

Q While going at a speed of $\frac{4}{5}$th the actual speed a man reaches 25 min late. If he travels at actual speed, what is the time taken?

Sol:- Let actual speed be 1

ratio = $\frac{\text{actual}}{\text{new}} = \frac{1}{4/5} = \frac{5}{4}$ ie 5:4

$\therefore$ ratio of time $\frac{1}{5} : \frac{1}{4} \Rightarrow 4:5$

$5x - 4x = 25$ min (given)

$x = 25$

$\therefore$ actual time = $4 \times 25 = 100$ min

Q If speed of a person is increased by 20% he reaches 15 min early. What is the initial time taken by the person?

Sol:- speed ratio $\Rightarrow$ actual : new

1 : 120/100

5 : 6

ratio of time $\Rightarrow$ 6 : 5

$6x - 5x = 15$

$x = 15$

$\therefore$ actual time = $6 \times 15 = 90$ min

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Q If a man goes at a speed of 20 km/hr he reaches 15 min late.
But if he goes at speed of 25 km/hr he reaches 10 min early.

find
a) distance
b) actual time
c) speed at which he reaches in actual time.

Sol:- to find time

ratio of speed ⇒ 20 : 25
4 : 5

ratio of time ⇒ 5 : 4

we take diff b/w the time

$5x - 4x = +15 - (-10)$
$x = 25$

∴ if speed is 20 km/hr $t = 5 \times 25 = 125$ min

125 min is 15 min more than the actual time.
∴ actual time = ~~110 min~~ 110 min

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to find distance

Let total distance be x km (actual distance)

$t_1 \rightarrow \left(\frac{x}{20}\right)$ $t_2 \rightarrow \left(\frac{x}{25}\right)$

$t_1 - t_2 = +15 - (-10)$

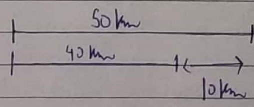
$\frac{x}{20} - \frac{x}{25} = 25$ min

$\frac{5x - 4x}{100} = \frac{25}{60}$ ⇒ $x = \frac{25}{60} \times 100 = \frac{125}{3}$ km

actual speed $= \frac{D}{t} = \frac{125}{3} \times \frac{60}{110} = \frac{250}{11}$ km/hr

(Important)

Q Without stoppages a train travels 50 km in an hour
With stoppages train travels 40 km in an hour.
What is the stoppage time in min per hour?

Sol: 

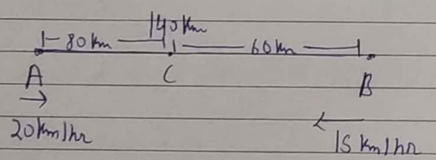
∴ Whatever time train has spent in stoppage, in same time it would have covered 10 km.

∴ time taken to cover 10 km = stoppage time.

$$t = \frac{10\,km}{50} = \frac{1}{5}\,hr = \frac{60}{5} = 12\,min$$

Relative speed Concept

1) 2 objects towards each other.



a) at what time will they meet?

b) at what distance from A will they meet?

Sol:- Relative speed concept:-

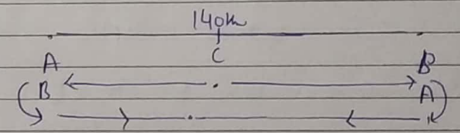
A & B will effectively have to reduce 140 km to 0 to meet each other

$$\therefore t = \frac{140}{(20+15)} = \frac{140}{35} = 4\,hr$$

∴ they meet after 4 hr

they meet at point C, its distance from A is $= 20 \times 4 = 80\,km$

c) after how much time will they meet for the second time?



effectively A & B when they meet second time would have travelled (140×3)

$$\therefore t = \frac{140 \times 3}{35} = 12\,hr$$

We can generalize this concept as:-

1st time meet = 4hr

2nd time meet = 4 + 4×2hr = 12hr

3rd time meet = 4 + 4×4hr = 20hr

4th time meet = 4 + 4×6hr = 28hr

(Important) (Train Questions)

Q

A B

A> Starts 6:00AM → reach : 12:00PM

reach : 11:AM ← start : 6:00 AM <B

a) At what time both trains will meet.?

Sol:

A (6hr) B (5hr)

In such questions where dist & speed not given take distance as multiple of both the times ie LCM.

$d = LCM(6,5)$

$\therefore d = 30km$ $s_A = \frac{30}{6} = 5km/hr$ $s_B = 6km/hr$

$\therefore$ they meet after $= \frac{30}{(6+5)} = \frac{30}{11}$ hr

b) At what distance from A's starting point do they meet?

Sol: $d = 5 \times \frac{30}{11} = \frac{150}{11}$ km $= 13.6$ km (approx)

Q

A B

A> Starts: 6:00AM → reach: 11:00 AM

reach: 12:00 PM ← Starts: 8:00AM <B

a) After how much time do they meet?

b) How far from A do they meet? $\frac{16}{3}$ km

Sol:-a) 20km

A (5hr) B (4hr)

$S_A = 4km/hr$ $d = LCM(5,4)$ $S_B = 5km/hr$

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∵ A starts early we have to bring both A & B to same starting time to ~~time~~ apply relative motion

In 2hr's A would have covered = $4 \times 2 = 8$ km

∴ t (meet time) = $\frac{12 \text{ km}}{9 \text{ km/hr}} = \frac{4}{3}$ hr

i.e. after 8:00 AM they meet after $\frac{4}{3}$ hr i.e. $1\frac{1}{3}$ hr i.e. at 9:20 AM.

II) Objects Moving in same direction (Thief & Police Questions)

Suppose thief escapes from prison

A
20 km/hr

After 2 hr, police came to know chases thief at speed of 25 km/hr

a) After how long will police be able to catch the thief?

b) How far from prison, will thief gets caught?

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Soln:- In 2hr thief had covered = 40 km/hr

40 km

P → 25 km/hr

T → 20 km/hr

In order to catch the thief police has to ~~reduce this gap~~ cover this gap of 40 km while chasing the thief.

$t = \frac{40}{(25-20)} = \frac{40}{5} = 8$ hr

distance from Jail = $25 \times 8 = 200$ km

Concept of Trains

I

200 m

20 m/s

after how much time train will cross a pole?

$t = \frac{200}{20} = 10$ sec

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II

200m

20m/s

300m (platform)

after how much time will it cross the platform?

$t = \frac{500}{20} = 25$ sec.

III

200m

20m/s

5m/s

time taken to cross?

Same

$t = \frac{200}{15}$

$= \frac{40}{3}$

$= 13.33$ sec

Opposite

$t = \frac{200}{25}$

$= 8$ sec

IV

200m

20m/s

100m

10m/s

time taken to cross?

Same

$t = \frac{300m}{10} = 30$ sec

Opposite

$t = \frac{300m}{30} = 10$ sec

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V

200m

20 m/s

100m

10 m/s

a) Time taken by faster train to cross a person sitting in a slower train?

Sol: Person can be sitting anywhere inside a train hence we will not consider the length of train in which person is sitting

Concept

$t = \frac{200}{10} = 20$ sec

b) Time taken by person in faster train to ~~cover~~ cross slower train?

Sol:- $t = \frac{100}{10} = 10$ sec

VI A train cross a pole in 15 sec. The same train cross a platform of length 300m in 25 sec. find?

i) speed of train?

ii) length of train?

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Sol: difference b/w the time = ~~18~~ 25 - 15 = 10 sec.

[∴ In this 10 sec train
∴ 300m has been covered by] X

Hence, this 300m must have been covered in 10 sec

∴ speed of train $S = \frac{300}{10} = 30$ m/sec

∴ length of train = $30 \times 15 = 450$m

VII 2 trains of same length takes 20 sec to (TOI) ~~opposition~~ opposite direction.
35 sec to (TOI) same direction. find ratio of speeds!

Sol:- let Length be L

$\frac{2L}{S_1+S_2} = 20$ $\frac{2L}{S_1-S_2} = 35$

$20(S_1+S_2) = 35(S_1-S_2)$

$\frac{S_1+S_2}{S_1-S_2} = \frac{35}{20}$

Using componendo & Dividendo

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$\frac{2S_1}{2S_2} = \frac{55}{15}$

Ans:- $\frac{S_1}{S_2} = \frac{11}{3}$

Boats/streams Concept

if speed of water $V = 0$ is still water

(D.S) down stream = $U + V$ U = speed of boat
V = speed of stream

(U.S) up stream = $U - V$

speed of boat (U) = $\frac{DS + US}{2}$

Speed of stream (V) = $\frac{DS - US}{2}$

Q A boat takes 10 hr in downstream travelling a distance of 200 km. it takes 20 hr travelling same distance upstream. find?

Speed of boat?
Speed of stream!

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Sol. $DS = \frac{200}{10} = 20$ km/hr

$UP = \frac{200}{20} = 10$ km/hr

Speed of boat (u) = $\frac{DS + UP}{2} = 15$ km/hr

speed of stream (v) = $\frac{DS - UP}{2} = 5$ km/hr

Q Speed boat is 10 km/hr. stream is 3 km/hr. A boat takes 20hr to travel a distance D.S and U.S. find distance?

Sol: Let distance be d

$\frac{d}{13} + \frac{d}{7} = 20$

$d = \frac{20 \times 91}{20}$

$d = 91$ km

Q If speed of Boat 10km/hr. A boat has to travel 91 km going D.S and then 91km going upstream. Total time 20 hr. speed of stream?

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Sol:- $\frac{91}{10+x} + \frac{91}{10-x} = 20$

Let speed of stream be x km/hr

$\frac{20}{100-x^2} = \frac{20}{91}$

$x^2 = 9$

$x = \pm 3$

∴ speed of stream is 3 km/hr.

Races (Linear & Circular)

Linear

1 In a race of 100m, A beats B by 20m. and C by 30m. Then in a race of 100m by how much metres will B beats C?

Sol:

A	B	C
100	80	70

ie when A covers 100 m, B covers 80m and C covers 70m

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We can say that in race is of 80m, B beats C by 10m

$80m\ B \longrightarrow 10m\ C$

$1m\ B \longrightarrow \frac{10}{80} m\ C$

$\therefore 100m\ B \longrightarrow \frac{10}{80} \times 100\ m\ C$

12.5 m

II In a race of 100m A beats B by 20m. In another race of 100m, B beats C by 30m. Then in a race of 100m, by how ~~much~~ many metres will A beat C?

Sol:

A	B		B	C
100	80		100m	70m

$\frac{A}{B} = \frac{100}{80}$ $\qquad$ $\frac{B}{C} = \frac{100}{70}$

If A covers 100m
then B covers 80m

$\frac{A}{B} \times \frac{B}{C} = \frac{100}{80} \times \frac{100}{70} \Rightarrow \frac{A}{C} = \frac{100}{56}$

i.e When A covers 100m, C covers 56m.
Hence A beats C by 44m.

(imp)
III In a race of 100m, A beats B either by 20m or 5sec & . find speed of A & B?

Sol:

A	B
100	80

given A beats B either by 20m or by 5sec means when A is at the finish line B ~~still~~ ~~has~~ has to lags behind by 5sec.
$\therefore$ to complete the race, B must have covered 20m in these 5sec.

$\therefore$ speed of B $= \frac{20}{5} = 4m/s$.

$\therefore$ time taken by B to cover 100m $\rightarrow \frac{100m}{4} = 25sec$.

This means A must have ~~cov~~ covered 100m in 20sec.

$\therefore$ speed of A $= \frac{100}{20} = 5m/sec$

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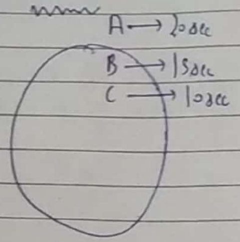
Circular Races



If all three started together from same point after how much time they meet for first time at starting point?

Sol: LCM(S_A, S_B, S_C)

LCM(20, 15, 10) = 60 sec

A → 20, 40, 60, ...

B → 15, 30, 45, 60, ...

C → 10, 20, 30, 40, ~~4~~50, 60, ...

Note: Whether they move in same direction or opposite direction, we will follow same approach ie LCM(S_A, S_B, S_C)

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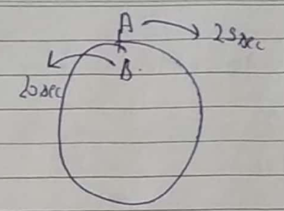
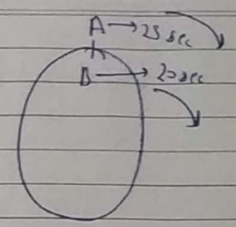
II





When will they meet for first time?

Sol: Let length of track be D

I) Same direction

$t = \dfrac{D}{\frac{D}{20} - \frac{D}{25}} = 100\text{ sec}$ $\qquad S_A = \dfrac{D}{25}$ $\quad S_B = \dfrac{D}{20}$

II) Opposite direction

$t = \dfrac{D}{\frac{D}{25} + \frac{D}{20}} = \dfrac{100}{9}\text{ sec.}$

$S_A = \dfrac{D}{25}$ $\qquad S_B = \dfrac{D}{20}$

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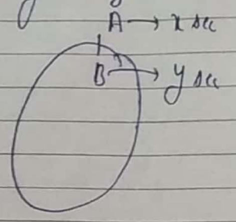
We can generalize the result.

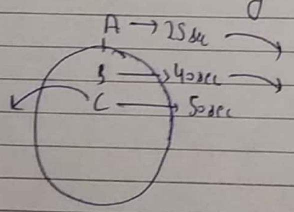


When will they meet for 1st time?

i) Same direction $\frac{xy}{x-y}$

ii) opposite direction $\frac{xy}{x+y}$

III)



When will they meet for 1st time?

Sol:- In such questions, fix any one

Lets say we fix A.

find when will A & B meet for first time (AB)

find when will A & C meet for first time. (AC)

All three meet for first time LCM (AB, AC)

$AB \rightarrow \frac{25 \times 40}{15} = \frac{200}{3}$ sec

$AC \rightarrow \frac{25 \times 50}{75} = \frac{50}{3}$ sec.

$ABC \rightarrow LCM\left(\frac{200}{3}, \frac{50}{3}\right) = \frac{LCM(200, 50)}{HCF(3,3)} = \frac{200}{3}$

∴ Ans: All will meet for first time after 66.66 sec

* Distinct point concept → (ME Aptitude book)

Time And Work

1) A → 10 days
B → 15 days
C → 20 days

a) if they work together, in how many days can they finish the job.?

Unitary method	
total work = 1 Here we assume ↑	total work = LCM(10, 15, 20) = 60
A's 1 day → $\frac{1}{10}$	A's 1 day → 6
B's 1 day → $\frac{1}{15}$	B's 1 day → 4
C's 1 day → $\frac{1}{20}$	C's 1 day → 3
(A+B+C)'s 1 day work	(A+B+C)'s 1 day → (6+4+3) = 13
$\left(\frac{1}{10}+\frac{1}{15}+\frac{1}{20}\right) = \frac{13}{60}$	13 units → 1 day ∴ 60 units → $\left(\frac{60}{13}\right)$ days
∴ (A+B+C) can complete work in $\left(\frac{60}{13}\right)$ days	∴ (A+B+C) can complete the work in $\left(\frac{60}{13}\right)$ days

b) They started working together. A left after 2 days of starting of work, B left 1 day before completion of work, find total time taken to complete the work?

Sol:- Lets say total work is completed in x days

A's 1 day → $\frac{1}{10}$	A's 1 day → 6
B's 1 day → $\frac{1}{15}$	B's 1 day → 4
C's 1 day → $\frac{1}{20}$	C's 1 day → 3
$2\times\frac{1}{10} + (x-1)\frac{1}{15} + x\times\frac{1}{20} = 1$ (total work)	$2\times 6 + (x-1)4 + x\cdot 3 = 60$
$\frac{2}{10} + \frac{(x-1)}{15} + \frac{x}{20} = 1$	$12 + 4x - 4 + 3x = 60$
$12 + 4x - 4 + 3x = 60$	$7x = 60 - 8$
$7x = 60 - 8$	$x = \frac{52}{7}$ days
$x = \frac{52}{7}$ days	

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c) They started working together. A left after 2 days of starting of work. B left after 1 day leaving of A.

find ?

i) Total time taken to complete the work?
ii) How much time will C take to do the remaining work?

Sol:- total work = 60

Let total time taken to complete the task be x days

A's 1 day → 6

B's 1 day → 4

C's 1 day → 3

$2 \times 6 + 3 \times 4 + \cancel{(x-3)} x \times 3 = 60$

$12 + 12 + 3x - \cancel{18} = 60$

$3x = 60 - 24$
$3x = 36$
$x = 12$ days

now, C worked for $12 - 3 = 9$ days alone

d) A started working alone. B joined A after 2 days. C joined then after 1 days of B joining. Find total time taken to complete the work?

Sol: total work = 60 units

A's 1 day → 6

B's 1 day → 4

C's 1 day → 3

Suppose total time taken to complete the work be x days

$x \times A + (x-2)B + (x-4)C = 60$

$x \times 6 + (x-2)4 + (x-4)3 = 60$

$6x + 4x - 8 + 3x - 12 = 60$

$13x = 80$

$x = \left(\frac{80}{13}\right)$ days

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Efficiency $\propto$ Work

III> a) 20 men can make 200 chairs working 8 hr per day in 25 days. 40 men will take how many days in making 1000 chairs working 10 hr per day?

Sol:

$$\boxed{\frac{\text{men} \times \text{days} \times \text{hr} \times \text{efficiency}}{\text{Total work}} = \frac{\text{men} \times \text{days} \times \text{hr} \times \text{efficiency}}{\text{Total work}}}$$

$$\frac{20 \times 25 \times 8}{200} = \frac{40 \times x \times 10}{1000}$$

$x = 50$ days

b) 30 men can do a work in 25 days working with 80% efficiency. 50 men will take how many days work with 100% efficiency?

Sol: Note:- total work done by both 30 men & 50 men is same.

$30 \times 25 \times 80 = 50 \times x \times 100$

$$x = \frac{30 \times 25 \times 80}{50 \times 100} = 12 \text{ days}$$

$x = 12$ days

c) A man takes 25 days 80% efficiency. How much will be taken by him if he works with 100% efficiency?

Sol:- $25 \times 80 = x \times 100$

$$x = \frac{25 \times 80}{100} = 20$$

$x = 20$ days

IV> In digging / making of wall

20 men takes 20 days in making wall of dimension $10 \times 2 \times 2$ ft. 30 men will take how many days in making wall with dimension $15 \times 3 \times 2$ ft?

Sol:- $$\frac{20 \times 20}{10 \times 2 \times 2} = \frac{30 \times x}{15 \times 3 \times 2}$$

$$x = \frac{20 \times 20 \times 15 \times 3 \times 2}{10 \times 2 \times 2 \times 30}$$

$x = 30$ days

III) Working in pairs

A+B ⟶ 10 days
B+C ⟶ 15 days
C+A ⟶ 20 days

a) if A, B & C work together, so in how many day they can finish the work?

Sol:- Total work = LCM (10, 15, 20)
= 60

A+B's 1 day ⟶ 6

B+C's 1 day ⟶ 4

C+A's 1 day ⟶ 3

(A+B)+(B+C)+(C+A) = 13

2(A+B+C) = 13

$A+B+C = 13/2$

60 Units ⟶ $\frac{60}{13/2} = \frac{120}{13}$ days

b) how much time is taken to complete the work by

i) A alone
ii) B alone
iii) C alone

Sol:- i) $A+B+C = 13/2$ B+C ⟶ 4 ⇒ A ⟶ $\frac{13}{2} - 4$

A ⟶ 2.5

∴ to complete 60 units ⟶ $\frac{60}{2.5}$ = 24 days

Sol: ii) $A+B+C = 13/2$ A+C ⟶ 3 ⇒ B ⟶ $\frac{13}{2} - 3 = \frac{7}{2}$

∴ to complete 60 units ⟶ $\frac{60}{3.5}$ ⇒ $\frac{120}{7}$ = $\cancel{17}$ days

Sol:- iii) $A+B+C = 13/2$ A+B ⟶ 6

C ⟶ $\frac{13}{2} - 6 = \frac{1}{2}$

∴ to complete 60 units ⟶ $\frac{60}{0.5}$ = 120 days

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IV) Work and Wages

Concept : $\boxed{\text{Salary} \propto \text{work}}$

a) A → 10 days
B → 15 days
C → 20 days

if A, B & C worked together to finish the job. They received total pay of Rs 5200.

find how much is received by each?

Sol: Total Work = LCM (10, 15, 20)
= 60

A's 1 day → 6
B's 1 day → 4
C's 1 day → 3

∴ ratio of their work = 6 : 4 : 3

A's Salary = $\frac{6}{13} \times 5200 = 2400$

B' salary = $\frac{4}{13} \times 5200 = 1600$

C's Salary = $\frac{3}{13} \times 5200 = 1200$

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b) A → 10 days
B → 20 days
with the help of C they completed the work in 5 days. They received a total salary of 20,000 find wages of C?

Sol: total work = LCM (10, 20, 5)
= 20

A's 1 day → 2
B's 1 day → 1
(A+B+C)'s 1 day → 4

∴ C's 1 day → 4 - (2+1) = 1

ratio of work = 2 : 1 : 1

C's wage = $\frac{1}{4} \times 20,000 = 5000$

(Imp)

V) Garrison / Army / Soldiers

a) Enough food is available for 200 soldiers for 50 days. After 20 days ~~50 soldiers~~

i) 50 soldiers left
ii) 50 soldiers added

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For how many days remaining food will lasts?

Sol: ~~It~~ We assume 1 soldier consumes 1 kg per day

$\therefore$ 200 soldier consumes (200×1) kg per day

$\therefore$ Total food we have $= 200 \times 50$ kg
$= 10,000$ kg

In 20 days food consumed $= 200 \times 20$
$= 4000$ kg

food left $= 10,000 - 4000 = 6000$ kg

i) 50 soldiers left.

now 150 soldier are left to consume food per day they will consume 150 kg

$\therefore$ 6000 kg food will last for $= \frac{6000}{150}$ kg $= 40$ days

ii) 50 soldiers added

250 soldiers will consume 250 kg per day
$\therefore$ 6000 kg food will last for $= \frac{6000}{250} = 24$ days

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(Imp)

VI) Efficiency based

A is twice as fast as B. (A is twice efficient as B)
B is thrice as fast as C. If A, B, C ~~worker~~ work together they take 30 days to complete the work?

In how many days work can be completed by

i) A alone
ii) B alone
iii) C alone

Sol: Concept:- The most efficient one is assumed to be x

Let say A takes x days to complete the work.

$\therefore$ B takes $= 2x$ days

C takes $= 6x$ days

$\frac{1}{x} + \frac{1}{2x} + \frac{1}{6x} = \frac{1}{30}$ (assume total work $= 1$)

$\frac{1}{x}\left(\frac{6+3+1}{6}\right) = \frac{1}{30} \Rightarrow x = \cancel{30}^{5} \times \frac{10}{\cancel{6}} = 50$ days

A alone will take = 50days
B " " " = 100 days
C " " " = 300 days

VII) Alternate Working Concept

A → 10 days
B → 15 days
C → 20 days

if they work on alternate days
i) Starting with A, followed by B and C.
ii) Starting with B, followed by C and A.
In how many days will the work be completed?

Sol:- Total Work = 60

i) $\underbrace{A \quad B \quad C}_{1 \text{ cycle}}, \underbrace{A \quad B \quad C}, \underbrace{A \quad B \quad C}, \ldots$

A's 1day → 6
B's 1day → 4
C's 1day → 3

~~(A+B+C)'s 1day work =~~ 1 cycle = 3 days

1 cycle work = (A+B+C) = 13

∴ To complete 60 units = $\frac{60}{13}$ cycles

ie 4 cycles (12 days) = 52 unit

remaining 8 unit work need to be completed

in 13^{th} day A works = 8 - 6 = 2 unit rem

14^{th} day B works = $\frac{2}{4} = \frac{1}{2}$ day

∴ Work is completed in $13\frac{1}{2}$ days

ii) $\underbrace{B \quad C \quad A}_{1 \text{ cycle}}, \underbrace{B \quad C \quad A}, \underbrace{B \quad C \quad A}, \ldots$

1 cycle = 3 days   Work done in 1 cycle = 13

∴ 60 units will take = $\frac{60}{13}$ cycles $4\frac{8}{13}$ cycles

4 cycles (12 days) = 52 units.

remaining 8 unit work need to be done.

13th day $\Rightarrow$ B works = 8-4 = 4 unit remaining

14th day C work = 4-3 = 1 unit remaining

15th day A works = $\frac{1}{6}$ days

$\therefore$ total work is done in $14\frac{1}{6}$ days

VIII) Equality Concept

a) 3 men or 5 boys $\longrightarrow$ 20 days

find 2 men and 2 boys to do same work $\longrightarrow$ total time ?

Sol:- 3m $\longrightarrow$ 20 days (assume total work = 1)

1m $\longrightarrow$ 3 X 20 days $\Rightarrow$ 60 days

1 men's 1 day $\longrightarrow \frac{1}{60}$

5b $\longrightarrow$ 20 days

1b $\longrightarrow$ 5 X 20 = 100 days

1 boy's 1 day $\longrightarrow \frac{1}{100}$

We have to find time taken by 2m + 2b

$2 \times \frac{1}{60} + 2 \times \frac{1}{100} = \frac{1}{30} + \frac{1}{50} = \frac{8}{150}$

$\therefore$ (2m + 2b) 1 day $\longrightarrow \frac{8}{150}$

$\therefore$ total time taken to complete work = $\frac{150}{8}$ days

b) 5m or 4B or 3G $\longrightarrow$ 10 days

find $\boxed{1 \text{ men} + 2B + 2G} \longrightarrow$?

Sol:- 5m — 10 4B $\rightarrow$ 10 3G — 10

1m — 5 X 10 = 50 days B — 10 X 4 = 40 days G — 30 days

1 day — $\frac{1}{30}$

$\left(1 \times \frac{1}{50} + 2 \times \frac{1}{40} + 2 \times \frac{1}{30}\right)$

$= \frac{1}{50} + \frac{1}{20} + \frac{2}{30} \Rightarrow \boxed{\frac{300}{6+15+20} \times \frac{300}{41}} \quad \frac{41}{300}$

total time = $\cancel{\frac{41}{300}}$ $\frac{300}{41}$ days

In such questions find association b/w m & B.

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(Imp)

(I) 10 m and 12 B ⟶ 5 days

6 m and 4 B ⟶ 10 days

How much time taken by (5m + 3B) ?

Sol: 10m + 12B — 5 days (given)

If we want to make them finish work in 1 day then their capacity need to be increased

10 × 5m + 12 × 5 B ⟶ 1 days

50 m + 60 B ⟶ 1 days — ①

6 m + 4 B — 10 days (given)

they can do the work in 1 day if their capacity increases

6m × 10 + 4B × 10 — 1 day

60 m + 40 B — 1 day — ②

from eqn ① and ② both the groups have same capacity, hence we can equate them

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50 m + 60 B = 60 m + 40 B

20 B = 10 m

2B = 1m

i.e. capacity of 1 m = capacity of 2B.

(5m + 3B) ⟶ ? ⇒ (10B + 3B) = 13 B ⟶ ?

10 m + 12 B ⟶ 5 days

10 × 2B + 12 B ⟶ 5 days

32 B ⟶ 5 days

∴ 1 B — 32 × 5 days

13 B ⟶ $\frac{32 \times 5}{13}$ days = $\frac{160}{13}$ days

(Imp)

X) Replacement Concept

10 m ⟶ 200 days.

They started working, after 20 days 10 men more joined them with their triple capacity. find time taken to complete the work

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Sol: We assume

1 man in 1 day can do 1 ~~work~~ unit work

∴ 10 men in 1 day can do 10 work

∴ (work done by) x

total work = 10 × 200 = 2000 work

work done by 10 m in 20 days = 200 work

(2000 − 200) = 1800 work remaining

+ 10 m joined with triple capacity
1 m in 1 day can do 3 work
∴ 10 m 1 day ~~work~~ can do 30 work

∴ total capacity of men/day = 10 work + 30 work
= 40 work

now they can do 40 work in 1 day

∴ 1800 work takes = $\frac{180\cancel{0}}{4\cancel{0}}$ = 45 days

∴ total time taken to complete work = 20 + 45 = 65 days

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Pipes & Cisterns

1) A ⟶ 10 hr
B ⟶ 15 hr
C ⟶ empty 20 hr

if all 3 taps are opened, after how much time tank is filled?

Sol: total capacity = LCM (10, 15, 20)
= 60 L

A 1 hr ⟶ 6 L
B 1 hr ⟶ 4 L
C 1 hr ⟶ −3 L

(A + B + C) ⟶ 6 + 4 − 3 = 7 L

∴ to fill 60 L ⟶ $\frac{60}{7}$ hr

b) they are opened together. Tap B was closed after 3 hr. Tap C was closed after 2 hr of tap B. find total time to fill the tank?

Sol: Let total time taken to fill tank is x hr

total capacity = 60 L

$x \times 6 + 3 \times 4 - 5 \times 3 = 60$

$6x + 12 - 15 = 60$

$6x = 63$

$x = \frac{21}{2}$ hr

or

total capacity = 1 l

$x \times \frac{1}{10} + 3 \times \frac{1}{15} + -\frac{1}{20} \times 5 = 1$

$x = \frac{21}{2}$ hr

c) A $\longrightarrow$ 4hr
B $\longrightarrow$ 5hr
C $\longrightarrow$ empty 6hr

Open tap A $\longrightarrow$ 6:00 PM
B $\longrightarrow$ 7:00 PM
C $\longrightarrow$ 8:00 PM

at what time tank is filled?

Sol: total capacity = LCM(4, 5, 6)
= 60 l

A 1hr $\longrightarrow$ 15 l
B 1hr $\longrightarrow$ 12 l
C 1hr $\longrightarrow$ -10 l

suppose "x" hr to fill the tank

$15 \times x + (x-1) \times 12 + (x-2)(-10) = 60$

$15x + 12x - 12 + (-10x) + 20 = 60$

$17x = 52$

$x = \frac{52}{17}$ hr $\quad 3\frac{1}{17}$ hr $\Rightarrow$ 3 hr 3 min

$\therefore$ tank was filled ~~after~~ at ~~3hr~~ 9:03 PM.

(Imp)

d) A fill $\longrightarrow$ 10hr
B fill $\longrightarrow$ 15hr
C empty

If they work together, tank was filled in 8hr. tap C empties 15 l / min.
find the capacity of tank

Sol: total capacity = LCM(10, 15, 8)
= 120

A 1hr — 12 l
B 1hr — 8 l
(A+B-C) 1hr — 15 l

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C 1hr — 20-15 = 5l

$\therefore \left[\begin{array}{l} 5l \text{ empty} \text{ — } 1hr \\ \text{?} \end{array}\right] x$

$\therefore$ C can empty 5l in 1hr

to empty 60l → $\frac{60}{5}$ = 24 hr

$\therefore$ given C empties 15l water / min

1 min → 15l
1hr → 60×15l
24 hr → 24×60×15l

$\therefore$ capacity of tank = 21600 l

(Imp)

i) A → 10hr
B → 12hr
C → empty 20hr

if all are opened alternatively ~~A~~, B, C starting from A followed by B & C.

find total time to fill the tank?

Sol:- $\underbrace{A\ B\ C}\ \underbrace{A\ B\ C}\ \underbrace{A\ B\ C}$ ....

total capacity = LCM(10, 12, 20)
= 60 l.

A 1hr → 6 l
B 1hr → 5l
C 1hr → -3l

1 cycle = 3hr

1 cycle work = 6+5-3 = 8l

60 l → $\frac{60}{8}$ cycles

7 cycle (21 hr) → 56l.

60-56l = 4 l remaining

22nd hr → $\frac{4}{6} = \frac{2}{3}$

$\therefore$ total time to fill tank = $21\frac{2}{3}$ hr

or 21 hr 40 min

(Imp)

1) Ratio of diameters of 3 taps is given 1:2:3 if tap with maximum diameter fills tank in 10hr. The find?

i) time taken by tap with minimum diameter

ii) time taken to fill the tank if all 3 are opened together.

Sol:- Concept 1: $\boxed{\text{time taken} \propto \frac{1}{\text{area of cross section}}}$

ratio of diameters: 1 : 2 : 3

ratio of area of crosssection : 1 : 4 : 9 → ratio of time $\frac{1}{1} : \frac{1}{4} : \frac{1}{9}$ → 36 : 9 : 4

∴ tap Area of cross 9 → 10hr

4 ≡ 10

∴ tap Area of cross section 1 → 90 hr

1 ≡ 10/4

36 ≡ $\frac{36 \times 10}{4}$

90hr

AC = area of cross section

tap of AC 9 → 10hr
1 → 90hr

When all three are open, then total AC = 9+4+1 = 14

tap of AC 14 — $\frac{90 hr}{14}$ = $\frac{45}{7}$ hr

Imp

Q 5m & 3B → 12 days & 1m + 3B → 60 days
How much time taken by 4B & 3M ?

Sol:- Let man capacity "m" & a boy capacity be b

∴ 5m + 3b = 12 days   total work = 60 units
1m + 3b = 60 days

∴ 5m + 3b = 5 unit/day
1m + 3b = 1 unit/day } → m = 1 unit/day, b = 0 unit/day

∴ 4b + 3m = 3 unit/day
∴ for 60 unit → $\frac{60}{3}$ = 20 days.

Q A is 20% less efficient than B. B → 40 days
U = 200 units   A+B → ?

B → 40
A → $\frac{40}{80\%}$ = $\frac{40 \times 5}{4}$ = 50

B → 5 U   A+B = 9U
A → 4 U
∴ 200U → $\frac{200}{9}$ ⇒ $22\frac{2}{9}$

Permutations & Combinations (Covered in Aptitude -1 Notebook)

Q $1\times1! + 2\times2! + 3\times3! + 4\times4! + \ldots\ldots + 20\times20!$

Sol: $(2-1)\times1! + (3-1)\times2! + (4-1)\times3! + \ldots\ldots + (21-1)20!$

$= \cancel{2!} - 1! + \cancel{3!} - \cancel{2!} + \cancel{4!} - \cancel{3!} + \cancel{5!} - \cancel{4!} + \ldots\ldots + 21! - \cancel{20!}$

$= 21! - 1$

Q find $^{50}C_4 + {}^{50}C_3 + {}^{51}C_3 + {}^{52}C_3$

Sol: $^{n}C_r + {}^{n}C_{r-1} = {}^{n+1}C_r$

$\underbrace{^{51}C_4 + {}^{51}C_3} + {}^{52}C_3$

$= {}^{52}C_4 + {}^{52}C_3$

$= {}^{53}C_4$

Imp Prop: $^{n}C_r = \frac{n}{r}\,{}^{n-1}C_{r-1}$

Q given a word MATHEMATICS

1) no of words containing all vowels together

Sol: $\boxed{AAEI}$ + 7 unit = 8 units

M - 2
T - 2
A - 2

no of words = $\frac{8!}{2!\,2!} \times \frac{4!}{2!}$

b) no of words in which all repeated alphabets are together

$\boxed{MMAATT}$ + 5 unit = 6 unit $\quad 6! \times \frac{6!}{2!2!2!}$

c) Vowels will come only at even places

_ 1 _ 2 _ 3 _ 4 _ 5 _ 6 _ 7 _ 8 _ 9 _ 10 _ 11

Vowels - AAEI $\quad {}^{5}C_4 \times \frac{4!}{2!} \times \frac{7!}{2!\times2!}$

Q given a word INSTITUTE

a) How many 5 letter words can be formed?

I - 2
T - 3
N
S
U
E

Here we have to consider all different cases

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I - 2 T-3 N S U E

I) all alphabets are distinct

${}^6C_5 \times 5!$

II) 2 rep + 3 distinct

For rep we have 2 choice.

${}^2C_1 \times {}^5C_3 \times \frac{5!}{2!}$

III) 2 rep + 2 rep + 1 distinct

${}^2C_2 \cdot 1 \cdot {}^4C_1 \times \frac{5!}{2!2!}$

IV) 3 rep + 2 distinct

$1 \cdot {}^5C_2 \times \frac{5!}{3!}$

V) 3 rep + 2 rep $\rightarrow$ $1 \cdot 1 \cdot \frac{5!}{3!2!} = 10$

Ans:- I + II + III + IV = 720 + 1200 + 120 + 200 + 10
= ~~1160~~ 2250

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RANK

- find rank of word PENCIL in dictionary

Sol:- PENCIL (C E I L N P)

C _ _ _ _ _ 5!
E _ _ _ _ _ 5!
I _ _ _ _ _ 5!
L _ _ _ _ _ 5!
N _ _ _ _ _ 5!
P C _ _ _ _ 4! (C E I L N ~~P~~)
P E C _ _ _ 3! (C ~~E~~ I L N ~~P~~
P E I _ _ _ 3!
P E L _ _ _ 3!
P E N C I L ~~4~~-1 (~~C~~ ~~E~~ ~~I~~ ~~L~~ ~~N~~ ~~P~~)
~~P~~ ~~E~~ ~~N~~ ~~C~~

rank = 5×120 + 24 + 6×3 + 1 =
= 600 + 24 + 18 + 1
= 643 th rank

ii) NESTLE - find rank

Here we have repetition

(EE L N S T) ← arrange lexicographically

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NESTLE (EELN ST)

E _ _ _ _ _ 5! ($\boxed{E}$ ELNST)
↳ Here one E is fixed
so, we have to arrange ELNST
here all distinct letters

L _ _ _ _ _ ($\frac{5!}{2!}$) → EENST → repetition

N E _ _ _ _ 4! (EEL~~N~~ST)

N E L _ _ _

NESTLE (EELNST)

E _ _ _ _ _ 5!
L _ _ _ _ _ 5!/2! (EENST)
N E E _ _ _ 3! (~~E~~EL~~N~~ST)
N E L _ _ _ 3!
N E S E _ _ 2! (~~E~~EL~~NS~~T)
N E S L _ _ 2!
N E S T E L - 1 (~~E~~EL~~NST~~)
N E S T L E - 1

120 + 60 + 12 + 4 + 2 = 198th rank

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finding Diagonals in a polygon

Given a polygon of n-sides, number of diagonals are

diagonals : ${}^nC_2 - n = \frac{n(n-3)}{2}$

Concept :- Since diagonal is also a line, we can get diagonal by joining 2 points.

In this ${}^nC_2$ we have also included sides (i.e lines forming the sides of a polygon) Hence we need to remove them

Q find the no of sides in a polygon whose diagonals are 3 times of sides?

Sol:- $\frac{n(n-3)}{2} = 3n$

$n = 6 + 3$

$n = 9$

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Number of Integral Solutions

1) $x + y = 5$

i) no of positive ~~soln~~ integral soln

$x + y = 5$

x	y
1	4
2	3
3	2
4	1

$\} \; 4$

ii) no of non-negative integral soln

$x + y = 5$

x	y
0	5
1	4
2	3
3	2
4	1
5	0

$\} \; 6$

2) $x + y + z = 10$

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i) +ve integral soln

${}^{n-1}C_{r-1}$ $\quad n = 10 \quad r = 3$

${}^{10-1}C_{3-1} = {}^{9}C_{2} = 36$

ii) non-negative integral soln

${}^{n+r-1}C_{r-1}$ $\quad n = 10 \quad r = 3$

${}^{10+3-1}C_{3-1} = {}^{12}C_{2} = 66$

2) $x + y + z = 10 \quad x \geq 2, \; y \geq 1, \; z \geq 1$

i) find the non-negative integral soln

$x = X + 2 \quad y = Y + 1 \quad z = Z + 1$

$\underbrace{x - 2}_{X} \geq 0 \quad \underbrace{y - 1}_{Y} \geq 0 \quad \underbrace{z - 1}_{Z} \geq 0$

$X \geq 0, \; Y \geq 0, \; Z \geq 0$

$X + Y + Z = 10 - 2 - 1 - 1$

$X + Y + Z = 6$

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$^{n-1}C_{r-1}$ $n=6$ $r=3$

$^{5}C_2 = 10$

3) $3x+y+z=12$

i) find positive integral soln

$x>0, y>0, z>0$

put the value of x until RHS becomes 0

$x=1$ $y+z=9 \rightarrow {}^{8}C_{1}=8$

$x=2$ $y+z=6 \rightarrow {}^{5}C_1=5$

$x=3$ $y+z=3 \rightarrow {}^{2}C_1=2$

$x=4$ $y+z=0$ no soln (bcz 2 +ve no's can never sum up to zero)

$\therefore$ no of +ve integral soln $=8+5+2=15$

ii) find non-negative integral soln

put value of x from 0

$x=0$ $y+z=12 \rightarrow {}^{13}C_1=13$

$x=1$ $y+z=9 \rightarrow {}^{10}C_1=10$

$x=2$ $y+z=6 \rightarrow {}^{7}C_1=7$

$x=3$ $y+z=3 \rightarrow {}^{4}C_1=4$

$x=4$ $y+z=0$ ~~no soln~~ 1 when $y=0$, $z=0$

no of non-negative soln $=13+10+7+4=34$ $+1$

Application of integral soln

$x+y \le 12$, $x\ge 0$, $y\ge 0$

Δ is formed by x-axis, y-axis & line.

i) find no of integral points that lie inside & on the side of the triangle

ii) no of integral pairs that lie inside the Δ.

iii) no of integral pairs that lie on side of Δ only

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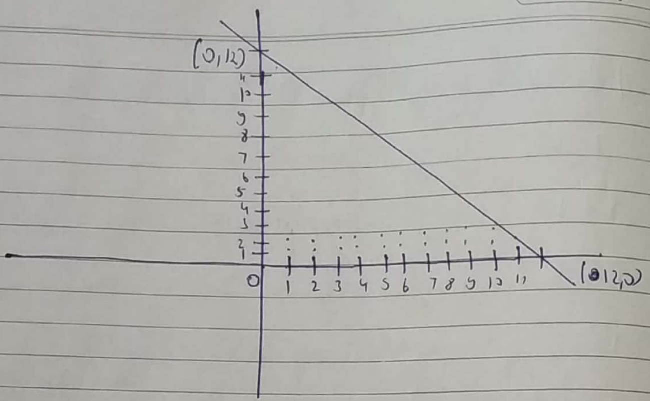

(i) point that lie on side or within Δ

$13 + 12 + 11 + 10 + \ldots + 1$

$= \frac{n(n+1)}{2} = \frac{13(14)}{2} = 91$

[13 → (0,0), (0,1), (0,2), ... (0,12)
12 → (·] ✗

13 → (0,0), (1,0), (2,0), (3,0) ..... (12,0)

12 → (0,1) (1,1) (2,1) (3,1) ...... (11,1)

11 → (0,2) (1,2) (2,2) (3,2) .... (10,2)

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this will not be considered bcz they lie on side

(ii) pairs which lie inside Δ

[13 − (0,0) (1,0) .... (11,0) (12,0) = 11]
X X

12 − (0,1) (1,1) (2,1) .... (10,1) (11,1) = 10
X X

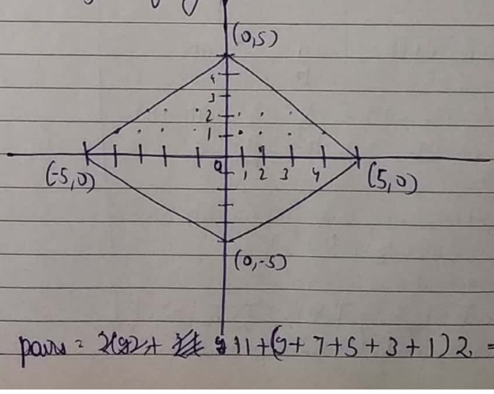
11 − (0,2) (1,2) (2,2) .... (9,2) (10,2) = 9
X X

$\therefore 10 + 9 + 8 + 7 + \ldots + 1 = \frac{10(11)}{2} = 55$

iii) pairs which lie on side Δ = 91 − 55 = 36

2) $|x| + |y| = 5$

no of integral pairs (x,y) that lie within or on the surface of graph



1 (0,1)
3 (1, 2)
5 (2+3)
7 (3+4)
9 (4+5)
11 (5+6)
9
7
5
3
1

total pairs = ~~2(9)+~~ 11 + (9+7+5+3+1)2 = 61

Progression (A·P / G·P / H·P)

A·P	G·P	H·P
1, 5, 9, 13, 17, ...	1, 2, 4, 16, 32, ...	$\frac{1}{3}, \frac{1}{5}, \frac{1}{7}, \frac{1}{9}, ...$
2, 8, 14, 20, 26, ...	2, 6, 18, 54, ...	$\frac{1}{13}, \frac{1}{17}, \frac{1}{21}, \frac{1}{25}, ...$
	(common ratio)	$\boxed{\text{reciprocal of terms AP}}$
a = first term	a = First term	
d = common difference	r = common ratio	
T_n = n^{th} term	$T_n = ar^{n-1}$	$T_n = \frac{1}{a+(n-1)d}$
$T_n = a+(n-1)d$	$S_n = \begin{cases} \frac{a(r^n-1)}{r-1} & r>1 \\ \frac{a(1-r^n)}{1-r} & r<1 \end{cases}$	
$S_n = \frac{n}{2}(T_1 + T_n)$		
$S_n = \frac{n}{2}(2a + (n-1)d)$		
Assume	Assume	
3 terms - $(a-d), a, (a+d)$	$\frac{a}{r}, a, ar$	
4 terms - $(a-3d), (a-d), (a+d), (a+3d)$	$\frac{a}{r^3}, \frac{a}{r}, ar, ar^3$	
5 term - $(a-2d), (a-d), a, (a+d), (a+2d)$	$\frac{a}{r^2}, \frac{a}{r}, a, ar, ar^2$	
$T_1 + T_n = T_2 + T_{n-1} = T_3 + T_{n-2} = ...$	$T_1 \times T_n = T_2 \times T_{n-1} = T_3 \times T_{n-2}$	
Same sum, terms equidistance from centre	Same product	

Important

$$AM \geq GM \geq HM$$

$$AM = \frac{a+b+c+ ..... + T_n}{n}$$

$$GM = (abc .... T_n)^{1/n}$$

$$HM \Rightarrow \frac{2}{b} = \frac{1}{a} + \frac{1}{c} \Rightarrow \frac{1}{\frac{(a+b+c+ ..... + T_n)}{n}}$$

Q $-6, -\frac{11}{2}, -5, ...$ $S_n = 0$

find $n = ?$

$a = -6$ $d = -\frac{11}{2} - (-6) = \frac{1}{2}$

$S_n = \frac{n}{2}(2a + (n-1)d)$

$0 = n\left(2 \times (-6) + (n-1) \times \frac{1}{2}\right)$

$0 = n\left(-12 + \frac{(n-1)}{2}\right) \Rightarrow \cancel{-12n + \frac{n(n-1)}{2} = 0}$

$\Rightarrow n = 0$ or $-12 + \frac{(n-1)}{2} = 0$

$\frac{-24 + (n-1)}{2} = 0$

$n = 25$

Q find sum of all 2 digit no when divided by 7 it leaves remainder 2.

Sol:- 16, 23, 30, ... 93

[last term $T_n = 16 + (n-1)7$] x

& $T_n = 16 + (n-1)7 = 93$

$16 + 7(n-1) = 93$

$7n - 7 = 93 - 16$

$7n = 84$

$n = 12$

$S_n = \frac{12}{2}[16 + 93]$

$= 6(109)$

$= 654$

Q Sum of n terms $S_n = n^2 + 2n$ find A.P ?

Sol: $S_n = n^2 + 2n$

$S_1 = T_1 = 1^2 + 2(1) = 3$

$S_2 = 4 + 4 = 8$

$T_2 = 8 - 3 = 5$

$T_3 = S_3 - (T_2 + T_1)$

$= (9 + 6) - (5 + 3)$

$= 15 - 8$

$= 7$

$T_1 = 3 \quad d = 2$

3, 5, 7, ...

Q given $T_7 = 30 \quad T_{13} = 54$, find $S_{30} = ?$

Sol: $T_7 = a + 6d = 30$ — (i)

$T_{13} = a + 12d = 54$ — (ii)

(ii) − (i) ⇒ $6d = 24$

$d = 4$

$30 = a + 24 \Rightarrow a = 6$

$S_{30} = \frac{30}{2}[2 \times 6 + (30-1)4]$

$= 15[12 + 116]$

$= 15(128)$

$= 1920$

Date : Page :

Q Ratio of sum of first n terms of 2 AP's is $\frac{2n-1}{3n+2}$. find the ratio of 10^{th} terms?.

Sol:- $\frac{S_n}{S_n} = \frac{2n-1}{3n+2} = \frac{\frac{n}{2}(2a_1+(n-1)d_1)}{\frac{n}{2}(2a_2+(n-1)d_2)}$

$\frac{S_1}{S_2} = \frac{2a_1+(n-1)d_1}{2a_2+(n-1)d_2} = \frac{a_1+\left(\frac{n-1}{2}\right)d_1}{a_2+\left(\frac{n-1}{2}\right)d_2}$ —— (i)

We have to find

$\frac{(a_1)_{10}}{(a_2)_{10}} = \frac{a_1+9d_1}{a_2+9d_2}$ —— (ii)

On comparing (i) & (ii)

$9 = \frac{n-1}{2} \Rightarrow n = 19$

$\frac{S_1}{S_2} = \frac{2(19)-1}{3(19)+2} = \frac{37}{59}$

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Q Sum of 3 terms of a GP is 31 and sum of their squares is 651

Sol: GP 3 terms → $\frac{a}{r}, a, ar$ ← better when product is given

$$\left[\begin{array}{l} \frac{a}{r}+a+ar=31 \\ \left(\frac{a}{r}\right)^2+a^2+(ar)^2=651 \end{array}\right]$$

Let us assume 3 terms a, ar, ar^2

$a+ar+ar^2=31$ $\quad$ $a^2+a^2r^2+a^2r^4=651$

$\frac{a^2+a^2r^2+a^2r^4}{a+ar+ar^2} = \frac{651}{31}$

$\frac{1+r^2+r^4}{1+r+r^2} = \frac{651}{31}$

$\frac{(1+r^2)^2-r^2}{1+r+r^2} = \frac{651}{31}$

$\frac{\cancel{(1+r+r^2)}(1-r+r^2)}{\cancel{(1+r+r^2)}} = \frac{\cancel{651}\,21}{\cancel{31}}$

$\cancel{31}(1-r+r^2) = \cancel{651}\,21$

$1 - r + r^2 = 21$
$r^2 - r - 20 = 0$
$r^2 - 5r + 4r - 20 = 0$
$(r-5)(r+4) = 0$
$r = 5$

$a + ar + ar^2 = 31$
$a(1+5+25) = 31$
$a = 1$

GP → ~~x,4~~ 1, 5, 25, ...

<u>Sequence & Series</u>

$\Sigma n = \frac{n(n+1)}{2}$

$\Sigma n^2 = \frac{n(n+1)(2n+1)}{6}$

$\Sigma n^3 = \left[\frac{n(n+1)}{2}\right]^2$

Sum of first "n" odd $= n^2$

Sum of first "n" even $= n(n+1)$

Concept :- Finding T_n term

Ex:- $S = 1\times2 + 2\times3 + 3\times4 + \ldots\ldots + 100\times101$

Sol: $S = 1\times2 + 2\times3 + 3\times4 + \ldots\ldots + 100\times101$

$T_n = n(n+1) = n^2 + n$

$S_n = \Sigma n^2 + \Sigma n$

$= \frac{n(n+1)(2n+1)}{6} + \frac{n(n+1)}{2}$

$= \frac{n(n+1)}{2}\left[\frac{(2n+1)}{3} + 1\right]$

$= \frac{n(n+1)}{2}\left[\frac{2n+4}{3}\right]$

$= \frac{n(n+1)(2n+4)}{6}$

put $n = 100$

$= \frac{\cancel{100}^{50}(101)(\cancel{204}^{68})}{\cancel{6}_{3}}$

$= 343400$

2525
× 203
7575
0000×
5050××
512575

101
× 25
505
202×
2525

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Ex: $1\times2^2 + 2\times3^2 + 3\times4^2 + \ldots\ldots + 20\times21^2$

Sol:- $S = 1\times2^2 + 2\times3^2 + 3\times4^2 + \ldots + 20\times21^2$

$T_n = n\times(n+1)^2$

$= n(n^2+2n+1)$

$= n^3+2n^2+n$

$S_n = \Sigma n^3 + 2\Sigma n^2 + \Sigma n$

$= \frac{n^2(n+1)^2}{4} + 2\frac{n(n+1)(2n+1)}{6} + \frac{n(n+1)}{2}$

$= \frac{n(n+1)}{2}\left[\frac{n(n+1)}{2} + \frac{2(2n+1)}{3} + 1\right]$

$= \frac{n(n+1)}{2}\left[\frac{n^2+n+8n+4+2}{6}\right]$

$= \frac{n(n+1)}{2}\left[\frac{n^2+9n+6}{2}\right]$

$= \frac{n(n+1)(n^2+9n+6)}{4}$

put $n=20$

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$S_n = \frac{20(21)(400+180+6)}{4}$

$= \frac{\cancel{20}^{10}(21)(\cancel{586})\ 293}{\cancel{4}_{2}}$

$2(10)(21)(293)$

$= 61530$

293
$\times 21$
293
$586\times$
6153

Imp

Ex:- $1\times3 + 3\times5 + 5\times7 + \ldots\ldots + 51\times53$

Sol: $T_n = (2n-1)(2n+1)$

$= 4n^2-1$

$S_n = 4\Sigma n^2 - \Sigma(1)$

$= 4\frac{n(n+1)(2n+1)}{6} - n$

$= \frac{2n(n+1)(2n+1)}{3} - n$

put $n=26$

$(2n-1) = 51$

$n=26$

$= \frac{2(26)(\cancel{27}^{9})(53)}{\cancel{3}} - 26$

$= 24804 - 26$

$= 24778$

52
$\times 53$
156
$260\times$
2756
$\times 9$
24804

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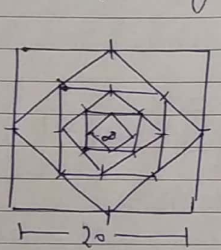
Infinite GP

$a, ar, ar^2, ar^3, \ldots \ldots \infty$

$S_\infty = \frac{a}{1-r} \qquad |r| < 1$

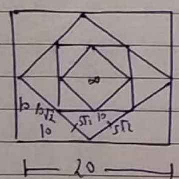
Ex. Squares are formed by joining the mid-points of square. This process goes on



i) find sum of areas of all such squares

ii) find ~~perimeter~~ of sum of perimeters of all such squares

Sol:- i)



Square 1 : $S_1 = 400$

$S_2 = 200$

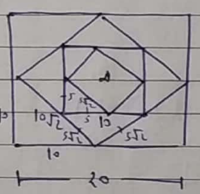
$S_3 = 100$

$400, 200, 100, \ldots \ldots$ <- forms a GP $\quad r = \frac{1}{2}$

GP $\quad S_\infty = \frac{a}{1-r} \qquad r < 1$

$S_\infty = \frac{400}{1-1/2} = 800$

ii)



$S_1 = 80$

$S_2 = 40\sqrt{2}$

$S_3 = 40$

$S_4 = 20\sqrt{2}$

$80, 40\sqrt{2}, 40, \ldots \ldots$ <- form a GP $\quad r = \frac{1}{\sqrt{2}} < 1$

$S_\infty = \frac{a}{1-r} = \frac{80}{1-\frac{1}{\sqrt{2}}} = \frac{80\sqrt{2}}{\sqrt{2}-1}$

$S_\infty = \frac{80\sqrt{2}}{\sqrt{2}-1}$

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Arithmetico - Geometric Progression

Ex $S_\infty = 1+2x+3x^2+4x^3+\ldots\ldots \infty$ $|x|<1$

Sol :- $S_\infty = 1+2x+3x^2+4x^3+\ldots \infty$ —(i)

multiply (i) by x (ie common ratio)

$xS_\infty = x+2x^2+3x^3+4x^4+\ldots \infty$ —(ii)

(i) − (ii)

$S_\infty(1-x) = 1+x+x^2+x^3+\ldots \infty$

$S_\infty(1-x) = \frac{1}{1-x}$

$S_\infty = \frac{1}{(1-x)^2}$

Ex. $S_\infty = 3+7x+11x^2+15x^3+\ldots \infty$

$xS_\infty = 3x+7x^2+11x^3+15x^4+\ldots \infty$

$S_\infty(1-x) = 3+4x+4x^2+4x^3+4x^4+\ldots \infty$

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$S_\infty(1-x) = 3+4\left[x+x^2+x^3+\ldots \infty\right]$

$(1-x)S_\infty = 3+4\left(\frac{x}{1-x}\right)$

$S_\infty = \frac{3}{(1-x)} + \frac{4x}{(1-x)^2}$

Concept :- Summation by Difference

Ex. $S_n = 3+7+13+21+31+\ldots\ldots + T_n$

Sol :- $S_n = 3+7+13+21+31+\ldots\ldots + T_{n-1} + T_n$

$0 = 3+4+6+8+10+\ldots\ldots + (T_n - T_{n-1}) - T_n$

$T_n = 3+4+6+8+10+\ldots\ldots + (T_n - T_{n-1})$

$T_n = 3 + [\underbrace{4+6+8+10+\ldots + (T_n - T_{n-1})}_{n-1 \text{ term}}]$

$T_n = \underbrace{2+[4+6+8+10+\ldots + (T_n-T_{n-1})]}_{n \text{ term}} + 1$

$T_n = \frac{n}{2}(2\times 2 + (n-1)2) + 1$

$T_n = n(2+n-1)+1$

$T_n = n(n+1)+1$
$T_n = n^2+n+1$

$S_n = \Sigma n^2 + \Sigma n + \Sigma(1)$

$S_n = \frac{n(n+1)(2n+1)}{6} + \frac{n(n+1)}{2} + n$

Ex:- $S_n = 5+7+13+31+85+\ldots\ldots + T_n$

Sol. $S_n = 5+7+13+31+85+\ldots\ldots T_{n-1} + T_n$

$0 = 5+2+6+18+54+\ldots(T_n - T_{n-1}) - T_n$

$T_n = 5 + [\underbrace{2+6+18+54+\ldots(T_n-T_{n-1})}_{n-1 \text{ terms}}]$ ← form GP

$T_n = 5 + \frac{(2)(3^{n-1}-1)}{3-1}$

$T_n = 5 + 3^{n-1} - 1$

$S_n = 5\Sigma(1) + \Sigma 3^{n-1} - \Sigma(1)$

$= 5n + \frac{1}{3}\Sigma 3^n - n$

$5n + \frac{1}{3}\frac{(3)(3^n-1)}{(3-1)} - n$

$S_n = 4n + \frac{3^n-1}{2}$

Miscellaneous

Ex:- $S = \frac{1}{1\times2} + \frac{1}{2\times3} + \frac{1}{3\times4} + \ldots\ldots + \frac{1}{50\times51}$

Sol:- Difference b/w terms in Denominator is 1

$S = \left(\frac{1}{1} - \frac{1}{2}\right) + \left(\frac{1}{2} - \frac{1}{3}\right) + \left(\frac{1}{3} - \frac{1}{4}\right) + \ldots \left(\frac{1}{50} - \frac{1}{51}\right)$

$S = \frac{1}{1} - \cancel{\frac{1}{2}} + \cancel{\frac{1}{2}} - \cancel{\frac{1}{3}} + \cancel{\frac{1}{3}} - \cancel{\frac{1}{4}} + \ldots + \cancel{\frac{1}{50}} - \frac{1}{51}$

$S = 1 - \frac{1}{51}$

$S = \frac{50}{51}$

Ex. $S = \frac{1}{1\times3} + \frac{1}{3\times5} + \frac{1}{5\times7} + \ldots\ldots + \frac{1}{99\times101}$

Sol: Difference b/w terms in Denominator is 2

We divide by 2 to nullify 2 in Numerator

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$$S = \frac{1}{2}\left[\left(\frac{1}{1}-\frac{1}{3}\right)+\left(\frac{1}{3}-\frac{1}{5}\right)+\left(\frac{1}{5}-\frac{1}{7}\right)+\cdots+\left(\frac{1}{99}-\frac{1}{101}\right)\right]$$

$$= \frac{1}{2}\left[\frac{1}{1}-\cancel{\frac{1}{3}}+\cancel{\frac{1}{3}}-\cancel{\frac{1}{5}}+\cancel{\frac{1}{5}}-\cancel{\frac{1}{7}}+\cdots+\cancel{\frac{1}{99}}-\frac{1}{101}\right]$$

$$= \frac{1}{2}\left[\frac{1}{1}-\frac{1}{101}\right]$$

$$= \frac{50}{101}$$

Imp
Ex: $\frac{1}{1\times2\times3}+\frac{1}{2\times3\times4}+\cdots\cdots+\frac{1}{10\times11\times12}$

Sol:- We have to select pairs such that Numerator is same.

$$S = \frac{1}{2}\left[\left(\frac{1}{1\times2}-\frac{1}{2\times3}\right)+\left(\frac{1}{2\times3}-\frac{1}{3\times4}\right)+\cdots\cdots+\left(\frac{1}{10\times11}-\frac{1}{11\times12}\right)\right]$$

$$= \frac{1}{2}\left[\frac{1}{(1\times2)}-\frac{1}{11\times12}\right]$$

$$= \frac{1}{2}\left(\frac{1}{2}-\frac{1}{132}\right) \Rightarrow \frac{1}{2}\left(\frac{66-1}{132}\right) \Rightarrow \frac{65}{264}$$

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Ex. (1), (2,3), (4,6,5), ...

find sum of all terms in 40^{th} bracket

Sol: 40^{th} bracket will have 40 terms

$\therefore n = 40$
$d = 1$ ($\because$ AP)

To find $a = ?$ ie first term

"a" for 2^{nd} bracket = (no of terms in 1^{st} bracket + 1)

"a" for 3^{rd} bracket = (no of term in 1^{st} bracket + no of terms in 2^{nd} bracket + 1)

$\therefore$ To find "a" in 40^{th} bracket

(Sum of no. of terms in 1 − 39 bracket) + 1

$1+2+3+4+\ldots\ldots+39 = \frac{39\cdot40}{2} = 780+1 = 781$

$a = 781$

Sum of all terms in 40^{th} bracket

$= \frac{n}{2}(2a+(n-1)d)$

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$= \frac{40}{2}(2 \times 781 + (40-1)1)$

$= 20(1562 + 39)$

$= 20(1601)$

$= 32020$

Ex. no of common terms b/w 2 APs.

3, 7, 11, 15, ... 403

5, 11, 17, 23, ... 505

Sol: first common term = 11

To find next common term = [LCM (d_1, d_2)]

$d_1 = 7-3 = 4$

$d_2 = 11-5 = 6$

LCM $(4, 6) = 12$

∴ Common terms

11, 23, 35, ...

the n^{th} common term (or the last common term) ≤ 403

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$11 + (n-1)12 \leq 403$

$12n - 1 \leq 403$

$n \leq \frac{\cancel{404}\ 202}{\cancel{12}\ 6}$

$n \leq 33.67$

$n = 33$

∴ There are 33 common terms

Ex: $\frac{1}{3^2-2^2} + \frac{1}{7^2-2^2} + \frac{1}{11^2-2^2} + \dots\dots + \frac{1}{39^2-2^2}$

$= \frac{1}{(1 \times 5)} + \frac{1}{(5 \times 9)} + \frac{1}{(9 \times 13)} + \dots\dots + \frac{1}{(37 \times 41)}$

$= \frac{1}{4}\left[\left(\frac{1}{1} - \cancel{\frac{1}{5}}\right) + \left(\cancel{\frac{1}{5}} - \cancel{\frac{1}{9}}\right) + \left(\cancel{\frac{1}{9}} - \cancel{\frac{1}{13}}\right) + \dots\dots + \left(\cancel{\frac{1}{37}} - \frac{1}{41}\right)\right]$

$= \frac{1}{4}\left[\frac{1}{1} - \frac{1}{41}\right]$

$= \frac{10}{41}$

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Ex: $5 + 55 + 555 + \ldots\ldots$ n terms

$S_n = 5[1 + 11 + 111 + \ldots\ldots \text{ n terms}]$

$= \frac{5}{9}[9 + 99 + 999 + \ldots\ldots \text{ n term}]$

$= \frac{5}{9}[(10-1) + (100-1) + (1000-1) + \ldots\ldots \text{ n terms}]$

$= \frac{5}{9}[(10 + 10^2 + 10^3 + \ldots + 10^n) - n]$

$= \frac{5}{9}\left[\frac{10(10^n - 1)}{(10-1)} - n\right]$

$= \frac{5}{9}\left[\frac{10}{9}(10^n - 1) - n\right]$

Ans $= \frac{5}{9}\left[\frac{10^{n+1} - 10 - 9n}{10}\right]$

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Probability

Ex. "APPLE"

What is the probability of word in which both P's are together?

Sol: $\text{Prob} = \frac{\text{no of favourable}}{\text{total cases}}$

favourable $= 4! \times 1$ total $= \frac{5!}{2!}$

$\text{Prob} = \frac{4!}{\frac{5!}{2!}} = \frac{2}{5}$

Ex: 3B + 4G arranged in a row

What is the probability that all boys sit together?

Sol: $\text{Prob} = \frac{5! \times 3!}{7!}$

$= \frac{\cancel{5!} \times \cancel{6}}{7 \times \cancel{6} \times \cancel{5!}}$

$= \frac{1}{7}$

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Odds in favour = $\frac{\text{favourable}}{\text{unfavourable}}$

odds against = $\frac{\text{unfavourable}}{\text{favourable}}$

Ex: Prob = $\frac{1}{4}$ favourable = 1 unfavourable = 3

Odds in favour = $\frac{1}{3}$

odds against = $\frac{3}{1}$

Dices

Ex: If 2 dices are thrown What is the probability that same no appears on both the dice?

Sol: Prob = $\frac{6}{36}$ (1,1) (2,2) (3,3) (4,4) (5,5) (6,6)

Ex. 2 dices are thrown What is prob that sum on faces ≤ 4

Sol. Sum = can be 2, 3, 4.

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2 – (1,1)
3 – (1,2) (2,1)
4 – (1,3) (3,1) (2,2)

Prob = $\frac{6}{36} = \frac{1}{6}$

Ex. 3 dices are thrown. What is prob that sum is 15.

Sol:- 15 – 5, 5, 5 – 1
6, 4, 5 – $3! = 6$
6, 6, 3 – $\frac{3!}{2!} = 3$

always start with max possible no

Prob = $\frac{10}{216}$

Ex:- 4 Dices are thrown. What is prob that sum is 20.

Sol. 20 – 6, 6, 6, 2 – $\frac{4!}{3!} = 4$
6, 6, 3, 5 – $\frac{4!}{2!} = 12$
6, 4, 6, 4 – $\frac{4!}{2!2!} = 6$
5, 5, 5, 5 – $\frac{4!}{4!} = 1$
~~5,5~~ 6, 4, 5, 5 → $\frac{4!}{2!} = 12$

Prob = $\frac{35}{1296}$

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Coins

Ex: 3 coins are tossed. What is prob of getting exactly 2 Heads?

Sol: H, H, T $\rightarrow \frac{3!}{2!} = 3$

$\text{Prob} = \frac{3}{8}$

(Imp)

Ex:- 10 coins are tossed. What is prob of getting 4 heads?

Sol:- H,H,H,H, T,T,T,T,T,T $= \frac{10!}{4!\,6!} = \frac{10 \times 9 \times 8 \times 7}{4 \times 6} = 210$

$\text{Prob} = \frac{210}{2^{10}} = \frac{210}{1024}$

Or

We can use concept of binomial distribution

Note: We can use this only when there are 2 outcomes possible. And both are complementary

$^{n}C_{r}\,(p)^{r}\,(1-p)^{n-r}$

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n = total no of coins

r = no of successful events

p = success   (1-p) = failure

$^{10}C_4 \left(\frac{1}{2}\right)^4 \left(1-\frac{1}{2}\right)^{10-4}$

$= {}^{10}C_4 \left(\frac{1}{2}\right)^{10}$

Cards

Cards (52)

Red (26) — (13) Diamond, Heart (13)

black (26) — club (13), Spade (13)

Honoured cards = A, K, Q, J $\rightarrow$ 16

face cards = K, Q, J $\rightarrow$ 12

Ex: 2 cards are drawn from deck. What is the prob. that they are king

Sol:- $\frac{^{4}C_2}{^{52}C_2}$

Different ways of drawing cards

1) random (means collective)

2) One by one

3) One by one with replacement

Ex. i) 2 cards are drawn one by one. What is prob that both are diamonds?

Sol: $\left(\frac{13}{52}\right) \times \left(\frac{12}{51}\right)$

Ex ii) 2 cards are drawn one by one with replacement:

Sol. $\left(\frac{13}{52}\right) \times \left(\frac{13}{52}\right)$

Ex: From a pack of 52 cards 4 cards are drawn such that they all are different suits and different no.

Sol: nothing is mentioned how they are drawn, so we use random

$\cancel{\left(\frac{52}{52}\right) \times \left(36\right.}$ $\frac{52 \times 36 \times 22 \times 10}{^{52}C_4}$

time

I Diamond — 13 ✓

II Club — 13 −1 = 12

III Heart — 13 −1 = 12

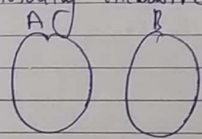
IV Spade — 13 −1 = 12

} the card no picked in diamond should not be considered again

ii) Cards are drawn one by one

$\left(\frac{52}{52}\right) \times \left(\frac{36}{51}\right) \times \left(\frac{22}{50}\right) \times \left(\frac{10}{49}\right)$

Mutually exclusive

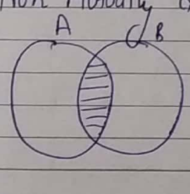


$P(A \text{ or } B) = P(A) + P(B)$

(or) $P(A \cup B)$

$P(A \cap B) = 0$

Non Mutually exclusive



$P(A \text{ or } B) = P(A) + P(B) - P(A \cap B)$

(or)

$P(A \cup B)$

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Q From a pack of cards, one card is drawn find prob. that it is red, king or ?

Sol:- A = red card
B = king

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$
$= \frac{26}{52} + \frac{4}{52} - \frac{2}{52}$

$= \frac{28}{52} = \frac{7}{13}$

ii) Prob that it is spade or diamond ?

Sol:- A = spade card
B = diamond card

$P(A \cup B) = P(A) + P(D)$
$= \frac{13}{52} + \frac{13}{52}$
$= \frac{13}{26} = \frac{1}{2}$

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Q Suppose 3 students appeared in a exam A, B, C, Prob that they pass is $P(A) = \frac{1}{3}$, $P(B) = \frac{1}{4}$, $P(C) = \frac{1}{5}$

Q i) Prob that all pass.

Sol: $P(A \cdot B \cdot C) = \frac{1}{3} \times \frac{1}{4} \times \frac{1}{5} = \frac{1}{60}$

ii) Prob that exactly 2 pass

Sol: $P(AB\bar{C} + \bar{A}BC + A\bar{B}C)$ $P(\bar{A}) = \frac{2}{3}$ $P(\bar{C}) = \frac{4}{5}$

$P(\bar{B}) = \frac{3}{4}$

$= \left(\frac{1}{3}\right)\left(\frac{1}{4}\right)\left(\frac{4}{5}\right) + \left(\frac{2}{3}\right)\left(\frac{1}{4}\right)\left(\frac{1}{5}\right) + \left(\frac{1}{3}\right)\left(\frac{3}{4}\right)\left(\frac{1}{5}\right)$

$= \frac{9}{60}$

iii) Prob that exactly 1 pass

$P(A\bar{B}\bar{C} + \bar{A}B\bar{C} + \bar{A}\bar{B}C)$

$\left(\frac{1}{3}\right)\left(\frac{3}{4}\right)\left(\frac{4}{5}\right) + \left(\frac{2}{3}\right)\left(\frac{1}{4}\right)\left(\frac{4}{5}\right) + \left(\frac{2}{3}\right)\left(\frac{3}{4}\right)\left(\frac{1}{5}\right)$

$= \frac{26}{60}$

iv) Prob that at least 1 pass ?

Sol:- $= 1 -$ at most 0 pass (or) 1 - all fail

$= 1 - \left(\frac{2}{3}\right)\left(\frac{3}{4}\right)\left(\frac{4}{5}\right)$

$= \frac{3}{5}$

Concept of Independent Events

$P(A \cap B) = P(A) \times P(B)$

Ex: Suppose a dice is thrown & a coin is tossed. What is the prob of getting an even no and a head on coin.

Sol:- $= \left(\frac{3}{6}\right) \times \left(\frac{1}{2}\right) \Rightarrow \frac{1}{4}$

Urns & Balls

I	II
4R + 5B	3R + 6B

i) One ball is drawn from I and put it into II. Now one ball is drawn from II. What is the prob that a red ball is drawn?

Sol:- RR + BR

$= \left(\frac{4}{9}\right) \times \left(\frac{4}{10}\right) + \left(\frac{5}{9}\right) \times \left(\frac{3}{10}\right)$

$= \frac{16}{90} + \frac{15}{90} = \frac{31}{90}$

ii) 2 balls are drawn. And same process repeated as in (i) question.

Sol:- RR R + BB R + BR R

$\frac{^4C_2}{^9C_2} \times \cancel{\frac{^5C_1}{^{11}C_1}}\left(\frac{5}{11}\right) + \frac{^5C_2}{^9C_2} \times \frac{3}{11} + \frac{^4C_1 \times {^5C_1}}{^9C_2} \times \frac{4}{11}$

$= \frac{6}{36} \times \frac{5}{11} + \frac{10}{36} \times \frac{3}{11} + \frac{20}{36} \times \frac{4}{11}$

$= \frac{\cancel{140}\ \cancel{70}\ 35}{\cancel{36 \times 11}\ \cancel{18}\ 9}$

$= \frac{35}{99}$

Date : Page :

Baye's Theorem

Ex:-

	B	R
I	5	3
II	4	4
III	2	6

one ball is selected. it is found to be Red. Prob that it is from II nd urn?

$$= \frac{\left(\frac{1}{3}\right)\left(\frac{4}{8}\right)}{\left(\frac{3}{8}\right)\times\left(\frac{1}{3}\right)+\left(\frac{4}{8}\right)\times\left(\frac{1}{3}\right)+\left(\frac{1}{3}\right)\left(\frac{6}{8}\right)}$$

$$= \frac{4}{13}$$

Ex. 3 Drivers

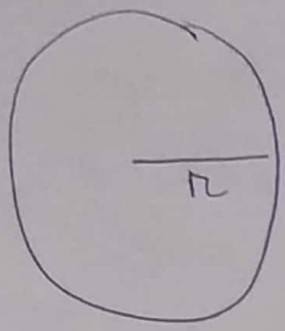
		Prob of accident
Scooter drivers	= 4000	1%
Car drivers	= 5000	2%
Truck drivers	= 6000	3%

one driver is selected it was found to be met with accident. What is prob that he is a truck driver?

Sol:-

$$\frac{\left(\frac{6}{15}\right)\left(\frac{3}{100}\right)}{\left(\frac{4}{15}\right)\times\left(\frac{1}{100}\right)+\left(\frac{5}{15}\right)\left(\frac{2}{100}\right)+\left(\frac{6}{15}\right)\left(\frac{3}{100}\right)} = \frac{18}{32} = \frac{9}{16}$$

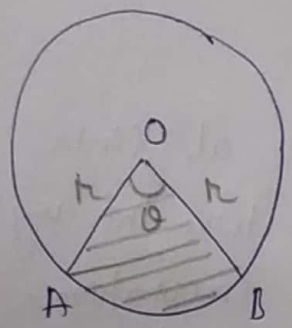
Circles



area $= \pi r^2$

Circumference $= 2\pi r$
(Perimeter)

Sector

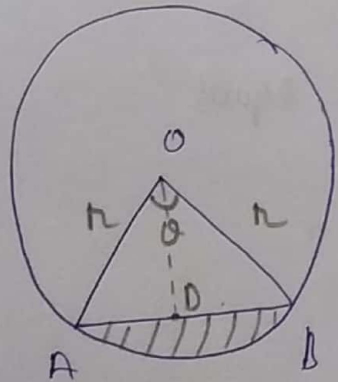


Area $= \frac{\theta}{360^\circ} \times \pi r^2$

length of arc $\widehat{AB} = \frac{\theta}{360} \times 2\pi r$

Perimeter of Sector $= \frac{\theta}{360} \times 2\pi r + 2r$

Segment



area = ar of sector − ar of Δ

$= \frac{\theta}{360} \times \pi r^2 - \frac{1}{2} \times r^2 \sin\theta$

Perimeter $= \frac{\theta}{360^\circ} \times 2\pi r + 2r\sin\frac{\theta}{2}$

AOB is isosceles Δ

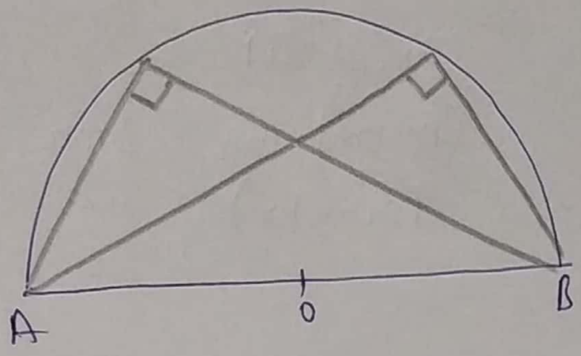
O is center, OD is ⊥ AB, OD ∠ bisector ∠AOB

AD = DB $\quad \sin\frac{\theta}{2} = \frac{DB}{r} = \frac{x}{r} \Rightarrow x = r\sin\frac{\theta}{2}$

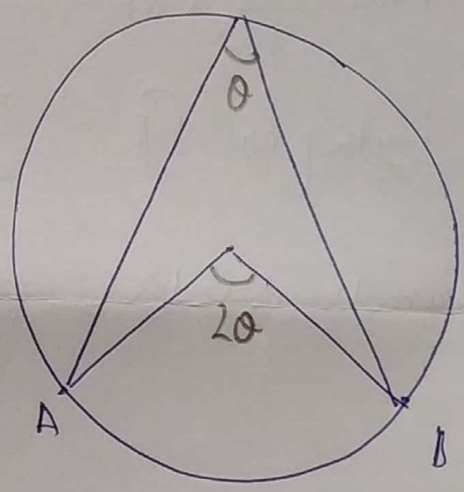
Let AD = DB = x $\quad AB = 2r\sin\frac{\theta}{2}$

Properties

1) Angle in semi circle is 90°

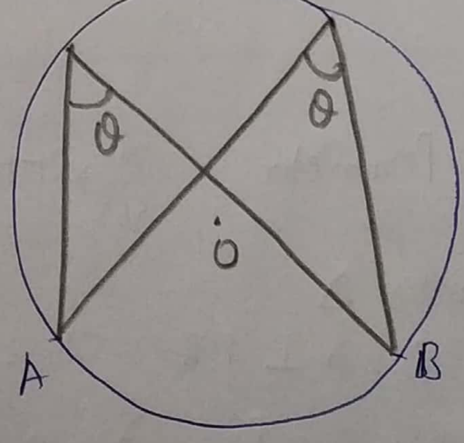


2)



angle at center is twice the angle at circumference made by the segment

3) Angles made by same segment are equal

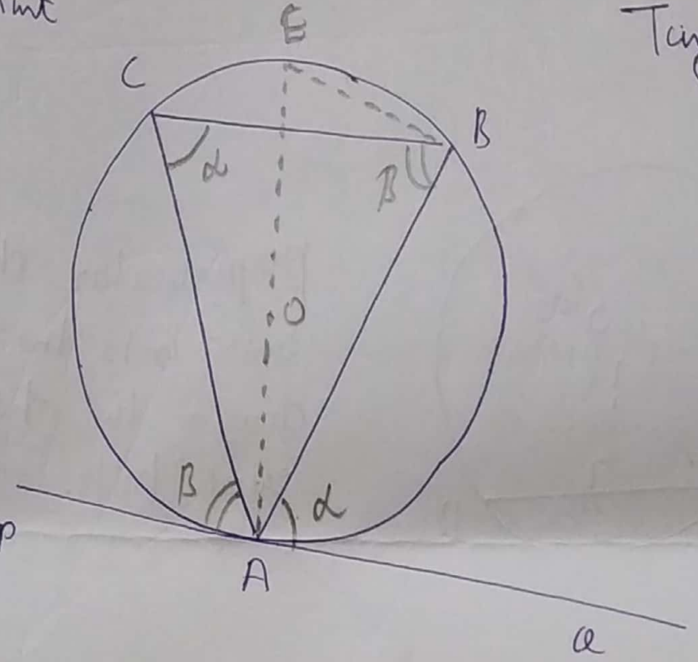


4) Angles in alternate segment are equal

(or)

Angle made by chord with tangent at point of contact is same as the angle in alternate segment

Tangent - chord Theorem



Prove: $\angle BAQ = \angle BCA$

Proof: $\angle EAQ = 90°$ [$\because$ tangent $\perp$ w.r.t to radius]

$\angle EAB + \angle BAQ = 90°$

$\angle EAB + \alpha = 90°$ — ①

$\angle ABE = 90°$ ($\angle$ in semi circle)

$\angle BEA + \angle EAB = 90°$ — ②

from ① and ②

$90° - \angle BEA + \alpha = 90°$

$\alpha = \angle BEA$

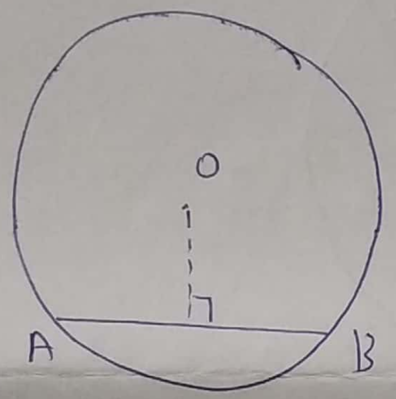
$\angle BEA = \angle BCA = \alpha$ ($\angle$ in same segment)

$\therefore \angle BAQ = \angle BCA = \alpha$

Hence Proved

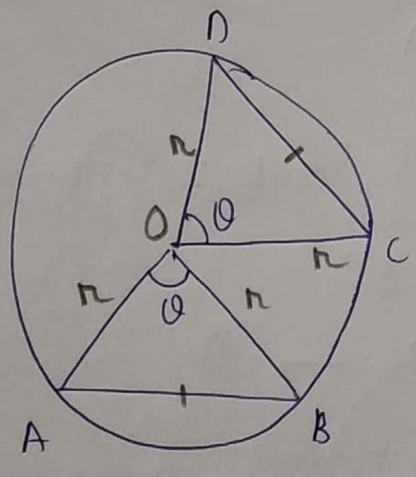
CHORD

1)



perpendicular drawn from center onto the ~~cord~~ chord divides the chord into equal halfs.

2)



Chords of equal length subtends equal angles at the center.

given $AB = CD$

$AOB \cong COD$

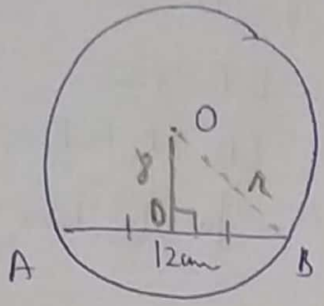
$AB = CD$

$AO = CO$

$OB = OD$

$\therefore$ by SSS $AOB \cong COD$

Ex:



OD = 8cm

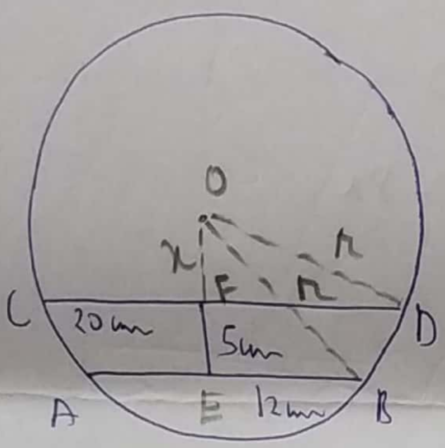
find r = ?

AD = DB = 6

$\sqrt{64+36} = r^2$

$r = 10$

Ex:



AB ∥ CD

distance b/w AB & CD is 5cm.

find r?

Sol. In Δ EOB

$r^2 = (x+5)^2 + (6)^2$ —— ①

In Δ ~~EO~~ FOD

$r^2 = x^2 + 10^2$ —— ②

$(x+5)^2 + 36 = x^2 + 100$

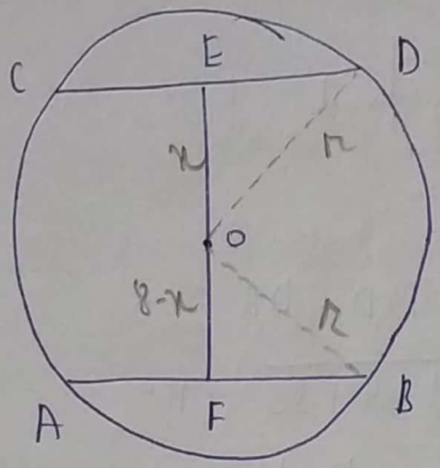
$\cancel{x^2} + 25 + 10x + 36 = \cancel{x^2} + 100$

$10x = 39$

$x = 3.9$ cm

$\therefore r = 10.73$

Ex:



$CD = 20\,cm$, $AB = 12\,cm$

$EF = 8\,cm$

$CD \parallel AB$

find r?

Sol:

$$100 + x^2 = r^2 \quad — (1)$$

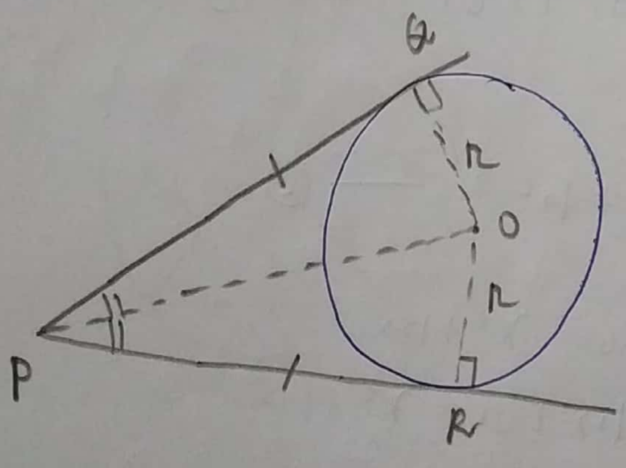
$$36 + (8-x)^2 = r^2 \quad — (2)$$

$$x^2 + 100 = 36 + 64 + x^2 - 8x$$

$$x = 0$$

$$\therefore r = 10\,cm$$

Tangent



Let $PO = x$

$PQ = \sqrt{x^2 - r^2}$

$PR = \sqrt{x^2 - r^2}$

Proof: $\Delta POQ \cong \Delta POR$

$PQ = PR$

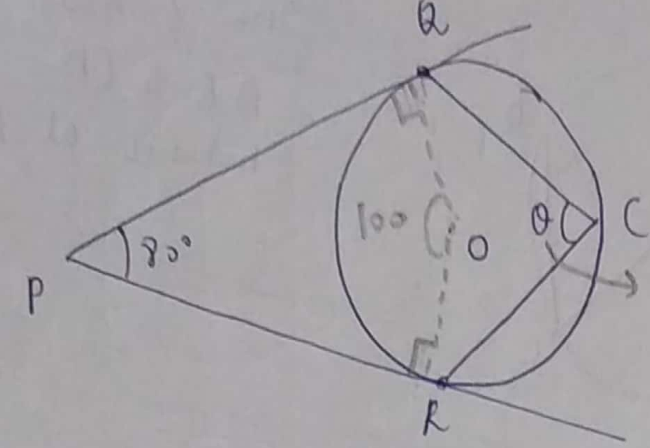
$OQ = OR$

$\angle PQO = \angle PRO$

by SAS

$PQ = PR \qquad \angle QPO = \angle RPO$

Ex:

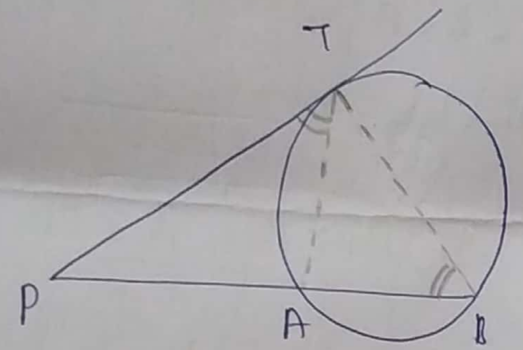


find $\angle QCR = ?$

$\theta = 50°$. (Angle in segment is half of angle at center)

Secant

A segment intersecting circle twice



$PT^2 = PA \times PB$

Proof: $\triangle PTA \sim \triangle PBT$

$\angle PTA = \angle PBT$

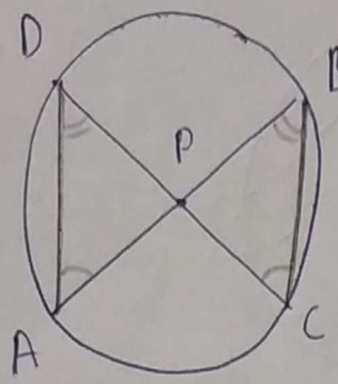
$\angle TPA = \angle BPT$

by AA similarity

$$\frac{PT}{PB} = \frac{PA}{PT}$$

$PT^2 = PA \times PB$

2)



given 2 chords AB & CD intersect at P.

$PA \times PB = PC \times PD$

Proof:- $\Delta APD \sim \Delta CPB$

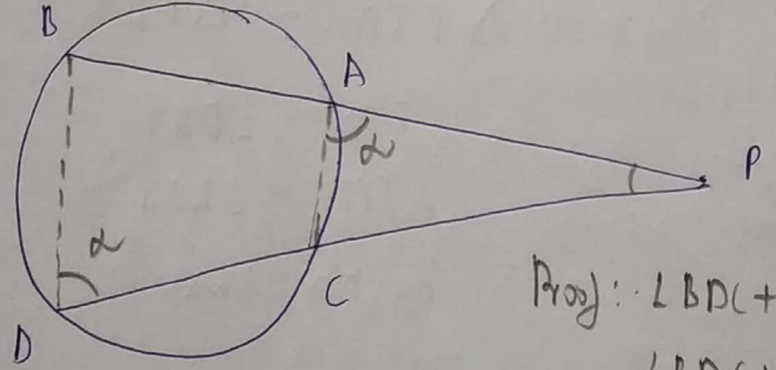
$\angle ADP = \angle CBP$

$\angle DAP = \angle BCP$

by AA similarity

$\frac{AP}{CP} = \frac{PD}{PB} \Rightarrow AP \times PB = PC \times PD$

3)



$PA \times PB = PC \times PD$

Proof:- $\angle BDC + \angle BAC = 180°$

$\angle BAC + \angle PAC = 180°$

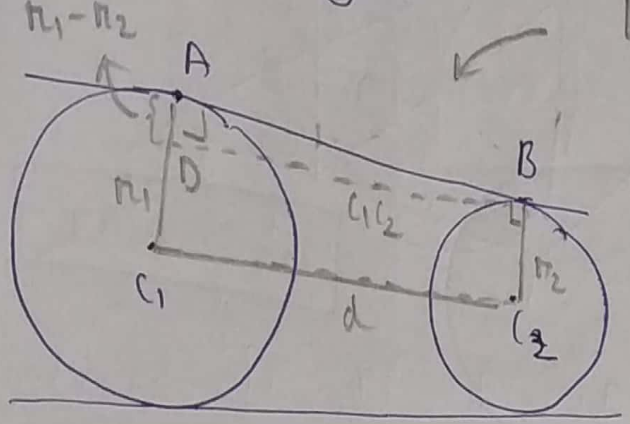
$\angle PAC = \angle BDC$

$\Delta PAC \sim \Delta PDB$

$\frac{PA}{PD} = \frac{PC}{PB}$

$PA \times PB = PC \times PD$

Common Tangent

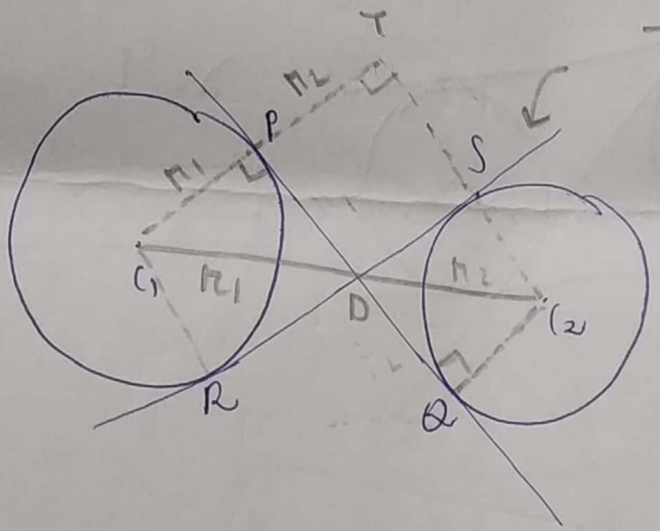


Direct Common tangent

find AB

In ΔADB

$$AB = \sqrt{C_1C_2^2 + (r_1 - r_2)^2} = \sqrt{d^2 + (r_1 - r_2)^2}$$



Transverse Common tangent

find length of TCT?

In ΔC_1TC_2

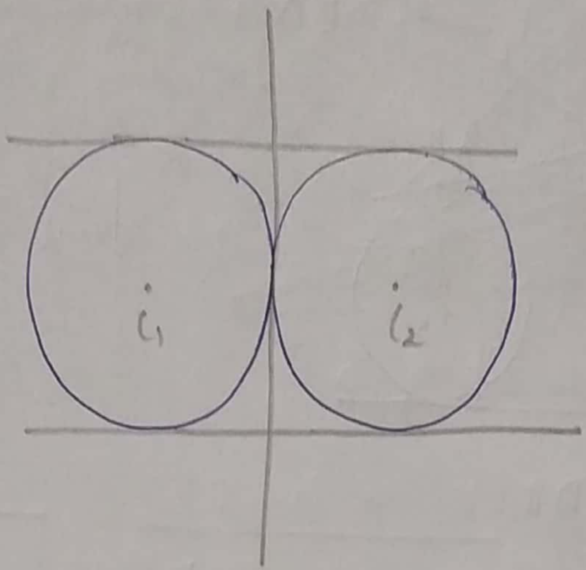
$$C_1C_2^2 = (r_1 + r_2)^2 + TC_2$$

$$\sqrt{C_1C_2^2 - (r_1 + r_2)^2} = TC_2 = PQ$$

$$PQ = \sqrt{d^2 - (r_1 + r_2)^2}$$

Number of Common Tangents

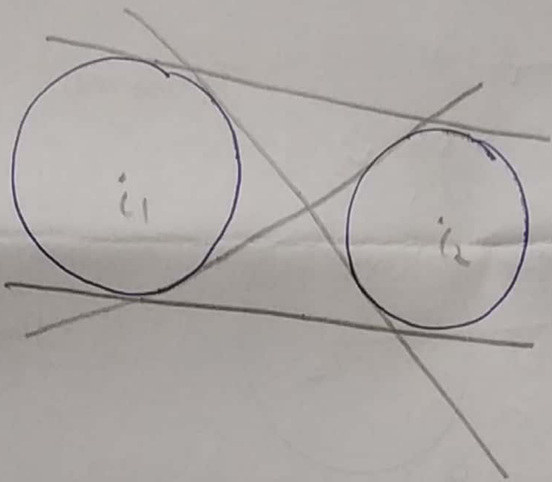
1)



$n = 3$

$\boxed{C_1C_2 = r_1 + r_2}$

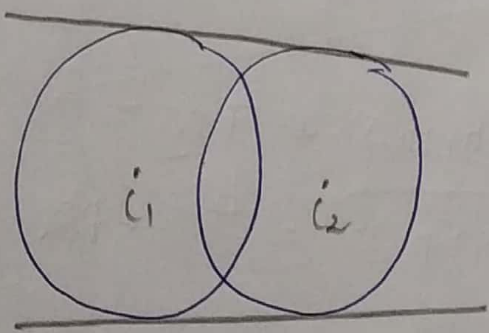
2)



$n = 4$

$\boxed{C_1C_2 > r_1 + r_2}$

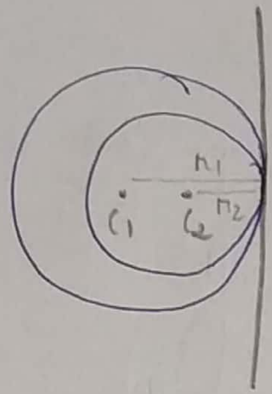
3)



$n = 2$

$\boxed{C_1C_2 < r_1 + r_2}$

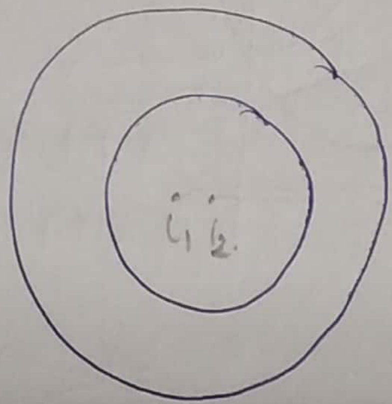
4)



$n = 1$

$C_1C_2 = r_1 - r_2$

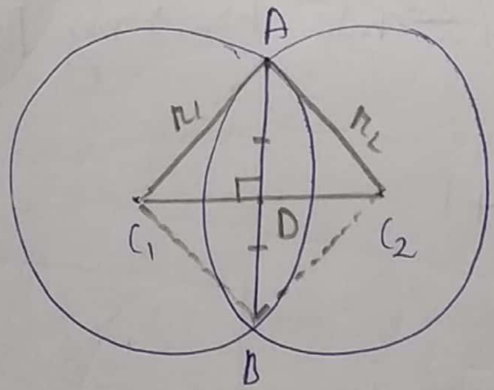
5)



$n = 0$

$C_1C_2 < r_1 - r_2$

Common Chord



$AD = \frac{2(\Delta C_1AC_2)}{C_1C_2}$

$BD = \frac{2(\Delta C_1BC_2)}{C_1C_2}$

$(\because \Delta C_1BC_2 \cong \Delta C_1AC_2)$

AB is divided equally by $\perp$ C_1C_2

$AD = DB$

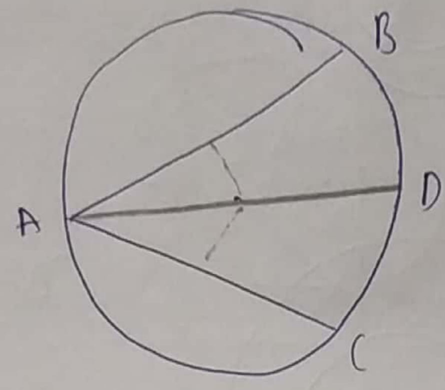
$\Delta = ar\ C_1AC_2$

Length of chord $= 2AD. = \frac{4(\Delta)}{C_1C_2}$

if ar of ΔAC_1C_2 is given then we can find AD

$ar = \frac{1}{2} \times C_1C_2 \times AD$

Chords equidistance from Center are equal



AB = AC

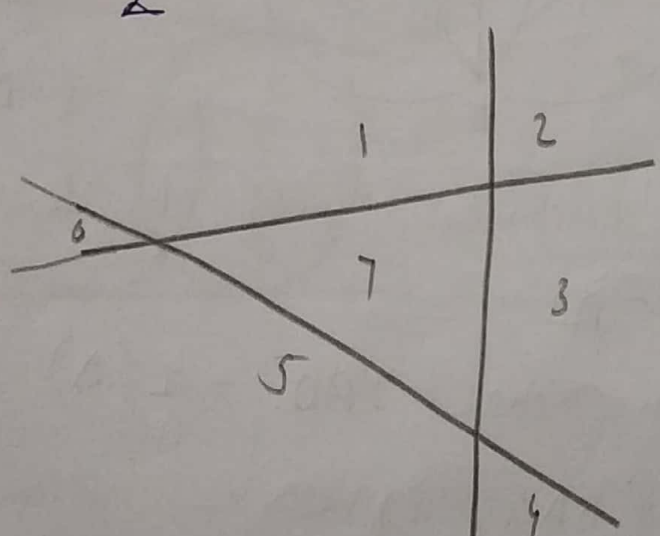
Concept

given unbounded plane, and N lines.

N lines divide unbounded plane into how many areas (or into how many parts)

$$= \frac{N(N+1)}{2} + 1$$

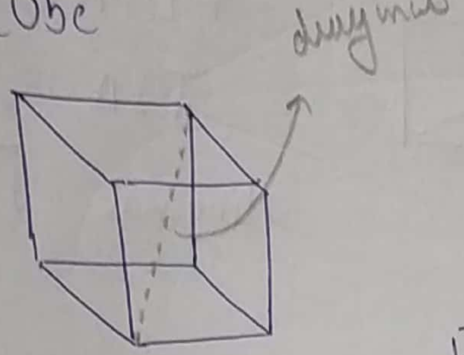
Ex.



Scanned by CamScanner

Mensuration

i) Cube



faces = 6
vertices = 8
edges = 12

$$\boxed{f + V = E + 2}$$

Euler formula

Volume = a^3

L.S.A = $4a^2$

T.S.A = $6a^2$

Diagonal (length of longest rod that can be placed inside a cube)

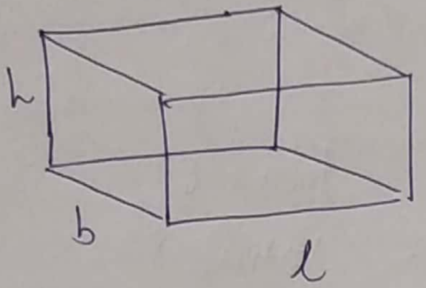
Diagonal = $\sqrt{3}a$

Ex: cube of side 8cm, melted. And smaller equal cubes of side 2cm are formed, How many such cubes are formed

$$\text{no of cubes} = \frac{(8)^3}{(2)^3} = 64$$

(bcz when melted volume does not change)

2) Cuboid



Volume = lbh

~~T~~. L.S.A = $2(l+b)h$

T.S.A = $2(lb+bh+hl)$

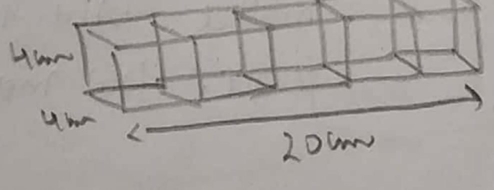
diagonal = $\sqrt{l^2+b^2+h^2}$

Ex: Cubes of ~~edge~~ side 4cm. are placed side by side 5 such cubes are placed to form a cuboid. find % change in T.S.A.

Sol: initially T.S.A = $6 \times (4)^2 = 96\ cm^2$

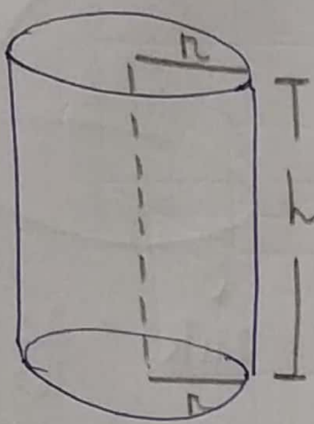
T.S.A of 5 cubes = $5 \times 96 = 480\ cm^2$

T.S.A of cuboid = $2(20\times4 + 4\times4 + 20\times4) = 192\ cm^2$



% change = $\frac{480-192}{480} \times 100 = \frac{288}{480} \times 100$ = 60% decrease

Cylinder



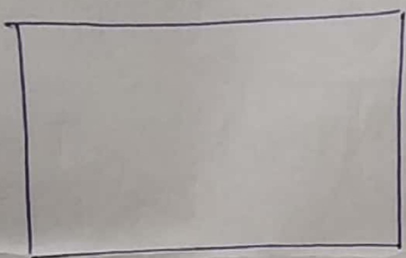
$$V = \pi r^2 h$$

$$C.S.A = 2\pi r h$$

$$T.S.A = 2\pi r h + 2\pi r^2$$

$$= 2\pi r(r+h)$$

Ex:

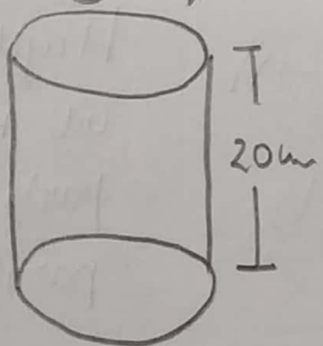


20cm

24cm

rectangular sheet is folded about breadth find V of cylinder formed?

⇓



$\rightarrow 2\pi r = 44cm$

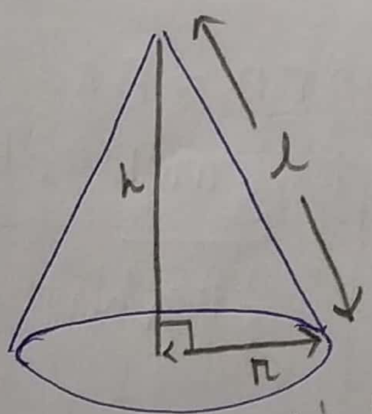
$r = 7cm$

$$V = \pi r^2 h$$

$$= \frac{22}{\cancel{7}} \times \cancel{7} \times 7 \times 20$$

$$= 3080 \text{ cm}^3$$

Cone



$$l = \sqrt{h^2 + r^2}$$

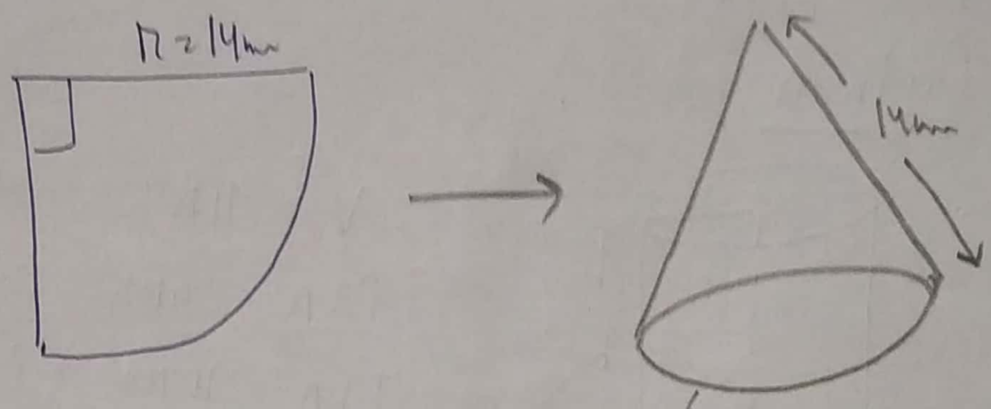
$$V = \frac{1}{3}\pi r^2 h$$

$$C.S.A = \pi r l$$

$$T.S.A = \pi r l + \pi r^2$$

$$= \pi r(r+l)$$

Ex:



$V = \frac{1}{3}\pi r^2 h$

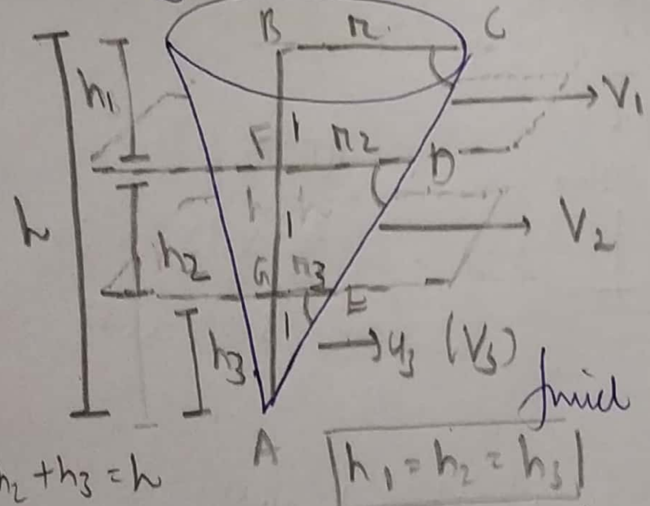
$179.66\,cm^3 \longleftarrow = \frac{1}{3} \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times \sqrt{14}$

$2\pi R = \frac{90}{360} \times 2\pi r$

$R = \frac{1}{4} \times 14 = \frac{7}{2}$

$h = \sqrt{196 - \frac{49}{4}} = \sqrt{14}\ (approx)$

Cutting a Cone



Height of a cone is cut into 3 equal parts by 2 planes parallel to base

find $V_1 : V_2 : V_3$

$\boxed{h_1 = h_2 = h_3}$

$h_1 + h_2 + h_3 = h$

$h_1 = \frac{h}{3} = h_2 = h_3$

$ABC \sim AGE$

$\frac{AB}{AG} = \frac{BC}{GE}$

$\frac{h}{h_3} = \frac{r}{r_3}$

$AFD \sim ABC$

$\frac{AF}{AB} = \frac{FD}{BC}$

$\frac{h_2 + h_3}{h} = \frac{r_2}{r}$

$\frac{V_3}{V} = \frac{\frac{1}{3}\pi r_3^2 h_3}{\frac{1}{3}\pi r^2 h} = \frac{r_3^2 h_3}{r^2 h} = \left(\frac{h_3}{h}\right)^3$

$\frac{V_3}{V} = \left(\frac{1}{3}\right)^3 = \frac{1}{27}$

$$\frac{V_2+V_3}{V} = \left(\frac{2}{3}\right)^3 = \frac{8}{27} \quad \left[\frac{\frac{1}{3}\pi r_2^2(h_2+h_3)}{\frac{1}{3}\pi r^2 h}\right]$$

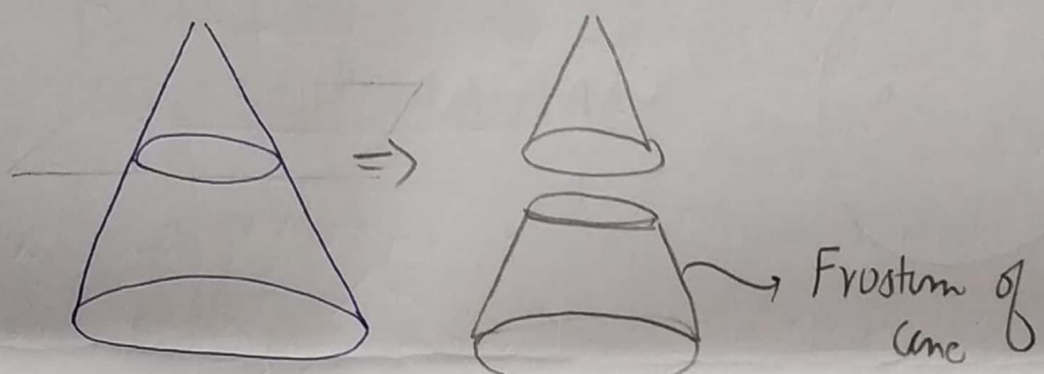
$$\frac{r_2^2\left(\frac{2h}{3}\right)}{r^2 h} = \left(\frac{2}{3}\right)^3$$

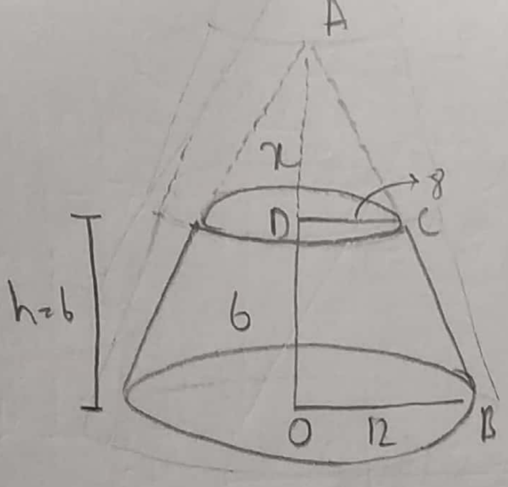
$$\left(\because \frac{r_2}{r} = \frac{h_2}{h}\right)$$

$V_1 : V_2 : V_3$

$19 : 7 : 1$

Frustum





Volume of Frustum = (volume of bigger cone) − (volume of smaller cone)

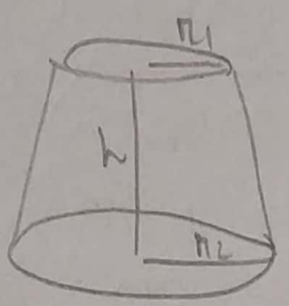
$AOB \sim ADC$

$$\frac{AO}{AD} = \frac{OB}{DC} \Rightarrow \frac{x+6}{x} = \frac{12}{8}$$

$x = 12$

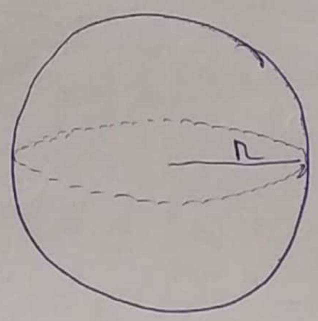
$$\text{Volume of Frustum} = \frac{1}{3}\pi(12)^2(18) - \frac{1}{3}\pi(8)^2 12$$

$$= \frac{1}{3}\pi(144\times18 - 64\times12) \Rightarrow 1909.12\ cm^3$$



Volume $= \frac{1}{3}\pi\left[r_1^2 + r_2^2 + r_1 r_2\right] \times h$

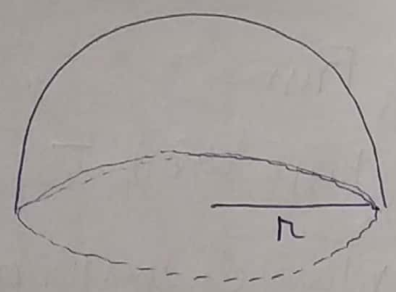
Sphere



$V = \frac{4}{3}\pi r^3$

C.S.A / T.S.A $= 4\pi r^2$

Hemisphere

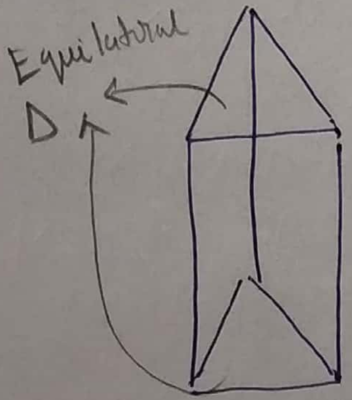


$V = \frac{2}{3}\pi r^3$

C.S.A $= 2\pi r^2$

T.S.A $= 3\pi r^2$

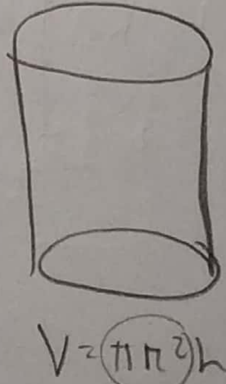
Prism



V = area of base x height

C.S.A = Perimeter of base x height

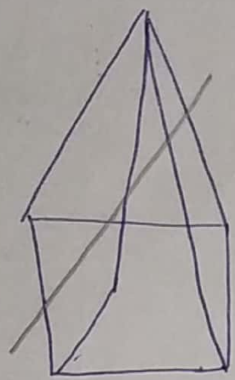
T.S.A = C.S.A + 2(area of base)



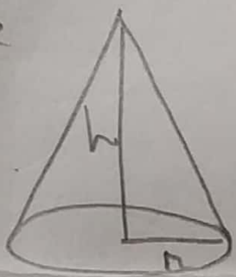
$V = \pi r^2 h$

C.S.A $= 2\pi r h$

Pyramid



We can use analogy of cone



$V = \frac{1}{3} \pi r^2 h$

$C.S.A = \pi r l$

$\left(\frac{2 \pi r}{2}\right) l$

$V = \frac{1}{3}$ (area of base) $\times$ height

$C.S.A = \frac{\text{perimeter of base}}{2} \times$ slant height

$T.S.A = C.S.A +$ area of base

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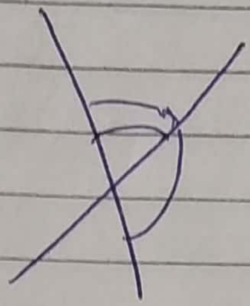
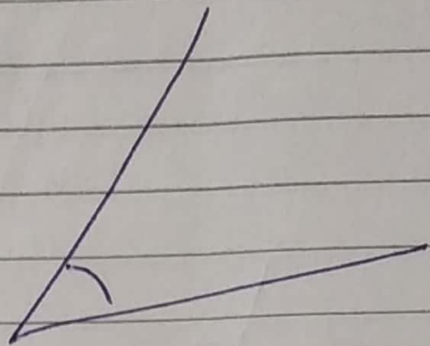
Geometry & Mensuration

Basics

Angles



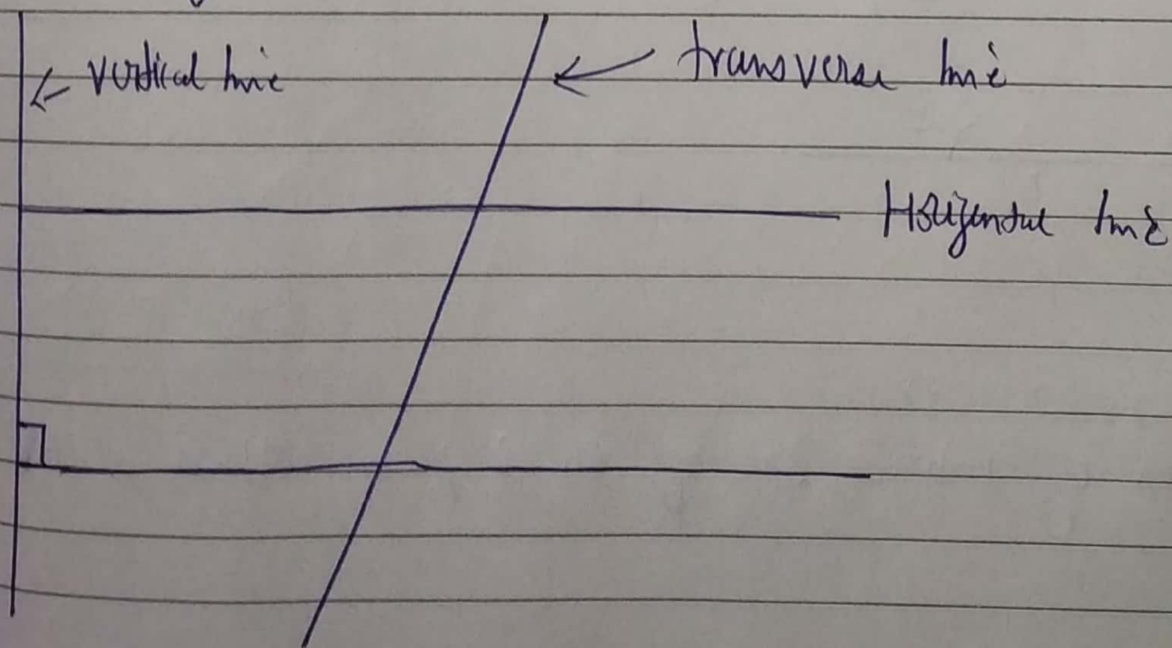


Acute angle $< 90^\circ$

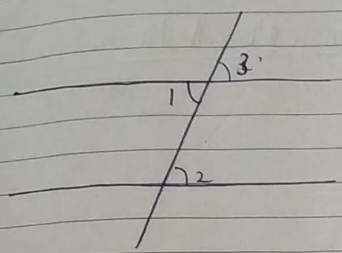
obtuse angle $90^\circ < \theta < 180^\circ$

right angle 90°

reflexive Angle $\theta > 180^\circ$



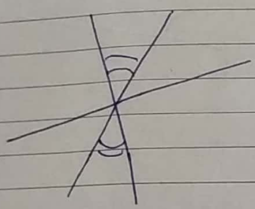
I



$\angle 1 = \angle 2$ (alternate $\angle$)

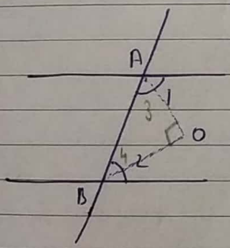
$\angle 1 = \angle 3$ (vertically opposite $\angle$)

$\angle 2 = \angle 3$ (corresponding $\angle$)



Vertically opposite $\angle$

II



i) $\angle 1 + \angle 2 = 180^\circ$

Sum of interior $\angle$'s of on 2 parallel lines is 180°

AO & BO are $\angle$ bisectors

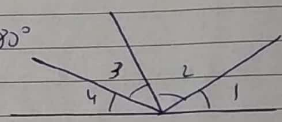
$\therefore \angle 3 + \angle 4 = \frac{1}{2}(\angle 1 + \angle 2) = \frac{1}{2}(180^\circ) = 90^\circ$

$\therefore \angle AOB = 90^\circ$

Complementary $a + b = 90^\circ$

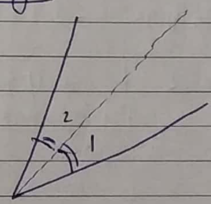
Supplimentary $a + b = 180^\circ$

Linear pair angles



$\angle 1 + \angle 2 + \angle 3 + \angle 4 = 180^\circ$

Adjucent Angles



Polygons

Enclosed figure with no. of sides is polygon

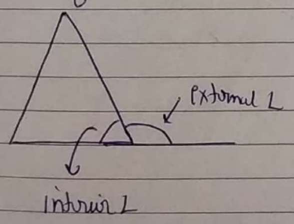
polygon

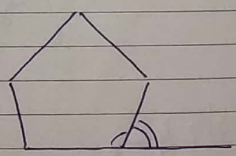
Concave	Convex
all ∠'s are less than 180°	~~all ∠'s are~~ at least one ∠ of polygon is reflexive

Regular Polygon	Irregular Polygon
→ all sides are equal → all ∠'s are equal.	→ all sides & angles are not equal

Note: In any polygon

Sum of internal ∠ + corresponding external ∠ = 180°





Whether polygon is regular or Irregular

Sum of all exterior ∠'s is 360°

Sum of all internal ∠'s is ~~180°~~ $(n-2)180°$

Note: For Regular Polygon

i) ~~Exterior~~ External angle = $\frac{360°}{n}$

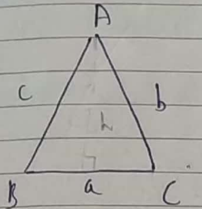
ii) Internal angle = $\frac{(n-2)180°}{n}$

Q If internal ∠ of a regular polygon is 3 times of external ∠. Then find total no. of sides?

Ans. $\frac{(n-2)\cancel{180°}}{\cancel{x}} = 3 \times \frac{\cancel{360}}{\cancel{x}}$

$n = 8$

Concept of Triangles



BC = a
AC = b
AB = c

Area $= \frac{1}{2} \times$ base $\times$ height

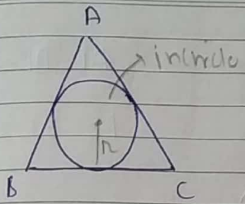
(Heron) formula $= \sqrt{s(s-a)(s-b)(s-c)}$

s = (semiperimeter) $= \frac{a+b+c}{2}$

$= \left[\frac{1}{2} \times a \times b \times \sin A \text{ (or) } \frac{1}{2} bc\right] \times$

$= \frac{1}{2} bc \sin A$ (or) $\frac{1}{2} ab \sin C$ (or) $\frac{1}{2} ac \sin B$

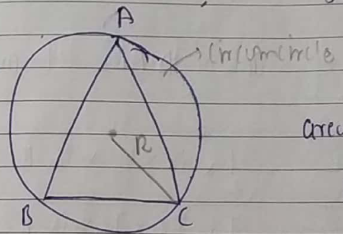
product of 2 sides; $\angle$ b/w them



$r \times s$ = area of ABC

r = incircle radius

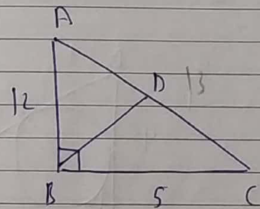
s = semi~~peter~~ perimeter



area of $\Delta = \frac{abc}{4R}$

R = radius of circumcircle.

Ex:



find length of BD

area of $\Delta = \frac{1}{2} \times 5 \times 12 = \frac{1}{2} \times BD \times AC$

$5 \times 12 = BD \times 13$

$BD = \frac{60}{13}$ cm

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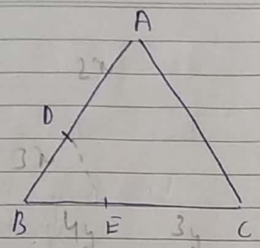
Ex:



AD : DB = 2 : 3

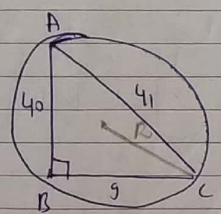
BE : EC = 4 : 3

find ratio of area of ΔBDE to area of ΔABC

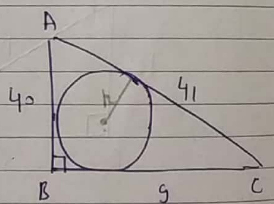
Sol: $\dfrac{\text{area of } \Delta BDE}{\text{area of } \Delta ABC} = \dfrac{\frac{1}{2} \times 3x \times 4y \times \operatorname{Sin} B}{\frac{1}{2} \times 5x \times 7y \times \operatorname{Sin} B} = \dfrac{12}{35}$

Ex:

i)



ii)



find R & r ?

Sol. i) area of Δ ABC = $\dfrac{abc}{4R} = \dfrac{1}{2} \times 9 \times 40$

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$\dfrac{40 \times 41 \times 9}{4 \times R} = \dfrac{1}{2} \times 9 \times 40$

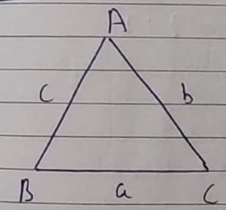
$\dfrac{41}{2} = R$

ii) area of ΔABC = $r \times s = \dfrac{1}{2} \times 9 \times 40$

$\Rightarrow r \times \left(\dfrac{9+40+41}{2}\right) = \dfrac{1}{2} \times 9 \times 40$

$\Rightarrow r = \dfrac{9 \times 40}{90} = 4$

Sine Rule



$\dfrac{a}{\operatorname{Sin} A} = \dfrac{b}{\operatorname{Sin} B} = \dfrac{c}{\operatorname{Sin} C} = 2R$

Cosine Rule

$\cos A = \dfrac{b^2 + c^2 - a^2}{2bc}$

$\cos B = \dfrac{a^2 + c^2 - b^2}{2ac}$

$\cos C = \dfrac{a^2 + b^2 - c^2}{2ab}$

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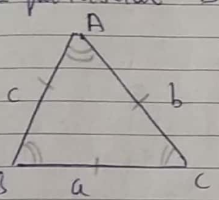
Types of Triangles

1) Equilateral Δ



area = $\frac{\sqrt{3}}{4} a^2$

height = $\frac{\sqrt{3}a}{2}$

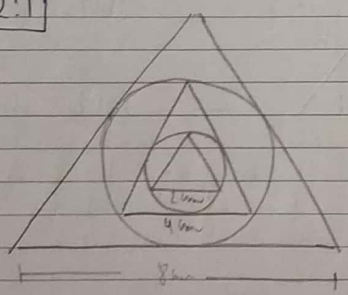
inradius = $\frac{a}{2\sqrt{3}}$

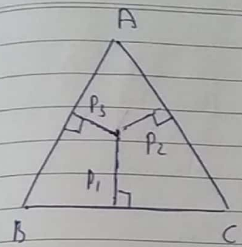
circumradius = $\frac{a}{\sqrt{3}}$

* → circumradius : inradius = 2 : 1

$\boxed{R : r = 2:1}$

* →





from any point inside equilateral Δ, if we draw ⊥'s on every side then

$p_1 + p_2 + p_3$ = height of equilateral Δ

$p_1 + p_2 + p_3 = \frac{\sqrt{3}}{2} a$

NOTE: of given perimeter of Δ's equilateral Δ will always have max area.

with given perimeter if we construct all possible Δ's, then among all of them equilateral Δ will have max area.

2) Isosceles Triangle

Any 2 sides are equal.

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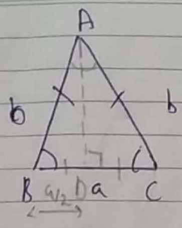


$AB = AC$

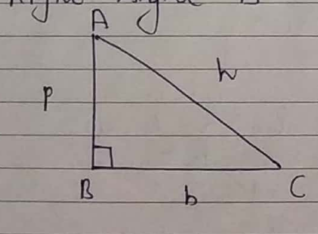
AD = ⊥ bisector, ⊥ bisector

$AD = \text{height} = \sqrt{b^2 - \frac{a^2}{4}} = \frac{1}{2}\sqrt{4b^2 - a^2}$

$\text{area} = \frac{1}{2} \times a \times \frac{1}{2}\sqrt{4b^2 - a^2}$

$= \frac{a}{4}\sqrt{4b^2 - a^2}$

Right Angled Δ



Pythagorean Triplets

3, 4, 5
5, 12, 13
7, 24, 25
8, 15, 17
9, 40, 41
11, 60, 61
13, 84, 85

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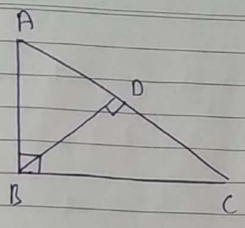
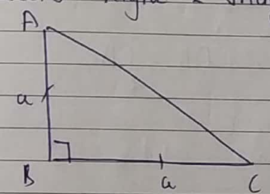
→



$\cancel{BD^2 = AB^2 + BC^2}$ $AB^2 + BC^2 = AC^2$

$BD^2 = AD \times DC$
$AB^2 = AD \times AC$
$BC^2 = CD \times AC$

Isosceles Right ∠ triangle.

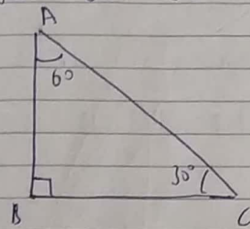


$\text{area} = \frac{1}{2} \times a \times a = \frac{a^2}{2}$

NOTE: ~~A~~ Among all the right ∠ triangle(s) with given perimeter (i.e. they all have same perimeter) isosceles right ∠ triangle has max area.

Special Case

30 – 60 – 90



Side opposite to 30° will always be half of hypotenuse

$\sin 30° = \frac{AB}{AC} \Rightarrow AB = \frac{1}{2} AC$

Properties of Triangle

1) Sum of internal ∠'s = 180°

2) Sum of 2 sides always greater than 3rd side

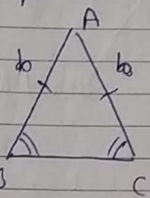
given 3 sides of Δ a, b, c

then $\boxed{a-b < c < a+b}$

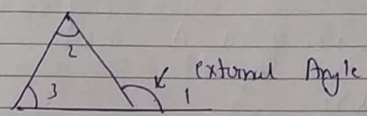
Ex. 5, 12 find 3rd side

$7 < c < 17$

3) If 2 sides are equal, ∠'s opposite to them will be equal

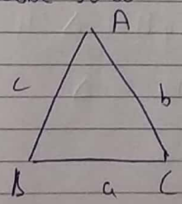


4)



$\angle 2 + \angle 3 = \angle 1$

5) The greatest angle will have greatest side opposite to it.



$A > B > C$

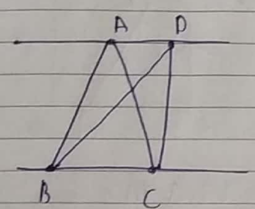
$a > b > c$

6) If ratio of ∠'s is given in Δ.

then ratio of sides $a : b : c$

$\sin A : \sin B : \sin C = a : b : c$

7)



area ABC = area BDC

8) given 3 sides of a Δ a, b, c

$a > b > c$

identify acute, Right, obtuse

$a^2 < b^2+c^2$ — acute Δ

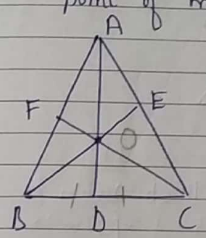
$a^2 = b^2+c^2$ — right Δ

$a^2 > b^2+c^2$ — obtuse Δ

Centers in Δ

i) Centroid

— point of intersection of median



to find AD

AD, BE, CF are median

$AB^2 + AC^2 = 2AD^2 + \frac{1}{2} BC^2$

$AB^2 + BC^2 = 2BE^2 + \frac{1}{2} AC^2$

$AC^2 + BC^2 + 2CF^2 + \frac{1}{2} AB^2$

D is mid point of BC

$3(AB^2 + BC^2 + AC^2) = 4(AD^2 + BE^2 + CF^2)$

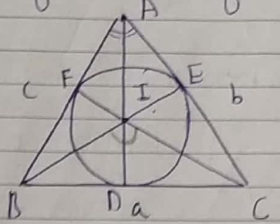
→ AD median divides area into 2 equal halfs

→ Centroid divides median in the ratio 2:1

ie $AO : OD = 2:1$

2) Incentre

→ point of intersection of ∠ bisectors



$r \times s$ = ar Δ ABC r = in radius
s = semi perimeter

→ $\angle BIC = 90° + \frac{1}{2} BAC$

$\angle AIC = 90° + \frac{1}{2} ABC$

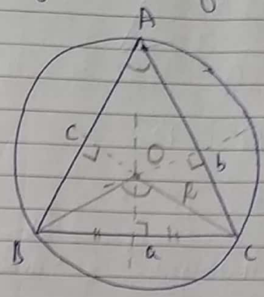
$\angle AIB = 90° + \frac{1}{2} ACB$

→ Incentre divides ∠ bisector in the ratio

$AI : ID = (b+c) : a$

3) Circumcenter

→ point of intersection of ⊥ bisectors.



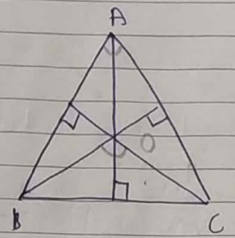
→ ar $\Delta ABC = \frac{abc}{4R}$

→ Circumcentre is at equidistance from all the 3 vertices of Δ.

→ $\angle BOC = 2 \angle BAC$

4) Orthocenter

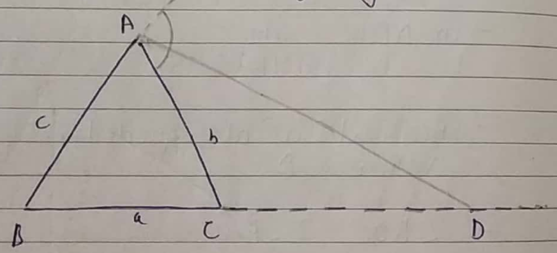
→ point of intersection of altitudes.



→ $\angle BOC + \angle BAC = 180°$

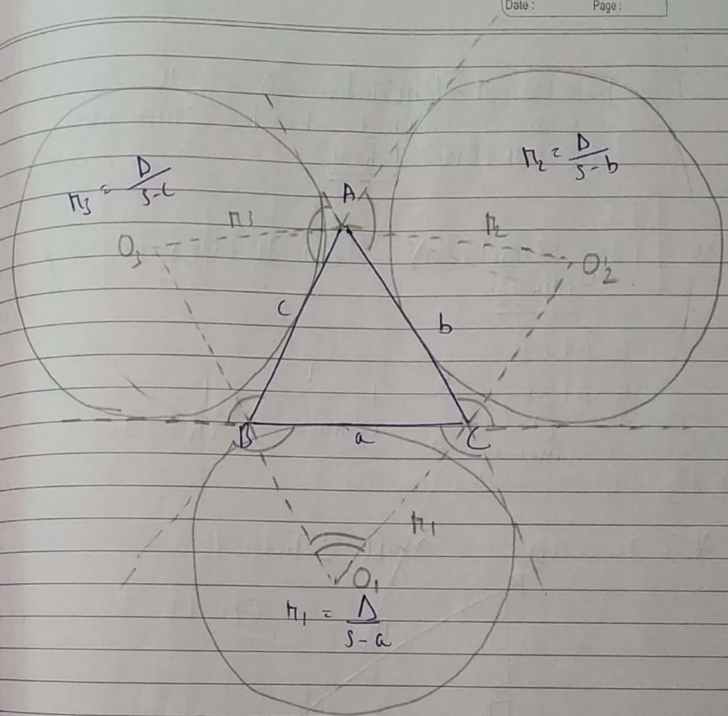
5) Ex-Center

→ point of intersection of external angle bisectors



AD is external $\angle$ bisector

$$\frac{AB}{AC} = \frac{BD}{CD}$$



→ For every Δ we will have 3 ex-centers and 3 ex-radius.

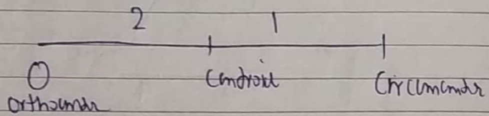
→ $\angle BO_1C = 90 - \frac{1}{2}\angle BAC$

$\angle CO_2A = 90 - \frac{1}{2}\angle ABC$

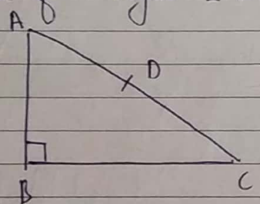
$\angle AO_3B = 90 - \frac{1}{2}\angle ACB$

* In case of Equilateral Δ, all centres centroid, incenter, circumcenter & orthocenter coincide

* In isosceles Δ, all 4 centers are collinear



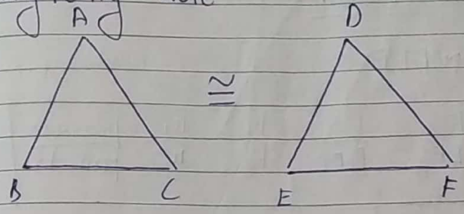
* In case of right ∠ triangle



* B → orthocenter
* Mid point of hypotenuse AC is circumcenter

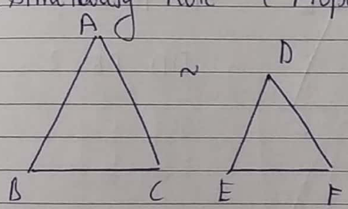
Circumcenter = $\frac{AC}{2}$

Congruency Rule



1) SSS	2) SAS	3) ASA	*4) AAS
AB = DE	AB = DE	∠A = ∠D	∠B = ∠E
BC = EF	BC = EF	∠B = ∠E	∠C = ∠F
AC = DF	∠B = ∠E	AB = DE	AC = DF

Similarity Rule (Proportional)



ABC ∼ DEF

i) SSS	ii) SAS	iii) AAA (or) AA
$\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$	$\frac{AB}{DE} = \frac{AC}{DF}$	∠A = ∠D
		∠B = ∠E
	∠A = ∠D	∠C = ∠F

Important Results

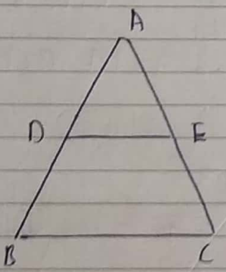
ABC ~ DEF

median

$$\boxed{\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF} = \frac{h_1}{h_2} = \frac{m_1}{m_2} = \frac{\text{perimeter}}{\text{perimeter}}} = a/b$$

$$\frac{ar\,\Delta ABC}{ar\,\Delta DEF} = \left(\frac{a}{b}\right)^2$$

Ex:



DE ∥ BC

AD : DB = 2 : 3

find $\frac{ar\,\Delta ADE}{ar\,\Delta ABC} = ?$

Ans: ADE ~ ABC

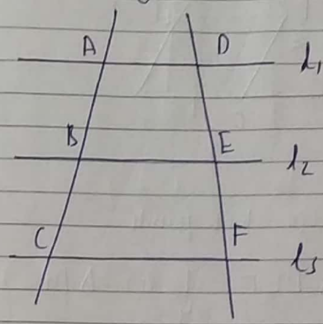
∠A = ∠A

∠D = ∠B

(by AA)

$$\frac{ar\,\Delta ADE}{ar\,\Delta ABC} = \left(\frac{AD}{AB}\right)^2 = \left(\frac{2}{5}\right)^2 = \frac{4}{25}$$

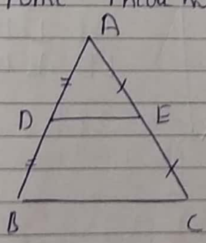
Basic Proportionality Theorem (B.P.T)



$l_1 \parallel l_2 \parallel l_3$

$$\boxed{\frac{AB}{BC} = \frac{DE}{EF}} \qquad \frac{AC}{BC} = \frac{DF}{EF}$$

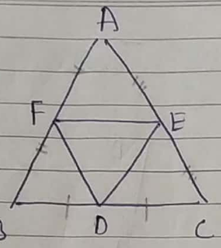
Mid Point Theorem (M.P.T)



D, E are mid points of AB & AC respectively

DE ∥ CB

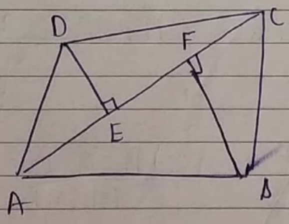
$$\boxed{DE = \frac{1}{2} BC}$$



D, E, F are mid points

* area of $\Delta DEF = \frac{1}{4}$ area ΔABC
* perimeter $\Delta DEF = \frac{1}{2}$ perimeter ΔABC

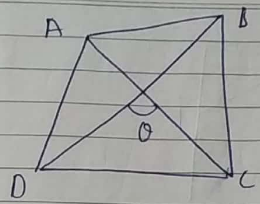
<u>Quadilaterals</u>



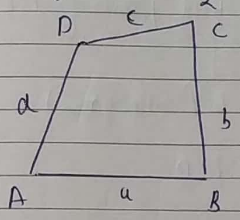
$DE \perp AC$
$BF \perp AC$

Sum of all $\angle$'s $= 360^\circ$

area of quadilateral $= \frac{1}{2} \times AC \times (DE + \cancel{BE})$ BF



area of $ABCD = \frac{1}{2} AC \times BD \sin\theta$



Sum of any two opposite $\angle$'s is given

$\angle A + \angle C$ or $\angle D + \angle B$

area of $ABCD = \sqrt{(s-a)(s-b)(s-c)(s-d) - abcd\cos^2\theta}$

$\theta = \frac{\angle A + \angle C}{2}$ (or) $\frac{\angle D + \angle B}{2}$

$s = \frac{a+b+c+d}{2}$

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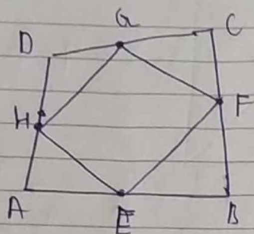
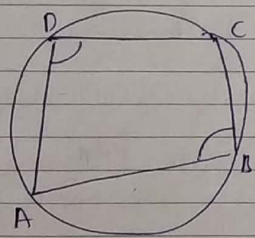


E, F, G, & H are mid points of AB, BC, CD, DA respectively

* EFGH is ||gm (Parallelogram)

* ar EFGH = $\frac{1}{2}$ ar ABCD

Cyclic Quadilateral



A, B, C, D are concyclic

$\angle D + \angle B = 180$

$\therefore \theta = 90°$

$\cos 90 = 0$

area = $\sqrt{(s-a)(s-b)(s-c)(s-d)}$

$s = \frac{a+b+c+d}{2}$

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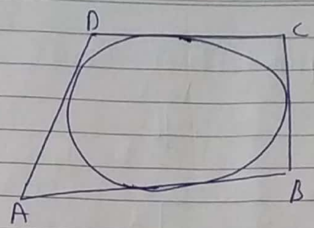
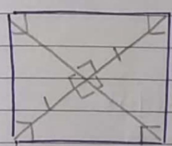


Quadilateral circumscribing a circle

$\boxed{AB + CD = AD + BC}$

Square



* all sides are equal
* all ∠'s are 90°
* Both diagonals are equal
* Both diagonal bisect each other perpendiculary

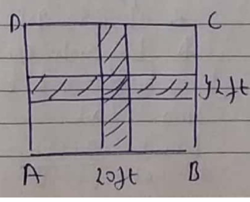
side = a

area = a^2

perimeter = 4a

diagonal = $\sqrt{2}a$

EX



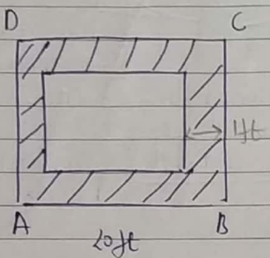
given a square park and path is laid down. find area of path?

Sol. area of path = $(20\times2) + (20\times2) - 4$ center square included twice

$= 80 - 4$

$= 76\ ft^2$

Q.



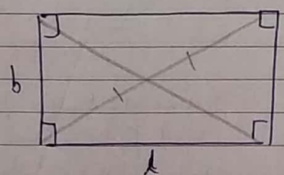
find area of path?

Sol: Area of path = $(20)^2 - (18)^2$

$= 400 - 324$

$= 76\ ft^2$

Rectangle



- opposite sides are equal
- all ∠'s 90°
- diagonals are equal and bisect each other

Area = $l \times b$

Perimeter = $2(l+b)$

diagonal = $\sqrt{l^2+b^2}$

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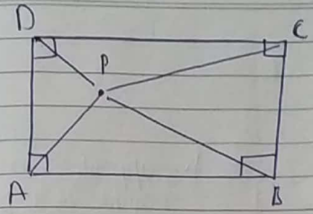
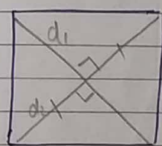


Let P be any point inside rectangle

$$\boxed{PA^2 + PC^2 = PD^2 + PB^2}$$

Rhombus

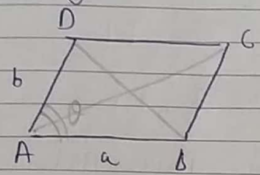


- all sides are equal
- diagonals bisect each other perpendicularly
- diagonals are unequal

area = $\frac{1}{2} \times d_1 \times d_1$

Perimeter = $4a$

Parallelogram

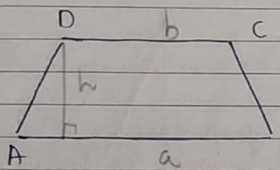


* opposite sides are equal and parallel
* diagonals bisect each other

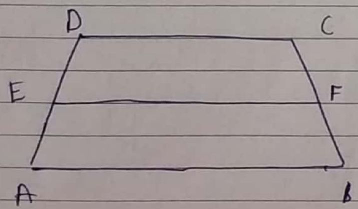
area = base X height

area = ab sin θ

Trapezium

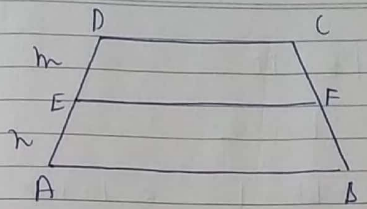


area = $\frac{1}{2}$ X (AB + CD)



E,F are mid points

EF = $\frac{1}{2}$(AB + CD)

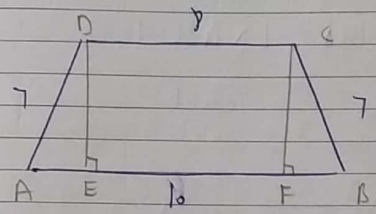


DC ∥ EF ∥ AB

DE : EA
m : n

$$EF = \frac{m(AB) + n(CD)}{m+n}$$

Ex.



find area = ?

Sol: given trapezium is isosceles trapezium bcz non parallel sides are equal.

△AED ≅ △BFC
∠A = ∠B
∠AED = ∠BFC
DE = FC
∴ by AAS

Hence $AE = BF$

$EF = 10 - 8 = 2$

$\therefore AE = BF = 1$ $\quad (\because AE + BF = 2)$

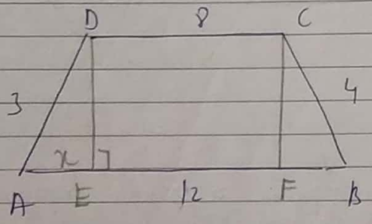
$\therefore DE = h = \sqrt{49 - 1} = \sqrt{48} = 4\sqrt{3}$

area $ABCD = \frac{1}{2} \times 4\sqrt{3}\,(10+8)$

$= 2\sqrt{3}\,(18)$

$= 36\sqrt{3}$

Ex



find area = ?

sol: $AE + BF = 12 - 8 = 4$

Let $AE = x$

$\therefore BF = \cancel{12-x}\ 4 - x$

$h = \sqrt{9 - x^2}$ (or) $h = \sqrt{16 - (4-x)^2}$

$\sqrt{9 - x^2} = \sqrt{16 - (4-x)^2}$

$9 - x^2 = 16 - (4-x)^2$

$9 - x^2 = 16 - 16 - x^2 + 8x$

$x = \frac{9}{8}$

$h = \sqrt{9 - \frac{81}{64}} = \frac{\sqrt{576 - 81}}{8} = \frac{\sqrt{495}}{8}$

area $ABCD = \frac{1}{2} \times \frac{\sqrt{495}}{8}\,(12+8) = \frac{5}{4}\sqrt{495}$

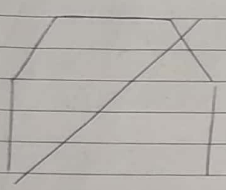
Regular Polygon

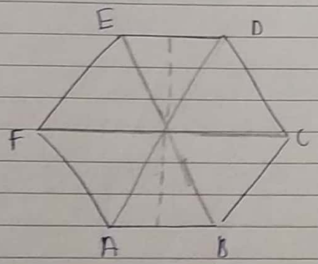
area of any regular polygon $n \frac{a^2}{4} \cot\left(\frac{180^\circ}{n}\right)$

n = no. of sides

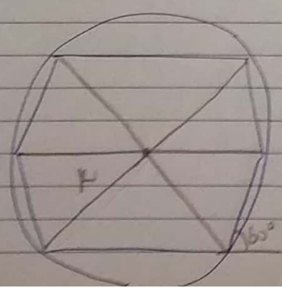
a = length of side

Hexagon



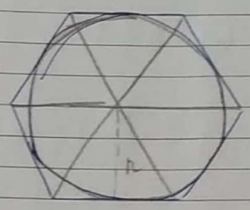


✯ area $= 6 \times \frac{\sqrt{3}}{4} a^2$ ✯ Height $= 2 \times \frac{\sqrt{3}}{2} a = \sqrt{3} a$



Consider a Hexagon circumscribed in a circle.

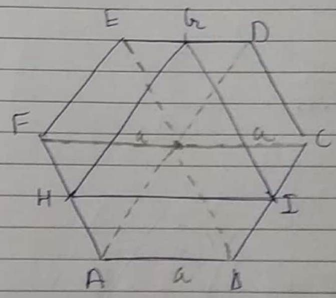
Radius $R = $ "a"
a = length of Hexagon



Hexagon circumscribing a circle

Radius of the circle $= \frac{\sqrt{3}}{2} a = \frac{\text{height}}{2}$

Ex



G, H, I are mid points. find $\frac{\text{ar } GHI}{\text{ar } ABCDEF} = ?$

ΔGHI is equilateral Δ

Sol:- ABCF is trapezium. Hence

$HI = \frac{1}{2}(AB + FC) = \frac{3a}{2}$

Date : Page :

$HI = \frac{3a}{2} = GI = HG$

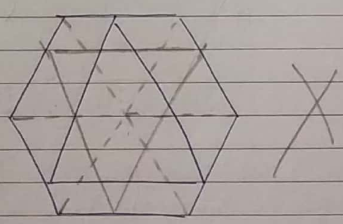
area of $GHI = \frac{\sqrt{3}}{4}\left(\frac{3a}{2}\right)^2$

$= \frac{\sqrt{3}}{4} \times \frac{9a^2}{4} = \frac{9\sqrt{3}}{16}a^2$

area of $ABCDEF = 6 \times \frac{\sqrt{3}}{4}a^2$

$$\frac{ar\ GHI}{ar\ ABCDEF} = \frac{\frac{9\sqrt{3}}{16}a^2}{6\frac{\sqrt{3}}{4}a^2} = \frac{9}{24}$$

II Method



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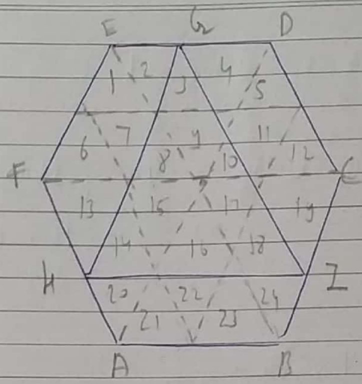
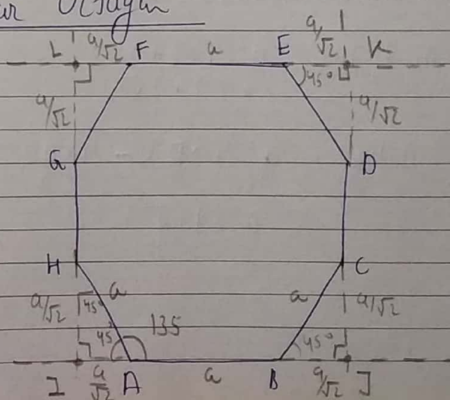


ar ABCDEF = 24 ar GHI = 9

~~Ratio~~ = $\frac{ar\ GHI}{ar\ ABCDEF} = \frac{9}{24}$

Regular Octagon



$\cos 45° = \frac{IA}{a}$

$\frac{a}{\sqrt{2}} = IA$

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Let side of Octagone be "a"
~~sides~~

Area of Octagon = (Area of square IJKL)
− (Area of 4 right angle isosceles Δ)

ar Δ = $\frac{1}{2} \times \cancel{\sqrt{2}a} \frac{a}{\sqrt{2}} \times \frac{a}{\sqrt{2}} = \frac{a^2}{4}$

ar Square = $(\text{side})^2$

side = $\frac{a}{\sqrt{2}} + a + \frac{a}{\sqrt{2}} = a + \sqrt{2}a$

$= a(1+\sqrt{2})$

ar IJKL = $a^2(1+2+2\sqrt{2})$
$= a^2(3+2\sqrt{2})$

area of octagon = $a^2(3+2\sqrt{2}) - 4 \times \frac{a^2}{4}$
$= 2a^2(1+\sqrt{2})$

Date
Page No.

Aptitude -3

8. Quadratic Equations
9. Trignometry
10. Height & Distance

$(\alpha^2+\beta^2)(\alpha^3+\beta^3) = \alpha^5+\beta^5+\alpha^2\beta^2(\alpha+\beta)$
$17 \times 65 = \alpha^5+\beta^5+16(5)$
$1105 - 80 = \alpha^5+\beta^5$
$1025 = \alpha^5+\beta^5$

Nature of Roots

$ax^2+bx+c=0$

$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$D = b^2-4ac$

i) if $D>0$ real and distinct roots

if $D=0$ real and equal roots

if $D<0$ imaginary roots

$ax^2+bx+c = a(x-\alpha)(x-\beta)$
$= x^2-(\alpha+\beta)x+\alpha\beta$

sum of roots product of roots

Formation

$x^2-3x-2=0$, α, β are roots of equations form an equation whose roots are

i) $2\alpha, 2\beta$ ii) $-\alpha, -\beta$ iii) $\frac{-\alpha}{2}, \frac{-\beta}{2}$

Sol. $x^2-3x-2=0$ — (i)
α, β roots.

i) Let $2\alpha = x$
$\alpha = \frac{x}{2}$

$\because \alpha$ is root of (i), Substitute α in (i)

$\left(\frac{x}{2}\right)^2 - 3\left(\frac{x}{2}\right) - 2 = 0$

$x^2-6x-8=0$

ii) Let $-\alpha = x$
$\alpha = -x$
On substituting $(-x)^2-3(-x)-2=0$
$x^2+3x-2=0$

iii) Let $\frac{-\alpha}{2}, \frac{-\beta}{2}$

$\frac{-\alpha}{2} = x$ $\alpha = -2x$

$(-2x)^2-3(-2x)-2=0$
$4x^2+6x-2=0$

Q. $2x^2-kx+5=0$
if one root is twice of other. Find k? $(k>0)$

Sol:- $2x^2 - kx + 5 = 0$

let α be root

Other root $= 2\alpha$

$\cancel{2\alpha^2 - k\alpha + 5 = 0}$

$\alpha + 2\alpha = \frac{k}{2}$

$\alpha = \frac{k}{6}$

$2\alpha^2 = \frac{5}{2}$

$\frac{2k^2}{36} = \frac{5}{2}$

$k^2 = 45$

$k = \pm\sqrt{45}$

$k = \sqrt{45}$ ($\because k > 0$)

Q $ax^2 + bx + c = 0$

if one root is square of other. find relation b/w a, b, c.

Sol:- α, α^2

$\alpha + \alpha^2 = \frac{-b}{a}$

$\alpha^3 = \frac{c}{a}$

$\sqrt[3]{\frac{c}{a}} + \left(\frac{c}{a}\right)^{2/3} = \frac{-b}{a}$

$\frac{c}{a} + \frac{c^2}{a^2} = \frac{-b^3}{a^3}$

$\frac{ac + c^2}{a^2} = \frac{-b^3}{a^3}$

$c(a+c) = \frac{-b^3}{a^3}$

Ans $a^4c + a^3c^2 + b^3 = 0$

Finding minimum & maximum value

$ax^2 + bx + c = 0$

$y = ax^2 + bx + c$

$y = a\left[x^2 + \frac{b}{a}x + \frac{c}{a}\right]$

$y = a\left[(x)^2 + 2x\left(\frac{b}{2a}\right) + \left(\frac{b}{2a}\right)^2 + \frac{c}{a} - \frac{b^2}{4a^2}\right]$

$y = a\left[\left(x + \frac{b}{2a}\right)^2 - \frac{b^2 - 4ac}{4a^2}\right]$

$y = a\left[\left(x + \frac{b}{2a}\right)^2 - \frac{D}{4a^2}\right]$

$\underbrace{y + \frac{D}{4a}}_{Y} = a\underbrace{\left(x + \frac{b}{2a}\right)}_{X}^2$

$Y = aX^2$

$X^2 = \frac{Y}{a}$ ← parabola

$y = f(x)$ is parabolic

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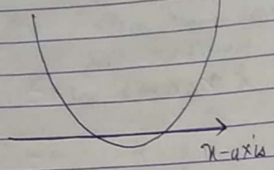
Case i) $a > 0$ $D > 0$

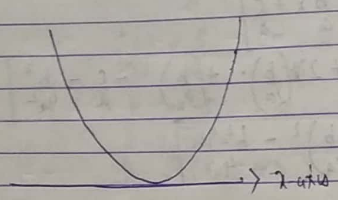
min value at $x = -\frac{b}{2a}$

↳ $-\frac{D}{4a}$

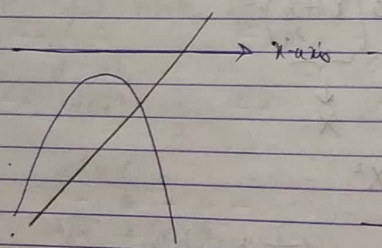
max value ∞

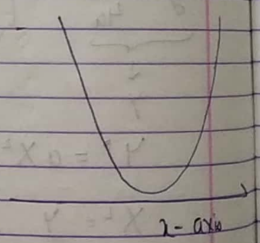


Case ii) $a > 0$ $D = 0$



Case iii) $a > 0$ $D < 0$

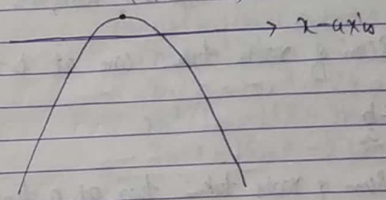




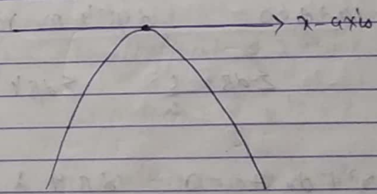
Case 4) $a < 0$ $D > 0$

$\left(-\frac{b}{2a}, \frac{-D}{4a}\right)$ ← max value

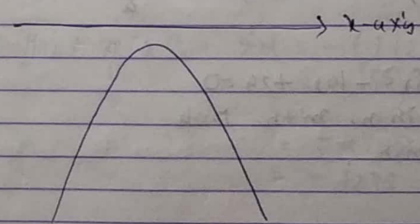
min value $-\infty$



Case 5) $a < 0$ $D = 0$



Case 6) $a < 0$ $D < 0$



High Order Equation

i) $ax^2+bx+c=0$ α,β are roots

$\Sigma\alpha$ = Sum of roots taken one at a time

$\Sigma\alpha = \frac{-b}{a}$

$\Sigma\alpha\beta$ = Sum of roots taken two at a time

$\Sigma\alpha\beta = \frac{c}{a}$

iii) $ax^3+bx^2+cx+d=0$ α,β,γ are roots

$\Sigma\alpha = \frac{-b}{a}$ $\Sigma\alpha\beta = \frac{c}{a}$ $\Sigma\alpha\beta\gamma = \frac{-d}{a}$

iv) $ax^4+bx^3+cx^2+dx+e=0$ $\alpha,\beta,\gamma,\lambda$ are roots

$\Sigma\alpha = \frac{-b}{a}$ $\Sigma\alpha\beta\gamma = \frac{-d}{a}$

$\Sigma\alpha\beta = \frac{c}{a}$ $\Sigma\alpha\beta\gamma\lambda = \frac{e}{a}$

Q $x^4-15x^2-10x+24=0$
$m-n, m, n, m+n$ roots
find m, n?

Sol- $\Sigma\alpha = m-n+m+n+m+n = 15$
$3m+n=15$

$\Sigma\alpha\beta\gamma\lambda = 24$

$(m-n)(m)(n)(m+n) = 24$

$(m^2-n^2)mn = 24$

$m^3n - mn^3 = 24$

$m^3(3m-15) - m(3m-15)^3 = 24$

$3m^4 - 15m^3 - 27m(m^3 - 3375 - 45m^2 + 675m) = 24$

$3m^4 - 15m^3 - 27m(m^3 - 45m^2 + 675m - 45m^2) = 24$

Solve for m.

Imp

Q $(x-1)(x-2)(x-3)\ldots\ldots\ldots(x-50)=0$
find i) coefficient of x^{50} = ?
ii) coefficient of x^{49} = ?
iii) coefficient of x^{48} = ?

Sol. $(x-1)(x-2)\ldots(x-50)$

coefficient of $x^{50} = 1$

coefficient of $x^{49} = \Sigma\alpha = -(1+2+3+\ldots+50)$

$= -\frac{(50)(51)}{2}$

$= -25\times51$

$= -1275$

Coefficient of $x^{49} = \sum \alpha\beta$

Concept : $\sum \alpha\beta$

$$(\sum \alpha)^2 = \sum \alpha^2 + 2(\sum \alpha\beta)$$

$$\sum \alpha\beta = \frac{(\sum \alpha)^2 - \sum \alpha^2}{2}$$

$$\sum \alpha\beta = \frac{(\sum \alpha)^2 - \sum \alpha^2}{2}$$

$$= \frac{(1275)^2 - (1^2+2^2+3^2+\ldots+50^2)}{2}$$

Coefficient of $x^{49} = \dfrac{(1275) - \frac{(50)(51)(101)}{6}}{2}$

Complex Equations

Q $12x + 17y = 157$, $x, y \in I \geq 0$

find all possible pairs of x, y.

Sol: $12x + 17y = 157$

$x = \frac{157 - 17y}{12}$ ← express this as multiple of 12

divisible by 12

$x = \frac{(156 - 12y) + 1 - 5y}{12}$

for x to be integer $1 - 5y$ must be divisible by 12.

put $y = 5$ $\quad x = \frac{157 - 17y}{12} = 6$

$y = 17$ $\quad x = \frac{157 - 17y}{12} < 0$ not possible

∴ Only 1 possible pair of (x, y) as solution.

Q $4x + 5y = 60$ $\quad x, y \geq 0$ $\quad x, y \in I$

Sol: $4x + 5y = 60$

$x = \frac{60 - 5y}{4} = \frac{(60 - 4y) - y}{4}$

put $y = 0$ $\quad x = 15$
$y = 4$ $\quad x = 10$
$y = 8$ $\quad x = 5$
$y = 12$ $\quad x = 0$
possible pairs

Q Suppose Shyam has Rs 68 and he wants to purchase some pen and pencils. Each pen costs Rs 3 & pencil costs Rs 8/each.

find ?

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i) How many ~~pen~~ pairs of pen, pencil he can purchase

ii) Maximum no of pens he can purchase.

Sol:- Let Shyam purchases x pencils & y pens

$8x + 3y = 68 \quad x, y \in I, \geq 0$

$y = \left(\frac{68 - 8x}{3}\right) = \frac{(66 - 6x) + 2 - 2x}{3}$

put $x = 1$, $y = 20$
$x = 4$, $y = 12$
$x = 7$, $y = 4$
} possible pen / pencil pairs

max no of pens he can purchase 20. ie When $x = 1$ (ie 1 pencil)

Q $x + y + z = 27 \quad x, y, z \geq 0 \quad x, y, z \in I$

$15x + 20y + 25z = 600$

Sol:-
$15x + 20y + 25z = 600 \qquad x + y + z = 27$
$3x + 4y + 5z = 120 \qquad z = 27 - (x + y)$
$3x + 4y + 5(27 - (x + y)) = 120$
$3x + 4y + 135 - 5x - 5y = 120$
$2x + y = 15$
$y = 15 - 2x$

$y = 15 - 2x \qquad z = 27 - (x + y)$

put $x = 0$	$y = 15$	$z = 12$
$x = 1$	$y = 13$	$z = 13$
$x = 2$	$y = 11$	$z = 14$
$x = 3$	$y = 9$	$z = 15$
$x = 4$	$y = 7$	$z = 16$
$x = 5$	$y = 5$	$z = 17$
$x = 6$	$y = 3$	$z = 18$
$x = 7$	$y = 1$	$z = 19$

Ans 8 possible pairs (x, y, z).

Trignometry

Measurement of Angle

i) Sexagesimal method,

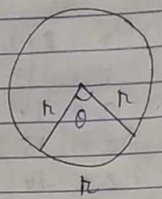
$1° = 60'$ Degree, Minutes, Seconds
$1' = 60''$

Ex: $\frac{180°}{7} = 25° \frac{5}{7} = 25° 42' 51''$

$\frac{5}{7} \times 60' = \frac{300'}{7} = 42 \frac{6}{7}'$

$\frac{6}{7}' \times 60'' = \frac{360''}{7} = 51$

2) Circular Method (radians)

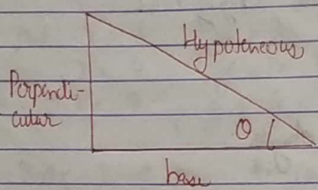


$\theta = 1^c$

$\theta = 1$ subtended by an arc of length "r".

$\pi^c = 180°$

$\boxed{1° = \frac{\pi}{180°}}$ $\boxed{1^c = \frac{180}{\pi}}$



$\sin\theta = \frac{p}{h}$ $\quad \text{cosec}\,\theta = \frac{h}{p}$

$\cos\theta = \frac{b}{h}$ $\quad \sec\theta = \frac{h}{b}$

$\tan\theta = \frac{p}{b}$ $\quad \cot\theta = \frac{b}{p}$

$\sin\theta \cdot \text{cosec}\,\theta = 1$

$\cos\theta \cdot \sec\theta = 1$

$\tan\theta \cdot \cot\theta = 1$

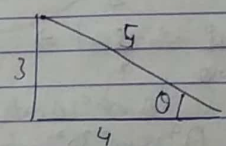
$\sin\theta = \frac{1}{\text{cosec}\,\theta}$ $\quad \cos\theta = \frac{1}{\sec\theta}$ $\quad \tan\theta = \frac{1}{\cot\theta}$

$\tan\theta = \frac{\sin\theta}{\cos\theta}$ $\quad \cot\theta = \frac{\cos\theta}{\sin\theta}$

Q given $\sin\theta = \frac{3}{5}$

find $2\tan\theta + \cos\theta$

Sol. $\sin\theta = \frac{3}{5}$



$2\tan\theta + \cos\theta$

$2\left(\frac{3}{4}\right) + \frac{4}{5} = \frac{3}{2} + \frac{4}{5} = \frac{23}{10}$

1. $\sin^2\theta + \cos^2\theta = 1$
2. $1 + \tan^2\theta = \sec^2\theta$
3. $1 + \cot^2\theta = \text{cosec}^2\theta$

tan

$1 + \tan^2\theta = \sec^2\theta \Rightarrow \sec^2\theta - \tan^2\theta = 1$

$\sec\theta + \tan\theta = \frac{1}{(\sec\theta - \tan\theta)}$

4. $\sec\theta + \tan\theta = \frac{1}{\sec\theta - \tan\theta}$

5. $\text{cosec}\,\theta + \cot\theta = \frac{1}{\text{cosec}\,\theta - \cot\theta}$

Q $\sec\theta + \tan\theta = 2$

find ~~$\tan\theta$~~ = ? $\cos\theta$ = ?

Sol: $\sec\theta + \tan\theta = 2$ —(i)

$\sec\theta - \tan\theta = \frac{1}{2}$ —(ii)

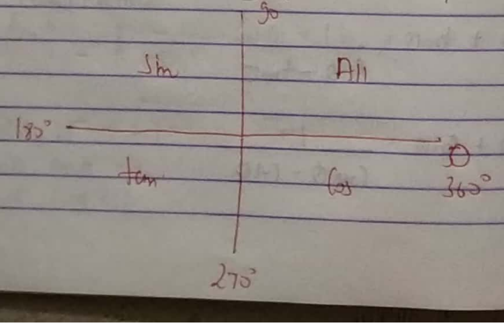
(i) + (ii)

$2\sec\theta = \frac{5}{2}$

$\frac{1}{\cos\theta} = \frac{5}{4} \Rightarrow \cos\theta = \frac{4}{5}$

	0°	30°	45°	60°	90°
$\sin\theta$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
$\tan\theta$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	∞

$30° = \frac{\pi}{6}$ $60° = \frac{\pi}{3}$ $45° = \frac{\pi}{4}$ $90° = \frac{\pi}{2}$



$180° \pm \theta$ | $360° \pm \theta$ $90° \pm \theta$ | $270° \pm \theta$

$\sin\theta \rightarrow \sin\theta$ $\sin \rightarrow \cos$

$\cos\theta \rightarrow \cos$ ~~cos~~

$\tan \rightarrow \tan$ $\tan \rightarrow \cot$

$\sec \rightarrow \operatorname{cosec}$

Q $\tan 9° \tan 27° \tan 45° \tan 63° \tan 81°$

Sol: $\tan 9° \tan 27° \tan 45° \tan 63° \tan 81°$

$\tan(90° - 9°)$ $\tan 27° = \tan(90° - 27°)$

$= \cot 81° \cot 63° \tan 45° \tan 63° \tan 81°$

$= \frac{1}{\tan 81°} \cdot \frac{1}{\tan 63°} (1) \tan 63° \tan 81°$

$= 1$

Q $\sin^2 5° + \sin^2 25° + \sin^2 45° + \sin^2 65° + \sin^2 85°$

Sol: $\cos^2 85° + \cos^2 65° + \sin^2 45° + \sin^2 65° + \sin^2 85°$

$= \cancel{8}$

$= 1 + 1 + \frac{1}{2} = \frac{5}{2}$

Min / Max Value

i) $-1 \le \sin\theta \le 1$

ii) $-1 \le \cos\theta \le 1$

iii) $-\infty < \tan\theta < \infty$

iv) $\sec\theta \ge 1$ && $\sec\theta \le -1$

v) $\operatorname{cosec}\theta \ge 1$ && $\operatorname{cosec}\theta \le -1$

vi) $0 \le \sin^2\theta \le 1$

vii) $0 \le \cos^2\theta \le 1$

$a\sin\theta + b\cos\theta$

max $= \sqrt{a^2+b^2}$ min $= -\sqrt{a^2+b^2}$

$a\sin\theta + b\cos\theta + c$

max $= c + \sqrt{a^2+b^2}$ min $= c - \sqrt{a^2+b^2}$

Q $3\sin\theta + 4\cos\theta$

max $= \sqrt{3^2+4^2} = 5$
min $= -\sqrt{3^2+4^2} = -5$

Q $5\sin\theta + 12\cos\theta - 3$

max $= -3 + \sqrt{25+144} = -3+13 = 10$

min $= -3 - \sqrt{25+144} = -3-13 = -16$

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Q $5\sin^2\theta - 4\cos^2\theta - 8$

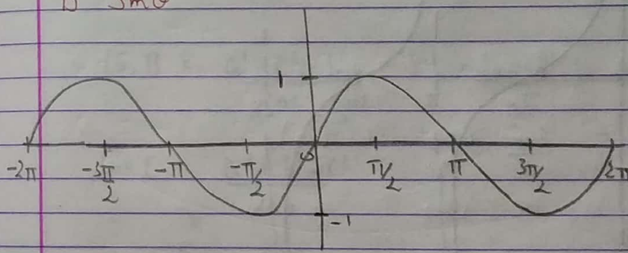
Sol) $5\sin^2\theta - 4\cos^2\theta - 8$
$5(1-\cos^2\theta) - 4\cos^2\theta - 8$
$5 - 5\cos^2\theta - 4\cos^2\theta - 8$
$5 - 9\cos^2\theta - 8 = -3 - 9\cos^2\theta$
$\cancel{\text{max} = 5-9(1)-8 = -12}$
$\cancel{\text{min} = 5-9(0)-8 = -3}$

$0 \le \cos^2\theta \le 1$

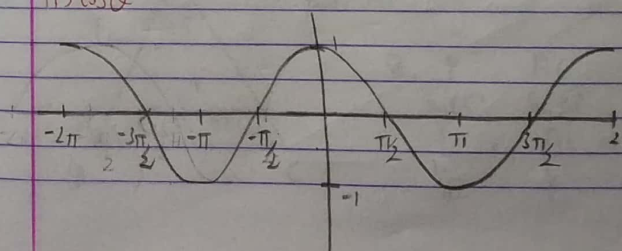
$-3 - 9(0) = -3$ max $= -3$
$-3 - 9(1) = -12$ min $= -12$

Graphs

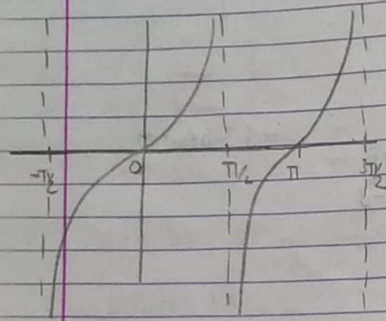
i) $\sin\theta$



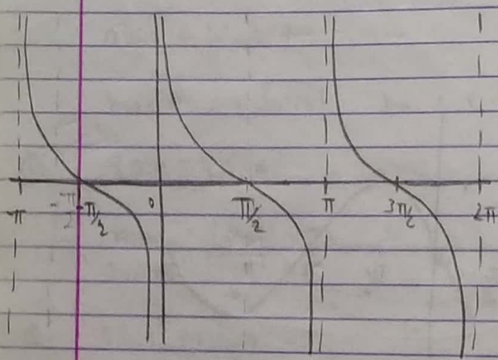
ii) $\cos\theta$



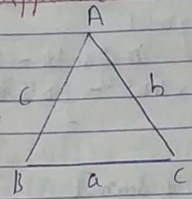
tanθ



Cot θ



Application



ar $\Delta ABC = \frac{1}{2} bc \sin A$

$= \frac{1}{2} ac \sin B$

$= \frac{1}{2} ab \sin C$

Sine Rule

$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R$ R = circum radius

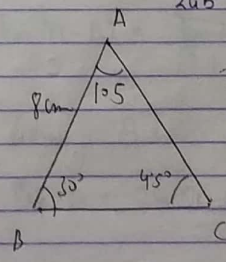
Cosign Rule

$\cos A = \frac{b^2+c^2-a^2}{2bc}$

$\cos B = \frac{a^2+c^2-b^2}{2ac}$

$\cos C = \frac{a^2+b^2-c^2}{2ab}$

Q



find AC = ?

$\frac{8}{\sin 45°} = \frac{AC}{\sin 30°}$

$\frac{8}{1/\sqrt{2}} = \frac{AC}{1/2} \Rightarrow AC = 4\sqrt{2}$ cm

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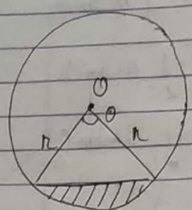
Segment



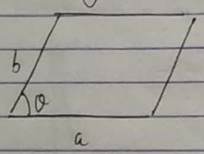
i) Ar of segment = Ar of sector − Ar of Δ

$$= \frac{\theta}{360^\circ} \times \pi r^2 - \frac{1}{2} r^2 \sin\theta$$

ii) Perimeter of Segment

$$= \frac{\theta}{360^\circ} \times 2\pi r + 2r\sin\frac{\theta}{2}$$

Parallelogram



Ar ||gm = $ab\sin\theta$

Imp

Area of Regular Polygon

area = $\frac{n \cdot a^2}{4} \cot\left(\frac{180^\circ}{n}\right)$

n = no of sides

a = length of side

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Important Formulae

$\sin(A+B) = \sin A\cos B + \cos A\sin B$

$\sin(A-B) = \sin A\cos B - \cos A\sin B$

$\cos(A+B) = \cos A\cos B - \sin A\sin B$

$\cos(A-B) = \cos A\cos B + \sin A\sin B$

$$\sin A\cos B = \frac{\sin(A+B) + \sin(A-B)}{2}$$

$$\cos A\sin B = \frac{\sin(A+B) - \sin(A-B)}{2}$$

$$\cos A\cos B = \frac{\cos(A+B) + \cos(A-B)}{2}$$

$$\sin A\sin B = \frac{\cos(A-B) - \cos(A+B)}{2}$$

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A\tan B}$$

$$\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A\tan B}$$

Imp When A = B

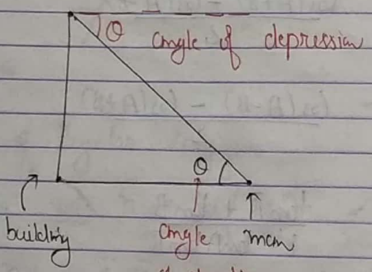
$$\sin 2A = 2\sin\theta\cos\theta = \frac{2\tan\theta}{1+\tan^2\theta}$$

ii) $\cos 2\theta = \cos^2\theta - \sin^2\theta$
$= \cancel{1-\sin^2\theta}\ 1 - 2\sin^2\theta$
$= 2\cos^2\theta - 1$
$= \dfrac{1-\tan^2\theta}{1+\tan^2\theta}$

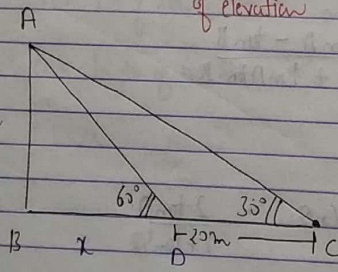
iii) $\tan 2\theta = \dfrac{2\tan\theta}{1-\tan^2\theta}$

Height And Distance

i) Angle of Elevation
ii) Angle of Depression



Q



He moves 20m towards building. What is the height of building?

Sol:- $\tan 30° = \dfrac{h}{BC}$

$\dfrac{1}{\sqrt{3}} = \dfrac{h}{BC}$

$\dfrac{1}{\sqrt{3}} = \dfrac{h}{x+20}$

$\tan 60° = \dfrac{h}{x}$

$\sqrt{3} = \dfrac{h}{x}$

$\sqrt{3}x = h$

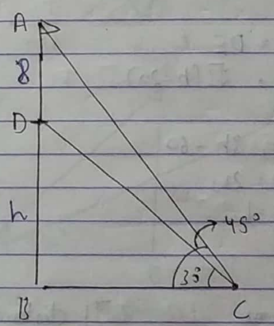
$\dfrac{x+20}{\sqrt{3}} = \sqrt{3}x$

$x + 20 = 3x$

$10 = x$

$h = 10\sqrt{3}\ m$

Q



If length of flag is 8ft. Find Height of building?

Sol:- $\tan 30° = \dfrac{h}{BC}$

$\dfrac{1}{\sqrt{3}} = \dfrac{h}{BC}$

$BC = h\sqrt{3}$

$\tan 45° = \dfrac{8+h}{BC}$

$BC = 8+h$

$h\sqrt{3} = 8+h$

$h = \dfrac{8(\sqrt{3}+1)}{2}$

$h = 4(\sqrt{3}+1)$ ft

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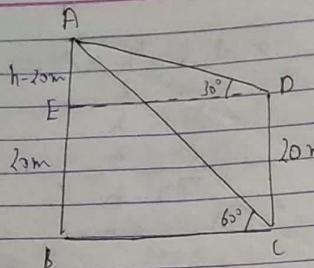
Q



find height of bigger building?

Sol:- $\tan 60^\circ = \frac{h}{BC}$ $\quad$ $\tan 30^\circ = \frac{h-20}{DE}$

$BC = \frac{h}{\sqrt{3}}$ $\quad$ $DE = (h-20)\sqrt{3}$

$BC = DE$

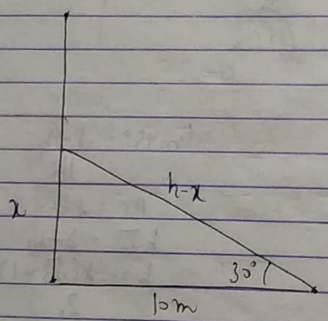
$\frac{h}{\sqrt{3}} = \sqrt{3}(h-20)$

$h = 3h - 60$

$60 = 2h$

$30 = h$

Q



If distance b/w top tree and base is 10ft actual height of tree?

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Sol:- ~~Sin 30°~~ $\tan 30^\circ = \frac{x}{10}$

$x = \frac{10}{\sqrt{3}}$

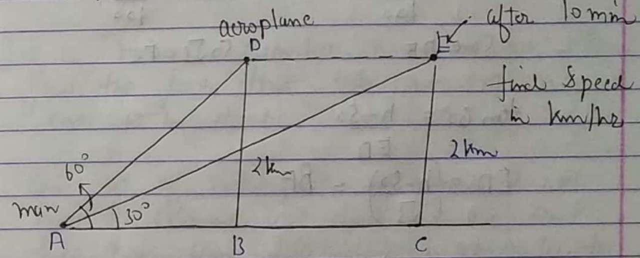
$\sin 30^\circ = \frac{x}{h-x}$

$\frac{1}{2} = \frac{10/\sqrt{3}}{h-10/\sqrt{3}}$

$h - \frac{10}{\sqrt{3}} = 2 \times \frac{10}{\sqrt{3}}$

$h = \frac{3 \times 10}{\sqrt{3}} \Rightarrow h = 10\sqrt{3}$ m

Q



Sol:- $\tan 60^\circ = \frac{2}{AB}$ $\quad$ $\tan 30^\circ = \frac{2}{AB+BC}$

$AB = \frac{2}{\sqrt{3}}$ $\quad$ $AB+BC = 2\sqrt{3}$

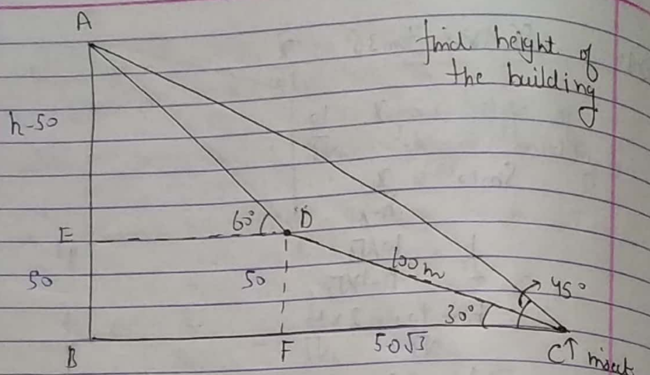
$BC = 2\sqrt{3} - \frac{2}{\sqrt{3}}$

distance travelled by aeroplane in 10 min

$\left(2\sqrt{3} - \frac{2}{\sqrt{3}}\right)$ m

Speed $= \frac{\left(2\sqrt{3} - \frac{2\sqrt{3}}{3}\right)}{\frac{10}{60}} = 2\sqrt{3}\left(\frac{2}{3}\right) \times 6 = 8\sqrt{3}$ km/hr

Q



find height of the building

Sol: $\sin 30° = \frac{DF}{100}$ $\cos 30° = \frac{FC}{100}$

$50 = DF$ $50\sqrt{3} = FC$

$\tan 60° = \frac{h-50}{ED}$

$ED = \left(\frac{h-50}{\sqrt{3}}\right) = BF$

$\tan 45° = \frac{AB}{BC} \Rightarrow AB = BC$

$h = 50\sqrt{3} + \frac{h-50}{\sqrt{3}}$

$h = 50\sqrt{3} + \frac{h\sqrt{3}}{3} - \frac{50\sqrt{3}}{3}$

$h\left(1 - \frac{\sqrt{3}}{3}\right) = \frac{100\sqrt{3}}{3}$

$h = \frac{100\sqrt{3}}{(3-\sqrt{3})}$ m

Probability

Numericals

Q Lets odds against India winning a match with Australia are 3:5. Then find the prob of India winning the match.

Sol: Pr E = India winning the match

$P(E) = \frac{5}{3+5} = \frac{5}{8}$

Q Let a natural number n is selected. Then find the prob. that 4th power of n ends in the digit "6".

Sol: Since set of natural no. is infinite & we have to examine unit digit so we can take nos from 1 to 10 and generalize the result for entire set.

last digit	I	I^2	I^4
	0	0	0
9,	1	1	1
8,	2	4	6
7,	3	9	1
6,	4	6	6
	5	5	5

I = set of nos b/w 1 to 10, we are considering only last digit.

I^2 = set of nos after squaring no's in set I

I^4 = set of nos after squaring nos in set I^2

Here we are considering only last digit.

I	I^2	I^4
0	0	0
1	1	1
2	4	6
3	9	1
4	6	6
5	5	5
6	6	6
7	9	1
8	4	6
9	1	1

We can not write sample space as

$\div \{0,1,5,6\}$ bcz $(n)^4 \to$ ending 0 prob $\frac{1}{10}$

$(n)^4 \to$ ending 1 prob $\frac{4}{10}$

Sample points are not equally likely

prob(ending with 6") = $\frac{4}{10} = \frac{2}{5} = 0.4$

Q There are n letter and n addressed envelopes. The probability that all the letters are not kept in the right envelope is

Sol:- $E \to$ all the letters are not kept in right envelope

$P(\bar{E}) = 1 - \frac{1}{n!}$

total Cases = distribute n letters among n envelops.

Q The letters of the word ASSASSIN are written down at random in a row. Probability that no two S occur together

Sol:- ASSASSIN

Concept:- Treat all the identical objects as distinct.

$A_1 S_1 S_2 A_2 S_3 S_4 I_1 N_1$

no two S occur together (condition on S)

S को अलग करलो

$|A_1|A_2|I_1|N| \rightarrow 4! \times {}^5C_4 \times 4!$

F.N.O.W = fav. no of ways

$= 4! \times {}^5C_4 \times 4!$

T.N.O.W = total no of ways

$= 8!$ (8 letter permutes)

$P(E) = \frac{4! \times 4! \times {}^5C_4}{8!} = \frac{4! \times 5!}{8!} = \frac{4 \times 6 \times 5!}{8 \times 7 \times 6 \times 5!}$

$= \frac{1}{14}$

Q Out of 21 tickets marked with numbers 1 to 21 three are drawn at random. The chance that no's on them are in AP.

Sol:- Let 3 no's drawn be a, b, c, if they are in AP

$2b = a + c$

even

e + e

(or)

o + o

Sim a+c is even only if both even or both odd.

Here we have to focus on selecting a & c by selecting a & c, b will automatically adjust.

# of ways of selecting a & c = ${}^{10}C_2 + {}^{11}C_2$

either both even or both odd

1 to 21 — 11 odd, 10 even

$P(E) = \frac{{}^{10}C_2 + {}^{11}C_2}{{}^{21}C_3} = \frac{45 + 55}{\frac{21 \cdot 20 \cdot 19}{6}} = \frac{100 \times 6}{21 \cdot 20 \cdot 19}$

$= \frac{10}{133}$

Q There are 4 apples and 3 oranges placed at random in a line. Then the chance of the extreme fruits being both oranges is

Sol:- 4A & 3O

T NOC = 7!

F NOC = O_1 _ _ _ _ _ O_2

${}^3C_2 \times 2! \times 5!$

$P(E) = \frac{{}^3C_2 \times 2! \times 5!}{7!} = \frac{3 \times 2}{7 \times 6} = \frac{1}{7}$

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Q. What is the probability that in a group of N people, at least two of them will have the same birthday? $(N < 365)$

Sol:- at least two
↳ always convert at least into exactly

exactly 2 + exactly 3 + exactly 4 + ...

1 − at most 1 (ie every body has birthday on different day)

TNOC = $(365)^N$

$P_1 \rightarrow$ का birthday 365 तरीके से आसकता है।

$P_2 \rightarrow 365$

$P_3 \rightarrow 365$

$P_N \rightarrow (365)$

FNOC = ${}^{365}C_N \times N!$

365 में से N days select करे

now we have to distribute these N days among N people

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$$P(E) = 1 - \frac{{}^{365}C_N \times N!}{(365)^N}$$

Q. Let 2 no's are selected from the set $\{1, 2, 3, 4, \ldots 51\}$; then find the probability

i) if sum of no. is even
ii) if product of no. is divisible by 2
iii) if product of no. is divisible by 3
iv) if square of none of no. ends in 4
v) if sum of no's is divisible by 5.
vi) sum of no's is divisible by 3.

Sol: TNOC = $51 \cdot 51 = 51^2$ (∵ 2 no's selected are not distinct)

i) $n_1 + n_2$ = even   25 − even
e + e   26 − odd
o + o

FNOC = $\dfrac{25 \cdot 25 + 26 \cdot 26}{\cancel{51^2}} = \dfrac{25^2 + 26^2}{\cancel{51^2}}$

$$P(E) = \frac{25^2 + 26^2}{51^2}$$

ii) $n_1 \cdot n_2 = 2k$

e & e
e & o
o & e

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$P(E) = \frac{25\cdot25 + 25\cdot26 + 26\cdot25}{51^2}$ (or) $1 - \left(\frac{26\cdot26}{51^2}\right)$

1 - Unfav

iii) $n_1 \cdot n_2 = 3$ $\qquad D(3) = \frac{51}{3} = 17$

$D\cdot D + D\bar{D} + \bar{D}\cdot D$ $\qquad \bar{D}(3) = 51-17 = 34$

FNOC = $17\cdot17 + 17\cdot34 + 34\cdot17$

$P(E) = \frac{17^2 + 17\cdot34 + 34\cdot17}{51^2}$ (or) $1 - \frac{34\cdot34}{51^2}$

iv) {1, 2, 3, ... 51}

N	N^2
0	0
1	1
2	4
3	9
4	6
5	5
6	6
7	9
8	4
9	1

nos whose square ends with 4.
2, 12, 22, 32, 42
8, 18, 28, 38, 48 } - 10

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nos whose square does not end in 4 = 51 - 10 = 41

$P(E) = \frac{41\cdot41}{51^2} = \left(\frac{41}{51}\right)^2$

iv) $n_1 + n_2 = 5k$

With respect to divisibility with 5,
I can divide numbers into 5 types

5n — 0, 5, 10, 15, ...

5n+1 — 1, 6, 11, 16, ...

5n+2 — 2, 7, 12, 17, ...

5n+3 — 3, 8, 13, 18, ...

5n+4 — 4, 9, 14, 19, ...

In every cycle of 5 nos, each type of no appears exactly once.
1 2 3 4 5

6 7 8 9 10

{1, 2, 3, ... 51} $\quad$ 5n - 10
10 cycles $\quad$ 5n+1 - 11 (10+1) bcz of 51
5n+2 - 10
5n+3 - 10
5n+4 - 10

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$n_1 + n_2 = 5k$

$5n \quad 5n$

$5n+1 \quad 5n+4$

$5n+4 \quad 5n+1$

$5n+3 \quad 5n+2$

$5n+2 \quad 5n+3$

$FNOC = 10\cdot 10 + 11\cdot 10\cdot 2 + 10\cdot 10\cdot 2$

$$P(E) = \frac{10^2 + (11\cdot 10)2 + (10)^2\cdot 2}{51^2}$$

vi) $n_1 + n_2 = 3k$

$\{1, 2, \ldots 51\}$
17 cycles

$3n \quad - 17$
$3n+1 \quad - 17$
$3n+2 \quad - 17$

$3n \,\&\, 3n$
$3n+1 \quad 3n+2$
$3n+2 \quad 3n+1$

$FNOC = 17\cdot 17 + 17\cdot 17\cdot 2$

$$P(E) = \frac{17^2 + (17)^2\cdot 2}{51^2}$$

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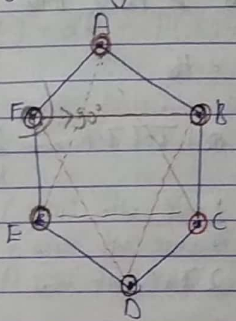
Q Let 3 vertices are selected at random from vertices of regular hexagon. Find probability that they form equilateral Δ.

Sol: Concept :- vertices of equi Δ they follow symmetry.



We can not select 3 consequitive vertices as b/c we get obtuse $\angle$. Similarly we can't select 2 consequitive vertices, and 3rd vertex non adjacent to selected 2 vertices, we will get corresponding acute $\angle$.

Only possible way to get form equi Δ is to select alternative vertices

$$P(E) = \frac{2}{{}^6C_3} = \frac{2}{20} = \frac{1}{10}$$

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Q Let a fair coin is tossed 4 times. Find the probability

i) all outcomes are identical
ii) the equal no of Heads & tails
iii) heads & tail occur alternatively
iv) no of heads > no of tails

Sol: TNOC = $2^4 = 16$

i) HHHH or TTTT = $\frac{2}{16} = \frac{1}{8}$

$\frac{1}{16} + \frac{1}{16}$

ii) 2H's & 2 T's

total arrangement = $\frac{4!}{2!2!} = 6$

$\frac{6}{16} = \frac{3}{8}$

iii) HTHT or THTH

$\left(\frac{1}{16}\right) + \left(\frac{1}{16}\right) = \frac{1}{8}$

iv)

H	T
0	4
1	3
2	2

$= \frac{1 + \frac{4!}{3!} + \frac{4!}{2!2!}}{16} = \frac{1+4+6}{16} = \frac{11}{16}$

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Q Let in a race of 3 persons A, B, C are participating. The chance of A winning is the race is twice as that of B & chance of B winning the race is thrice as that of C. Find odd against them winning the race, tie isn't possible

Sol: Let $P(C) = \alpha$ $P(B) = 3\alpha$ $P(A) = 6\alpha$

given tie isn't possible, hence exactly one of them has to win the race

E_1 = C win the race
E_2 = B win the race
E_3 = A win the race

$E_1 + E_2 + E_3$ are exhaustive events

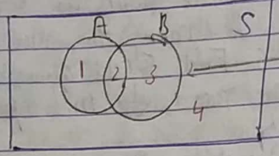
$\alpha + 3\alpha + 6\alpha = 1$
$\alpha = \frac{1}{10}$

$P(A) = \frac{6}{10}$ odd against = 4 : 6 = 2 : 3

$P(B) = \frac{3}{10}$ odd against = 7 : 3

$P(C) = \frac{1}{10}$ odd against 9 : 1

Set Theoritical Representation



this circle denotes event B

~~A = ...~~

$A = ① + ②$

$B = ② + ③$

$A \cap B = A$ and $B = ②$

$\bar{A} \cap \bar{B}$ = neither A nor B = none of A or B = ④

$A \cup B$ = at least one of A or B =

$① + ② + ③$

Imp $A \cap \bar{B}$ = A but not B
All elements present in ①

$$P(A) = \frac{n(A)}{n(S)}$$

$n(S)$ = Total no of elements in sample space
$n(A)$ = all elements present in A.

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(\bar{A} \cap \bar{B}) = 1 - P(A \cup B)$$

$$P(\bar{A} \cup \bar{B}) = 1 - P(A \cap B)$$

$$P(A \cap \bar{B}) = P(A) - P(A \cap B)$$

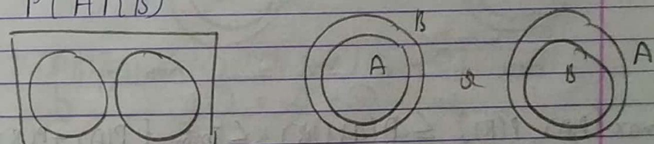
P(exactly one of A or B) = ~~$P(A \cap \bar{B})$~~

$= P(A) + P(B) - 2P(A \cap B)$

① ②   ② ③   2·②

Imp **Range of $P(A \cap B)$ And $P(A \cup B)$**

i) $P(A \cap B)$

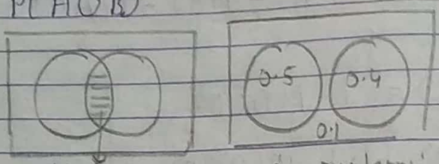


min 0 when
When no overlapping

When they completely overlap each other

$$\boxed{0 \le P(A \cap B) \le \min\{P(A), P(B)\}}$$

ii) $P(A\cup B)$



$P(A) = 0.5$
$P(B) = 0.4$

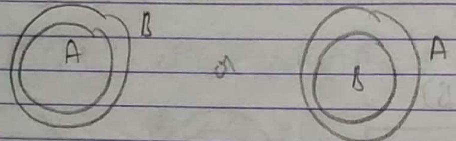
Here common area counted once

no overlapping area.

but if $P(A) = 0.8$ $P(B) = 0.4$

$P(A) + P(B) > 1$

There must be overlapping area
& $P(A\cup B) \leq 1$
↳ never exceed 1



Here we have maximum overlapping

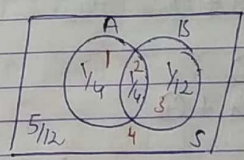
$\max\{P(A), P(B)\} \leq P(A\cup B) \leq \min\{P(A)+P(B), 1\}$

Q let $P(A) = \frac{1}{2}$ $P(B) = \frac{1}{3}$ ~~P(A∩B)~~

$P(\bar{A}\cap\bar{B}) = \frac{5}{12}$ then find

i) $P(A\cup\bar{B})$
ii) $P(\bar{A}\cap B)$
iii) $P(A/B)$
iv) $P(\bar{A}/B)$
v) $P\left(\frac{A\cup\bar{B}}{A\cup B}\right)$
vi) $P\left(\frac{\bar{A}\cap B}{\bar{A}\cup\bar{B}}\right)$

Sol:-



$P(A\cup B) = \cancel{\frac{5}{12}}\ 1 - \frac{5}{12} = \frac{7}{12}$

$P(A\cup B) = P(A) + P(B) - P(A\cap B)$

$\frac{7}{12} = \frac{1}{2} + \frac{1}{3} - P(A\cap B)$

$P(A\cap B) = \frac{5}{6} - \frac{7}{12} = \frac{3}{12} = \frac{1}{4}$

i) $P(A\cup\bar{B}) = \{1 + 2\} \cup \{1 + 4\}$
$= 1 + 2 + 4$
$= \frac{1}{4} + \frac{1}{4} + \frac{5}{12} = \frac{11}{12}$

ii) $P(\bar{A}\cap B) = \{3 + 4\} \cap \{2 + 3\} = 3 = \frac{1}{12}$
$= \cancel{\frac{1}{12} + \frac{5}{12} + \frac{1}{4} = \frac{1}{2} + \frac{1}{4} = \frac{3}{4}}$

iii) $P\left(A/B\right) = \dfrac{P(A\cap B)}{P(B)} = \dfrac{1/4}{1/3} = \dfrac{3}{4}$

iv) $P\left(\dfrac{\bar{A}}{B}\right) = \dfrac{1}{4}$

v) $P\left(\dfrac{A\cup \bar{B}}{A\cup B}\right) = \dfrac{1 + 2}{1 + 2 + 3} = \dfrac{1/2}{7/12} = \dfrac{6}{7}$

$A\cup B = 1 + 2 + 3$

$A\cup \bar{B} = 1 + 2 \cancel{+4} + 4$

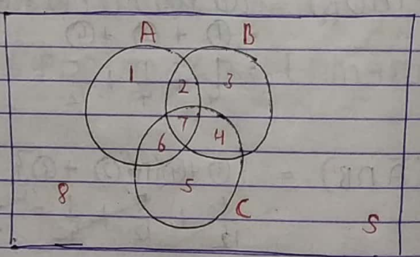
vi) $P\left(\dfrac{\bar{A}\cap B}{\bar{A}\cup \bar{B}}\right) = \dfrac{3}{1 + 3 + 4} = \dfrac{1/12}{3/4} = \dfrac{1}{9}$

$\bar{A}\cup \bar{B} = \{3 + 4\} \cup \{1 + 4\}$

$= \{1 + 3 + 4\}$

$\bar{A}\cap B = \{3 + 4\} \cap \{2 + 3\} = \{3\}$

<u>3 Events</u>



$A \rightarrow 1 + 2 + 6 + 7$

$B \rightarrow 2 + 3 + 4 + 7$

$C \rightarrow 4 + 5 + 6 + 7$

$A\cap B \rightarrow 2$

$B\cap C \rightarrow 4$

$C\cap A \rightarrow 6$

$A\cap B\cap C \rightarrow 7$

$\bar{A}\cap \bar{B}\cap \bar{C} \rightarrow 8$

$P(A\cup B\cup C) = P(A) + P(B) + P(C) - P(A\cap B)$
$\quad - P(B\cap C) - P(A\cap C) + P(A\cap B\cap C)$

P(A Note :- for n events

$P(A_1 \cup A_2 \cup A_3 \ldots \cup A_n) = \Sigma P(A_i) - \Sigma P(A_1\cap A_2)$
$\quad + \Sigma P(A_1\cap A_2\cap A_3)$
$\quad - \Sigma P(A_1\cap A_2 \cap A_3\cap A_4)$
$\quad + \ldots$

P(exactly one of A, B, C)

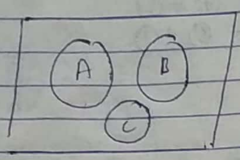
$= P(A) + P(B) + P(C) - 2\Sigma P(A\cap B) + 3P(A\cap B\cap C)$

$P(\text{none}) = P(\bar{A}\cap \bar{B}\cap \bar{C})$

$= 1 - P(A\cup B\cup C)$

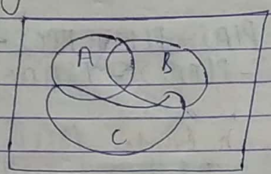
$P(\text{exactly two}) = \cancel{\Sigma P(A)} \ \Sigma P(A\cup B) - 3P(A\cap B\cap C)$

Pairwise ME



$P(A \cap B) = \phi$
$P(B \cap C) = \phi$
$P(C \cap A) = \phi$

Mutually Exclusive but not pairwise ME



$P(A \cap B \cap C) = \phi$ but $P(A \cap B) \neq \phi$

3-events are called pairwise independent

$P(A \cap B) = P(A) \cdot P(B)$
$P(B \cap C) = P(B) \cdot P(C)$
$P(A \cap C) = P(A) \cdot P(C)$

Note: for 3 events to be mutually ~~exclusive~~ independent if each possible subset of events is mutually independent

$P(A \cap B) = P(A) \cdot P(B)$
$P(B \cap C) = P(B) \cdot P(C)$
$P(A \cap C) = P(A) \cdot P(C)$
$P(A \cap B \cap C) = P(A) \cdot P(B) \cdot P(C)$

Similary for n-events to be Mutually independent.

Total no. of conditions required

$${}^nC_2 + {}^nC_3 + {}^nC_4 + {}^nC_5 + \dots + {}^nC_n = 2^n - {}^nC_0 - {}^nC_1$$
$$= 2^n - 1 - n$$

Q For 5-events to be Mutually independent, no of conditions required ?

Sol:- $2^5 - 1 - 5 = 32 - 6 = 26$

Q A box contains 50 defective & 50 non-defective bulbs and two are drawn one by one with replacement. Define event

A = 1st is defective
B = 2nd is ~~d~~ non-defective
C = either both defective or both non-defective

Find whether they are pair wise independent or mutually independent or both

Sol: S.S = {DD, DN, ND, NN}

A = {DD, DN}   C = {DD, NN}

B = {DN, NN}

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$P(A) = \frac{2}{4} = \frac{1}{2}$ $P(B) = \frac{2}{4} = \frac{1}{2}$

$P(C) = \frac{2}{4} = \frac{1}{2}$

Pair wise M I check ✓

$P(A \cap B) = P(A) \cdot P(B)$

$\frac{1}{4} = \frac{1}{2} \cdot \frac{1}{2}$

$\frac{1}{4} = \frac{1}{4}$

$P(B \cap C) = P(B) \cdot P(C)$

$\frac{1}{4} = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$

$P(A \cap C) = P(A) \cdot P(C)$

$\frac{1}{4} = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$

Mutually Independent X

$P(A \cap B \cap C) = P(A) \cdot P(B) \cdot P(C)$

$\phi = \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{8}$

Ans Pair wise M.I.

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Addition Rule

If A & B are M.I

$P(A \cup B) = P(A) + P(B) - P(A) \cdot P(B)$

If A & B are ME

$P(A \cup B) = P(A) + P(B)$

Multiplication Rule

$P(A \cap B) = P(A) \cdot P($ B occurs given A has already occurred $)$

$= P(A) \cdot P\left(\frac{B}{A}\right)$

For n-events

$P(A_1 \cap A_2 \cap \dots A_n) =$

$P(A_1) P\left(\frac{A_2}{A_1}\right) P\left(\frac{A_3}{A_1 \cap A_2}\right) \dots P\left(\frac{A_n}{A_1 \cap A_2 \dots A_{n-1}}\right)$

If events are M I

$P(A_1 \cap A_2 \dots \cap A_n) = P(A_1) \cdot P(A_2) \cdots P(A_n)$

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Probability of Strings

$\underbrace{c_1 c_1 c_1 \ldots c_1}_{n_1} \underbrace{c_2 c_2 c_2 \ldots c_2}_{n_2} \underbrace{c_3 c_3 c_3 \ldots c_3}_{n_3} \ldots\ldots$

$= \dfrac{n!}{n_1! n_2! \ldots n_n!} \left((p_1)^{n_1} (p_2)^{n_2} (p_3)^{n_3} \ldots \right)$

no of favourable strings of output

probability of favourable strings of output.

Q In a series of ~~7~~ 8 matches b/w India & Australia. Prob. of India winning, lossing or drawing a particular match is $\frac{1}{4}$, $\frac{7}{12}$, and $\frac{1}{6}$ respectively. then find the probability

i) India wins first 3 matches

ii) India wins 1^{st}, 2^{nd} and 5^{th} match. losses 3^{rd}, 4^{th}, & 6^{th} match. Draws 7^{th} match.

iii) India wins 3 matches, loses 3 matches and draws 1 match.

Sol:- i W W W A A A A

A = any thing W, L, D

$P(E) = \left(\frac{1}{4}\right)\left(\frac{1}{4}\right)\left(\frac{1}{4}\right) \cdot 1 \cdot 1 \cdot 1 \cdot 1 = \frac{1}{64}$

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ii) ∵ $W_1\ W_2\ L_3\ L_4\ W_5\ L_6\ D_7$

$P(E) = \left(\frac{1}{4}\right)\left(\frac{1}{4}\right)\left(\frac{7}{12}\right)\left(\frac{7}{12}\right)\left(\frac{1}{4}\right)\left(\frac{7}{12}\right)\left(\frac{1}{6}\right)$

iii) 3W 3L 1D

$\#$ of fav. strings $= \dfrac{7!}{3!\,3!}$

$P(E) = \dfrac{7!}{3!\,3!} \left[\left(\frac{1}{4}\right)^3 \left(\frac{7}{12}\right)^3 \left(\frac{1}{6}\right) \right]$

Total Probability Theorem

Q A person goes from home to office using train, bus or auto with respective probabilities 0.3, 0.5 and 0.2. If he uses train then the probability that he will be late is 0.2. and corresponding prob. for being late with bus and auto are 0.7 and 0.1 respectively. Now find the prob. that he reaches office in time.

Sol: $P(T) = 0.3$ $\quad P(B) = 0.5$ $\quad P(A) = 0.2$

$P\left(\frac{L}{T}\right) = 0.2$ $\quad P\left(\frac{L}{B}\right) = 0.7$ $\quad P\left(\frac{L}{A}\right) = 0.1$

$P(L) = P(T)P\left(\frac{L}{T}\right) + P(B)\cdot P\left(\frac{L}{B}\right) + P(A)\cdot P\left(\frac{L}{A}\right)$

$= 0.3 \times 0.2 + 0.5 \times 0.7 + 0.2 \times 0.1$

$= 0.06 + 0.35 + 0.02$

$= 0.43$

$P(\text{on time}) = 1 - P(L) = 1 - 0.43 = 0.57$

Q Bayes Theorem

Q Let a student attempts a MCQ (with one or more ans. correct) He either knows the ans or copies it or makes a guess. with respective probabilities as 0.3, 0.5 & 0.2. Also prob. that his answer is correct if he copies is 1/2. Now it was found that his answer is correct. find the probability that he knew the answer.

Sol:- $P(K) = 0.3 \quad P(C) = 0.5 \quad P(G) = 0.2$

$P\left(\frac{C}{K}\right) = 1 \quad P\left(\frac{C}{C}\right) = \frac{1}{2} \quad P\left(\frac{C}{G}\right) = \left(\frac{1}{15}\right)$

MCQ with 4 options A) B) C) D)

$\downarrow \quad \downarrow \quad \downarrow \quad \downarrow$

$2 \quad 2 \quad 2 \quad 2$

every option has 2 choices either it can be answer or it can not be answer

# of ways to answer $= 2^4 = 16 - 1 = 15$

Subtract that case where none option is considered in the answer.

(or)

3 options can be correct

# of ways to answer $= {}^4C_1 + {}^4C_2 + {}^4C_3 + {}^4C_4 = 2^4 - 1 = 15$

only 1 option is correct, 2 options correct, all 4 options can be correct

$P\left(\frac{C}{G}\right) = \frac{1/\cancel{6}}{\cancel{16}} \quad \frac{1}{15}$

$P\left(\frac{K}{C}\right) = \frac{P(K \cap C)}{P(C)} = \frac{P(K)\cdot P(C/K)}{P(C)} = \frac{0.3}{0.56}$

$P(C) = P(K)P\left(\frac{C}{K}\right) + P(C)P\left(\frac{C}{C}\right) + P(G)P\left(\frac{C}{G}\right)$

$= 0.3 \times 1 + 0.5 \times \frac{1}{2} + 0.2 \times \frac{1}{15}$

$= 0.3 + 0.25 + 0.013$

$= 0.3 + 0.263$

$= 0.563$

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Permutation & Combination (Numericals)

Q Ram has 3 shares in a lottery in which there are 3 prizes and 6 blanks. Mohan has 1 share in lottery in which there is 1 prize and 2 blanks. What is the ratio of Ram's chance of success to Mohan's chances of success?

Sol: Ram

3 shares in lottery

3 prizes + 6 blanks = total 9 tickets

no of ways to select 3 tickets = $^9C_3 = 84$

Ram's success

{W L L} {W W L} {W W W}

Winning ticket ↑ ↑ losing ticket

$^3C_1 \times {^6C_2} + {^3C_2} \times {^6C_1} + {^3C_3} \times {^6C_0} = 64$

$P(\text{Ram}) = \frac{64}{84}$

Mohan's success

no of ways to select 1 ticket = $^3C_1 = 3$

Mohan success = $\left(\frac{1}{3}\right)$

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$\text{Ratio} = \frac{\frac{64}{84}}{\frac{1}{3}} = \frac{64}{84} \times \frac{3}{1} = 16 : 7$

Q India plays 2 matches each with West Indies and Australia. In any match the probability of India getting point 0, 1, and 2 are 0.45, 0.05, 0.5 respectively. Assuming that the outcomes are independent, the probability of India getting at least 7 points is

Sol:

M_1	M_2	M_3	M_4	≥ 7
2	2	2	2	8
1	2	2	2	7

$\frac{4!}{3!} = 4$ ways

$P(\geq 7) = P(7) + P(8)$

$= 4 \times (0.5)^3 (0.05) + (0.5)^4 = \cancel{0.87}$

$= 0.0875$