Indian Institute of Technology Jammu

CSD001P5M | **Linear Algebra** | **Tutorial: 04**

1. Let $\mathbb{F}$ be a field, and let $a, b, c \in \mathbb{F}$. Prove that: if $a + b = a + c$, then $b = c$.
2. Let $\mathbb{F}$ be a field, and let $a, b, c \in \mathbb{F}$. Prove that: if $a \cdot b = a \cdot c$, then $b = c$.
3. Let $\mathbb{F}$ be a field, and let $a, b \in \mathbb{F}$. Prove that: if $a \cdot b = 0$, then $a = 0$ or $b = 0$. In particular, fields have no (nonzero) zero divisors.
4. Prove that the set $\mathbb{Q}(i) := \{a + ib \ : \ a, b \in \mathbb{Q}\}$ is field.
5. Let V be a vector space. Then:
   (a) The zero vector is unique.
   (b) The additive inverse vector of any vector $\mathbf{u}$ is unique; we use the notation $-\mathbf{u}$ for the inverse vector.
   (c) $0\mathbf{u} = \mathbf{0}$ for every vector $\mathbf{u}$.
   (d) $c\mathbf{0} = \mathbf{0}$ for every scalar c.
   (e) $-\mathbf{u} = (-1)\mathbf{u}$ for every vector $\mathbf{u}$.
6. Given any vector space V, show that if $c\mathbf{v} = \mathbf{0}$, where $\mathbf{v}$ is non-zero vector, then the scalar $c = 0$.
7. (a) Show that every vector space V satisfies the (additive) **cancellation law**, i.e. show that if **u** + **v** = **u** + **w**, for $\mathbf{u}, \mathbf{v}, \mathbf{w} \in V$, then $\mathbf{v} = \mathbf{w}$.
   (b) Give an example of a set X and an operation involving elements of X, which does not satisfy the cancellation law. Briefly justify your answer.
8. Verify the properties of a vector space for the space $\mathbb{R}^{m \times n}$ of $m \times n$ matrices with real entries using the field of real numbers as the underlying field of scalars.
9. Verify the properties of a vector space for the space $\mathbb{R}^{\infty}$ of real sequences using the field of real numbers as the underlying field of scalars.
10. Verify the properties of a vector space for the space $C[0, 1]$ of continuous real-valued functions defined on the closed interval $[0, 1]$ using the field of real numbers as the underlying field of scalars.

    Remark: Note that we can use any closed interval $[a, b]$ as the domain for the continuous functions under consideration.

11. Verify and write explicitly all the 10 properties of a vector space for all the examples discussed in Lecture 13.