Indian Institute of Technology Jammu

CSD001P5M | **Linear Algebra** | **Tutorial: 09**

1. Show that an $n \times n$ square matrix A is invertible iff Rank $A = n$.
   (Remark: this was stated in Version 2 of TIMT; given as an exercise to improve your understanding of TIMT.)

2. (a) Find the coordinates of the vectors $v_1 = (2,3,4)$ and $v_2 = (1,-1,2)$ with respect to the ordered basis $\beta = \{(1,1,1),(1,2,3),(1,3,6)\}$.
   (NB: the vectors have been written as 3-tuples, but should be regarded as column vectors.)

   (b) If $[v]_\beta = \begin{bmatrix} 2 \\ 3 \\ 2 \end{bmatrix}_\beta$, find $[v]_S$ where S is the standard basis for $\mathbb{R}^3$.

3. Find the matrix relative to the standard basis of the linear operator T on $\mathbb{R}^3$ given by:

$$T(x_1,x_2,x_3) = (x_1 + x_3, x_1 + 2x_2 + x_3, -x_1 + x_2).$$

4. Let $T : \mathbb{R}^3 \to \mathbb{R}^2$ be the linear transformation given by $T(x_1,x_2,x_3) = (x_1 + x_2, 2x_3 - x_1)$.

   (a) Find the matrix of T with respect to the standard bases for $\mathbb{R}^3$ and $\mathbb{R}^2$.

   (b) Verify that $\beta = \{(1,0,-1),(1,1,1),(1,0,0)\}$ is a basis for $\mathbb{R}^3$.

   (c) Now, determine the matrix of T with respect to the ordered bases β and $\beta' = \{(0,1),(1,0)\}$ for $\mathbb{R}^3$ and $\mathbb{R}^2$ respectively.

5. Let V be an n-dimensional space and let T be a linear operator on V such that Range $(T) =$ Kernel (T). Show that n must be even. Give an example of such an operator.
   (Note: a linear operator T on V is a linear transformation $T : V \to V$, i.e. the co-domain is the same as the domain.)