

Indian Institute of Technology Jammu

CSD001P5M

Linear Algebra

Tutorial: 01

1. Reduce the following matrix to an RREF matrix using elementary row operations:

$$A = \begin{bmatrix} 1 & 2 & -3 & 0 \\ 2 & 4 & -2 & 2 \\ 3 & 6 & -4 & 1 \end{bmatrix}$$

2. Reduce the following matrix to an RREF matrix using elementary row operations:

$$A = \begin{bmatrix} 1 & -2 & 3 & -1 \\ 2 & -1 & 2 & 2 \\ 3 & 1 & 2 & 3 \end{bmatrix}$$

3. Explicitly describe all non-zero 2×2 RREF matrices. You may also try to do this for 2×3 and 3×3 RREF matrices.
4. Define a relation T on the real number system \mathbb{R} by xTy if $y - x \in \mathbb{Z}$, the set of integers. Is T an equivalence relation? Justify your answer. If yes, can you find a special representative in each equivalence class, just as we could do for row-equivalence of matrices?
5. Prove that row-reduction is an equivalence relation on the set $\mathbb{R}^{m \times n}$ of all m by n matrices with real entries.
6. Show that if E is an equivalence relation on a set X , then any two distinct equivalence classes must be disjoint. Also, show that every element of X has to belong to an equivalence class.

Note: the equivalence class of any element $a \in X$ is the set of all elements of X which are related to a , the formal definition is:

$$[a] = \{x \in X : xEa, i.e. x \text{ is related to } a \text{ under the relation } E\}$$

7. Find the solution set in vector form for the homogeneous system $AX = 0$ given A below.

(Note: A must be row-reduced to an RREF matrix in order to give the solution in standard form.)

$$A = \begin{bmatrix} 1 & -2 & 3 & -1 \\ 2 & -1 & 2 & 2 \\ 3 & 1 & 2 & 3 \end{bmatrix}$$

8. (a) Row reduce the augmented matrix of the system given below to an RREF matrix:

$$\begin{aligned}3x + 2y + 7z + 9w &= 7 \\6x + 14y + 22z + 15w &= 13 \\x + 4y + 5z + 2w &= 2\end{aligned}$$

- (b) Express the solution (if the system is consistent) in the form of a vector \mathbf{u} which is a particular solution plus scalar multiples of vector(s) which are solutions of the associated homogeneous system.
9. Is it possible for a non-homogeneous system $AX = b, b \neq 0$, to be inconsistent when the associated homogeneous system $AX = 0$ has a unique solution (i.e. only the trivial solution) ? Answer YES or NO, and justify your answer. If YES, construct an example and verify. If NO, explain with reference to suitable propositions and theorems.