Linear Algebra

Sartaj Ul Hasan

Department of Mathematics
Indian Institute of Technology Jammu
Jammu, India - 181221

Email: sartaj.hasan@iitjammu.ac.in

Lecture 15

(Aug 28, 2019)

Subspaces (Conti . . .)

Test for Subspaces

- **Proposition 8:** A subset W of V is a subspace if and only if it satisfies the following three properties:
  1. The zero vector $\mathbf{0}$ is in W.
  2. W is closed under addition. That is, for each $\mathbf{u}$ and $\mathbf{v}$ in W, the sum $\mathbf{u} + \mathbf{v}$ is in W.
  3. W is closed under scalar multiplication. That is, for each $\mathbf{u}$ in W, and each scalar c, the scalar product $c\mathbf{u}$ is in W.

  Note: In some books, this is treated as the definition of a subspace. It is also possible to replace the condition 1 above by the condition $1'$ that W be non-empty. However, in practice, this is not so easy to use.

- **Proposition 9:** A non-empty subset W of V is a subspace if and only if for each $\mathbf{u}$ and $\mathbf{v}$ in W, and each scalar c, the sum $c\mathbf{u} + \mathbf{v}$ is in W.

  Remark: It is left as an exercise to show that the two tests are equivalent. In some books, the above is taken as the definition of a subspace. We may use either of the tests whichever is convenient.

Subspaces (Conti . . .)

Some More Examples of Subspaces

- For any vector space V, the subset consisting of the zero vector alone is a subspace of V, called the zero subspace.

  V is of course a subspace of itself. Subspaces other than V and $\{0\}$ are known as proper subspaces.

- The set $X = \{(x, y) \in \mathbb{R}^2 \; : \; x + y = 0\}$ is a subspace of $\mathbb{R}^2$–verify: exercise.

- The set Sym_n of all symmetric (square) matrices of size n is a subspace of the space $\mathbb{R}^{n\times n}$ of square $n \times n$ matrices.

Subspaces (Conti . . .)

But . . .

Something to think about: Is $\mathbb{R}^2$ is a subspace of $\mathbb{R}^3$?

Answer

Subspaces (Conti . . .)

But . . .

Something to think about: Is $\mathbb{R}^2$ is a subspace of $\mathbb{R}^3$?

Answer

However, $\mathbb{R}^2$ is not a subspace of $\mathbb{R}^3$! This is because $\mathbb{R}^2$ is not even a subset of $\mathbb{R}^3$. The set

$$W = \left\{ \begin{bmatrix} x \\ y \\ 0 \end{bmatrix} \ : \ x, y \in \mathbb{R} \right\}$$

is a subspace of $\mathbb{R}^3$, which behaves much like $\mathbb{R}^2$, but is logically distinct from $\mathbb{R}^2$.