Indian Institute of Technology Jammu

CSD001P5M | **Linear Algebra** | **Tutorial: 08**

1. Let $T : V_{\mathbb{F}} \to W_{\mathbb{F}}$ be a linear transformation.

   (a) Show that Ker $T = \{v \in V : Tv = 0\}$ is a subspace of V and Range $T = \{w \in W : w = Tv \text{ for some } v \in V\}$ is a subspace of W.

   (b) Show that Ker $T = \{0\}$ if and only if T is one-one.

2. Determine whether the following are linear transformations (yes or no). Justify your answers.

   (a) $T : \mathbb{R}^3 \to \mathbb{R}^2$ given by $T(x,y,z) = (x+y, x-z)$

   (b) $T : \mathbb{R}^3 \to \mathbb{R}^2$ given by $T(x,y,z) = (x+y, z^2)$

   (c) $U : \mathbb{M}_{n\times n}(\mathbb{R}) \to \mathbb{M}_{n\times n}(\mathbb{R})$ given by $U(A) = A^T$. Here A^T indicates the transpose of the matrix A.

   (d) $M : \mathbb{R}[x] \to \mathbb{R}[x]$ given by $M(p(x)) = xp(x)$ for all polynomials $p(x) \in \mathbb{R}[x]$.

3. Determine all linear transformations $T : \mathbb{R}^1 \to \mathbb{R}^1$.
   (Note: $\mathbb{R}^1$ is the vector space consisting of all 1-tuples with real entries; it is essentially the same as $\mathbb{R}$, however regarded as only a vector space rather than a field.)

4. Consider the field $\mathbb{C}$ of complex numbers.

   (a) Show that $\mathbb{C}$ is a vector space over the field $\mathbb{R}$.

   (b) Determine the dimension of $\mathbb{C}$ by finding a suitable basis.

   (c) Show that the function $\phi : \mathbb{C} \to \mathbb{C}$ given by $\phi(z) = \bar{z}$ is a linear transformation. Here $\bar{z}$ indicates the complex conjugate of z, i.e. if $z = a+bi$, then $\bar{z} = a-bi$.

   (d) Show that complex conjugation is actually a multiplicative function, i.e. if $w,z \in \mathbb{C}$, then $\phi(wz) = \phi(w)\phi(z)$.

   (e) Show that ϕ is the only multiplicative linear transformation on $\mathbb{C}$ to $\mathbb{C}$, other than the zero and identity linear transformations.

5. Consider the space $V = C[\mathbb{R}]$ of all continuous functions on $\mathbb{R}$ and consider the mapping $D_\varepsilon : V \to V$ given by $D_\varepsilon(f) = f(x+\varepsilon)$, where $f_\varepsilon(x) = f(x+\varepsilon)$ for all x. Here ε is an arbitrary but fixed real number. Is D_ε a linear transformation? Justify your answer.