Indian Institute of Technology Jammu

CSD001P5M **Linear Algebra** **Tutorial: 03**

1. Let $G = \mathbb{R}^n$ be the set of n-tuples (for any $n \geq 1$). These are often referred to as (column) vectors.

   If $\mathbf{u} = \begin{bmatrix} X_1 \\ X_2 \\ \vdots \\ X_n \end{bmatrix}$ and $\mathbf{v} = \begin{bmatrix} Y_1 \\ Y_2 \\ \vdots \\ Y_n \end{bmatrix}$ are any two column vectors in $\mathbb{R}^n$, then their sum is defined as

   $$\mathbf{u} + \mathbf{v} := \begin{bmatrix} X_1 + Y_1 \\ X_2 + Y_2 \\ \vdots \\ X_n + Y_n \end{bmatrix}$$

   Prove that G is an abelian group w. r. t. the operation defined above.

   Note: $\mathbb{R}^n$ is usually referred as **Euclidean space**.

2. Let $G = \mathbb{R}^{m \times n}$ be the set of all $m \times n$ matrices with real entries. Prove that G is an abelian group w. r. t. matrix addition.

3. Show that the set of all $m \times n$ invertible matrices with entries from $\mathbb{R}$ forms a group w. r. t. matrix multiplication. Is this group abelian?

4. Show that the set of all $m \times n$ invertible matrices with entries from $\mathbb{R}$ and with determinant 1 forms a group w. r. t. matrix multiplication.

5. Let $C[0,1]$ be the set of all continuous functions from the closed interval $[0,1]$ on the real line to $\mathbb{R}$, i.e.

   $$C[0,1] = \{f \ : \ f \text{ is a continuous function}, f \ : \ [0,1] \longrightarrow \mathbb{R}\}.$$

   Prove that $C[0,1]$ forms an abelian group w. r. t. addition of functions.

6. Let $\mathbb{R}^\infty$ be the set of all real sequences, i.e.,

   $$\mathbb{R}^\infty = \{< a_n > \ : \ < a_n > \text{ is a sequence with real number terms}\}.$$

   Show that $\mathbb{R}^\infty$ is an abelian group w. r. t. sequence addition.

7. Let c be set of all real convergent sequences, i.e.,

$$c = \{ < a_n > \ : \ < a_n > \text{ is a convergent sequence with real number terms} \} \subseteq \mathbb{R}^\infty.$$

Show that c is an abelian group w. r. t. sequence addition.

8. Let $\mathbb{R}_n[t]$ be the set of polynomials of degree $\leq n$ with real coefficients. Show that $\mathbb{R}_n[t]$ is an abelian group w. r. t. polynomial addition.

Note: The zero polynomial, which technically does not have any degree, is regarded as an element of $\mathbb{R}_n[t]$ for all $n = 0, 1, 2, \ldots$

9. Let $G = \mathbb{R}^+$ be the set of all positive real numbers. For $u, v \in \mathbb{R}^+$, define:

$$u + v = uv$$

Show that G is an abelian group w. r. t. "+".