Linear Algebra

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Lecture 42

(Nov 19, 2019)

Singular Value Decomposition (SVD)

Application of SVD

- We have already discussed 3 fundamental subspaces related to a matrix $A_{m\times n}$, namely: Row A=Col A^T, Col A= Row A^T, Nul A. We now bring into picture a 4th fundamental subspace of A, namely Nul A^T.
- The utility of SVD is that it identifies orthonormal bases for all fundamental subspaces. This is shown in the diagram on the next slide.
- Here we use the following facts:
  $(\text{Row } A)^{\perp} = \text{Nul } A$ and $(\text{Col } A)^{\perp} = \text{Nul } A^T$
  (For proof see Theorem 3 in Section 6.1 on page no 381)
- By Orthogonal Decomposition Theorem, we have:
  $\mathbb{R}^n = \text{Row } A \oplus \text{Nul } A$ and $\mathbb{R}^m = \text{Col } A \oplus \text{Nul } A^T$
- o.n.b of Row $A = \{v_1, \cdots, v_r\}$, where r is the rank of the matrix A; o.n.b. of Nul $A = \{v_{r+1}, \cdots, v_n\}$; o.n.b. of Col $A = \{u_1, \cdots, u_r\}$; and o.n.b of Nul $A^T = \{u_{r+1}, \cdots, u_m\}$

Application of SVD