Assignment Problem

$$\min(z) = x_1 - x_2$$

s.t,

$$2x_1 + x_2 \geq 2$$
$$-x_1 - x_2 \geq 1$$
$$x_1, x_2 \geq 0$$

Sol:- Writing the problem in standard form

$$\max(z) = -x_1 + x_2$$

s.t, $-2x_1 - x_2 \leq -2$

$$x_1 + x_2 \leq 1$$
$$x_1, x_2 \geq 0$$

Corresponding dual is

$$\max(w) = 2y_1 + y_2$$

s.t, $2y_1 - y_2 \leq 1$

$$y_1 - y_2 \leq -1 \quad — (2)$$
$$y_1, y_2 \geq 0$$

We know that b must be +ve, mul (2) by -1

$$\max(w) = 2y_1 + y_2$$

s.t,

$$2y_1 - y_2 \leq 1$$
$$-y_1 + y_2 \geq +1$$
$$y_1, y_2 \geq 0$$

We will use "big-M" Technique to solve above dual LP problem

~~C_B B Y_B b a_1 a_2~~ .

$$\max(w) = 2y_1 + y_2 + 0\cdot y_3 + 0\cdot y_4 - m y_5$$

s.t.,

$$2y_1 - y_2 + y_3 = 1$$
$$-y_1 + y_2 - y_4 + y_5 = 1$$
$$y_1, y_2, y_3, y_4, y_5 \geq 0$$

		C_j		2	1	0	0	$-m$	b/a_2	
C_B	B	Y_B	b	a_1	a_2	a_3	a_4	a_5	min. ratio	operation
0	a_3	y_3	1	2	-1	1	0	0	X	
$-m$	a_5	y_5	1	-1	1	0	-1	1	1	
	$Z_j - C_j$			$m-2$	$-m-1$	0	m	0	b/a_1	
0	a_3	y_3	2	1	0	1	-1	/	2	$R_1' \leftarrow R_1 + R_2'$
1	a_2	y_2	1	-1	1	0	-1		X	$R_2' \leftarrow R_2$
	$Z_j - C_j$			-3	0	0	-1		b/a_4	
2	a_1	y_1	2	1	0	1	-1		X	$R_1' \leftarrow R_1$
1	a_2	y_2	3	0	1	1	-2		X	$R_2' \leftarrow R_2 + R_1'$
	$Z_j - C_j$			0	0	3	-4			

Here a_4 wants to enter, but there is no leaving variable, hence this is case of unboundedness i.e. feasible region is not bounded

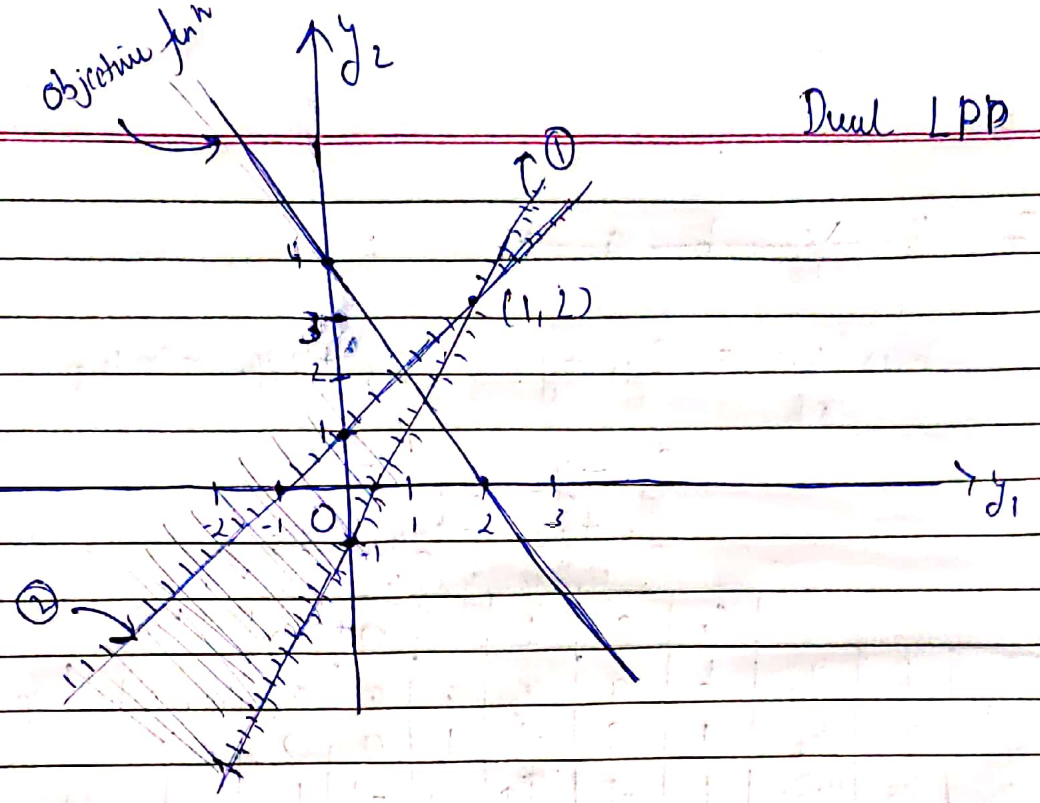
We can visualize this graphically as

① $2y_1 - y_2 = 1$ $\quad y_1 = 0 \quad y_2 = -1$
$\quad y_1 = 1/2 \quad y_2 = 0$

② $-y_1 + y_2 = 1$ $\quad y_1 = 0 \quad y_2 = 1$
$\quad y_1 = -1 \quad y_2 = 0$

$2y_1 + y_2 = 4$ $\quad y_1 = 0 \quad y_2 = 4$
$\quad y_1 = 2 \quad y_2 = 0$

Dual LPP



Graphically visualizing original LPP problem

$\cancel{\max}$ ($\min(Z) = x_1 - x_2$

s.t. $2x_1 + x_2 \geq 2$

$-x_1 - x_2 \geq 1$

$x_1, x_2 \geq 0$

① $2x_1 + x_2 = 2$ $\quad x_1 = 0 \quad x_2 = 2$

$x_1 = 1 \quad x_2 = 0$

② $x_1 - x_2 = 1$ $\quad x_1 = 0 \quad x_2 = -1$

$x_1 = -1 \quad x_2 = 0$

$x_1 - x_2 = 4$ $\quad x_1 = 0 \quad x_2 = -4$

$x_1 = 4 \quad x_2 = 0$

We can clearly see from the graph that, actual LPP is ① infeasible solution and corresponding dual has unboundedness.

Given LPP