201916006_PRML_Assignment7

May 26, 2020

1 Assignment 7 (PCA)

1.1 201916006

[41]:
```
import h5py
import numpy as np
import pandas as pd
path = 'faceimages.mat'
arrays = {}
input_file = h5py.File(path, 'r')
mat = {k:v for k,v in input_file.items() if k[0] != '_'}
mat1 = mat['data']
data = pd.DataFrame({k: pd.Series(v[0]) for k,v in mat.items()})
```

[42]:
```
df1 = pd.DataFrame(mat1).T
df1
```

[42]:

	0	1	2	3	4	5	6	7	8	9	\
0	48.0	46.0	47.0	48.0	47.0	47.0	49.0	53.0	51.0	54.0	
1	59.0	63.0	67.0	81.0	54.0	37.0	60.0	46.0	58.0	62.0	
2	40.0	50.0	51.0	52.0	64.0	73.0	68.0	60.0	49.0	47.0	
3	62.0	55.0	46.0	42.0	42.0	43.0	37.0	30.0	33.0	27.0	
4	62.0	64.0	61.0	59.0	56.0	42.0	27.0	23.0	36.0	55.0	
..	…	…	…	…	…	…	…	…	…	…	
395	125.0	125.0	126.0	127.0	127.0	126.0	125.0	125.0	120.0	119.0	
396	128.0	128.0	128.0	128.0	128.0	128.0	128.0	128.0	128.0	129.0	
397	121.0	122.0	123.0	123.0	123.0	121.0	119.0	118.0	124.0	120.0	
398	118.0	118.0	118.0	119.0	119.0	120.0	120.0	120.0	115.0	119.0	
399	122.0	123.0	124.0	126.0	127.0	127.0	126.0	126.0	128.0	128.0	

	…	10295	10296	10297	10298	10299	10300	10301	10302	10303	10304
0	…	43.0	47.0	48.0	48.0	48.0	46.0	45.0	46.0	47.0	0.0
1	…	39.0	33.0	33.0	33.0	35.0	35.0	34.0	33.0	34.0	0.0
2	…	35.0	33.0	31.0	31.0	31.0	30.0	28.0	27.0	28.0	0.0
3	…	40.0	35.0	33.0	28.0	23.0	23.0	26.0	28.0	27.0	0.0
4	…	43.0	41.0	42.0	42.0	41.0	39.0	38.0	38.0	39.0	0.0
..	…	…	…	…	…	…	…	…	…	…	…

```
395  …   36.0   40.0   45.0   43.0   40.0   41.0   41.0   39.0   41.0    0.0
396  …   93.0   93.0   93.0   93.0   93.0   93.0   93.0   93.0   93.0    0.0
397  …   41.0   39.0   44.0   44.0   41.0   41.0   40.0   39.0   39.0    0.0
398  …   93.0   96.0   92.0   94.0   94.0   90.0   90.0   92.0   89.0    0.0
399  …   37.0   37.0   37.0   34.0   34.0   37.0   34.0   31.0   35.0    0.0

[400 rows x 10305 columns]
```

[43]:
```
# Mean-centering the data
for c in df1:
    m = df1[c].mean()
    df1[c] = df1[c] - m
```

[44]:
```
df1
```

[44]:
```
        0      1        2        3        4        5        6        7      \
0   -37.7275 -39.91 -38.8825 -37.8925 -39.1125 -39.1775 -37.0975 -33.1425
1   -26.7275 -22.91 -18.8825  -4.8925 -32.1125 -49.1775 -26.0975 -40.1425
2   -45.7275 -35.91 -34.8825 -33.8925 -22.1125 -13.1775 -18.0975 -26.1425
3   -23.7275 -30.91 -39.8825 -43.8925 -44.1125 -43.1775 -49.0975 -56.1425
4   -23.7275 -21.91 -24.8825 -26.8925 -30.1125 -44.1775 -59.0975 -63.1425
..       …    …       …       …       …       …       …       …
395  39.2725  39.09  40.1175  41.1075  40.8875  39.8225  38.9025  38.8575
396  42.2725  42.09  42.1175  42.1075  41.8875  41.8225  41.9025  41.8575
397  35.2725  36.09  37.1175  37.1075  36.8875  34.8225  32.9025  31.8575
398  32.2725  32.09  32.1175  33.1075  32.8875  33.8225  33.9025  33.8575
399  36.2725  37.09  38.1175  40.1075  40.8875  40.8225  39.9025  39.8575

     8        9      …   10295    10296    10297   10298    10299   10300  \
0   -35.13 -32.2525  … -23.015 -19.8075 -19.8275 -21.285 -23.3525 -26.025
1   -28.13 -24.2525  … -27.015 -33.8075 -34.8275 -36.285 -36.3525 -37.025
2   -37.13 -39.2525  … -31.015 -33.8075 -36.8275 -38.285 -40.3525 -42.025
3   -53.13 -59.2525  … -26.015 -31.8075 -34.8275 -41.285 -48.3525 -49.025
4   -50.13 -31.2525  … -23.015 -25.8075 -25.8275 -27.285 -30.3525 -33.025
..     …       …  …      …       …       …       …       …      …
395  33.87  32.7475  … -30.015 -26.8075 -22.8275 -26.285 -31.3525 -31.025
396  41.87  42.7475  …  26.985  26.1925  25.1725  23.715  21.6475  20.975
397  37.87  33.7475  … -25.015 -27.8075 -23.8275 -25.285 -30.3525 -31.025
398  28.87  32.7475  …  26.985  29.1925  24.1725  24.715  22.6475  17.975
399  41.87  41.7475  … -29.015 -29.8075 -30.8275 -35.285 -37.3525 -35.025

      10301  10302    10303   10304
0   -27.105  -27.8 -28.3925 -0.2975
1   -38.105  -40.8 -41.3925 -0.2975
2   -44.105  -46.8 -47.3925 -0.2975
3   -46.105  -45.8 -48.3925 -0.2975
4   -34.105  -35.8 -36.3925 -0.2975
```

```
..       …    …       …      …
395 -31.105  -34.8 -34.3925 -0.2975
396  20.895   19.2  17.6075 -0.2975
397 -32.105  -34.8 -36.3925 -0.2975
398  17.895   18.2  13.6075 -0.2975
399 -38.105  -42.8 -40.3925 -0.2975

[400 rows x 10305 columns]
```

[45]:
```
df2 = np.array(df1)
```

[46]:
```
# Co-variance of the data matrix
cov = np.cov(df2.T)
```

[47]:
```
cov.shape
```

[47]:
```
(10305, 10305)
```

[48]:
```
print(cov)
```

```
[[ 1.27932656e+03  1.26883256e+03  1.25975737e+03 … -2.07218045e+00
  -5.86346303e+01 -3.87110902e+00]
 [ 1.26883256e+03  1.26408211e+03  1.25736784e+03 … -5.59197995e+00
  -6.27515539e+01 -3.77766917e+00]
 [ 1.25975737e+03  1.25736784e+03  1.25529694e+03 … -4.37694236e+00
  -5.91718108e+01 -3.70681078e+00]
 …
 [-2.07218045e+00 -5.59197995e+00 -4.37694236e+00 …  1.77720802e+03
   1.78067268e+03  2.15488722e+00]
 [-5.86346303e+01 -6.27515539e+01 -5.91718108e+01 …  1.78067268e+03
   1.93941699e+03  2.59722431e+00]
 [-3.87110902e+00 -3.77766917e+00 -3.70681078e+00 …  2.15488722e+00
   2.59722431e+00  2.09517544e-01]]
```

[49]:
```
#Eigen Values & Vectors from covariance matrix
eigenValues, eigenVectors = np.linalg.eig(cov)
print("Eigen Values:  {} \n".format(eigenValues))
print("Eigen Vectors: {} \n".format(eigenVectors))
```

```
Eigen Values:  [6.92968561e-310+6.92968091e-310j
6.92968713e-310+6.92968091e-310j
 3.50000000e+001+1.65000000e+002j … 7.40000000e+001+5.60000000e+001j
 4.60000000e+001+5.10000000e+001j 4.50000000e+001+1.36000000e+002j]

Eigen Vectors: [[0.+0.j 0.+0.j 0.+0.j … 0.+0.j 0.+0.j 0.+0.j]
 [0.+0.j 0.+0.j 0.+0.j … 0.+0.j 0.+0.j 0.+0.j]
 [0.+0.j 0.+0.j 0.+0.j … 0.+0.j 0.+0.j 0.+0.j]
 …
```

```
[0.+0.j 0.+0.j 0.+0.j … 0.+0.j 0.+0.j 0.+0.j]
[0.+0.j 0.+0.j 0.+0.j … 0.+0.j 0.+0.j 0.+0.j]
[0.+0.j 0.+0.j 0.+0.j … 0.+0.j 0.+0.j 0.+0.j]]
```

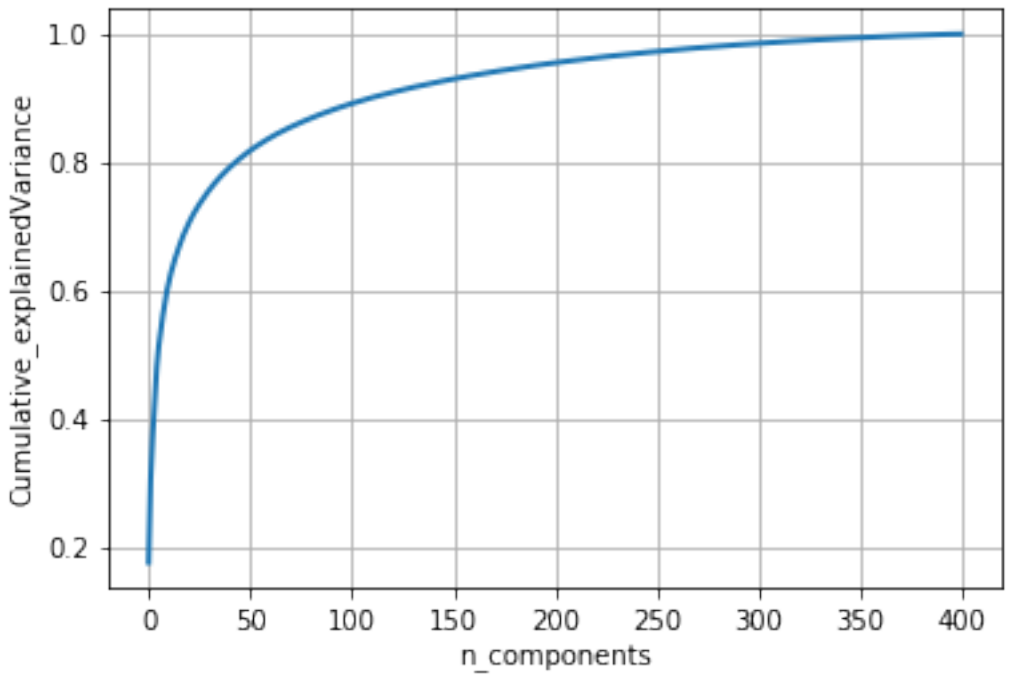
[50]:
```
# initializing the pca
import matplotlib.pyplot as plt
from sklearn import decomposition
pca = decomposition.PCA()
```

[51]:
```
# PCA for dimensionality redcution
pca_data = pca.fit_transform(df2)

percentage_var_explained = pca.explained_variance_ / np.sum(pca.
 ↪explained_variance_);
cum_var_explained = np.cumsum(percentage_var_explained)

# Plot the energy curve of PCA
plt.figure(1, figsize=(6, 4))
plt.clf()
plt.plot(cum_var_explained, linewidth=2)
plt.axis('tight')
plt.grid()
plt.xlabel('n_components')
plt.ylabel('Cumulative_explainedVariance')
plt.show()

# If we take 200-dimensions, approx. 95% of variance is explained.
```



[52]:
```
# Projection matrix with 300 pricipal components
pca.n_components = 300
lowdimdata = pca.fit_transform(df2)
lowdimdata_df = pd.DataFrame(data = lowdimdata)
lowdimdata_df
```

[52]:

	0	1	2	3	4	\
0	1528.387559	-1074.573843	-1865.264487	-263.049165	-690.306966	
1	2976.323373	-762.488394	464.182310	-1196.319087	-2510.476483	
2	2696.632841	-379.312931	-1003.866148	33.285337	-1032.559401	
3	3241.182438	-1161.562680	230.458352	-2717.415928	15.945138	
4	3644.144540	-812.920371	-118.612602	-2124.642132	-220.637392	
..	…	…	…	…	…	
395	654.897516	-1033.419834	1574.908024	478.806568	-3.403356	
396	1139.765012	-1264.504624	1058.023811	620.250051	-371.381449	
397	843.045986	-952.108778	1539.746484	238.296210	558.256754	
398	1400.073540	-1276.514206	807.532900	-116.193442	-292.718621	
399	534.447742	-476.970464	2059.007622	1045.951126	-922.502335	

	5	6	7	8	9	…	\
0	127.116011	-425.689089	374.924246	442.282992	-1029.678499	…	
1	-385.945395	-1232.637425	-330.966730	384.962802	72.781346	…	
2	1081.990970	-461.275125	-243.575701	1121.672729	-819.880722	…	
3	816.015917	673.378904	967.141201	-486.796328	-887.737103	…	

```
4     461.346840  -532.739364  -205.954092  -108.101633  -116.909140  …
..           …           …           …           …           …  …
395   478.892962   -51.154558 -1156.240781   905.215859  -467.886316  …
396   428.331697   380.717298   939.144049  -170.677750   -45.824020  …
397   191.497855  -104.740008  -796.628135  1069.359507  -274.597787  …
398   801.572719  1041.488890  1321.986966  -470.881288    40.059405  …
399   901.157724   280.846091  -615.234917   338.567669  -434.952521  …

            290         291         292         293         294         295  \
0     13.803655    7.445567  -62.572663   34.242013  -74.173855  -85.269952
1    -29.380933   32.044487   -2.924562    6.807622    3.301262    8.727468
2     89.194242   31.526028  -17.578819   15.881124   26.836475    0.709728
3      1.661426   35.554599  -53.435801   -6.957805  -30.224393  -19.862139
4    107.244521    3.779910   45.216871  -14.709315  -98.074092  -78.360280
..          …           …           …           …           …           …
395 -175.202042 -101.085796   46.979865  -82.915912  -66.910454  -19.248562
396  -78.494856   23.678209   63.403062  -29.652592   47.481255  -27.184858
397   51.546460   22.445629   -5.569787   41.072119  -35.093404  100.269436
398   52.396131  -77.100443   52.136462  -12.155128  -77.478737  -75.853962
399  -10.446103   -4.266720   13.453671   78.842166   85.129200  125.196360

            296         297         298         299
0    155.598643  -86.757440   46.087009    5.571634
1    -26.830734 -110.463819   22.463395  105.896784
2    -20.093514   17.536465  -14.780065  -23.749634
3      4.617532   24.847157   -0.341502   35.038673
4     60.682120   11.054009  -38.297258  -70.436449
..          …           …           …           …
395 -100.779310   73.983419  -30.156780   -5.720833
396   43.064234   16.277059   38.404939  -71.072874
397   57.432798 -132.304379   95.864579   50.040649
398   11.874895   -8.683287   12.441305   82.188874
399   -4.290333   -3.608014  -68.293575   22.628799

[400 rows x 300 columns]
```

[53]:
```
# Reconstruction error for 300 principal components
from sklearn.metrics import r2_score
from sklearn.metrics import mean_squared_error
from math import sqrt
import numpy as np
r2 = r2_score(df2[:,:300],lowdimdata)
rmse = sqrt(mean_squared_error(df2[:,:300],lowdimdata))
rmse
```

[53]: 232.09122960632587

[54]:
```
# Projection matrix with 5 principal components
pca.n_components = 5
lowdimdata = pca.fit_transform(df2)
lowdimdata_df = pd.DataFrame(data = lowdimdata)
lowdimdata_df
```

[54]:

	0	1	2	3	4
0	1528.387559	-1074.573843	-1865.264535	-263.049444	-690.305876
1	2976.323373	-762.488394	464.182121	-1196.319071	-2510.476658
2	2696.632841	-379.312931	-1003.866141	33.285059	-1032.557320
3	3241.182438	-1161.562681	230.458436	-2717.416335	15.946107
4	3644.144540	-812.920371	-118.612688	-2124.642020	-220.636651
..	…	…	…	…	…
395	654.897516	-1033.419832	1574.907791	478.806521	-3.405253
396	1139.765012	-1264.504625	1058.023853	620.250186	-371.380771
397	843.045986	-952.108776	1539.746370	238.296648	558.255395
398	1400.073540	-1276.514208	807.532971	-116.193507	-292.716576
399	534.447742	-476.970465	2059.007617	1045.951462	-922.500477

[400 rows x 5 columns]

[55]:
```
# Reconstruction error for 5 principal components
r2 = r2_score(data2[:,:5],lowdimdata)
rmse = sqrt(mean_squared_error(data2[:,:5],lowdimdata))
rmse
```

[55]: 1240.8493906288368

[56]:
```
# Projection matrix with 10 principal components
pca.n_components = 10
lowdimdata = pca.fit_transform(df2)
lowdimdata_df = pd.DataFrame(data = lowdimdata)
lowdimdata_df
```

[56]:

	0	1	2	3	4	\
0	1528.387559	-1074.573843	-1865.264426	-263.049165	-690.306671	
1	2976.323373	-762.488394	464.182299	-1196.319087	-2510.476534	
2	2696.632841	-379.312931	-1003.866122	33.285298	-1032.559284	
3	3241.182438	-1161.562679	230.458321	-2717.415755	15.944987	
4	3644.144540	-812.920370	-118.612653	-2124.641995	-220.637639	
..	…	…	…	…	…	
395	654.897516	-1033.419834	1574.908070	478.806637	-3.403127	
396	1139.765012	-1264.504624	1058.023793	620.250200	-371.381525	
397	843.045986	-952.108778	1539.746515	238.296246	558.256908	
398	1400.073540	-1276.514206	807.532906	-116.193476	-292.718600	
399	534.447742	-476.970464	2059.007636	1045.951167	-922.502249	

```
                5           6           7           8           9
0     127.120687  -425.609337   374.972883   441.748535 -1029.939249
1    -385.946021 -1232.636462  -330.963433   384.999819    72.838629
2    1081.993452  -461.223404  -243.539585  1121.397928  -819.971309
3     816.019778   673.261429   967.097873  -486.448339  -887.957155
4     461.347324  -532.850091  -205.999701  -107.649904  -116.946332
..           …           …           …           …           …
395   478.898414   -51.120765 -1156.213023   904.833313  -468.240520
396   428.334780   380.648152   939.125645  -170.509582   -46.035320
397   191.501653  -104.718088  -796.612164  1069.090631  -274.856878
398   801.572519  1041.502518  1321.991043  -470.959869    40.061546
399   901.158796   280.881599  -615.209051   338.389918  -435.043983

[400 rows x 10 columns]
```

[57]:
```
# Reconstruction error for 10 principal components
r2 = r2_score(data2[:,:10],lowdimdata)
rmse = sqrt(mean_squared_error(df2[:,:10],lowdimdata))
rmse
```

[57]:
```
980.3075784629849
```

[58]:
```
# Projection matrix with 50 principal components
pca.n_components = 50
lowdimdata = pca.fit_transform(df2)
lowdimdata_df = pd.DataFrame(data = lowdimdata)
lowdimdata_df
```

[58]:
```
                0           1           2           3           4   \
0    1528.387559 -1074.573843 -1865.264462  -263.049147  -690.307007
1    2976.323372  -762.488394   464.182317 -1196.319080 -2510.476518
2    2696.632841  -379.312932 -1003.866137    33.285330 -1032.559459
3    3241.182438 -1161.562680   230.458378 -2717.415934    15.945150
4    3644.144540  -812.920371  -118.612613 -2124.642141  -220.637354
..           …           …           …           …           …
395   654.897515 -1033.419835  1574.908024   478.806586    -3.403382
396  1139.765012 -1264.504624  1058.023811   620.250057  -371.381432
397   843.045986  -952.108777  1539.746495   238.296196   558.256732
398  1400.073540 -1276.514207   807.532888  -116.193439  -292.718676
399   534.447742  -476.970464  2059.007641  1045.951167  -922.502348

                5           6           7           8           9   …  \
0     127.115637  -425.688013   374.924229   442.283452 -1029.684285  …
1    -385.946443 -1232.636294  -330.964555   384.962067    72.774190  …
2    1081.990887  -461.274098  -243.576497  1121.674589  -819.881079  …
3     816.015492   673.377952   967.139840  -486.789341  -887.746452  …
4     461.346655  -532.740593  -205.954055  -108.099032  -116.908706  …
```

```
..           …           …           …           …          …  …
395   478.892850   -51.153072 -1156.239843   905.214787  -467.879819  …
396   428.331331   380.716942   939.143741  -170.678619   -45.824236  …
397   191.497288  -104.739898  -796.624249  1069.360494  -274.603624  …
398   801.572606  1041.489623  1321.986946  -470.885275    40.066049  …
399   901.158312   280.845270  -615.235583   338.569618  -434.947957  …

             40          41          42          43          44          45  \
0    162.514982 -359.810176 -193.437243  221.019356   29.173254   32.963458
1   -199.172489   15.099788 -217.227674 -219.420395   52.787365  270.143821
2    -96.779521 -209.614497  161.218635  174.506392  279.986947  386.465537
3   -143.767853  117.671967  190.733131 -111.239073  -84.082750  -85.667695
4    -92.349445 -220.446101 -246.363825 -433.386744  228.359817 -114.030282
..          …          …          …          …          …          …
395  129.196328 -274.834436 -271.301149 -217.890425  -68.288775   -1.080978
396   57.607374 -103.733124  -17.501518 -281.912467  405.722751 -137.179555
397   44.458583 -307.791083  -77.031363   10.577429   56.580969  288.288134
398  -95.818807 -307.255777 -292.336600 -220.046248  127.822732 -134.376841
399  110.305297 -175.857022 -213.204383 -119.578391 -204.690854  -87.558012

             46          47          48          49
0     47.764450 -425.976954  144.866158  -12.086949
1     19.424104    9.233334  -38.519236 -601.592284
2    354.077245 -219.882517  -65.736521 -364.797509
3   -167.020737   94.575321   17.781867   83.156645
4     34.166729 -105.895732  148.147185 -243.240575
..          …          …          …          …
395   15.113307  -23.755648   85.704211   72.753856
396  -22.983977  104.853969  322.566580  114.929344
397   22.484016    3.391290  -85.674704   90.014507
398 -360.152876  147.594718 -216.701652  298.001802
399   51.406285   58.419826  143.564394  171.546437

[400 rows x 50 columns]
```

[59]:
```
# Reconstruction error for 50 principal components
r2 = r2_score(data2[:,:50],lowdimdata)
rmse = sqrt(mean_squared_error(df2[:,:50],lowdimdata))
rmse
```

[59]:
```
511.8688612907902
```

[60]:
```
# Projection matrix with 100 principal components
pca.n_components = 100
lowdimdata = pca.fit_transform(df2)
lowdimdata_df = pd.DataFrame(data = lowdimdata)
lowdimdata_df
```

[60]:

	0	1	2	3	4	…	\
0	1528.387559	-1074.573843	-1865.264487	-263.049164	-690.306966		
1	2976.323373	-762.488394	464.182310	-1196.319088	-2510.476483		
2	2696.632841	-379.312931	-1003.866149	33.285337	-1032.559400		
3	3241.182438	-1161.562680	230.458351	-2717.415930	15.945140		
4	3644.144540	-812.920371	-118.612602	-2124.642131	-220.637393		
..	…	…	…	…	…		
395	654.897516	-1033.419834	1574.908024	478.806569	-3.403357		
396	1139.765012	-1264.504624	1058.023811	620.250052	-371.381450		
397	843.045986	-952.108778	1539.746484	238.296210	558.256754		
398	1400.073540	-1276.514206	807.532900	-116.193442	-292.718621		
399	534.447742	-476.970464	2059.007622	1045.951126	-922.502334		

	5	6	7	8	9	…	\
0	127.116003	-425.689127	374.924230	442.283045	-1029.678472	…	
1	-385.945401	-1232.637426	-330.966735	384.962812	72.781365	…	
2	1081.990963	-461.275037	-243.575680	1121.672684	-819.880795	…	
3	816.015905	673.378937	967.141133	-486.796252	-887.737126	…	
4	461.346851	-532.739362	-205.954042	-108.101718	-116.909169	…	
..	…	…	…	…	…	…	
395	478.892971	-51.154592	-1156.240736	905.215859	-467.886415	…	
396	428.331704	380.717304	939.144055	-170.677832	-45.824047	…	
397	191.497862	-104.740000	-796.628108	1069.359494	-274.597751	…	
398	801.572730	1041.488901	1321.986925	-470.881245	40.059419	…	
399	901.157718	280.846093	-615.234932	338.567719	-434.952629	…	

	90	91	92	93	94	95	\
0	-62.058102	5.860390	122.066160	-12.526518	-16.826543	-45.304852	
1	-172.686954	67.240489	-80.067378	-204.433538	247.512762	187.166336	
2	56.837094	27.463082	230.156853	224.196698	218.260850	-22.811461	
3	74.086795	85.692869	-14.245941	-60.091391	-136.735122	58.176707	
4	-130.561696	-92.634790	-5.522282	-141.455303	-44.210836	-47.508747	
..	…	…	…	…	…	…	
395	136.983270	-28.584509	-6.755030	138.981203	67.019575	14.633076	
396	68.199385	-16.610520	23.008956	-176.128834	-30.887950	-20.083761	
397	148.801101	-39.807355	22.023598	-136.855892	42.720241	78.232252	
398	-148.285446	91.879843	91.975136	83.774336	91.562298	59.635654	
399	-58.439243	42.696469	-247.440529	52.729412	-4.761532	-37.714476	

	96	97	98	99
0	-57.379266	-2.763592	35.915788	12.478858
1	11.060194	242.914352	-66.928183	182.049990
2	204.138705	231.757609	120.570987	87.513993
3	-37.708370	143.001392	-56.195746	-10.156159
4	20.878009	183.410979	59.542737	-23.233020
..	…	…	…	…
395	-57.205813	-52.384397	168.669134	168.778314

```
396   68.094754  -93.431161    8.460224  -84.794708
397  -13.852525  -39.404673   76.789657  297.350884
398 -188.245075  -27.113487 -108.139244  104.764233
399  150.055901  -80.245976   39.964291  209.016856

[400 rows x 100 columns]
```

[61]:
```
# Reconstruction error for 100 principal components
r2 = r2_score(data2[:,:100],lowdimdata)
rmse = sqrt(mean_squared_error(df2[:,:100],lowdimdata))
rmse
```

[61]:
```
378.8651002714159
```

[ ]:
```

```