1. The height of 10 people of a normal population are found to be

$$70, 67, 62, 67, 61, 68, 70, 64, 65, 66$$

   inches. Is it reasonable to believe that the average height is greater than 64 inches? Test at 5% at significance level assuming that for 9 degrees of freedom $P(t > 1.83) = 0.05$.

2. Of 400 mangoes selected at random from a large population, 53 were found to be bad. Test at 1% significance level that on the average 10% of the mangoes were bad. Given that

$$\frac{1}{\sqrt{2\pi}} \int_{2.58}^{\infty} e^{-\frac{x^2}{2}} = 0.005.$$

3. In a sample of 700 males drawn from a large city 400 are smokers and in a sample of 900 males drawn from another large city 400 are smokers. Do the two cities differ significantly in respect of smoking among males? Test at 1% level of significance. Given that

$$\frac{1}{\sqrt{2\pi}} \int_{2.575}^{\infty} e^{-\frac{x^2}{2}} \, dx = 0.005.$$

4. Given the population density function

$$f(x;\theta) = \begin{cases} \theta e^{-\theta x}, & 0 \le x < \infty, \theta > 0 \\ 0, & \text{else.} \end{cases}$$

   The null hypothesis $H_0 : \theta = 2$ against one sided alternative $H_1 : \theta > 2$ will be tested on the following procedure. $H_0$ should be rejected if a sample $x$ drawn from the population is greater than or equal to 6. Find the probability of Type I error and Type II error.

5. Let $p$ denote the probability of getting a head when a given coin is tossed once. Suppose that the hypothesis $H_0 : p = 0.5$ is rejected in favour of $H_1 : p = 0.6$ if 10 trials result in 7 or more heads. Calculate the probabilities of Type I and Type II error.

6. Let us consider a population with density,

$$f(x;\theta) = \begin{cases} \theta x^{\theta-1}, & 0 < x < 1, \\ 0, & \text{else.} \end{cases}$$

   A random sample $X_1, X_2$ of size 2 is drawn from the above population $f(x;\theta)$ for testing a null hypothesis $H_0 : \theta = 1$ against an alternative hypothesis $H_1 : \theta = 2$. Suppose that the critical region is defined by $C := \{(x_1 x_2) : x_1 x_2 \ge 3/4\}$. Calculate probability of (a) Type-I error, (b) type-II error, and (c) power of the test.

Answer key to few questions!

(1) $H_0$ is rejected and $H_1$ (greater than 64) is accepted.

(2) $H_0$ is accepted.

(3) $H_0$ is rejected.

(4) $e^{-12}, 1 - e^{-6\theta}, \theta > 2$.

(5) $0.172, 0.618$.

(6) $(a)$ 0.03424, $(b)$ 0.8861, $(c)$ 0.1139.