CSL003P1M: Probability and Statistics
©Indian Institute of Technology Jammu
Assignment -I
August 8, 2019

1. A die is biassed in such a way that the probability of getting an even number is twice that of an odd number. What is the probability of obtaining an even number? A prime number?

2. Prove that for any two events A and B,

$$P(B|A) \geq 1 - \frac{P(B^c)}{P(A)},$$

   where B^c denotes the complement of B.

3. A group of 40 students is made from 20 men and 20 women. If this group is then divided in two equal groups, what is the probability that each group has the same number of men and women?

4. A signal can be green or red with probability $\frac{4}{5}$ or $\frac{1}{5}$, respectively. The probability that it is received correctly by a station is $\frac{3}{4}$. Of the two stations A and B, the signal is first received correctly by A and then station A passes the signal to B. If the signal received at the station B is green, then find the probability that original signal was green?

5. Consider events A and B such that $P(A) = \frac{1}{4}, P(B|A) = \frac{1}{2}, P(A|B) = \frac{1}{4}$. Find $P(A^c|B^c)$ and $P(A|B) + P(A|B^c)$.

6. What is the probability that among a group of 25 people at least 2 have the same birthday? Assume that the year has 365 days and that all of them are equally likely to be somebody's birthday.

7. It is known that each of four people A, B, C, D tells the truth independently in a given instance with probability $\frac{1}{3}$. Suppose A makes a statement, then D says that C says that B says that A was telling the truth. What is the probability that A was actually telling the truth?

8. Let $A_1, A_2, A_3, ...$ be a sequence of events, which are 'monotonically decreasing',i.e., $A_{n+1} \subset A_n, \forall\, n$. Let $A = \cap_{n=1}^{\infty} A_n$. Show that

$$P(A) = \lim_{n \to \infty} P(A_n).$$

9. A secretary writes four letters and the corresponding address on envelopes. If he inserts the letters in the envelopes at random irrespective of address, then calculate the probability that all the letters are wrongly placed.

10. Assume that the events A_1, A_2, A_3, A_4 are independent and that $P(A_3 \cap A_4) > 0$. Show that

$$P(A_1 \cup A_2 | A_3 \cap A_4) = P(A_1 \cup A_2).$$

11. A jar contains two white balls and three black balls. The balls are drawn from the jar one by one and placed on the table in the order drawn. What is the probability that they are drawn in the order white, black, black, white, black?

12. We draw the top 7 cards from a well-shuffled standard 52-card deck. Find the probability that:

    (a) the 7 cards include exactly 3 aces;

    (b) the 7 cards include exactly 2 kings;

    (c) the probability that the 7 cards include exactly 3 aces, or exactly 2 kings, or both.

13. A M.Tech student must take on the same day a probability and an algebra exam. Let:

$$A := \text{"The student fails the probability exam"}.$$
$$B := \text{"The student fails the algebra exam"}.$$

Let $P(A) = 0.4, P(B) = 0.3$, and $P(A \cap B) = 0.2$. What is the probability that the student passes the algebra exam given that he/she passed the probability one? What is the probability that the student passes the probability exam given that he failed the algebra one?

14. Let A, B and C be independent events with $P(A) = P(B) = P(C) = \frac{1}{3}$. Find the probability that:

    (a) At least one event happens;

    (b) At least two events happen;

    (c) Exactly two of the events happen.