CSL3: Probability and Statistics
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Assignment -II
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1. Let us consider the following function:

$$f_\theta(x) = \begin{cases} \theta^2 x e^{-\theta x}, & x > 0, \\ 0, & x \leq 0, \end{cases}$$

   where $\theta > 0$.

   (i) Does $f_\theta(x)$ define a probability density function (PDF)?

   (ii) Find the distribution function (DF) associated with $f_\theta(x)$.

   (iii) If X is a random variable with PDF $f_\theta(x)$, find $P(X \geq 1)$.

2. Let us consider

$$p_k = p(1-p)^k, \quad k = 0, 1, 2, \ldots, \quad 0 < p < 1.$$

   (i) Does $\{p_k\}$ define the probability mass function (PMF) of some random variable (RV)?

   (ii) What is the distribution function of this RV?

   (iii) If X is a random variable with PMF $\{p_k\}$, find $P(n \leq X \leq N)$, where $n, N(N > n)$ are positive integers.

3. Show that the function

$$f(x) = \frac{1}{2} e^{-|x|}, \quad -\infty < x < \infty,$$

   is a PDF. Find its DF.

4. Let X be a random variable with density function given by:

$$f(x) = \begin{cases} 0.2, & -1 < x \leq 0, \\ 0.2 + cx, & 0 < x \leq 1, \\ 0, & \text{else.} \end{cases}$$

   (i) Determine the value of c.

   (ii) Obtain the DF of RV X.

   (iii) Calculate $P(0 \leq X < 0.5)$.

   (iv) Calculate the DF and PDF of the RV $Y := 2X + 3$.

5. The probability density function of a random variable X is symmetric about the origin. Prove that X and $-X$ have the same distribution.

6. The continuous random variable X has a PDF $f(x) = 3x^2$, $-1 \leq x \leq 0$. If b is a number satisfying $-1 < b < 0$, compute $P(X > b | X < \frac{b}{2})$.

7. Let X be an RV with PDF

$$f(x) = \begin{cases} 0, & x \leq 0, \\ \frac{1}{2}, & 0 < x \leq 1, \\ \frac{1}{2x^2}, & 1 < x < \infty. \end{cases}$$

Find the PDF of the RV $1/X$.

8. Let X be a random variable with distribution over the set of integers $\{-n, -(n-1), \ldots, -1, 0, 1, \ldots, (n-1), n\}$ given by:

$$P(X = x) = \begin{cases} \frac{1}{2n+1}, & x = 0, \pm 1, \pm 2, \ldots, \pm n \\ 0, & \text{otherwise.} \end{cases}$$

Find the distribution of   (i) $|X|$ and (ii) X^2.

9. Suppose that the RV X has a PDF $f(x) = \frac{1}{2}$, $-1 < x < 1$. Find the PDF of the following random variables:

(i) $Y = \sin(\pi/2)X$;

(ii) $Z = \cos(\pi/2)X$;

(iii) $W = |X|$.

10. Suppose that X is uniformly distributed over the interval $(0, 1)$. Find the PDF of the following random variables:

(i) $Y = X^2 + 1$;

(ii) $Z = 1/(X + 1)$.