CSL003P1M: Probability and Statistics

Assignment -IV

September 18, 2019

1. A batch of parts contains 100 parts from a local supplier of tubing and 200 parts from a supplier of tubing in the next state. If four parts are selected randomly and without replacement, then

   (i) what is the probability they are all from the local supplier?

   (ii) What is the probability that two or more parts in the sample are from the local supplier?

2. Let X be a random variable having a Poisson distribution with parameter λ. If $P(X = 0) = 0.4$, find $P(X \leq 3)$.

3. Each sample of water has a 10% chance of containing a particular organic pollutant. Assume that the samples are independent with regard to the presence of the pollutant. Find the probability that in the next 18 samples, exactly 2 contain the pollutant.

4. Find the characteristics function of the random variable having the following distributions:

   (i) Binomial distribution with parameters n and p.

   (ii) Poisson distribution with parameter λ.

5. Let X be a random variable having a geometric distribution with parameter p. Find $E\left(\frac{1}{X}\right)$.

6. For the case of the thin copper wire, suppose that the number of flaws follows a Poisson distribution with a mean of 2.3 flaws per millimeter. Then

   (i) Determine the probability of exactly 2 flaws in 1 millimeter of wire.

   (ii) Determine the probability of 10 flaws in 5 millimeters of wire.

   (iii) Determine the probability of at least 1 flaw in 2 millimeters of wire.

7. Suppose X is a discrete uniform random variable on the consecutive integers

   $$a, a+1, a+2, \ldots, b; \text{ for } a \leq b.$$

   Find the mean and variance of X.

8. A company employs 800 men under the age of 55. Suppose that 30% carry a marker on the male chromosome that indicates an increased risk for high blood pressure.

   (i) If 10 men in the company are tested for the marker in this chromosome, what is the probability that exactly 1 man has the marker?

   (ii) If 10 men in the company are tested for the marker in this chromosome, what is the probability that more than 1 has the marker?

9. Let X be a random variable having a binomial distribution with parameters n and p. Prove that

$$E\left(\frac{1}{X+1}\right) = \frac{1-(1-p)^{n+1}}{(n+1)p}.$$

10. A Web site contains three identical computer servers. Only one is used to operate the site, and the other two are spares that can be activated in case the primary system fails. The probability of a failure in the primary computer (or any activated spare system) from a request for service is 0.0005. Assuming that each request represents an independent trial, what is the mean number of requests until failure of all three servers? What is the probability that all three servers fail within five requests?

11. Let X be a random variable having a Poisson distribution with parameter λ. Show that

$$P(X \leq n) = \frac{1}{n!}\int_{\lambda}^{\infty} e^{-x}x^n \, dx, \quad n = 0, 1, 2, \ldots.$$

12. The probability that a wafer contains a large particle of contamination is 0.01. If it is assumed that the wafers are independent, what is the probability that exactly 125 wafers need to be analyzed before a large particle is detected?