CSL003P1M: Probability and Statistics

Assignment -VI (Solution Key)

November 2, 2019

Prob-1.

$$F(x,y)=\begin{cases}0, & x\le 0 \text{ or } y\le 0,\\ \frac{1}{2}(x^3y+xy^3), & 0<x<1, 0<y<1,\\ \frac{1}{2}(x^3+x), & 0<x<1, y\ge 1,\\ \frac{1}{2}(y^3+y), & 0<y<1, x\ge 1,\\ 1, & x\ge 1, y\ge 1.\end{cases}$$

Prob-2. (a) $P(X=0,Y=0)=\frac{1}{55},\ldots,P(X=2,Y=0)=\frac{6}{55}$, (b) $E(X)=\frac{8}{11}, E(Y)=\frac{10}{11}$.

Prob-3. $a=b=k=h=\frac{1}{18}, d=\frac{4}{5}, e=f=\frac{4}{45}$ and $E(XY)=0$.

Prob-4. $f_X(x)=3(1-x)^2, 0<x<1$, $f_Y(y)=3(1-y)^2,)0<y<1$, X and Y are not independent.

Prob-5. (i)

$$F(x,y)=\begin{cases}(1-e^{-x})(1-e^{-y}), & x\ge 0, y\ge 0,\\ 0, & \text{else.}\end{cases}$$

(ii) $f_X(x)=e^{-x}, x\ge 0$ and $f_Y(y)=e^{-y}, y\ge 0$.

(iii) $1-5e^{-4}$, (iv) e^{-1}, (v) $\frac{1}{2}$ (vi) $e^{-a}(a+1)-e^{-b}(b+1)$.

Prob-8. (a) $f(1,2)=f(1,3)=f(2,2)=f(2,3)=\frac{1}{6}, f(3,2)=0, f(3,3)=\frac{1}{3}$

(b) $f_{X|Y}(1|3)=f_{X|Y}(2|3)=\frac{1}{4}, f_{X|Y}(3|3)=\frac{1}{2}$, (c)$\frac{1}{6}$.

Prob-9. $\frac{p}{2-p}$.

Prob-10. (i) $c=6$, $f_X(x)=3(1-x)^2, 0<x<1, f_Y(y)=3(1-y)^2, 0<y<1$ and zero else.

(b) $f_{X|Y}(x|y)=\frac{2(1-x-y)}{(1-y)^2}, 0<x<1-y$, $E(X|\frac{1}{2})=\frac{1}{6}$, (c) $\rho_{X,Y}=-\frac{1}{3}$.

Prob-11. (i) $E(X|Y=2)=\frac{17}{15}, Var(X|Y=2)=\frac{146}{225}$ and $\rho_{X,Y}=-\frac{1}{\sqrt{345}}$.

Prob-12. (a)$\frac{2}{e}-\frac{3}{e^2}$, (b) $\frac{3}{4}$.

Prob-13. (a) $\frac{5}{12}$, conditional variance of X given $Y=1$ is $\frac{13}{162}$, (b) No.

Prob-14. (a) $f(x,y) = \binom{x}{y}(\frac{1}{2})^x \frac{x}{3}, x = 1,2; 0 \le y \le x$, zero else, (b) $\frac{5}{6}$.

Prob-15. $\frac{1}{3}$.

Prob-17. $\rho_{X,Y} = \frac{3}{8\sqrt{2}}$.

Prob-20. $f(1,1) = \frac{1}{36}, f(2,1) = f(2,2) = \frac{2}{36}, f(3,1) = f(3,3) = \frac{3}{36}, f(4,2) = \frac{4}{36}$, $f(6,2) = f(6,3) = \frac{6}{36}, f(9,3) = \frac{9}{36}$, zero elsewhere.

Prob-21. $f(y_1,y_2) = y_1^2 - y_2^2, 0 < y_2 < y_1, y_1 + y_2 < 2$, zero elsewhere.

Prob-22. (a) $f(y) = ye^{-y}, y > 0$, zero elsewhere, where $Y = X_1 + X_2$.
(b) $f(z) = \frac{1}{2}e^{-z/2}, z > 0$ and zero elsewhere.

Prob-24. $f_Y(y) = 15y^{14}, 0 < y < 1$.