CSL003P1M: Probability and Statistics

Assignment -VIII

November 18, 2019

1. Let $X_1, X_2, \ldots, X_n$ be a random sample from a Normal distribution with mean μ and variance σ^2. Obtain $100(1-\alpha)\%$ confidence interval for (a) μ (b) σ^2.

2. Let $X_1, X_2, \ldots, X_n$ be a random sample from a exponentially distributed population with unknown parameter θ. Find 95% confidence interval for θ when the sample size is large. ($Hint$: Use CLT).

3. Let $X_1, X_2, \ldots, X_n$ be a random sample from the uniform distribution $[0, \theta]$. Show that $100(1-\alpha)\%$ confidence interval for θ is given by

$$\left(X_{(n)}, \frac{X_{(n)}}{\alpha^{1/n}} \right),$$

   where $X_{(n)} = \max\{X_1, X_2, \ldots, X_n\}$.

4. Suppose that X has distribution $N(\mu, 4)$. A sample of size 25 yields a sample mean $\overline{X} = 78.3$. Obtain a 99-percent (two-sided) confidence interval for μ.

5. Suppose that X has distribution $N(\mu, \sigma^2)$. A sample of size 30, say $X_1, X_2, \ldots, X_n$, yields the following values: $\sum_{i=1}^{30} X_i = 700.8$, $\sum_{i=1}^{30} X_i^2 = 16,395.8$. Obtain a 95-percent (two-sided) confidence interval for μ.

6. Suppose that X has distribution $N(\mu, \sigma^2)$, where μ and σ^2 are unknown. A sample of size 15 has yielded the values $\sum_{i=1}^{15} X_i = 8.7$ and $\sum_{i=1}^{15} X_i^2 = 27.3$. Obtain a (two-sided) 95-percent confidence interval for σ^2.

7. The pdf of a population random variable X is given by

$$f(x;\alpha) = \begin{cases} \frac{2}{\alpha^2}(\alpha - x), & 0 < x < \alpha, \\ 0, & \text{else.} \end{cases}$$

   Obtain 95% confidence interval for the parameter α on the basis of a random sample x of unit size from the population of X by using the sampling distribution of the statistic $\frac{\alpha - x}{\alpha}$.

8. In a random sample of 400 articles 40 are found to be defective. Obtain 95% confidence interval for the true proportion of defectives in the population of such articles. Given that

$$\int_0^{1.96} \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} \, dx = 0.475$$

   .

9. 171 out of 300 voters picked at random from a large electorate said that they were going to vote a particular candidate. Find 95% confidence interval for the population of voters of the electorate who would vote in favour of the candidate.

10. The heights (in cm) were recorded for 10 students of IIT Jammu, chosen at random, as $160, 162, 169, 175, 172, 170, 178, 180, 177, 165$. Find 99% confidence interval for the mean of the population of heights of the students of the institute, assuming it to be normal.

11. The rainfall of a rainy season of Jammu is measured for few consecutive days and the measurements (in mm) are $9.4, 8.8, 10.6, 12.2, 11.8, 11.4, 9.9, 10.8, 12.1, 11.7$. Compute 99% confidence interval for mean and standard deviation of the population, assuming the population of measurements of rainfall is normal.