Q.2. Given,

$$g_1 = f_1 + h_2 * f_2$$

$$g_2 = h_1 * f_1 + f_2$$

Now,

Taking fourier transform,

$$F(g_1) = F(f_1) + F(h_2 * f_2)$$

$$= F(f_1) + F(h_2) \times F(f_2)$$

$$\Rightarrow G_1 = F_1 + H_2 \times F_2$$

Similarly,

$$G_2 = H_1 \times F_1 + F_2$$

Solving linear eqn gives,

$$F_1 = \frac{G_1 - H_2 \times G_2}{1 - H_1 H_2}$$

$$F_2 = \frac{G_2 - H_1 \times G_1}{1 - H_1 H_2}$$

PAGE No.
DATE

So,

$$f_1 = ifft(F_1) = ifft\left(\frac{G_1 - H_2 * G_2}{1 - H_1 * H_2}\right)$$

$$f_2 = ifft(F_2) = ifft\left(\frac{G_2 - H_1 * G_1}{1 - H_1 * H_2}\right)$$

Inherent problem :-

If $H_1 * H_2 = 1$, then denominator of F_1 and F_2 becomes 0 and f_1 and f_2 becomes undefined.

For some frequency u, v,
if $H_1(u, v) * H_2(u, v) = 1$
then,
$F_1(u, v)$ and $F_2(u, v)$ becomes undefined.

Noise :-

If there is noise in g_1 and g_2
then

$$g_1 = f_1 + h_2 * f_2 + n_1$$
$$g_2 = h_1 * f_1 + f_2 + n_2$$

In fourier domain,

$$G_1 = F_1 + H_2 \times F_2 + N_1$$
$$G_2 = H_2 * F_2 + F_2 + N_2$$

PAGE No.
DATE

$$F_1 = \frac{G_1 - H_2 * G_2}{1 - H_1 * H_2} - \left(\frac{N_1 - H_2 * N_2}{1 - H_1 * H_2}\right)$$

$$F_2 = \frac{G_2 - H_1 * G_1}{1 - H_1 * H_2} - \left(\frac{N_2 - H_1 N_1}{1 - H_1 H_2}\right)$$

Now,
at higher frequencies
$H_1 \to 0$
and $H_2 \to 0$
since,
h_1 and h_2 are blur kernels which becomes low-pass filter in fourier domain.
So,
at higher frequencies, noise added in F_1 is N_1 and in F_2 is N_2.
So,
noise does not get amplified.
So the ~~error~~ effect of noise on accuracy of solution won't be too high.