

Working principle involves detecting electrical conductivity when specific gases come in contact.

MQ 8 - H₂

MQ 131 - O₃

DSTL

Unit 1: Sets, Relations

set { Intro
Combination of sets

Relation { Definition
Operation on Relation
Properties of Relations
Composite Relations
Equality of Relations
Recursive definition
Order of Relation

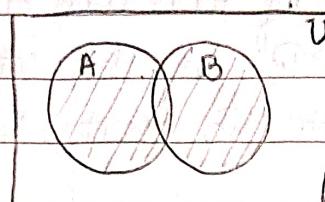
Poset & Lattice

→ Hasse diagram
→ Poset
→ Definition and properties of lattices
→ Bounded, complemented, distributed, modular and complete lattice.

• Operation on Sets

(i) Union

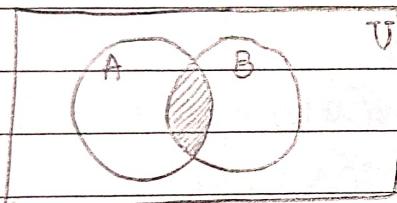
The union of two sets A and B, denoted by $A \cup B$, which is the set of elements which belongs to A or B.



$$A \cup B$$

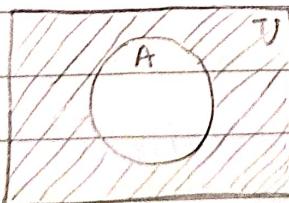
(ii) Intersection

$$A \cap B = \{x : x \in A \text{ and } x \in B\}$$



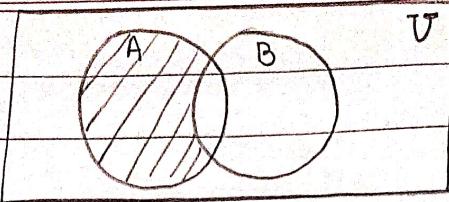
(iii) Complement

$$A' = \{x : x \notin A \text{ and } x \in U\}$$



Note: If A and B are two sets the relative complement of B w.r.t. A or simple difference of A and B denoted by $A - B$ is the set of elements which belongs to A but does not belong to B.

$$A - B = \{x : x \in A \text{ and } x \notin B\}$$

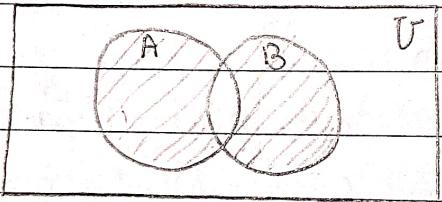


(iv) Symmetric Difference

The symmetric difference of two sets A and B denoted by $A \Delta B$ or $A \oplus B$ is the set of elements that belongs to A or to B but not to both A and B .

$$A \Delta B = (A - B) \cup (B - A)$$

$= \{x : x \text{ belongs to exactly one of } A \text{ and } B\}$



Ques: If $A = \{-3, 0, 1, 2\}$ and $B = \{1, 2, 3, 4\}$, then find $A - B$, $B - A$ and $A \Delta B$.

$$A - B = \{-3, 0\}$$

$$B - A = \{3, 4\}$$

$$A \Delta B = (A - B) \cup (B - A) = \{-3, 0, 3, 4\}$$

Ques

Cardinal Number of a set

The cardinal number of a set is the no. of elements in the set.
It is denoted by $|A|$ or $n(A)$.

Note: If A and B are two finite sets then,

- $|A \cup B| = |A| + |B| - |A \cap B|$

- $n(A - B) = n(A) - n(A \cap B)$ or $n(A \cap B') = n(A) - n(A \cap B)$

- $n(A \Delta B) = n[(A - B) \cup (B - A)] = n(A) + n(B) - 2n(A \cap B)$

number of elements which belongs to exactly one of A or B .

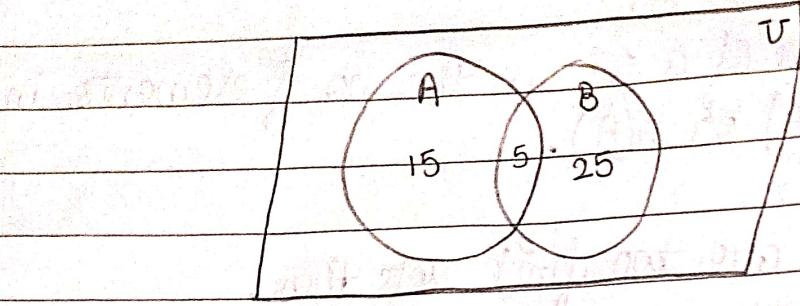
If A, B, C are finite sets then,

- $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(C \cap A) + n(A \cap B \cap C)$

- No. of elements in exactly two of the sets A, B, C .
 $= n(A \cap B) + n(B \cap C) + n(C \cap A) - 3n(A \cap B \cap C)$

- No. of elements in exactly one of the sets
 $= n(A) + n(B) + n(C) - 2n(A \cap B) - 2n(B \cap C) - 2n(C \cap A) + 3n(A \cap B \cap C)$

Ques: A computer company must hire 20 programmers to handle system programming jobs and 30 programmers for application programming. Of those hired, 5 are expected to perform job of both type. How many programmers must be hired?



Let A : Programming jobs (System)

and B : Application programming.

Given,

$$n(A) = 20$$

$$n(B) = 30$$

$$n(A \cap B) = 5$$

To find, $n(A \cup B)$

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

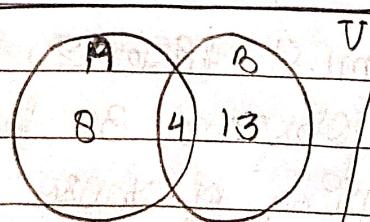
$$= 20 + 30 - 5$$

$$= 45$$

Therefore, company must hire 45 programmes.

Ques:

In a class of 25 students, 12 have taken mathematics, 8 have taken mathematics but not biology. Find the no. of students who have taken maths and biology and those who have taken biology but not maths.



Let A and B be the sets of students who have taken mathematics and biology respectively

We know that,

$$n(A \cap B') = n(A) - n(A \cap B)$$

$$B' = 12 - n(A \cap B)$$

$$n(A \cap B) = 4$$

Ans ∵ No. of students who have taken mathematics and biology = 4.

We have to find, $n(B \cap A') = n(B - A)$

$$n(A \cup B) = n(A - B) + n(B - A) + n(A \cap B)$$

$$25 = 8 + n(B - A) + 4$$

$$n(B - A) = 13$$

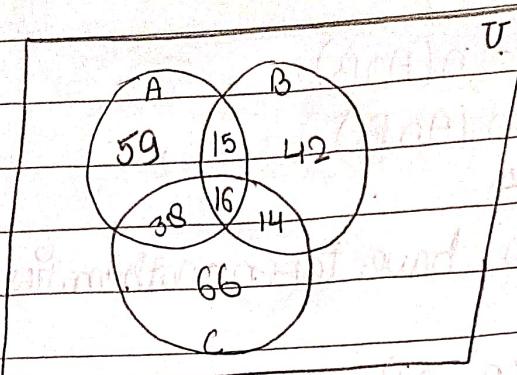
∴ No. of students who have taken biology but not maths = 13

*Ques: Out of 250 candidates who failed in an examination it was revealed that 128 failed in maths, 87 in physics and 134 in aggregate. 31 failed in maths and physics, 54 failed in aggregate and maths and 30 failed in aggregate and in physics. Find how many candidates failed.

- (i) in all 3 subjects
- (ii) in maths but not in physics
- (iii) in aggregate but not in maths
- (iv) in physics but not in aggregate or in maths.
- (v) in aggregate or in maths but not in physics.

Let A, B and C be the sets of students who failed in maths, physics and aggregate respectively.

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Given, $n(A \cup B \cup C) = 250$

$$n(A) = 128 \quad n(A \cap B) = 31$$

$$n(B) = 87 \quad n(B \cap C) = 30$$

$$n(C) = 134 \quad n(C \cap A) = 54$$

we know that,

$$(i) n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(C \cap A) + n(A \cap B \cap C)$$

$$250 = 128 + 87 + 134 - 31 - 30 - 54 + n(A \cap B \cap C)$$

$$n(A \cap B \cap C) = 365 - 349$$

$$n(A \cap B \cap C) = 16$$

\therefore No. of students who failed in all 3 subjects = 16

(ii) we have to find, $n(A \cap B')$

$$n(A \cap B') = n(A) - n(A \cap B)$$

$$= 128 - 31 = 97$$

$$(iii) n(C \cap A') = n(C) - n(C \cap A)$$

$$= 134 - 54 = 80$$

$$(iv) n[B \cap (A \cup C)'] = n(B) - n[B \cap (A \cup C)]$$

$$= n(B) - n[(B \cap A) \cup (B \cap C)]$$

$$= n(B) - [n(B \cap A) + n(B \cap C) - n(A \cap B \cap C)]$$

$$= 87 - [31 + 30 - 16]$$

$$= 87 - 45$$

$$= 42.$$

$$\begin{aligned}
 \text{(v) } n[(A \cup C) \cap B'] &= n(A \cup C) + n(B') - n(A \cup C \cup B') \\
 &= n(A \cup C) - n[(A \cup C) \cap B] \\
 &= n(A \cup C) - n[(A \cap B) \cup (C \cap B)] \\
 &= n(A) + n(C) - n(A \cap C) - [n(A \cap B) + n(B \cap C) - n(A \cap B \cap C)] \\
 &= 128 + 134 - 54 - 45 \\
 &= 262 - 99 \\
 &= 163
 \end{aligned}$$

Ques. Find how many integers between 1 and 60 that are not divisible by 2 nor by 3 and nor by 5. Also determine the number of integers divisible by 5 nor by 2, nor by 3.

$$\text{Let } S = \{1, 2, 3, 4, 5, \dots, 60\}$$

$$\begin{aligned}
 A &= \{x : x \in S \text{ and } x \text{ is divisible by 2}\} \\
 n(A) &= 30
 \end{aligned}$$

$$\begin{aligned}
 B &= \{y : y \in S \text{ and } y \text{ is divisible by 3}\} \\
 n(B) &= 20
 \end{aligned}$$

$$\begin{aligned}
 C &= \{z : z \in S \text{ and } z \text{ is divisible by 5}\} \\
 n(C) &= 12
 \end{aligned}$$

No. of integers that are not divisible by 2, nor by 3 and nor by 5

$$\begin{aligned}
 &= n(A' \cap B' \cap C') \\
 &= n[(A \cup B \cup C)'] \\
 &= n(S) - n(A \cup B \cup C) \\
 &= n(S) - [n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(C \cap A) + \\
 &\quad n(A \cap B \cap C)] \\
 &= 60 - [30 + 20 + 12 - 10 - 4 - 6 + 2] \\
 &= 16 \text{ Ans}
 \end{aligned}$$

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No. of integers divisible by 5 but not by 2 and not by 3.

$$= n[C \cap A \cap B']$$

$$= n[C \cap (A \cup B)']$$

$$= n(C) - n[C \cap (A \cup B)]$$

$$= n(C) - n[(C \cap A) \cup (C \cap B)]$$

$$= n(C) - [n(A \cap C) + n(B \cap C) - n(A \cap B \cap C)]$$

$$= 12 - [6 + 4 - 2] = 4$$

• Relations and Functions

Cartesian Product

Let A and B be two sets. Cartesian Product of A and B denoted by $A \times B$ is defined as

$$A \times B = \{(a, b) : a \in A \text{ and } b \in B\}$$

Definition of Relation

Let A and B be any two sets. A binary relation or simply relation from A to B is a subset of the cartesian product of A and B.

Suppose R is a relation from A to B, then R is a set of ordered pairs (a, b) where $a \in A$ and $b \in B$. Every such ordered pair is written as aRb and read as

'a is related to b by R'. If $(a, b) \in R$ then A is not related to b by R and is written as $a \not R b$.

Ques:

If $A = \{1, 2, 5\}$, $B = \{2, 4\}$ then

$$A \times B = \{(1, 2), (1, 4), (2, 2), (2, 4), (5, 2), (5, 4)\}$$

If we take the relationship $x < y$, then some ordered pairs are related and some are not.

The subset of $A \times B$ whose elements are related is the relation R and is given by

$$R = \{(1, 2), (1, 4), (2, 4)\}$$

Domain and Range

The set $\{a \in A : (a, b) \in R \text{ for some } b \in B\}$ is called the domain of R and denoted by $\text{Dom}(R)$.

The set $\{b \in B : (a, b) \in R \text{ for some } a \in A\}$ is called the range of R and denoted by $\text{Ran}(R)$.

Thus, the domain of a relation R is the set of all first elements of the ordered pairs which belongs to R and the range of R is the set of second elements.

Properties of Relation

Property of a relation tells the nature or type of relation.

(i) Reflexive relation

A relation R on a set A is reflexive if $\forall a \in A$ such that aRa for every $a \in A$ i.e. if $(a, a) \in R \forall a \in A$.

This simply means that each element a of H is related to itself.

e.g. $R = \{(x, y) \in R^2 : x \leq y\}$ is a reflexive relation since $x \leq x$ for any $x \in R$ (set of Real numbers).

Symmetric Relation

A relation R on a set A is symmetric if whenever $(a, b) \in R$ then $(b, a) \in R$. i.e. if $a R b \Rightarrow b R a$.

e.g. The relation $R = \{(1, 1), (1, 2), (1, 3), (2, 2), (2, 1), (3, 1)\}$ is a symmetric relation on set $A = \{1, 2, 3\}$.

Anti-symmetric Relation

A relation R on a set A is said to be anti-symmetric if $(a, b) \in R$ and $(b, a) \in R \Rightarrow (a, b) \neq (b, a)$. Thus, R is anti-symmetric if $(a, b) \in R \Rightarrow (b, a) \notin R$ unless $a = b$.

e.g. $R = \{(x, y) \in N : x \text{ is a divisor of } y\}$ is an anti-symmetric relation since x divides y and y divides $x \Rightarrow x = y$.

Transitive Relation

A relation R on a set A is said to be transitive if whenever $(a, b) \in R$ and $(b, c) \in R$ then $(a, c) \in R$. i.e. $a R b$ and $b R c \Rightarrow a R c$.

Ques. Give an example of a relation which is

- Reflexive and transitive but not symmetric
- Symmetric and transitive but not reflexive
- Reflexive and symmetric but not transitive
- Reflexive and transitive but neither symmetric nor anti-symmetric

Let $A = \{1, 2, 3\}$

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(i) $R_1 = \{(1,1), (2,2), (3,3), (1,2)\}$

(ii) $R_2 = \{(1,1), (3,3), (1,3), (3,1)\}$

(iii) $R_3 = \{(1,1), (2,2), (3,3), (1,2), (2,1), (2,3), (3,2)\}$

$(1,2) \in R_3, (2,3) \in R_3$ But $(1,3) \notin R_3$

$\Rightarrow R_3$ is not transitive.

(iv) Let Z^+ be the set of all non-zero integers and R be the relation on Z^+ defined by $(a,b) \in R$ if a divides b . (or $a|b$)

(a) $\because a|a \quad \forall a \in Z^+$

$\therefore R$ is reflexive.

(b) $a|b$ and $b|c \Rightarrow a|c$

Hence R is transitive.

(c) ~~$\frac{2}{2} \text{ div } 2|4$ but $4 \not| 2$ ($4|2$ is not true)~~

Hence R is not symmetric.

(d) $5|-5$ and $-5|5$

but $5 \neq -5$. Hence R is not anti-symmetric.

Equivalence relation.

A relation R on a set A is called an equivalence relation if it is reflexive, symmetric and transitive.

i.e. R is an equivalence relation on a set A if it has the following properties.

(i) $(a,a) \in R \quad \forall a \in A$ (Reflexivity)

(ii) $(a,b) \in R \Rightarrow (b,a) \in R$ (Symmetric)

(iii) $(a,b) \in R$ and $(b,c) \in R \Rightarrow (a,c) \in R$ (Transitivity)

Ans: If R be the relation in the set of integers \mathbb{Z} defined by
 $R = \{(x,y) : x \in \mathbb{Z}, y \in \mathbb{Z}, (x-y) \text{ is divisible by } 6\}$, then prove that R is an equivalence relation.

i) Given, $R = \{(x,y) : x \in \mathbb{Z}, y \in \mathbb{Z}, (x-y) \text{ is divisible by } 6\}$

To prove:

R is an equivalence relation.

ii) Reflexive

Let $x \in \mathbb{Z}$. Then $x-x=0$ and 0 is divisible by 6 .

Hence, $xRx \forall x \in \mathbb{Z}$

i.e. R is reflexive.

iii) Symmetric

$xRy \Rightarrow (x-y) \text{ is divisible by } 6$.

$\Rightarrow -(x-y) \text{ is divisible by } 6$.

$\Rightarrow (y-x) \text{ is divisible by } 6$.

$xRy \Rightarrow yRx$

Hence R is symmetric.

iv) Transitive

xRy and $yRz \Rightarrow (x-y) \text{ is divisible by } 6$ & $(y-z) \text{ is divisible by } 6$

$\Rightarrow (x-y+y-z) \text{ is divisible by } 6$

$\Rightarrow (x-z) \text{ is divisible by } 6$.

$\Rightarrow xRz$.

Hence R is transitive.

Therefore R is an equivalence relation.

Ans:

Consider the following relation on $\{1, 2, 3, 4, 5, 6\}$

$$R = \{(i, j) \mid |i-j| = 2\}$$

Is R transitive? Is R reflexive? Is R symmetric?

$$\text{Let } A = \{1, 2, 3, 4, 5, 6\}$$

$$\text{Then } R = \{(i, j) \mid |i-j| = 2\} \text{ on } A.$$

$$= \{(1, 3), (2, 4), (3, 1), (3, 5), (4, 2), (4, 6), (5, 3), (6, 4)\}$$

i) R is not reflexive since $(1, 1) \notin R$.

ii) R is symmetric since $(i, j) \in R \Rightarrow (j, i) \in R$.

iii) R is not transitive since $(1, 3) \in R$ and $(3, 1) \in R$ but $(1, 1) \notin R$

Composition of Relations

Let A, B, C be three sets. Let R be a relation from A to B and S be a relation from B to C; i.e. R is a subset of $A \times B$ and S is a subset of $B \times C$.

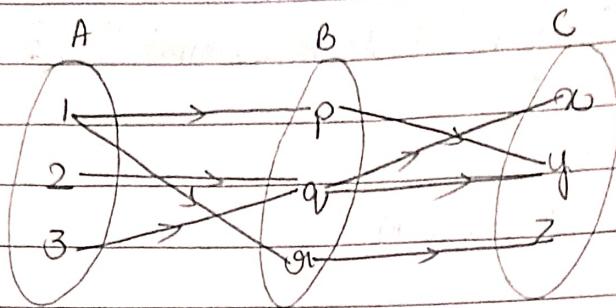
The composite of R and S, denoted by $R \circ S$ is the relation consisting of the ordered pairs (a, c) . When $a \in A$ and $c \in C$ and for which there exists an element $b \in B$ such that $(a, b) \in R$ and $(b, c) \in S$. Thus,

$$R \circ S = \{(a, c) : \text{for some } b \in B, (a, b) \in R \text{ and } (b, c) \in S\}$$

Ques: Let $A = \{1, 2, 3\}$, $B = \{p, q, r\}$, $C = \{x, y, z\}$ and let
 $R = \{(1, p), (1, q), (2, q), (3, q)\}$ and $S = \{(p, x), (q, x), (q, y), (q, z)\}$
Compute $R \circ S$.

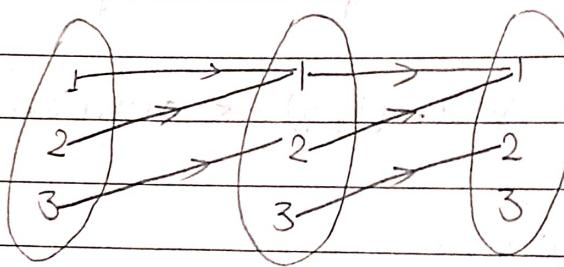
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$$R \circ S = \{(1,y), (1,z), (2,x), (2,y), (3,x), (3,y)\}$$



Q.2. Let $R = \{(1,1), (2,1), (3,2)\}$. Compute R^2 .

$$R^2 = R \circ R = \{(1,1), (2,1), (3,1)\}$$



Q.3. Let $X = \{4, 5, 6\}$, $Y = \{a, b, c\}$ and $Z = \{l, m, n\}$. Consider the relation R_1 from X to Y and R_2 from Y to Z .

$$R_1 = \{(4,a), (4,b), (5,c), (6,a), (6,c)\}$$

$$R_2 = \{(a,l), (a,n), (b,l), (b,m), (c,l), (c,m), (c,n)\}$$

Find the composition relation (i) $R_1 \circ R_2$ (ii) $R_2 \circ R_1^{-1}$

$$R_1 \circ R_2 = \{(4,l), (4,n), (4,m), (5,l), (5,m), (5,n), (6,l), (6,n), (6,m)\}$$

Inverse Relation: Let R be a relation from A to B , the inverse of R , denoted by R' is the relation from B to A which consists of those ordered pairs which when reversed belongs to R . i.e. $R' = \{(b,a) : (a,b) \in R\}$
 Consequently, $xRy \Rightarrow yR'x$.

If R and S are equivalence relations on the set A . Prove that R'' is an equivalence relation and $R \cap S$ is an equivalence relation.

i) To prove R' is an equivalence relation.

Since R is an equivalence relation.

∴ It is reflexive, symmetric and transitive.

a) Reflexive.

∴ $(a,a) \in R \forall a \in A$, therefore $(a,a) \in R'$
 ∴ R' is reflexive.

b) Symmetric.

Let $(a,b) \in R'$

$$\begin{aligned} \therefore (a,b) \in R' &\Rightarrow (b,a) \in R \\ &\Rightarrow (a,b) \in R \quad [\because R \text{ is symmetric}] \\ &\Rightarrow (b,a) \in R' \end{aligned}$$

∴ R' is symmetric.

c) Transitive.

Let $(a,b), (b,c) \in R'$

$$\Rightarrow (b,a), (c,b) \in R$$

$$\Rightarrow (c,b), (b,a) \in R$$

$$\Rightarrow (c,a) \in R \quad [\because R \text{ is transitive}]$$

$$\Rightarrow (a,c) \in R'$$

∴ R' is transitive. Hence R' is equivalence relation.

(ii)

To prove $R \cap S$ is an equivalence relation.

@ Reflexive.

$\forall a \in A$, $(a, a) \in R$ and $(a, a) \in S$, since R and S are equivalence relations.

Therefore for all $a \in A$, $(a, a) \in R \cap S$.

$\therefore R \cap S$ is reflexive.

(b)

Symmetric.

Let $(a, b) \in R \cap S$

$\Rightarrow (a, b) \in R$ and $(a, b) \in S$

$\Rightarrow (b, a) \in R$ and $(b, a) \in S$ [$\because R$ and S are symmetric being an equivalence]

$\Rightarrow (b, a) \in R \cap S$

$\Rightarrow R \cap S$ is symmetric.

(c)

Transitive.

Let $(a, b) \in R \cap S$ and $(b, c) \in R \cap S$

$\Rightarrow [(a, b) \in R \text{ and } (a, b) \in S] \text{ and } [(b, c) \in R \text{ and } (b, c) \in S]$

$\Rightarrow [(a, b) \in R \text{ and } (b, c) \in R] \text{ and } [(a, b) \in S \text{ and } (b, c) \in S]$

$\Rightarrow (a, c) \in R$ and $(a, c) \in S$ [$\because R$ and S are transitive]

$\Rightarrow (a, c) \in R \cap S$

$\Rightarrow R \cap S$ is transitive.

Hence $R \cap S$ is an equivalence relation.

Operation on Relation

If R and S be two relations, then $R \cap S$ known as intersection of R and S defines a relation such that

$$x(R \cap S)y = (xRy) \wedge (xSy)$$

Similarly,

$$x(R \cup S)y = (xRy) \vee (xSy)$$

$$x(R - S)y = (xRy) \wedge (\neg xSy), R - S \text{ is known as}$$

difference of R and S .

$xR'y = xRy$, where R' is the complement of R .

Ques:

If $A = \{x, y, z\}$, $B = \{X, Y, Z\}$, $C = \{x, y\}$ and $D = \{Y, Z\}$.
 R is a relation from A to B defined by $R = \{(x, X), (x, Y), (y, Z)\}$
and S is a relation from C to D defined by $S = \{(x, Y), (y, Z)\}$. Find R^o , $R \cup S$, $R \cap S$ and $R - S$.

Sol: (i) To find R^o

The complement of R consists of all pairs of the cardinal product $A \times B$ that are not in R .

We have, $A = \{x, y, z\}$, $B = \{X, Y, Z\}$

$\therefore A \times B = \{(x, X), (x, Y), (x, Z), (y, X), (y, Y), (y, Z), (z, X), (z, Y), (z, Z)\}$

Given $R = \{(x, X), (x, Y), (y, Z)\}$

$R^o = \{(x, Z), (y, X), (y, Y), (z, X), (z, Y), (z, Z)\}$

Sol: (ii) To find $R \cup S$,

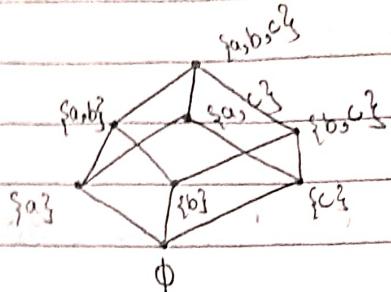
$R \cup S = \{(x, X), (x, Y), (y, Z)\} = R$

(iii) $R \cap S = \{(x, Y), (y, Z)\} = S$.

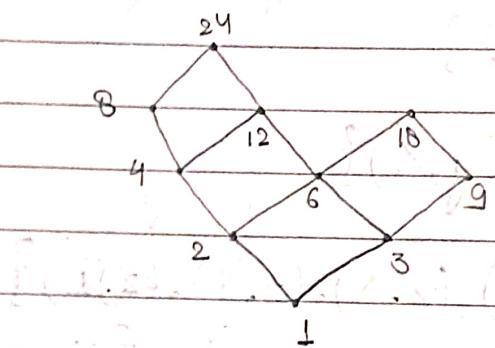
(iv) $R - S = \{(x, X)\}$

Partial Order Relation

A relation R on a set S is called a partial order if it is reflexive, anti-symmetric and transitive.



- Q. Let $A = \{1, 2, 3, 4, 6, 8, 9, 12, 18, 24\}$ be ordered by the relation ' x divides y '. Draw the Hasse diagram.



- Q. Let D_m denote the positive divisors of m ordered by divisibility. Draw the Hasse diagram of

- (i) D_{30} (ii) D_{12} (iii) D_{15} (iv) D_{16} (v) D_{42}

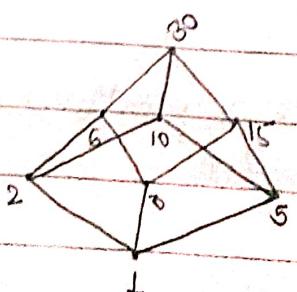
$$D_{30} = \{1, 2, 3, 5, 6, 10, 15, 30\}$$

No. of the divisors of 30 = $T(30)$

$$= T(2^2 \times 3^1 \times 5^1)$$

$$= (1+1)(1+1)(1+1) \quad [+1 \text{ in powers}]$$

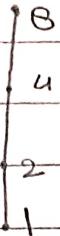
$$= 2 \cdot 2 \cdot 2 = 8.$$



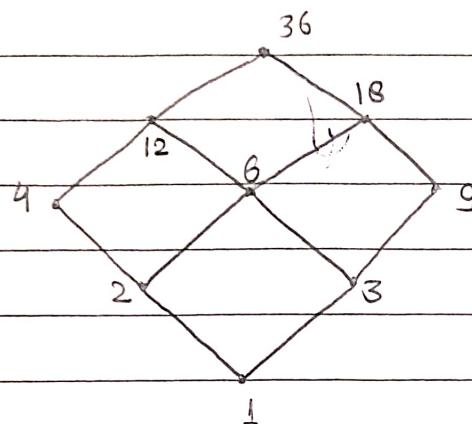
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$$\underline{D_8} = \{1, 2, 4, 8\}$$



$$\text{④ } D_{36} = \{1, 2, 3, 4, 6, 9, 12, 18, 36\}$$



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1	1	2	3	4	6	9	12	18	36
1	1	1	1	1	1	1	1	1	1
2	1	2	1	2	2	1	2	2	2
3	1	1	3	1	3	3	3	3	3
4	1	2	1	4	2	1	4	2	4
6	1	2	3	2	6	3	6	6	6
9	1	1	3	1	3	9	3	9	9
12	1	2	3	4	6	3	12	6	12
18	1	2	3	2	6	9	6	18	18
36	1	2	3	4	6	9	12	18	36

$\downarrow n(M) = 32$

$n(P) = 20$

$n(B) = 45$

$n(M \cap B) = 15$

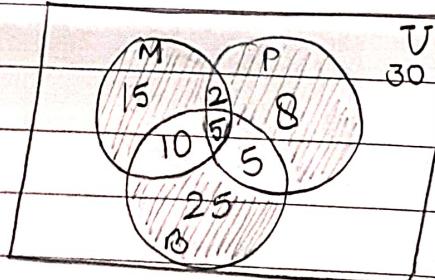
$n(M \cap P) = 7$

$n(P \cap B) = 10$

$n(M' \cap P' \cap B') = 30 = n(M \cup P \cup B)$

$n(U) = 100$

(Q)



$$\begin{aligned}
 \text{(ii)} \quad n(M \cup P \cup B) &= n(U) - n(M \cap P \cap B)' \\
 &= n(U) - n(M' \cap P' \cap B') \\
 &= 100 - 30 = 70
 \end{aligned}$$

$$\begin{aligned}
 n(M) + n(P) + n(B) - n(M \cap P) - n(M \cap B) - n(P \cap B) + n(M \cap P \cap B) &= 70 \\
 32 + 20 + 45 - 7 - 15 - 10 + n(M \cap P \cap B) &= 70 \\
 97 - 32 + n(M \cap P \cap B) &= 70 \\
 n(M \cap P \cap B) &= 5.
 \end{aligned}$$

No. of students studying exactly one of those subjects = $15 + 8 + 25 = 48$

$$\text{Q: } n(U) = 500$$

A : divisible by 2

B : divisible by 3

C : divisible by 5.

$$n(A) = 250$$

$$n(B) = 166$$

$$n(C) = 100$$

$$n(A \cap B) = 83$$

$$n(B \cap C) = 33$$

$$n(A \cap C) = 50$$

$$n(A \cap B \cap C) = 16$$

$$\begin{aligned} n(A \cup B \cup C) &= n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C) \\ &= 250 + 166 + 100 - 83 - 33 - 50 + 16 \\ &= 366 \end{aligned}$$

i) Integers neither divisible by 2, nor by 3 and nor by 5 = $n(A' \cap B' \cap C')$

$$= n(A \cup B \cup C)$$

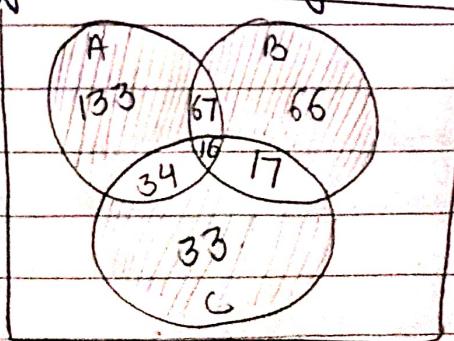
$$= n(U) - n(A \cup B \cup C)$$

$$= 500 - 366$$

$$= 134 \text{ Ans}$$

ii) Integers exactly divisible by one of them = $133 + 66 + 33$

$$= 232$$



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5. Reflexive:

$$\because \forall a \in A$$

$$\therefore (a,a) \in R$$

$\Rightarrow R$ is reflexive.

Anti-symmetric:

$$\text{If } aRb, bRa, \forall a, b \in A$$

$$\therefore a = b$$

$\Rightarrow R$ is anti-symmetric

Transitive:

$$\because (a,b) \in R \text{ and } (b,d) \in R$$

$\Rightarrow (a,d) \in R$, which is true.

Hence R is transitive.

Hence R is partial ordering.

6. Reflexive:-

For every $a \in A$, $(a,a) \in R$

$\Rightarrow R$ is reflexive.

Anti-symmetric:-

For every $a, b \in A$,

if aRb and bRa

then $a = b$.

Hence R is anti-symmetric.

Transitive:

$$\because (1,3) \in R \text{ and } (3,2) \in R$$

$\Rightarrow (1,2) \in R$, which is true

Hence R is transitive.

Therefore R is partial ordering.

Saathi

Date / /

10. $R = \{(1, 2), (1, 3), (1, 4), (1, 6), (1, 8), (1, 9), (1, 12), (1, 18), (1, 24) \dots\}$

