# Unit - 3 Theory of Logic

- Propositional
- Truth tables
- Tautology
- Satisfiability
- Contradiction
- Algebra of propositions
- Theory of Inference
- Predicate logic
- First order predicate
- Well ordered formula of predicate
- Quantifier
- Inference theory of predicate logic

## Logic

**Proposition calculus**
(which deals with statements with values true or false and is concerned with analysis of proposition).

**Predicate calculus**
(which deals with the predicates which are propositions containing variables.)

## Proposition

A proposition or statement is a declarative sentence that is either true or false. e.g.

$3+3=6$ and $3+3=7$ are both statements.

Similarly, $x+y>1$ is not a statement because for some values of $x$ and $y$ the sentence is true whereas for some it is false.

The truth ~~an~~ or falsity of a statement is called its truth value.

It is customary to represent simple statements by letter p, q, r..... known as propositional variables.

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There are two propositional constants T and F that represent true and false respectively.

## Compound Propositional

A proposition obtained from the combination of two or more proposition by means of logical operators or connectives is referred a compound proposition.

| Symbols used | Nature of the compound statement formed by the connective | Connective word | Symbolic form | Negation |
|---|---|---|---|---|
| $\sim, \neg, ⌉$ | Negation | not | $\sim p$ | $\sim(\sim p)$ |
| $\wedge$ | Conjunction | and | $p \wedge q$ | $\sim(p \wedge q) = \sim p \vee \sim q$ |
| $\vee$ | disjunction | or | $p \vee q$ | $\sim(p \vee q) = \sim p \wedge \sim q$ |
| $\Rightarrow, \rightarrow$ | Implication ~~If ... then~~ (conditional) | If ... then | $p \Rightarrow q$ | $\sim(p \Rightarrow q) = p \wedge (\sim q)$ |
| $\Leftrightarrow, \leftrightarrow$ | Equivalence (or biconditional) | If and only If | $p \Leftrightarrow q$ | $\sim(p \Leftrightarrow q)$ $(p \wedge \sim q) \vee (\sim p \wedge q)$ |

- **Truth Table**

A truth table is a table that shows a truth value of a compound proposition for all possible cases.

- **Negation**

If $p$ is any proposition, the negation of $p$ denoted by $\sim p$ and read as 'not $p$' is a proposition which is false when $p$ is true and true when $p$ is false.

| $p$ | $\sim p$ |
|---|---|
| T | F |
| F | T |

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Consider the statement:

(i) $p$: Paris is in France.

$\sim p$: Paris is not in France.

The negation of $p$ is the statement

$\sim p$: It is not the case that Paris is in France.

<u>Note</u>: The following propositions have the same meaning.

1. $p$: All people are intelligent.

   $q$: Every person is intelligent.

   $r$: Each person is intelligent.

   $s$: Any person is intelligent.

2. The negation of the proposition.

   $p$: All students are intelligent

   $\sim p$: Some students are not intelligent.

   $\sim p$: There exist a student who is not intelligent

   $\sim p$: There exists students who are not intelligent

   $\sim p$: Atleast one student is not intelligent.

3. The negation of

   $q$: No student is intelligent.

   $\sim q$: Some students are intelligent.

→ ~~$q$~~ note that 'no student is intelligent' is not the negation of $p$.

→ "All students are intelligent" is not the negation of $q$.

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# Conjunction

If p and q are two statements, then conjunction of p and q is the compound statement, denoted by $p \wedge q$ [$p \wedge q$] and read as p and q.

| $p$ | $q$ | $p \wedge q$ |
|---|---|---|
| T | T | T |
| T | F | F |
| F | T | F |
| F | F | F |

# Disjunction $p \vee q$

| $p$ | $q$ | $p \vee q$ |
|---|---|---|
| T | T | T |
| T | F | T |
| F | T | T |
| F | F | F |

# Conditional propositional.

If p and q are propositions. the compound proposition "if p then q" denoted by $p \Rightarrow q$ is called a conditional proposition or implication.

The proposition p is called antecedent or hypothesis and proposition q is called consequent or conclusion.
The connective 'if.....then' can also be read as.

- p implies q
- p is sufficient for q.
- q is necessary for p.
- p only if q.

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| $p$ | $q$ | $p \Rightarrow q$ |
|---|---|---|
| T | T | T |
| T | F | F |
| F | T | T |
| F | F | T |

Biconditional

| $p$ | $q$ | $p \Leftrightarrow q$ |
|---|---|---|
| T | T | T |
| T | F | F |
| F | T | F |
| F | F | T |

**Ques** Calculate truth table for

(i) $p \vee \sim q \Rightarrow p$

(ii) $(\sim(p \wedge q) \vee r) \Rightarrow \sim p$

(i)

| $p$ | $q$ | $\sim q$ | $p \vee \sim q$ | $(p \vee \sim q) \Rightarrow p$ |
|---|---|---|---|---|
| T | T | F | T | T |
| T | F | T | T | T |
| F | T | F | F | T |
| F | F | T | T | F |

(ii)

| $p$ | $q$ | $r$ | $\sim p$ | $p \wedge q$ | $\sim(p \wedge q)$ | $\sim(p \wedge q) \vee r$ | $(\sim(p \wedge q) \vee r) \Rightarrow \sim p$ |
|---|---|---|---|---|---|---|---|
| T | T | T | F | T | F | T | F |
| T | T | F | F | T | F | F | T |
| T | F | T | F | F | T | T | F |
| T | F | F | F | F | T | T | F |
| F | T | T | T | F | T | T | T |
| F | T | F | T | F | T | T | T |
| F | F | T | T | F | T | T | T |
| F | F | F | T | F | T | T | T |

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***Q. Use TT as a result in numerical problem to show that

$$p \Rightarrow q \equiv \sim p \vee q$$

| $p$ | $q$ | $p \Rightarrow q$ | $\sim p$ | $\sim p \vee q$ |
|---|---|---|---|---|
| T | T | T | F | T |
| T | F | F | F | F |
| F | T | T | T | T |
| F | F | T | T | T |

$\equiv$

The logical equivalence is established by the third and fifth column of the truth table which are identical.

***Q. Use as a result in numerical problem:
Show that $p \Leftrightarrow q \equiv (p \Rightarrow q) \wedge (q \Rightarrow p)$

| $p$ | $q$ | $p \Leftrightarrow q$ | $p \Rightarrow q$ | $q \Rightarrow p$ | $(p \Rightarrow q) \wedge (q \Rightarrow p)$ |
|---|---|---|---|---|---|
| T | T | T | T | T | T |
| T | F | F | F | T | F |
| F | T | F | T | F | F |
| F | F | T | T | T | T |

$\equiv$

The logical equivalence is established by the third and sixth column which are identical.

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# Algebra of propositions

- Idempotent laws: $p \vee p \equiv p$ , $p \wedge p \equiv p$
- Associative law: $(p \vee q) \vee r \equiv p \vee (q \vee r)$ , $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$
- Commutative law: $p \wedge q \equiv q \wedge p$ , $p \vee q \equiv q \vee p$
- Distributive law: $p \vee (q \wedge r) = (p \vee q) \wedge (p \vee r)$
  $p \wedge (q \vee r) = (p \wedge q) \vee (p \wedge r)$
- Identity laws: $p \vee F \equiv p$ , $p \wedge F \equiv F$
  $p \vee T \equiv T$ , $p \wedge T \equiv p$
- Complement law: $p \vee \sim p \equiv T$ , $p \wedge \sim p \equiv F$
- Involution law: $\sim(\sim p) \equiv p$
- De-morgan's law: $\sim(p \vee q) \equiv \sim p \wedge \sim q$ , $\sim(p \wedge q) \equiv \sim p \vee \sim q$.
- Absorption law: $p \vee (p \wedge q) \equiv p$ , $p \wedge (p \vee q) \equiv p$.

Ques Show that $(p \Leftrightarrow q) \equiv (p \vee q) \Rightarrow (p \wedge q)$ using
(i) truth table

| $p$ | $q$ | $p \Leftrightarrow q$ | $p \vee q$ | $p \wedge q$ | $(p \vee q) \Rightarrow (p \wedge q)$ |
|---|---|---|---|---|---|
| T | T | T | T | T | T |
| T | F | F | T | F | F |
| F | T | F | T | F | F |
| F | F | T | F | F | T |

$\equiv$

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(ii) algebra of propositions.

$$p \Leftrightarrow q \equiv (p \Rightarrow q) \wedge (q \Rightarrow p)$$
$$= (\sim p \vee q) \wedge (\sim q \vee p) \qquad [\because p \Rightarrow q \equiv \sim p \vee q]$$
$$= [(\sim p \vee q) \wedge \sim q] \vee [(\sim p \vee q) \wedge p] \qquad \text{By distributive law}$$
$$= [\sim q \wedge (\sim p \vee q)] \vee [p \wedge (\sim p \vee q)] \qquad \text{By commutative law}$$
$$= [(\sim q \wedge \sim p) \vee (\sim q \wedge q)] \vee [(p \wedge \sim p) \vee (p \wedge q)]$$
$$= [(\sim q \wedge \sim p) \vee F] \vee [F \vee (p \wedge q)]$$
$$= (\sim q \wedge \sim p) \vee (p \wedge q)$$
$$= \sim (p \vee q) \vee (p \wedge q)$$
$$= (p \vee q) \Rightarrow (p \wedge q)$$

Que Prove the following equivalencies by laws of propositions

(a) $(p \Rightarrow q) \Rightarrow q \equiv p \vee q$

(b) $p \rightarrow (q \vee r) \equiv (p \rightarrow q) \vee (p \rightarrow r)$

(c) $(\sim p \vee q) \wedge (p \wedge (p \wedge q)) \equiv p \wedge q$

(a) L.H.S. $= (p \Rightarrow q) \Rightarrow q$
$$= (\sim p \vee q) \Rightarrow q \qquad [\because p \Rightarrow q \equiv \sim p \vee q]$$
$$= [\sim (\sim p \vee q) \vee q]$$
$$= [(p \wedge \sim q) \vee q] \qquad [\because \sim (p \vee q) = \sim p \wedge \sim q]$$
$$= [q \vee (p \wedge \sim q)] \qquad \text{By commutative law}$$
$$= [(q \vee p) \wedge (q \vee \sim q)] \qquad \text{By distributive law}$$
$$= (q \vee p) \wedge T$$
$$= q \vee p$$
$$= p \vee q \qquad \text{By commutative law.}$$

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(b) L.H.S. $= p \rightarrow (q \vee r)$

$= \sim p \vee (q \vee r)$

$= (\sim p \vee q) \vee (\sim p \vee r)$

$= (p \rightarrow q) \vee (p \rightarrow r)$

(c) L.H.S. $= (\sim p \vee q) \wedge (p \wedge (p \wedge q))$

$= (\sim p \vee q) \wedge ((p \wedge p) \wedge q)$ [Associative law]

$= (\sim p \vee q) \wedge \underline{(p \wedge q)}$

$\cancel{= ((\sim p \vee q) \wedge p)}$

$= (\sim p \wedge (p \wedge q)) \vee (q \wedge (p \wedge q))$ [Distributive law]

$= [(\sim p \wedge p) \wedge q] \vee ((q \wedge q) \wedge p)$ [Associative law]

$= [F \wedge q] \vee (q \wedge p)$ [Complement law]

$= F \vee (p \wedge q)$ [Identity law]

$= p \wedge q$

$=$ R.H.S.

Q. Prove that $\sim (p \wedge q) \Rightarrow (\sim p \vee (\sim p \vee q)) \equiv \sim p \vee q$

L.H.S. $= \sim (p \wedge q) \Rightarrow (\sim p \vee (\sim p \vee q))$

$= \sim (p \wedge q) \Rightarrow [(\sim p \vee \sim p) \vee q]$ Associative law

$= \sim (p \wedge q) \Rightarrow (\sim p \vee q)$

$= \sim [\sim (p \wedge q)] \vee (\sim p \vee q)$ [$\because p \Rightarrow q = \sim p \vee q$]

$= (p \wedge q) \vee \underline{(\sim p \vee q)}$

$= (\sim p \vee q) \vee (p \wedge q)$ Commutative law

$= [(\sim p \vee q) \vee p] \wedge [(\sim p \vee q) \vee q]$

$= [(q \vee \sim p) \vee p] \wedge [\sim p \vee (q \vee q)]$

$= [q \vee (\sim p \vee p)] \wedge [\sim p \vee q]$ Associative

$= [q \vee T] \wedge [\sim p \vee q]$

$= T \wedge (\sim p \vee q)$

$= \sim p \vee q$

$=$ R.H.S.

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## Tautology

A compound proposition that is always true for all possible truth values of the variables or in other words, contains only (T) in the last column of its truth table is called a tautology.

## Contradiction

A compound statement that is always false for all possible values of its variables or in other words contain only F in the last column in its truth table is called contradiction.

## Contingency

A proposition ie neither tautology nor a contradiction is called a contingency.

Q. Prove that the following propositions are tautology.

(i) $p \vee \sim p$

| $p$ | $\sim p$ | $p \vee \sim p$ |
|---|---|---|
| T | F | T |
| F | T | T |

Hence given proposition is tautology.

(ii) $\sim(p \wedge q) \vee q$

| $p$ | $q$ | $p \wedge q$ | $\sim(p \wedge q)$ | $\sim(p \wedge q) \vee q$ |
|---|---|---|---|---|
| T | T | T | F | T |
| T | F | F | T | T |
| F | T | F | T | T |
| F | F | F | T | T |

Since the truth table is ~~true~~ TRUE for all possible values of the propositional variables which can be seen in the last column of the truth table. Hence, the given proposition is tautology.

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(iii) $p \Rightarrow (p \vee q)$

| $p$ | $q$ | $p \vee q$ | $p \Rightarrow (p \vee q)$ |
|---|---|---|---|
| T | T | T | T |
| T | F | T | T |
| F | T | T | T |
| F | F | F | T |

Since the truth table is true for all possible values of propositional variables which can be seen in the last column of truth table. Hence, given proposition is tautology.

**Q** Verify that the proposition $p \wedge (q \wedge \sim p)$ is a contradiction.

| $p$ | $\sim p$ | $q$ | $q \wedge \sim p$ | $p \wedge (q \wedge \sim p)$ |
|---|---|---|---|---|
| T | F | T | F | F |
| T | F | F | F | F |
| F | T | T | T | F |
| F | T | F | F | F |

Since the truth table is FALSE for all possible values of propositional variables which can be seen in the last column of truth table. Hence, given proposition is contradiction.

**Q** Show that $(p \vee q) \wedge (\sim p \wedge \sim q)$ is a contradiction.

| $p$ | $q$ | $\sim p$ | $\sim q$ | $p \vee q$ | $\sim p \wedge \sim q$ | $(p \vee q) \wedge (\sim p \wedge \sim q)$ |
|---|---|---|---|---|---|---|
| T | T | F | F | T | F | F |
| T | F | F | T | T | F | F |
| F | T | T | F | T | F | F |
| F | F | T | T | F | T | F |

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$$(p \vee q) \wedge (\sim p \wedge \sim q) = [p \wedge (\sim p \wedge \sim q)] \vee [q \wedge (\sim p \wedge \sim q)]$$ Distributive

$$= [(p \wedge \sim p) \wedge \sim q] \vee [(q \wedge \sim q) \wedge \sim p]$$ Associative

$$= (F \wedge \sim q) \vee (F \wedge \sim p)$$ Complement

$$= F \vee F$$ Identity.

$$= F$$

which is a contradiction.

Q. Show that $[(p \vee q) \wedge \sim (\sim p \wedge (\sim q \vee \sim r))] \vee [(\sim p \wedge \sim q) \vee (\sim p \wedge \sim r)]$ is a tautology by using law of logic.

Firstly we have, $[(p \vee q) \wedge \sim(\sim p \wedge (\sim q \vee \sim r))]$

$$= [(p \vee q) \wedge (p \vee (q \wedge r))]$$
$$= [(p \vee q) \wedge ((p \vee q) \wedge (p \vee r))]$$
$$= [(p \vee q) \wedge (p \vee q)] \wedge (p \vee r)$$
$$= (p \vee q) \wedge (p \vee r)$$

Secondly we have, $[(\sim p \wedge \sim q) \vee (\sim p \wedge \sim r)]$

$$= [\sim (p \vee q) \vee \sim (p \vee r)]$$
$$= \sim [(p \vee q) \wedge (p \vee r)]$$

Now, $[(p \vee q) \wedge (p \vee r)] \vee \sim [(p \vee q) \wedge (p \vee r)]$ = Tautology (T)

Valid arguments: An argument is a sequence of statements. All statements except the final one are called premises (hypothesis) or assumption.

The final statement is called conclusion.

The symbol $\therefore$ read therefore is normally placed just before the conclusion.

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To test an argument follows for validity.

Step 1: Identify the premises and the conclusion of the argument.

Step 2: Construct a truth table showing the truth value of all premises and the conclusion.

Step 3: Find the rows called critical rows in which all the premises are true.

~~Step 4: In each critical row, the conclusion is also true then the argument is valid.~~

Step 4: In each critical row, determine whether the conclusion of the argument is also true.

(a) If in each critical row, the conclusion is also true then the argument is valid.

(b) In which conclusion is false. If the statement p is assumed as true and also the statement $p \Rightarrow q$ is accepted as true then it is true. It is written in the form

$$\left.\begin{array}{l} p \Rightarrow q \\ \underline{\quad p \quad} \end{array}\right\} \text{Premises}$$
$$\therefore q \rightarrow \text{conclusion.}$$

This rule is known as 'Modus Ponens or rule of detachment'.

| p | q | $p \Rightarrow q$ | $(p \Rightarrow q) \wedge p$ | $[(p \Rightarrow q) \wedge p] \Rightarrow q$ |
|---|---|---|---|---|
| T | T | T | T | T |
| T | F | F | F | T |
| F | T | T | F | T |
| F | F | T | F | T |

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An argument is said to be valid if and only if the conjunction of all the premises implies the conclusion. That means if the premises are all true, then the conclusion is also true. However if one or more of the premises is called, so that the conjunction of all the premises is false. Then the conclusion may be either true or false.

From table we see that there is only one case in which both premises are true namely the first case and in this case conclusion is also true, hence the argument is valid.

The above argument is valid is that.

$$[(p \Rightarrow q) \wedge p] \Rightarrow q \text{ is a tautology.}$$

as

Q. Represent the argument : If I study hard then I get A.S. I study hard. I get A.S.

let $p$ : I study hard

$q$ : I get A.S.

The argument may be written symbolically as

$$\begin{array}{l} p \Rightarrow q \\ \underline{p} \\ \therefore q \end{array}$$

by Modus Ponens.

Hence by modus ponens law, argument is valid.

Method 2:

| $p$ | $q$ | $p \Rightarrow q$ | $(p \Rightarrow q) \wedge p$ | $[(p \Rightarrow q) \wedge p] \Rightarrow q$ |
|---|---|---|---|---|
| T | T | T | T | T |
| T | F | F | F | T |
| F | T | T | F | T |
| F | F | T | F | T |

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| S.No. | Rule of Inference | Tautological form | Name |
|---|---|---|---|
| 1. | $\dfrac{p}{\therefore p \vee q}$ or $\dfrac{q}{p \vee q}$ | $p \Rightarrow (p \vee q)$ | Addition. |
| 2. | $\dfrac{p \wedge q}{\therefore p}$ | $(p \wedge q) \Rightarrow p$ | Simplification |
| 3. | $\begin{array}{c} p \\ q \\ \hline \therefore p \wedge q \end{array}$ | $[(p) \wedge (q)] \Rightarrow p \wedge q$ | Conjunction |
| 4. | $\begin{array}{c} p \Rightarrow q \\ p \\ \hline \therefore q \end{array}$ | $[(p \Rightarrow q) \wedge p] \Rightarrow q$ | Modus Ponens |
| 5. | $\begin{array}{c} p \Rightarrow q \\ \sim q \\ \hline \therefore \sim p \end{array}$ | $[(p \Rightarrow q) \wedge \sim q] \Rightarrow \sim p$ | Modus Tollens |
| 6. | $\begin{array}{c} p \Rightarrow q \\ q \Rightarrow r \\ \hline \therefore p \Rightarrow r \end{array}$ | $[(p \Rightarrow q)(q \Rightarrow r)] \Rightarrow (p \Rightarrow r)$ | Hypothetical syllogism. |
| 7. | $\begin{array}{c} p \vee q \\ \sim p \\ \hline \therefore q \end{array}$ | $[(p \vee q) \wedge (\sim p)] \Rightarrow q$ | Disjunction syllogism. |
| 8. | $\begin{array}{c} (p \Rightarrow q) \wedge (r \Rightarrow s) \\ p \vee r \\ \hline \therefore q \vee s \end{array}$ | $[(p \Rightarrow q) \wedge (r \Rightarrow s)] \wedge (p \vee r) \Rightarrow (q \vee s)$ | Conjunctive Dilemma. |
| 9. | $\begin{array}{c} (p \Rightarrow q) \wedge (r \Rightarrow s) \\ \sim q \vee \sim s \\ \hline \therefore \sim p \vee \sim r \end{array}$ | $[(p \Rightarrow q) \wedge (r \Rightarrow s)] \wedge (\sim q \vee \sim s) \Rightarrow (\sim p \vee \sim r)$ | Disjunction Dilemma. |

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Prove the validity of the following argument

'If I get the job and work hard, then I will get promoted. If I get promoted then I will be happy. I will not be happy therefore, either I will not get ~~if I get if~~ the job or I will not work hard.'

let $p$ : I get the job
$q$ : I work hard
$r$ : I get promoted
$s$ : I will be happy.

Then the above argument can be written in symbolic form as

$$\left.\begin{array}{c}(p \wedge q) \Rightarrow r \\ r \Rightarrow s \\ \sim s\end{array}\right\} \text{Premises given}$$

$\therefore \sim p \vee \sim q$ (conclusion)

1. $(p \wedge q) \Rightarrow r$
2. $r \Rightarrow s$
3. $\sim s$
4. $(p \wedge q) \Rightarrow s$ (Hypothetical syllogism using 1 and 2)
5. $\sim s$
6. $\sim(p \wedge q)$ (Modus tollens using 4 and 5)
7. $\sim p \vee \sim q$ (By deMorgan's law)

Hence above argument is valid.

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**Q.** Show that $t$ is a valid conclusion from the premises $p \Rightarrow q$, $q \Rightarrow t$, $t \Rightarrow s$, $\sim s$ and $p \vee t$.

**Sol** We have,

1. $p \Rightarrow q$
2. $q \Rightarrow t$
3. $t \Rightarrow s$ — Premises (Given)
4. $\sim s$
5. $p \vee t$
6. $p \Rightarrow t$ (Hypothetical syllogism using 1 and 2)
7. $p \Rightarrow s$ ( " " " 6 and 3)
8. $\sim p$ (Modus Tollens using 7 and 4)
9. $p \vee t$
10. $t$ (Disjunction syllogism using 8 and 9)

Prove that $s$ is a valid conclusion from the premises $p \Rightarrow q$, $p \Rightarrow r$, $\sim(q \wedge r)$ and $s \vee p$

We have,

1. $p \Rightarrow q$
2. $p \Rightarrow r$
3. $\sim(q \wedge r) \equiv \sim q \vee \sim r$
4. $s \vee p$
5. $(p \Rightarrow q) \wedge (p \Rightarrow r)$ [conjunction using 1 and 2]
6. $\sim q \vee \sim r$
7. $\sim p \vee \sim p$ [Disjunctive dilemma using 5 and 6]
8. $\sim p$
9. $s \vee p$
10. $s$ [Disjunction syllogism using 8 and 9]

Hence given argument is valid.

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** Determine whether the conclusion C is valid in the following premises.

$H_1$ : $P \Rightarrow (Q \Rightarrow R)$
$H_2$ : $P \wedge Q$
C : R.

Method 1: By truth table.

The given argument will be valid if the implication $[P \Rightarrow (Q \Rightarrow R)] \wedge [P \wedge Q] \Rightarrow R$. is a tautology.

| P | Q | R | $Q \Rightarrow R$ | $P \Rightarrow (Q \Rightarrow R)$ | $P \wedge Q$ | $P \Rightarrow (Q \Rightarrow R) \wedge (P \wedge Q)$ | $P \Rightarrow (Q \Rightarrow R) \wedge (P \wedge Q) \Rightarrow R$ |
|---|---|---|---|---|---|---|---|
| T | T | T | T | T | T | T | T |
| T | T | F | F | F | T | F | T |
| T | F | T | T | T | F | F | T |
| T | F | F | T | T | F | F | T |
| F | T | T | T | T | F | F | T |
| F | T | F | F | T | F | F | T |
| F | F | T | T | T | F | F | T |
| F | F | F | T | T | F | F | T |

Method 2:

1. $P \Rightarrow (Q \Rightarrow R)$
2. $P \wedge Q$
3. P (Simplification using 2)
4. $Q \Rightarrow R$ (Modus Ponens using 1 and 3)
5. Q (Simplification using 2)
6. R (Modus Ponens using 4 and 5)

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Q. Show that $R \wedge (P \vee Q)$ is a valid conclusion from the premises $P \vee Q$, $Q \Rightarrow R$, $P \Rightarrow M$ and $\sim M$.

1. $P \vee Q$
2. $Q \Rightarrow R$
3. $P \Rightarrow M$
4. $\sim M$

$\}$ Premises (Given)

5. $\sim P$ [Modus tollens using 3 and 4]
6. $Q$ [Disjunctive syllogism using 1 and 5]
7. $R$ [Modus Ponens using 2 and 6]
8. $R \wedge (P \vee Q)$ [Conjunction using 1 and 7]

Q. Using rule of inference, determine whether the following inference patterns are valid or not.

$\sim t \Rightarrow \sim r$

$\sim s$

$t \Rightarrow w$

$r \vee s$

$\therefore w$

1. $\sim t \Rightarrow \sim r$
2. $\sim s$
3. $t \Rightarrow w$
4. $r \vee s$
5. $r$ [Disjunction syllogism using 2 and 4]
6. $\sim(\sim t) = t$ [Modus Tollens using 1 and 5]
7. $w$ [Modus ponens using 3 and 6]

Hence argument is valid.

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# Fallacies

A fallacy is an error in reasoning that results in an invalid argument.

We discuss two types of fallacies :-

1. The fallacy of affirming the consequent (or affirming the converse)
If $p \Rightarrow q$
then $q \Rightarrow p$ $\Big\}$ X

2. The fallacy of denying the hypothesis (or assuming the inverse)
If $p \Rightarrow q$
then $\sim p \Rightarrow \sim q$ $\Big\}$ X

① $p \Rightarrow q$
$\underline{\quad q \quad}$ X
$\therefore p$

② $p \Rightarrow q$
$\underline{\quad \sim p \quad}$ X
$\therefore \sim q$

Imp. Ques. Show that the following argument is invalid :-
If Siddhartha solved this problem, then he obtained the answer 5.
Siddhartha obtained the answer 5.

Therefore, Siddhartha solved this problem correctly.

Sol. Let $p$: Siddhartha solved this problem.
$q$: Siddhartha obtained the answer 5.
$\therefore$ Symbolically the above argument may be written as

$p \Rightarrow q$
$\underline{\quad q \quad}$
$\therefore p$.

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This argument is faulty because the conclusion can be false even though $p \Rightarrow q$ and $q$ are true i.e. the implication b/w ~~p and q and~~ q $[(p \Rightarrow q) \wedge q] \Rightarrow p$ is not a tautology.

| $p$ | $q$ | $p \Rightarrow q$ | $(p \Rightarrow q) \wedge q$ | $[(p \Rightarrow q) \wedge q] \Rightarrow p$ |
|---|---|---|---|---|
| T | T | T | T | T |
| T | F | F | F | T |
| F | T | T | T | F |
| F | F | T | F | T |

Hence given argument is invalid.

Q. Test the validity of the following argument :-

If two sides of a triangle are equal, then the opposite angles are equal.
Two sides of a triangle are not equal.
Therefore, the opposite angles are not equal.

Let $p$ and $q$ be the prepositions as :
$p$ : Two sides of a triangle are equal
$q$ : The opposite angles are equal.
∴ Symbolically it can represented as

$$\begin{array}{l} p \Rightarrow q \\ \underline{\sim p} \\ \therefore \sim q \end{array}$$

The fallacy underline this invalid argument is called the fallacy of denying the hypothesis assuming the inverse.

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The preposition $[(p \Rightarrow q) \wedge \sim p] \Rightarrow \sim q$ is not a tautology.

| $p$ | $q$ | $p \Rightarrow q$ | $\sim p$ | $(p \Rightarrow q) \wedge \sim p$ | $[(p \Rightarrow q) \wedge \sim p] \Rightarrow \sim q$ |
|---|---|---|---|---|---|
| T | T | T | F | F | T |
| T | F | F | F | F | T |
| F | T | T | T | T | F |
| F | F | T | T | T | T |

Que. Prove the validity of the following argument using truth table as well as inference rule.

"If the market is free then there is no inflation. If there is no inflation then there are price controls. Since there are price controls, therefore the market is free."

Let $p$: The market is free.
$q$: There is no inflation
$r$: There are price controls.

Symbolically it can be represented as:-

$$\begin{array}{c} (p \Rightarrow q) \wedge (q \Rightarrow r) \\ r \\ \hline \therefore p \end{array}$$

1. $p \Rightarrow q$
2. $q \Rightarrow r$
3. $r$
4. $p \Rightarrow r$ [Hypothetical syllogism using ① and ②]

This argument is faulty because the conclusion ~~can~~ is faulty. The preposition $[(p \Rightarrow q) \wedge \sim p] \Rightarrow \sim q$ is not a tautology.

| $p$ | $q$ | $r$ | $p \Rightarrow q$ | $q \Rightarrow r$ | $\overbrace{(p \Rightarrow q) \wedge (q \Rightarrow r)}^{P_1}$ | $P_1 \wedge r$ | $(P_1 \wedge r) \Rightarrow$ |
|---|---|---|---|---|---|---|---|
| T | T | T | T | T | T | T | T |
| T | T | F | T | F | F | F | T |
| T | F | T | F | T | F | F | T |
| T | F | F | F | T | F | F | T |
| F | T | T | T | T | T | T | F |
| F | T | F | T | F | F | F | F |
| F | F | T | T | T | T | T | F |
| F | F | F | T | T | T | F | T |

**Q. Predicate logic or First order logic**

First order logic generalises propositional logic by involving quantifiers (for all, there exists). These quantifiers can only range over individual elements in your domain of discourse.

e.g. "for all $x$, $P(x)$" or "there exists $x$ $Q(x)$"

P and Q are just propositions from propositional logic.

imp **Quantifiers.**

Quantifiers are words that refer to quantities such as 'some' or 'all' and indicate how frequently a certain statement is true.

**Universal quantifier.**

The phrase "for all" (denoted by $\forall$) is called universal quantifier.

e.g. Consider the sentence "All human beings are mortal".

Let $P(x)$ denote "$x$ is mortal".

Then the above sentence can be written as $(\forall x \in S) P(x)$ or $\forall x P(x)$

where S denotes the set of all human beings.

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## Existential quantifier:

The phrase 'there exists' (denoted by $\exists$) is called existential quantifier.

e.g. Consider the sentence: "There exists $x$ such that $x^2 = 5$"

This sentence can be written as $(\exists x \in R)\, P(x)$ or $\exists x P(x)$

where $P(x) =$ "$x^2 = 5$"

## Translating sentences into logical expressions.

- Every person is precious.

We translate this

For every $x$, if $x$ is a person then $x$ is precious.

$\forall x,\ P(x) \Rightarrow Q(x)$

where, $P(x)$:- $x$ is person

$Q(x)$:- $x$ is precious.

- Consider the sentence: some student of this college passed MCA entrance exam.

We translate this,

There exist a student of this college who have passed MCA entrance exam.

let P be the property: passed MCA entrance exam.

$\therefore$ given sentence can be written as $\exists x\, P(x)$

**Que:** let $K(x)$: $x$ is man ; $L(x)$: $x$ is mortal ; $M(x)$: $x$ is an integer ; $N(x)$: either positive or negative.

express the following using quantifier.

(a) All man are mortal $\forall x$, if $x$ is man then $x$ is mortal.

$\forall x,\ K(x) \Rightarrow L(x)$

(b) Any integer is either positive or negative.

$\forall x$, if $x$ is an integer then it is either positive or negative.

$\forall x,\ M(x) \Rightarrow N(x)$

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Que Let $K(x)$: $x$ is student; $M(x)$: $x$ is clever; $N(x)$: $x$ is successful
Express the following using quantifier.

(a) There exists a student $\exists x K(x)$

(b) Some students are clever. There exist $x$ such that $x$ is a student and $x$ is clever. $\exists x (K(x) \wedge M(x))$

(c) Some students are not successful. There exist $x$ such that $x$ is a student and $x$ is not successful.
$\exists x (K(x) \wedge \sim N(x))$.

Negation of quantified statement.

i) $\sim \forall x P(x) \equiv \exists x \sim P(x)$
ii) $\sim \exists x Q(x) \equiv \forall x \sim Q(x)$

Que Negate the statement: For all real numbers $x$, if $x > 3$ then $x^2 > 9$.
Let $P(x)$ and $Q(x)$ denote $x > 3$ and $x^2 > 9$ respectively
then the given sentence can be written as:-
$(\forall x \in R)(P(x) \Rightarrow Q(x))$.
which is equivalent to $\forall x, (\sim P(x) \vee Q(x))$

The negation of statement is $\exists x, (P(x) \wedge \sim Q(x))$.

There exists a real number $x$ such that $x > 3$ and $x^2 \leq 9$.

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## Universal Modus Ponens.

The rule of universal 'modus ponens' can be combined with modus ponens.

$$\begin{array}{l} p \Rightarrow q \\ \underline{p} \\ \therefore q \end{array}$$

$\forall x$, if $P(x)$ then $Q(x)$.
$\underline{\quad P(a) \text{ for a particular } a.}$
$\therefore \quad Q(a)$.

## Universal Modus Tollens.

$$\begin{array}{l} p \Rightarrow q \\ \underline{\sim q} \\ \therefore \sim p. \end{array}$$

$\forall x$, if $P(x)$ then $Q(x)$
$\underline{\quad \sim Q(a) \text{ for a particular } a}$
$\therefore \quad \sim P(a)$

Rewrite the following argument using quantifiers, variables and predicate symbol. Prove the validity of argument:-

(i) If a number is odd, then its square is odd.
$\underline{\text{K is a particular number that is odd}}$
$\therefore$ $K^2$ is odd.

Let $P(x)$: $x$ is odd.
$Q(x)$: $x^2$ is odd.
and K stands for a particular number that is odd then the given argument has following form:-

$\forall x$, if $P(x)$ then $Q(x)$
$\underline{\quad P(K) \text{ for a particular } K}$
$\therefore \quad Q(K)$ [By Universal modus ponens]
the given argument is valid.

(ii) All healthy people eat an apple a day.
Ram does not eat apple a day.
Ram is not a healthy person.

Let $P(x)$: $x$ is a healthy person.
$Q(x)$: $x$ eat an apple a day.
and $K$ stands for Ram.
∵ Given argument can be written as:-
$\forall x$, if $P(x)$ then $Q(x)$
$\sim Q(K)$ for a particular $K$
∴ $\sim P(K)$ [By Modus Tollens given argument is valid]

Ques. Analyse the argument's validity:
Premises:
If a person is happy, they smile.
John is smiling.
Conclusion: John is happy.
Let $P(x)$: $x$ is a happy person.
$Q(x)$: $x$ smiles.
and $K$ stands for John.
∵ Given argument can be written as:-
$\forall x$, if $P(x)$ then $Q(x)$
$Q(K)$ for a particular $K$
∴ $Q(K)$ (X) ~~[By Modus Ponens given argument is valid]~~
Hence given argument is invalid.

Q.2. Justify that the following premises are inconsistent:-
(i) If Alex misses many classes through illness then he fails high school
(ii) If Alex fails high school, then he is uneducated.
(iii) If Alex reads a lot of books then he is not uneducated.
(iv) Alex misses many classes through illness and reads a lots of books

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Q.3. Show the validity of the following argument:

Hypothesis:- 'It is not sunny this afternoon and it is colder than yesterday.'
We will go swimming only if it is sunny. If we do not go swimming, then we will take a canoe trip. If we take a canoe trip, then we will be home by sunsets.

Conclusion: "We will be home by sunset."

~~Let P: It is sunny this afternoon.~~
~~Q: It is colder than yesterday.~~
~~Q: We will go swimming.~~
~~R: We will take a canoe trip.~~

Sol.2. Let P: Alex misses many classes through illness.
Q: He fails high school.
R: He is uneducated.
S: Alex reads a lot of books.

1. $P \Rightarrow Q$
2. $Q \Rightarrow R$
3. $S \Rightarrow \sim R \equiv R \Rightarrow \sim S$
4. $P \wedge S$
5. $P \Rightarrow R$ [From 1 and 2 by Hypothetical syllogism]
6. $P \Rightarrow \sim S$ [From 3 and 5 using hypothetical syllogism]
7. $\sim P \vee \sim S$ [From 6 by property]
8. $F$ [From 4 and 7 by complement law].

Hence the given premises are inconsistent.

Let $p$ : It is sunny this afternoon.
$q$ : It is colder than yesterday
$r$ : We will go to swimming.
$s$ : We will take a canoe trip
$t$ : We will be home by sunset.

1. $\sim p \wedge q$
2. $r \Rightarrow p$
3. $\sim r \Rightarrow s$
4. $s \Rightarrow t$
5. $\sim p$ [From 1 using simplification]
6. $\sim r$ [From 2 & 5 using modus tollens]
7. $s$ [From 3 & 6 using modus ponens]
8. $t$ [From 4 & 7 " " "]