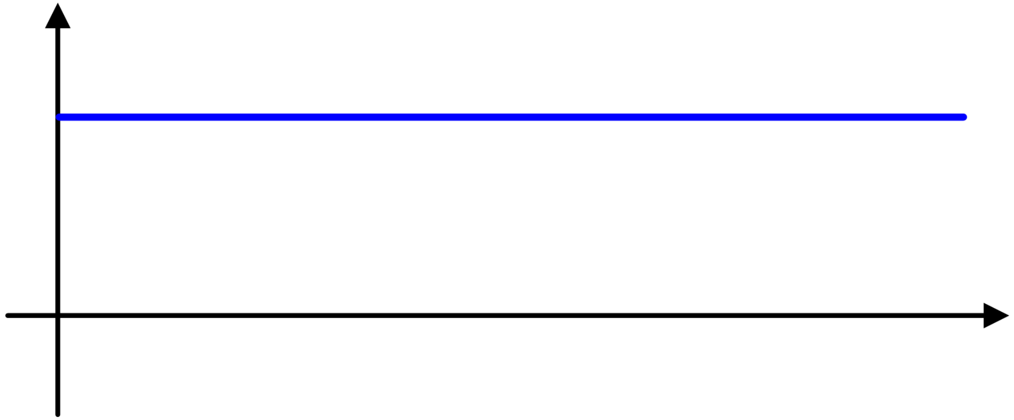
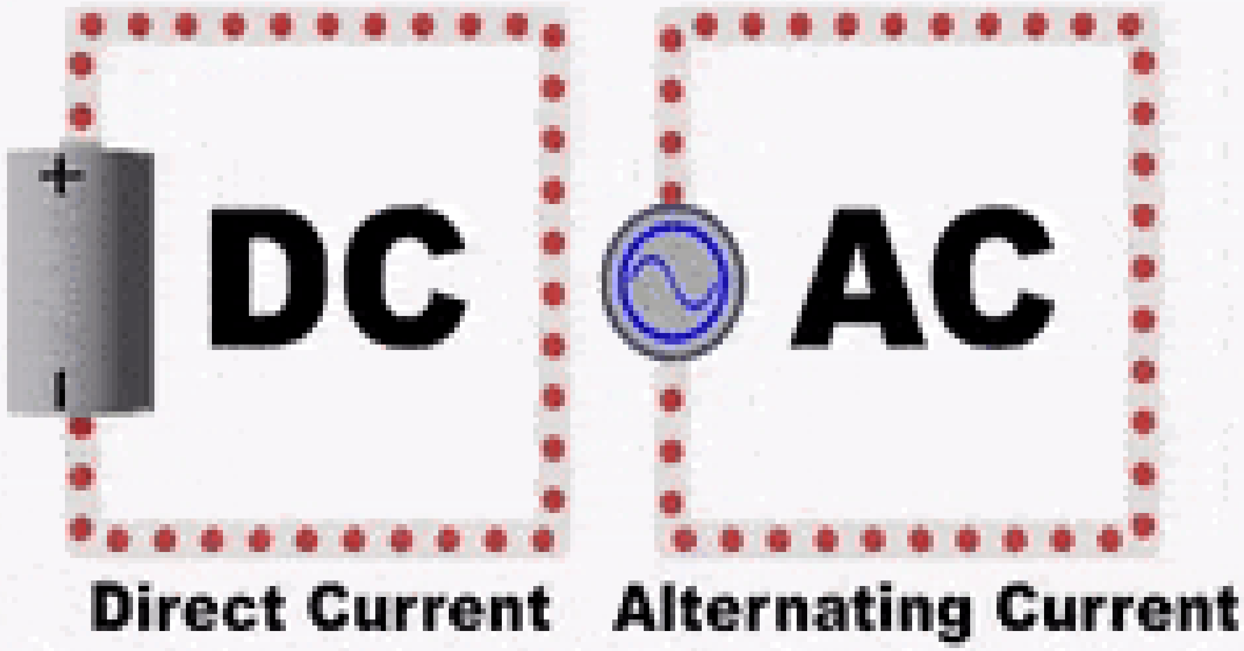
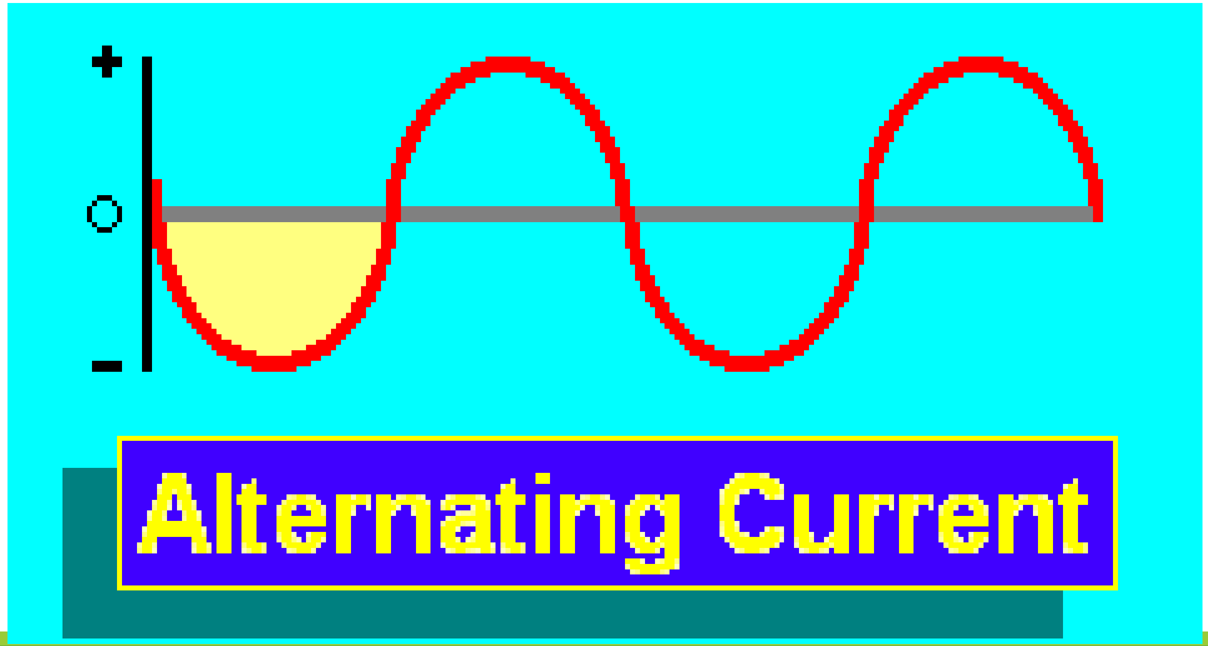
Module 2. A.C.Fundamentals

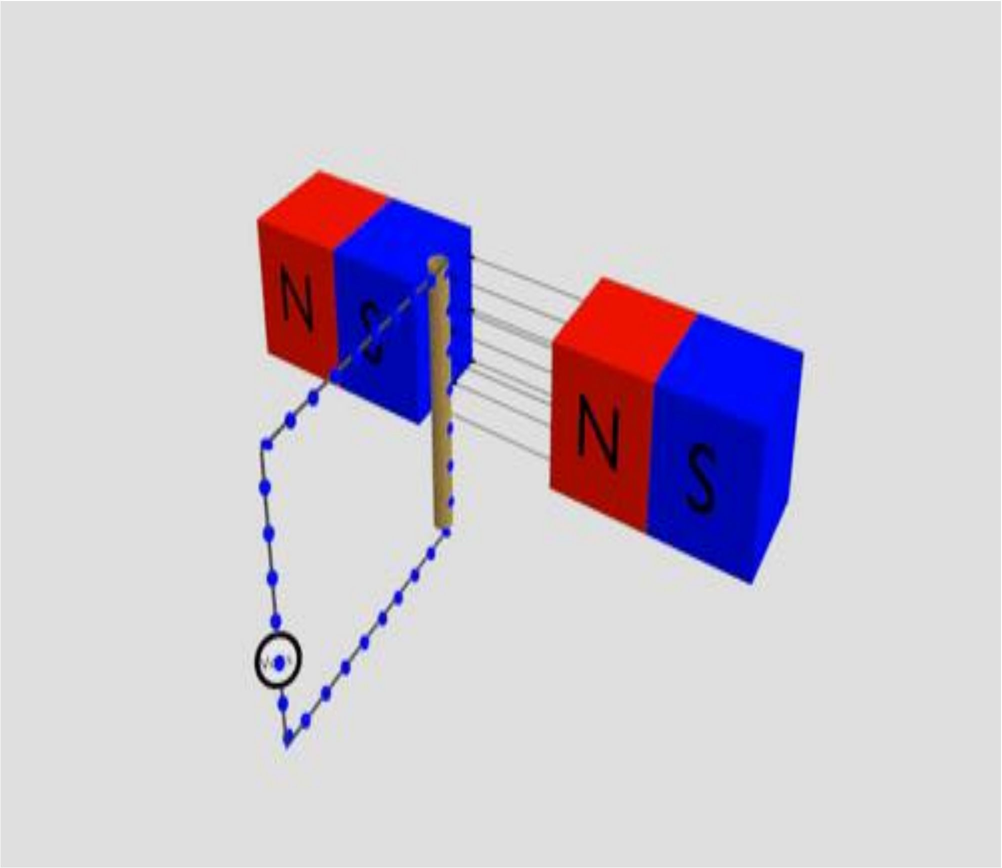
- Alternating quantity
- Instantaneous signal
- Sinusoidal signal







Faradays laws of Electromagnetic Induction



To generate an EMF

1. Magnetic Field
2. Conductor
3. Relative motion between the conductor.

Whenever a conductor is placed in a varying magnetic field, an EMF is induced in a conductor.

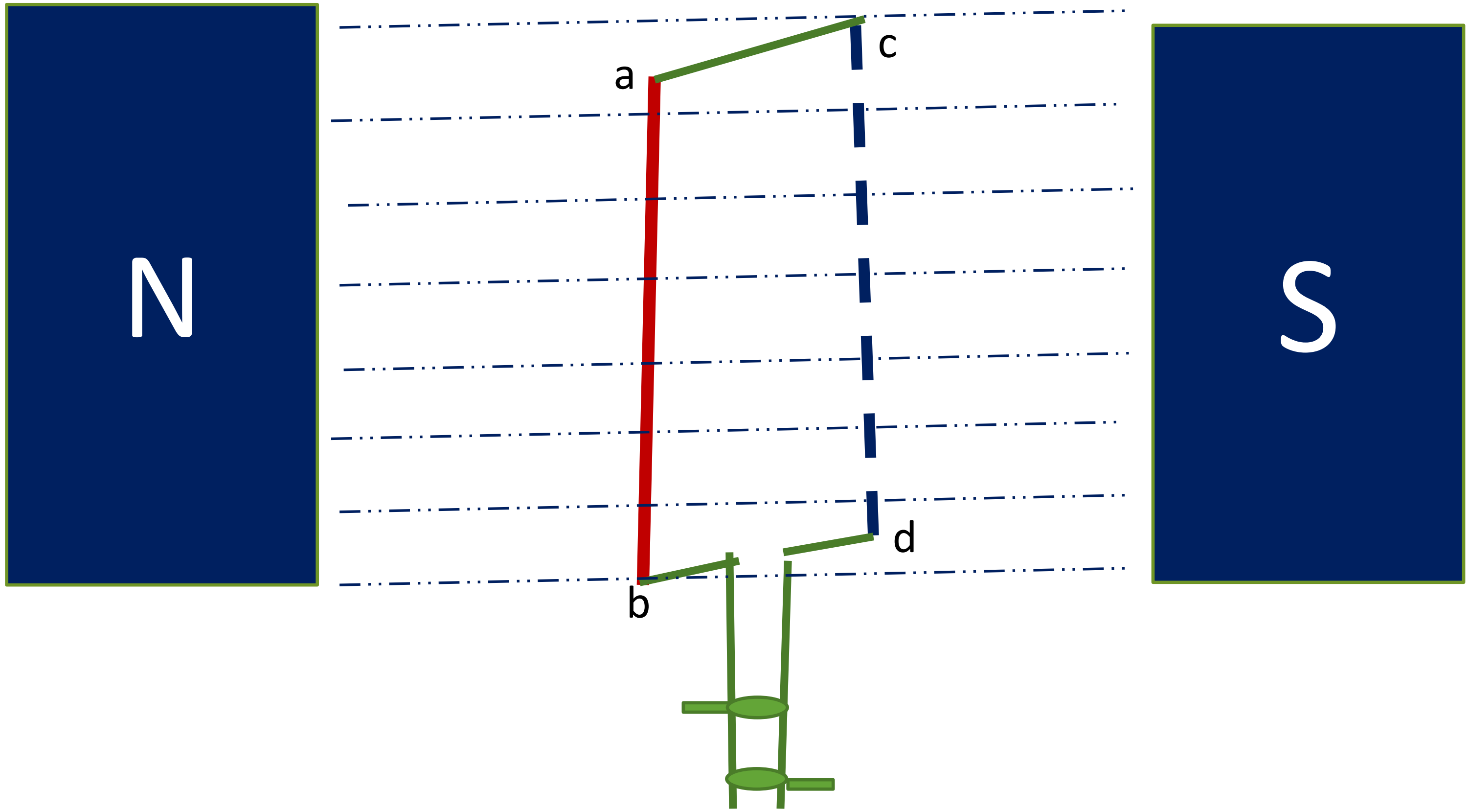
Dynamically Induced Emf

$$e = B\, l\, v\, \sin\theta \quad \text{volts}$$

- **where B - Flux density of the magnetic field in wb/m^2**
- **l - Length of the conductor in m**
- **v - velocity of the conductor in m/s**
- **θ - angle between the relative velocity of the conductor and the plane of the flux.**

Generation of AC voltage

- Consider a rectangular coil placed in a uniform magnetic field produced by two poles as shown in the figure.

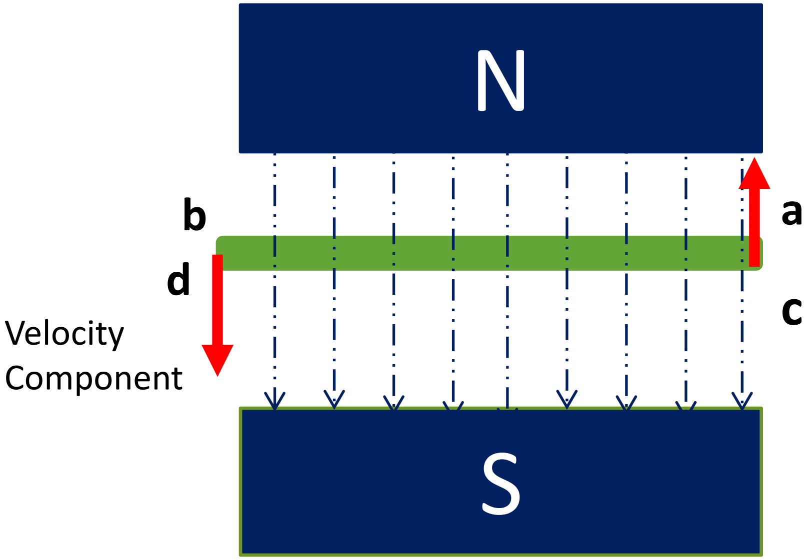


- **Rotate the coil in the anticlockwise direction ,while rotating the conductors will cut the magnetic flux due to which an emf is Induced in the conductor . The magnitude of the Induced Emf is depends upon the position of the conductor in an magnetic Field.**
- **We know the from the faraday's laws of electromagnetic Induction**
- **The dynamically Induced Emf is given by**

$$e = B\,l\,v\,\mathrm{Sin}\theta \quad \text{volts}$$

- **where B - Flux density of the magnetic field in wb/m^2**
- **l - Length of the conductor in m**
- **v - velocity of the conductor in m/s**
- **θ - angle between the relative velocity of the conductor and the plane of the flux.**

- Let the Initial position of the coil be as shown in fig a.

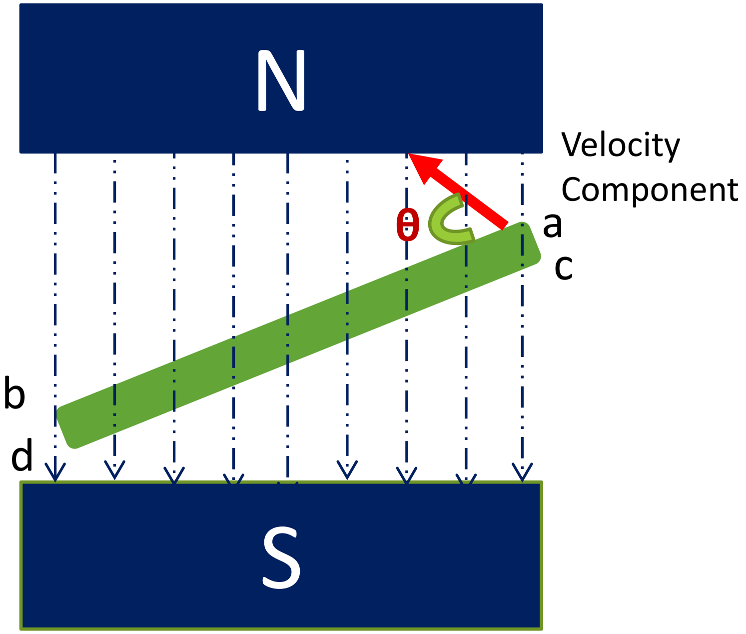


(a) θ = 0

- The angle between the magnetic flux and velocity component is zero I,e θ =0.
- Hence the Emf Induced

e = B l v Sin0 =0 --- zero Emf

- **When the coil is rotated in anticlockwise direction through some angle θ as shown in fig b.**



(b) (0<θ<90)

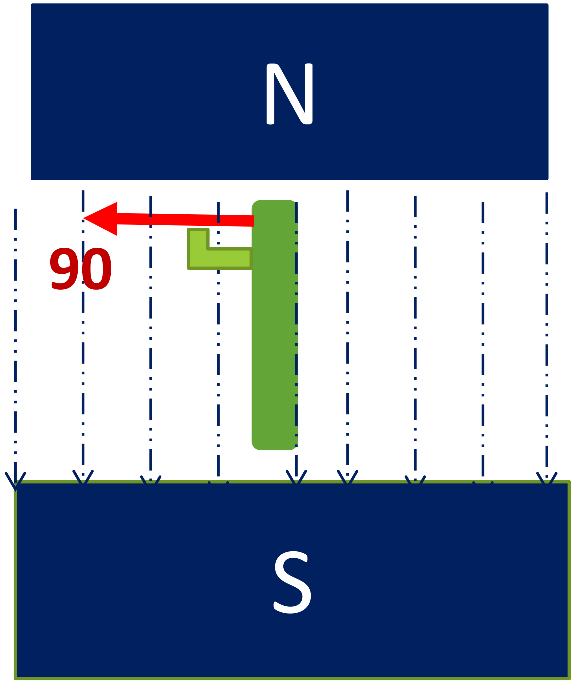
- **The instantaneous component of velocity of conductors is cutting the flux with an angle θ.**

- Hence the Emf Induced

$e = B\, l\, v\, \sin\theta$ ----- (0<θ<90)

ex: θ is 30 then e = 0.5 Blv

When the coil is further rotate in an anticlockwise direction ,the instantaneous component of velocity acting perpendicular to the line of the flux as shown in fig c.



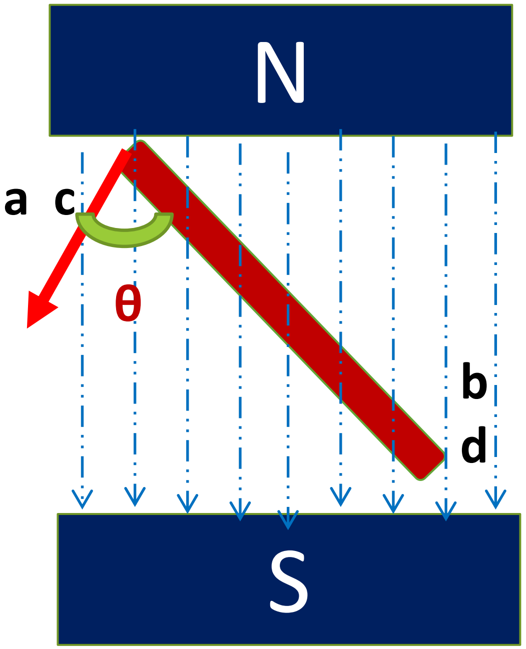
(c) θ =90

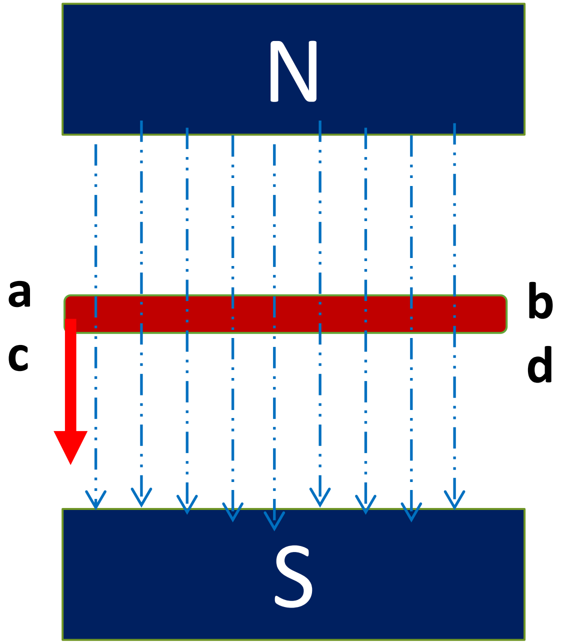
- Therefore the angle between the magnetic flux and velocity component is 90 I,e θ =90.
- Hence the Emf Induced

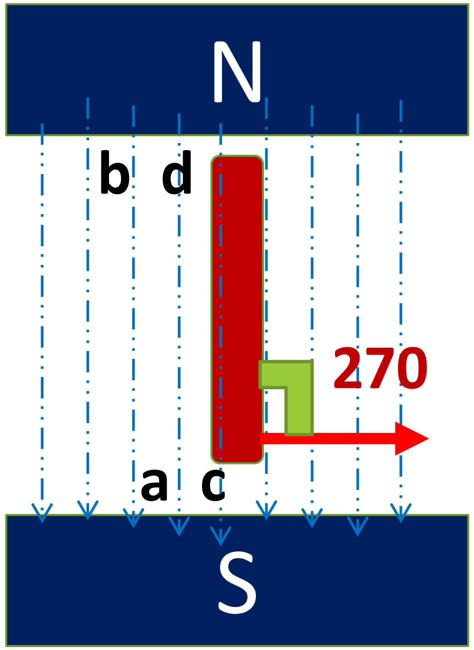
$$e = B\, l\, v \, \mathrm{Sin}90 \;\; = Blv$$

- **So the Induced emf in this position is at its maximum Value.**

Lets consider other Instants of coil as shown in Fig d,e and f .







(d) (90<θ<180)

$e = B\ l\ v\ Sin\theta$

Ex : if θ =150

e= 0.5Blv

(e) θ=180

e = 0

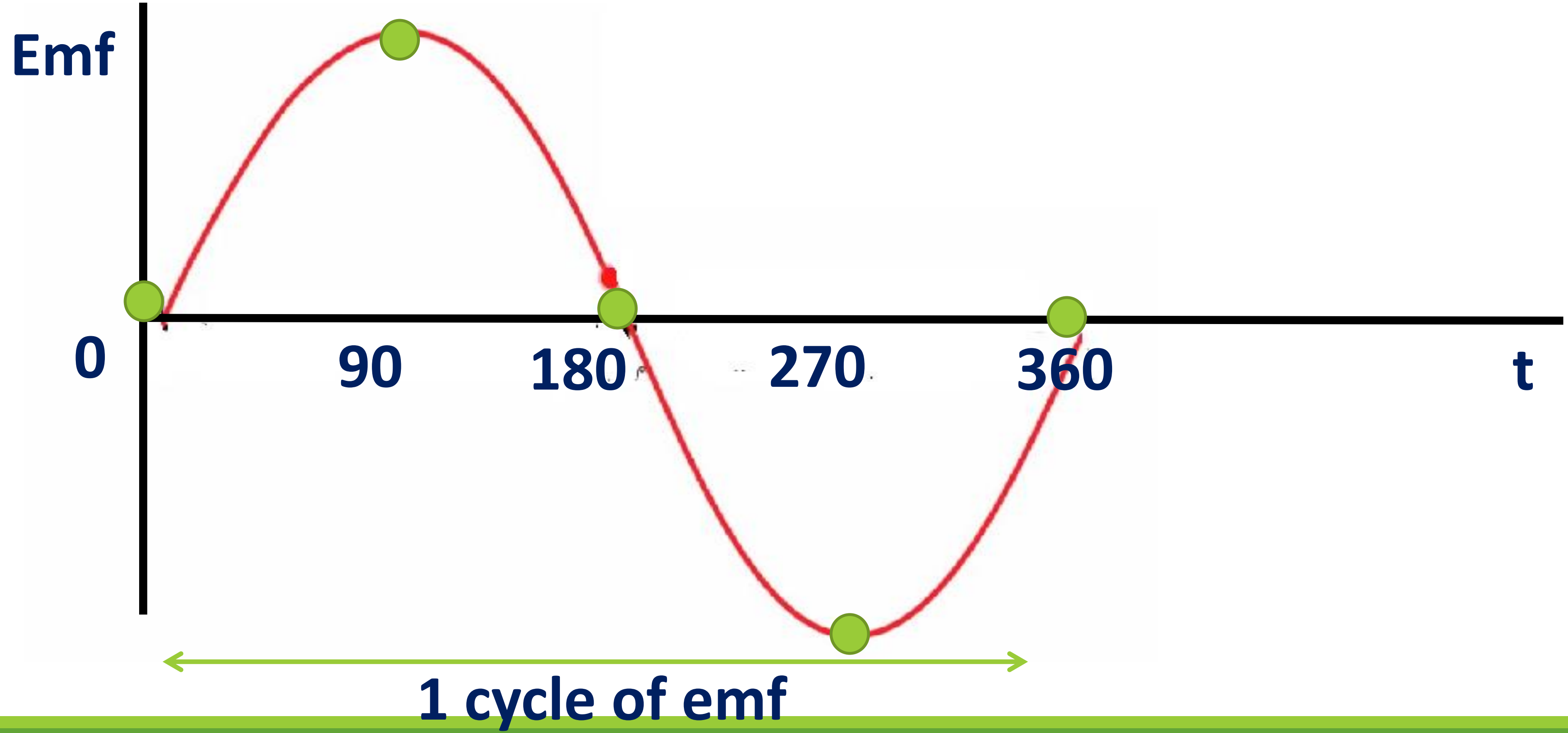
(f) θ=270

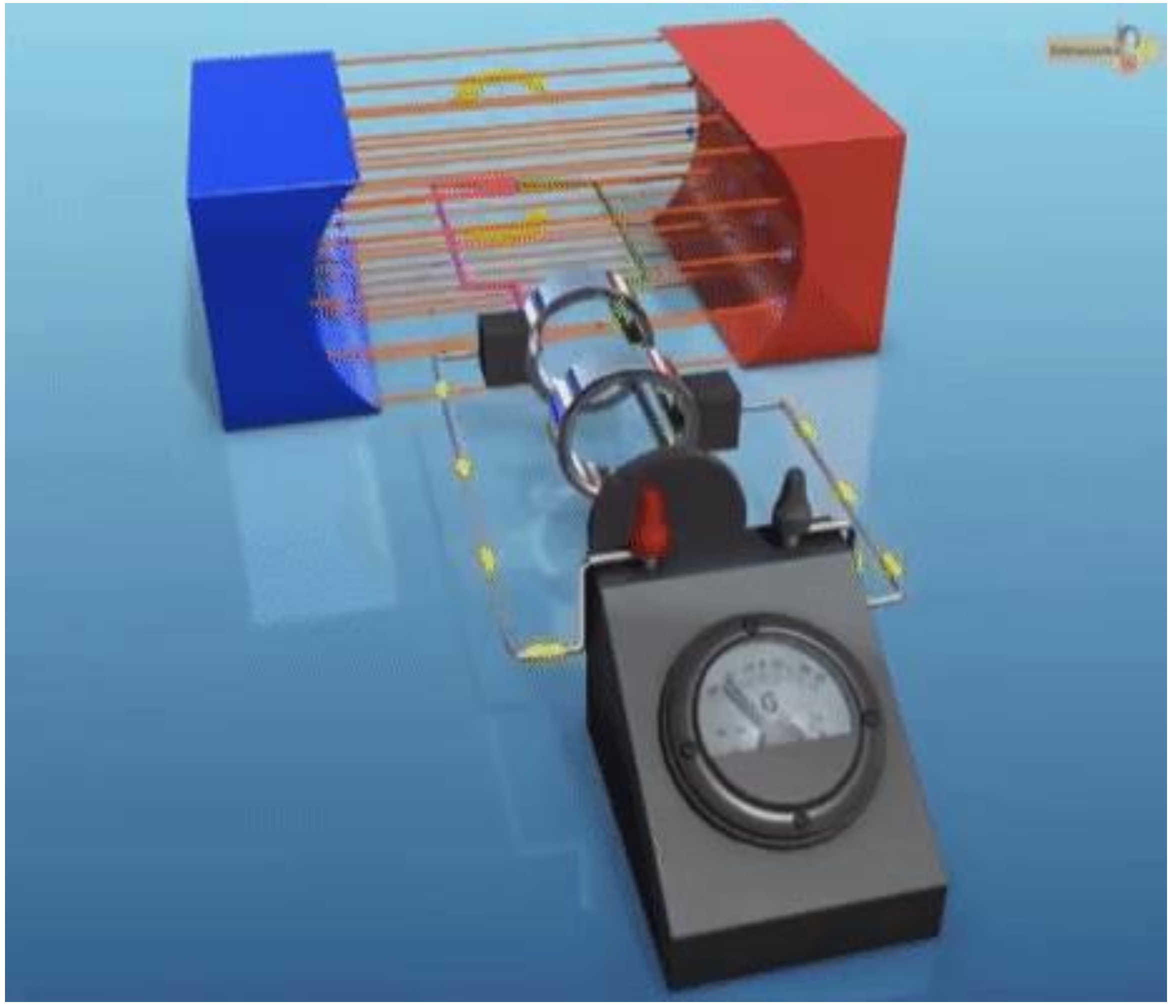
e = -Blv

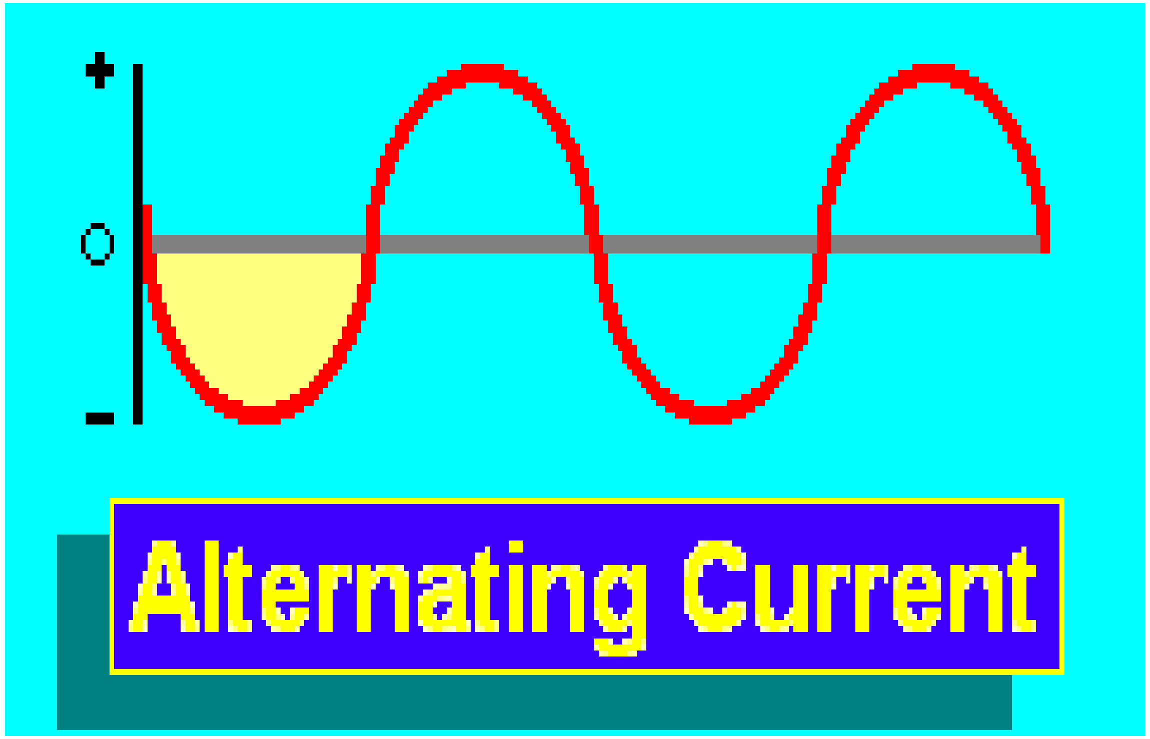
Similarly when θ=360 then e=0

So, when θ varies from 0 to 360 the emf in an conductor varies in an alternating manner I,e from zero to maximum in one direction ,decreasing to zero, then achieving maximum in another direction and again decreasing to zero.

Therefore by rotating conductor from 0 to 360 we get one cycle of emf






+
0
-
Alternating Current

Advantages of AC system over DC system

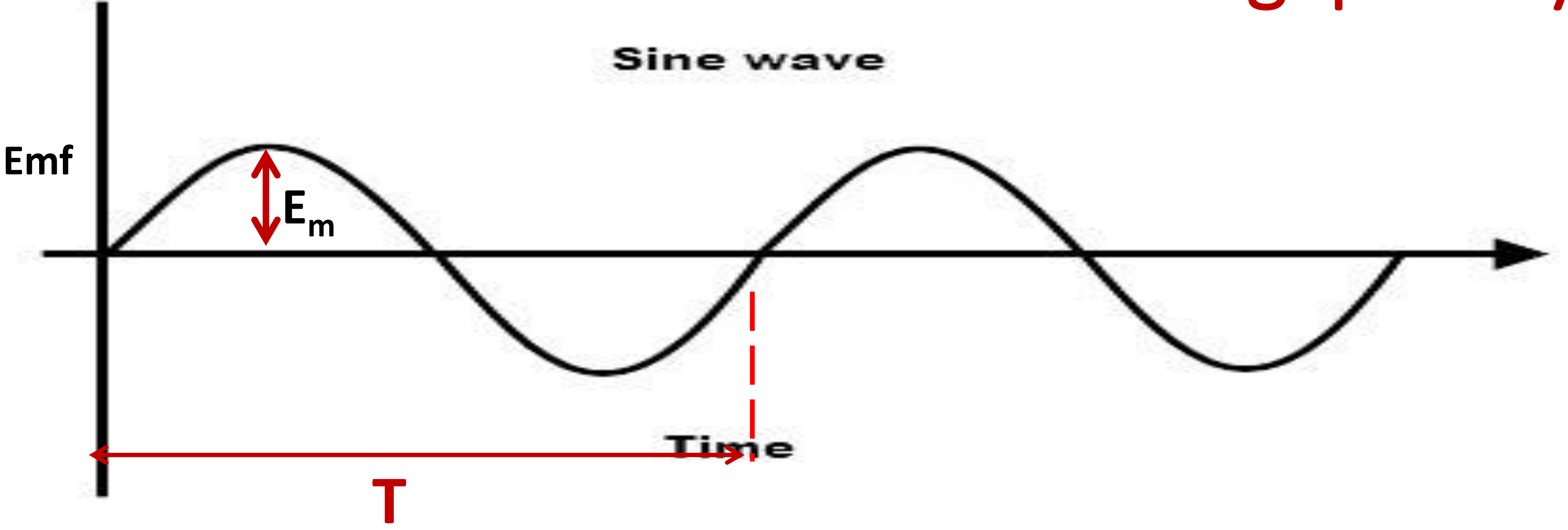
1. AC voltages can be efficiently stepped up/down using a device transformer.

2. By increasing the transmission voltage the losses can be reduced.

3.AC machines are simpler in construction than DC machines.

4. The cost and maintenance of Ac machines are less compare to Dc machines.

Important terms associated with an alternating quantity



Amplitude(E_m or I_m): It is the maximum value attained by an alternating quantity. Also called as maximum or peak value.

Time Period (T): It is the Time Taken to complete one cycle of an alternating quantity

Instantaneous Value (e or i): It is the value of the quantity at any instant.

$e = Blv\sin\theta$ wkt $E_m = Blv$

$e = E_m \sin\theta$ or $E_m \sin\omega t$ and $i = I_m \sin\theta$ or $I_m \sin\omega t$

4. Frequency (f): It is the number of cycles completed by alternating quantity in one second.

The unit for frequency is **Hz or cycles/sec.**

$$\boxed{T = 1/f \text{ ----sec}}$$

5. Angular Frequency (ω): Angular frequency is defined as the number of radians covered in one second.

The unit of angular frequency is rad/sec.

$$\boxed{\omega = 2\pi f \quad \text{------r/s}}$$

6. Average Value(E_{av} or I_{av}): The arithmetic average of all the values of an alternating quantity over one cycle is called its average value.

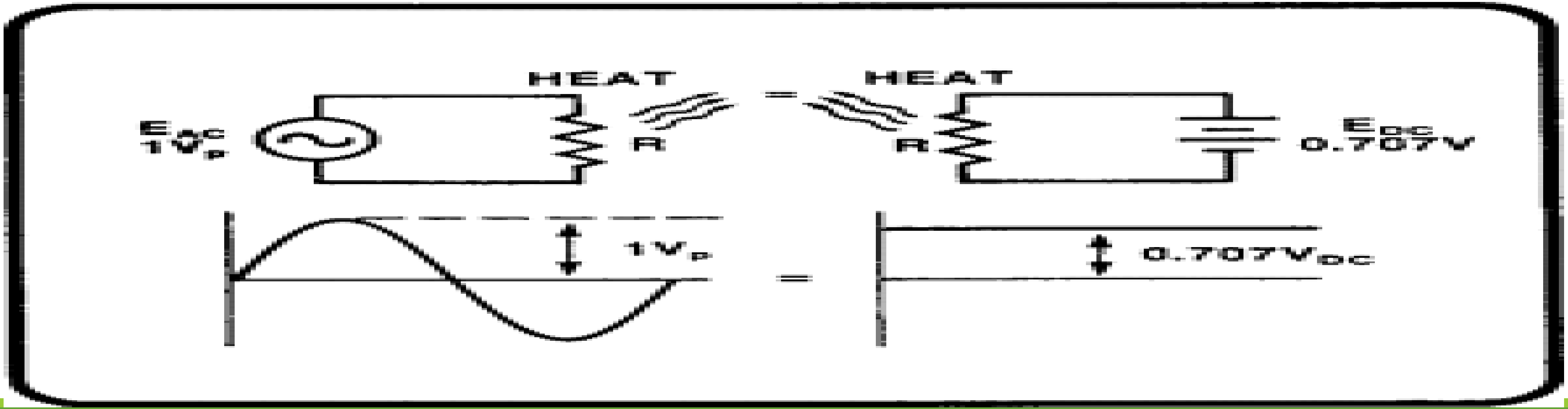
$$\text{Average value} = \frac{\text{Area under curve half cycle}}{\text{length of base over half cycle}}$$

$$I_{av} = \frac{2I_m}{\pi} = 0.637I_m$$

$$E_{av} = \frac{2E_m}{\pi} = 0.637E_m$$

RMS(Root mean square) or Effective Value(E_{rms} or I_{rms})

Definition: The effective or RMS value of an alternating quantity is that steady current (dc) which when flowing through a given resistance for a given time produces the same amount of heat produced by the alternating current flowing through the same resistance for the same time.



$$I_{rms} = \frac{I_m}{\sqrt{2}} = 0.707I_m$$

$$E_{rms} = \frac{E_m}{\sqrt{2}} = 0.707E_m$$

Form Factor (K_f): The Form factor of an alternating quantity is defined as the ratio of Rms value to the average value.

$$K_f = \frac{RMS\ value}{Average\ value}$$

The form factor of alternating current can be obtained as

$$K_f = \frac{0.707Im}{0.637Im} = 1.11$$

Crest or Peak Factor (K_p): The peak factor of an alternating quantity is defined as the ratio of maximum value to the RMS value.

$$\boxed{K_p = \frac{Maximum\ value}{RMS\ value}}$$

The peak factor of alternating current can be obtained as

$$K_p = \frac{I_m}{0.707 Im} = 1.414$$

Formulas

1. **Instantaneous Value** $e = E_m \sin\theta$ and $i = I_m \sin\theta$

$e = E_m \sin\omega t$ and $i = I_m \sin\omega t$

2. **Time period** $T = 1/f$ sec

3. **Angular Frequency** $\omega = 2\pi f$ rad/sec

4. **Average Value** $$I_{av} = \frac{2I_m}{\pi} = 0.637 I_m$$

5. **Rms value** $$I_{rms} = \frac{I_m}{\sqrt{2}} = 0.707 I_m$$

6. **Form factor** $$K_f = \frac{RMS\ value}{Average\ value}$$

7. **crest or peak factor** $$K_p = \frac{Maximum\ value}{RMS\ value}$$

Problems

1.The equation of an alternating current is given by $i = 42.42 \sin 568\, t$. Calculate its i) Maximum Value ii) Frequency iii) Rms value iv) Average value v) form factor vi) peak factor.

Solution: Given - $i = 42.42 \sin 568\, t$

Compare the equation with $i = I_m \sin \omega t$

We get Maximum value $\boxed{I_m = 42.42\ A}$

and $\omega = 568$

Wkt $\omega = 2\pi f$

ii) $f = \frac{\omega}{2\pi} = \frac{568}{2x3.14}$ =90.42Hz

$\boxed{\text{Frequency } f = 90.42\text{Hz}}$

iii) Rms value $I_{rms} = 0.707 I_m = 0.707 \times 42.42$

$$\boxed{I_{rms} = 30A}$$

iv) Average value $I_{av} = 0.637\, I_m = 0.637 \times 42.42 = 27.02\, A$

$$\boxed{I_{av} = 27.02A}$$

v) Form factor $K_f = \dfrac{RMS\ value}{Average\ value} = \dfrac{30}{27.02} = 1.11$

vi) peak factor $K_p = \dfrac{Maximum\ value}{RMS\ value} = \dfrac{42.42}{30} = 1.414$

The equation of an alternating current is given by $i = 414 \sin 628t$

find i) Max value ii) frequency iii) RMS value iv) Average value V) Instantaneous value of current at t= 5ms

The equation for an AC voltage is given as $V = 0.04\sin(2000t + 60°)$V. Determine the frequency, the angular frequency, instantaneous voltage when t = 160μs. What is the time represented by a 60° phase angle. (06 Marks)

Solution: $v = 0.04 \sin(2000t + 60)$ $= V_m \sin(\omega t + \phi)$

$\omega = 2000$ r/s $V_m = 0.04$ v

$f = \omega/2\pi = 2000/2*3.142 = 318.47 = 319$Hz

Instantenous value v=? At $t = 160 \times 10^{-6}$

$v = 0.04 \sin(2000t + 60)$

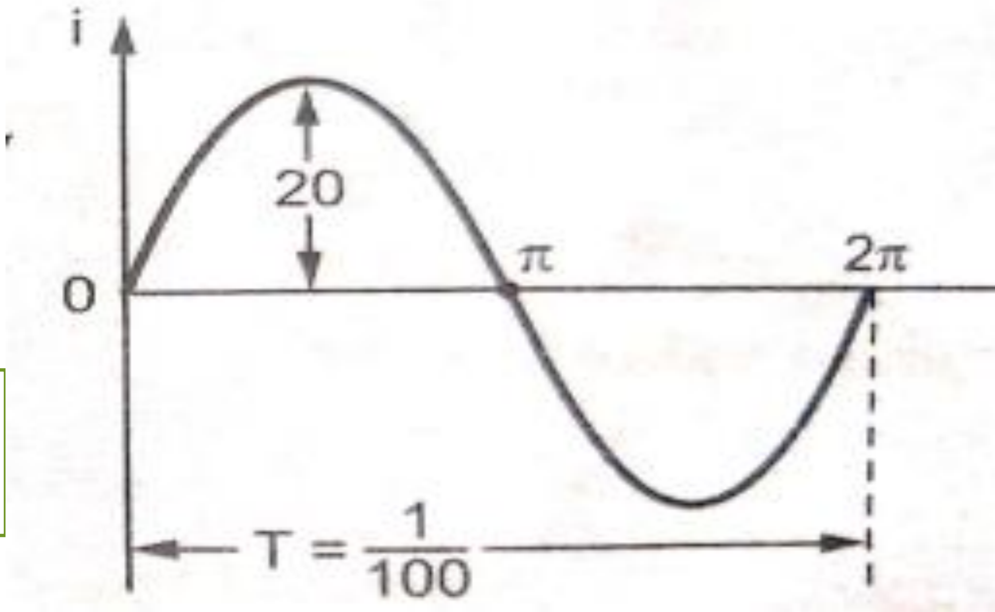
$v = 0.04 \sin(2000 \times 160 \times 10^{-6} + 1.047)$ $= 0.00095$V

t=? $\phi = \omega t$ $60 = 2000\, t$ $t = 0.03$s

3.For the current wave shown in the fig

Find its i) Peak Value ii)Periodic time iii) Frequency iii) Rms value iv) Average value v) form factor v) peak factor vi) instantaneous value at t=3ms

Solution: From the fig



i) Peak Value $\boxed{I_m = 20A}$

ii) Periodic Time $\boxed{T = 1/100 = 0.01 \text{ sec}}$

iii) Frequency f =1/T = 1/0.01 = 100Hz

iv)Rms value $I_{rms} = 0.707 I_m = 0.707 \times 20 = 14.14\ A$

iv) **Average value** $I_{av} = 0.637\, I_m = 0.637 \times 20 = 12.74\, A$

$$\boxed{I_{av} = 12.74A}$$

v) **Form factor** $K_f = \dfrac{RMS\ value}{Average\ value} = \dfrac{14.14}{12.74} = 1.10$

vi) **peak factor** $K_p = \dfrac{Maximum\ value}{RMS\ value} = \dfrac{20}{14.14} = 1.41$

instantaneous value at $t=3ms$ $i = ?$

$i = I_m \sin\omega t$

$\omega = 2\pi f = 2 x 3.14 x 100 = 628 r/s$

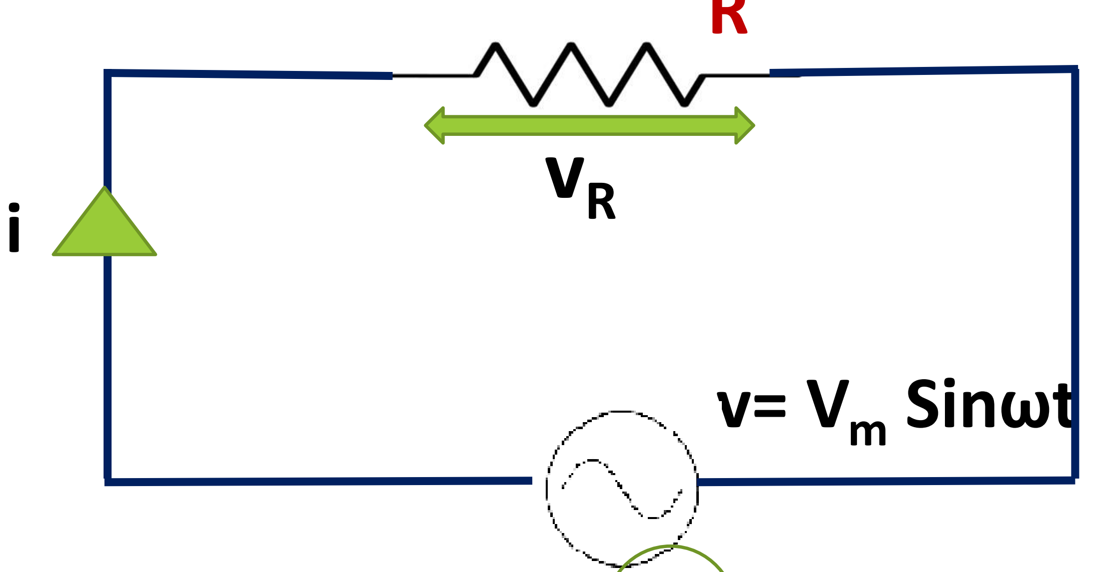
therefore $i = 20 \sin 628 \times 3\times10^{-3}$

$i = 0.657A$

To determine the behavior of the pure resistive, pure inductive and pure capacitive circuit and to find the relationship between voltage and current

Pure resistive circuit (R- circuit)

Consider an circuit consists of pure resistance 'R'Ω connected across an A C supply of 'V' volts.



the supply voltage **$v = V_m$ Sinωt ---------1**

Since the voltage across resistance is same as the supply voltage

$$v = v_R = iR$$

$$i = \frac{v}{R}$$

$$i = \frac{V_m \, \text{Sin}\omega t}{R}$$

WKT , $I_m = \frac{V_m}{R}$

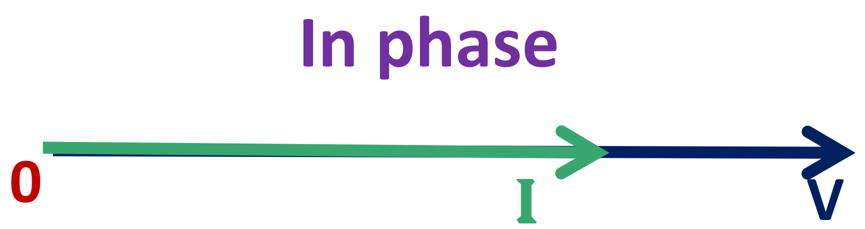
Therefore

$$\boxed{i = I_m \, \text{Sin}\omega t} \text{ ------------------- } 2$$

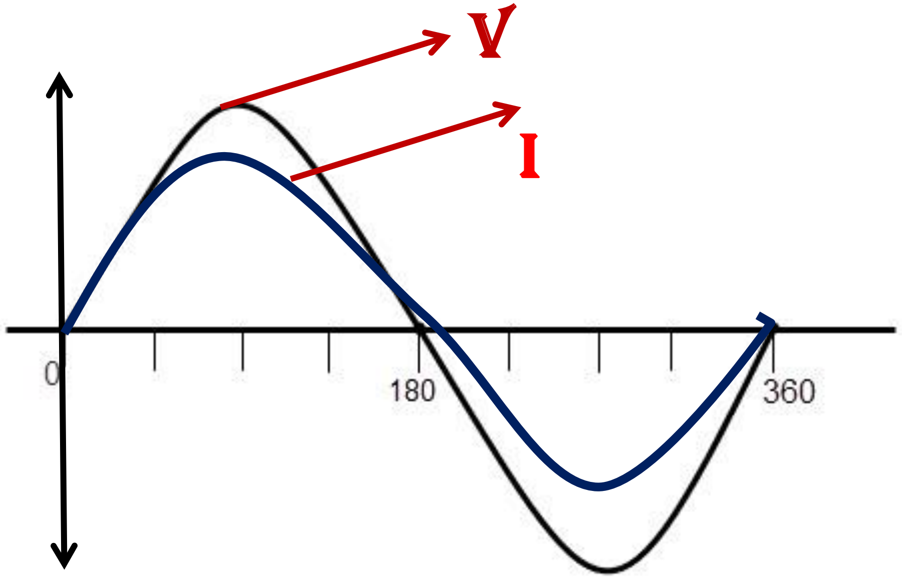
Comparing equation 1 and 2

In Pure Resistive circuit the Voltage and current are in Inphase

Phasor Diagram or vector diagram



Waveforms



Power Factor (cosϕ): It is defined as the cosine of angle between Voltage and the current.

For pure resistive circuit angle between voltage and current is zero

I,e $\phi = 0$

Therefore pf = **cosϕ = cos 0 = 1 (unity power factor)**

Power

$p = vi$

$p = V_m \sin\omega t \times I_m \sin\omega t$

$p = V_m I_m \sin^2\omega t \quad = V_m I_m [\frac{1-\cos 2\omega t}{2}]$

Wkt Average power $= \frac{\text{area under curve over a cycle}}{\text{length of base over a cycle}}$

$$P_{av} = \frac{\int_0^{2\pi} V_m I_m [\frac{1-\cos 2\omega t}{2}]}{2\pi}$$

$$= \frac{V_m I_m}{4\pi} \int_0^{2\pi} 1 - \cos 2\omega t \, d\omega t$$

$$P_{av} = \frac{V_m I_m}{4\pi}\left[\omega t - \frac{\sin 2\omega t}{2}\right]_{0}^{2\pi}$$

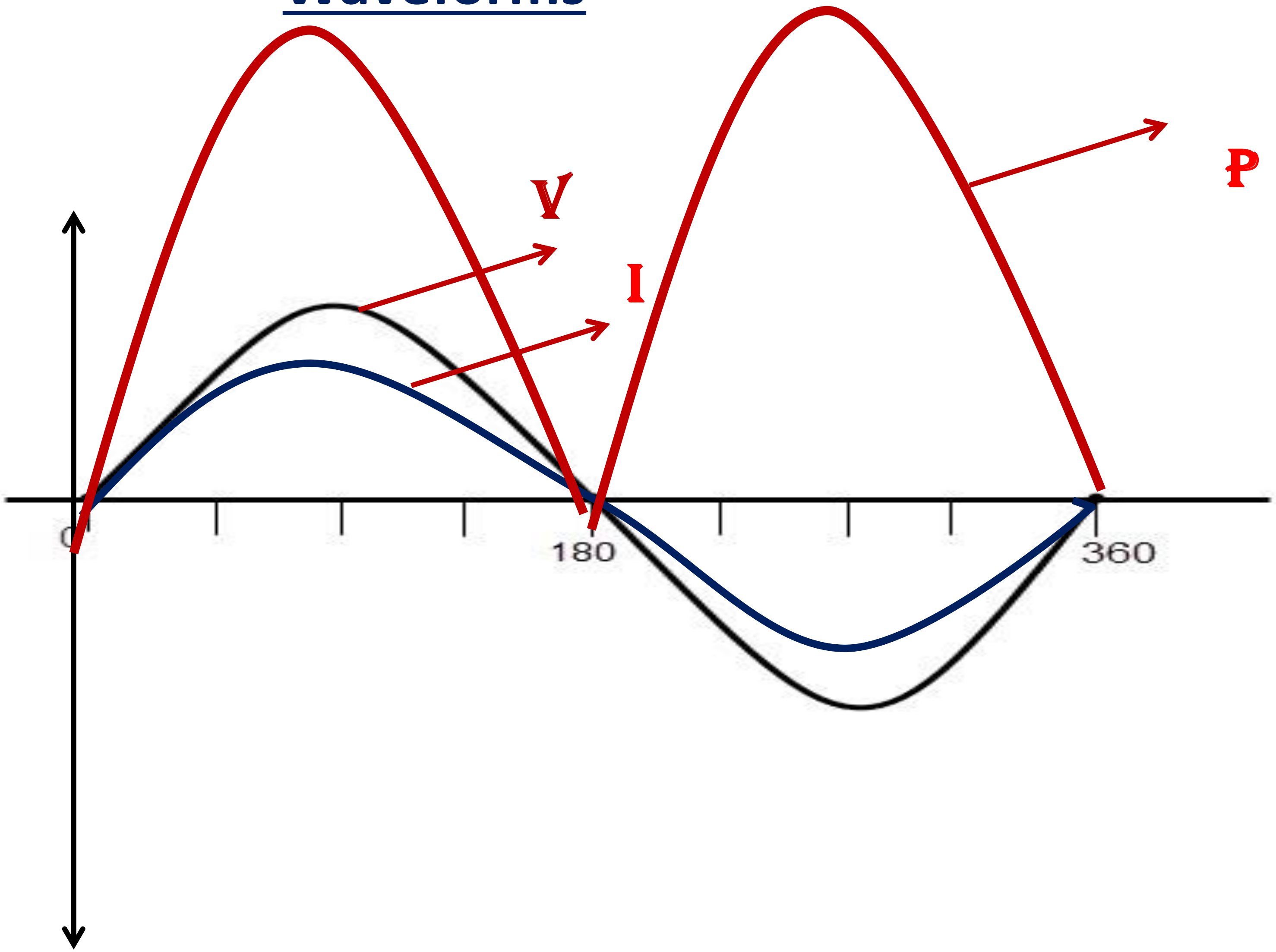
$$= \frac{V_m I_m}{4\pi}\left[(2\pi - 0) - \frac{1}{2}(\sin 2x2\pi - \sin 2x0)\right]$$

$$= \frac{V_m I_m}{4\pi}\left[(2\pi) - \frac{1}{2}(0 - 0)\right]$$

$$= \frac{V_m I_m}{\cancel{4\pi}}\cancel{(2\pi)}$$

$$P = \frac{V_m I_m}{2} = \frac{V_m}{\sqrt{2}} \; \frac{I_m}{\sqrt{2}} = V_{rms} \; I_{rms}$$

Waveforms



Impedance(z) : The total opposition offered by an AC circuit to the flow of an alternating current is called Impedance. The unit is ohms

$$Z = V/I \quad \text{ohms}(\Omega)$$

$$Z = R \pm jX = z \angle \phi$$

Note: For a pure resistive circuit (R circuit)

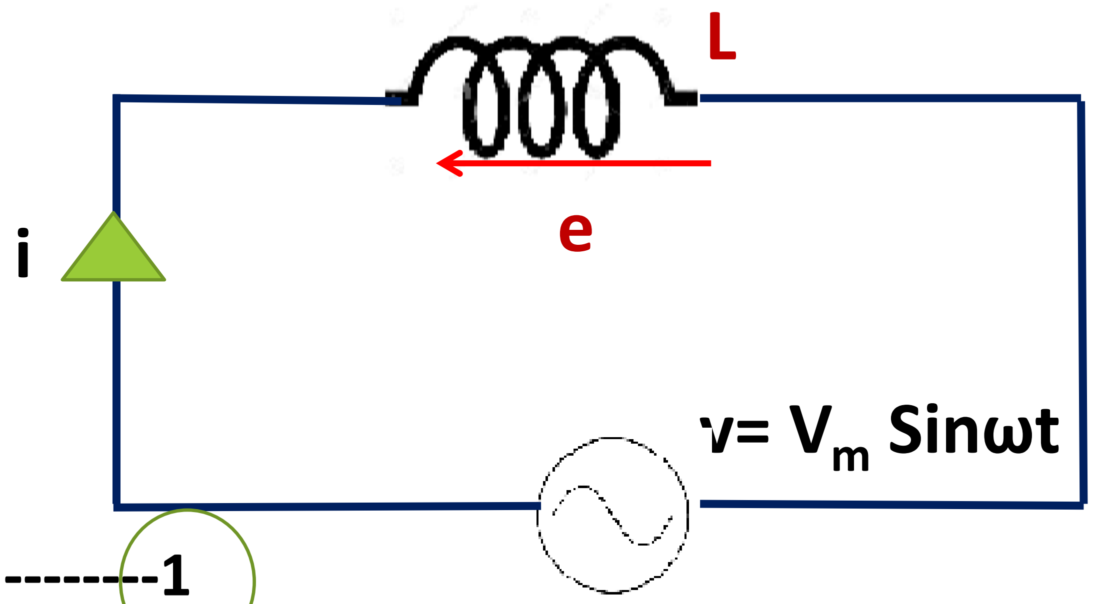
$$Z = R + j0 = R \quad \Omega$$

$v = V_m \sin wt \qquad i = I_m \sin wt$ and $P = VI$

In polar form- $V\angle 0$ and $I \angle 0$

Pure Inductive circuit (L- circuit)

Consider an circuit consists of pure Inductor of 'L' Henry connected across an A C supply of 'V' volts.



v= V_m Sinωt --------1

By Faraday's law

$$e = -L\frac{dI}{dt}$$

According to the Lenz's Law **e=-v**

$$v = L\frac{dI}{dt}$$

$$dI = \frac{v\,dt}{L} = \frac{V_m \,\mathrm{Sin}\omega t\, dt}{L}$$

Intergrating both sides $\int dI = \int \frac{V_m \,\mathrm{Sin}\omega t\, dt}{L}$

$$i = \frac{V_m}{L} \int sin\omega t$$

$$i = \frac{V_m}{L} \left[\frac{-cos\omega t}{\omega}\right]$$

$$i = \frac{V_m}{\omega L}\left[-\sin\left(\frac{\pi}{2} - \omega t\right)\right]$$

$$i = \frac{V_m}{\omega L}\left[\sin\left(\omega t - \frac{\pi}{2}\right)\right]$$

Wkt Inductive Reactance $X_L = \omega L = 2\pi f L \ \Omega$

$$i = \frac{V_m \sin\left(\omega t - \frac{\pi}{2}\right)}{X_L}$$

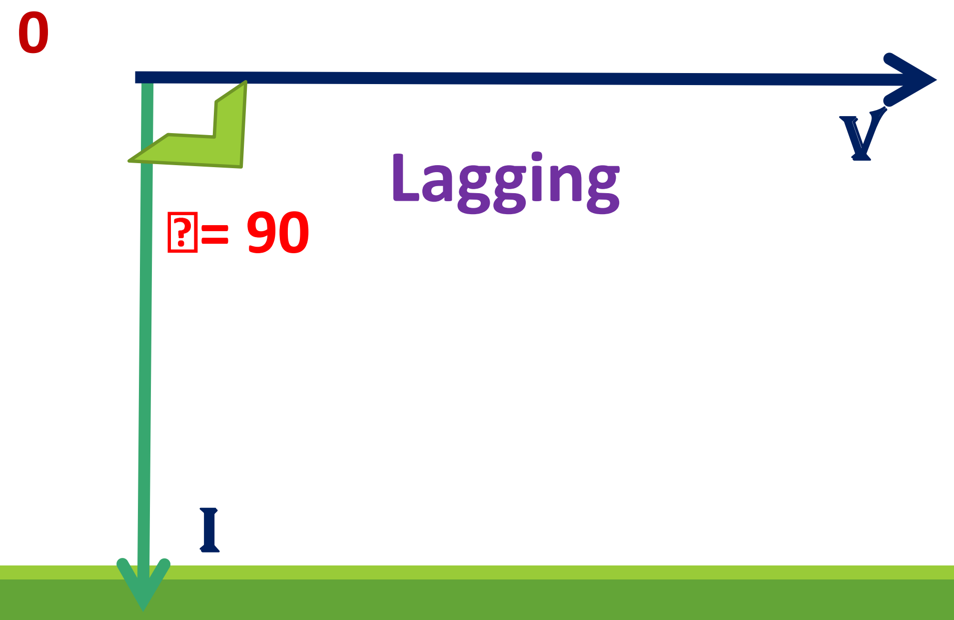
WKT, $I_m = \frac{V_m}{X_L}$

Therefore $\boxed{i = I_m \sin(\omega t - \pi/2)}$ ------------------- 2

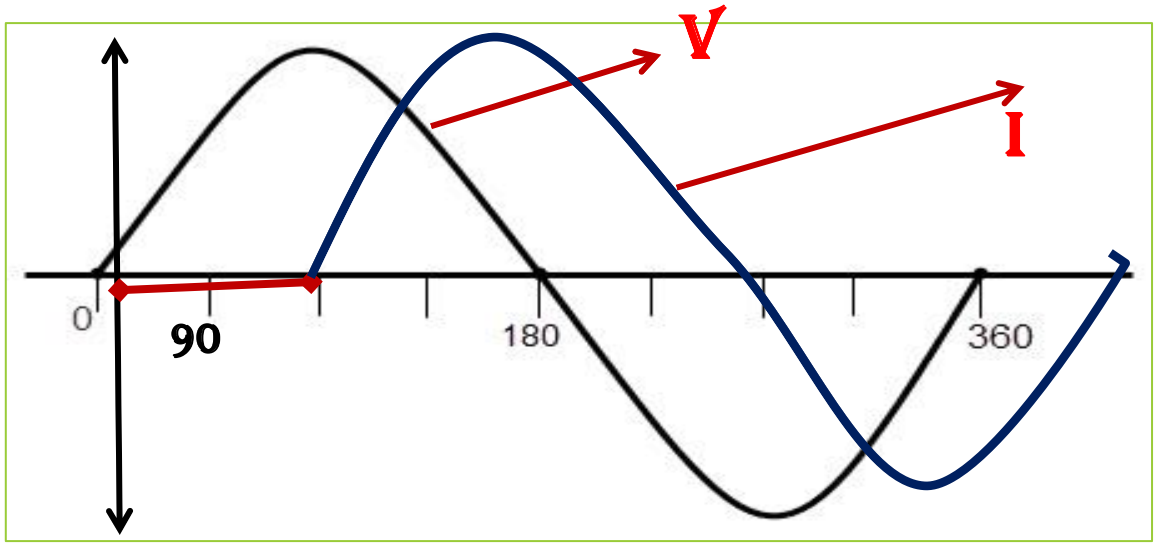
Comparing equation 1 and 2

In Pure Inductive circuit the current lag the voltage exactly by 90 .

Phasor diagram



waveforms



Power of pure Inductive circuit

$P = vi \qquad = V_m \operatorname{Sin}\omega t \times I_m \operatorname{Sin}(\omega t - \pi/2)$

$P = V_m I_m \sin\omega t(-\cos\omega t)$

$P = -V_m I_m \dfrac{\sin 2\omega t}{2}$

As 2sintωtcosωt =sint2ωt

$$P_{av} = \int_0^{2\pi} \frac{-V_m I_m}{2} \sin 2\omega t \; d\omega t \qquad = \frac{-V_m I_m}{2} \left[-\frac{\cos 2\omega t}{2} \right]_0^{2\pi}$$

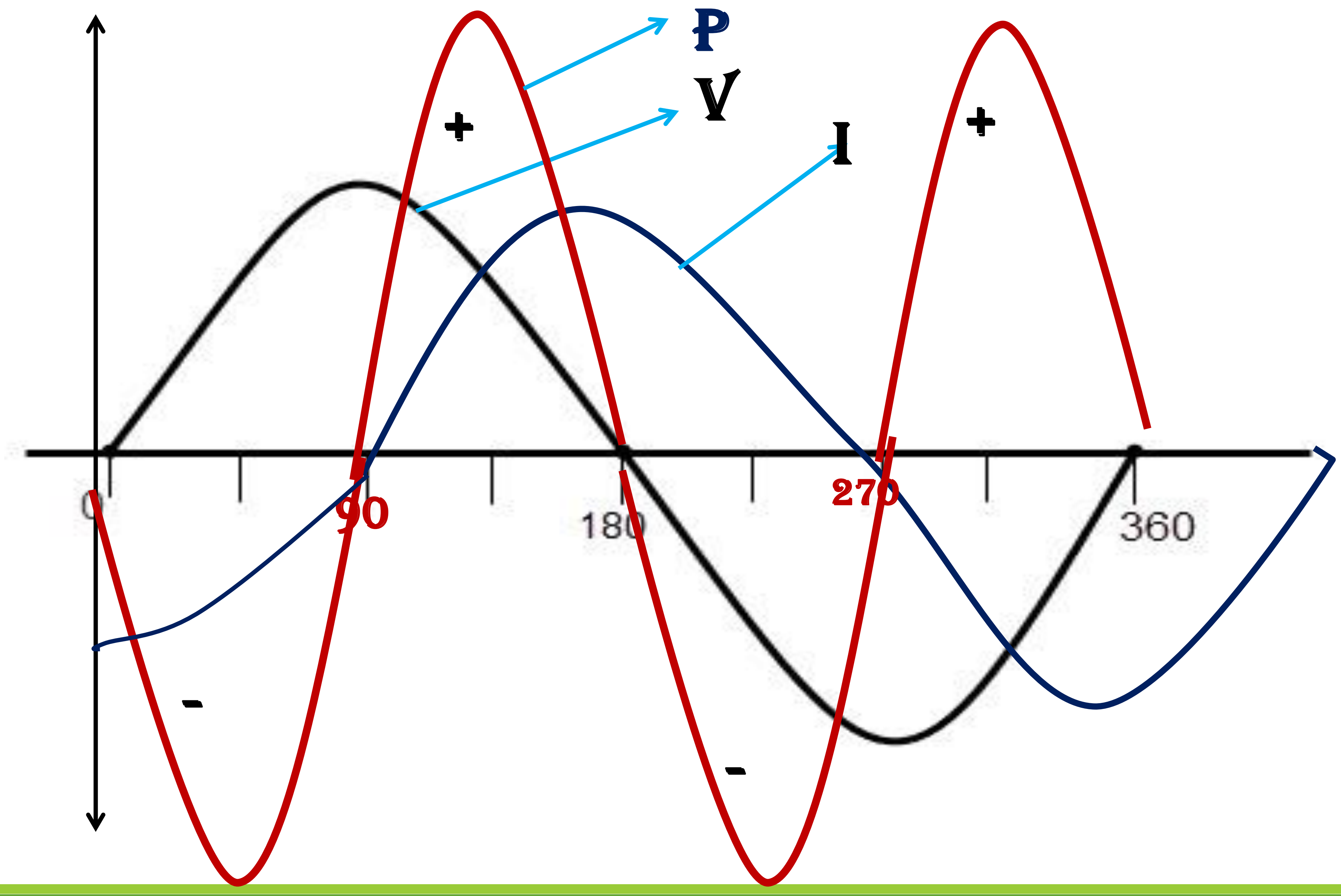
$$P_{av} = \frac{V_m I_m}{4} [\cos 2(2\pi) - \cos(0)]$$

$$P_{av} = \frac{V_m I_m}{4} [1 - 1] = 0$$

$$P_{av} = 0$$

The average value over a cycle of the fluctuating sine component is zero.

So the average power consumed by an pure inductor is zero



P
V
I
+
+
-
-
0
90
180
270
360

Note: For a pure Inductive circuit (L circuit)

$v = V_m \sin\omega t$ $i = I_m \, Sin(\omega t - \pi/2)$ and $P_{av} = 0$

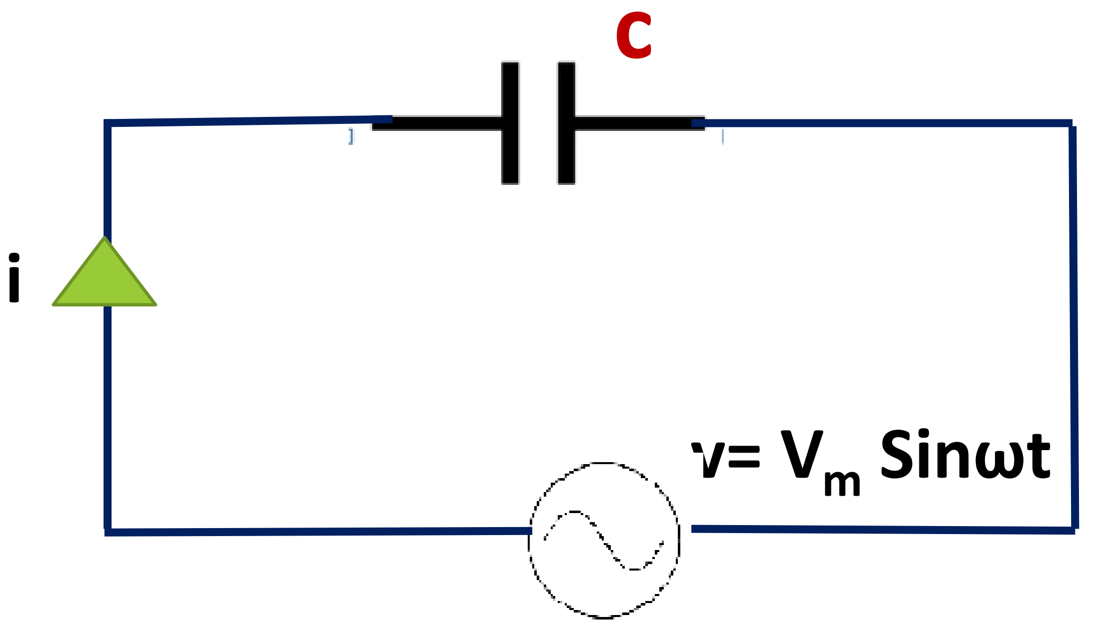
In polar form $V∟0$ and $I∟-90$

Impedance :

$$Z = \frac{V\angle 0}{I\angle -90} = X\angle 90 = 0 + jX_L = jX_L$$

Pure Capacitive circuit (C- circuit)

Consider an circuit consists of pure capacitor of 'C' farad's connected across an A C supply of 'V' volts.



$v = V_m \sin\omega t$ --------1

wkt $q = CV$

where q= charge in coloumb's and C- capacitance of a capacitor in farad's (F)

$q = c\, V_m \sin\omega t$

differentiating both sides w.r.t 't'

$$\frac{dq}{dt} = \frac{d(cV_m \sin\omega t)}{dt}$$

$$= cV_m \frac{d(\sin\omega t)}{dt}$$

wkt $i = \frac{dq}{dt}$

Therefore, $i = cV_m(\omega . \cos\omega t)$

$$i = c\omega V_m(\cos\omega t)$$

$$i = \omega c\, V_m \sin(\pi/2 + \omega t)$$

$$i = \frac{V_m}{1/\omega C} \sin(\pi/2+\omega t)$$

wkt capacitive reactance , $X_c = 1/\omega c = \frac{1}{2\pi f C}$

$$i = \frac{V_m}{X_c} \sin(\pi/2+\omega t)$$

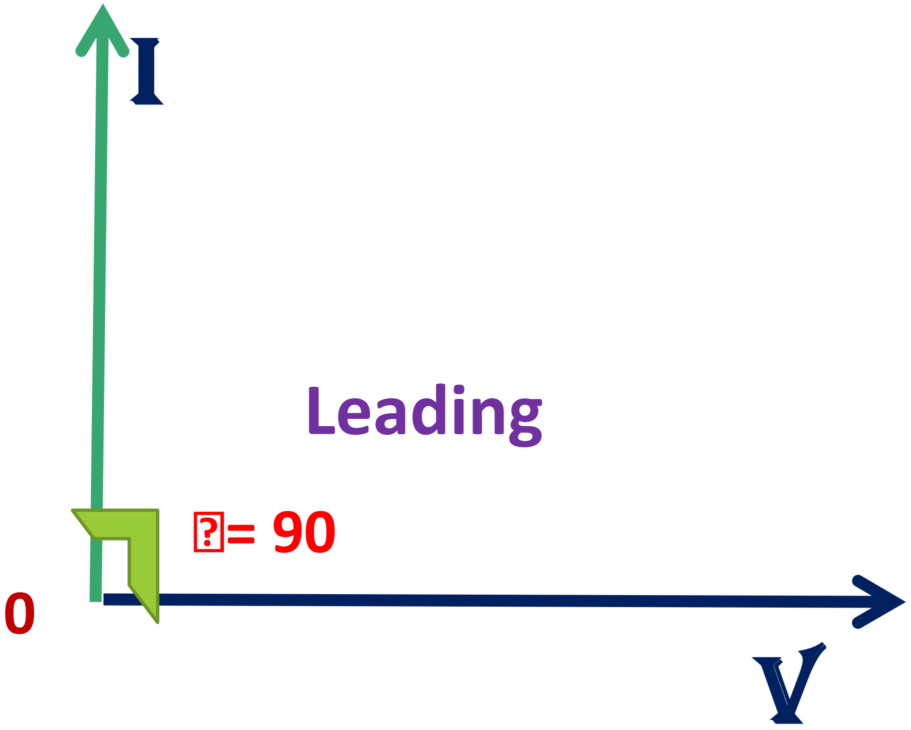
wkt , $I_m = \frac{V_m}{X_c}$

$$\boxed{i = I_m \sin(\omega t+\pi/2)} \text{------------------} 2$$

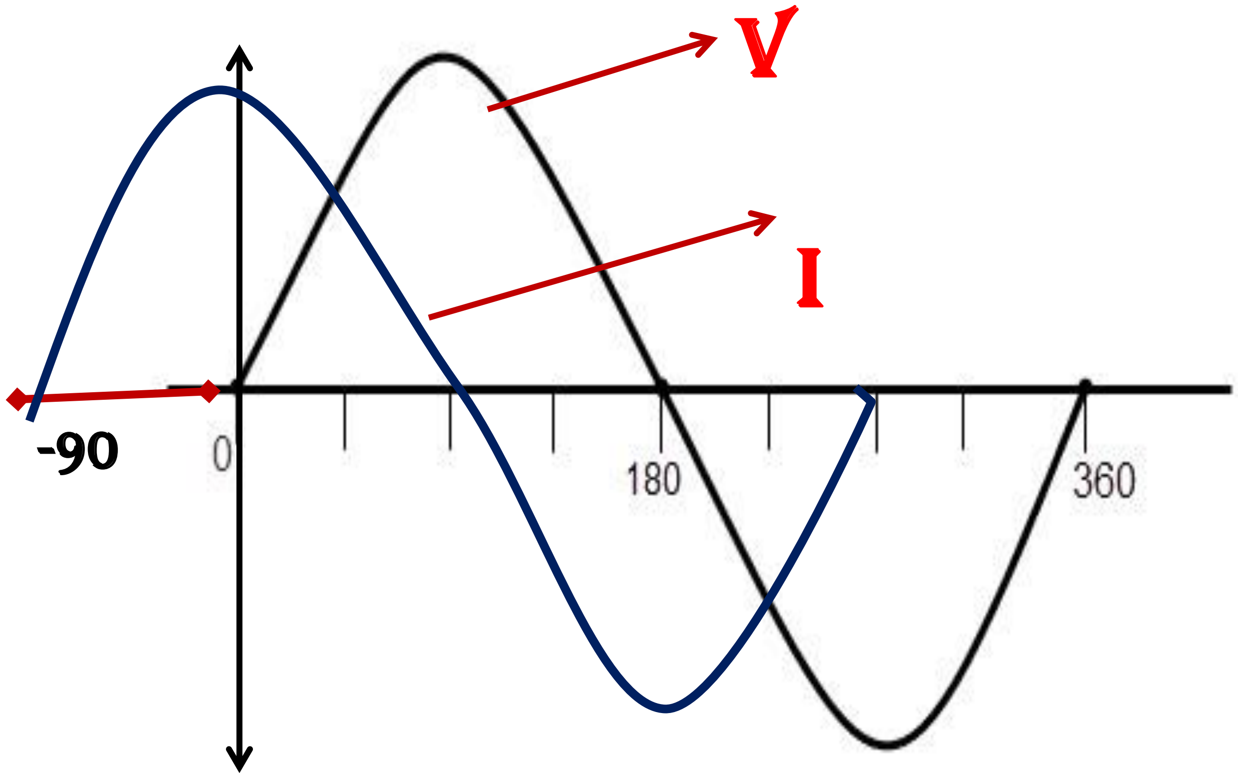
Comparing equation 1 and 2

In Pure Capacitive circuit the current **leads** the voltage exactly by 90 .

Phasor diagram



waveforms



Power of pure capacitive circuit

$P = vi \quad = V_m \sin\omega t \times I_m \sin(\pi/2 + \omega t)$

$P = V_m I_m \sin\omega t(\cos\omega t)$

$P = V_m I_m \dfrac{\sin 2\omega t}{2}$

As $2\sin\omega t\cos\omega t = \sin 2\omega t$

The average power consumed over a cycle

$$P_{av} = \int_0^{2\pi} \frac{V_m I_m}{2} \sin 2\omega t \, d\omega t \qquad = \frac{V_m I_m}{2}\left[-\frac{\cos 2\omega t}{2}\right]_0^{2\pi}$$

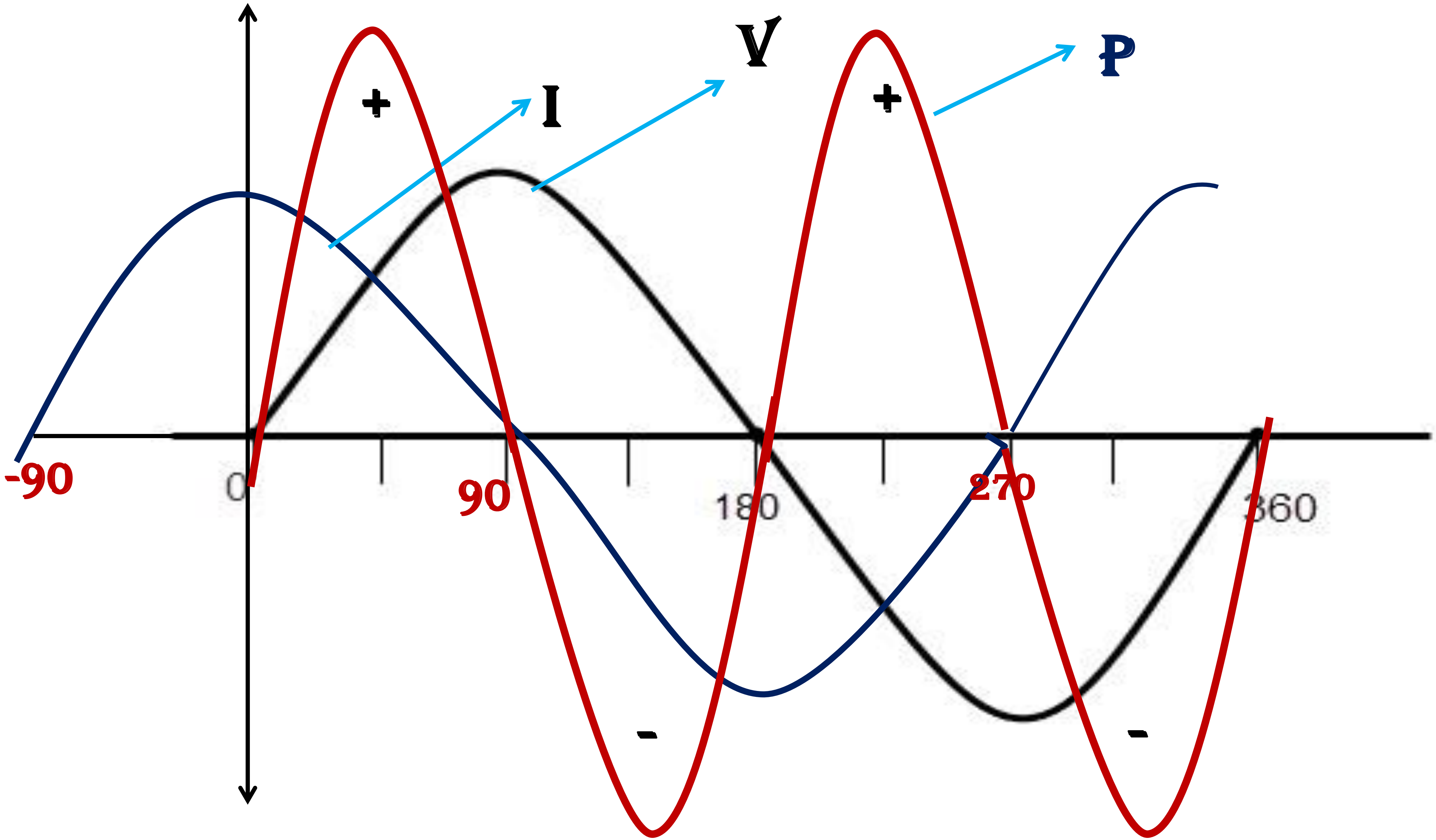
$$P_{av} = \frac{-V_m I_m}{4}[\cos 2(2\pi) - \cos(0)]$$

$$P_{av} = \frac{-V_m I_m}{4}[1 - 1] = 0$$

$$P_{av} = 0$$

The average value over a cycle of the fluctuating sine component is zero.

So the average power consumed by an pure capacitor is zero



I
V
P
+
+
-
-
-90
0
90
180
270
360

Note: For a pure capacitor circuit (C circuit)

$v = V_m \sin\omega t$ $i = I_m \sin(\omega t + \pi/2)$ and $P_{av} = 0$

In polar form- $V\llcorner 0$ and $I\llcorner 90$

Impedance: $z = R \pm jX$

$$Z = \frac{V\angle 0}{I\angle 90} = X_c\angle -90 = 0 - jX_c = -jX_c$$

Problems on R,L, and C circuit

Formulas

- **Impedance:** $Z = \frac{V}{I}$
- **For pure Resistive circuit :** v= $V_m$sinwt i = $I_m$sinwt and

  P= V_{rms} X I_{rms} =VI

  In polar form- V∟0 and I∟0 and Z = R$\angle 0 = R + j0$
- **For pure *Inductive* circuit :** v= $V_m$sinωt i = I_m Sin(ωt $-\pi/2$)

  P_{av} =0 **X∟ =2πfL** ohm's

  Z = X∟$\angle 90 =$ jX∟
- **For pure *capacitive* circuit :** v= $V_m$sinωt i = I_m Sin(ωt $+ \pi/2$)
- P_{av} =0 **Xc= 1/2πfc** ohm's
- Z = $X_c\angle - 90$ = $-jX_c$

Problems

1. A pure Inductor of Inductance 20mH is connected across an ac supply of 230V, 50Hz . Find the current drawn by an Inductor.

Solution: $L = 20 \times 10^{-3}$ H , V=230∟0 V and f=50Hz I=?

w.k.t $X_L = 2\pi f L = 2*3.142*50*20 \times 10^{-3}$

$Z = jX_L = j6.284\ \Omega$

Therefore $I = \frac{V}{Z} = \frac{230∟0}{j6.284} = -j36.60\ A = 36.6∟-90\ A$

a

2. The current drawn by pure capacitor 20μF is 1.382 A connected across an ac supply of 220V. Find the supply frequency.

Solution: $C=20 \times 10^{-6}$ F , V=220∟0V and I= 1.382A f=?

w.k.t for pure capacitor circuit

$$Z=-jX_c = \frac{V}{I} = \frac{220∟0}{1.382}$$

$$X_c = 159.18\Omega$$

Therefore $X_c = \dfrac{1}{2\pi f C}$

$$f = \frac{1}{2\pi C X_c} = \frac{1}{2*3.142*159.18*20 \times 10^{-6}} = 49.99 = 50\text{Hz}$$

3. The current drawn by pure Inductor of 30H is 2A. connected across an ac supply of 220V. Find the supply frequency.

Solution :L= 30 , I=2 A v=220 f=?

z=jX_L = V/I =220/2=110Ω and

X_L =2*3.142*f*L 110 =2*3.142*f*30

f =0.58HZ =1Hz

4. A pure capacitor of 50F is connected to a Ac source of 230V,50Hz .find current drawn by the circuit and also capacitive reactance.

Solution: c= 50 V=230 v f=50Hz I=? Xc=?

Xc=1/2*3.142*f*C = $6.3 \text{x} 10^{-5}$ Ω

Z= - jXc = -j $6.3 \text{x} 10^{-5}$

I=V/z= 230/ $-j6.3 \text{x} 10^{-5}$ = j3.6x 10^{6} A

4. A pure capacitor of 50F is connected to a Ac source of 230V,50Hz .find current drawn by the circuit and also capacitive reactance.

Solution: c= 50   V=230 v   f=50Hz   I=?   Xc=?

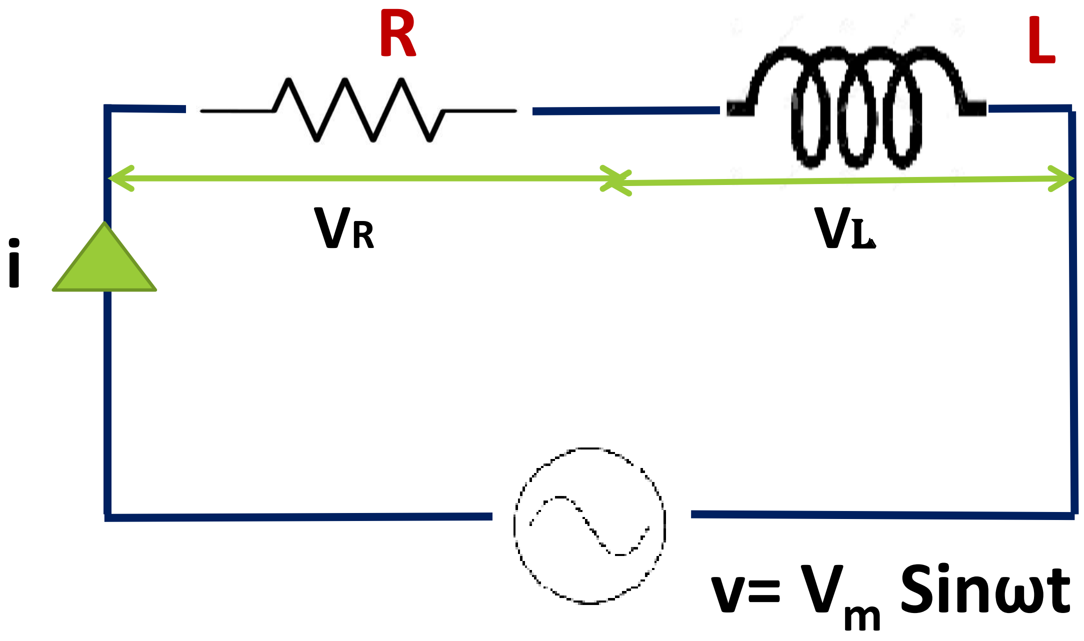
Xc=1/2*3.142*f*C = $6.3x10^{-5}$ Ω

Z= - jXc = -j $6.3x10^{-5}$

I=V/z= 230/ $-j6.3x10^{-5}$ = j3.6x 10^6 A

R-L Circuit

Consider an circuit of resistance 'R'Ω connected in series with an Inductor of 'L' Henry across an A C supply of 'V' volts.



$v = V_m \sin\omega t$ --------1

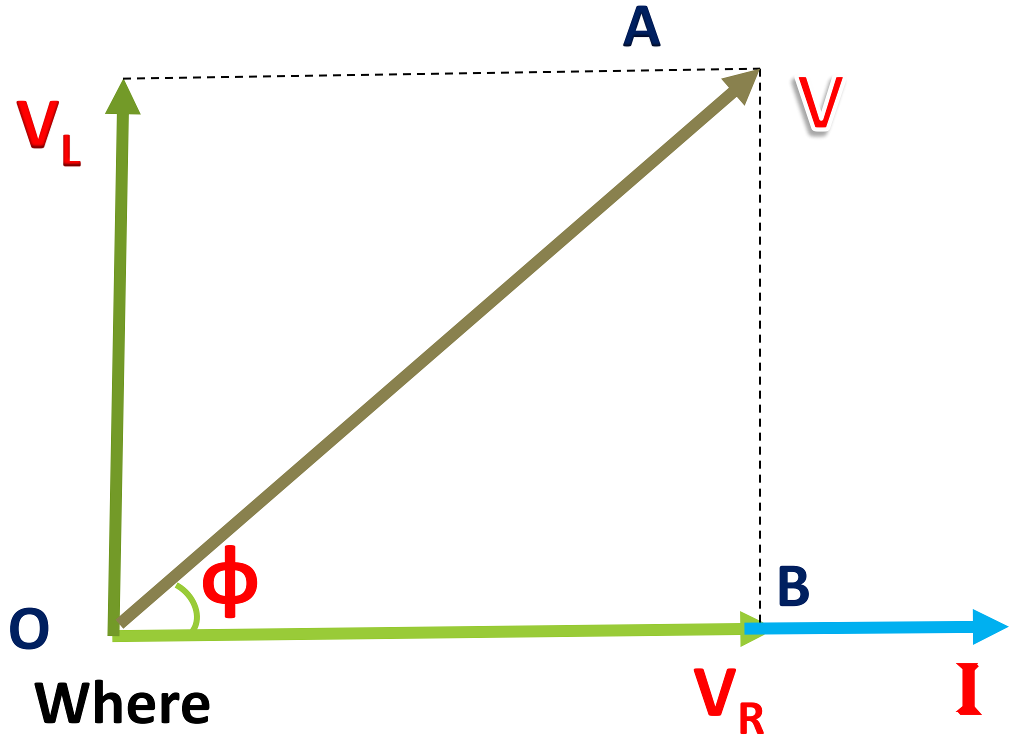
V_R = Voltage drop across 'R' and V_L= Voltage drop across 'L'

$V_R = IR$

$V_L = IX_L$,where $X_L = 2\pi fL\ \Omega$

The vector sum

$$\overline{V} = \overline{V}_R + \overline{V}_L$$



Where

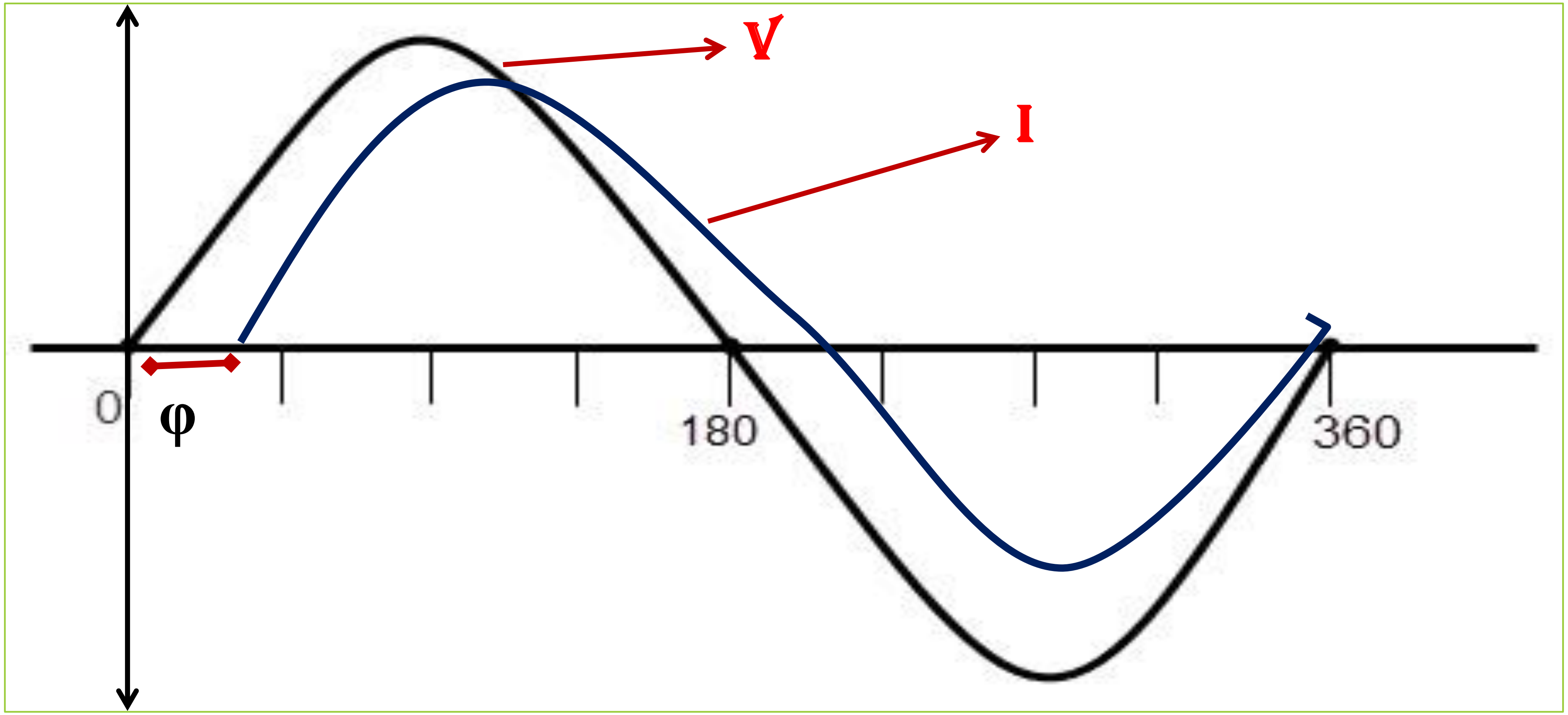
V_R is in phase with current

V_L leads current by angle of 90

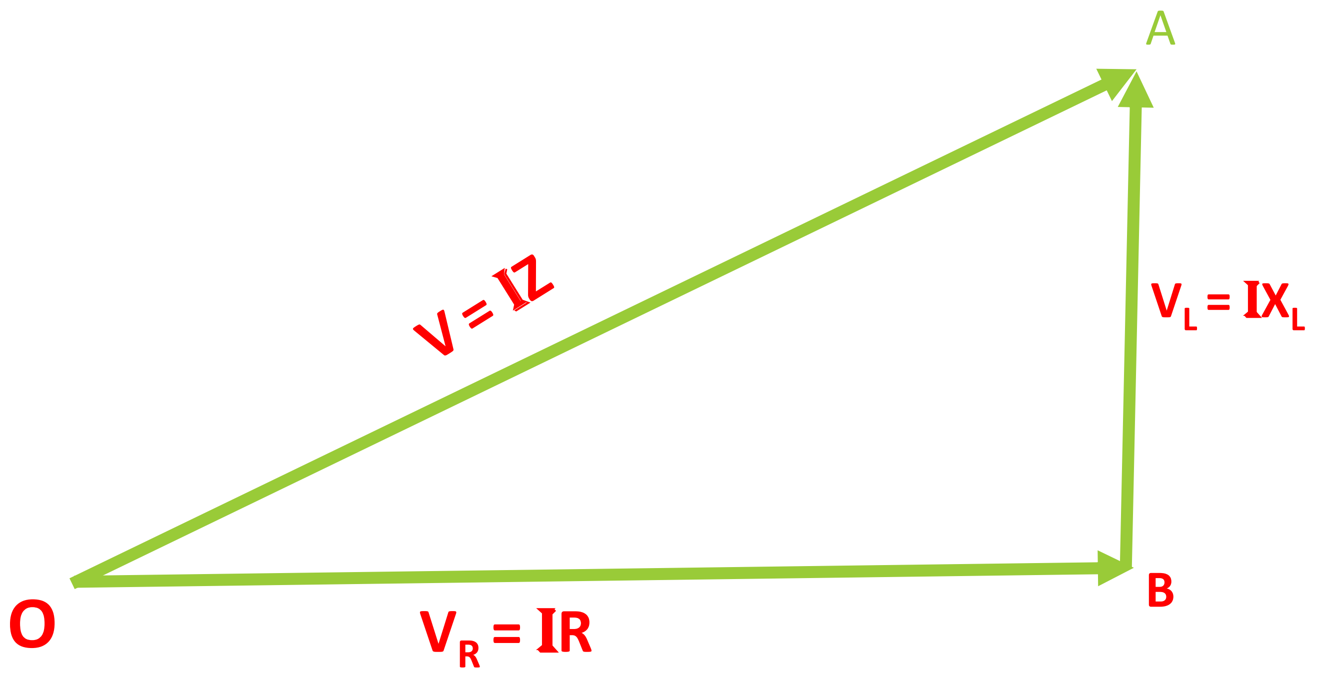
phasor diagram

$i = I_m \sin(\omega t - \phi)$------------ 2

by comparing equation 1 and 2 in R-L the current lags the voltage by an of ϕ


V
I
0
φ
180
360

Considering a triangle OAB



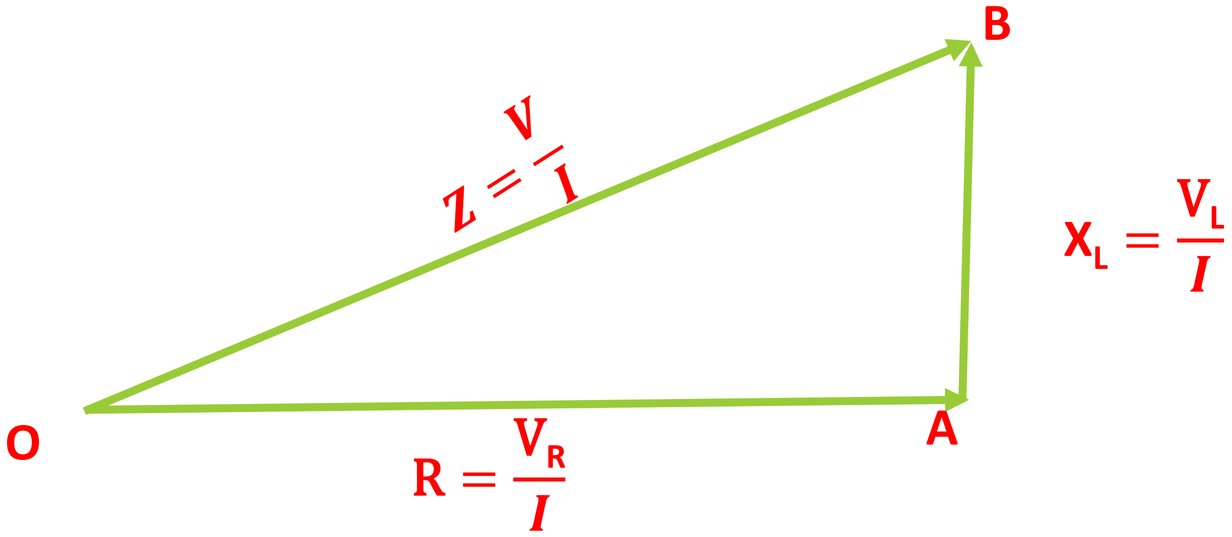
$$V = \sqrt{V_R^2 + V_L^2}$$

$$V = \sqrt{(IR)^2 + (I\,X_L)^2}$$

$$V = I\sqrt{R^2 + X_L^2}$$

Where $\sqrt{R^2 + X_L^2} = Z$**, impedance of the circuit**

Impedance Triangle



Z= R+jXL Ω $= \sqrt{R^2 + X_L{}^2}$

Power factor $\text{Cos}\,\phi = \frac{R}{Z}$

Power of R-L circuit

Instantenuous power **p=vi** $= V_m \sin\omega t \times I_m \sin(\omega t - \varphi)$

$$P = V_m I_m \sin\omega t \, \sin(\omega t - \varphi)$$

$$\sin A \sin B = \frac{\cos(A-B)}{2} - \frac{\cos(A+B)}{2}$$

$$P = V_m I_m \left[\frac{\cos(\omega t - (\omega t - \varphi))}{2} - \frac{\cos(\omega t + (\omega t - \varphi))}{2}\right]$$

$$P = \frac{V_m I_m}{2}[\cos(\omega t - \omega t + \varphi) - \cos(\omega t + \omega t - \varphi)]$$

$$P = \frac{V_m I_m}{2}[\cos(\varphi) - \cos(2\omega t - \varphi)]$$

The average power consumed over a cycle

$$P_{av} = \frac{\int_0^{2\pi} \frac{V_m I_m}{2}[\cos(\varphi) - \cos(2\omega t - \varphi)]\, d\omega t}{2\pi}$$

$$P_{av} = \frac{V_m I_m}{4\pi}\left[\int_0^{2\pi} \cos\varphi \, d\omega t - \int_0^{2\pi} \cos(2\omega t - \varphi)\, d\omega t\right] \xrightarrow{} 0$$

{ the average value of cos (2wt-ϕ) =0}.

$$P_{av} = \frac{V_m I_m \cos\varphi}{4\pi}[\omega t]_0^{2\pi} = \frac{V_m I_m \cos\varphi}{4\pi}[2\pi - 0]$$

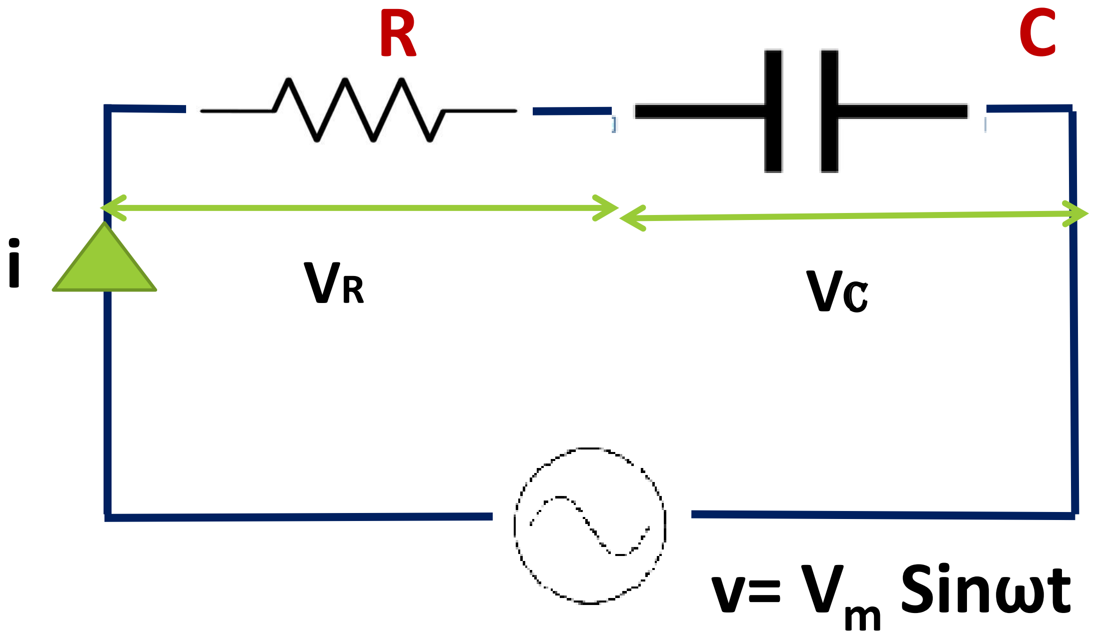
$$P_{av} = \frac{V_m I_m \cos\varphi}{4\pi}[2\pi] = \frac{V_m I_m \cos\varphi}{2}$$

$$P_{av} = \frac{V_m I_m \cos\varphi}{\sqrt{2}\sqrt{2}} = V_{rms} I_{rms} \cos\varphi$$

$$P = V\, I \cos\varphi$$

R-C Circuit

Consider an circuit of resistance ‘R’Ω connected in series with an Capacitor of ‘C’ Farads across an A C supply of ‘V’ volts.



$V = V_m \sin\omega t$ --------1

V_R = Voltage drop across ‘R’ and **V_C= Voltage drop across ‘C’**

$V_R = IR$

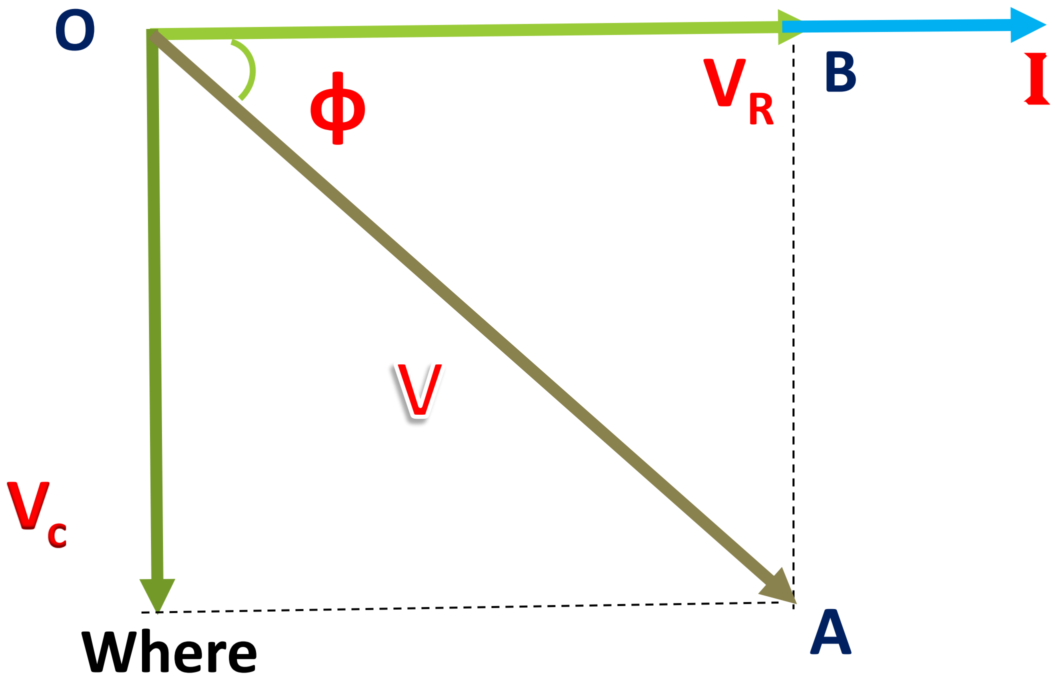
$V_c = IX_c$

where $X_c = 1/2\pi fc\ \Omega$

The vector sum

$$\overline{V} = \overline{V}_R + \overline{V}c$$

Phasor diagram



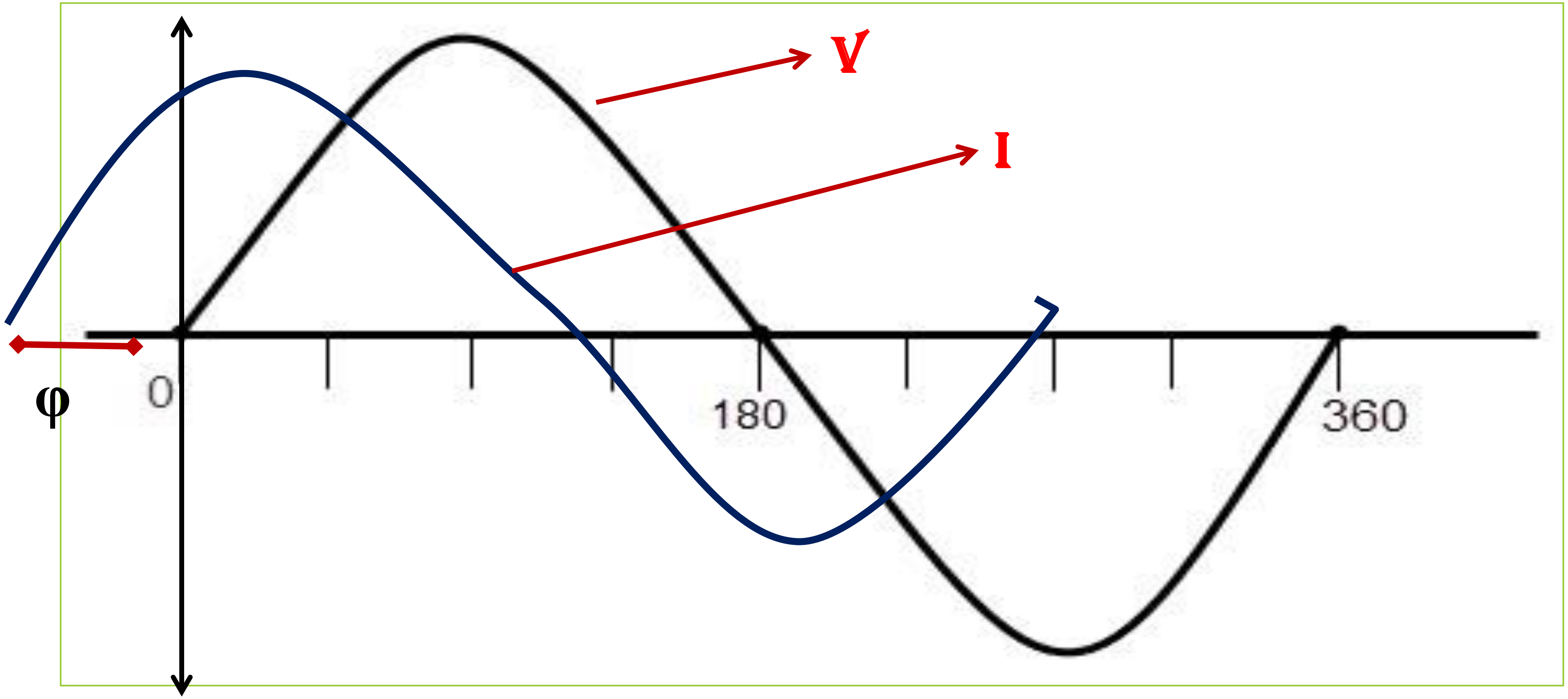
Where

V_R is in phase with current

V_c lags current by angle of 90

$$i = I_m \sin(\omega t + \phi) \text{---------}\ 2$$

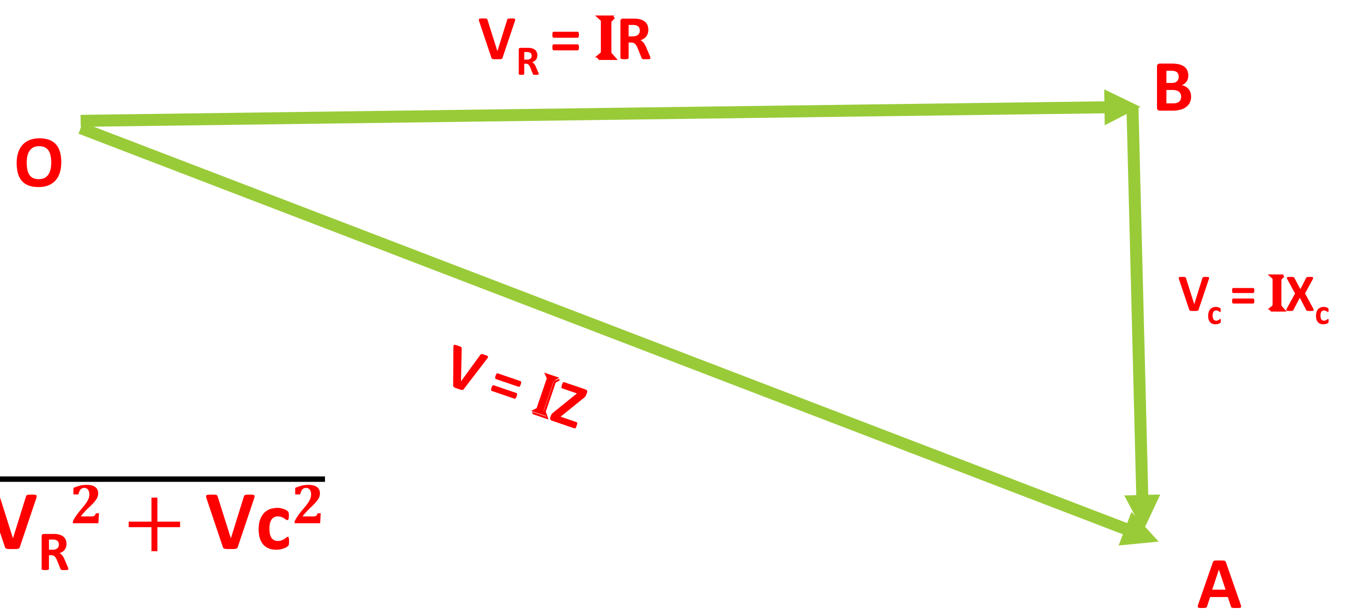
by comparing equation 1 and 2 in R-C the current leads the voltage by an of ϕ



V
I
φ
0
180
360

A

Considering a triangle OAB



$$V = \sqrt{V_R^{\,2} + Vc^2}$$

$$V = \sqrt{(IR)^2 + (I\,X_c)^2} \qquad V = I\sqrt{R^2 + X_c^{\,2}}$$

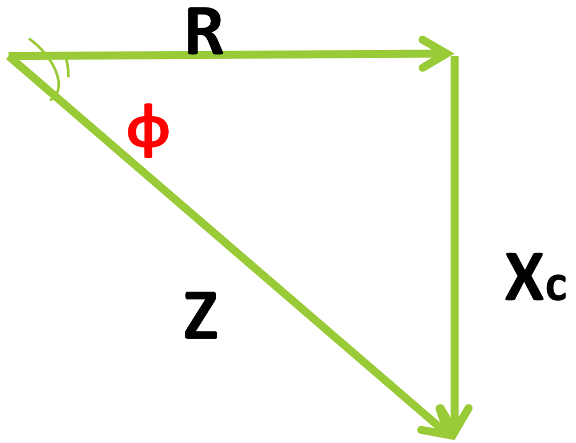
Where $\sqrt{R^2 + xc^{\,2}} = Z,$ **impedance of the circuit**

Impedance triangle

Z= R-jXc Ω $=\sqrt{R^2+Xc^2}$

Z = $\frac{V\angle 0}{I\angle \varphi}$ = Z $\angle -\varphi$

Power factor Cos ϕ = $\frac{R}{z}$



Power of circuit

Instantenuous power $p = vi$ $= V_m \sin\omega t \times I_m \sin(\omega t + \varphi)$

$$P = V_m I_m \sin\omega t \,\sin(\omega t + \varphi)$$

$$\sin A \sin B = \frac{\cos(A-B)}{2} - \frac{\cos(A+B)}{2}$$

$$P = V_m I_m \left[\frac{\cos(\omega t - (\omega t + \varphi))}{2} - \frac{\cos(\omega t + (\omega t + \varphi))}{2}\right]$$

$$P = \frac{V_m I_m}{2}[\cos(\omega t - \omega t - \varphi) - \cos(\omega t + \omega t + \varphi)]$$

$$P = \frac{V_m I_m}{2}[\cos(-\varphi) - \cos(2\omega t + \varphi)]$$

The average power consumed over a cycle

$$P_{av} = \frac{\int_0^{2\pi} \frac{V_m I_m}{2}[\cos(\varphi) - \cos(2\omega t - \varphi)]\, d\omega t}{2\pi}$$

$$P_{av} = \frac{V_m\, I_m}{4\pi}\left[\int_0^{2\pi} \cos\varphi \, d\omega t - \int_0^{2\pi} \cancel{\cos(2\omega t + \varphi)\, d\omega t}^{\,0}\right]$$

{ the average value of cos (2wt-ϕ) =0}.

$$P_{av} = \frac{V_m\, I_m \cos\varphi}{4\pi}\left[\omega t\right]_0^{2\pi} = \frac{V_m\, I_m \cos\varphi}{4\pi}\left[2\pi - 0\right]$$

$$P_{av} = \frac{V_m\, I_m \cos\varphi}{\cancel{4\pi}}\left[\cancel{2\pi}\right] = \frac{V_m\, I_m \cos\varphi}{2}$$

$$P_{av} = \frac{V_m\, I_m \cos\varphi}{\sqrt{2}\,\sqrt{2}} = V_{rms}\, I_{rms} \cos\varphi$$

$$P = V\, I \cos\varphi$$

Power factor (pf):

It is defined as the cosine of angle between applied voltage and current. I,e **cosϕ**

where φ – is the angle between applied voltage and current

Significance :-

1) The Power factor gives the nature of the circuit by which we can decide whether it is resistive capacitive or inductive
2) The nature of the Power factor is always determined by the position of current w.r.t to voltage
3) The Power factor varies between zero to one.

Power

<u>**Real or True or Active power(P)**</u>: It is defined as the product of applied voltage an active component of the current and the unit is **Watts(W)**

$$P = V\,I\cos\phi \quad \textbf{watts}$$

Apparent Power (S): It is defined as the product of RMS value of voltage and current and unit is **VA(volt ampere)**

$$S = V\,I \quad \textbf{VA}$$

<u>**Reactive power(Q)**</u>: It is defined as the product of applied voltage an reactive component of the current and the unit is **VAR(Volt ampere reactive**

$$Q = V\,I\sin\phi \quad \textbf{VAR}$$

Formulas of R-L and R- C circuit

$V = \frac{V_m}{\sqrt{2}}$ $I = \frac{I_m}{\sqrt{2}}$ $\omega = 2\pi f$

R-L circuit (current lags voltage by an angle of ϕ)

$V = V_m \sin\omega t$ and $I = I_m \sin(\omega t - \phi)$

$Z = R + jX_L \ \Omega = \sqrt{R^2 + X_L^2}$

R-C circuit((current leads voltage by an angle of ϕ)

$V = V_m \sin\omega t$ and $I = I_m \sin(\omega t + \phi)$

$Z = R - jX_c \ \Omega = \sqrt{R^2 + X_c^2}$

Power factor $\cos\phi = \frac{R}{z}$

Active power	$P = V\,I\cos\phi$	watts
Apparent power	$S = V\,I$	VA
Reactive power	$Q = V\,I\sin\phi$	VAR

1. A circuit consists of resistance of 20Ω and an Inductance of 50mH is connected in series across the supply of 250V ,50Hz . Determine the current ,power consumed by the circuit and power factor of the circuit.

Given : **R= 20Ω , L= 50mH = 50 $x10^{-3}$ and V=250$\angle 0$ f=50Hz**

I=? P =? Pf=?

Wkt The Impeadance of R –L circuit is

$$Z = R + jX_L$$

Therefore $X_L = 2\pi fL = 2 * 3.142 * 50 * 50 \ x10^{-3} = 15.7\Omega$

$$Z = R + jX_L = 20 + j15.7\ \Omega = 25.42 \angle 38.18$$

$$I = \frac{V}{Z} = \frac{250\angle 0}{25.42\angle 38.18} = 9.83\angle 0 - 38.18$$

$$\boxed{I = 9.83\angle -38.18 \text{ A}}$$ (-ve indicates lagging)

The current lags the voltage by an angle of

Φ = voltage angle –current angle = 0 - (-38.18) = 38.18

$$Pf = \cos\phi = \cos(38.18) = 0.78 \text{ lagging}$$

$$P = VI\cos\phi = 250 \times 9.83 \times 0.78 =$$ **1916.85 W**

5. A non-inductive resistor of 10 Ω is in series with 100 μF capacitor across 250 V 50 Hz AC circuit. Determine the current taken by the capacitor & Power factor of the circuit

Given:- $R = 10\Omega$

$C = 100\mu F = 100 \times 10^{-6} F$

$V = 250V$

$f = 50Hz$

$I = ?$ $P = ?$

The circuit elements are R & C

$\therefore Z = R - jX_C$

$$X_C = \frac{1}{2\pi fC} = \frac{1}{2 \times 3.142 \times 50 \times 100 \times 10^{-6}}$$

$$\boxed{X_C = 31.18\ \Omega}$$

$$\boxed{\begin{aligned} Z &= 10 - j31.18 \\ &= 33.37 \angle -72.56 \end{aligned}}$$

$$I = \frac{V}{Z} = \frac{250\angle 0}{33.37\angle -72.56} = 7.49\angle 0-(-72.56)$$

$$\boxed{I = 7.49\angle 72.56\ A}$$

The current is leading by an angle of

$\phi =$ (0-72.56) = -72.56

$\cos\phi = \cos 72.56 = 0.3$ leading

$P = VI\cos\phi = 250 \times 7.49 \times 0.3$ = 561.8W

3.The Alternating Voltage of V=160+j120 V is applied to a circuit and the current is given by I =6- j2 A .Find the values of circuit elements assuming supply frequency as 50Hz and also determine the Power factor

Solution : Given **V= 160+j120 V** **I = 6-j2 A**

Z=? , f=50Hz And cosϕ =?

V=160+j120 = 200∟36.85 V

I =6- j2 = 6.32∟-18.43 A **(‘-’ negative signs indicates lagging)**

The current lags the voltage by an angle of

Φ = 36.86- (-18.43)= 55.29

Therefore this the R-L circuit

$$Z = \frac{V}{I} = \frac{200\angle 36.85}{6.32\angle -18.43} = 31.64\angle 55.29 = 18+j26\ \Omega$$

$$Z = 18+j26\ \Omega = R + jX_L$$

The circuit elements are R and L

$\boxed{R = 18\Omega}$ **and** $X_L = 26\ \Omega$

$$X_L = 2\pi f L$$

$$L = \frac{X_L}{2\pi f} = \frac{26}{2 * 3.142 * 50} \quad \boxed{=0.082H}$$

Power factor pf= cosϕ= cos(55.29) = 0.56 Lagging

2.The Alternating Voltage of V=160+j120 V is applied to a circuit and the current I =6+ j8 A drawn from the supply .Find the values of circuit elements assuming supply frequency as 50Hz and also determine the Power consumed

Solution : Given **V= 160+j120 V** **I = 6+j8 A**

Z=? , f=50Hz And P =?

V=160+j120 = **$200\angle 36.85$ V**

I =6+ j8 = **$10\angle 53.13$ A** **('+' positive signs indicates leading)**

The current leads the voltage by an angle of

$\Phi = 36.85 - 53.13 = -16.27$

Therefore this the R-c circuit

$$Z = \frac{V}{I} = \frac{200\angle 36.85}{10\angle 53.13} = \quad 20\angle -16.27 \quad = 19.9 - j\,5.6\Omega$$

$$Z = 19.9 - j5.6\ \Omega = R - jX_c$$

The circuit elements are R and C

$\boxed{R = 19.9\Omega}$ and $X_c = 5.6\ \Omega$

$$X_c = 1/2\pi f C$$

$$C = \frac{1}{2\pi f X_c} = \frac{1}{2 * 3.142 * 50 * 5.6} \quad \boxed{= 0.56mF}$$

Power factor pf= cosϕ= cos(16.27) = 0.95 Leading

P = V I cosϕ= 200 X 10 x cos(16.27) = 1919.90 Watts

5. Given V= 200 sin 377t volts and I = 8 sin (377t-30^0)A for an AC circuit determine :i) frequency ii) power factor iii)True power iv) Apparent power v) Reactive power.

Given : V= 200 sin 377t volts I = 8 sin (377t-30^0)

From above equation $V = V_m \sin\omega t$ $I = I_m \sin(\omega t - \phi)$

V_m=200 V , I_m = 8 and $\omega = 2\pi f = 377$

$$f = \frac{377}{2\pi} = \frac{377}{2*3.142} = 60\text{Hz}$$

ϕ is an angle between voltage and current

Therefore ϕ = 0-(-30) = 30^0

Pf= cos ϕ = cos(30) = 0.866 lagging

$V_m = 200\ V$, $I_m = 8A$

$$V = \frac{V_m}{\sqrt{2}} = \frac{200}{\sqrt{2}} = 141.42\ V$$

$$I = \frac{I_m}{\sqrt{2}} = \frac{8}{\sqrt{2}} = 5.656\ A$$

True power $P = V\ I\cos\phi$ = 141.42 * 5.656* cos(30) = 692.8 Watts

Apperent power $S = VI$ = 141.42 * 5.656 = 800 VA

Reactive Power $Q = V\ I\sin\phi$ = 141.42 * 5.656* sin(30) = 400 VAR

6. Given V= 200 sin (377t -30) volts and I = 8 sin (377t+90)A for an AC circuit determine :i) frequency ii) power factor iii)True power iv) Apparent power v) Reactive power

Given : **V= 200 sin (377t -30) volts I = 8 sin (377t$+90^0$)**

From above equation V= $V_m\sin(\omega t - \phi)$ I = $I_m\sin(\omega t + \phi)$

V_m=200 V , I_m = 8 $\omega = 2\pi f = 377$

$$f = \frac{377}{2\pi} = \frac{377}{2*3.142} = 60Hz$$

ϕ is an angle between voltage and current

Therefore $\phi = -30 -90 = -120^0$

Pf= cos ϕ = cos(120) = **0.5 leading**

$V_m = 200\ V, \quad I_m = 8A$

$$V = \frac{V_m}{\sqrt{2}} = \frac{200}{\sqrt{2}} = 141.42$$

$$I = \frac{I_m}{\sqrt{2}} = \frac{8}{\sqrt{2}} = 5.656$$

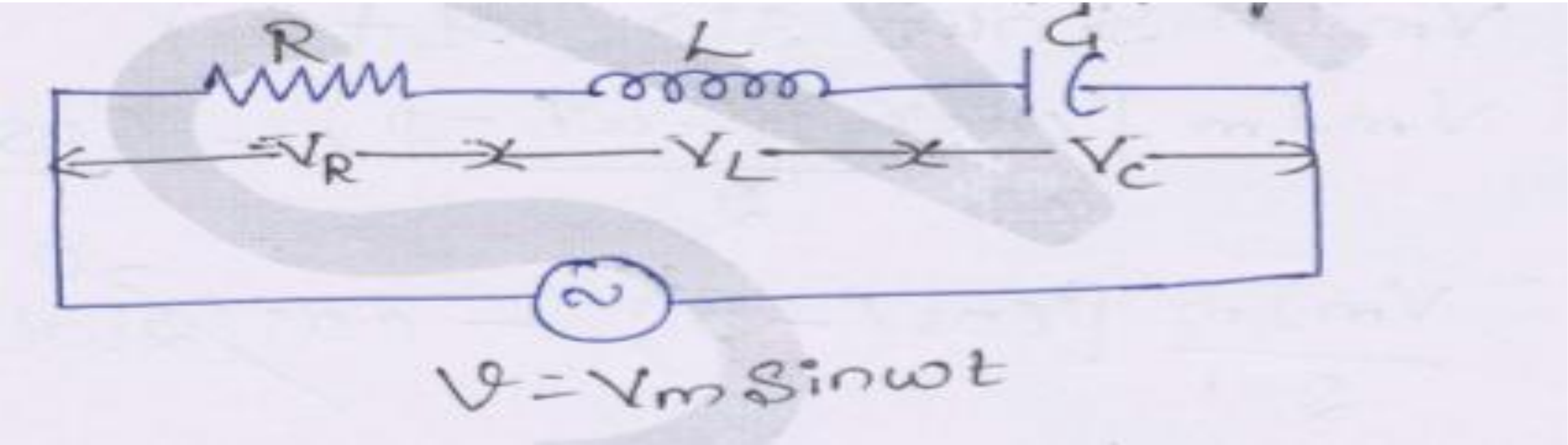
True power $P = V\, I \cos\phi$ **= 141.42 * 5.656* cos(120) = 399.9Watts**

Apperent power $S = VI$ **= 141.42 * 5.656 = 800 VA**

Reactive Power $Q = V\, I \sin\phi$ **= 141.42 * 5.656* sin(120) = 692.70VAR**

R-L- C Circuit

Consider an circuit of resistance 'R'Ω connected in series with an Inductance of 'L' Henry and capacitance of 'c' farads across an A C supply of 'V' volts.



$v = V_m \sin\omega t$ --------1

V_R = Voltage drop across 'R' ,V_L = Voltage drop across 'L' and

V_C= Voltage drop across 'C'

$V_R = IR$

$V_L = IX_L$ where $X_L = 2\pi fL\ \Omega$

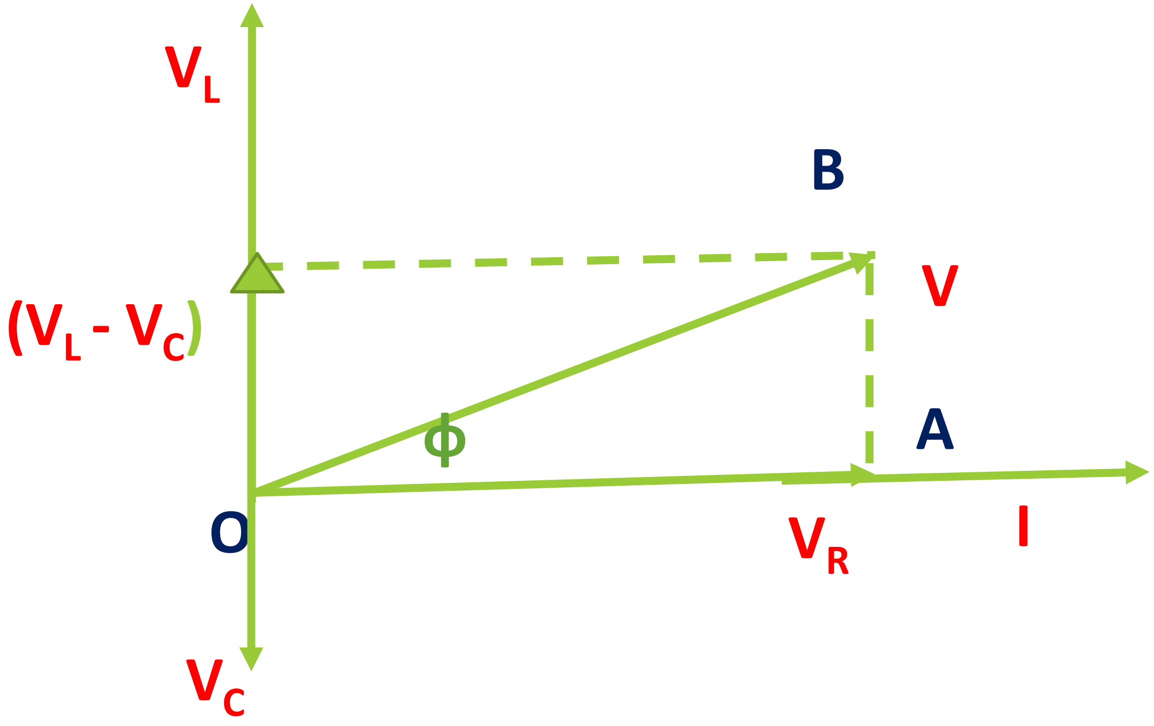
$V_c = IX_c$ where $X_c = 1/2\pi fc\ \Omega$

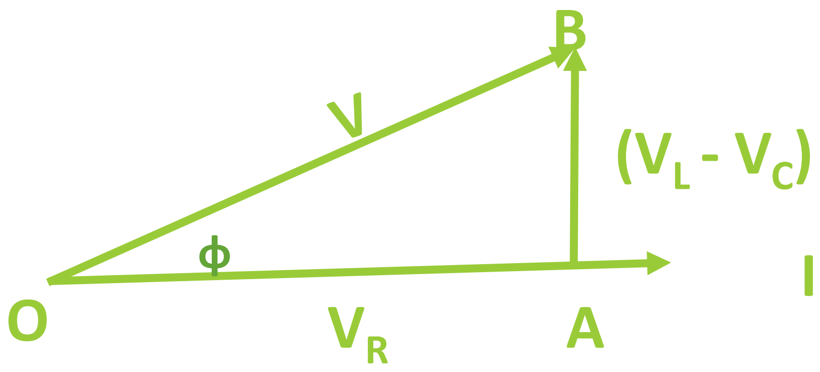
The vector sum

$$\bar{V} = \bar{V}_R + \bar{V}_L + \bar{V}_c$$

Phasor diagram

Case i) when $X_L > X_C$ and $V_L > V_C$





By phasor diagram the current lags the voltage by an angle of ‘ϕ’ .
Therefore when $X_L > X_C$ the circuit behaves like inductive circuit.

Consider a triangle OAB

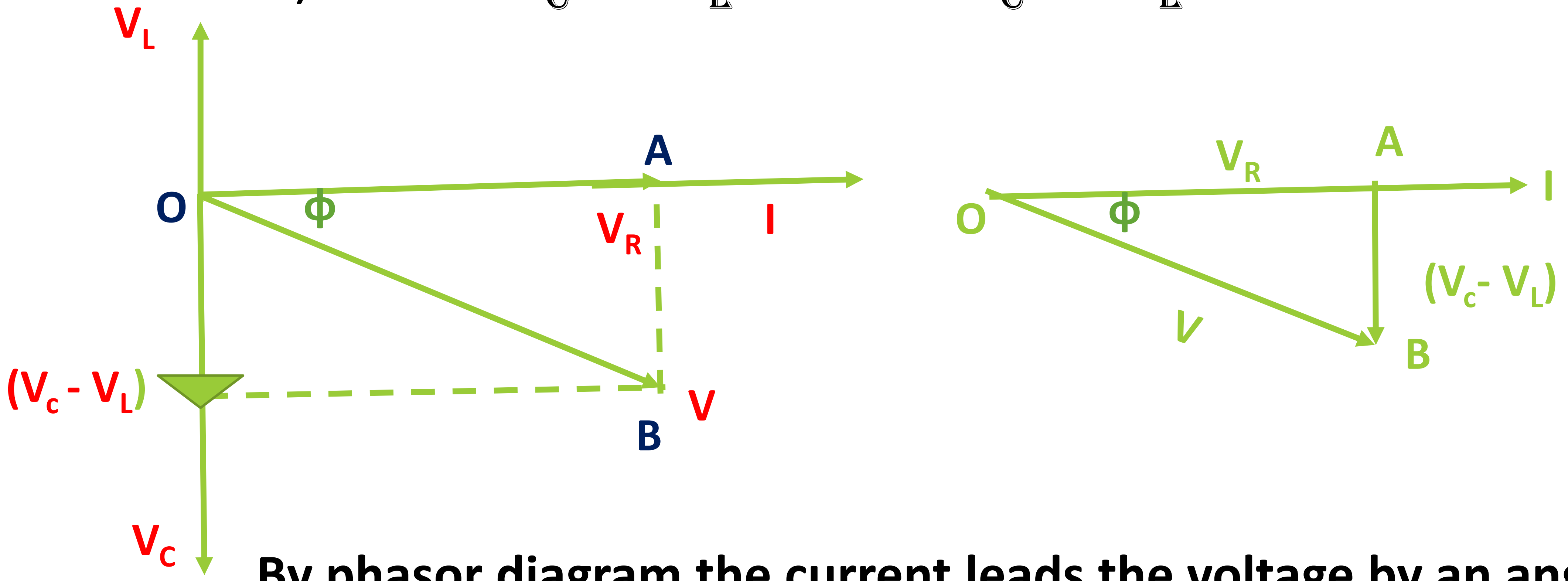
$$V = \sqrt{V_R^2 + (V_L - V_C)^2} = \sqrt{(IR)^2 + (IX_L - IX_C)^2}$$

$$= I\sqrt{R^2 + (X_L - X_C)^2}$$

$$\boxed{V = IZ} \quad \text{where} \quad Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$Z = R + jX_L - jX_C \ \Omega$

Case ii) when $X_C > X_L$ and $V_C > V_L$



By phasor diagram the current leads the voltage by an angle of 'ϕ' .

Therefore when $X_C > X_L$ the circuit behaves like capacitive circuit.

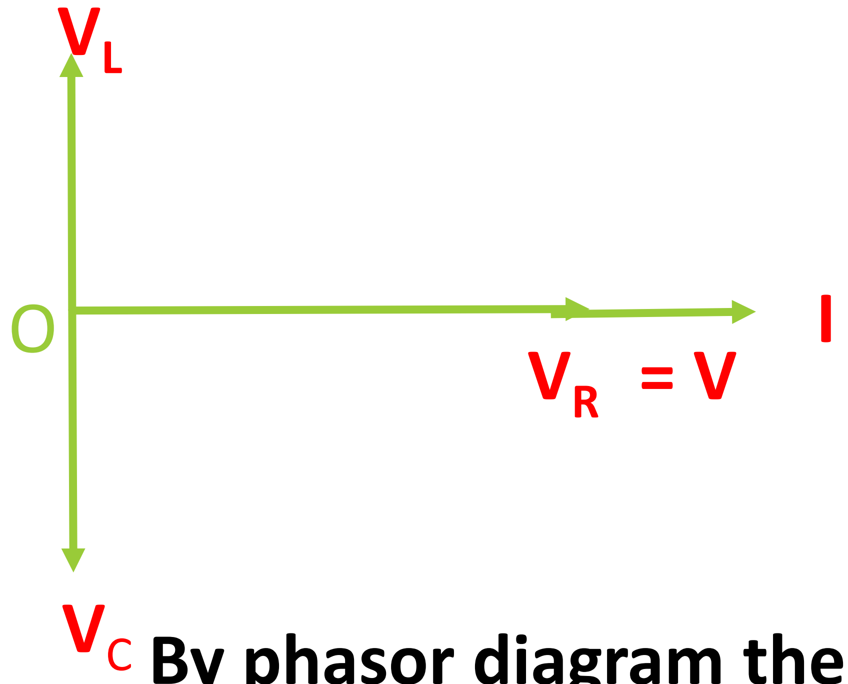
$$V = \sqrt{V_R^2 + (V_C - V_L)^2} = \sqrt{(IR)^2 + (IX_C - IX_L)^2}$$

$$V = I\sqrt{R^2 + (X_C - X_L)^2}$$

$$\boxed{V = IZ} \quad \text{where } Z = \sqrt{R^2 + (X_C - X_L)^2}$$

$Z = R + jX_L - jX_C \;\; \Omega$

Caseiii) when $Xc=X_L$ and $V_C = V_L$



$V = V_R = IR$ $\boxed{Z=R}$

By phasor diagram the current is in phase with the voltage .

Therefore when $Xc=X_L$ the circuit behaves like Resistive circuit.

∴ Case i) when $X_L > X_C$ it is inductive circuit

Case ii) when $X_C > X_L$ it is capacitive circuit

case iii) when $X_C = X_L$ resistive circuit

Formulas on R-L-C Circuit

R-L-C Circuit When $X_L > X_c$ $Z = R + jX_L - jX_c \ \Omega$

$$= \sqrt{R^2 + (XL - Xc)^2}$$

When $X_c > X_L$ $Z = R + jX_L - jX_c \ \Omega$ $= \sqrt{R^2 + (Xc - XL)^2}$

When $X_L = X_c$ $Z = R$

$Z = V/I \ \Omega$

Power factor $\cos\phi = \frac{R}{z}$

Active power $P = V\,I\cos\phi$ **watts**

Apparent power $S = V\,I$ **VA**

Reactive power $Q = V\,I\sin\phi$ **VAR**

7. A single phase voltage of 220+j0 V,50Hz applied across the circuit of resistance of 20Ω and an Inductance of 20mH and capacitance of 120µF are connected in series. Determine the current ,power consumed by the circuit , power factor of the circuit and energy stored in an Inductor and draw phasor diagram.

Given: V=220+j0 = $220\angle 0$, f=50Hz

R= 20Ω , L= 20mH = 20 $\times 10^{-3}$, C= 120µF = 120 $\times 10^{-6}$ F and I=? P =? Pf=? and energy stored in an Inductor E=?

The circuit elements are R, L and C

Therefore Z= R +jX_L – jXc

$$X_L = 2\pi f L = 2 \times 3.142 \times 50 \times 20 \times 10^{-3} = 6.28\Omega$$

$$X_c = \frac{1}{2\pi f C} = \frac{1}{2 * 3.142 * 50 * 120 \times 10^{-6}} = 26.52\Omega$$

$Z = R + jX_L - jXc = 20 + j\,6.28 - j26.52$

$Z = 20 - j20.24\ \Omega$ $= \boxed{28.45\angle -45.3}$

$I = \frac{V}{Z} = \frac{220\angle 0}{28.45\angle -45.3} =$ $\boxed{= 7.7\angle 45.3\ \ A}$

The current is leading the voltage by an angle of

$\phi = 0 - 45.3 = -45.3$

Power factor pf= $\cos\phi = \cos(45.3) = 0.703$ Leading

$P = V\,I\cos\phi = 220 \times 7.7 \times \cos(45.3) =$ 1191.55Watts

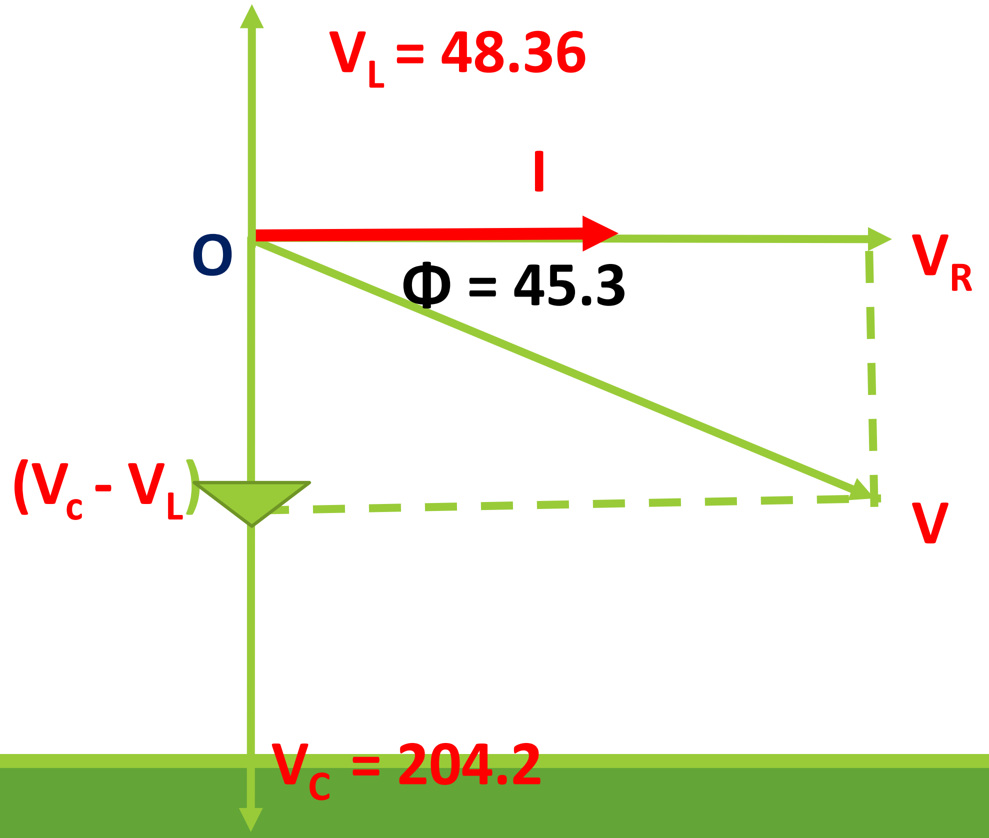
$E = \frac{1}{2} L I^2 = \frac{1}{2} \times 20 \times 10^{-3} \times 7.7^2 =$ 0.59 Joules

Phasor diagram

$V_R = IR = 7.7x20 = 154V$

$V_L = IXL = 7.7x\ 6.28 = 48.36V$

$V_c = Ixc = 7.7X\ 26.52 = 204.2V$



7) A Series RLC circuit is composed of 100Ω Resistance, 1 H inductance & $5\mu F$ capacitance. A voltage $V(t) = 1414.1 \sin 377t$ volts is applied to the circuit determine the current impedance & the voltage drop across each element

- Given:- $V(t) = 1414.1 \sin 377t$ volts

$V_m \sin \omega t$

$R = 100\Omega$ $L = 1H$ $C = 5\mu F = 5\times10^{-6} F$

$V_m = 1414.1 V.$ $\omega = 2\pi f = 377$

$377 = 2\pi \times 3.142 \times f$

$f = 377 / 2\pi \times 3.142$

$\boxed{f = 60 Hz}$

$V_{rms} = \frac{V_m}{\sqrt{2}} = \frac{1414.1}{\sqrt{2}}$

$\boxed{V_{rms} = V = 1000 V}$

$$Z = R + jX_L - jX_C$$

$$Z = 100 + j376.8 - j530.78$$

$$Z = 100 - j246 -$$

$$\boxed{Z = 183.6\angle{-56.99}}\ \Omega$$

$$X_L = 2\pi f L$$
$$= 2 \times 3.142 \times 60 \times 1$$

$$\boxed{X_L = 376.8\,\Omega}$$

$$X_C = \frac{1}{2\pi f c} = \frac{1}{2 \times 3.142 \times 60 \times 5 \times 10^{-6}}$$

$$\boxed{X_C = 530.78\,\Omega}$$

$$I = \frac{V}{Z} = \frac{1000\angle 0}{183.6\angle{-56.99}} = 5.44\angle 56.99$$

$$\phi = 56.99$$

$$V_R = IR = 5.44 \times 100 = 544\,V$$

$$V_L = IX_L = 5.44 \times 376.8 = 2034.79\,V$$

$$V_C = IX_C = 5.44 \times 530.78 = 2866.21\,V.$$

Module -2b : Three Phase AC Circuit

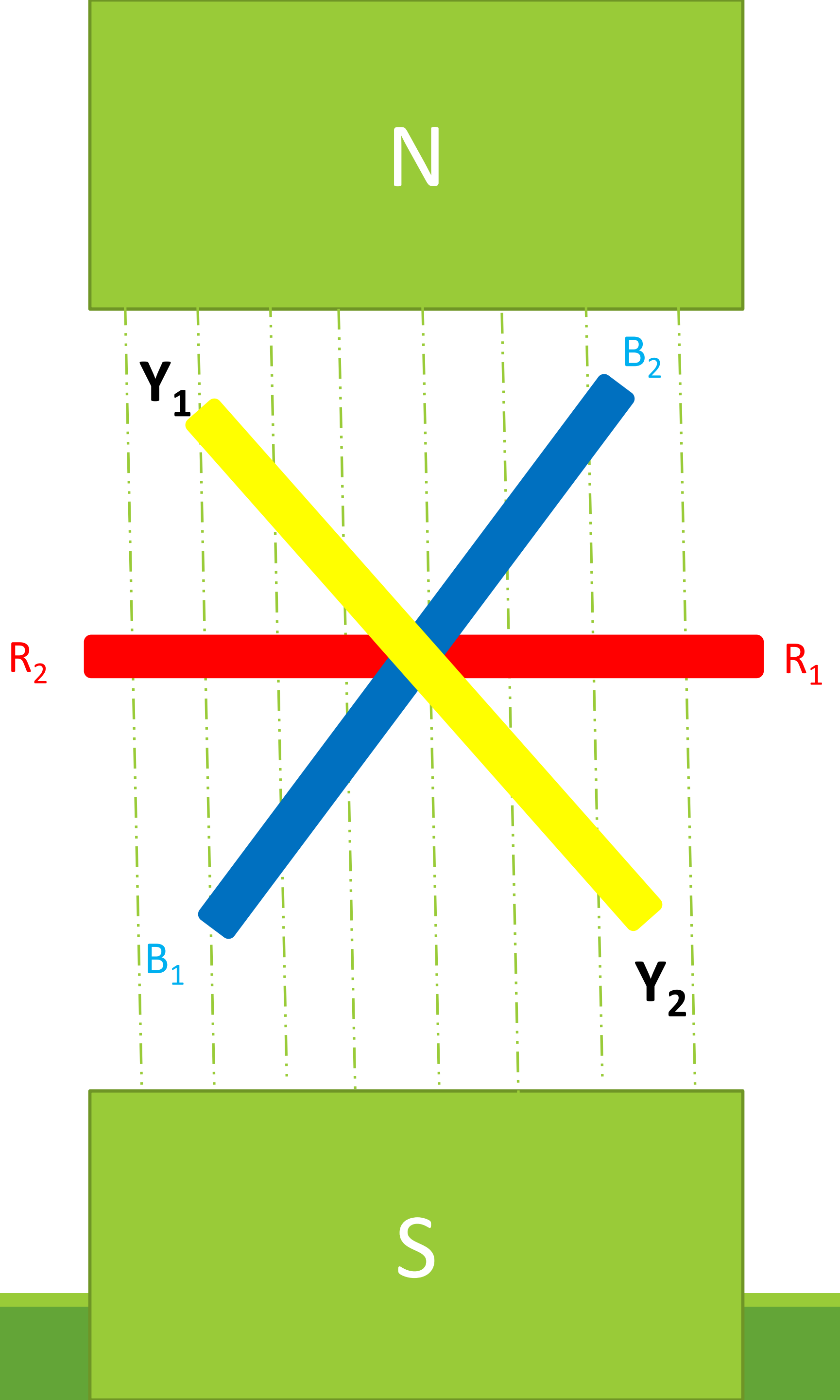
Advantages of 3- phase over single phase

1. **The output of the three phase machine is always greater than single phase machine of the same size.**
2. **The 3-ϕ system gives a steady output.**
3. **The three phase motor is self started where as single phase motors are not self started.**
4. **The single phase can be obtained by 3-phase but 3-phase cannot be obtained by 1-ϕ.**
5. **For transmission and distribution , three phase system requires less copper or conducting material compare to 1-phase .**
6. **The power factor of the single phase motor is poor than 3-phase motors.**

Generation of 3-phase voltage system

- Consider an armature winding are divided into 3 groups of an alternator they are displaced by an angle of **120 ˚ from each other , three alternating voltages are developed** as shown in fig.

- The coils are named as R1-R2, Y1-Y2 and B1-B2



- Let e_R , e_y and e_B are voltages induced in the coil R1-R2 ,Y1-Y2 and B1-B2 respectively

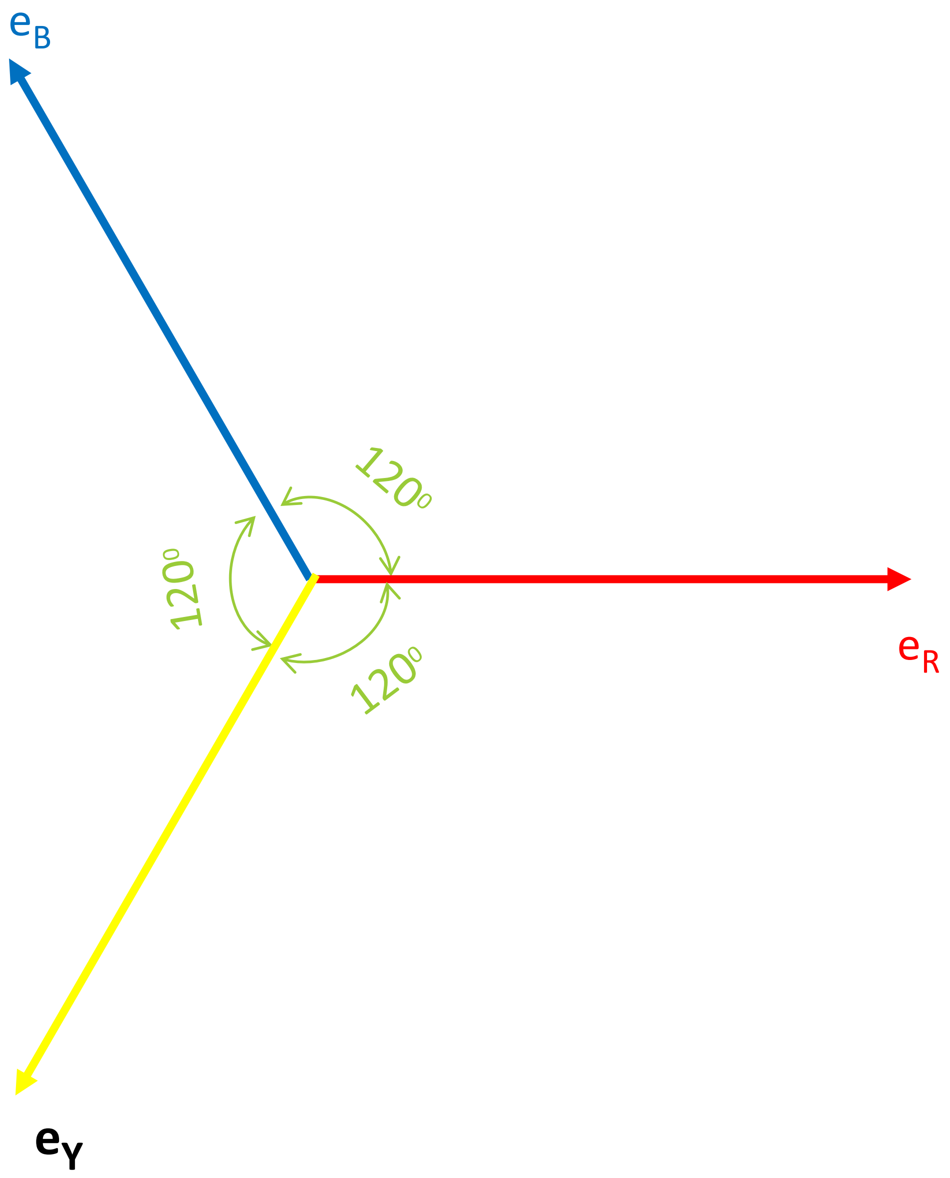
- **All the voltages are of same magnitude and frequency but they displaced by 120˚ apart from each other.**

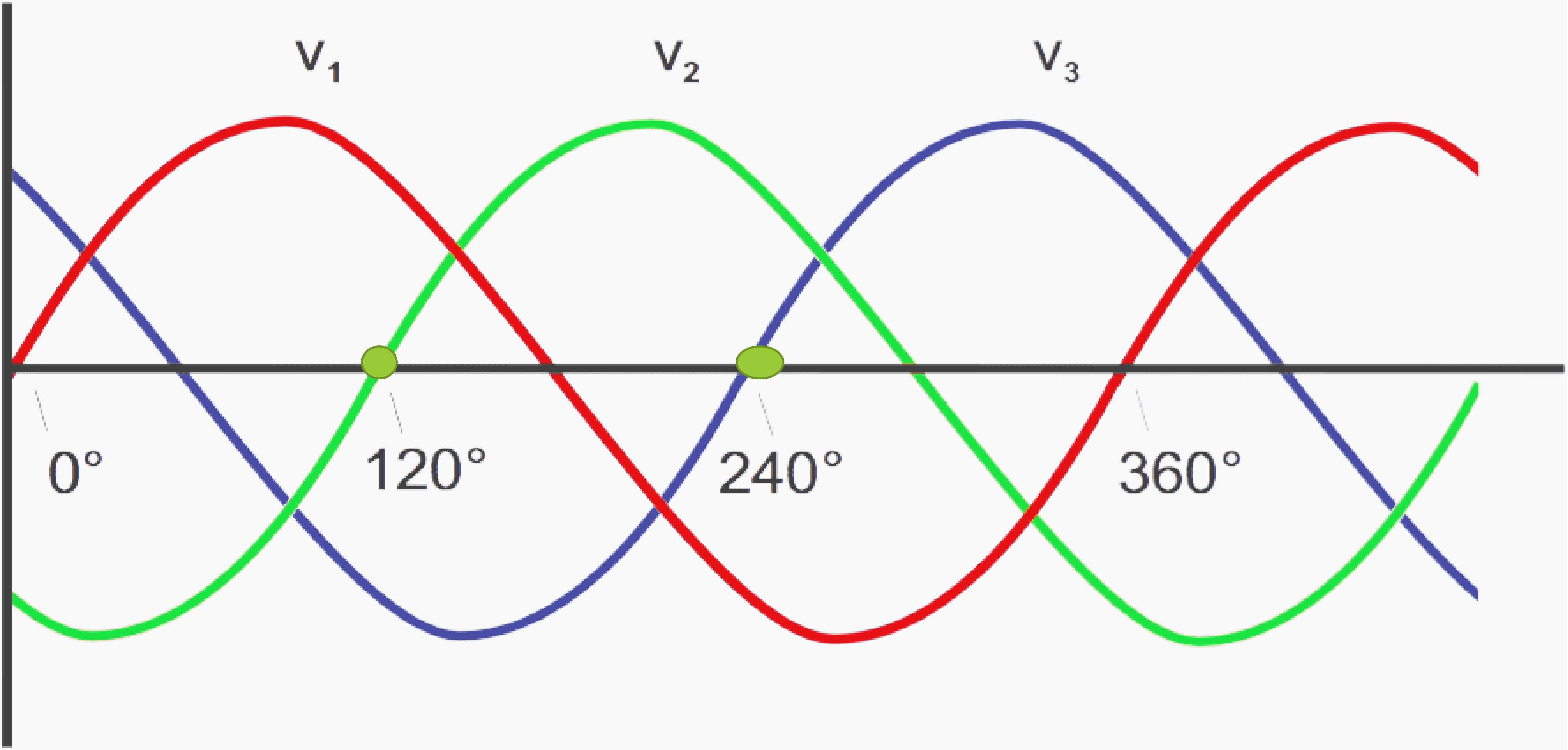
- **Let us assume e_R are the reference the equations of the induced emf's are**

- **$e_R = Em \sin\omega t$**

- **$e_y = Em \sin(\omega t - 120)$**

- **$e_B = Em \sin(\omega t - 240) = Em (\sin\omega t + 120)$**

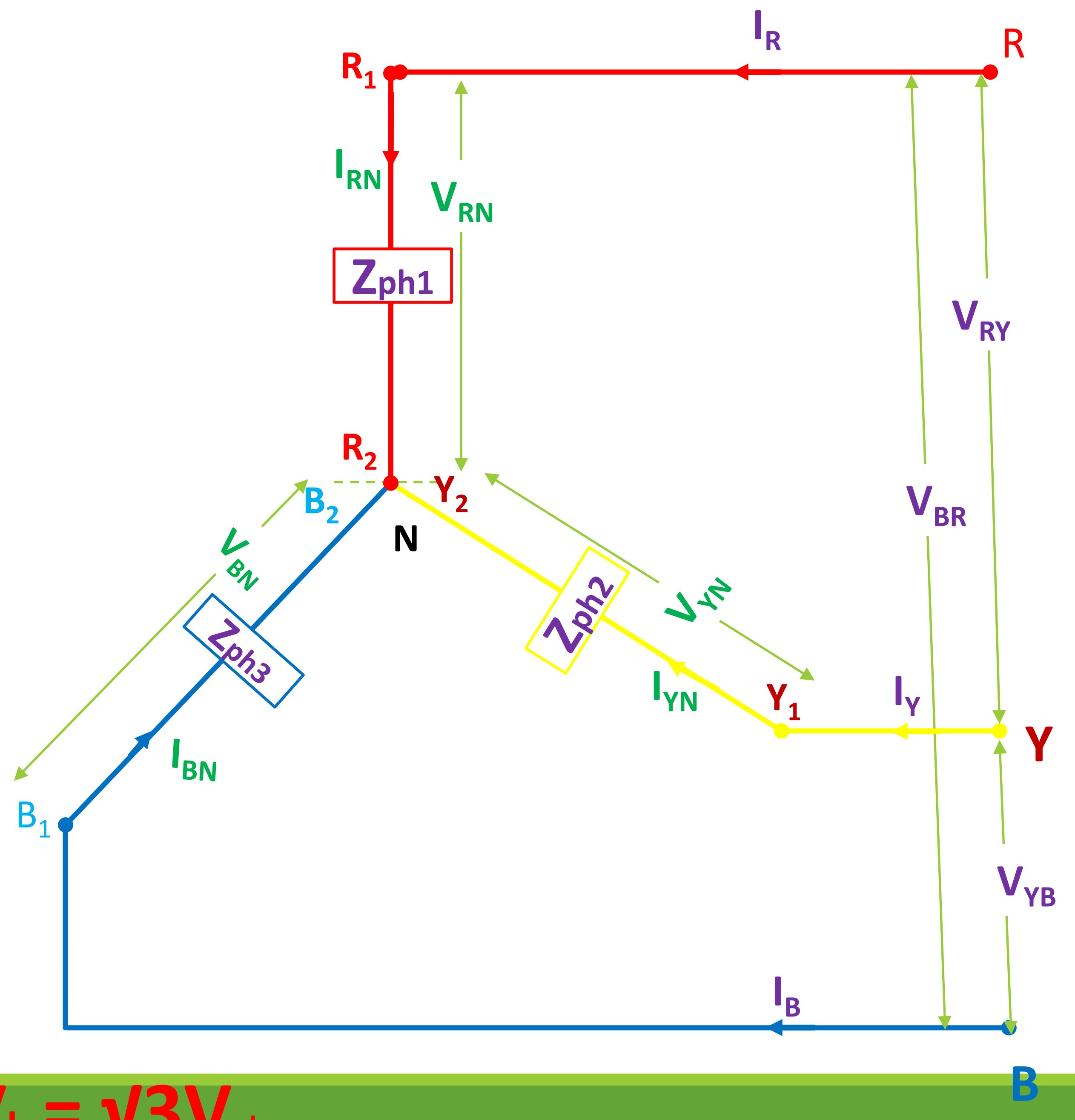




V1
V2
V3
0°
120°
240°
360°

Three phase Load Connection
STAR CONNECTION

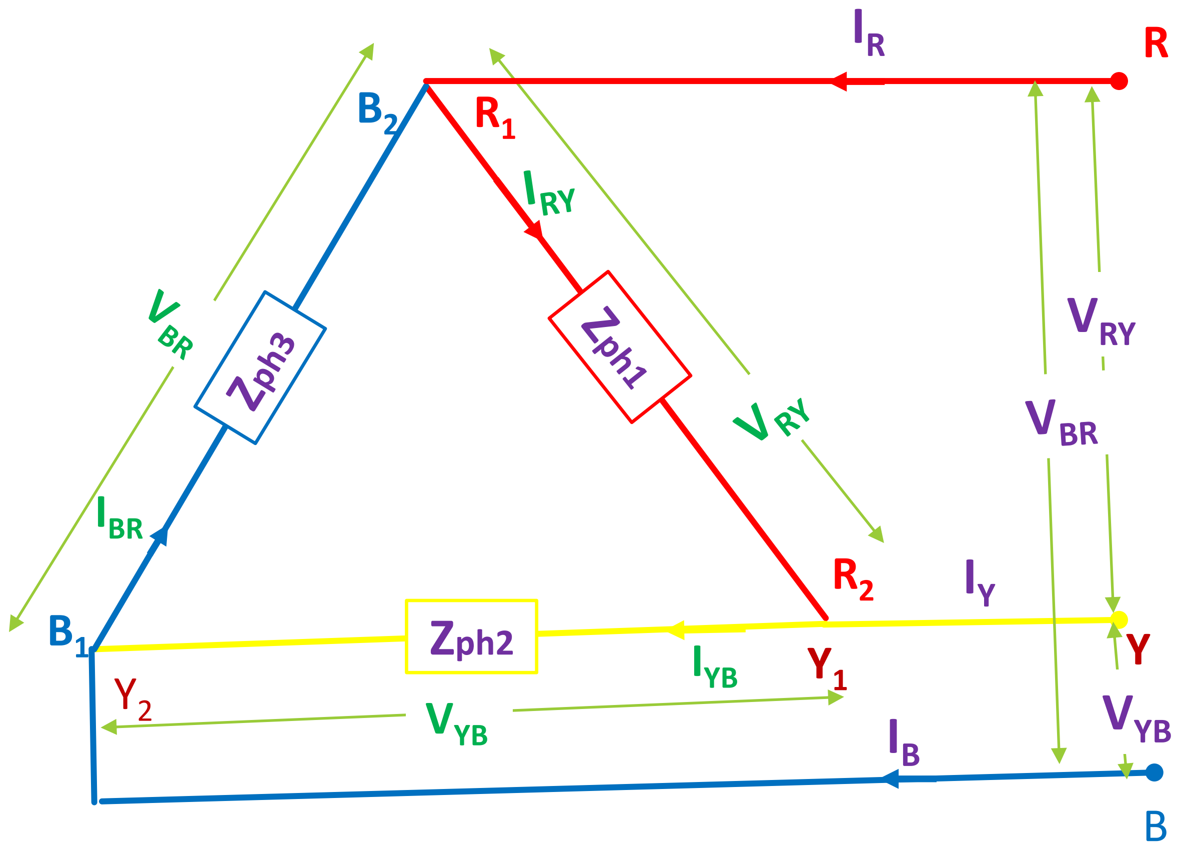
- The star connection is formed by connecting terminating ends of all the three windings together at a common point called **Netural (N).**
- **I,e R2,Y2 and B2 together.**
- The remaining ends brought together for supply connections **I,e to R-Y-B.**
- The connection is as shown in the fig.
- V_{RY} , V_{YB} and V_{BR} are line voltages
- I_R , I_Y and I_B are line currents.
- V_{RN} , V_{YN} and V_{BN} are Phase voltages
- I_{RN} , I_{YN} and I_{BN} are phase currents



In star connection $I_L = I_{ph}$ and $V_L = \sqrt{3}V_{ph}$

DELTA CONNECTION

- **The delta connection is formed by connecting the terminating end of one winding to starting end of the another winding to from a closed loop.**
- **The supply connection is take out from the each junction.**
- **The connection is as shown in the fig.**
- V_{RY} , V_{YB} and V_{BR} are line voltages
- **I_R , I_Y and I_B are line currents.**
- V_{RY} , V_{YB} and V_{BR} are Phase voltages
- I_{RY} , I_{YB} and I_{BR} are phase currents



In delta connection $I_L = \sqrt{3} I_{ph}$ and $V_L = V_{ph}$

List of formulas

1. In star connection $I_L = I_{ph}$ and $V_L = \sqrt{3}V_{ph}$
2. In delta connection $I_L = \sqrt{3}Iph$ and $V_L = Vph$
3. $P = \sqrt{3}\ V_L\ I_L\ Cos\ \Phi$ Watt or $P = 3\ Vph\ Iph\ Cos\ \Phi$
4. $Q = \sqrt{3}\ V_L\ I_L\ Sin\ \Phi$ VAR
5. $S = = \sqrt{3}\ V_L\ I_L$ VA

Formulas

For R-L circuit: Z= R+jXL where XL = 2πfL Ω

For R-C circuit: Z= R-jXc where Xc = 1/2πfc Ω

For R- L -C circuit: Z= R+jXL -jXc

Zph= $\frac{Vph}{Iph}$

[Note: Until unless the voltage is mentioned as phase voltage take it as a line voltage]

Star connection is also denoted as - **Y**

Delta connection is also denoted as -**Δ**

1. A balanced star connected load of (8+j6)Ω is connected to a 3-ϕ 230V supply. Find the line current, phase current, power factor, active power, reactive volt ampere, total volt ampere.

Given : Zph = 8+j6 = $10\angle 36.86$ Ω(star connected)

$V_L = 230\angle 0$

I_L = ? I_{ph} = ? pf= =? P= ? Q=? S=?

In star connection : $I_L = I_{ph}$ and $V_L = \sqrt{3}V_{ph}$

$$V_{ph} = \frac{V_L}{\sqrt{3}} = \frac{230\angle 0}{\sqrt{3}} = 132.79\angle 0 \text{ V}$$

$$I_{ph} = \frac{V_{ph}}{Z_{ph}} = \frac{132.79\angle 0}{10\angle 36.86} = 13.27\angle -36.86\ A$$

$I_L = I_{ph} = 13.27\angle - 36.86\ A$

Φ = 0-(-36.86) = 36.86

Cos ϕ = Cos(36.86) = 0.8 lagging

$P = \sqrt{3}\, V_L I_L \cos\Phi = \sqrt{3} * 230 * 13.27 * 0.8 = 4.2$ KW

$Q = \sqrt{3}\, V_L I_L \sin\Phi = \sqrt{3} * 230 * 13.27 * \sin(36.86) = 3.1$ KVAR

$S = \sqrt{3}\, V_L I_L = \sqrt{3} * 230 * 13.27 = 5.2$ KVA

2. A balanced Delta connected load of (8+j6)Ωis connected to a 3-ϕ 230V supply. Find the line current, phase current, power factor, active power, reactive volt ampere, total volt ampere.

Given : Zph = 8+j6 = $10\angle 36.86$ Ω(Delta connected)

V_L =$230\angle 0$

I_L = ? I_{ph} = ?   pf= =?   P= ?   Q=?   S=?

In Delta connection : $V_L = V_{ph}$ and $I_L = \sqrt{3} I_{ph}$

$V_L = V_{ph} = 230\angle 0V$

$$Iph = \frac{Vph}{Zph} = \frac{230\angle 0}{10\angle 36.86} = 23\angle -36.86A$$

$I_L = \sqrt{3} I_{ph} = \sqrt{3} * 23\angle -36.86 = 39.83 \angle -36.86\ A$

$\Phi = 0-(-36.86) = 36.86$

$\cos \phi = \cos(36.86) = 0.8$ lagging

$P = \sqrt{3}\ V_L\ I_L \cos \Phi = \sqrt{3} * 230 * 39.83 * 0.8 = 12.6$ KW

$Q = \sqrt{3}\ V_L\ I_L \sin \Phi = \sqrt{3} * 230 * 39.83 * \sin(36.86) = 9.3$ KVAR

$S = \sqrt{3}\ V_L\ I_L = \sqrt{3} * 230 * 39.83 = 15.86$ KVA

3. A resistance of 100 ohms is connected in series with an inductance of 1mH in each phase of a 400V star connected 50Hz three phase supply. Calculate i) impedance per phase (ii) Total current (iii) power factor (iv) active power (v) reactive power (vi) apparent power.

Given: R = 100, L = 1mH = $1\text{x}10^{-3}$H ,

$V_L = 400\angle 0$ V, f= 50Hz

Zph = ? , I_L = ? , Cos Φ =? , P = ?, Q=?,S =?

The phase consists of R in series with L   Zph = R + j XL

$X_L = 2\pi fL = 2 * 3.142 * 50 * 1\text{x}10^{-3} = 0.314\Omega$

Zph = R + j XL = 100+j0.314 = $100\angle 0.179\Omega$

In star connection : $I_L = I_{ph}$ and $V_L = \sqrt{3}V_{ph}$

$$V_{ph} = \frac{V_L}{\sqrt{3}} = \frac{400\angle 0}{\sqrt{3}} = 230.94\angle 0\ \text{V}$$

$$Iph = \frac{Vph}{Zph} = \frac{230.94\angle 0}{100\angle 0.179} = 2.3\angle -0.179\ A$$

$$I_L = I_{ph} = 2.3\angle -0.179\ A$$

Φ =0- (-0.179) = 0.179

Cos Φ = Cos (0.179) = 0.99 lagging

P =√3 V_L I_L Cos Φ = √3*400*2.3 *0.99 = 1577.55 =1.5KW

Q =√3$V_L I_L$sinΦ = √3 * 400*2.3* sin(0.179) = 4.978VAR,

S = √3 VL IL = √3 *400*2.3= 1593.48VA

4. A resistance of 50 ohms is connected in series with a capacitance of 150µF in each phase of a 450V delta connected 50Hz three phase supply. Calculate capacitive reactance (ii) impedance per phase (iii) line current and phase current (iv) power factor (v) active power (vi) reactive power (vii) apparent power.

Given : R =50 Ω C = 150 µ F = 150 $X10^{-6}$ F VL= $450\angle 0$

$X_C = 1 / 2\prod fC = \frac{1}{2 * \prod * 50 * 150 * 10^{-6}} = 21.22\ \Omega$

Z ph= R – j Xc =50-j21.22 = 54.31∠-22.99

In Delta connection : $V_L = V_{ph}$ and $I_L = \sqrt{3} I_{ph}$

$V_L = V_{ph} = 450\angle 0$

$I\ ph = \frac{V\ ph}{Zph} = \frac{450\angle 0}{54.31\angle -22.99} = 8.28\ \angle 22.99\ A$

$I_L = \sqrt{3} I_{ph} = \sqrt{3} * 8.28\angle 22.99 = 14.34 \angle 22.99\ A$

$\Phi = 0 - 22.99 = -22.99$

$\cos \Phi = \cos(-22.99) = 0.92$ leading

$P = \sqrt{3}\, V_L I_L \cos \Phi = \sqrt{3} * 450 * 14.34 * 0.92 = 10282.76W = 10.28KW$

$Q = \sqrt{3}\, V_L I_L \sin \Phi = \sqrt{3} * 450 * 14.34 * \sin(22.99) = 4365.37VAR$

$S = \sqrt{3}\, V_L I_L = \sqrt{3} * 450 * 14.34 = 11176.92VA$

5. Arms of a 3-ϕ load consists of an inductor of 0.15H and a resistance of 25Ω and capacitance of 120μF are connected series the supply voltage is 415V, 50Hz ,the load is connected in delta. Calculate the line current ,phase current, power and reactive power.

Given : R =25 Ω, L= 0.15 H ,C = 120 μ F = 120 X10^{-6} F , V_L= 415$\angle 0$, f=50Hz

I_L = ? I_{ph} = ? P= ? Q=?

The phase consists of R in series with L and C Zph = R + j X_L -jXc

X_L = 2π fL = 2* 3.142* 50* 0.15= 47.13Ω

X_C = 1 / 2∏fC = $\frac{1}{2 * \prod * 50 * 120 * 10^{-6}}$ = 26.56 Ω

Zph = R + j X_L –jXc = 25+ j47.13- j26.56

Zph = 25+j20.57= 32.37$\angle$39.44 Ω

In Delta connection : $V_L = V_{ph}$ and $I_L = \sqrt{3} I_{ph}$

$V_L = V_{ph} = 415\angle 0V$

$I_{ph} = \dfrac{Vph}{Zph} = \dfrac{415\angle 0}{32.37\angle 39.44\ \Omega} = 12.81\ \angle{-39.44}\ A$

$I_L = \sqrt{3} I_{ph} = \sqrt{3} * 12.81\angle -39.44 = 22.18\ \angle -39.44\ A$

$\Phi = (0-(-39.44)) = 39.44$

$\cos\ \Phi = \cos(39.44) = 0.77$ lagging

$P = \sqrt{3}\ V_L\ I_L \cos\Phi = \sqrt{3} * 415 * 22.18 * 0.77 = 12312.6W = 12.312KW$

$Q = \sqrt{3}\ V_L\ I_L \sin\Phi = \sqrt{3} * 415 * 22.18 * \sin(39.44) = 10128.12VAR$